Optimal Monetary Policy In the Presence of Sticky Prices

Background

- Showed that Volatile Prices Can be Part of an Optimal Fiscal Program.
- Environment Lacked Price Setting Frictions which Might Make Price Volatility Costly.
- Today:
  (a) Examine the Impact of Price Frictions on Optimal Monetary Policy.
  (b) Work with a Model that Abstracts from Public Finance Considerations.
- Results:
  (a) Presence of Price Setting Frictions Sharply Reduce Degree of Price Volatility that is Desirable (Siu; Schmitt-Grohe and Uribe)
  (b) May Prevent Friedman Rule from Being Optimal (Khan, King, Wolman).

Outline:

- Optimal Monetary Policy in Simple Environment with No Price Frictions.
- Introduce Frictions in a Particular Way.

Model

- Households:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

Cash constraint:

$$P_t c_t \leq M_t - D_t + W_t l_t - T_t,\ 0 \leq D_t \leq M_t$$

Cash Evolution equation:

$$M_{t+1} = (1+R_t)(X_t+D_t)+M_t-D_t+W_t l_t-P_t c_t-T_t+\text{Profits}_t$$

Necessary and sufficient conditions for household optimization:

$$\begin{aligned} u_{c,t} &= \beta E_t u_{c,t+1} \frac{1+R_t}{\pi_{t+1}} \\ \frac{-u_{l,t}}{u_{c,t}} &= \frac{W_t}{P_t} \end{aligned}$$

$$\begin{aligned} P_t c_t &\leq M_t - D_t + W_t l_t - T_t, \\ P_t c_t &= M_t - D_t + W_t l_t - T_t, \text{ if } R_t > 0. \end{aligned}$$

plus a transversality condition.

Firms

- Final Good Firms

  Technology:

$$Y_t = \left[\int_0^1 \left(y_t^i\right)^{\kappa} di\right]^{\frac{1}{\kappa}},\ 0 < \kappa \leq 1.$$

Profit Maximization Problem:

$$\max P_t Y_t - \int_0^1 P_t^i y_t^i di$$

First Order Condition and Consistency Condition:

$$\left(\frac{P_t}{P_t^i}\right)^{\frac{1}{1-\kappa}} = \frac{y_t^i}{Y_t},\ P_t = \left[\int_0^1 \left(P_t^i\right)^{\frac{\kappa}{\kappa-1}} di\right]^{\frac{\kappa-1}{\kappa}}$$

- Intermediate Good Firms:
  Technology:

$$y_t^i = z_t l_t^i$$

Cost Function:

$$C(W_t, P_t, R_t, y_t^i) = \frac{W_t\,(1+R_t)}{z_t} y_t^i$$

Profits:

$$\max_{y_t^i, P_t^i} \; P_t^i y_t^i - C(W_t, P_t, R_t, y_t^i) \text{ subject to } \left(\frac{P_t}{P_t^i}\right)^{\frac{1}{1-\kappa}} = \frac{y_t^i}{Y_t},$$

or,

$$\max_{P_t^i} \left\{ P_{it} \left(\frac{P_t}{P_t^i}\right)^{\frac{1}{1-\kappa}} Y_t - \frac{W_t\,(1+R_t)}{z_t} \left(\frac{P_t}{P_t^i}\right)^{\frac{1}{1-\kappa}} Y_t \right\}$$

or,

$$\max_{P_t^i} P_t^{\frac{1}{1-\kappa}} Y_t \left\{ \left(P_t^i\right)^{1-\frac{1}{1-\kappa}} - \frac{W_t\left(1+R_t\right)}{z_t}\left(P_t^i\right)^{-\frac{1}{1-\kappa}} \right\}$$

Differentiating:

$$P_t^i = \frac{1}{\kappa}\frac{W_t\left(1+R_t\right)}{z_t},$$

'Price Equals Fixed Markup Over Marginal Cost'

Symmetric Equilibrium In Firm Sector

$$P_t = P_t^i,\ y_t^i = Y_t,\ i \in (0,1).$$

- Static Euler Equation:

$$\frac{W_t(1+R_t)}{P_t} = \kappa z_t.$$

- Aggregate Technology:

$$Y_t = z_t l_t.$$

- Aggregate Resource Constraint:

$$c_t + g \leq z_t l_t.$$

Money Market Clearing

$$
\begin{aligned}
W_t l_t &\leq D_t + X_t, \\
W_t l_t &= D_t + X_t, \text{ if } R_t > 0
\end{aligned}
$$

Monetary and Fiscal Policy

Policy Authority Chooses a State Contingent Sequence,

$$X_0, X_1, \ldots$$

$$M_{t+1} = M_t + X_t.$$

Fiscal Authority Operates a Balanced Budget:

$$P_t g = T_t,\ t = 0, 1, 2, \ldots$$

Equilibrium

- Collecting Equations....

$$u_{c,t} = \beta E_t u_{c,t+1} \frac{1 + R_t}{\pi_{t+1}}$$

$$\begin{aligned} P_t \left(c_t + g \right) &\leq M_t + X_t, \\ P_t \left(c_t + g \right) &= M_t + X_t, \text{ if } R_t > 0. \end{aligned}$$

$$\frac{-u_{l,t}}{u_{c,t}}(1 + R_t) = \kappa z_t.$$

$$c_t + g \leq z_t l_t.$$

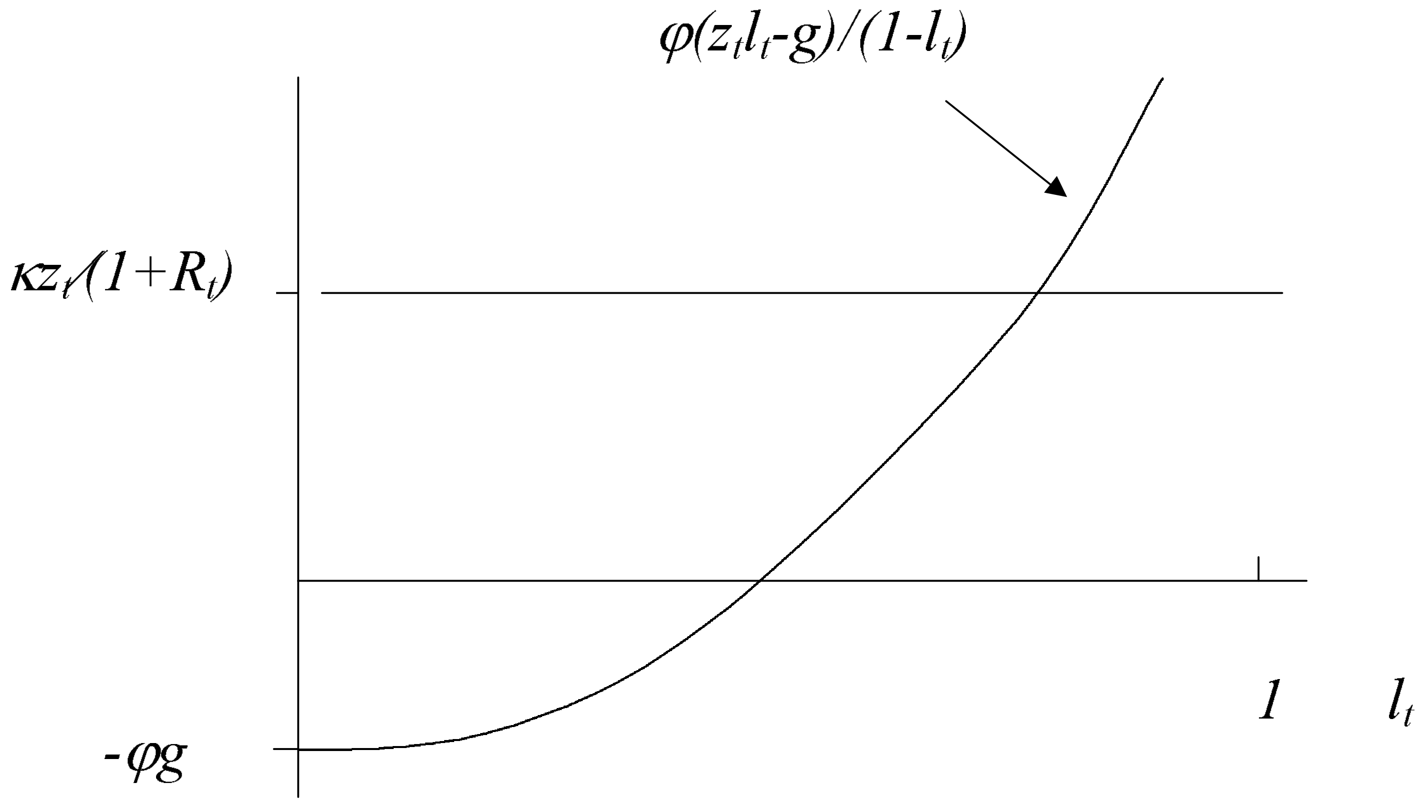
- Note: Set of Equilibria 'Indexed' by R_t.
  - Consider a Sequence, R_0, $R_1, \ldots.$
    As Long as These are Non-Negative (and, Possibly, other Constraints on the R_t's are Satisfied Too), Can Fill Out Remaining Objects in Equilibrium...
  - Solve Static Euler Equation and Resource Constraint for $c(R_t)$, $l(R_t)$.
    (i) Example 1: $u(c,l) = \left[c(1-l)^{\varphi}\right]^{1-\sigma}/(1-\sigma)$, so that

$$\frac{\varphi(z_t l_t - g)}{1 - l_t} = \frac{\kappa z_t}{1 + R_t}.$$

Can Always find $l_t \geq 0$ that solves this for any $R_t \geq 0$.

(ii) Example 2: ZIL Preferences: $u(c,l) = u\left[c - \psi_0 l^{1+\psi}/(1+\psi)\right]$.
Can Always solve for $l_t \geq 0$,

$$\psi_0 l_t^{\psi} = \frac{\kappa z_t}{1 + R_t}$$



φ(z_t l_t-g)/(1-l_t)
κz_t/(1+R_t)
-φg
1
l_t

– Need to Establish that $D_0, D_1, ..., X_0, X_1, ...,$ $P_0, P_1, \ldots, W_0, W_1, ...$ Can Be Found that Satisfy Remaining Equilibrium Conditions and Restrictions. This May Involve More Restrictions on the R_t's. We Ignore These For Now.

- Optimal Monetary Policy 'Chooses' the Best R_t :

$$\max_{R_t \geq 0} \sum_{t=0}^{\infty} \beta^t u(c(R_t), l(R_t)).$$

or with ZIL Preferences:

$$\max_{R_t \geq 0} E_0 \sum_{t=0}^{\infty} \beta^t u[z_t \left(\frac{\kappa z_t}{\psi_0 (1 + R_t)} \right)^{\frac{1}{\psi}} - g - \frac{\psi_0}{1+\psi} \left(\frac{\kappa z_t}{\psi_0 (1 + R_t)} \right)^{\frac{1+\psi}{\psi}}].$$

- Optimal Policy Corresponds to Solving:

$$\max_{R \geq 0} \left\{ \left(\frac{1}{1+R} \right)^{\frac{1}{\psi}} - \frac{1}{1+\psi} \kappa \left(\frac{1}{1+R} \right)^{\frac{1+\psi}{\psi}} \right\}$$

- First Order Condition is, (after multiplying by positive constants):

$$\kappa - (1+R) = 0.$$

which is negative for $R \geq \kappa - 1$. So, the unique solution that satisfies $R \geq 0$ is

$$R = 0$$

To Verify That This Truly is Optimal, Must Verify that $D_0, D_1, ..., X_0, X_1, ..., P_0, P_1, ...$, $W_0, W_1, ...$ Can Be Found that Satisfy Remaining Equilibrium Conditions and Restrictions.

- *Really* would like $R < 0$! This Would Subsidize Monopolists and Help Undo Monopoly Distortion. Since $R \geq 0$ and Objective Monotonically Declining, It Follows that $R = 0$.
- Conclude: Friedman Rule is Optimal (Not Surprising Here).