Formulation and Estimation of a Standard General Equilibrium Macro Model: A Shock-Based Approach

- Shock-Based Strategy for Estimating a Dynamic GE Model

- Estimating the Dynamic Effects of Monetary Policy and Technology Shocks.

- Fit a General Equilibrium Model to these Effects
  (a) Nominal Part of Economy Important
    - Sticky Prices, Sticky Wages, etc.

  (b) Real Part of Economy Important
    - Adjustment Costs in Investment, Variable Capital Utilization, Habit Persistence in Preferences.

  (c) What Nominal Frictions and Real Features are Necessary for a Model to Conform Well with the Facts?

Shock-Based Strategy for Estimating a Dynamic GE Model

- Vector Autoregression (VAR):

$$Y_t = B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t,\ E u_t u_t' = V$$

  B_l's, u_t's and V Easily Obtained by OLS Regressions.

- Fundamental Economic Shocks, e_t:

$$u_t = C e_t,\ E e_t e_t' = I,\ CC' = V.$$

Shock-Based Strategy...

- Impulse Responses ($p = 2$):

$$\begin{aligned}
Y_t - E_{t-1}Y_t &= Ce_t \\
E_tY_{t+1} - E_{t-1}Y_{t+1} &= B_1Ce_t \\
E_tY_{t+2} - E_{t-1}Y_{t+2} &= B_1^2Ce_t + B_2Ce_t \\
E_tY_{t+3} - E_{t-1}Y_{t+3} &= \left[B_1\left(B_1^2 + B_2\right) + B_1B_2 + B_2B_1\right]Ce_t
\end{aligned}$$

- Suppose Want Dynamic Response of Y_t to i^{th} Element of e_t.
  Need B_l's and i^{th} Column of C.

Shock-Based Strategy...

- Problem:
  N^2 Unknown Elements in C,
  (a) Only $N(N+1)/2$ Equations in:

  $$CC' = V$$

  (b) Need to Make (Identifying) Assumptions!

Shock-Based Strategy...

- Moving Average Representation:

$$
\begin{aligned}
Y_t &= c_1(L)e_{1t} + c_2(L)e_{2t} + ... + c_p(L)e_{pt} \\
c_i(L) &= c_i^0 + c_i^1 L + c_i^2 L^2 ...
\end{aligned}
$$

- A Dynamic GE Model Implies:

$$c_i(L;\gamma), \text{ each } i.$$

- Estimate γ by making

$$c_i(L;\gamma) - c_i(L)$$

  small, $i = 1, 2, 3...$

- With Enough Shocks, Have a Model that Fits the Data As Well as a VAR.

Alternative (More Traditional) Strategy for Estimating A Model:

- Compute Unconditional Moments of Data
- Estimate Model Based on All Moments (Maximum Likelihood)
- Disadvantage of This Approach:
  - Need to Determine All Shocks in the Model

Advantages of Shock-Based Strategy

- Avoid Need to Specify All the Shocks Right Away
- The Economics of a Model Lie in its Impulse Responses to Shocks.
  - Analysis and Diagnosis of Models Made Transparent.

Advantages, cont'd...

- Sometimes, Interesting Puzzles and Questions Posed Directly in Terms of Shocks.
  (a) Monetary Policy Shock.
    - Why So Much Inflation Inertia, Output Persistence?
    - Other Issues (Price 'Puzzle')

  (b) Technology Shock.
    - How Important In Business Cycle?
    - Response of Employment and Other Variables to Technology?
    - What is Role of Monetary Policy in Propagation of Technology Shocks?

Outline of Remainder of Analysis

(1) The Vector Autoregression Used in the Analysis.

(2) Estimated Impulse Responses to:
   (a) Monetary Policy Shock.
   (b) Persistent Shock to Technology. Reconciliation With the Literature.

(3) Our GE Model.

(4) Fitting the GE Model to VAR Impulse Responses.

Vector Autoregression

- VAR:

$$\begin{aligned} Y_t \ &= \ B(L)Y_{t-1} + Ce_t, \\ & \quad e_t \sim \text{fundamental shocks} \end{aligned}$$

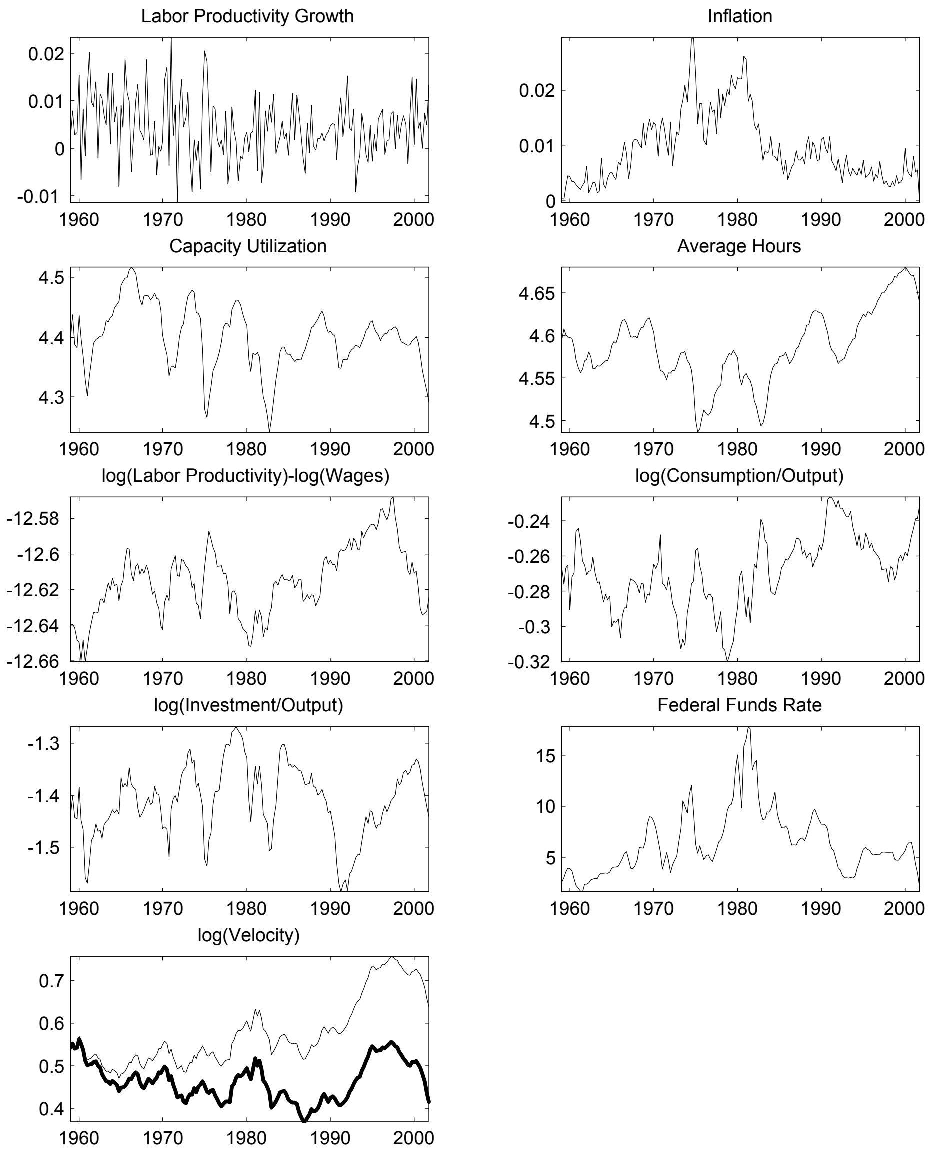
- Vector, Y_t, Set Up to Satisfy Integration and Co-Integration Properties of Model.

$$\underbrace{Y_t}_{9\times 1} = \left(\begin{array}{c} \Delta \ln(GDP_t/\text{Hours}_t) \\ \Delta \ln(GDP \text{ deflator}_t) \\ \text{capacity utilitzation}_t \\ \ln(GDP_t/\text{Hours}_t) - \ln(W_t/P_t) \\ \ln(\text{Hours}_t) \\ \ln(C_t/GDP_t) \\ \ln(I_t/GDP_t) \\ \text{Federal Funds Rate}_t \\ \ln(GDP \text{ deflator}_t) + \ln(GDP_t) - \ln(M2_t) \end{array} \right)$$

- Estimation Period: 1960QI - 2001QIV

Figure 1

Data Used In VAR



Identification Assumptions for Monetary Policy Shock

- Monetary Policy Rule:

$$
\begin{aligned}
R_t &= f(\Omega_t) + \varepsilon_t \\
\varepsilon_t &\sim \text{ Monetary Policy Shock}
\end{aligned}
$$

- Identification Assumptions:
  - $\Omega_t = \{$aggregate activity$_t$, aggregate prices and wages$_t$, lagged variables$\}$
  - ε_t orthogonal with Ω_t
- Our GE Model is Consistent with These Assumptions.

What Is A Monetary Policy Shock?

○Shocks to Preferences of Monetary Authority
○Technical Factors Like Measurement Error (Bernanke-Mihov):

$$x_t(0) = x_t + v_t,\ x_t(1) = x_t + u_t$$

$$S_t = \beta_0 S_{t-1} + \beta_1 x_t(0) + \beta_2 x_{t-1}(1)$$

or

$$\begin{aligned} S_t &= \beta_0 S_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t \\ \varepsilon_t &= \beta_1 v_t + \beta_2 u_{t-1}. \end{aligned}$$

Recursiveness Assumption: $\beta_0 = 0$, or $\beta_0 \neq 0$, $u_t = 0$.

What Is a Monetary Policy Rule?

○Literal Interpretation: Structural Policy Rule of Central Bank.

○Combination of Structural Policy Rule and Other Stuff

– Example 1:

$$\text{'True' Policy Rule: } S_t = \alpha e_t + \varepsilon_t,$$

where

$$\begin{aligned} e_t &\sim \text{ 'innovation' in some variable} \\ &= \sum_{i=0}^{\infty} \beta_i x_{t-i}. \end{aligned}$$

Then

$$\text{Estimated Policy Rule: } S_t = \alpha \sum_{i=0}^{\infty} \beta_i x_{t-i} + \varepsilon_t$$

Here, α's are not structural.

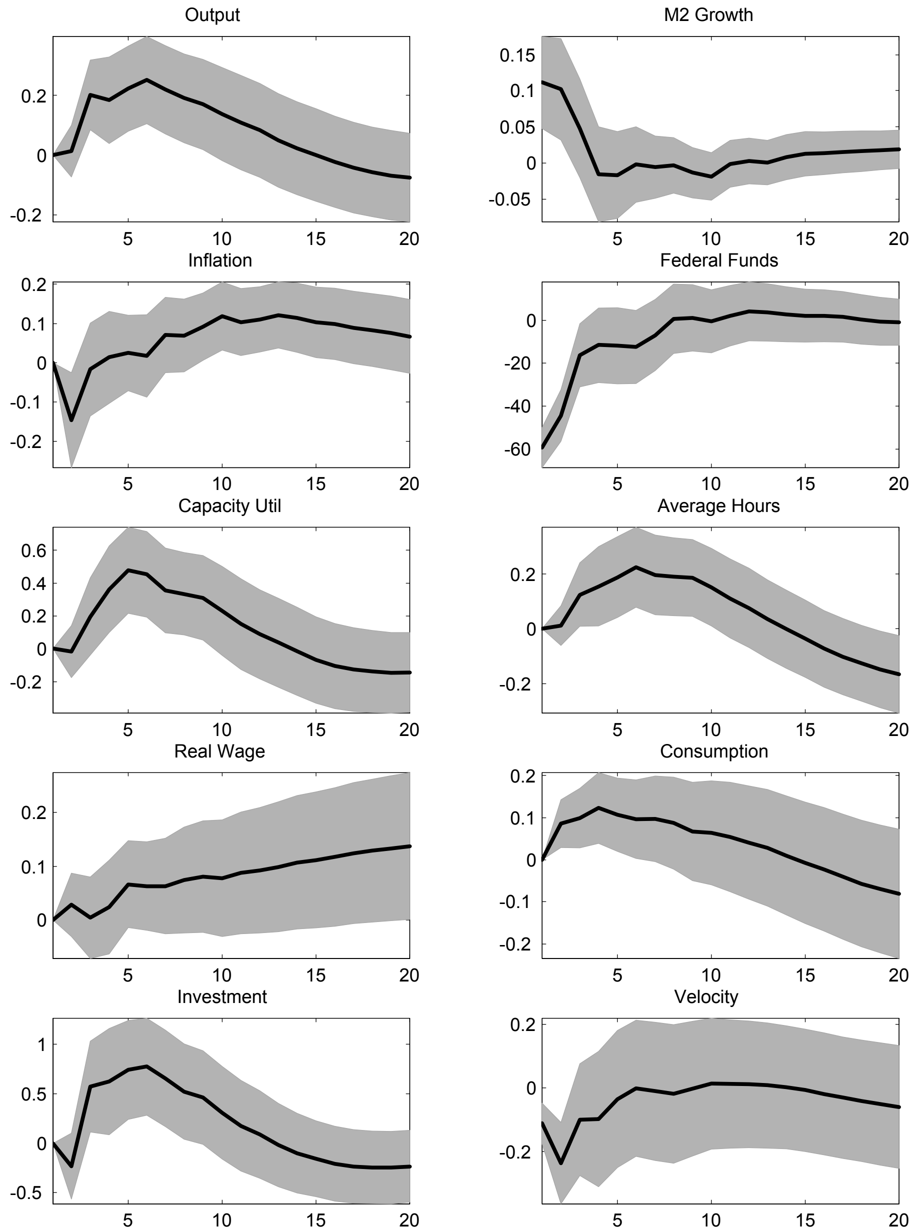
– Example 2 (Clarida-Gertler):

$$\text{'True' Policy Rule: } S_t = \alpha E_t x_{t+1} + \varepsilon_t$$

Dynamic Effects of a Monetary Policy Shock

- After a Positive Monetary Shock:
  (a) hump-shaped response of output, consumption, investment, labor with peak effect after roughly 1 year.
  (b) hump-shaped response inflation, with peak response after about 2 years ('Inflation Inertia')
  (c) interest rate down for one year.
  (d) money growth up for 2-3 quarters.
  (e) real wage up slightly.
- Lots of Internal Propagation
- Strong Liquidity Effect

Figure: Impulse Responses to a Monetary Policy Shock



Identification Assumptions for Technology Shock

- Identification Assumption:
  Technology Shock is *Only* Shock that Has Long-Run Impact on (Forecast of) Level of Labor Productivity:

$$\lim_{j\to\infty}\left[E_t y_{t+j} - E_{t-1} y_{t+j}\right] = \text{f(technology shock only)}$$

$$y_t = \frac{\text{output}}{\text{hour}}$$

- Blanchard-Quah/Jordi Gali:
  This Assumption Makes it Possible to Estimate Technology Shock, Even Without Direct Observations on Technology

- Our GE Model is Consistent with This Assumption.

Technology Identification

- Simple VAR:

$$
\begin{aligned}
Y_t &= BY_{t-1} + u_t,\ Eu_t u_t' = V \\
u_t &= Ce_t \\
Y_t &= \begin{pmatrix} \Delta y_t \\ x_t \end{pmatrix},\ C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},\ e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \\
& e_{1t} \text{ ~ Technology Shock.}
\end{aligned}
$$

- From Applying OLS To Both Equations in VAR, We Know:

$$B,\ V$$

- Problem: $CC' = V$ Provides only Three Equations in Four Unknowns in C.
- Result: Assumption that e_{2t} Has No Long Run Impact on y_t Can Be Used to Identify *All* of C

Technology Identification

- Note #1:

$$\begin{aligned} & E_t\left[\Delta y_{t+1}+\Delta y_t\right]-E_{t-1}\left[\Delta y_{t+1}+\Delta y_t\right] \\ = \; & E_t\left[y_{t+1}-y_t+y_t-y_{t-1}\right]-E_{t-1}\left[y_{t+1}-y_t+y_t-y_{t-1}\right] \end{aligned}$$

Technology Identification

- Note #1:

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\begin{aligned}
& E_t\left[\Delta y_{t+1}+\Delta y_t\right]-E_{t-1}\left[\Delta y_{t+1}+\Delta y_t\right] \\
= & E_t\left[y_{t+1}-y_t+y_t-y_{t-1}\right]-E_{t-1}\left[y_{t+1}-y_t+y_t-y_{t-1}\right] \\
= & E_t[y_{t+1}]-y_{t-1}-E_{t-1}[y_{t+1}]+y_{t-1}
\end{aligned}
$$

Technology Identification

- Note #1:

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Technology Identification

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= \; & E_t[y_{t+1}]-E_{t-1}[y_{t+1}]
\end{aligned}
$$

- Note #2:

$$
\begin{aligned}
& (1,0)E_t\left[Y_{t+1}+Y_t\right]-(1,0)E_{t-1}\left[Y_{t+1}+Y_t\right] \\
= \; & E_t\left[\Delta y_{t+1}+\Delta y_t\right]-E_{t-1}\left[\Delta y_{t+1}+\Delta y_t\right]
\end{aligned}
$$

Technology Identification

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- Note #3:

$$\begin{aligned} & (1,0)E_t\left[Y_{t+1}+Y_t\right]-(1,0)E_{t-1}\left[Y_{t+1}+Y_t\right] \\ = \; & (1,0)\left\{E_t\left[Y_{t+1}\right]-E_{t-1}\left[Y_{t+1}\right]+E_t\left[Y_t\right]-E_{t-1}\left[Y_t\right]\right\} \end{aligned}$$

Technology Identification

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= \; & (1,0)\left\{E_t\left[Y_{t+1}\right]-E_{t-1}\left[Y_{t+1}\right]+E_t\left[Y_t\right]-E_{t-1}\left[Y_t\right]\right\} \\
= \; & (1,0)\left[BCe_t+Ce_t\right]
\end{aligned}
$$

Technology Identification

- Note #1:

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- Conclude:

$$E_t[y_{t+1}]-E_{t-1}[y_{t+1}]=(1,0)\left[B+I\right]Ce_t$$

Technology Identification

- Result for Two Period-Ahead Forecast of y_t:

$$E_t[y_{t+2}] - E_{t-1}[y_{t+2}] = (1,0)\left[B^2 + B + I\right]Ce_t$$

Technology Identification

- Result for Two Period-Ahead Forecast of y_t:

$$E_t[y_{t+2}] - E_{t-1}[y_{t+2}] = (1,0)\left[B^2 + B + I\right] Ce_t$$

- Result for j Period-Ahead Forecast of y_t:

$$E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\left[B^j + B^{j-1} + ... + B^2 + B + I\right] Ce_t$$

Technology Identification

- Result for Two Period-Ahead Forecast of y_t:

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$$E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\left[B^j + B^{j-1} + ... + B^2 + B + I\right] Ce_t$$

- As $j \to \infty$:

$$\lim_{j\to\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\left[... + B^j + B^{j-1} + ... + B^2 + B + I\right] Ce_t$$

Technology Identification

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- As $j \to \infty$:

$$\lim_{j\to\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\left[... + B^j + B^{j-1} + ... + B^2 + B + I\right] Ce_t$$
$$= (1,0)\left[I - B\right]^{-1} Ce_t$$

Technology Identification

- As $j \rightarrow \infty$:

$$\lim_{j\to\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\,[I-B]^{-1}\,Ce_t$$

- Identification Assumption About Technology:

$$[I-B]^{-1}\,Ce_t = \begin{bmatrix} \text{number} & 0 \\ \text{number} & \text{number} \end{bmatrix} \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

Technology Identification

- As $j \to \infty$:

$$\lim_{j\to\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\,[I-B]^{-1}\,Ce_t$$

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[I-B]^{-1}\,Ce_t &= \begin{bmatrix} \text{number} & 0 \\ \text{number} & \text{number} \end{bmatrix} \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \\
&= \begin{bmatrix} \text{number} \times e_{1t} + 0 \times e_{2t} \\ \text{number} \times e_{1t} + \text{number} \times e_{2t} \end{bmatrix}
\end{aligned}$$

Technology Identification

- As $j \rightarrow \infty$:

$$\lim_{j\rightarrow\infty} E_t[y_{t+j}] - E_{t-1}[y_{t+j}] = (1,0)\,[I-B]^{-1}\,Ce_t$$

- Identification Assumption About Technology:

$$[I-B]^{-1}\,C = \begin{bmatrix} \text{number} & 0 \\ \text{number} & \text{number} \end{bmatrix}$$

- Final Result (?!)
  Solve for C Using

$$1,2 \text{ element of } [I-B]^{-1}\,C \text{ is } zero$$
$$CC' = V$$

Conclusion of Technology Identification With Long-Run Restrictions

- Example (Taken from Blanchard-Quah) Gives Flavor of Identification When There are Long-Run Restrictions
- In Practice, An Alternative Computational Strategy, Based on Instrumental Variables Estimation, Also Useful.
- Other Applications of Long-Run Restrictions:
  - Labor Supply Shock Has Long-Run Impact on Hours Worked, No Long Run Impact on Productivity
  - Money Supply Shock Has No Impact on Real Variables in Long Run, Money Demand Shock Affects Real Balances in Long Run

Instrumental Variables (Shapiro-Watson) Approach

- IV Regression:

$$\begin{aligned}\Delta y_t &= a_{\Delta y}(L)\Delta y_{t-1} + a_1(L)Y_t + a_R(L)R_{t-1}\\ &\quad + a_V(L)Velocity_{t-1} + \varepsilon_{xt}\end{aligned}$$

- All Non-ε_{xt} Shocks Affect Δy_t Via Y_t, R_t, $Velocity_t$.
- To Have No Long-Run Effect of y_t Requires:

$$\begin{aligned}a_1(L) &= \tilde{a}_1(L)(1-L),\\ a_R(L) &= \tilde{a}_R(L)(1-L),\\ a_V(L) &= \tilde{a}_V(L)(1-L).\end{aligned}$$

- IV Regression With LR Restrictions:

$$\begin{aligned}\Delta y_t &= a_{\Delta y}(L)\Delta y_{t-1} + \tilde{a}_1(L)\Delta Y_t + \tilde{a}_R(L)\Delta R_{t-1}\\ &\quad + \tilde{a}_V(L)\Delta Velocity_{t-1} + \varepsilon_{xt}\end{aligned}$$

Shapiro-Watson Approach, Cont'd....

- First Stage: Estimate IV Regression (IV is Needed Because Technology Shocks, ε_t^x, *Also* Affect Y_t)
- Second Stage: Regress VAR Reduced-Form Disturbances, u_t, on $\hat{\varepsilon}_{xt}$, To Obtain Relevant C_i.

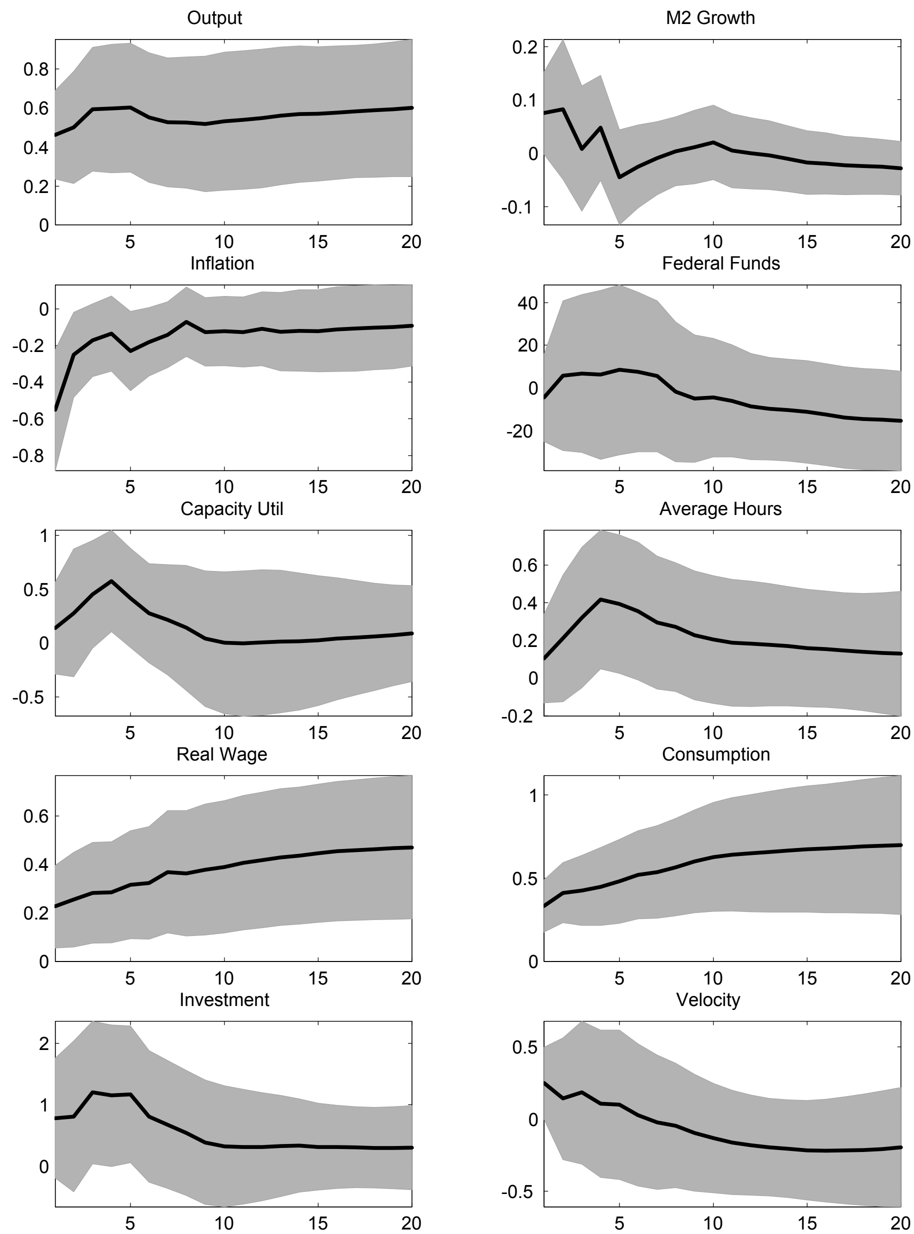
$$Y_t = B(L)Y_{t-1} + u_t,\ u_t = Ce_t,$$

- Practical Problem: Method Does Not Impose Overindentifying Restrictions on $B(L)$ (see ACEL).

Results for Technology Identification

- (a) Labor and Capital Utilization Rise.
  (b) Inflation Low.
  (c) Money Growth Accommodates.

Figure: Impulse Responses to an Innovation in Technology



Relation to the Literature:

- Our Finding That Employment Rises Persistently After Technology Shock Contrasts with Results in Literature
- Literature: Employment *Falls* Persistently After Technology Shock
(Gali, Francis-Ramey, Basu).
- We Use Same Long-Run Identifying Restrictions Used in Literature.

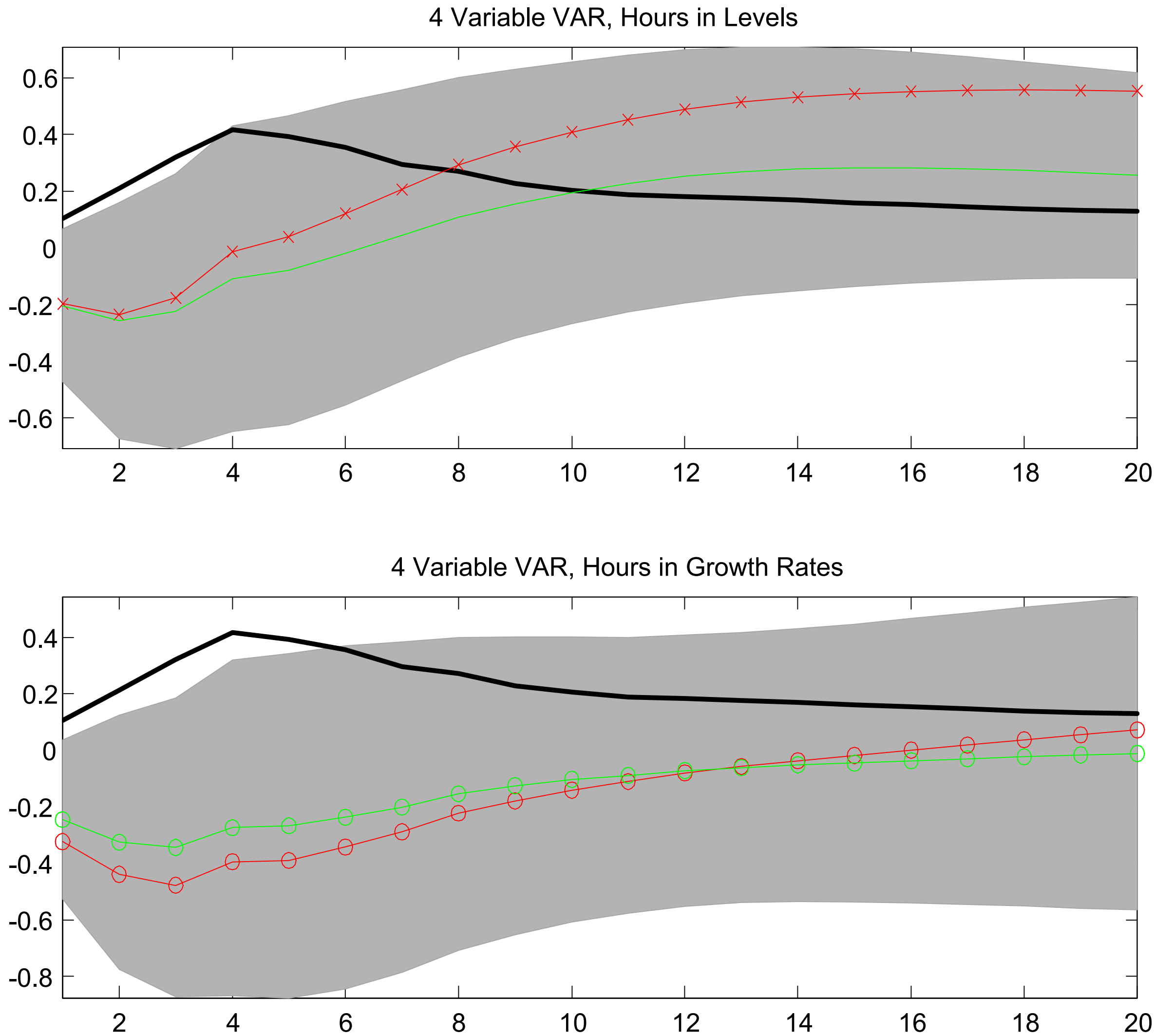
Relation to the Literature, Cont'd...

- Our Analysis Suggests Results in Literature Reflect Two Forms of Specification Error:
  - Working With VAR's With Too Few Variables.
  - Overdifferencing the Hours Data.
- How Do We Establish These Conclusions?
  - These Are The Implications of Our VAR Model
    - Model Passes a Basic Encompassing Test
  - Our VAR Model is More Plausible than Alternatives
    - 'We can explain their results more easily than they can explain our results' (Christiano-Ljungqvist, 1988.)

Analysis Conditional on Our VAR Being True

- Omitted Variables:
  - Drop $Cap.\ Ut.$, C_t, I_t, W_t/P_t, $M2_t$ From Our System to 4 - Variable Model with Levels Hours.
  - This Model Implies: Hours *Fall* After Positive Technology Shock.
  - This Outcome is Predicted by Our (9-variable) Model.
- Over-Differencing:
  - Also Measure Hours in First Differences
  - With This Further Modification, Hours Fall *a Lot* After Positive Technology Shock.
  - This Outcome is Predicted by Our (9-variable) Model.

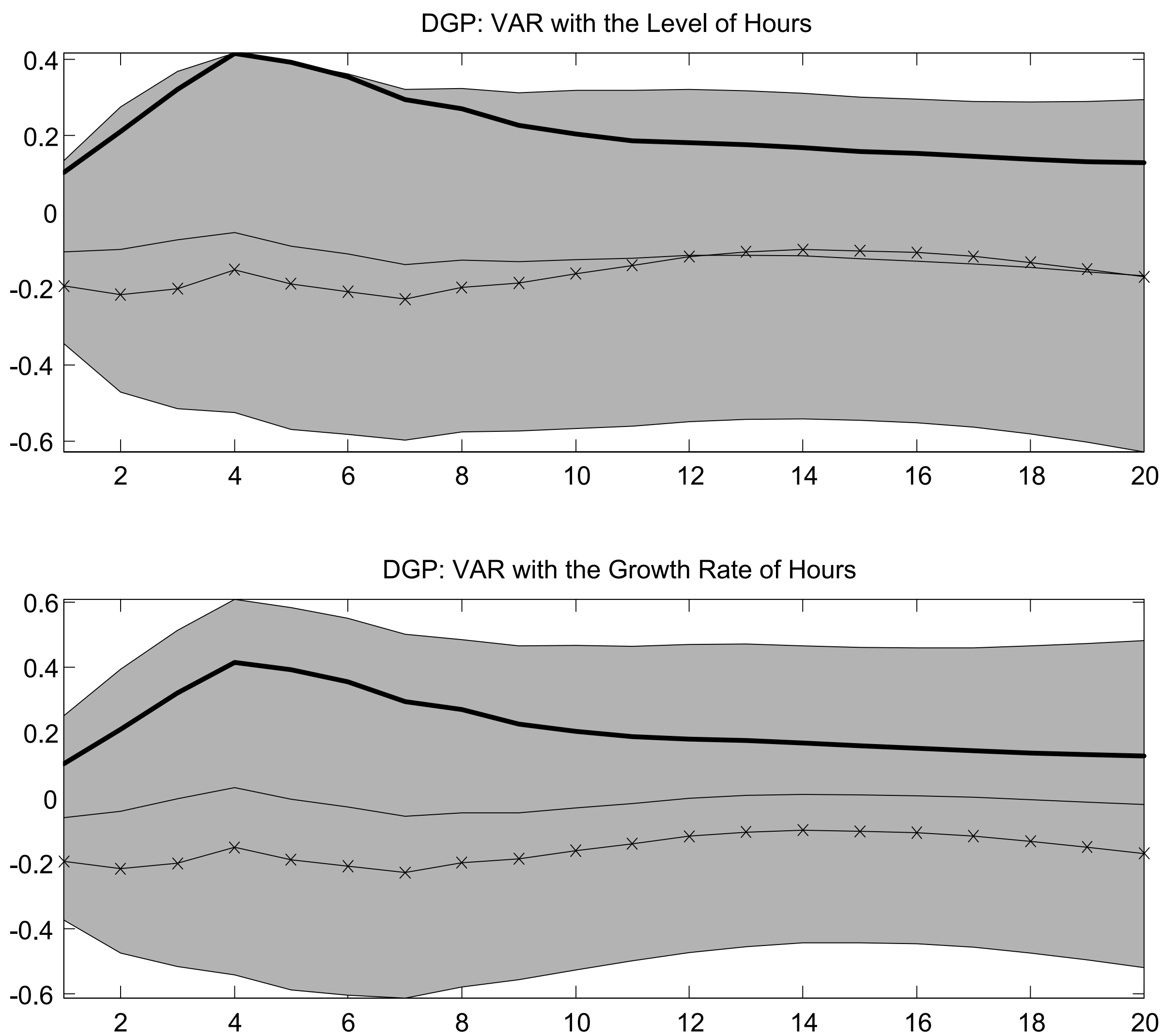
Figure 10: Analysis of Two Versions of Four-Variable VAR



Our VAR Specification is More Plausible than Alternatives

- Alternative to Our VAR to Worry About: Version with First Difference In Hours.
- This Version Predicts Hours *Fall* After a Positive Shock To Technology.
- According to a Limited Information Bayesian Procedure:
  - Odds are 1.7 to 1 in Favor of Our VAR Over Alternative.

Effect of Differencing Hours Worked on our Benchmark Model



Limited Information Bayesian Analysis

- Define: $Q=\{$Event that Hours Rise After Positive Technology Shock in 'Our' Model And Hours Fall After Positive Technology Shock in 'Their' Model$\}$
- Results of Simulation Analysis:

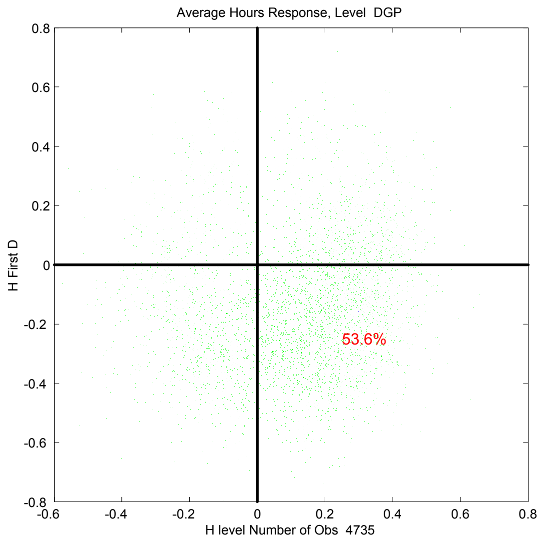
$$\begin{aligned} P(Q|\text{'Our'}) &= 54\% \\ P(Q|\text{'Their'}) &= 32\% \end{aligned}$$

- Using Bayesian Updating Formula (With Flat Priors):

$$\frac{P(\text{'Our'}|Q)}{P(\text{'Their'}|Q)}=\frac{54}{35}=1.7$$

- The Odds Favor 'Our' Model Nearly 2 to 1.

Computing $P(Q|\text{'Our'})$ Computing $P(Q|\text{'Their'})$



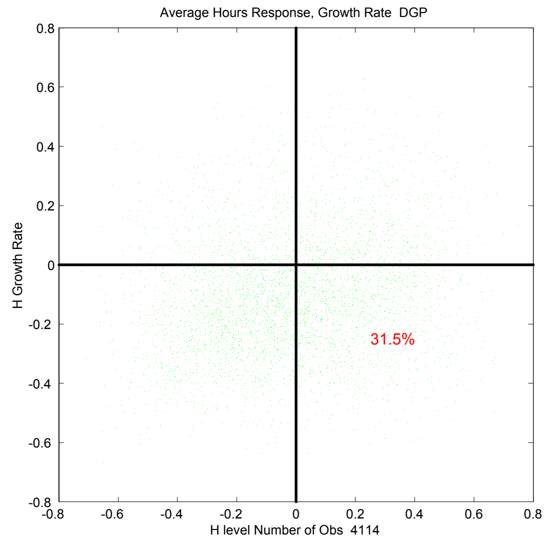


Figure: Impulse Responses to a Monetary Policy Shock

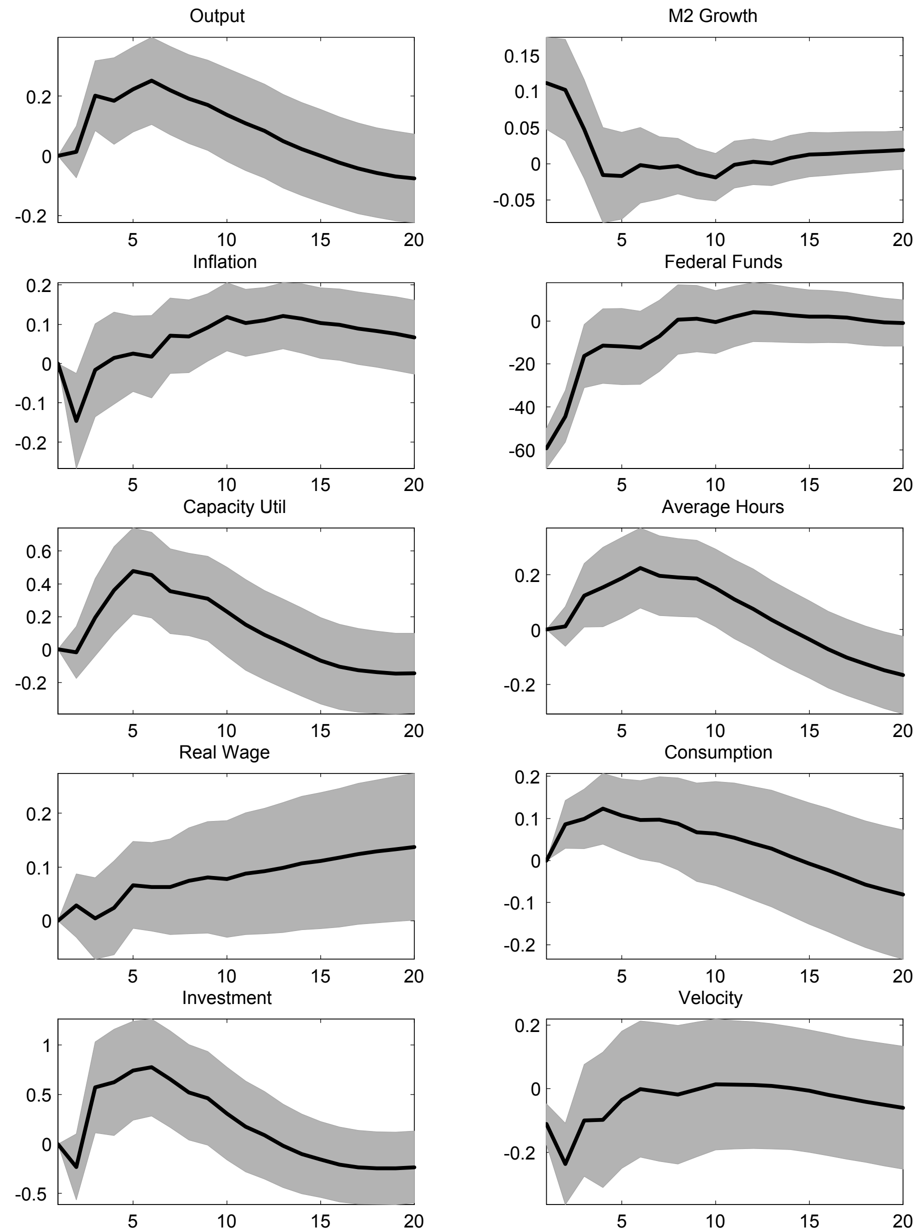
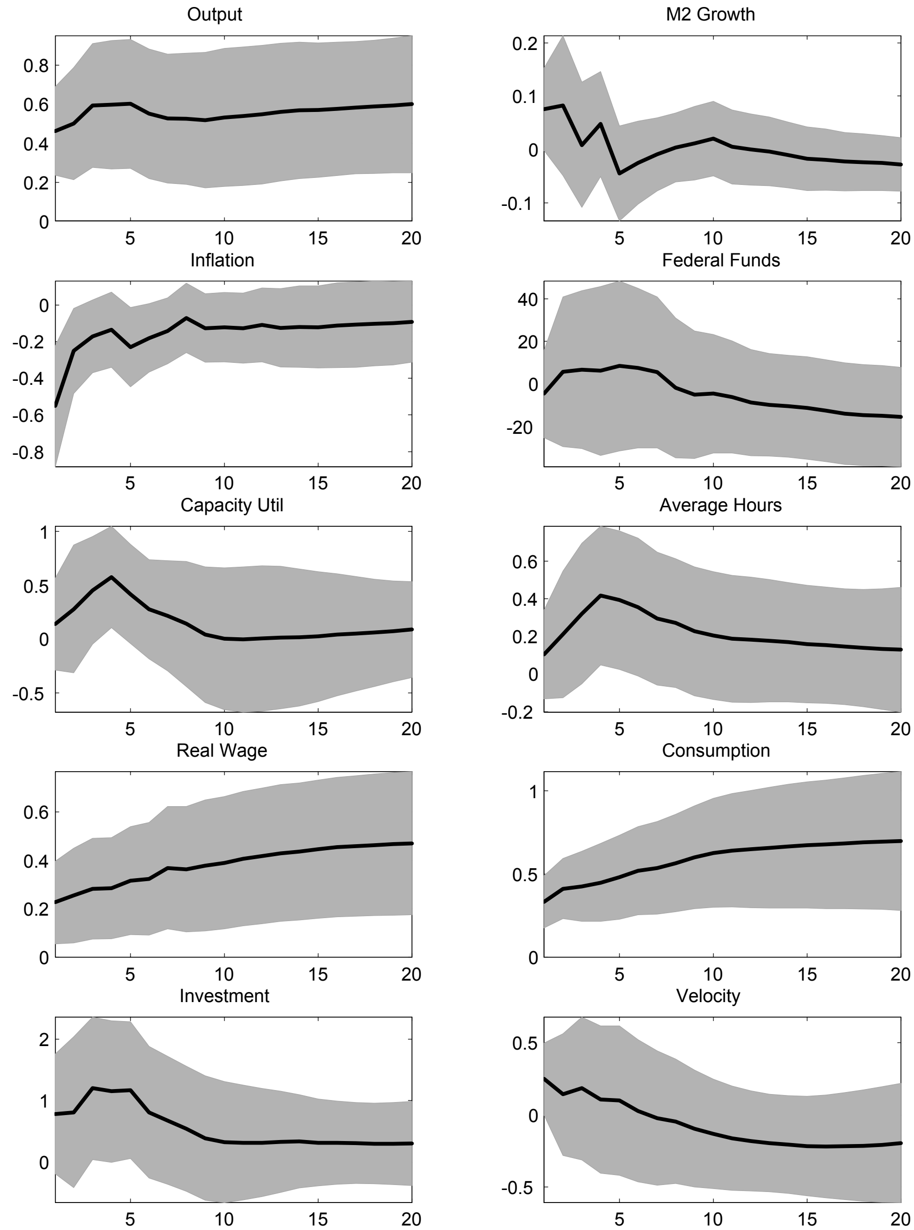


Figure: Impulse Responses to an Innovation in Technology



Is there Other Useful Information We can Get From our VAR?

- Decomposition of Variance: What Fraction of Variance in Data is Due to this or That Identified Shock
- What Shocks Account Most for Various Episodes in the Data?
- Policy Analysis:
  - What Is the Impact on the Distribution of the Variables in the VAR, If Policy over the Next 6 Months Keeps the Interest Rate 25 Basis Points Lower than it is Today?
  - Formally:

$$\begin{aligned}\text{Event} &: Q = R \uparrow \text{by 25 Basis Points for 6 Months}\\ &E[\text{Variables of Interest}|Q]\\ &95 \text{ Confidence Interval on Variables of Interest}|Q\end{aligned}$$

- Original Contributions on Policy Analysis in VAR: Doan-Litterman-Sims, 1980s; Sims, Minneapolis Fed Quarterly Review, 1980s (see RATS manual). See also recent work by Sims, Zha, Leeper.

Variance Decomposition Results

- Monetary Policy Shocks Account for Very Little Variance, Even in Money.
- Technology Shocks Account for Over 50 Percent of Overall Variance of Output Horizon.
- Technology Shocks Account for Only a Small Part of Business Cycle Fluctuations.

Percent of Overall Variance At Various Horizons, Due to Monetary Policy Shocks

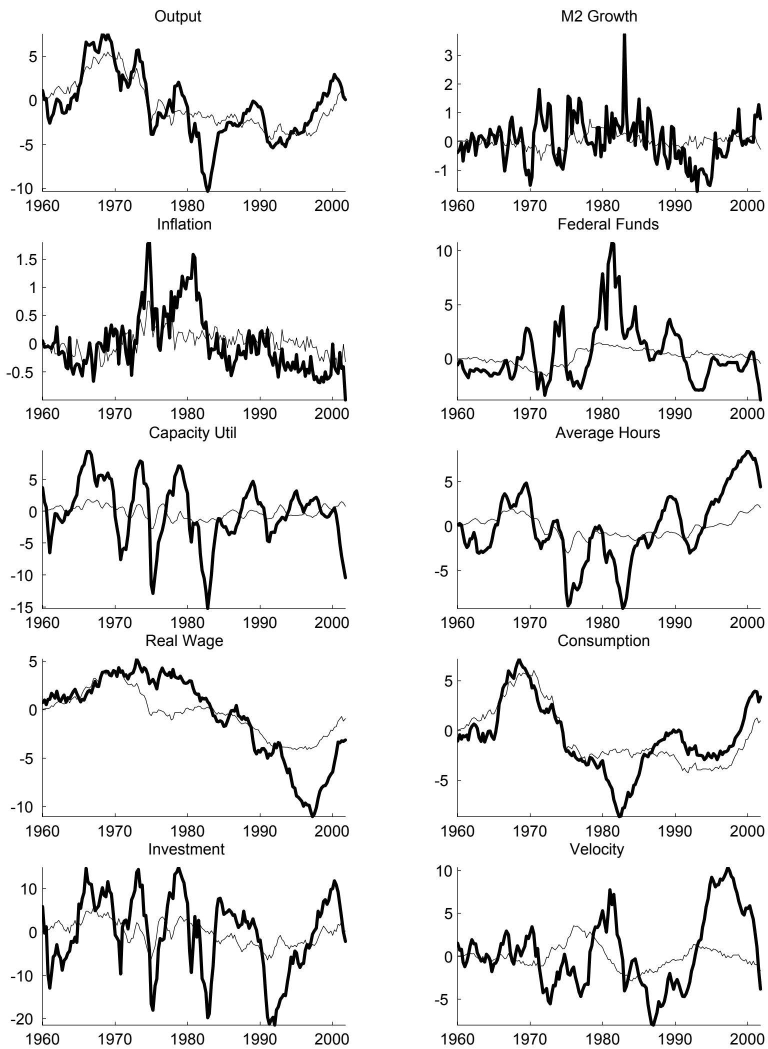
Table 1: Contribution of Policy Shocks to Variance						
Variable	Forecast Variance					B. C.
	at Indicated Horizons					Freq's
	1	4	8	12	30	HP
Output	0.0	3.1	5.4	4.4	2.5	7.7
M2 Growth	6.2	6.5	6.0	5.4	5.1	7.0
Inflation	0.0	1.7	1.8	3.4	4.4	4.3
Fed Funds	65.2	21.0	12.5	11.0	9.1	20.3
Capacity Util	0.0	2.6	7.2	5.6	4.4	6.7
Average Hours	0.0	1.9	5.0	5.1	6.0	7.2
Real Wage	0.0	0.2	0.9	1.3	3.2	1.1
Consumption	0.0	3.3	2.6	1.6	1.4	6.0
Investment	0.0	2.8	5.4	4.8	4.6	7.3
Velocity	2.3	2.1	1.1	0.9	0.8	3.3

Percent of Overall Variance At Various Horizons, Due to Technology Shocks

Table 2: Contribution of Tech. Shocks to Variance						
Variable	Forecast Variance					B. C.
	at Indicated Horizon					Freq's
	1	4	8	12	30	HP
Output	48.4	48.8	47.1	45.7	62.7	13.7
M2 Growth	2.8	3.8	4.1	3.8	5.7	4.6
Inflation	41.1	32.1	28.5	25.8	17.4	20.9
Fed Funds	0.4	0.5	0.6	0.8	5.7	1.1
Capacity Util	1.4	9.7	8.2	5.3	4.5	3.2
Average Hours	4.1	15.6	19.3	17.2	11.2	5.0
Real Wage	27.7	32.1	35.6	36.9	44.8	16.4
Consumption	61.0	67.4	65.3	65.3	69.9	24.7
Investment	9.8	14.5	14.1	11.8	12.2	5.5
Velocity	11.4	3.0	1.7	2.2	3.7	4.7

Role of Technology Shocks: Big in Low Frequencies,

Small in Business Cycles



Thick Line - Actual Historical Data, Thin Line - Technology Only

Next Step:

- Construct a Model that is Consistent With Identifying Assumptions in VAR
- Estimate the Combination of Frictions Needed for Model Responses to Resemble Estimated Responses.

Key Features of the Model

- Consistent with Identifying Assumptions Used in Estimating Response of Economy to Shocks.
- Two Forms of Nominal Rigidities: Calvo-style Nominal Price and Wage Contracts.
- Real Side of Model - Three Departures from Standard Textbook Growth Model
  - Habit Persistence in Consumption
  - Adjustment Costs in Investment
  - Variable Capacity Utilization
- Model Will Do Well Empirically, with Reasonable Degree Price and Wage Stickiness.

Model

- Timing Assumptions.
- Firms.
- Households.
- Monetary Authority.

Timing

(1) Technology Shock Realized.

(2) Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.

(3) Monetary Policy Shock Realized.

(4) Household Money Demand Decision Made.

(5) Production, Employment, Purchases Occur, and Markets Clear.

- Note: Wages, Prices and Output Predetermined Relative to Policy Shock.

Firms

Final Good Firms

- Technology:

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \ 1 \leq \lambda_f < \infty$$

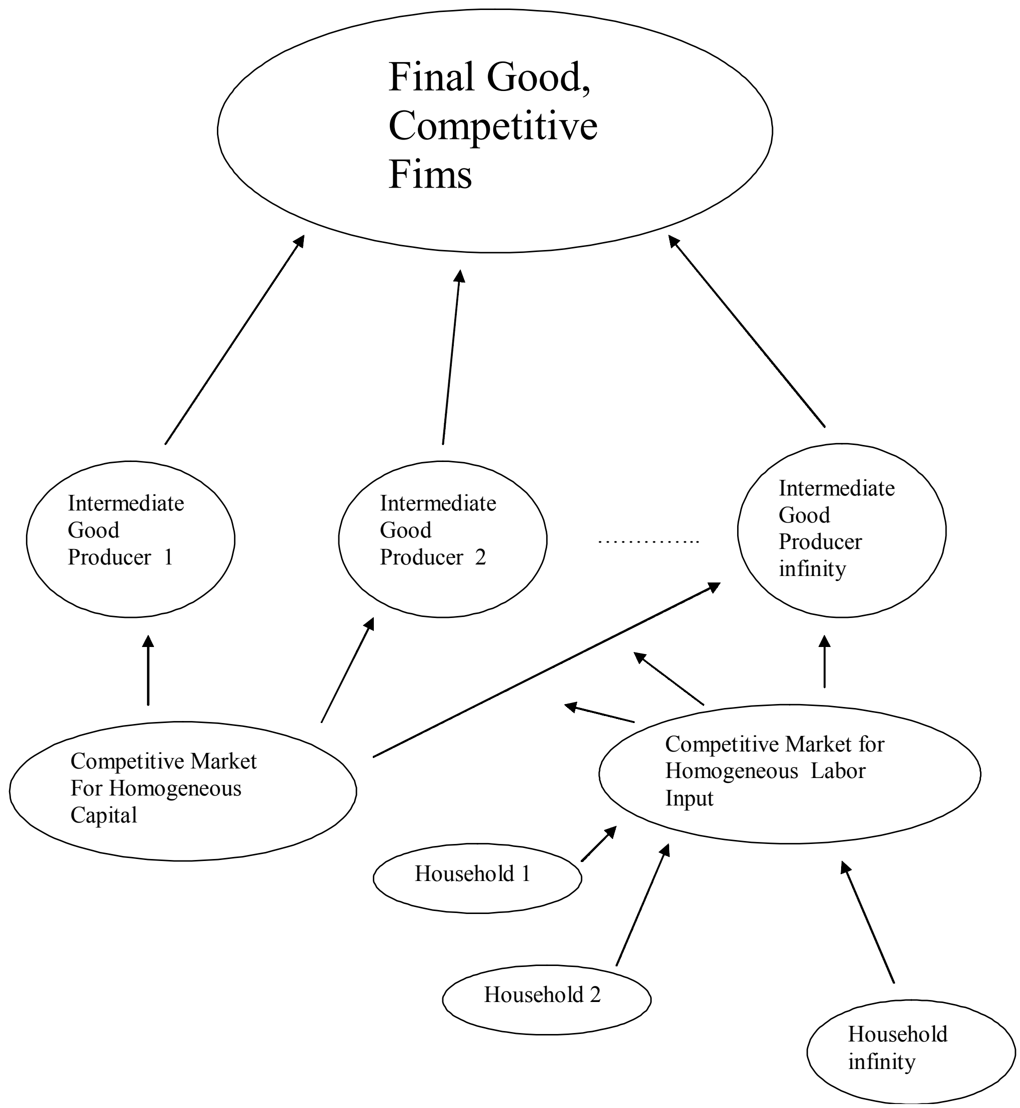
- Objective:

$$\max P_t Y_t - \int_0^1 P_{it} Y_{it} di$$

- Foncs and Prices:

$$\left(\frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}, \ P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)}.$$

Firm Sector



1.

Intermediate Good Firms -

- Each Y_{it} Produced by a Monopolist, With Demand Curve:

$$\left(\frac{P_t}{P_{it}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$

- Technology:

$$Y_{it} = \begin{cases} k_{it}^{\alpha}\left(z_t L_{it}\right)^{1-\alpha} - \phi z_t & \text{if } k_{it}^{\alpha}\left(z_t L_{it}\right)^{1-\alpha} \geq \phi z_t \\ 0, & \text{otherwise} \end{cases}.$$

- Here, z_t is a Technology Shock:

$$x_t = \log z_t - \log z_{t-1},\ x_t = (1-\rho_x)x + \rho_x x_{t-1} + \varepsilon_{xt}.$$

- $\phi > 0$:
  - Ensures Zero Profits in Steady State
  - Helps Generate Rise in Y/L After Positive Monetary Policy Shock.

- Calvo Price Setting:
  - With Probability $1 - \xi_p$, i^{th} Firm Sets Price, P_{it}, Optimally, to $\tilde{P}_t$.
  - With Probability ξ_p, Do Not Optimize Current Price. Instead:

$$P_{it} = \pi_{t-1} P_{i,t-1}, \ \pi_t = \frac{P_t}{P_{t-1}}.$$

- Firms Setting Prices Optimally at t
  Choose $\tilde{P}_t$ to max:

$$\begin{aligned}
&\upsilon_t \left[\tilde{P}_t Y_{it} - MC_t Y_{it}\right] \\
&+\beta\xi_p \upsilon_{t+1}\left[\tilde{P}_t \pi_t Y_{i,t+1} - MC_{t+1} Y_{i,t+1}\right] \\
&+\left(\beta\xi_p\right)^2 \upsilon_{t+2}\left[\tilde{P}_t \pi_t \pi_{t+1} Y_{i,t+2} - MC_{t+2} Y_{i,t+2}\right] \\
&+...
\end{aligned}$$

  subject to:

$$\left(\frac{P_t}{\tilde{P}_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.$$

υ_t ˜ value of a dividend at t
MC_t ~given

- Scaling:

$$\begin{aligned}
\tilde{p}_t &= \frac{\tilde{P}_t}{P_t},\ w_t = \frac{W_t}{P_t} \\
r_t^k &= \frac{\text{rental rate on capital}}{P_t} \\
s_t &= \frac{MC_t}{P_t}.
\end{aligned}$$

- Real Marginal Cost:

$$s_t = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)} \left(\frac{1}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} (w_t R_t)^{1-\alpha} \frac{1}{z_t}$$

- Linear approximation:

$$\hat{x}_t \equiv \frac{x_t - x}{x}.$$

- Approximate (Linearized) Solution:

$$
\begin{aligned}
\widehat{\tilde{p}}_t \;=\; & \hat{s}_t + \sum_{l=1}^{\infty} \left(\beta \xi_p\right)^l \left(\hat{s}_{t+l} - \hat{s}_{t+l-1}\right) \\
& + \sum_{l=1}^{\infty} \left(\beta \xi_p\right)^l \left(\hat{\pi}_{t+l} - \hat{\pi}_{t+l-1}\right)
\end{aligned}
$$

- Front-Loading:
  - $\widehat{\tilde{p}}_t > \hat{s}_t$ if $\hat{s}_{t+l} > \hat{s}_t$ and/or $\hat{\pi}_{t+l} > \hat{\pi}_t$.

- Aggregate Price Level:

$$
\begin{aligned}
P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di\right]^{(1-\lambda_f)} \\
&= \left[(1-\xi_p)\tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left(\pi_{t-1} P_{t-1}\right)^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}
\end{aligned}
$$

- Scale:

$$
1 = \left[(1-\xi_p)\tilde{p}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left(\frac{\pi_{t-1}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}
$$

- Approximately

$$
\widehat{\tilde{p}}_t = \frac{\xi_p}{1-\xi_p}\left[\hat{\pi}_t - \hat{\pi}_{t-1}\right].
$$

- Combining:

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_{t-1}\hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta)\,\xi_p}E_{t-1}\hat{s}_t,$$

- Or:

$$\hat{\pi}_t = \hat{\pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}E_{t-1}\sum_{j=0}^{\infty}\beta^j\hat{s}_{t+j}$$

- Damped Inflation Response Requires Damped Marginal Cost Response.
- Econometric Estimates Likely to Emphasize Model Features That Mute Response of Marginal Cost to Shocks.

- Under Standard Price-Updating Scheme:

$$P_{it} = \bar{\pi} P_{i,t-1}.$$

Associated Reduced Form:

$$\hat{\pi}_t = \beta E_{t-1} \hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} E_{t-1} \hat{s}_t.$$

- According to Fuhrer-Moore (1995), Gali-Gertler (1997), Casares-McCallum (2000), Mankiw (2000), Walsh (1998):
  - Standard Approach Fits Data Badly.
  - Need Lagged Inflation.
- For Any Process Describing $\hat{s}_t$, Inflation Will Be More Inertial with Dynamic Price-Updating.

Households

- Wage Decision.
- Consumption Decision.
- Investment Decision.
- Capital Utilization Decision.
- Portfolio Decision.

- State Contingent Securities
  - Allow Household to Insulate Consumption, Asset Holdings from Realization of Idiosyncratic Calvo Uncertainty.
  - This Simplifies Computation of Equilibrium.
  - Ignore State Contingent Securities in the Presentation.
  - Households Are Different With Respect to Wages and Employment.

- Preferences:

$$E_{t-1}^{h}\sum_{l=0}^{\infty}\beta^{l-t}\left[u(c_{t+l}-bc_{t+l-1})-z(h_{j,t+l})+v(q_{t+l})\right].$$

b ~habit parameter

$$\begin{aligned} q &= \frac{Q}{P} \\ u(\cdot) &= \log(\cdot) \\ z(\cdot) &= \frac{\psi_0}{2}(\cdot)^2 \\ v(\cdot) &= \psi_q\frac{(\cdot)^{1-\sigma_q}}{1-\sigma_q} \end{aligned}$$

Habit Persistence and Response of Consumption

- Recall that After an Expansionary Monetary Policy Shock, we see
  - hump-shaped rise in consumption
  - decline in real interest rate.
- Euler Equation in Standard Model:

$$\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1}}{\beta c_t} = \frac{g_{t+1}}{\beta} = \frac{R_t}{\pi_{t+1}},\ \pi_{t+1} = \frac{P_{t+1}}{P_t}.$$

- Problem: Can't Have g_t High and $\frac{R_t}{\pi_{t+1}}$ Simultaneously!

- Habit Persistence in Preferences (example):

$u(c_t - b\bar{c}_{t-1}),\ \bar{c}_{t-1}$ ~ aggregate consumption

- Euler Equation:

$$\frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1} - bc_t}{\beta\left(c_t - bc_{t-1}\right)} = \frac{g_{t+1} - b}{\beta\left(1 - \frac{b}{g_t}\right)}$$
$$\approx \frac{g_{t+1} - bg_t}{\beta(1-b)}$$

- Result:
  - g_{t+1} and g_t Can Both be High, as Long as $g_{t+1} < bg_t$.
  - Consistent with Simultaneous Hump-Shape c Response and Low Real Rate.
- Habit Persistence Also Helpful for Understanding Asset Prices

Flow Budget Constraint of j^{th} Household (Ignoring Insurance Considerations):

$$\begin{aligned} M_{t+1} &= R_t \left[M_t - Q_t + (\mu_t - 1) M_t^a \right] \\ &\quad + Q_t + W_{jt} h_{jt} + P_t r_t^k u_t \bar{k}_t + D_t \\ &\quad - P_t \left(c_t + i_t + a(u_t) \bar{k}_t \right) \end{aligned}$$

$$k_t = u_t \bar{k}_t, \text{ capital services}$$

$$\bar{k}_{t+1} = (1 - \delta) \bar{k}_t + F(i_t, i_{t-1})$$

Variables:

Q_t ~cash balances
M_t ~beginning-of-period t Household Money
M_t^a ~beginning-of-period t Aggregate Money
D_t ~profits
μ_t ~gross money growth rate
$M_t - Q_t + (\mu_t - 1)M_t^a$ ~deposits at financial intermediary
$a(\cdot)$ ~costs of utilizing capital more intensively
u_t ~utilization rate of capital
$F(i_t, i_{t-1})$ ~cost of adjusting investment
k_t ~capital services
$\bar{k}_t$ ~physical capital.

Structure of the Labor Market

- Intermediate Good Firms Use Labor Aggregate:

$$L_t = \left[\int_0^1 h_{j,t}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}.$$

- Price of L_t :

$$W_t = \left[\int_0^1 W_{it}^{\frac{1}{1-\lambda_w}} di \right]^{1-\lambda_w}.$$

- Demand for Household Labor Service, $h_{j,t}$:

$$h_{jt} = \left(\frac{W_t}{W_{jt}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} L_t, \ 1 \leq \lambda_w < \infty.$$

W_{jt} ~wage set by household
L_t ~homogeneous aggregate labor
W_t ~wage rate of aggregate labor

Calvo-style Wage Setting:

- With Probability $1-\xi_w$, i^{th} Household Sets Wage, W_{it}, Optimally, to $\tilde{W}_t$.
- With Probability ξ_w,

$$W_{it}=\pi_{t-1}W_{i,t-1},\ \pi_t=\frac{P_t}{P_{t-1}}.$$

- First Order Condition:

$$E_{t-1}\sum_{l=0}^{\infty}\left(\xi_w\beta\right)^l h_{jt+l}\left[\psi_{t+l}\frac{\tilde{W}_t X_{t,l}}{P_{t+l}}-\lambda_w z_{h,t+l}\right]=0.$$

$\dfrac{\psi_t}{P_t}$ value of one dollar (Multiplier on Budget Constraint)

Cash Balance Decision, Q_t

- Households Set Q_t To Maximize Utility

$$v'\left(\frac{Q_t}{P_t}\right)\frac{1}{P_t}+\frac{\psi_t}{P_t}=\frac{\psi_t}{P_t}R_t,$$

- Q_t/P_t Decreasing in R_t.
- Liquidity Effect Lives In This Equation.
  - c_t, i_t, Y_t, L_t, P_t, W_t Predetermined Relative to Monetary Shock
  - Loan Market Clearing:

$$W_t L_t = \mu_t M_t - Q_t$$

  - Q_t Must Absorb all Money Injections.
  - Can Only Happen With Fall in R_t.
- This is a ‘Limited Participation Story’
  - But With A Different Twist

Consumption Decision

$$E_{t-1}\frac{u_{c,t}}{P_t} = \beta E_{t-1}\frac{u_{c,t+1}}{P_{t+1}}R_{t+1}.$$

Capital Utilization Decision

$$E_{t-1}u_{c,t}\left[r_t^k - a'(u_t)\right] = 0$$

- Reduces Upward Pressure On Rental Rate of Capital and, hence, on Marginal Costs After Expansionary Monetary Policy Shock.
- Helps Account for Rise in Y/L After Expansionary Monetary Policy Shock.
  - Suppose Fixed Cost, ϕ, is Zero.
  - Standard Model: $L \uparrow \Rightarrow \frac{Y}{L} = \left(\frac{\bar{k}}{L}\right)^{\alpha} \downarrow$.
  - Our Model: $L \uparrow \Rightarrow \frac{Y}{L} = \left(\frac{u\bar{k}}{L}\right)^{\alpha} \uparrow$.

Investment and Adjustment Costs

- Rate of Return on Capital:

$$R_t^k = \frac{r_{t+1}^k + P_{k',t+1}(1-\delta)}{P_{k',t}},$$

$$\begin{aligned}
& P_{k',t} \text{ ~ consumption price of installed capital} \\
\delta & \in (0,1) \text{ ~depreciation rate.} \\
r_{t+1}^k & = s_{t+1} MP_{t+1}^k, \text{ rental rate on capital} \\
& MP_t^k \text{ ~marginal product of capital} \\
s_{t+1} & = \frac{MC_t}{P_t} = \frac{1}{\text{markup}}
\end{aligned}$$

- Almost Any Model,

$$\frac{R_t}{\pi_{t+1}} \approx R_t^k = \frac{r_{t+1}^k + P_{k',t+1}(1-\delta)}{P_{k',t}}.$$

- So, If a Positive Money Shock Drives Down Real Rate, Then

$$R_t^k \downarrow$$

- This is Trouble For Standard Models ($P_{k',t} = 1$, $s_t = 1$):

R_t^k down requires MP_t^k down

- Problem:

MP^k down Requires Surge in Investment, especially with employment up.

- With Adjustment Costs, No Surge in Investment
- Cost-of-Change Adjustment Costs:

$$k' = (1-\delta)k + F(\frac{i}{i_{-1}})i$$

  Good for 'Hump-shaped Investment Response'

- Other Reasons for Interest in Adjustment Costs:
  - Important for Understanding Asset Prices
  - Necessary for Movements in Price of Capital

Investment Decision

- Household Owns the Capital Stock and Carries Out Capital Accumulation.
- Technology for Capital Accumulation:

$$\bar{k}_{t+1} = (1-\delta)\bar{k}_t + F(i_t, i_{t-1}),$$

$$F(i_t, i_{t-1}) = (1 - S\left(\frac{i_t}{i_{t-1}}\right))i_t.$$

- Euler Equation for $\bar{k}_{t+1}$:

$$E_{t-1}\psi_t = \beta E_{t-1}\psi_{t+1}\frac{u_{t+1}r_{t+1}^k - a(u_{t+1}) + P_{k',t+1}(1-\delta)}{P_{k',t}}.$$

$P_{k',t}$ ~marginal cost, in units of consumption goods, of installed, physical capital

- Euler Equation for i_t:

$$E_{t-1}\psi_t = E_{t-1}\left[\psi_t P_{k',t} F_{1,t} + \beta\psi_{t+1} P_{k',t+1} F_{2,t+1}\right].$$

- After linearization:

$$\hat{\imath}_t = \hat{\imath}_{t-1} + \frac{1}{S''}\sum_{j=0}^{\infty} \beta^j E_{t-1}\hat{P}_{k',t+j}.$$

- Monetary Growth, μ_t:

$$\mu_t = \mu + \mu_{p,t} + \mu_{x,t},$$

$$\begin{aligned} \text{`Exogenous Component'}\ \mu_{p,t} &= \rho_{\mu_p}\mu_{p,t-1} + \varepsilon_{\mu_p,t} \\ \text{`Endogenous Component'}\ \mu_{x,t} &= \rho_{\mu_x}\mu_{x,t-1} + c_{\mu_x}\varepsilon_{x,t} \end{aligned}$$

Econometric Estimation

- Two Types of Parameters:
  (a) Parameters Set Without Reference to VAR Data.
  (b) Parameters Estimated by Making Model Impulse Responses Look Like The Estimated Impulse Responses.

Parameter Set 1: Parameters that Don't Enter Formal Estimation Criterion		
discount factor	β	$1.03^{-.25}$
capital's share	α	0.36
capital depreciation rate	δ	0.025
markup, labor suppliers	λ_w	1.05
mean, money growth	μ	1.017
labor utility parameter	ψ_0	set to imply $L = 1$
real balance utility parameter	ψ_q	set to imply $Q/M = 0.44$
fixed cost of production	ϕ	set to imply ss profits $=$ 0

- γ ~ 13 free parameters to be estimated:

$$\gamma \equiv (\lambda_f,\ \xi_w,\ \xi_p,\ \sigma_q,\ S'',\ b,\ \sigma_a,$$
$$\text{6 parameters governing exogenous shocks}).$$

λ_f	Steady State Markup of Firms
ξ_w	Degree of Stickiness in Wages
ξ_p	Degree of Stickiness in Prices
b	Habit Persistence Parameter

- Estimation Criterion:

$$J = \min_{\gamma}(\hat{\psi} - \psi(\gamma))' V^{-1}(\hat{\psi} - \psi(\gamma)),$$

- $\psi(\gamma)$ ~ 353 model impulse responses
- $\hat{\psi}$ ~ 353 estimated VAR impulse responses

Estimation Results:

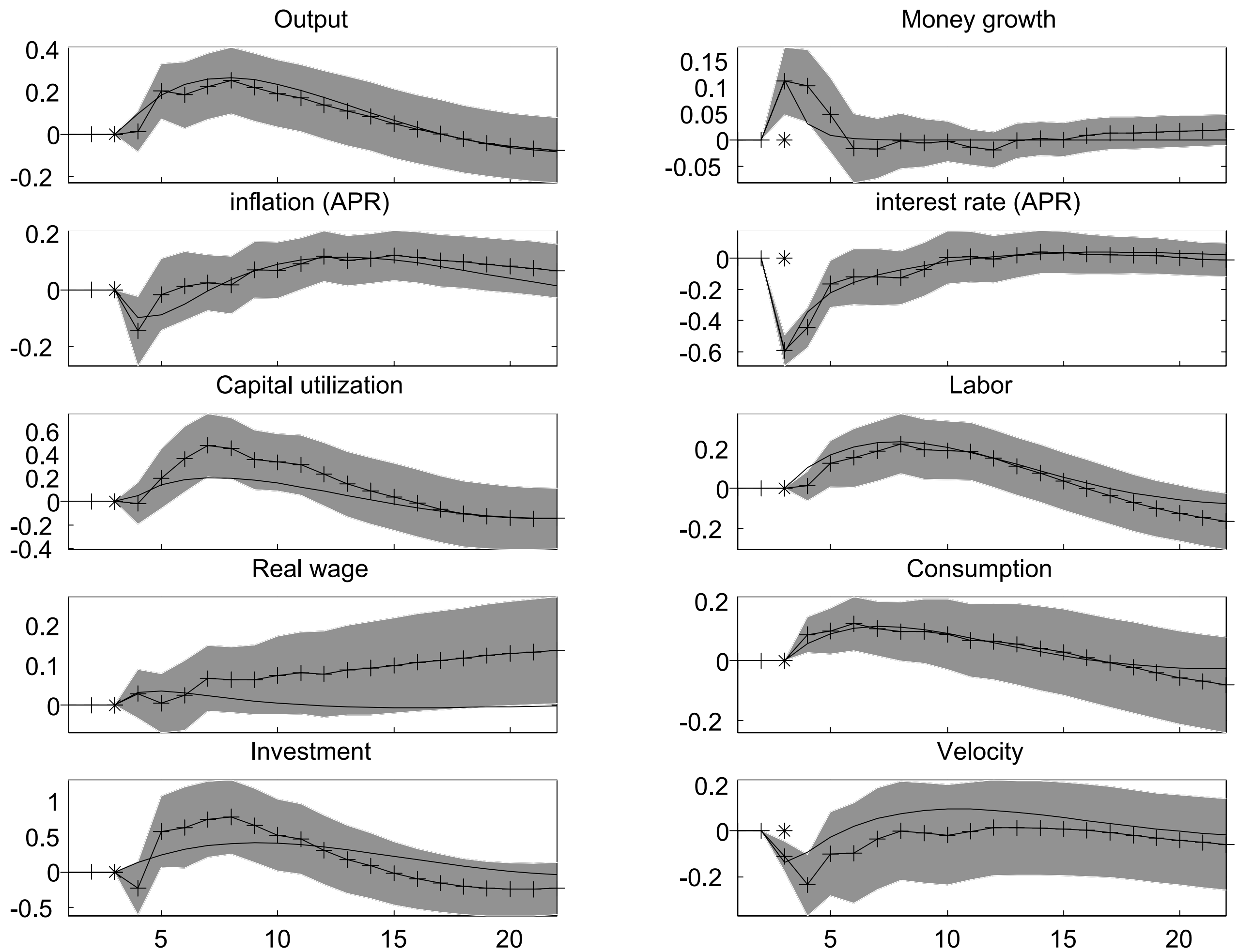
Table: Economic Parameters		
	Estimation Based on Estimated Responses to:	
	Policy and Technology	Policy
Parameter	Shocks Simultaneously	Shocks Only
λ_f	1.14 (0.10)	1.15 (0.13)
ξ_w	0.78 (0.04)	0.73 (0.04)
ξ_p	0.42 (0.30)	0.45 (0.19)
σ_q	14.13 (4.25)	12.33 (3.19)
S''	7.69 (3.04)	9.97 (3.90)
b	0.73 (0.07)	0.77 (0.05)
σ_a	0.05 (0.007)	0.03 (0.005)

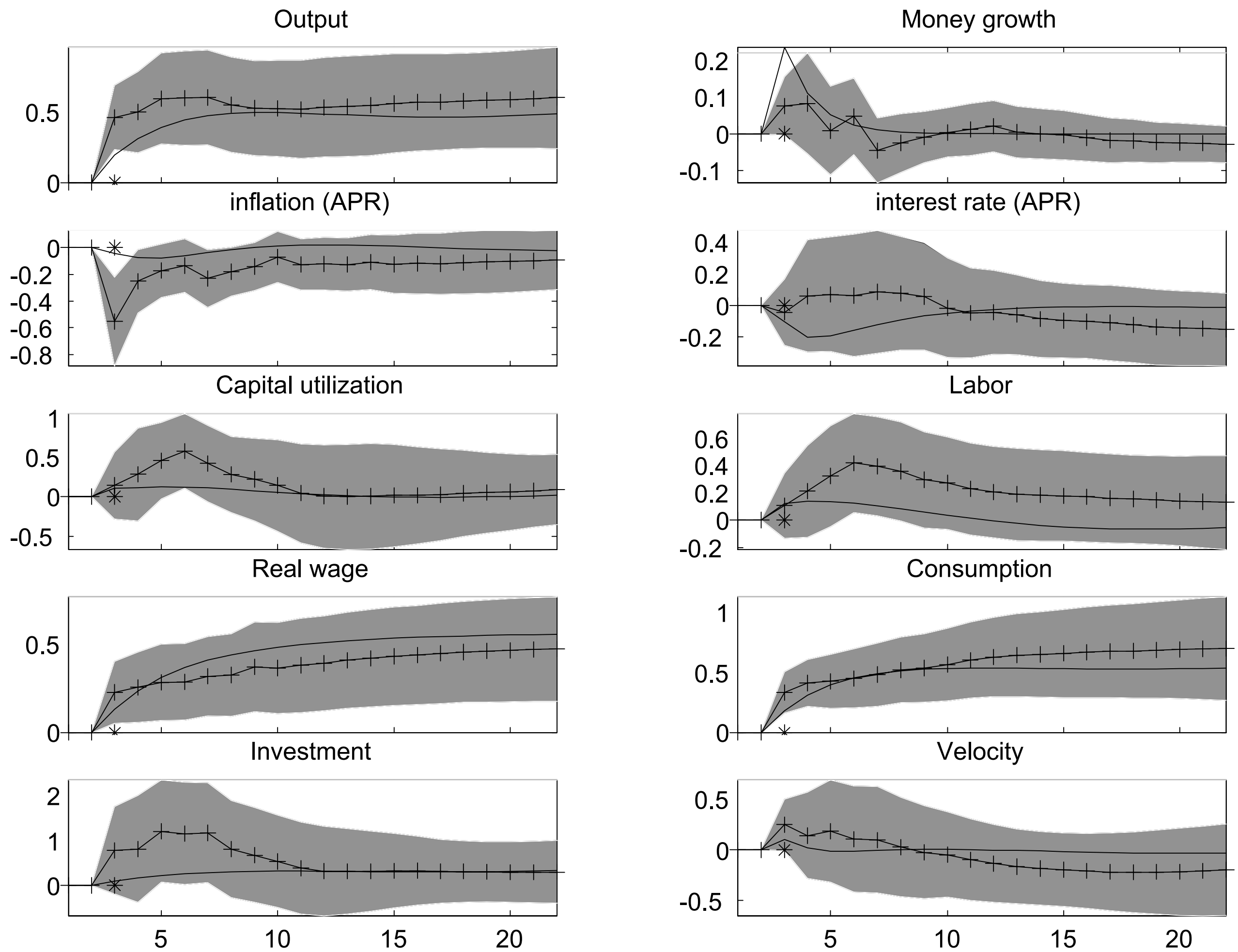
Parameters that Match Money Impulse Responses Work Well With Technology Too.

Estimation Results, Cont'd.....

Table: Exogenous Shock Processes		
	Estimation Based on Estimated Responses to:	
	Policy and Technology	Policy
Parameter	Shocks Simultaneously	Shocks Only
ρ_x	0.80 (0.05)	na
σ_{ε_x}	0.12 (0.03)	na
ρ_{μ_x}	0.47 (0.15)	na
c_{μ_x}	2.07 (0.63)	na
ρ_{μ_p}	0.27 (0.04)	0.27 (0.06)
$\sigma_{\varepsilon_{\mu_p}}$	0.11 (0.04)	0.13 (0.04)

Figure 1: Model and Data Impulse Response Functions to a Policy Shock



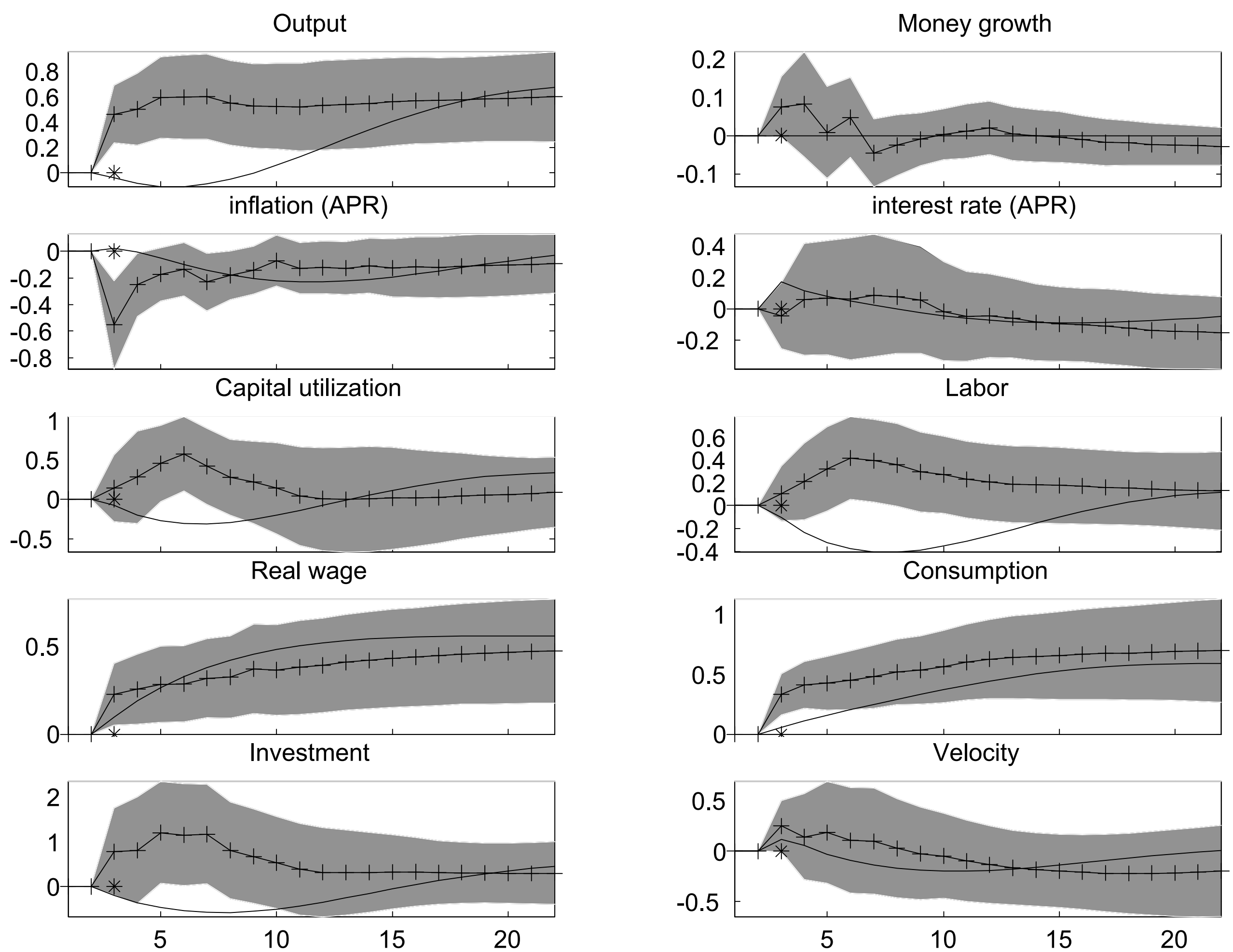

Output
0.5
0
Money growth
0.2
0.1
0
-0.1
inflation (APR)
0
-0.2
-0.4
-0.6
-0.8
interest rate (APR)
0.4
0.2
0
-0.2
Capital utilization
1
0.5
0
-0.5
Labor
0.6
0.4
0.2
0
-0.2
Real wage
0.5
0
Consumption
1
0.5
0
Investment
2
1
0
Velocity
0.5
0
-0.5
5
10
15
20
5
10
15
20

Model Diagnostics

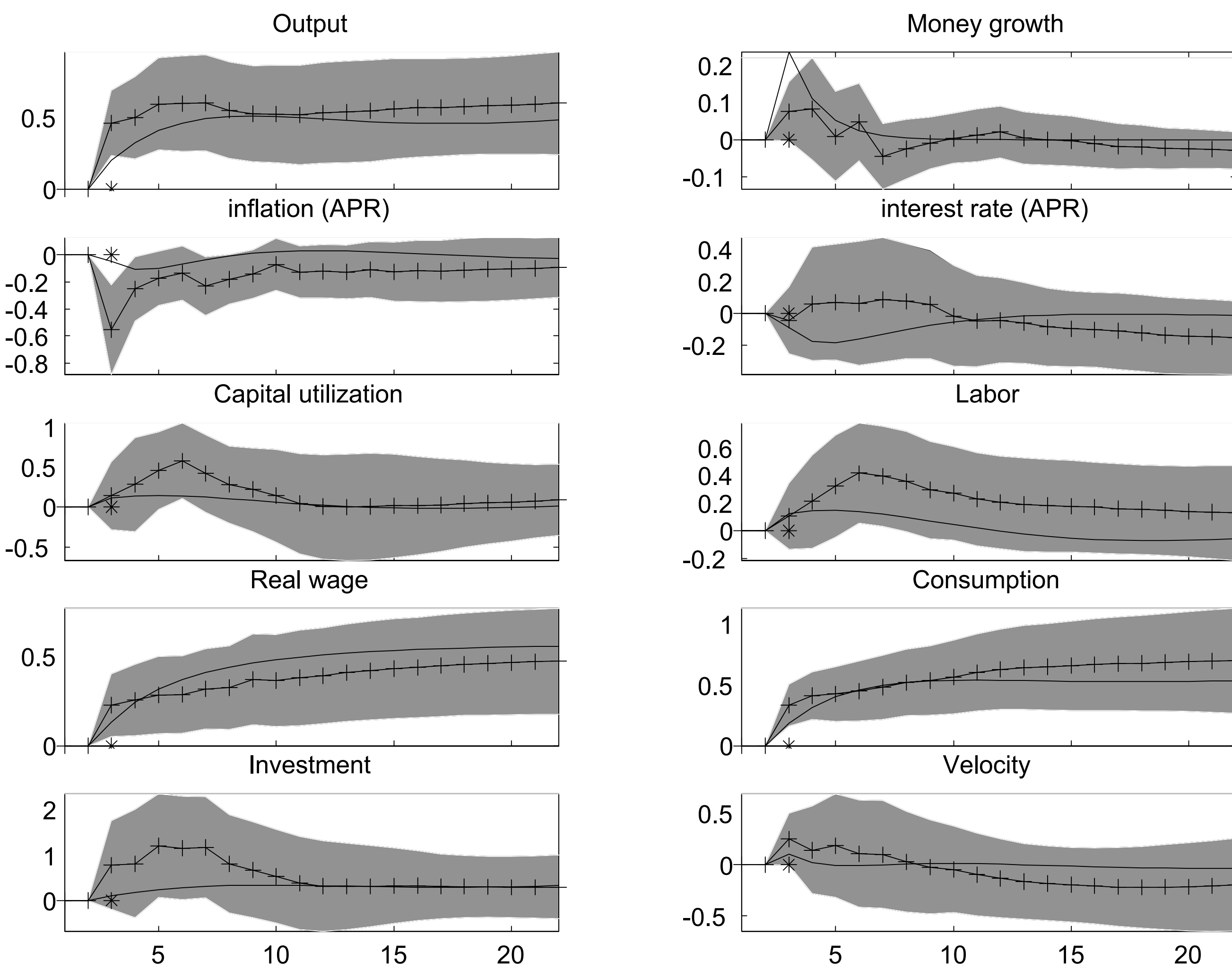
- Employment Rises in Response to Technology Because Monetary Policy is Accommodative.
- Sticky Prices Not Important for Transmission of Monetary Policy or Technology Shocks.

Diagnostics: Monetary Accommodation Important for Transmission of

Technology Shocks



Diagnostics: Sticky Prices *Not* Important for Transmission of Technology Shocks



Conclusion

- We Presented Empirical Estimates of the Dynamic Effects of Monetary Policy and Technology Shocks.
  - A Model Was Found that Can Roughly Reproduce These Effects.
  - According to Model, Endogenous Monetary Policy Important for Understanding Propagation of Technology Shock.

- Finding: Technology Shocks Seem to Have Little to do with Business Cycles
  - Need to Bring Other Technology Shocks Into the Analysis.
  - Stationary Shocks to Disembodied Technology.
  - Stationary Investment-Specific Shocks.
  - These Shocks May Account for Business Cycle Fluctuations.
  - Paper Describes a ‘Model-Based’ Identification Strategy For Doing This.
- Hope: Model with Additional Shocks Capable of Accounting for Details of Quarterly Data.
- Such a Model is Ready for Policy Analysis!