The Great Depression and the Friedman-Schwartz Hypothesis

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1. Introduction

Was the US Great Depression of the 1930s due to bungling at the Fed? In their classic analysis of US monetary history, Friedman and Schwartz (1963) conclude that the answer is 'yes'. To be sure, they do admit that if the Fed had not been part of the problem we would have seen recessions. But, they would have been the usual garden-variety slowdowns, not the spectacular collapse that actually occurred. The Friedman and Schwarz answer is a comforting one. Under the assumption that the Fed is smarter now than it was then, we don't have to worry about the possibility of a repeat.

Or do we? Is there anything the Fed can do that has consequences on the order of magnitude of the Great Depression? A recent analysis by Sims (1999) concludes 'no'. He argues that if Alan Greenspan had somehow been transported back into the 1930s and made chairman of the Fed, the Great Depression would have unfolded pretty much the way it did. For example, using a similar style of reasoning as Sims, Christiano (1999) argued that it would have made little difference if the Fed had acted to prevent the fall in $M1$. This seems inconsistent with a centerpiece of Friedman and Schwartz's argument: that the Great Depression was so severe, in part because the Fed allowed $M1$ to collapse. Although this argument creates a doubt, it is at best only suggestive because it is made by manipulating a subset of equations in a vector autoregression, without worrying about the possible consequences for other equations.

Our purpose is to do the relevant experiment 'right'. For this, we require a structural model of the economy that captures the essential features emphasized by Friedman and Schwartz. There is a variety of elements that this model must incorporate, to be interesting. First, there must be some model of credit market frictions that allow us to capture the effects of the enormous fall in stock market value that occurred. For this, we incorporate the credit market frictions described in Bernanke, Gertler and Gilchrist (1999) (BGG).[1] Second, an important component of the Friedman and Schwartz argument is that the Fed did not act to prevent the decline in M1 that occurred as people converted demand deposits into currency. Also, Friedman and Schwartz argue that later in the depression, the Fed failed to appreciate the fact that banks wanted to hold excess reserves in conducting monetary policy. Thinking that the high levels of reserves the banks held were potentially inflationary, they increased reserve requirements. This was highly contractionary, when it turned out that the excess reserves banks were holding were desired. To model these features of the time, we need to incorporate a banking sector with demand deposits, currency, bank reserves and bank excess reserves. For this, we use the banking model of Chari, Christiano and Eichenbaum (1995)

[1] This work builds on Townsend (1979), Gale and Hellwig (1985), Williamson (1987). Other recent contributions to this literature include Fisher (1996) and Carlstrom and Fuerst (1997, 2000).

(CCE). Finally, we incorporate these banking and net worth considerations into the model environment described in Altig, Christiano and Linde (2002) (ACEL). This model seems appropriate for the task, since it captures key features of aggregate data, as well as of the monetary transmission mechanism.

This draft provides a description of the model and the solution method. In addition, a set of preliminary parameter values are reported, together with the associated steady state properties as well as some impulse responses. The full analysis will appear in the next draft.

2. The Model Economy

In this section we describe our model economy and display the problems solved by intermediate and final good firms, entrepreneurs, producers of physical capital, banks and households. Final output is produced using the usual Dixit-Stiglitz aggregator of intermediate inputs. Intermediate inputs are produced by monopolists who set prices using a variant of the approach described in Calvo (1983). These firms use the services of capital and labor. We assume that a fraction of these variable costs ('working capital') must be financed in advance by banks. Capital services are supplied by entrepreneurs who own the physical capital and determine its rate of utilization. They finance their acquisition of physical capital partially using their own net worth and partially using the variant on the costly state verification (CSV) technology described in BGG. As is standard in the CSV literature with net worth, we need to make assumptions to guarantee that entrepreneurs do not accumulate enough net worth to make the CSV technology irrelevant. We accomplish this by assuming that a part of net worth is exogenously destroyed in each period. Physical capital is produced by firms which combine old capital and investment goods to produce new, installed, capital.

The model has banks which are the entities that make working capital loans to intermediate good firms and which provide the standard CSV debt contracts to entrepreneurs to help them finance their acquisition of new capital.

The timing of decisions during a period is important in the model. At the beginning of the period, the shocks to the various technologies in the model are realized. Then, wage, price, consumption, investment and capital utilization decisions are made. After this various financial market shocks are realized and the monetary action occurs. Finally, goods and asset markets meet and clear. See Figure 1 for reference.

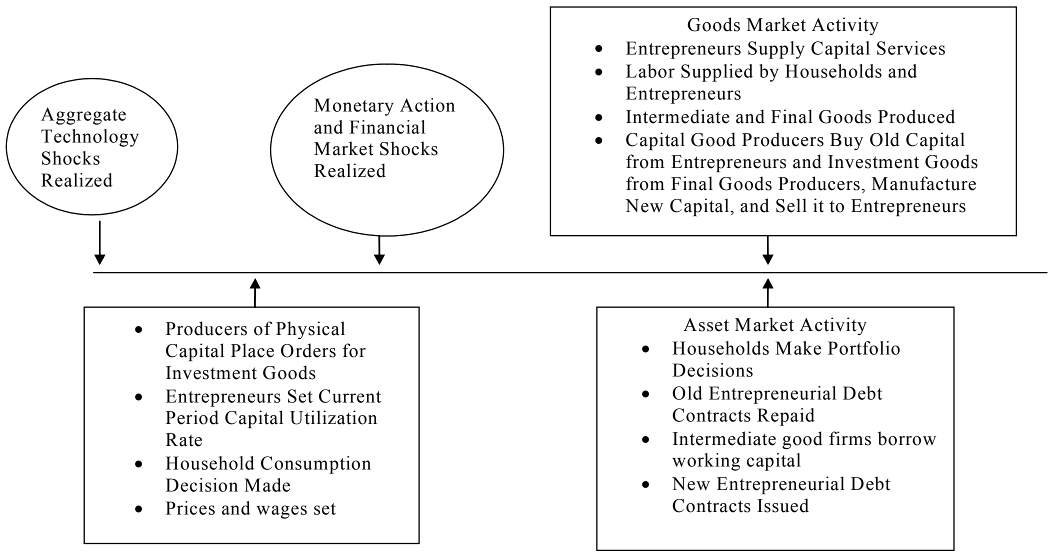


Figure 1: Timing in Model

2.1. Information

We divide up the shocks in the model into financial market shocks - money demand (by banks, households and firms) and monetary policy shocks - and non-financial market shocks (technology, government spending, preference for leisure, elasticities of demand for differentiated products and labor, etc.). The time t information set which includes period $t-s$, $s>0$, and period t observations on the non-financial shocks is denoted Ω_t. The information set which includes Ω_t plus the current period financial market shocks is denoted Ω_t^μ. Also,

$$\begin{aligned} E\left[X_t|\Omega_t\right] &= E_t X_t \\ E\left[X_t|\Omega_t^\mu\right] &= E_t^\mu X_t. \end{aligned}$$

2.2. Firm Sector

We adopt the variant on the standard Dixit-Stiglitz setup for our firm sector that was used in CEE. At time t, a final consumption good, Y_t, is produced by a perfectly competitive firm. The firm does so by combining a continuum of intermediate goods, indexed by $j \in [0,1]$, using the technology

$$Y_t = \left[\int_0^1 Y_{jt}{}^{\frac{1}{\lambda_f}} dj\right]^{\lambda_f}$$

where $1 \leq \lambda_f < \infty$, and Y_{jt} denotes the time t input of intermediate good j. Let P_t and P_{jt} denote the time t price of the consumption good and intermediate good j, respectively. Profit maximization implies the Euler equation

$$\left(\frac{P_t}{P_{jt}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{jt}}{Y_t}, \tag{2.1}$$

which leads to the following relationship between the aggregate price level and individual prices:

$$P_t = \left[\int_0^1 P_{jt}^{\frac{1}{1-\lambda_f}} dj\right]^{(1-\lambda_f)}. \tag{2.2}$$

The j^{th} intermediate good is produced by a monopolist who sets its price, P_{jt}, after the realization of non-financial market shocks, but before the realization of financial market shocks. In addition to this information constraint, there are also Calvo-style frictions in setting prices that we will describe shortly. The intermediate good producer is assumed to satisfy whatever demand materializes at its posted price. Once prices have been set, and after the realization of current period uncertainty, the intermediate good producer selects inputs to minimize costs. The production function of the j^{th} intermediate good firm is:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha} \left(z_t l_{jt}\right)^{1-\alpha} - \Phi z_t & \text{if } \epsilon_t K_{jt}^{\alpha} \left(z_t l_{jt}\right)^{1-\alpha} > \Phi z_t \\ 0, & \text{otherwise} \end{cases}, \; 0 < \alpha < 1,$$

where Φ is a fixed cost and K_{jt} and l_{jt} denote the services of capital and labor. The variable, z_t, is a shock to technology, which has a covariance stationary growth rate, μ_{zt}, where

$$\mu_{zt} = \frac{z_t}{z_{t-1}}.$$

The variable, ϵ_t, is a stationary shock to technology. The time series representations for z_t and ϵ_t are discussed below. Firms are competitive in factor markets, where they confront a rental rate, Pr_t^k, on capital services and a wage rate, W_t, on labor services. Each of these is expressed in units of money. Also, each firm must finance a fraction, $\psi_{k,t}$, of its capital services expenses in advance. Similarly, it must finance a fraction, $\psi_{l,t}$, of its labor services in advance. The interest rate it faces is R_t. Working capital includes the wage bill, $W_t l_{jt}$, and the rent on capital services, $P_t r_t^k K_t$. As a result, the marginal cost - after dividing by P_t - of producing one unit of Y_{jt} is:

$$s_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(r_t^k \left[1 + \psi_{k,t} R_t\right]\right)^{\alpha} \left(w_t \left[1 + \psi_{l,t} R_t\right]\right)^{1-\alpha}}{\epsilon_t}, \tag{2.3}$$

where

$$w_t = \frac{W_t}{z_t P_t}.$$

Efficient input choice by firms also leads to the following condition:

$$s_t = \frac{r_t^k \left[1 + \psi_{k,t} R_t\right]}{\alpha \epsilon_t \left(\frac{z_t l_t}{K_t}\right)^{1-\alpha}}, \tag{2.4}$$

where ν is the share of aggregate labor and capital services in the intermediate good sector. The complementary share, $1 - \nu$, is used in the banking sector. We impose equality of the share of capital and labor in their respective aggregates to save notation and because this is a property of equilibrium, given that we adopt the same production function for the intermediate good and banking sectors. Finally, l_t and K_t are the unweighted integrals of employment and capital services hired by individual intermediate good producers.

We adopt the variant of Calvo pricing proposed in CEE. In each period, t, a fraction of intermediate good firms, $1 - \xi_p$, can reoptimize its price. The complementary fraction must set its price equal to what it was in period $t - 1$, scaled up by the inflation rate from $t - 2$ to $t - 1$. After linearizing (2.2) and the optimizing firms' first order condition about steady state, we obtain the following law of motion for aggregate inflation:

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta)\,\xi_p}\left[E_t\left(\hat{s}_t\right) + \hat{\lambda}_{f,t}\right]. \tag{2.5}$$

In the usual way, $\hat{x}_t = (x_t - x)/x_t$, where x is the value of x_t in nonstochastic steady state, and x_t is a small deviation from that steady state. Also, π_t denotes the aggregate inflation rate, $\pi_t = P_t/P_{t-1}$. Finally, the stochastic process, $\hat{\lambda}_{f,t}$, is a shock to the parameter, λ_f, in the final good production function. In the linearization of our economy, the only place this shock shows up is (2.5).

2.3. Capital Producers

There is a large, fixed, number of identical capital producers, who take prices as given. They are owned by households and any profits or losses are transmitted in a lump-sum fashion to households. The capital producer must commit to a level of investment, I_t, before the period t realization of the monetary policy shock and after the period t realization of the other shocks. Investment goods are actually purchased in the goods market which meets after the monetary policy shock. The price of investment goods in that market is P_t, and this is a function of the realization of the monetary policy shock. The capital producer also purchases old capital in the amount, x, at the time the goods market meets. Old capital and investment goods are combined to produce new capital, x', using the following technology:

$$x' = x + F(I_t, I_{t-1}),$$

where the presence of lagged investment reflects that there are costs to changing the flow of investment. We denote the price of new capital by $Q_{\bar{K}',t}$, and this is a function of the realized value of the monetary policy shock. Since the marginal rate of transformation from old capital into new capital is unity, the price of old capital is also $Q_{\bar{K}',t}$. The firm's time t profits, after the realization of the monetary policy shock are:

$$\Pi_t^k = Q_{\bar{K}',t}\left[x + F(I_t, I_{t-1})\right] - Q_{\bar{K}',t}x - P_t I_t.$$

This expression for profits is a function of the realization of the period t monetary policy shock, because $Q_{\bar{K}',t}$, x, and P_t are. Since the choice of I_t influences profits in period $t+1$, the firm must incorporate that into the objective as well. But, that term involves I_{t+1} and x_{t+1}. So, state contingent choices for those variables must be made for the firm to be able to select I_t and x_t. Evidently, the problem choosing x_t and I_t expands into the problem of solving an infinite horizon optimization problem:

$$\max_{\{I_{t+j}, x_{t+j}\}} E\left\{\sum_{j=0}^{\infty}\beta^j\lambda_{t+j}\left(Q_{\bar{K}',t+j}\left[x_{t+j} + F(I_{t+j}, I_{t+j-1})\right] - Q_{\bar{K}',t+j}x_{t+j} - P_{t+j}I_{t+j}\right)|\Omega_t\right\},$$

where it is understood that I_{t+j} is a function of all shocks up to period $t+j$ except the $t+j$ financial market shocks and x_{t+j} is a function of all the shocks up to period $t+j$. Also, Ω_t includes all shocks up to period t, except the period t financial market shocks. These are composed of shocks to monetary policy and to money demand.

From this problem it is evident that any value of x_{t+j} whatsoever is profit maximizing. Thus, setting $x_{t+j} = (1-\delta)\bar{K}_{t+j}$ is consistent with both profit maximization by firms and with market clearing.

The first order necessary condition for maximization of I_t is:

$$E\left[\lambda_t P_t q_t F_{1,t} - \lambda_t P_t + \beta\lambda_{t+1}P_{t+1}q_{t+1}F_{2,t+1}|\Omega_t\right] = 0,$$

where q_t is Tobin's q :

$$q_t = \frac{Q_{\bar{K},t}}{P_t}.$$

The physical stock of capital evolves as follows

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t,$$

where S is a function that is concave in the neighborhood of steady state.

2.4. Entrepreneurs

There is a large population of entrepreneurs. Consider the j^{th} entrepreneur (see Figure 2). During the period t goods market, the j^{th} entrepreneur accumulates net worth, N_{t+1}^{j}. This abstract purchasing power, which is denominated in units of money, is determined as follows. The sources of funds are the rent earned as a consequence of supplying capital services to the period t capital rental market, the sales proceeds from selling the undepreciated component of the physical stock of capital to capital goods producers. The uses of funds include repayment on debt incurred on loans in period $t-1$ and expenses for capital utilization. Net worth is composed of these sources minus these uses of funds.

At this point, $1-\gamma$ entrepreneurs die and γ survive to live another day. The newly produced stock of physical capital is purchased by the γ entrepreneurs who survive and $1-\gamma$ newly-born entrepreneurs. The surviving entrepreneurs finance their purchases with their net worth and loans from the bank. The newly-born entrepreneurs finance their purchases with a transfer payment received from the government and a loan from the bank. We actually allow γ to be a random variable, but we delete the time subscript here to keep from cluttering the notation too much.

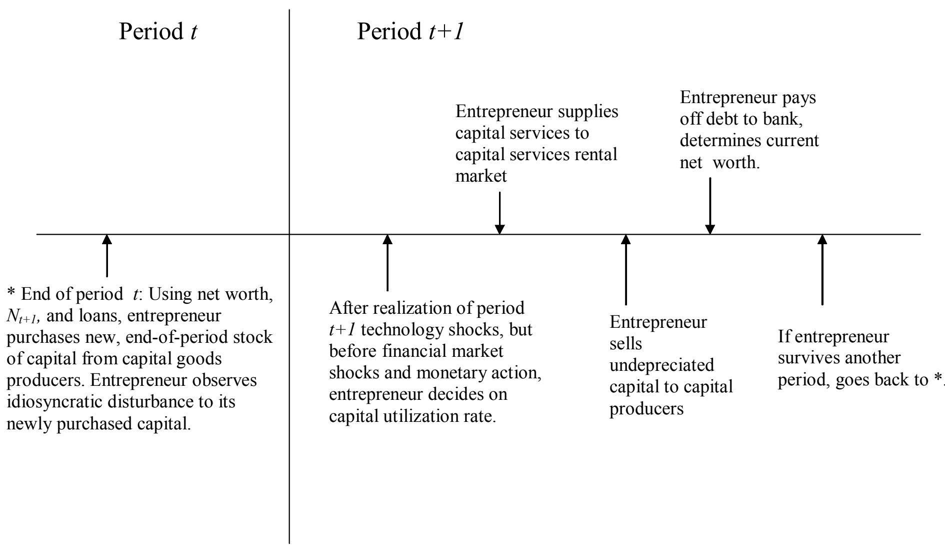
The j^{th} entrepreneur who purchases capital, $\bar{K}_{t+1}^{j}$, from the capital goods producers at the price, $Q_{\bar{K}',t}$ in period t experiences an idiosyncratic shock to the size of his purchase. Just after the purchase, the size of capital changes from $\bar{K}_{t+1}^{j}$ to $\omega\bar{K}_{t+1}^{j}$. Here, ω is a unit mean, non-negative random variable distributed independently across entrepreneurs. After observing the realization of the non-financial market shocks, but before observing the financial market shock, the j^{th} entrepreneur decides on the level of capital utilitzation in period $t+1$, and then rents capital services. At the end of the period $t+1$ goods market, the entrepreneur sells its undepreciated capital. At this point, the entrepreneur's net worth, N_{t+2}^{j}, is the rent earned in period $t+1$, minus the utilization costs on capital, minus debt repayment, plus the proceeds of the sale of the undepreciated capital, $(1-\delta)\omega\bar{K}_{t+1}^{j}$. As indicated above, the entrepreneur then proceeds to die with probability $1-\gamma$, and to survive to live another day with the complementary probability, γ.

The $1-\gamma$ entrepreneurs who are born and the γ who survive receive a subsidy, W_t^e. There is a technical reason for this. The standard debt contract in the entrepreneurial loan market has the property that entrepreneurs with no net worth receive no loans. If newborn entrepreneurs received no transfers, they would have no net worth and would therefore not be able to purchase any capital. In effect, without the transfer they could not enter the population of entrepreneurs. Regarding the surviving entrepreneurs, in each period a fraction loses everything, and they would have no net worth in the absence of a transfer. Absent a transfer, these entrepreneurs would in effect leave the population of entrepreneurs. Absent transfers, the population of entrepreneurs would be empty. The transfers are designed to avoid this. They are financed by a lump sum tax on households.

Entrepreneurial death in the model is a device to ensure that net worth does not grow to the point where the CSV setup becomes irrelevant. Presumably, this corresponds to the real-world observation that enormous concentrations of wealth, for various reasons, do not survive for long.

We need to allocate the net worth of the entrepreneurs who die. We assume that a fraction, Θ, of a dead entrepreneur's net worth is used to finance the purchase of C_t^e of final output. The complementary fraction is redistributed as a lump-sum transfer to the household. In practice, Θ will be small or zero.

FIGURE 2: A Day in the Life of an Entrepreneur



2.4.1. The Production Technology of the Entrepreneur

We now go into the details of the entrepreneur's situation. The j^{th} entrepreneur produces capital services, K_{t+1}^j, from physical capital using using the following technology:

$$K_{t+1}^j = u_{t+1}^j \omega \bar{K}_{t+1}^j,$$

where u_{t+1}^j denotes the capital utilization rate chosen by the j^{th} entrepreneur. Here, ω is drawn from a distribution with mean unity and distribution function, F :

$$\Pr[\omega \leq x] = F(x).$$

Each entrepreneur draws independently from this distribution immediately after $\bar{K}^j_{t+1}$ has been purchased. Capital services are supplied to the capital services market in period $t+1$, where they earn the rental rate, r^k_{t+1}.

The capital utilization rate chosen by the j^{th} entrepreneur, u^j_{t+1}, must be chosen before period $t+1$ financial market shocks, and after the other period $t+1$ shocks. Higher rates of utilization are associated with higher costs as follows:

$$P_{t+1}a(u^j_{t+1})\omega\bar{K}^j_{t+1},\ a', a'' > 0.$$

As in BGG, we suppose that the entrepreneur is risk neutral. As a result, the j^{th} entrepreneur chooses u^j_{t+1} to solve:

$$\max_{u^j_{t+1}} E\left\{\left[u^j_{t+1}r^k_{t+1} - a(u^j_{t+1})\right]\omega\bar{K}^j_{t+1}P_{t+1}|\Omega_{t+1}\right\}.$$

The first order necessary condition for optimization is:

$$E_t\left[r^k_t - a'(u_t)\right] = 0.$$

This reflects that $\bar{K}^j_{t+1}$ P_{t+1} are contained in Ω_{t+1}. After the capital has been rented in period $t+1$, the j^{th} entrepreneur sells the undepreciated part, $(1-\delta)\omega\bar{K}^j_{t+1}$, to the capital goods producer.

Below we introduce taxation on capital income. This does not enter into the above first order condition because capital income taxation affects rental income and the cost of utilization symmetrically. In addition, the capital income tax rate that applies to the utilization rate at time $t+1$ is contained in the information set, Ω_{t+1}.

2.4.2. Taxation of Capital Income

We adopt the following simple, tractable treatment of taxation on capital income. We suppose that the after-tax rate of return to capital, for an entrepreneur with productivity ω, is:

$$\begin{aligned} 1+R^{k,\omega}_{t+1} &= \left\{\frac{(1-\tau^k_t)\left[u_{t+1}r^k_{t+1} - a(u_{t+1})\right] + (1-\delta)q_{t+1}}{q_t}\frac{P_{t+1}}{P_t} + \tau^k_t\delta\right\}\omega \\ &= (1+R^k_{t+1})\omega. \end{aligned}$$

Note how after tax rate of return on capital for an individual entrepreneur is proportional to ω. A drawback of this specification is the implication that one cannot depreciate the full amount of the initial capital purchase, when ω is low. An interpretation is that depreciation allowances are lost when the level of income is too low to deduct the full amount.

2.4.3. The Financing Arrangement for the Entrepreneur

How is the j^{th} entrepreneur's level of capital, $\bar{K}_{t+1}^{j}$, determined? At the moment the entrepreneur enters the loan market, it's state variable is its net worth. It is has nothing else. It owns no capital, for example. Apart from net worth, no other aspect of the entrepreneur's history is relevant at this point.

There are many entrepreneurs, all with different amounts of net worth. We imagine that corresponding to each possible value of net worth, there are many entrepreneurs. They participate in a competitive loan market with banks. That is, there is a competitive loan market corresponding to each different level of net worth, N_{t+1}. In the usual CSV way, the contracts traded in the loan market specify an interest rate and a loan amount. The contracts are competitively determined. This means that they must satisfy a zero profit condition on banks and they must be utility maximizing for entrepreneurs. Equilibrium is incompatible with positive profits because of free entry and incompatible with negative profits because of free exit. In addition, contracts must be utility maximizing (subject to zero profits) for entrepreneurs because of competition. Equilibrium is incompatible with contracts that fail to do so, because in any candidate equilibrium like this, an individual bank could offer a better contract, one that makes positive profits, and take over the market.

The CSV contracts that we study are known to be optimal when there is no aggregate uncertainty. However, the way we have set up our environment, there is such uncertainty. We do this in part because we are interested exploring phenomena like the 'debt deflation hypothesis' discussed by Irving Fisher. We interpret this hypothesis as corresponding to a situation in which a shock (in this case, to the price level) occurs after entrepreneurs have borrowed from banks, but before they have paid back what they owe. A problem with what we do is that the contract we study is not known to be the optimal one. However, we suspect that in fact the contract is optimal, at least for sufficiently risk averse households. This is because the contract has the property that uncertainty associated with an aggregate shock is absorbed by entrepreneurs, while households receive a state-noncontingent rate of return on their loans to entrepreneurs (these loans actually are intermediated by banks). The reason this arrangement may not be optimal is as follows. We have not ruled out the possibility that there could be a return for households which is state contingent but compensates them for this, and which permits a CSV loan contract to entrepreneurs that increases their welfare. An alternative interpretation of our results is that there are other, nonmodeled reasons for assuming that the rate of return paid to households by banks are non-statecontingent. Subject to this restriction, the contracts we work with are optimal.

We now discuss the contracts offered in equilibrium to entrepreneurs with level of net worth, N_{t+1}. Denote the level of capital purchases by such an entrepreneur by $\bar{K}_{t+1}^{N}$. To finance such a purchase an $N_{t+1}-$type entrepreneur must borrow

$$B_{t+1}^N = Q_{\bar{K}',t}\bar{K}_{t+1}^N - N_{t+1}. \tag{2.6}$$

The standard debt contract specifies a loan amount, B_{t+1}^N, and a gross rate of interest, Z_{t+1}^N, to be paid if ω is high enough that the entrepreneur can do so. Entrepreneurs who cannot pay this interest rate, because they have a low value of ω must give everything they have to the bank. The parameters of the $N_{t+1}-$type standard debt contract, B_{t+1}^N Z_{t+1}^N, imply a cutoff value of ω, $\bar{\omega}_{t+1}^N$, as follows:[2]

$$\bar{\omega}_{t+1}^N\left(1+R_{t+1}^k\right)Q_{\bar{K}',t}\bar{K}_{t+1}^N = Z_{t+1}^N B_{t+1}^N. \tag{2.7}$$

The amount of the loan, B_{t+1}^N, extended to an $N_{t+1}-$type entrepreneur is obviously not dependent on the realization of the period $t+1$ shocks. For reasons explained below, the interest rate on the loan, Z_{t+1}^N, is dependent on those shocks. Since R_{t+1}^k and Z_{t+1}^N are dependent on the period $t+1$ shocks, it follows from the previous expression that $\bar{\omega}_{t+1}^N$ is in principle also dependent upon those shocks.

For $\omega < \bar{\omega}_{t+1}^N$, the entrepreneur pays all its revenues to the bank:

$$\left(1+R_{t+1}^k\right)\omega Q_{\bar{K}',t}\bar{K}_{t+1}^N,$$

which is less than $Z_{t+1}^N B_{t+1}^N$. In this case, the bank must monitor the entrepreneur, at cost

$$\mu\left(1+R_{t+1}^k\right)\omega Q_{\bar{K}',t}\bar{K}_{t+1}^N.$$

We now describe how the parameters, B_{t+1}^N and Z_{t+1}^N, of the standard debt contract that is offered in equilibrium to entrepreneurs with net worth N_{t+1} are chosen.

We suppose that banks have access to funds at the end of the period t goods market at a nominal rate of interest, R_{t+1}^e. This interest rate is contingent on all shocks realized in period t, and is not contingent on the realization of the idiosyncratic shocks to individual $N_{t+1}-$type entrepreneurs, and is also not contingent on the $t+1$ aggregate shocks. Banks obtain these funds for lending to entrepreneurs by issuing time deposits at the end of the

[2]With the alternative treatment of depreciation, this expression becomes:

$$\left(\left[1+\tilde{R}_{t+1}^k\right]\bar{\omega}_{t+1}^N + \tau_t^k\delta\right)Q_{\bar{K}',t}\bar{K}_{t+1}^N = Z_{t+1}^N B_{t+1}^N.$$

goods market in period t, which is when the entrepreneurs need funds for the purchase of $\bar{K}_{t+1}^N$. Zero profits for banks implies:

$$\left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] Z_{t+1}^N B_{t+1}^N + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega) \left(1 + R_{t+1}^k\right) Q_{\bar{K}',t} \bar{K}_{t+1}^N = \left(1 + R_{t+1}^e\right) B_{t+1}^N, \tag{2.8}$$

or,

$$\left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] \bar{\omega}_{t+1}^N + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega) = \frac{1 + R_{t+1}^e}{1 + R_{t+1}^k} \frac{B_{t+1}^N}{Q_{\bar{K}',t} \bar{K}_{t+1}^N}. \tag{2.9}$$

BGG argue that, given a mild regularity condition on F, the expression on the left of the equality has an inverted U shape. There is some unique interior maximum, $\bar{\omega}^*$. It is increasing for $\bar{\omega}_{t+1}^N < \bar{\omega}^*$ and decreasing for $\bar{\omega}_{t+1}^N > \bar{\omega}^*$. Conditional on a given ratio, $B_{t+1}^N / \left(Q_{\bar{K}',t} \bar{K}_{t+1}^N\right)$, the right side fluctuates with R_{t+1}^k. The setup resembles the usual Laffer-curve setup, with the right side playing the role of the financing requirement and the left the role of tax revenues as a function of function of the 'tax rate', $\bar{\omega}_{t+1}^N$. So, we see that, generically, there are two $\bar{\omega}_{t+1}^N$'s that solve the above equation for given $B_{t+1}^N / \left(Q_{\bar{K}',t} \bar{K}_{t+1}^N\right)$. Between these two, the smaller one is preferred to entrepreneurs, so this is a candidate CSV. The implication is that in a CSV, $\bar{\omega}_{t+1}^N \leq \bar{\omega}^*$. Since, for $\bar{\omega}_{t+1}^N < \bar{\omega}^*$ the left side is increasing in a CSV, we conclude that any shock that drives up R_{t+1}^k will simultaneously drive down $\bar{\omega}_{t+1}^N$.

From (2.8), it is possible to see why Z_{t+1}^N must be dependent upon the realization of the period $t+1$ shocks. Substitute out for $\left(1 + R_{t+1}^k\right) Q_{\bar{K}',t} \bar{K}_{t+1}^N$ using (2.7), to obtain:

$$\left[1 - F(\bar{\omega}_{t+1}^N) + \frac{1-\mu}{\bar{\omega}_{t+1}^N} \int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega)\right] Z_{t+1}^N = \left(1 + R_{t+1}^e\right),$$

after dividing both sides by B_{t+1}^N. Recall our specification that R_{t+1}^e is not dependent on the period $t+1$ realization of shocks. The last expression then implies that if Z_{t+1}^N is not dependent on the period $t+1$ shocks, then $\bar{\omega}_{t+1}^N$ must not be either. In this case, it is impossible for (2.7) to hold for all date $t+1$ states of nature. So, Z_{t+1}^N must be dependent on the period $t+1$ shocks.[3] Of course, if R_{t+1}^e were state dependent, then perhaps we could specify Z_{t+1}^N to be period $t+1$ state independent.

[3]This may appear implausible, at first glance. In practice, when banks extend loans the rate of interest that is to be paid is specified in advance. One interpretation of the fact that Z_t^N is contingent on the realization of the aggregate shock is that banks are unwilling to extend loans whose duration spans the whole period of the entrepreneur's project. Instead, they extend the loan for a part of the period, and that allows them to back out before too many funds are commited, in case it looks like the project is going bad. This is closely related to the interpretation offered in Bernanke, Gertler and Gilchrist (1999, footnote 10).

Substituting out for $Z_{t+1}^{N}B_{t+1}^{N}$ from (2.7) in the bank's zero profit condition, we obtain:[4]

$$\begin{aligned}\left(1+R_{t+1}^{e}\right)B_{t+1}^{N} &= \left[1-F(\bar{\omega}_{t+1}^{N})\right]\bar{\omega}_{t+1}^{N}\left(1+R_{t+1}^{k}\right)Q_{\bar{K}',t}\bar{K}_{t+1}^{N} \\ &\quad +\int_{0}^{\bar{\omega}_{t+1}^{N}}(1-\mu)\left(1+R_{t+1}^{k}\right)\omega Q_{\bar{K}',t}\bar{K}_{t+1}^{j}dF(\omega) \\ &= \left[\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})\right]\left(1+R_{t+1}^{k}\right)Q_{\bar{K}',t}\bar{K}_{t+1}^{N},\end{aligned} \tag{2.10}$$

where $\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})$ is the expected share of profits, net of monitoring costs, accruing to the bank and

$$\begin{aligned}G(\bar{\omega}_{t+1}^{N}) &= \int_{0}^{\bar{\omega}_{t+1}^{N}}\omega dF(\omega). \\ \Gamma(\bar{\omega}_{t+1}^{N}) &= \bar{\omega}_{t+1}^{N}\left[1-F(\bar{\omega}_{t+1}^{N})\right]+G(\bar{\omega}_{t+1}^{N})\end{aligned}$$

It is useful to work out the derivative of Γ :

$$\begin{aligned}\Gamma'(\bar{\omega}_{t+1}^{N}) &= 1-F(\bar{\omega}_{t+1}^{N})-\bar{\omega}_{t+1}^{N}F'(\bar{\omega}_{t+1}^{N})+G'(\bar{\omega}_{t+1}^{N}) \\ &= 1-F(\bar{\omega}_{t+1}^{N})>0.\end{aligned} \tag{2.11}$$

Dividing both sides of (2.10) by $Q_{\bar{K}',t}\bar{K}_{t+1}^{N}\left(1+R_{t+1}^{k}\right)$:

$$\frac{1+R_{t+1}^{e}}{1+R_{t+1}^{k}}\left(1-\frac{N_{t+1}}{Q_{\bar{K}',t}\bar{K}_{t+1}^{N}}\right)=\left[\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})\right]$$

[4] Under the alternative treatment of depreciation,

$$\begin{aligned}\left(1+R_{t+1}^{e}\right)B_{t+1}^{N} &= \left[1-F(\bar{\omega}_{t+1}^{N})\right]\left[(1+\tilde{R}_{t+1}^{k})\bar{\omega}_{t+1}+\tau_{t}^{k}\delta\right]Q_{\bar{K}',t}\bar{K}_{t+1}^{N} \\ &\quad +\int_{0}^{\bar{\omega}_{t+1}^{N}}(1-\mu)\left[(1+\tilde{R}_{t+1}^{k})\omega+\tau_{t}^{k}\delta\right]Q_{\bar{K}',t}\bar{K}_{t+1}^{j}dF(\omega) \\ &= \left[1-F(\bar{\omega}_{t+1}^{N})\right]\left[(1+\tilde{R}_{t+1}^{k})\bar{\omega}_{t+1}+\tau_{t}^{k}\delta\right]Q_{\bar{K}',t}\bar{K}_{t+1}^{N} \\ &\quad +G(\bar{\omega}_{t+1}^{N})(1-\mu)(1+\tilde{R}_{t+1}^{k})Q_{\bar{K}',t}\bar{K}_{t+1}^{j}+F(\bar{\omega}_{t+1}^{N})(1-\mu)\tau_{t}^{k}\delta Q_{\bar{K}',t}\bar{K}_{t+1}^{j} \\ &= \left[\left(1-F(\bar{\omega}_{t+1}^{N})\right)\bar{\omega}_{t+1}+G(\bar{\omega}_{t+1}^{N})(1-\mu)\right](1+\tilde{R}_{t+1}^{k})Q_{\bar{K}',t}\bar{K}_{t+1}^{N}+\tau_{t}^{k}\delta Q_{\bar{K}',t}\bar{K}_{t+1}^{N}\left[1-F(\bar{\omega}_{t+1}^{N})\mu\right] \\ &= \left[\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})\right](1+\tilde{R}_{t+1}^{k})Q_{\bar{K}',t}\bar{K}_{t+1}^{N}+\tau_{t}^{k}\delta Q_{\bar{K}',t}\bar{K}_{t+1}^{N}\left[1-F(\bar{\omega}_{t+1}^{N})\mu\right]\end{aligned}$$

or, after dividing:

$$\frac{\left(1+R_{t+1}^{e}\right)B_{t+1}^{N}}{(1+\tilde{R}_{t+1}^{k})Q_{\bar{K}',t}\bar{K}_{t+1}^{N}}=\left[\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})\right]+\frac{\tau_{t}^{k}\delta\left[1-F(\bar{\omega}_{t+1}^{N})\mu\right]}{(1+\tilde{R}_{t+1}^{k})}$$

Multiply this expression by $\left(Q_{\bar{K}',t}\bar{K}^N_{t+1}/N_{t+1}\right)(1+R^k_{t+1})/(1+R^e_{t+1})$, to obtain:

$$\frac{Q_{\bar{K}',t}\bar{K}^N_{t+1}}{N_{t+1}}-1=\frac{Q_{\bar{K}',t}\bar{K}^N_{t+1}}{N_{t+1}}\frac{1+R^k_{t+1}}{1+R^e_{t+1}}\left[\Gamma(\bar{\omega}^N_{t+1})-\mu G(\bar{\omega}^N_{t+1})\right].$$

Let

$$\tilde{u}_{t+1}\equiv\frac{1+R^k_{t+1}}{E\left(1+R^k_{t+1}|\Omega^\mu_t\right)},\ s_{t+1}\equiv\frac{E\left(1+R^k_{t+1}|\Omega^\mu_t\right)}{1+R^e_{t+1}}.$$

Then, the non-negativity constraint on bank profits is:

$$\frac{Q_{\bar{K}',t}\bar{K}^N_{t+1}}{N_{t+1}}-1\leq\frac{Q_{\bar{K}',t}\bar{K}^N_{t+1}}{N_{t+1}}\tilde{u}_{t+1}s_{t+1}\left[\Gamma(\bar{\omega}^N_{t+1})-\mu G(\bar{\omega}^N_{t+1})\right], \tag{2.12}$$

From this we can see that $\bar{\omega}^N_{t+1}$ is a function of the capital to net worth ratio and $\left(1+R^e_{t+1}\right)/\left(1+R^k_{t+1}\right)$ only:

$$\bar{\omega}^N_{t+1}=g\left(\frac{1+R^e_{t+1}}{1+R^k_{t+1}}\left(1-\frac{N_{t+1}}{Q_{\bar{K}',t}\bar{K}^N_{t+1}}\right)\right). \tag{2.13}$$

As noted above, competition implies that the loan contract is the best possible one, from the point of view of the entrepreneur. That is, it maximizes the entrepreneur's 'utility' subject to the zero profit constraint just stated. The entrepreneur's expected revenues over the period in which the standard debt contract applies is:

$$\begin{aligned}&E\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left[\left(1+R^k_{t+1}\right)\omega Q_{\bar{K}',t}\bar{K}^N_{t+1}-Z^N_{t+1}B^N_{t+1}\right]dF(\omega)|\Omega^\mu_t\right\}\\=\;&E\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left[\omega-\bar{\omega}^N_{t+1}\right]dF(\omega)\left(1+R^k_{t+1}\right)|\Omega^\mu_t\right\}Q_{\bar{K}',t}\bar{K}^N_{t+1}.\end{aligned}$$

Note that[5]

$$1=\int_0^\infty\omega dF(\omega)=\int_{\bar{\omega}^N_{t+1}}^{\infty}\omega dF(\omega)+G(\bar{\omega}^N_{t+1}),$$

[5] Under the alternative treatment of depreciation,

$$\begin{aligned}&E\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left[\left((1+\tilde{R}^k_{t+1})\omega+\tau^k_t\delta\right)Q_{\bar{K}',t}\bar{K}^N_{t+1}-\left(\left[1+\tilde{R}^k_{t+1}\right]\bar{\omega}^N_{t+1}+\tau^k_t\delta\right)Q_{\bar{K}',t}\bar{K}^N_{t+1}\right]dF(\omega)|\Omega_t,X_t\right\}\\=\;&E\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left[\omega-\bar{\omega}^N_{t+1}\right]dF(\omega)\left(1+\tilde{R}^k_{t+1}\right)|\Omega_t,X_t\right\}Q_{\bar{K}',t}\bar{K}^N_{t+1}.\end{aligned}$$

so that the objective can be written:

$$E\left\{\left[1-\Gamma(\bar{\omega}_{t+1}^{N})\right]\left(1+R_{t+1}^{k}\right)|\Omega_{t}^{\mu}\right\}Q_{\bar{K}',t}\bar{K}_{t+1}^{N},$$

or, after dividing by $(1+R_{t+1}^{e})N_{t+1}$ (which is constant across realizations of date $t+1$ uncertainty), and rewriting:

$$E\left\{\left[1-\Gamma(\bar{\omega}_{t+1}^{N})\right]\tilde{u}_{t+1}|\Omega_{t}^{\mu}\right\}s_{t+1}\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^{N}}{N_{t+1}},\ \tilde{u}_{t+1}=\frac{1+R_{t+1}^{k}}{E\left(1+R_{t+1}^{k}|\Omega_{t}^{\mu}\right)},\ s_{t+1}=\frac{E\left(1+R_{t+1}^{k}|\Omega_{t}^{\mu}\right)}{1+R_{t+1}^{e}}, \tag{2.14}$$

where Ω_{t}^{μ} denotes all period t shocks. From this expression and the fact, $\Gamma'>0$, it is evident that the objective is decreasing in $\bar{\omega}_{t+1}^{N}$ for given $Q_{\bar{K}',t}\bar{K}_{t+1}^{N}/N_{t+1}$. This property of the objective was alluded to above.

The debt contract selects $Q_{\bar{K}',t}\bar{K}_{t+1}^{N}/N_{t+1}$ and $\bar{\omega}_{t+1}^{N}$ to optimize (2.14) subject to (2.12). It is convenient to denote:

$$k_{t+1}^{N}=\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^{N}}{N_{t+1}}.$$

Writing the CSV problem in Lagrangian form,

$$\max_{\bar{\omega}^{N},k^{N}}E\left\{\left[1-\Gamma(\bar{\omega}^{N})\right]\tilde{u}_{t+1}s_{t+1}k^{N}+\lambda^{N}\left[k^{N}\tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}^{N})-\mu G(\bar{\omega}^{N})\right)-k^{N}+1\right]|\Omega_{t}^{\mu}\right\}.$$

The single first order condition for k^{N} is:

$$E\left\{\left[1-\Gamma(\bar{\omega}_{t+1}^{N})\right]\tilde{u}_{t+1}s_{t+1}+\lambda_{t+1}^{N}\left[\tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})\right)-1\right]|\Omega_{t}^{\mu}\right\}=0. \tag{2.15}$$

The first order conditions for $\bar{\omega}^{N}$ are, after dividing by $\tilde{u}_{t+1}s_{t+1}k_{t+1}^{N}$:

$$\Gamma'(\bar{\omega}_{t+1}^{N})=\lambda_{t+1}^{N}\left[\Gamma'(\bar{\omega}_{t+1}^{N})-\mu G'(\bar{\omega}_{t+1}^{N})\right]. \tag{2.16}$$

Finally, there is the complementary slackness condition, $\lambda^{N}\left[k^{N}\tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}^{N})-\mu G(\bar{\omega}^{N})\right)-k^{N}+1\right]=0$. Assuming the constraint is binding, so that $\lambda^{N}>0$, this reduces to:

$$k_{t+1}^{N}\tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}_{t+1}^{N})-\mu G(\bar{\omega}_{t+1}^{N})\right)-k_{t+1}^{N}+1=0. \tag{2.17}$$

It should be understood that λ_{t+1}^{N} in (2.15) is defined by (2.16). We can think of (2.15)-(2.17) as defining functions relating k_{t+1}^{N} and $\bar{\omega}_{t+1}^{N}$to s_{t+1}. Remember, k_{t+1}^{N} is not indexed by $\tilde{u}_{t+1}$, while $\bar{\omega}_{t+1}^{N}$ is. So, we think of $\bar{\omega}_{t+1}^{N}$ as a family of functions of s_{t+1}, each function being indexed by a different realization of $\tilde{u}_{t+1}$. Note that N_{t+1} does not appear in the equations that define k_{t+1}^N and $\bar{\omega}_{t+1}^N$. This establishes that the values of these variables in the CSV contract is the same for each value of N_{t+1}. For this reason, we can drop the superscript notation, N. That is, the functions we are concerned with are k_{t+1} and $\bar{\omega}_{t+1}$.

We find it convenient to drop time subscripts to keep the notation simple, and because it should entail no confusion. The equations that concern us are:

$$E\left\{[1-\Gamma(\bar{\omega})]\,\tilde{u}s+\lambda\left[\tilde{u}s\left(\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right)-1\right]\right\}=0, \tag{2.18}$$

$$\Gamma'(\bar{\omega})=\lambda\left[\Gamma'(\bar{\omega})-\mu G'(\bar{\omega})\right], \tag{2.19}$$

$$k\tilde{u}s\left(\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right)-k+1=0. \tag{2.20}$$

It is understood that the expectation operator is over different values of $\tilde{u}$, and k is constant across $\tilde{u}$ while λ and $\bar{\omega}$ vary with $\tilde{u}$. These three equations are used used to help characterize the equilibrium of the model.

2.4.4. Aggregating Across Entrepreneurs

We now discuss the evolution of the aggregate net worth of all entrepreneurs. In terms of the previous notation, if $f_{t+1}(N)$ is the density of entrepreneurs having net worth N_{t+1}, then aggregate net worth, $\bar{N}_{t+1}$, is:

$$\bar{N}_{t+1}=\int_0^\infty N f_{t+1}(N)dN.$$

We now discuss the law of motion of aggregate net worth. Suppose $\bar{N}_t$ is given. Let V_t^N denote the average of profits of N_t-type entrepreneurs, net of repayments to banks:

$$V_t^N=\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t^N-\Gamma(\bar{\omega}_t)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t^N.$$

The aggregate capital stock is:

$$\bar{K}_t=\int_0^\infty f_t(N)\bar{K}_t^N dN$$

Given that R_t^k and $\bar{\omega}_t$ are independent of N_t, we have:

$$V_t\equiv\int_0^\infty f_t(N)V_t^N dN=\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t-\Gamma(\bar{\omega}_t)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t$$

Writing this out more fully:

$$\begin{aligned}
V_t &= \left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t-\left\{[1-F(\bar{\omega}_t)]\,\bar{\omega}_t+\int_0^{\bar{\omega}_t}\omega dF(\omega)\right\}\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t \\
&= \left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t \\
&\quad -\left\{[1-F(\bar{\omega}_t)]\,\bar{\omega}_t+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF(\omega)+\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\right\}\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t.
\end{aligned}$$

Notice that the first two terms in braces correspond to the net revenues of the bank, which must equal $(1+R_t^e)\left(Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t\right)$. Substituting:

$$V_t = \left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t - \left\{1+R_t^e + \frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t}\right\}\left(Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t\right). \tag{2.21}$$

Since entrepreneurs are selected randomly for death, the integral over entrepreneurs' net profits is just γV_t. So, the law of motion for $\bar{N}_t$ is:

$$\begin{aligned}\bar{N}_{t+1} &= \gamma\left\{\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t - \left[1+R_t^e + \frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t}\right]\left(Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t\right)\right\} \\ &\quad + W_t^e,\end{aligned} \tag{2.22}$$

where W_t^e is the transfer payment to entrepreneurs. The $(1-\gamma)$ entrepreneurs who are selected for death, consume:

$$P_t C_t^e = \Theta(1-\gamma)V_t.$$

Finally, there seem to be at least two objects that could be called the 'external finance premium'. One is the ratio involving μ in square brackets above. The other is $Z_t - (1+R_t^e)$. Either one is straightforward to compute. The former appears to correspond to the 'average' external finance premium, while the latter is only the external finance premium for entrepreneurs who are able to repay Z_t. In the case of the former, the 'premium' paid by some is actually negative. For those entrepreneurs with ω sufficiently small, they are paying essentially nothing, and so in particular they pay less than R_t^e and so they have an ex post negative premium. BGG refer to s as the external finance premium.

2.5. Banks

We assume that there is a continuum of identical, competitive banks. Each operates a technology to convert capital, K_t^b, labor, l_t^b, and excess reserves, E_t^b, into real deposit services, D_t/P_t. The production function is:

$$\frac{D_t}{P_t} = a^b x_t^b\left(\left(K_t^b\right)^\alpha\left(z_t l_t^b\right)^{1-\alpha}\right)^{\xi_t}\left(\frac{E_t^r}{P_t}\right)^{1-\xi_t} \tag{2.23}$$

Here a^b is a positive scalar, and $0<\alpha<1$. Also, x_t^b is a unit-mean technology shock that is specific to the banking sector. In addition, $\xi_t\in(0,1)$ is a shock to the relative value of excess reserves, E_t^r. The stochastic process governing these shocks will be discussed later. We include excess reserves as an input to the production of demand deposit services as a reduced form way to capture the precautionary motive of a bank concerned about the possibility of unexpected withdrawals.

We now discuss a typical bank's balance sheet. The bank's assets consist of cash reserves and loans. It obtains cash reserves from two sources. Households deposit A_t dollars and the monetary authority credits households' checking accounts with X_t dollars. Consequently, total time t cash reserves of the banking system equal $A_t + X_t$. Bank loans are extended to firms and other banks to cover their working capital needs, and to entrepreneurs to finance purchases of capital.

The bank has two types of liabilities: demand deposits, D_t, and time deposits, T_t. Demand deposits, which pay interest, R_{at}, are created for two reasons. First, there are the household deposits, $A_t + X_t$ mentioned above. We denote this by D_t^h. Second, working capital loans made by banks to firms and other banks are granted in the form of demand deposits. We denote firm and bank demand deposits by D_t^f. Total deposits, then, are:

$$D_t = D_t^h + D_t^f.$$

Time deposit liabilities are issued by the bank to finance the standard debt contracts offered to entrepreneurs and discussed in the previous section. Time and demand deposits differ in three respects. First, demand deposits yield transactions services, while time deposits do not. Second, time deposits have a longer maturity structure. Third, demand deposits are backed by working capital loans and reserves, while time deposits are backed by standard debt contracts to entrepreneurs.

We now discuss the demand deposit liabilities. We suppose that the interest on demand deposits that are created when firms and banks receive working capital loans, are paid to the recipient of the loans. Firms and banks just sit on these demand deposits. The wage bill isn't actually paid to workers until a settlement period that occurs after the goods market.

We denote the interest payment on working capital loans, net of interest on the associated demand deposits, by R_t. Since each borrower receives interest on the deposit associated with their loan, the gross interest payment on loans is $R_t + R_{at}$. Put differently, the spread between the interest on working capital loans and the interest on demand deposits is R_t.

The maturity of period t working capital loans and the associated demand deposit liabilities coincide. A period t working capital loan is extended just prior to production in period t, and then paid off after production. The household deposits funds into the bank just prior to production in period t and then liquidates the deposit after production.

We now discuss the time deposit liabilities. Unlike in the case of demand deposits, we assume that the cost of maintaining time deposit liabilities is zero. Competition among banks in the provision of time deposits and entrepreneurial loans drives the interest rate on time deposits to the return the bank earns (net of expenses, including monitoring costs) on the loans, R_t^e. The maturity structure of time deposits coincides with that of the standard debt contract, and differs from that of demand deposits and working capital loans. The maturity structure of the two types of assets can be seen in Figure 3. Time deposits and entrepreneurial loans are created at the end of a given period's goods market. This is the time when newly constructed capital is sold by capital producers to entrepreneurs. Time deposits and entrepreneurial loans pay off at the end of next period's goods market, when the entrepreneurs sell their undepreciated capital to capital producers (who use it as a raw material in the production of next period's capital). The payoff on the entrepreneurial loan coincides with the payoff on time deposits. Competition in the provision of time deposits guarantees that these payoffs coincide.

The maturity difference between demand and time deposits implies that the return on the latter in principle carries risks not present in the former. In the case of demand deposits, no shocks are realized between the creation of a deposit and its payoff. In the case of time deposits, there are shocks whose value is realized between creation and payoff (see Figure 3). Since time deposits finance assets with an uncertain payoff, someone has to bear the risk. We follow BGG in focusing on equilibria in which the entrepreneur bears all the risk. The ex post return on time deposits is know with certainty to the household at the time the deposit decision is made.

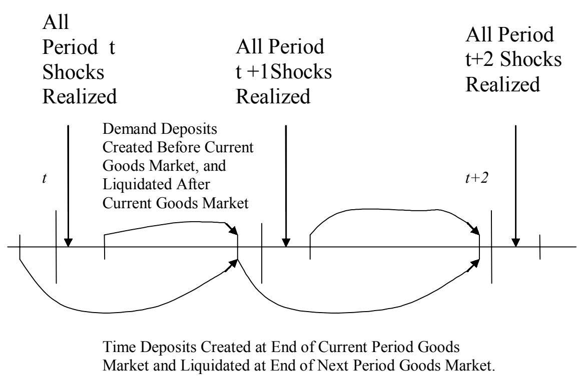


Figure 3: Maturity Structure of Time and Demand Deposits

We now discuss the assets and liabilities of the bank in greater detail. We describe the banks' books at two points in time within the period: just before the goods market, when the market for working capital loans and demand deposits is open, and just after the goods market. At the latter point in time, the market for time deposits and entrepreneurial loans is open. Liabilities and assets just before the goods market are:

$$D_t + T_{t-1} = A_t + X_t + S_t^w + B_t, \tag{2.24}$$

where S_t^w denotes working capital loans. The monetary authority imposes a reserve requirement that banks must hold at least a fraction τ of their demand deposits in the form of currency. Consequently, nominal excess reserves, E_t^r, are given by

$$E_t^r = A_t + X_t - \tau_t D_t. \tag{2.25}$$

The bank's 'T' accounts are as follows:

Assets	Liabilities
Reserves	
A_t	D_t
X_t	
Short-term Working Capital Loans	
S_t^w	
Long-term, Entrepreneurial Loans	
B_t	T_{t-1}

After the goods market, demand deposits are liquidated, so that $D_t = 0$ and $A_t + X_t$ is returned to the households, so this no longer appears on the bank's balance sheet. Similarly, working capital loans, S_t^w, and 'old' entrepreneurial loans, B_t, are liquidated at the end of the goods market and also do not appear on the bank's balance sheet. At this point, the assets on the bank's balance sheet are the new entrepreneurial loans issued at the end of the goods market, B_{t+1}, and the bank liabilities are the new time deposits, T_t.

At the end of the goods market, the bank settles claims for transactions that occurred in the goods market and that arose from it's activities in the previous period's entrepreneurial loan and time deposit market. The bank's sources of funds at this time are: net interest from borrowers and $A_t + X_t$ of high-powered money (i.e., a mix of vault cash and claims on the central bank).[6] Working capital loans coming due at the end of the period pay R_t in interest and so the associated principal and interest is

$$(1 + R_t)S_t^w = (1 + R_t)\left(\psi_{l,t} W_t l_t + \psi_{k,t} P_t r_t^k K_t\right).$$

Loans to entrepreneurs coming due at the end of the period are the ones that were extended in the previous period, $Q_{\bar{k}',t-1}\bar{K}_t - N_t$, and they pay the interest rate from the previous period, after monitoring costs:

$$(1 + R_t^e)\left(Q_{\bar{K}',t-1}\bar{K}_t - N_t\right)$$

[6] Interest is not paid by the central bank on high-powered money.

The bank's uses of funds are (i) interest and principle obligations on demand deposits and time deposits, $(1+R_{at})D_t$ and $(1+R_t^e)T_{t-1}$, respectively, and (ii) interest and principal expenses on working capital, i.e., capital and labor services. Interest and principal expenses on factor payments in the banking sector are handled in the same way as in the goods sector. In particular, banks must finance a fraction, $\psi_{k,t}$, of capital services and a fraction, $\psi_{l,t}$, of labor services, in advance, so that total factor costs as of the end of the period, are $(1+\psi_{k,t}R_t)\,P_t r_t^k K_t^b$. The bank's net source of funds, Π_t^b, is:

$$\begin{aligned}\Pi_t^b \;=\;& (A_t+X_t)+(1+R_t+R_{at})S_t^w-(1+R_{at})D_t \\ & -\left[(1+\psi_{k,t}R_t)\,P_t r_t^k K_t^b\right]-\left[(1+\psi_{l,t}R_t)\,W_t l_t^b\right] \\ & +\left[1+R_t^e+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t-N_t}\right]B_t \\ & -\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t-\left(1+R_t^e\right)T_{t-1} \\ & +T_t-B_{t+1}\end{aligned} \tag{2.26}$$

Because of competition, the bank takes all wages and prices and interest rates as given and beyond its control.

We now describe the bank's optimization problem. The bank pays Π_t^b to households in the form of dividends. It's objective is to maximize the present discounted value of these dividends. In period 0, its objective is:

$$E_0\sum_{t=0}^{\infty}\beta^t\lambda_t\Pi_t^b,$$

where λ_t is the multiplier on Π_t^b in the Lagrangian representation of the household's optimization problem. It takes as given its time deposit liabilities from the previous period, T_{-1}, and its entrepreneurial loans issued in the previous period, B_0. In addition, the bank takes all rates of return and λ_t as given. The bank optimizes its objective by choice of $\left\{S_t^w,\ B_{t+1},\ D_t,\ T_t,\ K_t^b,\ E_t^r;t\geq 0\right\}$, subject to (2.23)-(2.25).

In the previous section, we discussed the determination of the variables relating to entrepreneurial loans. There is no further need to discuss them here, and so we take those as given. To discuss the variables of concern here, we adopt a Lagrangian representation of the bank problem which uses a version of (2.26) that ignores variables pertaining to the entrepreneur. The Lagrangian representation of the problem that we work with is:

$$\max_{A_t,S_t^w,K_t^b,l_t^b}\{R_tS_t^w-R_{at}\left(A_t+X_t\right)-\left[(1+\psi_{k,t}R_t)\,P_t r_t^k K_t^b\right]-\left[(1+\psi_{l,t}R_t)\,W_t l_t^b\right]\}$$

$$+\lambda_t^b \left[h(x_t^b, K_t^b, l_t^b, \frac{A_t + X_t - \tau_t \left(A_t + X_t + S_t^w\right)}{P_t}, \xi_t, x_t^b, z_t) - \frac{A_t + X_t + S_t^w}{P_t} \right]$$

where

$$\begin{aligned} h(x_t^b, K_t^b, l_t^b, e_t^r, \xi_t, x_t^b, z_t) &= a^b x_t^b \left(\left(K_t^b\right)^\alpha \left(z_t l_t^b\right)^{1-\alpha} \right)^{\xi_t} \left(e_t^r\right)^{1-\xi_t} \\ e_t^r &= \frac{E_t^r}{P_t} = \frac{A_t + X_t - \tau_t \left(A_t + X_t + S_t^w\right)}{P_t} \end{aligned}$$

The first order conditions are, for A_t, S_t^w, K_t^b, l_t^b, respectively:

$$-R_{at} + \lambda_t^b \frac{1}{P_t} \left[\left(1 - \tau_t\right) h_{e^r,t} - 1 \right] = 0 \tag{2.27}$$

$$R_t - \lambda_t^b \frac{1}{P_t} \left[\tau_t h_{e^r,t} + 1 \right] = 0 \tag{2.28}$$

$$-\left(1 + \psi_{k,t} R_t\right) P_t r_t^k + \lambda_t^b h_{K^b,t} = 0 \tag{2.29}$$

$$-\left(1 + \psi_{l,t} R_t\right) W_t + \lambda_t^b h_{l^b,t} = 0 \tag{2.30}$$

Substituting for λ_t^b in (2.29) and (2.30) from (2.28), we obtain:

$$\left(1 + \psi_{k,t} R_t\right) r_t^k = \frac{R_t h_{K^b,t}}{1 + \tau_t h_{e^r,t}},$$

and

$$\left(1 + \psi_{l,t} R_t\right) \frac{W_t}{P_t} = \frac{R_t h_{l^b,t}}{1 + \tau_t h_{e^r,t}}.$$

These are the first order conditions associated with the bank's choice of capital and labor. Each says that the bank attempts to equate the marginal product - in terms of extra loans - of an additional factor of production, with the associated marginal cost. The marginal product in producing loans must take into account two things: an increase in S^w requires an equal increase in deposits and an increase in deposits raises required reserves. The first raises loans by the marginal product of the factor in h, while the reserve implication works in the other direction.

Taking the ratio of (2.28) to (2.27), we obtain:

$$R_{at} = \frac{\left(1 - \tau_t\right) h_{e^r,t} - 1}{\tau_t h_{e^r,t} + 1} R_t. \tag{2.31}$$

This can be thought of as the first order condition associated with the bank's choice of A_t. The object multiplying R_t is the increase in S^w the bank can offer for one unit increase in A.

The term on the right indicates the net interest earnings from those loans. The term on the left indicates the cost. Recall that R_t represents *net* interest on loans, because the actual interest is $R_t + R_{at}$, so that R_t represents the spread between the interest rate charged by banks on their loans and the cost to them of the underlying funds. Since loans are made in the form of deposits, and deposits earn R_{at} in interest, the net cost of a loan to a borrower is R_t.

The clearing condition in the market for working capital loans is:

$$S_t^w = \psi_{l,t} W_t l_t + \psi_{k,t} P_t r_t^k K_t \tag{2.32}$$

Here, S_t^w represents the supply of loans, and the terms on the right of the equality in (2.32) represent total demand.

2.6. Households

There is a continuum of households, indexed by $j \in (0,1)$. Households consume, save and supply a differentiated labor input. They set their wages using the variant on the Calvo (1983) technology used in CEE.[7]

The sequence of decisions by the household during a period is as follows. First, it makes its consumption decision after the non-financial shocks are realized. Second, it purchases securities whose payoffs are contingent upon whether it can reoptimize its wage decision. Third, it sets its wage rate after finding out whether or not it can reoptimize. Fourth, the current period monetary action is realized. Fifth, after the monetary action, and before the goods market, the household decides how much of its financial assets to hold in the form of currency and demand deposits. At this point, the time deposits purchased by the household in the previous period are fixed and beyond its control. Sixth, the household goes to the goods market, where labor services are supplied and goods are purchased. Seventh, after the goods market, the household settles claims arising from its goods market experience and makes its current period time deposit decision.

Since the uncertainty faced by the household over whether it can reoptimize its wage is idiosyncratic in nature, households work different amounts and earn different wage rates. So, in principle they are also heterogeneous with respect to consumption and asset holdings. A straightforward extension of arguments in Erceg, Henderson and Levin (2000) and Woodford (1996), establish that the existence of state contingent securities ensures that in equilibrium households are homogeneous with respect to consumption and asset holdings. Reflecting this result, our notation assumes that households are homogeneous with respect to consumption and asset holdings, and heterogeneous with respect to the wage rate that they earn and

[7]CEE build on Erceg, Henderson and Levin (2000).

hours worked. The preferences of the j^{th} household are given by:

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) - \upsilon_{t+l} \frac{\left[\left(\frac{P_{t+l}C_{t+l}}{M_{t+l}} \right)^{\theta_{t+l}} \left(\frac{P_{t+l}C_{t+l}}{D_{t+l}^h} \right)^{1-\theta_{t+l}} \right]^{1-\sigma_q}}{1-\sigma_q} \right\}, \tag{2.33}$$

where E_t^j is the expectation operator, conditional on aggregate and household j idiosyncratic information up to, and including, time $t-1$; C_t denotes time t consumption; and h_{jt} denotes time t hours worked. In order to help assure that our model has a balanced growth path, we specify that u is the natural logarithm. When $b > 0$, (2.33) allows for habit formation in consumption preferences. Various authors, such as Fuhrer (2000), and McCallum and Nelson (1998), have argued that this is important for understanding the monetary transmission mechanism. In addition, habit formation is useful for understanding other aspects of the economy, including the size of the premium on equity. Finally, the term in square brackets captures the notion that currency and demand deposits contribute to utility by providing transactions services. Those services are an increasing function of the level of consumption.

We now discuss the household's period t uses and sources of funds. Just before the goods market in period t, after the realization of all shocks, the household has M_t^b units of high powered money which it splits into currency, M_t, and deposits with the bank:

$$M_t^b - (M_t + A_t) \geq 0. \tag{2.34}$$

The household deposits A_t with the bank, in exchange for a demand deposit. Demand deposits pay the relatively low interest rate, R_{at}, but offer transactions services.

The central bank credits the household's bank deposit with X_t units of high powered money, which automatically augments the household's demand deposits. So, household demand deposits are D_t^h :

$$D_t^h = A_t + X_t.$$

As noted in the previous section, the household only receives interest on the non-wage component of its demand deposits, since the interest on the wage component is earned by intermediate good firms.

The household also can acquire a time deposit. This can be acquired at the end of the period t goods market and pays a rate of return, $1+R_{t+1}^e$, at the end of the period $t+1$ goods market. The rate of return, R_{t+1}^e, is known at the time that the time deposit is purchased. It is not contingent on the realization of any of the period $t+1$ shocks.

The household also uses its funds to pay for consumption goods, P_tC_t and to acquire high powered money, Q_{t+1}, for use in the following period. Additional sources of funds include profits from producers of capital, Π_t^k, from banks, Π_t^b, from intermediate good firms, $\int \Pi_t^j dj$, and $A_{j,t}$, the net payoff on the state contingent securities that the household purchases to insulate itself from uncertainty associated with being able to reoptimize its wage rate. Households also receive lump-sum transfers, $1-\Theta$, corresponding to the net worth of the $1-\gamma$ entrepreneurs which die in the current period. Finally, the households pay a lump-sum tax to finance the transfer payments made to the γ entrepreneurs that survive and to the $1-\gamma$ newly born entrepreneurs. These observations are summarized in the following asset accumulation equation:

$$\begin{aligned}&\left[1+\left(1-\tau_t^D\right)R_{at}\right]\left(M_t^b-M_t+X_t\right)-T_t \\ &-\left(1+\tau_t^c\right)P_tC_t+\left(1-\Theta\right)\left(1-\gamma\right)V_t-W_t^e+Lump_t \\ &+\left[1+\left(1-\tau_t^T\right)R_t^e\right]T_{t-1}+\left(1-\tau_t^l\right)W_{j,t}h_{j,t}+M_t+\Pi_t^b+\Pi_t^k+\int\Pi_t^f df+A_{j,t}-M_{t+1}^b\geq 0.\end{aligned} \tag{2.35}$$

The household's problem is to maximize (2.33) subject to the timing constraints mentioned above, the various non-negativity constraints, and (2.35).

We consider the Lagrangian representation of the household problem, in which $\lambda_t \geq 0$ is the multiplier on (2.35). The consumption and the wage decisions are taken before the realization of the financial market shocks. The other decisions, M_{t+1}^b, M_t and T_t are taken after the realization of all shocks during the period. The period t multipliers are functions of all the date t shocks. We now consider the first order conditions associated with C_t, M_{t+1}^b, M_t and T_t. The Lagrangian representation of the problem, ignoring constant terms in the asset evolution equation, is:

$$\begin{aligned}&E_0^j\sum_{t=0}^{\infty}\beta^t\{u(C_t-bC_{t-1})-\zeta_t z(h_{j,t})-\upsilon_t\frac{\left[P_tC_t\left(\frac{1}{M_t}\right)^{\theta_t}\left(\frac{1}{M_t^b-M_t+X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q}}{1-\sigma_q} \\ &+\lambda_t[\left[1+\left(1-\tau_t^D\right)R_{at}\right]\left(M_t^b-M_t\right)-T_t-\left(1+\tau_t^c\right)P_tC_t \\ &+\left[1+\left(1-\tau_t^T\right)R_t^e\right]T_{t-1}+\left(1-\tau_t^l\right)W_{j,t}h_{j,t}+M_t-M_{t+1}^b]\}\end{aligned}$$

We now consider the various first order conditions associated with this maximization problem.

The first order condition with respect to T_t is:

$$E\left\{-\lambda_t+\beta\lambda_{t+1}\left[1+\left(1-\tau_{t+1}^T\right)R_{t+1}^e\right]|\Omega_t^\mu\right\}=0$$

The first order condition with respect to M_t is:

$$\upsilon_t\left[\left(\frac{P_tC_t}{M_t}\right)^{\theta_t}\left(\frac{P_tC_t}{M_t^b-M_t+X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q}[\frac{\theta_t}{M_t}-\frac{(1-\theta_t)}{M_t^b-M_t+X_t}] \tag{2.36}$$

$$-\lambda_t\left(1-\tau_t^D\right)R_{at}=0$$

The first order condition with respect to M_{t+1}^b is:

$$E\{\beta\upsilon_{t+1}\left(1-\theta_{t+1}\right)\left[P_{t+1}C_{t+1}\left(\frac{1}{M_{t+1}}\right)^{\theta_{t+1}}\left(\frac{1}{M_{t+1}^b-M_{t+1}+X_{t+1}}\right)^{(1-\theta_{t+1})}\right]^{1-\sigma_q}\frac{1}{M_{t+1}^b-M_{t+1}+X_{t+1}}$$
$$+\beta\lambda_{t+1}\left[1+\left(1-\tau_{t+1}^D\right)R_{a,t+1}\right]-\lambda_t|\Omega_t^\mu\}=0$$

The first two terms on the left of the equality capture the discounted value of an extra unit of currency in base in the next period. The last term captures the cost, which is the multiplier on the current period budget constraint.

We now consider C_t. It is useful to define $u_{c,t}$ as the derivative of the present discounted value of utility with respect to C_t :

$$E\left\{u_{c,t}-u'(C_t-bC_{t-1})+b\beta u'(C_{t+1}-bC_t)|\Omega_t^\mu\right\}=0.$$

The first order condition associated with C_t is:

$$E_t\left\{u_{c,t}-\upsilon_tC_t^{-\sigma_q}\left[\left(\frac{P_t}{M_t}\right)^{\theta_t}\left(\frac{P_t}{M_t^b-M_t+X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q}-\left(1+\tau_t^c\right)P_t\lambda_t\right\}=0.$$

The wage rate set by the household that has the option to reoptimize in period t is $\tilde{W}_t$. The household takes into account that if it cannot reoptimize in period $t+1$, its wage rate then is

$$W_{t+1}=\pi_t\mu_{z,t+1}\tilde{W}_t.$$

Note the slight difference in timing between inflation and the technology shock. The former reflects that indexing is lagged. The latter reflects that indexing to the technology shock is contemporaneous.

The demand curve that the individual household faces is:

$$h_{t+j}=\left(\frac{\tilde{W}_{t+j}}{W_{t+j}}\right)^{\frac{\lambda_w}{1-\lambda_w}}l_{t+j}=\left(\frac{\tilde{W}_t\mu_{z,t+1}\times\cdots\times\mu_{z,t+l}}{w_{t+j}z_{t+j}P_t}X_{t,j}\right)^{\frac{\lambda_w}{1-\lambda_w}}l_{t+j}, \tag{2.37}$$

where $\tilde{W}_t$ denotes the nominal wage set by households that reoptimize in period t, and W_t denotes the nominal wage rate associated with aggregate, homogeneous labor, l_t. Also,

$$X_{t,l}=\frac{\pi_t\times\pi_{t+1}\times\cdots\times\pi_{t+l-1}}{\pi_{t+1}\times\cdots\times\pi_{t+l}}=\frac{\pi_t}{\pi_{t+l}}.$$

The homogeneous labor is related to household labor by:

$$l = \left[\int_0^1 (h_j)^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty.$$

The contractor that produces homogeneous labor is competitive in the relevant output market, where labor is sold for the wage rate, W_t, and in the input market. Optimization leads to the following restrictions:

$$W_t = \left[(1 - \xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\pi_{t-1} \mu_{z,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{2.38}$$

The j^{th} household that reoptimizes its wage, $\tilde{W}_t$, does so to optimize (neglecting irrelevant terms in the household objective):

$$E_t \sum_{l=0}^{\infty} (\beta \xi_w)^{l-t} \{ -\zeta_{t+l} z(h_{j,t+l}) + \lambda_{t+l} (1 - \tau_{t+l}^l) W_{j,t+l} h_{j,t+l} \},$$

where

$$z(h) = \psi_L \frac{h_t^{1+\sigma_L}}{1 + \sigma_L}$$

The presence of ξ_w by the discount factor reflects that in optimizing its wage rate, the household is only concerned with the future states of the world in which it cannot reoptimize.

Linearizing the household's first order condition associated with the wage decision, as well as (2.38), and combining the result produces the following equilibrium relationship between the aggregate wage rate, inflation, employment, the technology shock, the labor supply shock and the labor income tax:

$$E_t \left\{ \eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_{\bar{3}} \hat{\pi}_{t-1} + \eta_3 \hat{\pi}_t + \eta_4 \hat{\pi}_{t+1} + \eta_5 \hat{l}_t + \eta_6 \left[\hat{\lambda}_{z,t} - \frac{\tau^l}{1 - \tau^l} \hat{\tau}_t^l \right] + \eta_7 \hat{\zeta}_t \right\} = 0$$

where

$$\eta = \begin{pmatrix} b_w \xi_w \\ -b_w \left(1 + \beta \xi_w^2\right) + \sigma_L \lambda_w \\ \beta \xi_w b_w \\ b_w \xi_w \\ -\xi_w b_w (1 + \beta) \\ b_w \beta \xi_w \\ -\sigma_L (1 - \lambda_w) \\ 1 - \lambda_w \\ -(1 - \lambda_w) \end{pmatrix} = \begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_{\bar{3}} \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \end{pmatrix}.$$

2.7. Monetary Policy

We consider a representation of monetary policy in which base growth feeds back on the shocks. The law of motion for the base is:

$$M_{t+1}^{b} = M_{t}^{b}(1 + x_t),$$

where x_t is the net growth rate of the monetary base. (Above, we have also used the notation, X_t, where $x_t = X_t/M_t^b$.) Monetary policy is characterized by a feedback from $\hat{x}_t$ $(= (x_t - x)/x)$ to an innovation in monetary policy and to the innovation in all the other shocks in the economy. Let the $p-$ dimensional vector summarizing these innovations be denoted $\hat{\varphi}_t$, and suppose that the first element in $\hat{\varphi}_t$ is the innovation to monetary policy. Then, monetary policy has the following representation:

$$\hat{x}_t = \sum_{i=1}^{p} x_{it},$$

where x_{it} is the component of money growth reflecting the i^{th} element in $\hat{\varphi}_t$. Also,

$$x_{it} = \rho_i x_{i,t-1} + \theta_i^0 \hat{\varphi}_{it} + \theta_i^1 \hat{\varphi}_{i,t-1}, \tag{2.39}$$

for $i = 1, ..., p$, with $\theta_1^0 \equiv 1$.

2.8. Final Goods Market Clearing

We now develop the aggregate resource constraint for this economy, relating the use of final goods to the quantity of aggregate labor and capital. Our derivation takes into account that it is not just the aggregate quantity of factor inputs that matters, but also its distribution across sectors, and proceeds in the style of Tak Yun ( ).

Define Y^* as the unweighted integral of output of the intermediate good producers:

$$Y^* = \int_0^1 Y(f)df = \int_0^1 F(\epsilon, z, K(f), l(f))df,$$

where, assuming production is positive for each f,

$$F(\epsilon, z, K(f), l(f)) = \epsilon z^{1-\alpha} K(f)^{\alpha} l(f)^{1-\alpha} - z\phi.$$

Here, by $l(f)$ we mean homogeneous labor hired by the f^{th} intermediate good firm, $f \in (0, 1)$. Recall that all firms confront the same wage rate and rental rate on capital. As a result, they all have the same capital-labor ratio, $K(f)/l(f)$. Moreover, this ratio coincides with the ratio of the aggregate inputs:

$$\frac{K^f}{l^f},\ K^f = \int_0^1 K(f)df,\ l^f = \int_0^1 l(f)df,$$

where K^f and l^f are aggregate capital and labor used in the goods producing sector, respectively. Then, it is easy to see that $Y^* = F(\epsilon, z, K^f, l^f)$.

Unweighted integration of the demand curve for $Y(f)$, (2.1), yields

$$Y^* = YP^{\frac{\lambda_f}{\lambda_f - 1}} (P^*)^{\frac{\lambda_f}{1-\lambda_f}}$$

where

$$P^* = \left[\int_0^1 P(f)^{\frac{\lambda_f}{1-\lambda_f}} df\right]^{\frac{1-\lambda_f}{\lambda_f}}.$$

Then,

$$Y = (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[z^{1-\alpha}\epsilon (\nu K)^\alpha (\nu l)^{1-\alpha} - z\phi\right],\ p^* = \frac{P^*}{P},$$

where

$$K^f = \nu K,\ l^f = \nu l.$$

Note that l is the integral of all employment of the labor 'produced' by the representative labor contractor. It is not necessarily the simple sum over all the labor supplied by households. Let the unweighted integral of the differentiated labor supplied by households be denoted by L :

$$L = \int_0^1 h^j dj.$$

Evaluating the unweighted integral of the demand curve for differentiated household labor, (2.37), we obtain:

$$L = l\left(\frac{W}{W^*}\right)^{\frac{\lambda_w}{\lambda_w - 1}},$$

where

$$W^* = \left[\int_0^1 W_j^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

We conclude that the total output of final goods, Y, is related to total factor inputs by the following relationship:

$$Y = (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[z^{1-\alpha}\epsilon (\nu K)^\alpha \left(\nu (w^*)^{\frac{\lambda_w - 1}{\lambda_w}} L\right)^{1-\alpha} - z\phi\right],\ w^* = \frac{W^*}{W}.$$

Note the presence in the last expression of two efficiency wedges, p^* and w^*. Productive efficiency and our symmetry assumptions require that all forms of specialized labor be employed at the same rate, and that each intermediate good producer use an equal amount of resources. In this case, $p^* = w^* = 1$. However, the presence of wage and price frictions implies that one or both of these conditions may not be satisfied. In this case, p^* and/or w^* are less than unity. In this sense, the standard sticky price framework that we adopt here has the potential to provide the 'theory of TFP' called for by, among others, Chari, Kehoe and McGrattan ( ). Unfortunately, the evidence so far is that the sticky price mechanism is unlikely to provide a basis for a quantitatively successful theory of TFP. This can be seen in two ways. Tak Yun ( ) showed that because we adopt assumptions which have the implication that $p^* = w^* = 1$ in steady state, it follows that, to a first approximation, this is true near steady state too.[8] Of course, the sort of shocks experienced in the Great Depression are hardly local deviations from a steady state. Still, for plausible parameter values deviations must be truly enormous to produce much of a fall in TFP. Consider the following simple example. Suppose final goods use intermediate inputs from just two types of sectors, Y^1 and Y^2, according to the following production function:

$$Y = \left[\frac{1}{2}\left(Y^1\right)^{\frac{1}{\lambda_f}} + \frac{1}{2}\left(Y^2\right)^{\frac{1}{\lambda_f}}\right]^{\lambda_f}$$

A large number for λ_f is 1.4. This implies a markup of 40 percent. Consider two scenarios. In each case, the same amount of resources are used. In one, $Y^1 = Y^2 = 1$. In this case, obviously, $Y = 1$. In the other there is an enormous deviation from equality of inputs: $Y^1 = 0.5$ and $Y^2 = 1.5$. Then,

$$\begin{aligned} &= \left[\frac{1}{2}(1.5)^{\frac{1}{1.4}} + \frac{1}{2}(0.5)^{\frac{1}{1.4}}\right]^{1.4} \\ &= 0.962, \end{aligned}$$

implying only a 4 percent reduction in efficiency.

[8] The assumptions which have this implication concern the prices and wages set by firms and households which do not have the opportunity to reoptimize. The crucial assumption in the case of firms is that their price is indexed to past inflation. In the case of wages it is crucial that the wage be indexed to past inflation *and* that it be indexed to aggregate productivity. Any deviation from these assumptions, and there will be dispersion in wages and/or prices across agents in steady state. This will have numerous effects on the steady state. First, the expressions for aggregate inflation and wages change in basic ways, by including additional variables. Second, the efficiency expressions, p^* and w^*, will deviate from unity and be quided by their own laws of motion over time. These are substantial qualitative changes. We suspect that they do not represent substantial quantitative changes, however.

A more substantial drop in efficiency could be had by setting λ_f to a higher number, say 4. Of course, the monopolistic competition assumption would not be so plausible in this case, because it implies a markup of 300 percent. But, we could assume that intermediate good producers cannot charge a price above marginal cost because they are surrounded by a competitive fringe. In this case, $Y = 0.90$. It is not clear whether even this 10 percent drop in efficiency is enough, given the enormous misallocation of resources in the example. In addition, note that the swing in relative prices associated with such a large deviation from efficiency when substitutability is so low is quite large. In particular, $P_1/P_2 = (Y_2/Y_1)^{\left[(\lambda_f - 1)/\lambda_f\right]}$, which is 0.44 in this case.

In any case, from here on we set $p^* = w^* = 1$, since this is correct to a first order approximation. To complete our discussion, final goods are allocated to monitoring for banks, utilization costs of capital, last meals of entrepreneurs slated for death, government consumption, household consumption and investment. So, the goods market clearing condition is:

$$\begin{aligned} & \mu \int_0^{\bar{\omega}_t} \omega dF(\omega) \left(1 + R^k\right) Q_{\bar{K}',t-1} \bar{K} + a(u)\bar{K} + \Theta(1-\gamma) v_t z_t + G_t + C_t + I_t \\ \leq & \left[z^{1-\alpha} \epsilon \left(\nu K\right)^{\alpha} \left(\nu L\right)^{1-\alpha} - z\phi\right], \end{aligned} \tag{2.40}$$

Here, government consumption is modeled as in Christiano and Eichenbaum (1992):

$$G = zg,$$

where g is an exogenous process.

3. Model Calibration

We adopted the following parameter values: $\mu = 0.06$, $x = 0.015$, $\mu_z = 1.015\hat{}.25$, $\lambda_f = 1.2$, $\lambda_w = 1.05$, $\alpha = 0.36$, $\psi_k = 0.5$, $\psi_l = 1$, $\delta = 0.02$, $\tau^k = .28$, $\tau^l = 0.25$, $\tau^c = 0.25$, $\gamma = 1 - .04$, $\tau^T = \tau^D = 0$, $\sigma_L = 1$, $\zeta = 1$, $\theta = 0.8$, $v = 0.005$, $\beta = 1.03^{-.25}$, $\xi_w = 0.7$, $\xi_p = 0.5$, $\sigma_a = 0.01$, $\sigma_q = 0.3$, $b = 0.63$, $w^e = 0.15$, $m = 0.8$, $\nu = 0.98$, $S'' = 7.69$, $\tau = 0.025$, $\psi_L = 0.01$, $\xi = 0.65$, $x^b = 9.33$, $\varepsilon = 1$.

Some steady state properties of the model are reported in Table 1:

Table 1: Properties of the Model

$\frac{k}{y}$	8.14
$\frac{i}{y}$	0.19
$\frac{c}{y}$	0.62
$\frac{g}{y}$	0.18
r^k	0.045
Bankruptcy Rate (per quarter)	3%
$\frac{N}{K-N}$ ('Equity to Debt')	1.29
$\frac{W^e}{py}$	0.06
Fraction of Goods Output Devoted to Monitoring	0.006
Fraction of Aggregate Labor and Capital in Banking	2%
Inflation	4.6%

The steady state table of bank assets and liabilities is reported in Table 2, and various steady state interest rates are reported in Table 3.

Responses of model variables to a monetary policy shock are reported in Figures 1 and 2. The money shock takes the form of an increase in the money growth rate from a steady state value of 0.015 (6 percent per year) to 0.0167 (6.68 percent per year) in the impact period. Responses of model variables to a drop in γ by 0.01 from 0.97 to 0.96 with persistence 0.90 are reported in Figures 3 and 4. In Figures 1 and 3, interest rates are reported at annual percent rates. Other variables are reported as percent deviations from steady state, in percent terms. That is, 0.001 means 'one-tenths of one percent'. Figures 2 and 4, depict ratios.

Consider the responses to a monetary policy shock first. Qualitatively, interest rates drop and output rises. There is substantial inertia in the inflation response and persistence in the output response. The response of employment, capacity utilization, labor, output, consumption and investment are all hump-shaped. These match, at a qualitative level, the responses reported in ACEL. Also, the currency to deposit ratio rises, and velocity generally falls. The bankruptcy rate falls, the price of capital is high an falling, after one period the rate of return on capital is low (presumably, due to the capital losses), net worth is high (presumably because of the high price level on capital). These responses seem reasonable. However, the responses that overlap with those reported in ACEL do not do well quantitatively. Interest rate effects are too small, and this may account for the fact that the output effect is too small too. Figure 2 shows that when the money supply is increased, some of it goes into excess reserves, while some of it goes towards an increase in working capital loans. Entrepreneurial loans fall, presumably because of the low rate of return on capital.

Now consider the responses to a bad net worth shock. After this shock, investment, output, employment, inflation, interest rates, and the capital stock fall. Bankruptcies rise. These responses seem reasonable. However, there is a rise in consumption. This presumably reflects a relative shift in wealth towards households.

4. Analysis of the Great Depression

Here, we will report a simulation of the Great Depression. We will then do a counterfactual simulation in which the Fed prevents M1 from falling.

5. Conclusion

6. Appendix A: Nonstochastic Steady State for the Model

We now develop equations for the steady state of our benchmark model. For purposes of these calculations, the exogenously set variables are:

$$\tau^l,\ \tau^c,\ \beta,\ F(\bar{\omega}),\ \mu,\ x,\ \mu_z,\ \lambda_f,\ \lambda_w,\ \alpha,\ \psi_k,\ \psi_l,\ \delta,\ \upsilon,$$
$$\tau^k,\ \gamma,\ \tau,\ \tau^T,\ \tau^D,\ \sigma_L,\ \zeta,\ \sigma_q,\ \theta,\ \upsilon,\ w^e,\ \nu^l,\ \nu^k,\ m,\ \eta_g,\ r^k$$

The variables to be solved for are

$$q,\ \pi,\ R^e,\ R_a,\ h_{e^r},\ R,\ R^k,\ \bar{\omega},\ k,\ n,\ i,\ w,\ l,\ c,\ u_c^z,\ m^b,\ \lambda_z,\ \psi_L,\ e_z^r,\ e_v,\ a^b x^b,\ \xi,\ h_{K^b},\ y,\ g$$

The equations available for solving for these unknowns are summarized below. The first three variables are trivial functions of the structural parameters, and from here on we treat them as known. There remain 22 unknowns. Below, we have 22 equations that can be used to solve for them.

The algorithm proceeds as follows. Solve for R_a using (6.17); h_{e^r} using (6.12).

We now compute R to enforce (6.8). This equation is a nonlinear function of R. For a given R, evaluate (6.8) as follows. Solve for R^k using (6.4); solve for $\bar{\omega}$ using (6.5); solve for k and n using (6.6) and (6.7); solve for i using (6.3); solve for w using (6.1); solve for l using (6.2); solve for c using (6.20); solve (6.22) and (6.23) for g and y; solve for u_c^z using (6.18); solve for m^b and λ_z using (6.15) and (6.16); solve (6.19) for ψ_L; solve for e_z^r using (6.14); solve ξ from (6.13); solve e_v from (6.11); solve $a^b x^b$ from (6.10); h_{K^b} from (6.9). Vary R until (6.8) is satisfied. In these calculations, all variables must be positive, and:

$$0 \leq m \leq 1 + x,\ 0 \leq \xi \leq 1,\ \lambda_z > 0,\ k > n > 0.$$

6.1. Firm Sector

From the firm sector, and the assumption that there are no price distortions in a steady state, we have

$$s = \frac{1}{\lambda_f}.$$

Also, evaluating (2.3) in steady state:

$$\frac{1}{\lambda_f} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r^k \left[1 + \psi_k R\right]\right)^{\alpha} \left(w \left[1 + \psi_l R\right]\right)^{1-\alpha}, \tag{6.1}$$

Combining (2.3) and (2.4):

$$\frac{r^k \left[1 + \psi_k R\right]}{w \left[1 + \psi_l R\right]} = \frac{\alpha}{1-\alpha} \frac{\mu_z l}{\bar{k}} \tag{6.2}$$

6.2. Capital Producers

From the capital producers,

$$\lambda_{zt} q_t F_{1,t} - \lambda_{z,t} + \frac{\beta}{\mu_{z,t+1}} \lambda_{z,t+1} q_{t+1} F_{2,t+1} = 0$$

or, since $F_{1,t} = 1$ and $F_{2,t} = 0$,

$$q = 1.$$

Also,

$$\bar{k}_{t+1} = (1-\delta) \frac{1}{\mu_{z,t}} \bar{k}_t + \left[1 - S\left(\frac{i_t \mu_{z,t}}{i_{t-1}}\right)\right] i_t,$$

so that in steady state, when $S = 0$,

$$\frac{i}{\bar{k}} = 1 - \frac{1-\delta}{\mu_z}. \tag{6.3}$$

6.3. Entrepreneurs

From the entrepreneurs:

$$r^k = a'.$$

Also,

$$u = 1.$$

The after tax rate of return on capital, in steady state, is:

$$R^k = \left[(1-\tau^k)r^k + (1-\delta)\right]\pi + \tau^k\delta - 1 \tag{6.4}$$

Conditional on a value for R^k, R^e, the steady state value for $\bar{\omega}$ may be found using the following equation:

$$[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R^e} + \frac{1}{1-\mu\bar{\omega}h(\bar{\omega})}\left[\frac{1+R^k}{1+R^e}\left(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right) - 1\right] = 0, \tag{6.5}$$

where the hazard rate, h, is defined as follows:

$$h(\omega) = \frac{F'(\omega)}{1-F(\omega)}.$$

This equation has two additional parameters, the two parameters of the lognormal distribution, F. These two parameters, however, are pinned down by the assumption, $E\omega = 1$, and the fact that we specify $F(\bar{\omega})$ exogenously. With these conditions, the above equation forms a basis for computing $\bar{\omega}$. Note here that when $\mu = 0$,(6.5) reduces to $R^k = R^e$. Then, combining (6.4) with the first order condition for time deposits, we end up with the conclusion that r^k is determined as it is in the neoclassical growth model.

Conditional on $F(\bar{\omega})$ and $\bar{\omega}$, we may solve for k using (2.20):

$$\frac{\bar{k}}{n} = \frac{1}{1-\frac{1+R^k}{1+R^e}\left(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right)}. \tag{6.6}$$

The law of motion for net worth implies the following relation in steady state:

$$n = \frac{\frac{\gamma}{\pi\mu_z}\left[R^k - R^e - \mu G(\bar{\omega})\left(1+R^k\right)\right]\bar{k} + w^e}{1-\gamma\left(\frac{1+R^e}{\pi}\right)\frac{1}{\mu_z}}. \tag{6.7}$$

6.4. Banks

The first order condition associated with the bank's capital decision is:

$$\left(1+\psi_k R\right)r^k = \frac{Rh_{K^b}}{1+\tau h_{e^r}}. \tag{6.8}$$

The first order condition for labor is redundant given (6.1), (6.2), and (6.8), and so we do not list it here. In the preceding equations,

$$h_{K^b} = \alpha\xi a^b x^b \left(e_v\right)^{1-\xi}\left(\frac{\mu_z l}{k}\right)^{1-\alpha}, \tag{6.9}$$

$$h_{e^r}=(1-\xi)\,a^b x^b\,(e_v)^{-\xi}\,, \tag{6.10}$$

and

$$e_v=\frac{(1-\tau)\,m^b\,(1-m+x)-\tau\left(\psi_l wl+\frac{1}{\mu_z}\psi_k r^k\bar{k}\right)}{\left(\frac{1}{\mu_z}(1-\nu^k)\bar{k}\right)^{\alpha}\left((1-\nu^l)l\right)^{1-\alpha}}. \tag{6.11}$$

Another efficiency condition for the banks is (2.31). Rewriting that expression, we obtain:

$$1+\frac{R}{R_a}=h_{e^r}\left[(1-\tau)\,\frac{R}{R_a}-\tau\right] \tag{6.12}$$

Substituting out for $a^b x^b\,(e_v)^{-\xi}$ from (6.10) into the scaled production function, we obtain:

$$\frac{h_{e^r}}{(1-\xi)}e_z^r=m^b\,(1-m+x)+\psi_l wl+\psi_k r^k\frac{\bar{k}}{\mu_z}, \tag{6.13}$$

where

$$e_z^r=(1-\tau)m^b\,(1-m+x)-\tau\left(\psi_l wl+\psi_k r^k\frac{\bar{k}}{\mu_z}\right). \tag{6.14}$$

6.5. Households

The first order condition for T :

$$1+\left(1-\tau^T\right)R^e=\frac{\mu_z\pi}{\beta}$$

The first order condition for M :

$$\begin{aligned}\upsilon\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}&[\frac{\theta}{m}-\frac{1-\theta}{1-m+x}]\left(m^b\right)^{\sigma_q-2}\\&-\lambda_z\left(1-\tau^D\right)R_a=0\end{aligned} \tag{6.15}$$

The first order condition for M^b

$$\begin{aligned}\upsilon\,(1-\theta)\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}&\left(\frac{1}{m^b}\right)^{2-\sigma_q}\left(\frac{1}{1-m+x}\right)\\&=\pi\lambda_z\left\{\frac{\mu_z}{\beta}-\frac{\left[1+\left(1-\tau_t^D\right)R_a\right]}{\pi}\right\}\end{aligned}$$

Under the ACEL specification of preferences, c in the previous two expressions are replaced by unity. The first order condition for consumption corresponds to:

$$u_c^z - (1+\tau^c)\,\lambda_z = \upsilon c^{-1}\left(m^b\right)^{\sigma_q - 1}\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}, \tag{6.16}$$

Under the ACEL specification, the expression to the right of the equality in (6.16) is replaced by zero.

Taking the ratio of (6.15) and the first order conditions for m^b, and rearranging, we obtain:

$$\begin{aligned} R_a &= \frac{\frac{(1-m+x)}{m}\theta - (1-\theta)\left(\frac{\pi\mu_z}{\beta} - 1\right)}{\frac{(1-m+x)}{m}\theta}\frac{}{(1-\tau^D)} \\ &= \left[1 - \frac{m}{1-m+x}\frac{(1-\theta)}{\theta}\right]\frac{1-\tau^T}{1-\tau^D}R^e \end{aligned} \tag{6.17}$$

The marginal utility of consumption is:

$$cu_c^z = \frac{\mu_{z,}}{\mu_z - b} - b\beta\frac{1}{\mu_z - b} = \frac{\mu_z - b\beta}{\mu_z - b} \tag{6.18}$$

The first order condition for households setting wages is:

$$w\frac{\lambda_z(1-\tau^l)}{\lambda_w} = \zeta\psi_L l^{\sigma_L} \tag{6.19}$$

6.6. Monetary Authority

$$\pi = \frac{(1+x)}{\mu_z}.$$

6.7. Resource Constraint

After substituting out for the fixed cost in the resource constraint using the restriction that firm profits are zero in steady state, and using $g = \eta_g y$, we obtain:

$$c = (1-\eta_g)\left[\frac{1}{\lambda_f}\left(\frac{1}{\mu_z}\nu^k\bar{k}\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha} - \mu G(\bar{\omega})(1+R^k)\frac{k}{\mu_z\pi}\right] - i. \tag{6.20}$$

Here, we have made use of the facts,

$$y = \frac{1}{\lambda_f}\left(\frac{1}{\mu_z}\nu^k\bar{k}\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha} - \mu G(\bar{\omega})(1+R^k)\frac{k}{\mu_z\pi},$$

and $g = \eta_g y$, so that $c = (1-\eta_g)y - i$.

We now develop the condition on ϕ to assure that intermediate good firm profits in steady state are zero. If we loosely write their production function as $F - \phi z$, then the total cost of labor and capital inputs to the firm are sF, where s is real marginal cost, or the (reciprocal of the) markup (at least, in steady state when aggregate price and the individual intermediate good firm prices coincide). We want sF to exhaust total revenues, $F - \phi z$, i.e., we want $sF = F - \phi z$, or, $\phi = F(1-s)/z = (F/z)(1 - 1/\lambda_f)$, or

$$\phi = \left(\frac{z_{t-1}\nu^k K_t}{z_{t-1}z_t}\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha}(1-\frac{1}{\lambda_f}) = \left(\frac{\nu^k k}{\mu_z}\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha}(1-\frac{1}{\lambda_f}) \tag{6.21}$$

We obtain (6.20) by substituting from the last equation into the resource constraint:

$$y = \left(\frac{1}{\mu_z}\nu^k\bar{k}\right)^{\alpha}\left(\nu^l l\right)^{1-\alpha} - \phi - \mu G(\bar{\omega})(1+R^k)\frac{k}{\mu_z\pi}, \tag{6.22}$$

We obtain g from output from:

$$g = \eta_g y. \tag{6.23}$$

7. Appendix B: Linearly Approximating the Model Dynamics

There are 24 endogenous variables whose values are determined at time t. We load them into a vector, z_t. The elements in this vector are reported in the following table. In addition, there is an indication about which shocks the variable depends on. If it depends on the realization of all period t shocks (i.e., the information set, Ω_t^{μ}, then we indicate a, for 'all'. If it depends only on the realization of the current period non-financial shocks, Ω_t, then we indicate p, for 'partial'. The table also indicates the information associated with each of the 24 equations used to solve the model. These equations are collected below from the preceding discussion. Note that the number of equations and elements in z_t is the same. Note also, in each case, the third and fourth columns always have the same entry. In several cases, z_t contains variables dated $t+1$. In the case of $\hat{\bar{k}}_{t+1}$, for example, the presence of a p in the third column indicates that $\hat{\bar{k}}_{t+1}$ is a function of the realization of the period t non-financial shocks, and is not a function of the realization of period t financial shocks, or later period shocks. In the case of $\hat{R}^e_{t+1}$, the presence of an a indicates that this variable is a function of all period t shocks, but not of any period $t+1$ shocks.

$$
\begin{array}{cccc}
 & z_t & \text{information, } z & \text{information, equation} \\
1 & \hat{\pi}_t & p & p \\
2 & \hat{s}_t & a & a \\
3 & \hat{r}^k_t & a & a \\
4 & \hat{\imath}_t & p & p \\
5 & \hat{u}_t & p & p \\
6 & \hat{\bar{\omega}}_t & a & a \\
7 & \hat{R}^k_t & a & a \\
8 & \hat{n}_{t+1} & a & a \\
9 & \hat{q}_t & a & a \\
10 & \hat{\nu}^l_t & a & a \\
11 & \hat{e}_{\nu,t} & a & a \\
12 & \hat{m}^b_t & a & a \\
13 & \hat{R}_t & a & a \\
14 & \hat{u}^z_{c,t} & a & a \\
15 & \hat{\lambda}_{z,t} & a & a \\
16 & \hat{m}_t & a & a \\
17 & \hat{R}_{a,t} & a & a \\
18 & \hat{c}_t & p & p \\
19 & \hat{w}_t & p & p \\
20 & \hat{l}_t & a & a \\
21 & \hat{\bar{k}}_{t+1} & p & p \\
22 & \hat{R}^e_{t+1} & a & a \\
23 & \hat{x}_t & a & a
\end{array}
\qquad (7.1)
$$

The last of these variables is money growth, $\hat{x}_t$. As we show below, this is simply a trivial function of the underlying shocks. In addition, recall (**??**), in which the 10^{th} and 11^{th} variables are the same. A combination of the efficiency conditions for labor and capital in the firm sector, equations (1) and (2) below, are redundant with the efficiency conditions for labor and capital in the banking sector, (11) and (12). We deleted equation (11) below from our system.

In fact, we have 25 equations and unknowns in our model. The system we work with is one dimension less because we set $\Theta \equiv 0$, so that $\hat{v}_t$ disappears from the system. When we want $\Theta > 0$, we can get our 25^{th} equation by linearizing (2.21), and $\hat{v}_t$ is then our 25^{th} variable.

7.1. Firms

The inflation equation, when there is indexing to lagged inflation, is:

$$(1)\ E\left[\hat{\pi}_t - \frac{1}{1+\beta}\hat{\pi}_{t-1} - \frac{\beta}{1+\beta}\hat{\pi}_{t+1} - \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta)\,\xi_p}\left(\hat{s}_t + \hat{\lambda}_{f,t}\right)|\Omega_t\right] = 0$$

The linearized expression for marginal cost is:

$$(2)\ \alpha\hat{r}_t^k + \frac{\alpha\psi_k R}{1+\psi_k R}\hat{\psi}_{k,t} + (1-\alpha)\,\hat{w}_t + \frac{(1-\alpha)\,\psi_l R}{1+\psi_l R}\hat{\psi}_{l,t}$$
$$+\left[\frac{\alpha\psi_k R}{1+\psi_k R} + \frac{(1-\alpha)\,\psi_l R}{1+\psi_l R}\right]\hat{R}_t - \hat{\epsilon}_t - \hat{s}_t = 0$$

Another condition that marginal cost must satisfy is that it is equal to the marginal cost of one unit of capital services, divided by the marginal product of one unit of services. After linearization, this implies:

$$(3)\ \hat{r}_t^k + \frac{\psi_k R\left(\hat{\psi}_{k,t} + \hat{R}_t\right)}{1+\psi_k R} - \hat{\epsilon}_t - (1-\alpha)\left(\hat{\mu}_{z,t} + \hat{l}_t - \left[\hat{\bar{k}}_t + \hat{u}_t\right]\right) - \hat{s}_t = 0$$

7.2. Capital Producers

The 'Tobin's q' relation is:

$$(4)\ E\left\{\hat{q}_t - S''\mu_z^2(1+\beta)\hat{\imath}_t - S''\mu_z^2\hat{\mu}_{z,t} + S''\mu_z^2\hat{\imath}_{t-1} + \beta S''\mu_z^2\hat{\imath}_{t+1} + \beta S''\mu_z^2\hat{\mu}_{z,t+1}|\Omega_t\right\} = 0$$

7.3. Entrepreneurs

The variable utilization equation is

$$(5)\ E\left[\hat{r}_t^k - \sigma_a\hat{u}_t|\Omega_t\right] = 0,$$

where $\hat{r}_t^k$ denotes the rental rate on capital. The date t standard debt contract has two parameters, the amount borrowed and $\hat{\bar{\omega}}_{t+1}$. The former is not a function of the period $t+1$ state of nature, and the latter is not. Two equations characterize the efficient contract. The first order condition associated with the quantity loaned by banks in period t in the optimal contract is:

$$(6)\ E\{\lambda\left(\frac{R^k\hat{R}_{t+1}^k}{1+R^k} - \frac{R^e\hat{R}_{t+1}^e}{1+R^e}\right)$$
$$-\left[1-\Gamma(\bar{\omega})\right]\frac{1+R^k}{1+R^e}\left[\frac{\Gamma''(\bar{\omega})\bar{\omega}}{\Gamma'(\bar{\omega})} - \frac{\lambda\left[\Gamma''(\bar{\omega}) - \mu G''(\bar{\omega})\right]\bar{\omega}}{\Gamma'(\bar{\omega})}\right]\hat{\bar{\omega}}_{t+1}|\Omega_t^\mu\} \quad = \quad 0.$$

Note that this is not a function of the period $t+1$ uncertainty. Also, note that when $\mu = 0$, so that $\lambda = 1$, then this equation simply reduces to $E\left[\hat{R}^k_{t+1}|\Omega^\mu_t\right] = \hat{R}^e_{t+1}$. The linearized zero profit condition is:

$$
\begin{aligned}
(7)\quad & \left(\tfrac{\bar{k}}{n}-1\right)\tfrac{R^k}{1+R^k}\hat{R}^k_t - \left(\tfrac{\bar{k}}{n}-1\right)\tfrac{R^e}{1+R^e}\hat{R}^e_t + \left(\tfrac{\bar{k}}{n}-1\right)\tfrac{(\Gamma'(\bar{\omega})-\mu G'(\bar{\omega}))}{(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))}\bar{\omega}\hat{\bar{\omega}}_t \\
& - \left(\hat{q}_{t-1} + \hat{\bar{k}}_t - \hat{n}_t\right) = \qquad 0.
\end{aligned}
$$

The law of motion for net worth is:

$$
(8)\quad -\hat{n}_{t+1} + a_0\hat{R}^k_t + a_1\hat{R}^e_t + a_2\hat{\bar{k}}_t + a_3\hat{w}^e_t + a_4\hat{\gamma}_t + a_5\hat{\pi}_t + a_6\hat{\mu}_{z,t} + a_7\hat{q}_{t-1} + a_8\hat{\bar{\omega}}_t + a_9\hat{n}_t = 0
$$

The definition of the rate of return on capital is:

$$
(9)\ \hat{R}^k_{t+1} - \frac{(1-\tau^k)r^k + (1-\delta)q}{R^k q}\pi\left[\frac{\left(1-\tau^k\right)r^k\hat{r}^k_{t+1} - \tau^k r^k\hat{\tau}^k_t + (1-\delta)q\hat{q}_{t+1}}{(1-\tau^k)r^k + (1-\delta)q} + \hat{\pi}_{t+1} - \hat{q}_t\right] - \frac{\delta\tau^k\hat{\tau}^k_t}{R^k}
$$

7.4. Banking Sector

In the equations for the banking sector, it is capital services, k_t, which appears, not the physical stock of capital, $\bar{k}_t$. The link between them is:

$$
\hat{k}_t = \hat{\bar{k}}_t + \hat{u}_t.
$$

An expression for the ratio of excess reserves to value added in the banking sector is:

$$
\begin{aligned}
&(10)\ \ -\hat{e}_{v,t} + n_\tau\hat{\tau}_t + n_{m^b}\hat{m}^b_t + n_m\hat{m}_t + n_x\hat{x}_t + n_{\psi_l}\hat{\psi}_{l,t} \\
&+n_{\psi k}\hat{\psi}_{k,t} + (n_k - d_k)\left[\hat{\bar{k}}_t + \hat{u}_t\right] + n_{r^k}\hat{r}^k_t + n_w\hat{w}_t \\
&+(n_l - d_l)\,\hat{l}_t + (n_{\mu_z} - d_{\mu_z})\,\hat{\mu}_{z,t} - d_{\nu^k}\hat{\nu}^k_t - d_{\nu^l}\hat{\nu}^l_t = 0
\end{aligned}
$$

where m^b_t is the scaled monetary base, m_t is the currency-to-base ratio, x_t is the growth rate of the base

$$
\begin{aligned}
n_\tau &= \frac{-\tau m^b(1-m+x) - \tau\left(\psi_l wl + \frac{1}{\mu_z}\psi_k r^k k\right) - \tau\frac{1}{\mu_z}\psi_k r^k k}{n}, \\
n &= (1-\tau)\,m^b(1-m+x) - \tau\left(\psi_l wl + \frac{1}{\mu_z}\psi_k r^k k\right), \\
n_{m^b} &= (1-\tau)\,m^b(1-m+x)/n
\end{aligned}
$$

$$\begin{aligned}
n_m &= -(1-\tau)\, m^b m/n \\
n_x &= (1-\tau)\, m^b x/n \\
n_{\psi_l} &= n_w = n_l = -\tau \psi_l w l/n \\
n_{\psi_k} &= n_{r^k} = n_k = -\tau \frac{1}{\mu_z} \psi_k r^k k/n \\
n_{\mu_z} &= \tau \frac{1}{\mu_z} \psi_k r^k k/n
\end{aligned}$$

and

$$\begin{aligned}
d &= \left(\frac{1}{\mu_z}(1-\nu^k)k\right)^{\alpha} \left((1-\nu^l)l\right)^{1-\alpha} \\
d_{\mu_z} &= \frac{-\alpha \left(\frac{1}{\mu_z}(1-\nu^k)k\right)^{\alpha} \left((1-\nu^l)l\right)^{1-\alpha}}{\left(\frac{1}{\mu_z}(1-\nu^k)k\right)^{\alpha} \left((1-\nu^l)l\right)^{1-\alpha}} = -\alpha \\
d_k &= \alpha \\
d_{\nu^k} &= -\alpha \frac{\nu^k}{1-\nu^k} \\
d_l &= 1-\alpha \\
d_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l}
\end{aligned}$$

The first order condition for capital in the banking sector is:

$$\begin{aligned}
0 &= k_R \hat{R}_t + k_\xi \hat{\xi}_t - \hat{r}_t^k + k_x \hat{x}_t^b + k_e \hat{e}_{v,t} + k_\mu \hat{\mu}_{z,t} \\
&\quad + k_{\nu^l} \hat{\nu}_t^l + k_{\nu^k} \hat{\nu}_t^k + k_l \hat{l}_t + k_k \left[\hat{\bar{k}}_t + \hat{u}_t\right] + k_\tau \hat{\tau}_t + k_{\psi_k} \hat{\psi}_{k,t}
\end{aligned}$$

$$\begin{aligned}
k_R &= \left[1 - \frac{\psi_k R}{1+\psi_k R}\right],\ k_\xi = 1 - \log(e_v)\,\xi + \frac{\tau h_{e^r}\left[\frac{1}{1-\xi} + \log(e_v)\right]\xi}{1+\tau h_{e^r}} \\
k_x &= \frac{1}{1+\tau h_{e^r}},\ k_e = 1 - \xi + \frac{\tau h_{e^r} \xi}{1+\tau h_{e^r}},\ k_\mu = (1-\alpha) \\
k_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l},\ k_{\nu^k} = (1-\alpha)\frac{\nu^k}{1-\nu^k},\ k_l = (1-\alpha),\ k_k = -(1-\alpha) \\
k_\tau &= -\frac{\tau h_{e^r}}{1+\tau h_{e^r}},\ k_{\psi_k} = -\frac{\psi_k R}{1+\psi_k R}.
\end{aligned}$$

The latter equation was deleted from our system, because it is redundant given the two firm Euler equations and the following equation.

The first order condition for labor in the banking sector is:

$$
\begin{aligned}
(11)\ 0 &= l_R\hat{R}_t + l_\xi\hat{\xi}_t - \hat{w}_t + l_x\hat{x}_t^b + l_e\hat{e}_{v,t} + l_\mu\hat{\mu}_{z,t} \\
&\quad + l_{\nu^l}\hat{\nu}_t^l + l_{\nu^k}\hat{\nu}_t^k + l_l\hat{l}_t + l_k\left[\hat{\bar{k}}_t + \hat{u}_t\right] + l_\tau\hat{\tau}_t + l_{\psi_l}\hat{\psi}_{l,t},
\end{aligned}
$$

where

$$
\begin{aligned}
l_i &= k_i \text{ for all } i, \text{ except} \\
l_R &= \left[1 - \frac{\psi_l R}{1+\psi_l R}\right],\ l_{\psi l} = -\frac{\psi_l R}{1+\psi_l R} \\
l_\mu &= k_\mu - 1,\ l_{\nu^l} = k_{\nu^l} + \frac{\nu^l}{1-\nu^l},\ l_l = k_l - 1, \\
l_{\nu^k} &= k_{\nu^k} - \frac{\nu^k}{1-\nu^k},\ l_k = k_k + 1.
\end{aligned}
$$

The production function for deposits is:

$$
\begin{aligned}
(12)\qquad & \hat{x}_t^b - \xi\hat{e}_{v,t} - \log\left(e_{v,t}\right)\xi\hat{\xi}_t - \frac{\tau\left(m_1+m_2\right)}{(1-\tau)m_1 - \tau m_2}\hat{\tau}_t \\
= &\left[\frac{m_1}{m_1+m_2} - \frac{(1-\tau)m_1}{(1-\tau)m_1 - \tau m_2}\right]\left[\hat{m}_t^b + \frac{-m\hat{m}_t + x\hat{x}_t}{1-m+x}\right] \\
& + \left[\frac{m_2}{m_1+m_2} + \frac{\tau m_2}{(1-\tau)m_1 - \tau m_2}\right] \\
& \times [\frac{\psi_l w l}{\psi_l w l + \psi_k r^k k/\mu_z}\left(\hat{\psi}_{l,t} + \hat{w}_t + \hat{l}_t\right) + \frac{\psi_k r^k k/\mu_z}{\psi_l w l + \psi_k r^k k/\mu_z}\left(\hat{\psi}_{k,t} + \hat{r}_t^k + \hat{k}_t - \hat{\mu}_{zt}\right)].
\end{aligned}
$$

The expression for $\hat{R}_{at}$ is:

$$
\begin{aligned}
&(13)\ \hat{R}_{at} - \left[\frac{h_{e^r} - \tau h_{e^r}}{(1-\tau)\,h_{e^r} - 1} - \frac{\tau h_{e^r}}{\tau h_{e^r} + 1}\right]\left[-\left(\frac{1}{1-\xi} + \log\left(e_v\right)\right)\xi\hat{\xi}_t + \hat{x}_t^b - \xi\hat{e}_{v,t}\right] \\
&+ \left[\frac{\tau h_{e^r}}{(1-\tau)\,h_{e^r} - 1} + \frac{\tau h_{e^r}}{\tau h_{e^r} + 1}\right]\hat{\tau}_t - \hat{R}_t = 0
\end{aligned}
$$

7.5. Household Sector

The definition of u_c^z is:

$$(14)\ E\{u_c^z\hat{u}_{c,t}^z - \left[\frac{\mu_z}{c(\mu_z - b)} - \frac{\mu_z^2 c}{c^2(\mu_z - b)^2}\right]\hat{\mu}_{z,t} - b\beta\frac{\mu_z c}{c^2(\mu_z - b)^2}\hat{\mu}_{z,t+1}$$

$$+\frac{\mu_z^2 + \beta b^2}{c^2(\mu_z - b)^2}c\hat{c}_t - \frac{b\beta\mu_z}{c^2(\mu_z - b)^2}c\hat{c}_{t+1} - \frac{b\mu_z}{c^2(\mu_z - b)^2}c\hat{c}_{t-1}|\Omega_t^\mu\} \quad = \quad 0.$$

The household's first order condition for time deposits is:

$$(15)\ E\left\{-\hat{\lambda}_{z,t} + \hat{\lambda}_{z,t+1} - \hat{\mu}_{z,t+1} - \hat{\pi}_{t+1} - \frac{R^e\tau^T}{1 + (1-\tau^T)R^e}\hat{\tau}_{t+1}^T + \frac{R^e\left(1-\tau^T\right)}{1+(1-\tau^T)R^e}\hat{R}_{t+1}^e|\Omega_t^\mu\right\} = 0.$$

The household's first order condition for capital is:

$$(24)\ E\left\{-\hat{\lambda}_{zt} + \left[\frac{R^k}{1+R^k}\hat{R}_{t+1}^k + \hat{\lambda}_{z,t+1} - \hat{\pi}_{t+1} - \hat{\mu}_{z,t+1}\right]|\Omega_t\right\}$$

The first order condition for currency, M_t :

$$(16)\ \hat{\upsilon}_t + (1-\sigma_q)\hat{c}_t + \left[-(1-\sigma_q)\left(\theta - (1-\theta)\frac{m}{1-m+x}\right) - \frac{\frac{\theta}{m} + \frac{1-\theta}{(1-m+x)^2}m}{\frac{\theta}{m} - \frac{1-\theta}{1-m+x}}\right]\hat{m}_t$$

$$-\left[\frac{(1-\sigma_q)(1-\theta)x}{1-m+x} - \frac{\frac{1-\theta}{(1-m+x)^2}x}{\frac{\theta}{m} - \frac{1-\theta}{1-m+x}}\right]\hat{x}_t$$

$$+\left[-(1-\sigma_q)(\log(m) - \log(1-m+x)) + \frac{1+x}{\theta(1+x)-m}\right]\theta\hat{\theta}_t$$

$$-(2-\sigma_q)\hat{m}_t^b - \left[\hat{\lambda}_{z,t} + \frac{-\tau^D}{1-\tau^D}\hat{\tau}_t^D + \hat{R}_{a,t}\right] \quad = \quad 0$$

The household's first order condition for currency, M_{t+1}^b, is:

$$(17)\ E\{\frac{\beta}{\pi\mu_z}\upsilon(1-\theta)\left[c\left(\frac{1}{m}\right)^\theta\right]^{1-\sigma_q}\left(\frac{1}{1-m+x}\right)^{(1-\theta)(1-\sigma_q)+1}\left(\frac{1}{m^b}\right)^{2-\sigma_q}$$

$$\times\{\hat{\upsilon}_{t+1} - \frac{\theta\hat{\theta}_{t+1}}{1-\theta} + (1-\sigma_q)\hat{c}_{t+1} - (1-\sigma_q)\log(m)\theta\hat{\theta}_{t+1} - \theta(1-\sigma_q)\hat{m}_{t+1}$$

$$
\begin{aligned}
&-\left[(1-\theta)\left(1-\sigma_q\right)+1\right]\left(\frac{1}{1-m+x}\right)\left[x\hat{x}_{t+1}-m\hat{m}_{t+1}\right] \\
&+\left(1-\sigma_q\right)\log\left(1-m+x\right)\theta\hat{\theta}_{t+1}-\left(2-\sigma_q\right)\hat{m}^b_{t+1}\} \\
&+\frac{\beta}{\pi\mu_z}\lambda_z\left[1+\left(1-\tau^D\right)R_a\right]\hat{\lambda}_{z,t+1} \\
&+\frac{\beta}{\pi\mu_z}\lambda_z\left[\left(1-\tau^D\right)R_a\hat{R}_{a,t+1}-\tau^D R_a\hat{\tau}^D_{t+1}\right]-\lambda_z\left[\hat{\lambda}_{zt}+\hat{\pi}_{t+1}+\hat{\mu}_{z,t+1}\right]|\Omega^\mu_t\} \\
&= 0.
\end{aligned}
$$

The first order condition for consumption is:

$$
\begin{aligned}
&(18)\ E\{u^z_c\hat{u}^z_{c,t}-\upsilon c^{-\sigma_q}\left[\frac{1}{m^b}\left(\frac{1}{m}\right)^\theta\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q} \\
&\times[\hat{\upsilon}_t-\sigma_q\hat{c}_t+\left(1-\sigma_q\right)\left(-\hat{m}^b_t-\theta_t\hat{m}_t-\left(1-\theta_t\right)\left(\frac{-m}{1-m+x}\hat{m}_t+\frac{x}{1-m+x}\hat{x}_t\right)\right) \\
&+\left(1-\sigma_q\right)\left[\log\left(\frac{1}{m}\right)-\log\left(\frac{1}{1-m+x}\right)\right]\theta\hat{\theta}_t] \\
&-\left(1+\tau^c\right)\lambda_z\left[\frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t+\hat{\lambda}_{z,t}\right]|\Omega_t\}=0
\end{aligned}
$$

The reduced form wage equation is:

$$
(19)\ E\left\{\eta_0\hat{w}_{t-1}+\eta_1\hat{w}_t+\eta_2\hat{w}_{t+1}+\eta_{\bar{3}}\hat{\pi}_{t-1}+\eta_3\hat{\pi}_t+\eta_4\hat{\pi}_{t+1}+\eta_5\hat{l}_t+\eta_6\left[\hat{\lambda}_{z,t}-\frac{\tau^l}{1-\tau^l}\hat{\tau}^l_t\right]+\eta_7\hat{\zeta}_t|\Omega_t\right\}=0
$$

where

$$
\eta=\begin{pmatrix}
b_w\xi_w \\
-b_w\left(1+\beta\xi_w^2\right)+\sigma_L\lambda_w \\
\beta\xi_w b_w \\
b_w\xi_w \\
-\xi_w b_w\left(1+\beta\right) \\
b_w\beta\xi_w \\
-\sigma_L\left(1-\lambda_w\right) \\
1-\lambda_w \\
-\left(1-\lambda_w\right)
\end{pmatrix}=\begin{pmatrix}
\eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_{\bar{3}} \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7
\end{pmatrix}.
$$

7.6. Aggregate Restrictions

The resource constraint is:

$$(20)\ 0 = \quad d_y \left[\frac{G'(\bar{\omega})}{G(\bar{\omega})} \bar{\omega} \widehat{\bar{\omega}}_t + \frac{R^k}{1+R^k} \hat{R}_t^k + \hat{q}_{t-1} + \widehat{\bar{k}}_t - \hat{\mu}_{z,t} - \hat{\pi}_t \right] + u_y \hat{u}_t + g_y \hat{g}_t + c_y \hat{c}_t + \bar{k}_y \frac{i}{\bar{k}} \hat{\imath}_t$$
$$+\Theta(1-\gamma) v_y \hat{v}_t - \alpha \left(\hat{u}_t - \hat{\mu}_{z,t} + \widehat{\bar{k}}_t + \hat{\nu}_t^k \right) - (1-\alpha) \left(\hat{l}_t + \hat{\nu}_t^l \right) - \hat{\epsilon}_t$$

where

$$\bar{k}_y = \frac{\bar{k}}{y + \phi + d},$$

and the object in square brackets corresponds to the resources used up in monitoring.

$$(21)\ \widehat{\bar{k}}_{t+1} - \frac{1-\delta}{\mu_z} \left(\widehat{\bar{k}}_t - \hat{\mu}_{z,t} \right) - \frac{i}{\bar{k}} \hat{\imath}_t = 0.$$

Monetary policy is represented by:

$$(22)\ \hat{m}_{t-1}^b + \frac{x}{1+x} \hat{x}_{t-1} - \hat{\pi}_t - \hat{\mu}_{z,t} - \hat{m}_t^b = 0$$

7.7. Monetary Policy

Monetary policy has the following representation:

$$(23)\ \hat{x}_t = \sum_{i=1}^{p} x_{it},$$

where the x_{it}'s are functions of the underlying shocks.

7.8. Collecting the Equations

We can write the 24 equations listed above in matrix form as follows:

$$\mathcal{E}_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0,$$

where z_t is defined above and $\mathcal{E}_t$ is the expectation operator which takes into account the information set associated with each equation. Also, s_t is constructed from the vector of shocks, Ψ_t, that impact on agents' environment, and it has the following representation:

$$s_t = P s_{t-1} + \tilde{\varepsilon}_t. \tag{7.2}$$

We now discuss the construction of the elements, s_t and P, of this time series representation.

There are $N = 20$ basic exogenous shocks, ς_t, in the model:

$$\begin{aligned}&\hat{\lambda}_{f,t},\ \hat{\tau}_t,\ \hat{\psi}_{l,t},\ \hat{\psi}_{k,t},\ \hat{\xi}_t,\ \hat{x}_t^b,\ \hat{\tau}_t^T,\ \hat{\theta}_t,\ \hat{\tau}_t^D,\ \hat{\tau}_t^l,\\ &\hat{\tau}_t^k,\ \hat{\zeta}_t,\ \hat{g}_t,\ \hat{\upsilon}_t,\ \hat{w}_t^e,\ \hat{\mu}_{z,t},\ \hat{\gamma}_t,\ \hat{\epsilon}_t,\ \hat{x}_{pt},\ \hat{\tau}_t^C\end{aligned} \tag{7.3}$$

Here, λ_f is the steady state markup for intermediate good firms; τ is the reserve requirement for banks; ψ_l is the fraction of the wage bill that must be financed in advance; ψ_k is the fraction of the capital services bill that must be financed in advance; x_t is the growth rate of the monetary base; ξ_t is a shock influencing bank demand for reserves; x_t^b is a technology shock to the bank production function; τ_t^T is the tax rate on household earnings of interest on time deposits; θ_t is a shock to the relative preference for currency versus deposits; τ_t^D is the tax rate on household earnings of interest on deposits; τ_t^l is the tax rate on wage income; τ_t^k is the tax rate paid by entrepreneurs on their earnings of rent on capital services; ζ_t is a preference shock for household leisure; g_t is a shock to government consumption; $\hat{\upsilon}_t$ is a shock to the household demand for transactions services; $\hat{w}_t^e$ is a shock to the transfers received by entrepreneurs; $\hat{\mu}_{z,t}$ is a shock to the growth rate of technology; $\hat{\gamma}_t$ is a shock to the rate of survival of entrepreneurs; $\hat{\epsilon}_t$ is a stationary technology shock to intermediate good production; $\hat{x}_{pt}$ is the monetary policy shock; $\hat{\tau}_t^C$ is the tax on consumption.

In each case, we give the shock an ARMA(1,1) representation. In addition, we suppose that monetary policy corresponds to the innovation in a shock according to an ARMA(1,1), as in (2.39). Consider, for example, the first shock $\hat{\lambda}_{f,t}$. The following vector first order autoregression captures in its first row, the ARMA(1,1) representation of $\hat{\lambda}_{f,t}$ and in the third row the ARMA(1,1) representation of the response of monetary policy to the shock:

$$\begin{pmatrix} \hat{\lambda}_{f,t} \\ \epsilon_{f,t} \\ x_{f,t} \end{pmatrix} = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & \phi_f^1 & \phi_f^2 \end{bmatrix} \begin{pmatrix} \hat{\lambda}_{f,t-1} \\ \epsilon_{f,t-1} \\ x_{f,t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{f,t} \\ \epsilon_{f,t} \\ \phi_f^0 \epsilon_{f,t} \end{pmatrix}.$$

There are 6 parameters associated with $\hat{\lambda}_{f,t} : \rho_f,\ \eta_f,\ \phi_f^2,\ \phi_f^1,\ \phi_f^0$ and the standard deviation of $\epsilon_{f,t}$, σ_f. The parameters ϕ_f^0 and σ_f are only needed when the model is simulated, such as for computing impulse response functions or obtaining second moments. It is not required for computing the model solution. In this way, there are 6 parameters associated with each of the first 18 shocks, and the 20^{th}. Since logically there is no monetary policy response to a monetary policy shock, there are only three parameters for that shock. So, the total number of parameters associated with the exogenous shocks is $19 \times 6 + 3 = 117$.

We now discuss the construction of (7.2) in detail. Define the $3N \times 1$ vector Ψ_t as follows:

$$\Psi_t = \begin{pmatrix} \Psi_{1,t} \\ \vdots \\ \Psi_{N,t} \end{pmatrix}.$$

Here, $\Psi_{i,t}$ is 3×1 for $i = 1, ..., N$:

$$\begin{aligned}
\Psi_{1,t} &= \begin{pmatrix} \hat{\lambda}_{f,t} \\ \epsilon_{f,t} \\ x_{f,t} \end{pmatrix}, \ \Psi_{2,t} = \begin{pmatrix} \hat{\tau}_t \\ \epsilon_{\tau,t} \\ x_{\tau,t} \end{pmatrix}, \ \Psi_{3,t} = \begin{pmatrix} \hat{\psi}_{l,t} \\ \epsilon_{l,t} \\ x_{l,t} \end{pmatrix}, \ \Psi_{4,t} = \begin{pmatrix} \hat{\psi}_{k,t} \\ \epsilon_{k,t} \\ x_{k,t} \end{pmatrix} \\
\Psi_{5,t} &= \begin{pmatrix} \hat{\xi}_t \\ \epsilon_{\xi,t} \\ x_{\xi,t} \end{pmatrix}, \ \Psi_{6,t} = \begin{pmatrix} \hat{x}_t^b \\ \epsilon_{b,t} \\ x_{b,t} \end{pmatrix}, \ \Psi_{7,t} = \begin{pmatrix} \hat{\tau}_t^T \\ \epsilon_{T,t} \\ x_{T,t} \end{pmatrix}, \ \Psi_{8,t} = \begin{pmatrix} \hat{\theta}_t \\ \epsilon_{\theta,t} \\ x_{\theta,t} \end{pmatrix}, \\
\Psi_{9,t} &= \begin{pmatrix} \hat{\tau}_t^D \\ \epsilon_{D,t} \\ x_{D,t} \end{pmatrix}, \ \Psi_{10,t} = \begin{pmatrix} \hat{\tau}_t^l \\ \epsilon_{\tau^l,t} \\ x_{\tau^l,t} \end{pmatrix}, \ \Psi_{11,t} = \begin{pmatrix} \hat{\tau}_t^k \\ \epsilon_{\tau^k,t} \\ x_{\tau^k,t} \end{pmatrix}, \ \Psi_{12,t} = \begin{pmatrix} \hat{\zeta}_t \\ \epsilon_{\zeta,t} \\ x_{\zeta,t} \end{pmatrix}, \\
\Psi_{13,t} &= \begin{pmatrix} \hat{g}_t \\ \epsilon_{g,t} \\ x_{g,t} \end{pmatrix}, \ \Psi_{14,t} = \begin{pmatrix} \hat{\upsilon}_t \\ \epsilon_{\upsilon,t} \\ x_{\upsilon,t} \end{pmatrix}, \ \Psi_{15,t} = \begin{pmatrix} \hat{w}_t^e \\ \epsilon_{w^e,t} \\ x_{w^e,t} \end{pmatrix}, \ \Psi_{16,t} = \begin{pmatrix} \hat{\mu}_{z,t} \\ \epsilon_{\mu_z,t} \\ x_{\mu_z,t} \end{pmatrix}, \\
\Psi_{17,t} &= \begin{pmatrix} \hat{\gamma}_t \\ \epsilon_{\gamma,t} \\ x_{\gamma,t} \end{pmatrix}, \ \Psi_{18,t} = \begin{pmatrix} \hat{\epsilon}_t \\ \epsilon_{\epsilon,t} \\ x_{\epsilon,t} \end{pmatrix}, \ \Psi_{19,t} = \begin{pmatrix} \hat{x}_{pt} \\ \epsilon_{p,t} \\ \hat{\tau}_{t-1}^k \end{pmatrix}, \ \Psi_{20,t} = \begin{pmatrix} \hat{\tau}_t^C \\ \epsilon_{\tau^C,t} \\ \hat{x}_{\tau^C,t} \end{pmatrix}
\end{aligned}$$

The non-financial market shocks are

$$\hat{\lambda}_{f,t}, \ \hat{\tau}_t, \ \hat{x}_t^b, \ \hat{\tau}_t^T, \ \hat{\tau}_t^D, \ \hat{\tau}_t^l, \ \hat{\tau}_t^k, \ \hat{\zeta}_t, \ \hat{g}_t, \ \hat{w}_t^e, \ \hat{\mu}_{z,t}, \ \hat{\epsilon}_t, \ \hat{\tau}_t^C$$

The financial market shocks are:

$$\hat{\psi}_{l,t}(7-9), \ \hat{\psi}_{k,t}(10-12), \ \hat{\xi}_t(13-15), \ \hat{\theta}_t(22-24), \ \hat{\upsilon}_t(40-42), \ \hat{\gamma}_t(49-51), \ \hat{x}_{pt}(55-56)$$

Numbers in parentheses correspond to the associated entries in Ψ_t.

The time series representation of Ψ_t is:

$$\Psi_t = \rho \Psi_{t-1} + \varepsilon_t^{\Psi},$$

where ρ is a $3N \times 3N$ matrix. With one exception, it is block diagonal in a way that is conformable with the partitioning of Ψ_t. Each block is 3×3. Thus, with one exception, ρ

has the following structure:

$$\rho = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \rho_N \end{bmatrix},$$

with the partitioning being conformable with the partitioning of Ψ_t. The exception is the 31^{st} entry in the 57^{th} row of ρ, which is unity. For example,

$$\rho_1 = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & \phi_f^2 & \phi_f^1 \end{bmatrix}, \; \rho_{19} = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In general, ρ_i is 3×3 for $i = 1, ..., 18$, with zeros in the middle row and in the 1,3 and 3,1 elements. Similarly, we partition

$$\varepsilon_t^{\Psi} = \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{20t} \end{bmatrix},$$

where ε_{it} is 3×1 for $i = 1, ..., 20$, and the last element of $\varepsilon_{19,t}$ is zero. The first two entries of ε_{it} are equal and represent the innovation in the associated exogenous shock variable. The last entry is proportional to the second, where the factor of proportionality characterizes the contemporaneous response of monetary policy to the shock.

We now discuss the relation between s_t and Ψ_t. In the 'standard case' we assume that the information set in each equation is Ω_t^{μ}. In this case,

$$s_t = \theta_t, \; P = \rho, \; \tilde{\varepsilon}_t = \varepsilon_t^{\Psi}.$$

If any one of the information sets in any one of the equations contains less information than Ω_t^{μ}, then s_t is constructed slightly differently:

$$s_t = \begin{pmatrix} \Psi_t \\ \Psi_{t-1} \end{pmatrix}, \; P = \begin{bmatrix} \rho & 0 \\ I & 0 \end{bmatrix}, \; \tilde{\varepsilon}_t = \begin{pmatrix} \varepsilon_t^{\Psi} \\ 0 \end{pmatrix}. \tag{7.4}$$

The matrices, β_0 and β_1 provided to the computational algorithm are the ones that are suitable for the standard case. If the algorithm detects that some information sets are small, then it makes appropriate adjustments to the β's.

Monetary policy is a function of Ψ_t, according to equation (24) and (2.39):

$$\hat{x}_t = \left[\sum_{i=1, \text{ i}\neq 19}^{20} (0, 0, 1)\Psi_{it} \right] + (1, 0, 0)\Psi_{19,t}.$$

A solution to the model is a set of matrices, A and B, in:

$$z_t = Az_{t-1} + Bs_t,$$

where B is restricted to be consistent with our information set assumptions. The computation of the A and B matrices is discussed in Christiano ( )[9].

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Table 2: Consolidated Banking Sector Balance Sheet					
Assets (Fraction of GNP)	4.40		Liabilities (Fraction of GNP)	4.40	
Reserves	0.018		Demand Deposit	0.205	
Required		0.005	Firms		0.187
Excess		0.013	Households		0.018
Working Capital Loans	0.187				
Capital Expenditures		0.042	Time Deposits	0.795	
Labor Expenditures		0.015			
'Long Term', Entrepreneurial Loans	0.795				
Note: Unless Otherwise Indicated, Numbers are Ratios to Total Bank Assets.					

Table 3: Money and Interest Rates				
Velocity	Model	US Data	Interest Rates (APR)	
Monetary Base	11.	13	Demand Deposits	0.63
M1	3.36	6.0	Time Deposits	9.32
M3	0.85	1.2	Rate of Return on Capital	12.28
Currency to Demand Deposit Ratio	0.32	0.60	Loans to Entrepreneurs	11.81
Currency to Monetary Base Ratio	0.80	0.80	Interest Rate on Working Capital Loans	10.42

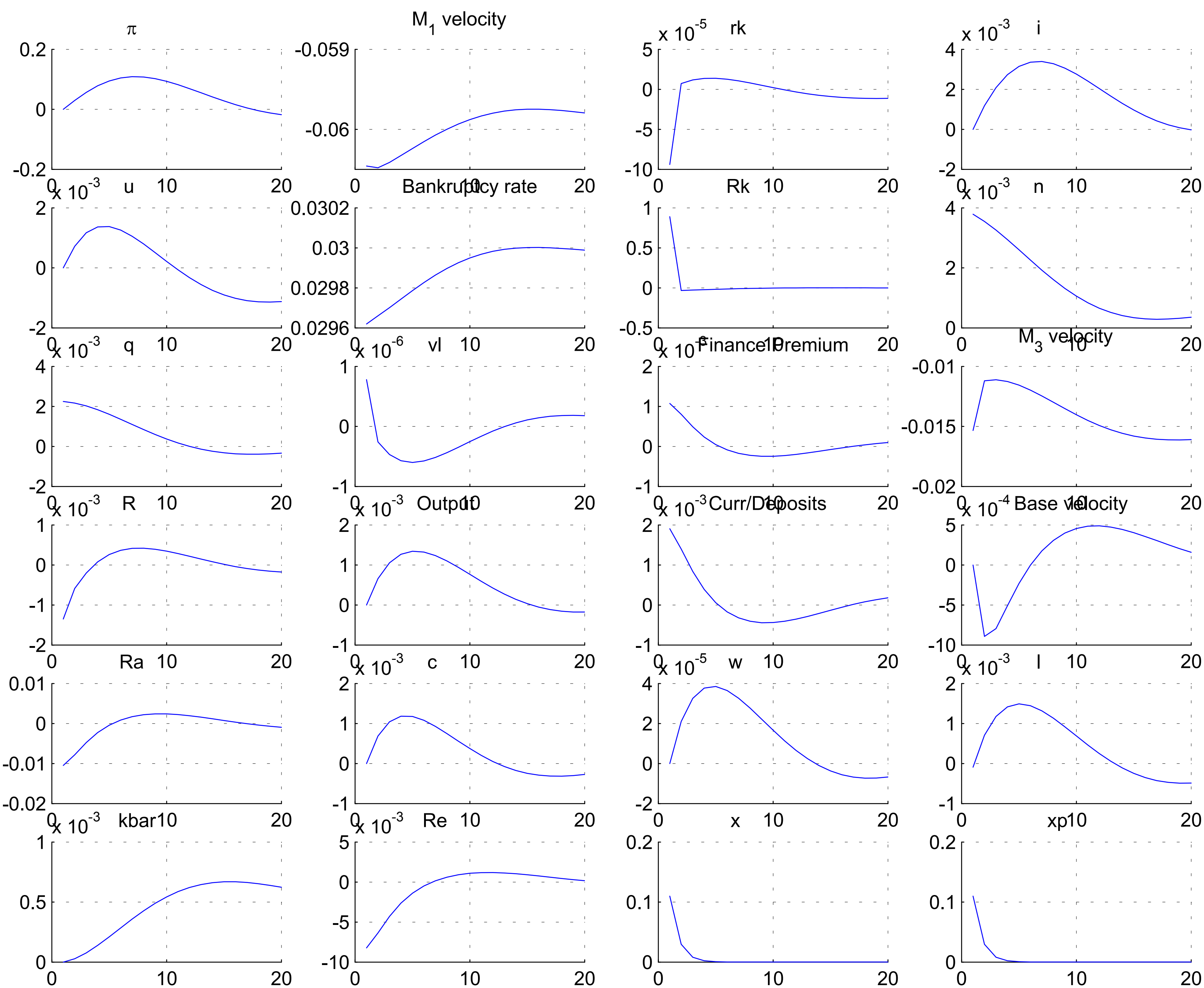
π
M1 velocity
x 10-5 rk
x 10-3 i
x 10-3 u
Bankruptcy rate
Rk
x 10-3 n
x 10-3 q
x 10-6 vl
x 10-3 Finance Premium
M3 velocity
x 10-3 R
x 10-3 Output
x 10-3 Curr/Deposits
x 10-4 Base velocity
Ra
x 10-3 c
x 10-5 w
x 10-3 I
x 10-3 kbar
x 10-3 Re
x
xp

FIG 1

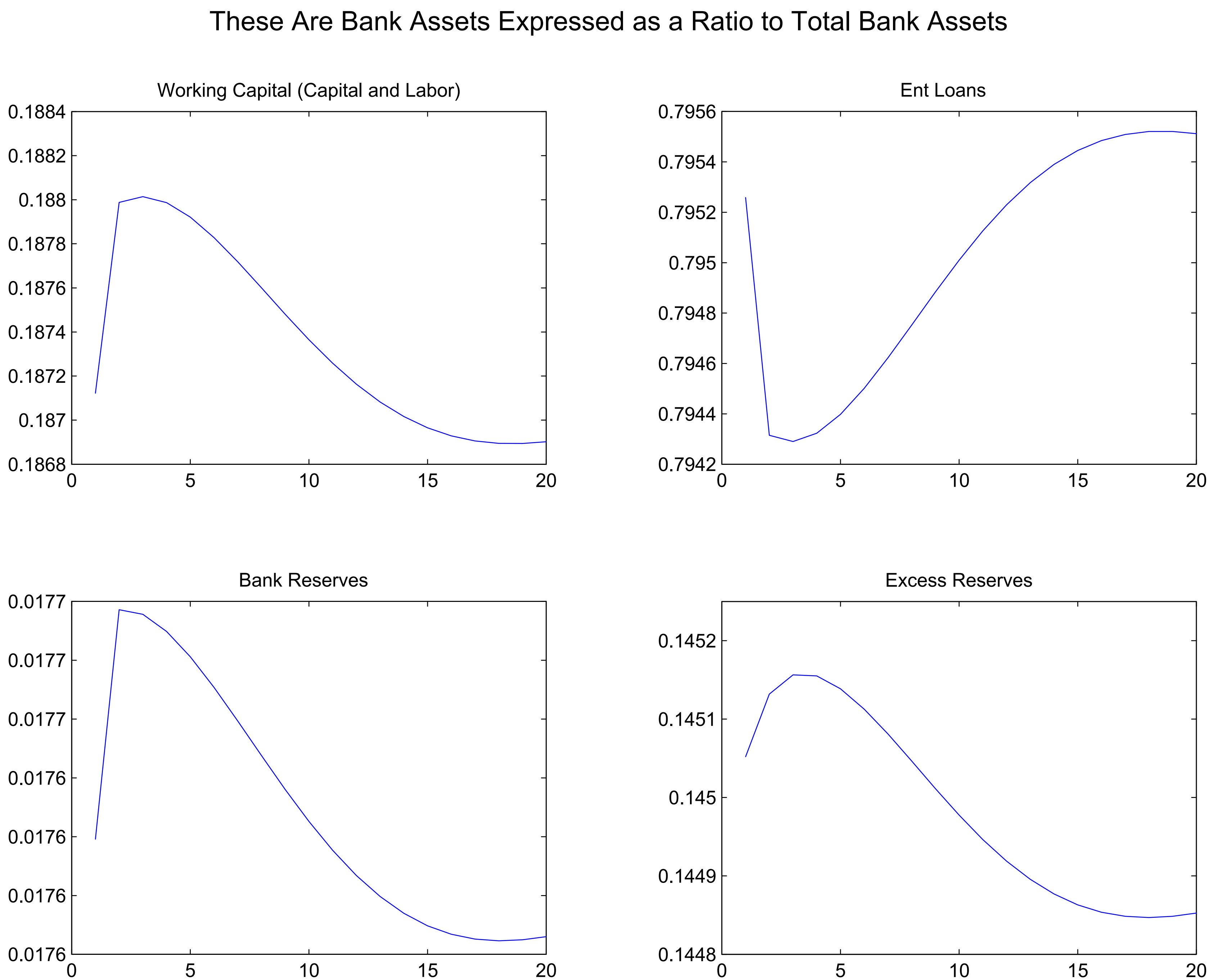
These Are Bank Assets Expressed as a Ratio to Total Bank Assets
Working Capital (Capital and Labor)
0.1884
0.1882
0.188
0.1878
0.1876
0.1874
0.1872
0.187
0.1868
0
5
10
15
20
Ent Loans
0.7956
0.7954
0.7952
0.795
0.7948
0.7946
0.7944
0.7942
Bank Reserves
0.0177
0.0176
Excess Reserves
0.1452
0.1451
0.145
0.1449
0.1448

FIG 2

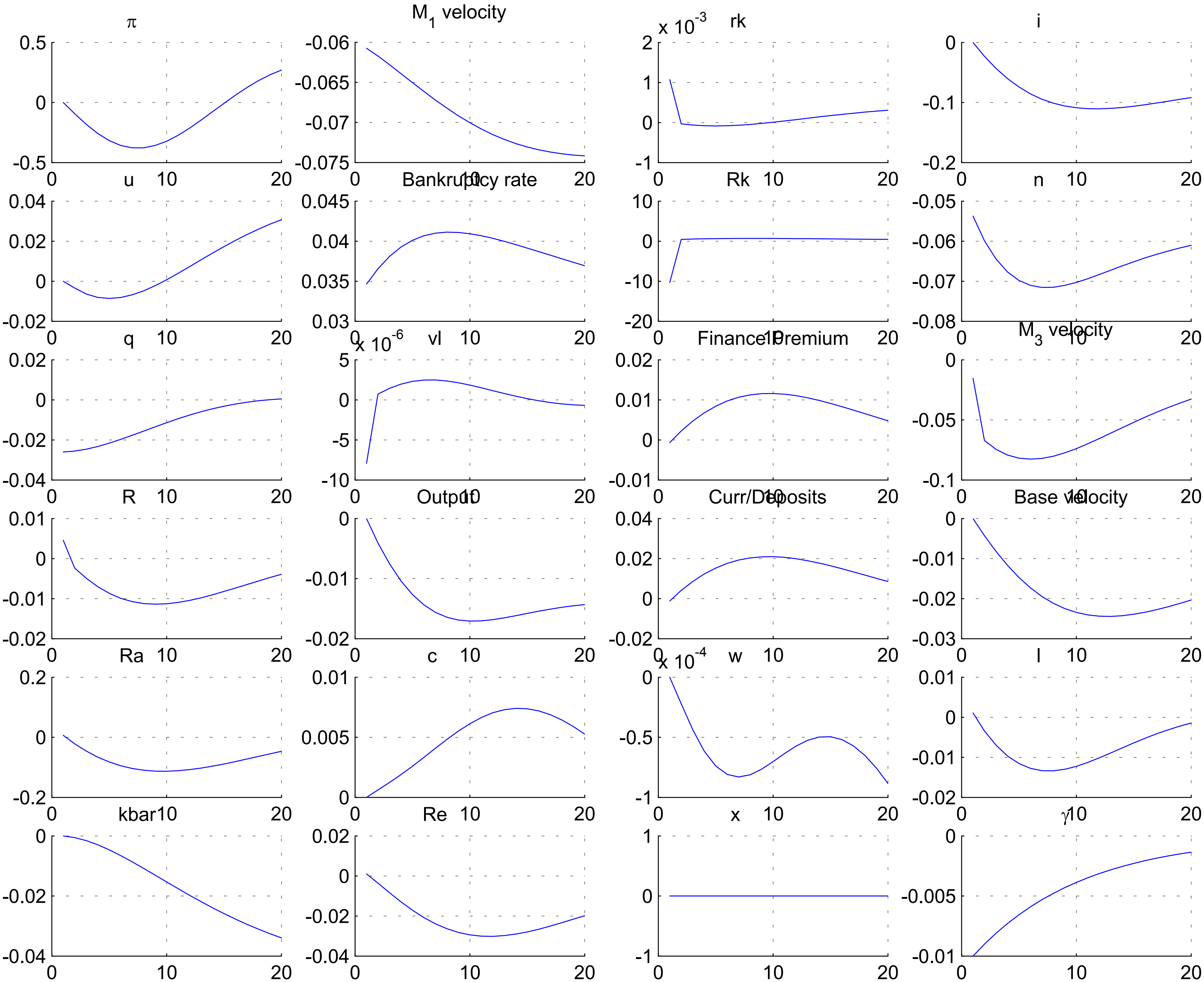
π
M1 velocity
rk
i
u
Bankruptcy rate
Rk
n
q
vl
Finance Premium
M3 velocity
R
Output
Curr/Deposits
Base velocity
Ra
c
w
l
kbar
Re
x
γ

FIG 3

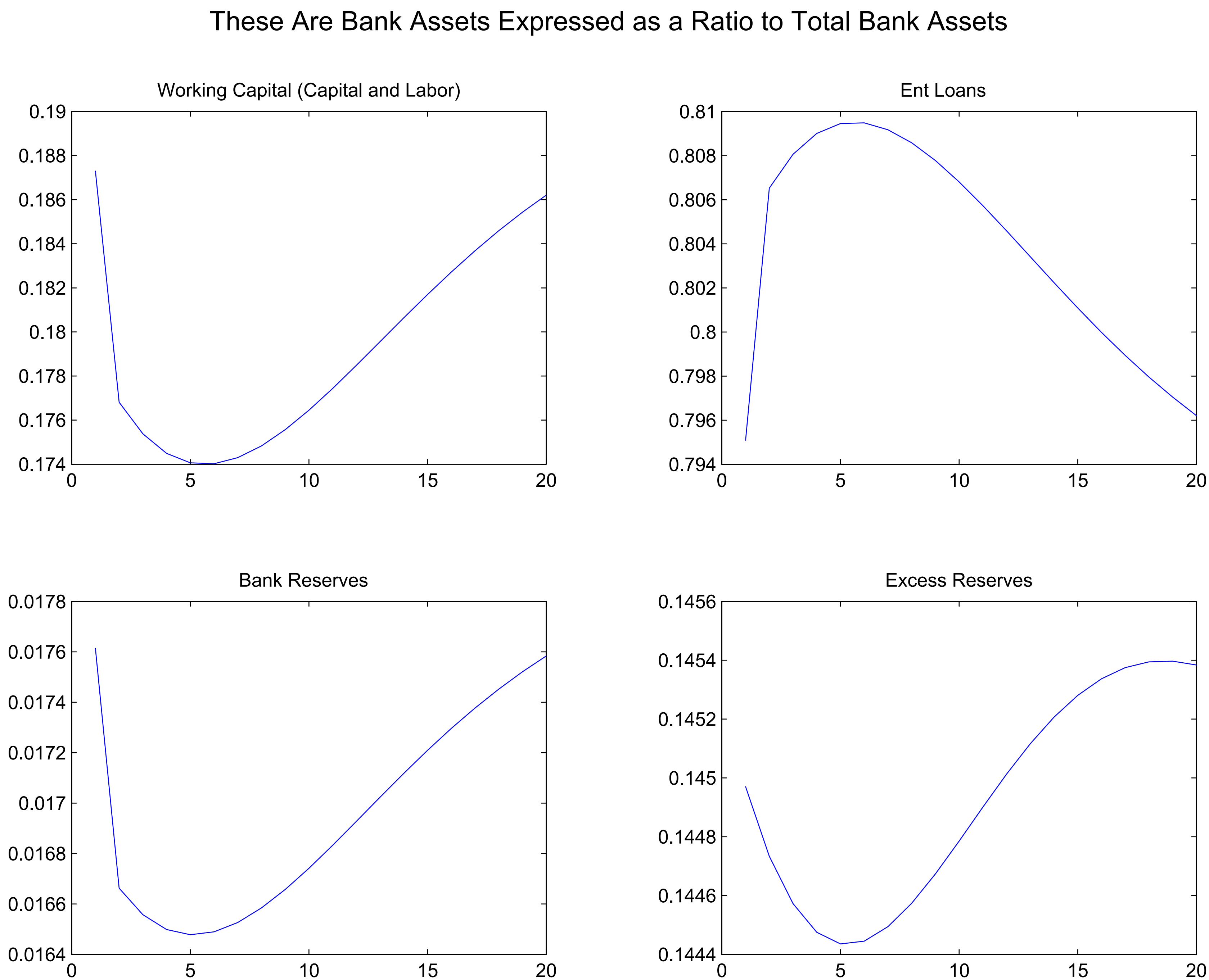
These Are Bank Assets Expressed as a Ratio to Total Bank Assets
Working Capital (Capital and Labor)
Ent Loans
Bank Reserves
Excess Reserves

FIG 4