- State at the beginning of the period:

$$s = p_{-1}.$$

p_{-1} price set by those who set prices last period, scaled by current aggregate M

- State after Policy Maker Takes Current Action, μ:

$$(s, \mu),$$

where

$$\mu = \frac{M'}{M}.$$

- For Any Monetary Policy Function, $\sigma(s)$, Define Private Sector Equilibrium Functions:

$$c(s, \mu; \sigma),\ \ l(s, \mu; \sigma),\ \ p(s, \mu; \sigma).$$

  Functions, c, l, p conditioned on current monetary action and 'expectation' that future monetary actions determined by σ.

- Private Sector Period Utility:

$$u(s, \mu; \sigma) = U\left(c\left(s, \sigma(s); \sigma\right), l\left(s, \sigma(s)\right)\right)$$

- Next period's state:

$$s' = H(s, \mu; \sigma) \equiv \frac{p(s, \mu; \sigma)}{\mu}.$$

- Value associated with Following Equilibrium Functions Today and Forever:

$$V(s; \sigma) = u\left(s, \sigma(s); \sigma\right) + \beta V(H(s, \mu; \sigma); \sigma).$$

- Policy Problem:

$$\begin{aligned}&\tilde{\sigma}(s;\sigma)\\ =\ &\arg\max_{\mu} u\,(s,\mu;\sigma)+\beta V(H(s,\mu;\sigma);\sigma).\end{aligned}$$

- A Markov equilibrium:

  – $\sigma^*(s)$ such that

$$\sigma^*(s)=\tilde{\sigma}(s;\sigma^*).$$

  – A Fixed Point of Operator, $\tilde{\sigma}$.

- Hence:

$$V(s)=\max_{\mu} u\,(s,\mu)+\beta V(H(s,\mu)),$$

Where Absence of σ Argument Signifies $\sigma=\sigma^*$,

i.e.,

$$u(s,\mu)\equiv u(s,\mu;\sigma^*).$$

- FE Corresponds to the Following 'Sequence Problem':

$$\max_{\{\mu_t, s_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(s_t, \mu_t\right),$$

subject to:

$$\begin{aligned} s_{t+1} &= H(s_t, \mu_t),\ t = 0, 1, 2, \ldots \\ & s_0 \text{ given.} \end{aligned}$$

- Alternative Representation:

$$\max_{\substack{(\mu_0, s_1), \{\mu_t, s_{t+1}\}_{t=1}^{\infty} \\ s_{t+1} = H(s_t, \mu_t)}} u(s_0, \mu_0) + \beta u(H(s_0, \mu_0), \mu_1) + ..$$

- First Order Condition for μ_t (Klein-Krusell-Rios Rull 'Generalized Euler Equation'):

$$\begin{aligned} u_2(s, \sigma^*(s)) \\ +\beta u_1(H(s, \sigma^*(s)), \sigma^*(H(s, \sigma^*(s)))) \\ \times H_2(s, \sigma^*(s)) = 0 \end{aligned}$$

Finding Equation That Characterizes $p(p_{-1}, \mu)$

Final Good Firms

- State:

$$s_2 = (p_{-1}, \mu)$$

- Technology :

$$\begin{aligned} c(s_2) &= \left[\int_0^1 (y_i(s_2))^\lambda \, di\right]^{\frac{1}{\lambda}} \\ &= \left[\frac{1}{2} y(s_2)^\lambda + \frac{1}{2} y_{-1}(s_2)^\lambda\right]^{\frac{1}{\lambda}}. \end{aligned}$$

$y(s_2)$ output of intermediate
good firms who set prices after s_2
$y_{-1}(s_2)$ output of intermediate
good firms who set price in
previous period

- Final Good Problem:

$$\max_{c(s_2),\{y_i(s_2)\}} \bar{p}(s_2)c(s_2) - \int_0^1 p_i(s_2)y_i(s_2)di,$$

$\bar{p}(s_2)$ price of the final good

$p_i(s_2)$ price of i^{th} intermediate good

Both prices scaled by Beginning-of-Period $\mathbf{M}$

- First Order Conditions:

$$y_i(s_2) = c(s_2)\left(\frac{\bar{p}(s_2)}{p_i(s_2)}\right)^{\frac{1}{1-\lambda}}.$$

- Aggregate Price Level:

$$\bar{p}(s_2) = \left[\int_0^1 p_i(s_2)^{\frac{\lambda}{\lambda-1}}di\right]^{\frac{\lambda-1}{\lambda}} = \left[\frac{1}{2}p(s_2)^{\frac{\lambda}{\lambda-1}} + \frac{1}{2}p_{-1}^{\frac{\lambda}{\lambda-1}}\right]^{\frac{\lambda-1}{\lambda}}$$

Intermediate Good Firms

- Technology:

$$y_i(s_2) = zl_i(s_2).$$

- Pricing:

Each Firm Sets its Price for Two Periods
$\frac{1}{2}$ Set Price in Even Periods
$\frac{1}{2}$ Set Price in Odd Periods

- Period t Profits (Divided by Beginning of t Aggregate Money):

$$\pi_i(s_2; p_i) = p_i(s_2)y_i(s_2) - \left[\frac{R(s_2)w(s_2)}{z}\right] y_i(s_2),$$

- From Demand Curve:

$$
\begin{aligned}
p_i y_i &= c\,(\bar{p})^{1/(1-\lambda)}\,(p_i)^{\lambda/(\lambda-1)}\,, \\
y_i &= c\,(\bar{p})^{1/(1-\lambda)}\,(p_i)^{-1/(1-\lambda)}
\end{aligned}
$$

So, Period t Profits:

$$
\pi_i(s_2; p_i) = c(s_2)\,(\bar{p}(s_2))^{1/(1-\lambda)}
$$
$$
\times \left\{ (p_i(s_2))^{\frac{\lambda}{\lambda-1}} - \left[\frac{R(s_2)w(s_2)}{z} \right] (p_i(s_2))^{\frac{1}{\lambda-1}} \right\}
$$

- Present Value of a Dollar Paid to Households at the End of Current Period:

$$\frac{\beta u_c(s_2')}{M\mu\bar{p}(s_2')}.$$

- Value to Household, of Period t Profits:

$$\begin{aligned} & \frac{\beta u_c(s_2')}{M\mu\bar{p}(s_2')} \times \pi_i(s_2; p_i) M \\ = \quad & \frac{\beta u_c(s_2')}{\mu\bar{p}(s_2')} \times \pi_i(s_2; p_i) \\ = \quad & q(s_2)\pi_i(s_2; p_i), \end{aligned}$$

where

$$\begin{aligned} q(s_2) &= \frac{\beta u_c(s_2')}{\mu\bar{p}(s_2')} \\ &= \frac{\beta u_c(H(s_2), \sigma(H(s_2)))}{\mu\bar{p}(H(s_2), \sigma(H(s_2)))} \end{aligned}$$

- Recall:

$$s_2' = (s', \mu') = (H(s_2), \sigma(H(s_2)))$$

- Then,

$$\begin{aligned} q(s_2') &= q(s', \mu') \\ &= q(H(s_2), \sigma(H(s_2))) \end{aligned}$$

- Objective of a Price-Setting Firm:

$$\begin{aligned} &\max_{p_i(s_2)} \{\pi(s_2) q(s_2) \\ &+\beta q(H(s_2), \sigma(H(s_2)))\pi(H(s_2), \sigma(H(s_2)))\} \end{aligned}$$

- Straightforward Differentiation yields:

$$p(s_2) = \frac{a(s_2)}{b(s_2)},$$

where

$$\begin{aligned} a(s_2) &= q(s_2)\bar{p}(s_2)^{\frac{2-\lambda}{1-\lambda}}\frac{w}{\bar{p}}(s_2)\frac{R(s_2)}{z}c(s_2) \\ &+\frac{\beta}{\mu}q(s_2')\left(\bar{p}(s_2')\mu\right)^{\frac{2-\lambda}{1-\lambda}}\frac{w}{\bar{p}}(s_2')\frac{R(s_2')}{z'}c(s_2') \end{aligned}$$

$$\begin{aligned} b(s_2) &= \lambda[q(s_2)\bar{p}(s_2)^{\frac{1}{1-\lambda}}c(s_2) \\ &+\frac{\beta}{\mu}q(s_2')\left(\bar{p}(s_2')\mu\right)^{\frac{1}{1-\lambda}}c(s_2')] \end{aligned}$$

Households

- Labor Euler Equation:

$$\frac{w}{\bar{p}}(s_2) = -\frac{u_l(s_2)}{u_c(s_2)},$$

- Intertemporal Euler Equation for Saving:

$$\frac{u_c}{\bar{P}} = R\frac{\beta u_c'}{\bar{P}'},$$

or

$$\frac{u_c c}{c\bar{P}} = R\frac{\beta u_c' c'}{\bar{P}' c'},$$

or

$$\frac{u_c c}{\mu M} = R\frac{\beta u_c' c'}{\mu \sigma' M},$$

or

$$R(s_2) = \frac{c(s_2)u_c(s_2)}{\beta\frac{c(s_2')u_c(s_2')}{\sigma(s')}}.$$

Equilibrium Conditions

- Resource Constraint:

$$c(s_2) = zl(s_2)\frac{g(s_2)}{h(s_2)},$$

where

$$g(s_2) = \left[\frac{1}{2} + \frac{1}{2}\left(\frac{p(s_2)}{p_{-1}}\right)^{\frac{\lambda}{1-\lambda}}\right]^{\frac{1}{\lambda}}$$

$$h(s_2) = \frac{1}{2} + \frac{1}{2}\left(\frac{p(s_2)}{p_{-1}}\right)^{\frac{1}{1-\lambda}}.$$