Time-Consistent Policy

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Preliminary

Motivation

- A pervasive problem: a decision maker at time t cares about the future, disagrees with the decision maker at $t+1$, but has no direct influence over it (e.g. the optimal taxation problem without commitment).

- Early literature (Kydland-Prescott (1977)) focused on finding the Markov equilibrium that is a limit of the corresponding finite-horizon economies.

- Later: use of "reputation mechanisms" (using Abreu, Pearce, & Stacchetti (1990), as in Chari & Kehoe (1990)).

- We are interested in the (differentiable) Markov equilibrium in models with state variables such as capital, debt, income distribution, etc.

- The Markov equilibrium is interesting as a benchmark where no reputation mechanism is operative: it is *fundamental* in emphasizing the basic economics dictated by the state variables.

Earlier Work On Markov Equilibria

- Lots of work on finite-horizon models. Solution procedure: solve it backwards. See, e.g., Basar and Olsder (1982).

- With infinite horizon, some linear-quadratic models can be solved explicitly (e.g., Basar and Olsder (1983), Cohen and Michel (1988) and Currie and Levine (1993)).

- Some work on differential games in various literatures (imperfect altruism, resource extraction problems); we have not yet digested these papers.

- Numerical approach: Krusell, Quadrini & Ríos-Rull (1997) and related papers; more recently, e.g., Klein and Ríos-Rull (2001). Problem here: these methods are of the "black-box" type and they did not deliver controlled accuracy.

Contributions · · ·

- We show how to characterize and solve for the Markov equilibrium:

1. We derive a "generalized Euler equation"—GEE—allowing us to interpret the incentives facing the decision maker; this equation does not appear in the existing literature, and it allows qualitative and quantitative interpretations.

2. We show how to solve this functional equation; a much harder problem than that of solving a standard Euler equation. Reason: to solve for a steady state, one needs to solve jointly for dynamics; to solve for first-order dynamics, one needs to solve for second-order dynamics, and so on . . .

. . . Contributions

3. We study a simple, canonical problem in public economics: how to optimally provide public goods over time. We compare the predictions of the Pareto, Ramsey, and Markov allocations and find (among other things)

   - Governments without commitment may use capital taxation "strategically" and not just as a lump sum.

   - Often the Markov allocation has lower taxes.

   - The difference between Ramsey and Markov may be large.

The methods are entirely general and seem widely applicable: optimal fiscal and monetary policy, dynamic political economy, dynamic I.O. (durable goods monopoly, dynamic oligopoly), impure intergenerational altruism.

Our Economy: Public Goods Provision And Finance

• Standard growth model with a non-committing benevolent government, a period-by-period balanced budget, and proportional taxation. In the presentation, the tax base is total income and leisure is not valued.

Households maximize $$\sum_{t=0}^{\infty} \beta^t \, u(c_t, g_t)$$

s.t. $$c_t + k_{t+1} = k_t + (1 - \tau_t) \left[w_t + (r_t - \delta) k_t \right].$$

Resource constraint: $$C_t + K_{t+1} + G_t = f(K_t, 1) + (1 - \delta) K_t$$

Balanced budget constraint: $$G_t = \tau_t \left[f(K_t, 1) - \delta K_t \right].$$

Analysis

In a subgame-perfect equilibrium, the government compares the effects of any current policy choice, τ, on endogenous variables given any current value of the state, K. Thus we need to find the two key equilibrium objects:

$$
\begin{aligned}
K' &= \mathcal{H}(K, \tau) \\
\tau &= \Psi(K).
\end{aligned}
$$

The idea: the government and private agents expect the future governments to obey the *rule* Ψ, but the current tax rate is free. This is a *one-period deviation* from Ψ.

$\mathcal{H}$ and Ψ are unknown—they are the **key equilibrium objects**.

- $\mathcal{H}$ is determined so as to satisfy the FOC's for the household.
- Ψ is determined by the government's FOC (GEE).

There are two other, auxiliary functions that are convenient to define:

$$C = \mathcal{C}(K, \tau)$$
$$G = \mathcal{G}(K, \tau)$$

satisfying

$$\begin{aligned} \mathcal{G}(K, \tau) &= \tau\left[f(K, 1) - \delta K\right] \\ \mathcal{C}(K, \tau) &= f(K, 1) + (1 - \delta)K - \mathcal{H}(K, \tau) - \mathcal{G}(K, \tau). \end{aligned}$$

The private sector's first-order conditions

$\mathcal{H}$ satisfies the functional-eqtn version of the FOC for savings $\forall t$:

$$u_c(C_t, G_t) = \beta\, u_c(C_{t+1}, G_{t+1})\; [1 + (1 - \tau_{t+1})(f_k(K_{t+1}, 1) - \delta)].$$

It is obtained by using $\mathcal{H}$ and Ψ in this equation: for all (K, τ),

$$u_c\,[\mathcal{C}(K,\tau), \mathcal{G}(K,\tau)] =$$
$$\beta\, u_c\,\{\mathcal{C}[\mathcal{H}(K,\tau), \Psi(\mathcal{H}(K,\tau))]\mathcal{G}[\mathcal{H}(K,\tau), \Psi(\mathcal{H}(K,\tau))]\} \cdot$$
$$\{1 + [1 - \Psi(\mathcal{H}(K,\tau))][f_K(\mathcal{H}(K,\tau), 1) - \delta]\}\,.$$

Note: Ψ is a determinant of $\mathcal{H}$: the expectations of future government behavior influence how consumers save.

The government's problem

Gov't problem: $$\max_{\tau} \; u\,[\mathcal{C}(K,\tau),\mathcal{G}(K,\tau)] + \; \beta\, v[\mathcal{H}(K,\tau)],$$

where $$v(K) \equiv u\,[\mathcal{C}(K,\Psi(K)),\mathcal{G}(K,\Psi(K))] + \beta\, v[\mathcal{H}(K,\Psi(K))].$$

Optimal policy: $$\Psi(K) = \arg\max_{\tau} \{u\,[\mathcal{C}(K,\tau),\mathcal{G}(K,\tau)] + \beta\, v(\mathcal{H}(K,\tau))\}$$

Hence $$v(K) = \max_{\tau} \; u\,[\mathcal{C}(K,\tau),\mathcal{G}(K,\tau)] + \beta\, v[\mathcal{H}(K,\tau)].$$

This is a *recursive problem*!

A sequential formulation: derivation of the GEE

Because the government's problem is recursive, we can characterize the optimal policy sequence, $\{\tau_t\}_{t=0}^{\infty}$, in sequential form

$$\max_{\{\tau_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \, u[\mathcal{C}(K_t, \tau_t), \mathcal{G}(K_t, \tau_t)]$$

$$\text{subject: to} \qquad \mathcal{H}(K_t, \tau_t).$$

• This problem is not in terms of primitives: $\mathcal{H}$ (and $\mathcal{C}$ and $\mathcal{G}$) are endogenous, and depend on Ψ; *there is still a fixed-point problem to solve*.

• To derive the GEE is now easy: just differentiate, like in a standard optimal growth context. The tax sequence here is the "control" sequence: it plays the role of consumption. We will assume that the GEE is sufficient for an optimum.

The GEE

Differentiation yields

$$u_c \left[-\mathcal{H}_\tau - \mathcal{G}_\tau\right] + u_g\, \mathcal{G}_\tau +$$

$$\beta\, \mathcal{H}_\tau \left\{ u'_c \left[f'_K + 1 - \delta - \mathcal{H}'_K - \mathcal{G}'_K\right] + u'_g\, \mathcal{G}'_K + \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau} \left(u'_c[\mathcal{H}'_\tau + \mathcal{G}'_\tau] + u'_\tau\, \mathcal{G}'_\tau\right) \right\} = 0,$$

It holds for all K, and is a *functional equation* in $\Psi(K)$, given $\mathcal{H}(K, \tau)$.

Equilibrium: A *time-consistent policy equilibrium* is a set of differentiable functions Ψ and $\mathcal{H}$ (and $\mathcal{C}$ and $\mathcal{G}$) such that

- $\mathcal{H}(k, \tau)$ solves the functional FOC of the private sector; and
- $\Psi(K)$ solves the functional FOC of the government.

Interpretations

• The **"raw"** version of the GEE (same as above). Look at the marginal utility effects of changing the level of savings today, using τ:

$$u_c\,[-\mathcal{H}_\tau - \mathcal{G}_\tau] + u_g\,\mathcal{G}_\tau +$$

$$\beta\,\mathcal{H}_\tau \left\{ u'_c\,[f'_K + 1 - \delta - \mathcal{H}'_K - \mathcal{G}'_K] + u'_g\,\mathcal{G}'_K + \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau}\,(u'_c[\mathcal{H}'_\tau + \mathcal{G}'_\tau] + u'_\tau\,\mathcal{G}'_\tau) \right\} = 0,$$

The term $\mathcal{H}'_K/\mathcal{H}'_\tau$ reflects the **variational** nature of the GEE: How to vary optimally τ and τ' subject to keeping K and K'' unchanged. Thus, $-\mathcal{H}'_K/\mathcal{H}'_\tau$ is the increase in τ' needed in order not to change K''.

The **"public economics"** version of the GEE. Trade off wedges:

$$\mathcal{G}_\tau\,[u_g - u_c] + \mathcal{H}_\tau\,\{-u_c + \beta u'_c(1 + f'_K - \delta)\} + \beta\mathcal{H}_\tau \left(1 - \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau}\right) \mathcal{G}'_\tau\,[u'_g - u'_c] = 0.$$

Computation of a steady state

•• We need to find functions that jointly satisfy the two functional FOCs.

•• Can we find a steady state by just evaluating at $\bar{K}$ and $\bar{\tau}$? No: 2 equations and 4 unknowns: $\mathcal{H}_k$ and $\mathcal{H}_\tau$ appear.

•• Can we specify flexible functional forms and require the FOCs to hold over some grid? No—this method doesn't work (see Krusell and Smith (2001)).

•• Instead, we use a Perturbation method.

- STEP 1: Make $\mathcal{H}$ a linear function (3 coefficients) and Ψ a constant. Use the FOC for savings **and its 2 derivatives** and use the GEE. Solve a nonlinear equation system. It delivers the steady state.

- STEP 2: Make $\mathcal{H}$ a quadratic function (6 coefficients) and Ψ a linear one (2 coefficients). Use the FOC for savings and its 1st– and 2nd–order derivatives (1+2+3 eqn's) and the GEE and its derivative (1+1 eqn's). Solve a nonlinear equation system. It delivers a new steady state.

- Go on until the steady state level, and perhaps the low-order derivatives, do not change.

•• Notice that differentiability is used critically.

•• Notice also that the method can be used in a standard model (where lack of commitment is not binding). Here it is *necessary* though. Also, for a standard model the steady state does not change across iterations.

Quantitative analysis: Baseline example

We specify the period utility function as

$$u(c, \ell, g) = (1 - \alpha_p)\alpha_c \ln c + (1 - \alpha_p)(1 - \alpha_c) \ln \ell + \alpha_p \ln g$$

The production function is

$$f(K, L) = A \cdot K^{\theta} L^{1-\theta}.$$

Parameter Values		
$\theta = .36$	$\alpha_c = .30$	$\alpha_p = .13$
$\beta = .96$	$\delta = .08$	

Table 1: Parameterization of the Baseline Model Economy.

Labor taxes only

Statistic	Pareto	Ramsey	Markov
Y	1.000	0.700	0.719
K/Y	2.959	2.959	2.959
C/G	2.005	2.005	3.017
L	0.350	0.245	0.252
τ	–	0.397	0.297

No intertemporal distortion. Ramsey has the right ratio between C and G. Markov does not: it ignores the positive effect of a higher τ_t on L_{t-1}.

Capital taxes only

Statistic	Pareto	Ramsey	Markov
Y	1.000	0.588	0.488
K/Y	2.959	1.734	1.193
C/G	2.005	4.779	3.211
L	0.350	0.278	0.255
τ	–	0.673	0.812

Very large taxes. Small expenditures in Ramsey. Also small in Markov, even though τ is lump-sum: a decrease in τ increases K'.

Total Income taxes

Statistic	Pareto	Ramsey	Markov
Y	1.000	0.669	0.693
K/Y	2.959	2.527	2.649
C/G	2.005	2.005	2.928
L	0.350	0.256	0.258
τ	–	0.334	0.255

Markov again taxes less; it does not take into account the effect of the tax on yesterday work effort, and it uses τ strategically.

Conclusions

•1 We derived a GEE to interpret the decision of time inconsistent agents; a new equation that allows qualitative and quantitative interpretations.

•2 We show how to solve this functional equation; a much harder problem than that of solving a standard Euler equation. The numerical methods seem to work very well.

•3 We document some interesting properties for the problem of optimal provision of public goods: Markov does not necessarily tax more heavily than Ramsey, and the difference between the two is nontrivial.

•4 The class of problems for which these methods are relevant seems large.

•5 Remaining issues: existence.