Notes on a General Strategy for Linearly Approximating Model Solutions

Outline

(1) Define a Model ‘Solution’

(2) Motivate the Need to Somehow Approximate Model Solutions
Example #1: Nonstochastic RBC Model.

(3) Describe Basic Idea Behind Linearization Approximation Method

(4) A Canonical Form For Linear (or, Linearized) Models

(5) Example #2: Putting the Stochastic RBC Model into General Canonical Form

(6) Example #3: Stochastic RBC Model with Hours Worked

(7) Example #4: Example #3 with ‘Exotic’ Information Sets.

(8) Solving the General Canonical Model.

Example #1: Nonstochastic RBC Model

$$\text{Maximize}_{\{c_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma},$$

subject to:

$$C_t + K_{t+1} - (1-\delta)K_t = K_t^{\alpha}, \; K_0 \text{ given}$$

First order condition:

$$C_t^{-\sigma} - \beta C_{t+1}^{-\sigma} \left[\alpha K_{t+1}^{\alpha-1} + (1-\delta) \right],$$

or, after substituting out resource constraint:

$$v(K_t, K_{t+1}, K_{t+2}) = 0,$$

for $t = 0, 1, ...,$ with K_0 given.

'Solution' is a function, $K_{t+1} = g(K_t)$, such that

$$v(K_t, g(K_t), g[g(K_t)]) = 0,$$

for all K_t.

Problem:

This is an Infinite Number of Equations
(one for each possible K_t)
in an Infinite Number of Unknowns
(a value for g for each possible K_t)

With Only a Few Exceptions (e.g., $\sigma = 1$, $\delta = 1$) this is very hard to Solve. Must Approximate.

Approximation Method Based on Linearization

- Replace v by Linear Expansion about About Steady State, $K_t = K_{t+1} = K_{t+2} = K^*$:

$$v_1(K_t - K^*) + v_2(K_{t+1} - K^*) + v_3(K_{t+2} - K^*) = 0,$$

or, in conventional notation:

$$v_1 K\hat{K}_t + v_2 K\hat{K}_{t+1} + v_3 K\hat{K}_{t+2} = 0,$$
$$\hat{K}_t \equiv \frac{K_t - K^*}{K^*}.$$

- Policy Rule by Rule:

$$\hat{K}_{t+1} = \lambda \hat{K}_t.$$

- Find λs By 'Method of Undetermined Coefficients'. By Substitution,

$$v_1 + v_2\lambda_i + v_3\lambda_i^2 = 0,\ i = 1, 2.$$

- With RBC Example, One λ_i is Explosive. There Are Theorems (see Stokey-Lucas, chap. 6) That Say You Should Ignore the Explosive λ_i.
- In General Class of Models (Especially Monetary Models) There are No Such Theorems. So, Explosive λ_i May Possibly Correspond to a Legitimate Equilibrium Path (see ex. in Stokey-Lucas, chap. 6 for an example).
- Still, Linearization Methods are Not Suited to Studying Equilibria that Diverge Away from K^*, Because Validity of Approximation Falls Apart.

- If Both Eigenvalues Are Explosive, There is Nothing Useful for Linearization Methods to Do.
- If Both Eigenvalues Are Non-Explosive, You Have Found Multiple Equilibria For Your Model.
- A Unique Solution Requires Just the Right Number of Explosive Roots.

Canonical Form

- Suppose z_t is a Vector of Endogenous Variables, Determined at t.
- Many Models Imply that z_t Satisfies:

$$E\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] = 0,$$

$$s_t = P s_{t-1} + \epsilon_t,\ \epsilon_t \sim \text{ iid}$$

- Simple RBC Model:

$$\alpha_0 = v_3,\ \alpha_1 = v_2,\ \alpha_2 = v_1,\ \beta_0 = \beta_1 = \epsilon_t = s_t = 0.$$

Example #2: RBC Model With Uncertainty

- Maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t),$$

subject to

$$C_t + K_{t+1} - (1-\delta)K_t = f(K_t, \theta_t),$$

$$\theta_t = \rho\theta_{t-1} + e_t,\ e_t \tilde{}\, N(0, \sigma_e^2).$$

- First Order Condition:

$$\begin{aligned} & v(K_{t+2}, K_{t+1}, K_t, \theta_{t+1}, \theta_t) \\ = \ & U_c\left(f(K_t, \theta_t) + (1-\delta)K_t - K_{t+1}\right) \\ & -\beta U_c\left(f(K_{t+1}, \theta_{t+1}) + (1-\delta)K_{t+1} - K_{t+2}\right) \\ & \times \left[f_K(\theta_{t+1}) + 1 - \delta\right]. \end{aligned}$$

- Solution is a $g(K_t, \theta_t)$, Such That

$$E\{v\left(g(g(K_t,\theta_t),\theta_{t+1}),g(K_t,\theta_t),K_t,\theta_{t+1},\theta_t\right)|\theta_t\} = 0,$$

For All K_t, θ_t.

- Linearization Strategy: Replace v by Linear Taylor Series Expansion About $K_t = K^*$, $\theta_t = 0$. Can Write this In Canonical Form,

$$E_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] = 0,$$

$$s_t = P s_{t-1} + \epsilon_t,\ \epsilon_t = e_t$$

with

$$z_t = \hat{K}_{t+1},\ s_t = \theta_t,\ P = \rho.$$

- As Before, Look for a Linear Solution:

$$z_t = Az_{t-1} + Bs_t.$$

- Using Method of Undetermined Coefficients, Substitute Solution (With As Yet Undetermined Coefficient Values) into Linearized First Order Condition, to Obtain:

$$\alpha(A)z_{t-1}+Fs_t+E_t\beta_0\varepsilon_{t+1} = \alpha(A)z_{t-1}+Fs_t = 0,$$

where

$$\begin{aligned}\alpha(A) &= \alpha_0 A^2 + \alpha_1 A + \alpha_2, \\ F &= (\beta_0 + \alpha_0 B)P + [\beta_1 + (A\alpha_0 + \alpha_1)B].\end{aligned}$$

- Strategy: Solve for A First, Using Requirement

$$\alpha(A) = 0.$$

Then, solve for B Using the Requirement

$$F = 0.$$

Note: Finding B Involves Solving a Linear Equation, Conditional on Having A in Hand.

Example #3 RBC Model With Hours Worked and Uncertainty

- Maximize

$$E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$C_t + K_{t+1} - (1-\delta)K_t = f(K_t, N_t, \theta_t)$$

  - First Order Conditions:

$$0 = E[v_K(K_{t+2}, N_{t+1,} K_{t+1}, N_t, K_t, \theta_{t+1}, \theta_t)|\theta_t]$$

and

$$0 = v_N(K_{t+1}, N_t, K_t, \theta_t).$$

where

$$
\begin{aligned}
& v_K(K_{t+2}, N_{t+1,} K_{t+1}, N_t, K_t, \theta_{t+1}, \theta_t) \\
= \; & U_c\left(f(K_t, N_t, \theta_t) + (1-\delta)K_t - K_{t+1}, N_t\right) \\
& -\beta U_c\left(f(K_{t+1}, N_{t+1,}\theta_{t+1}) + (1-\delta)K_{t+1} - K_{t+2}, N_{t+1}\right) \\
& \times\left[f_K(K_{t+1}, N_{t+1}, \theta_{t+1}) + 1 - \delta\right]
\end{aligned}
$$

and,

$$
\begin{aligned}
& v_N(K_{t+1}, N_t, K_t, \theta_t) \\
= \; & U_N\left(f(K_t, N_t, \theta_t) + (1-\delta)K_t - K_{t+1}, N_t\right) \\
& +U_c\left(f(K_t, N_{t,}\theta_t) + (1-\delta)K_t - K_{t+1}, N_t\right) \\
& \times f_N(K_t, N_t, \theta_t).
\end{aligned}
$$

– After Linearizing, Obtain Canonical Form:

$$E_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] = 0,$$

$$s_t = P s_{t-1} + \epsilon_t,\ \epsilon_t \sim \text{ iid}$$

– Here,

$$\alpha_0 = \begin{bmatrix} v_{K,1}K^* & v_{K,2} \\ 0 & 0 \end{bmatrix},\ \alpha_1 = \begin{bmatrix} v_{K,3}K^* & v_{K,4}N^* \\ v_{N,1}K^* & v_{N,2}N^* \end{bmatrix},$$
$$\alpha_2 = \begin{bmatrix} v_{K,5}K^* & 0 \\ v_{N,3}K^* & 0 \end{bmatrix},$$

$$\beta_0 = \begin{pmatrix} v_{K,6} \\ 0 \end{pmatrix},\ \beta_1 = \begin{pmatrix} v_{K,7} \\ v_{N,4} \end{pmatrix}.$$

$$z_t = \left(\hat{K}_{t+1}, \hat{N}_t\right)'.$$

– Again, Look for Solution:

$$z_t = Az_{t-1} + Bs_t.$$

– Now, A is Root of *Matrix* Polynomial:

$$\alpha(A) = \alpha_0 A^2 + \alpha_1 A + \alpha_0 I = 0.$$

– Also, B Satisfies Same System of Linear Equations as Before:

$$F = (\beta_0 + \alpha_0 B)P + [\beta_1 + (A\alpha_0 + \alpha_1)B].$$

Example #4: Example #3 With 'Exotic' Information Set

- Suppose the Date t Investment Decision is Made Before the Current Realization of the Technology Shock, While the Hours Decision is Made Afterward.
- Now, Canonical Form Must Be Written Differently:

$$\mathcal{E}_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] = 0,$$

where

$$\mathcal{E}_t X_t = \begin{bmatrix} E\left[X_{1t}|\theta_{t-1}\right] \\ E\left[X_{2t}|\theta_t\right] \end{bmatrix}.$$

- Must Change s_t:

$$s_t = \begin{pmatrix} \theta_t \\ \theta_{t-1} \end{pmatrix},\ P = \begin{bmatrix} \rho & 0 \\ 1 & 0 \end{bmatrix},\ \epsilon_t = \begin{pmatrix} e_t \\ 0 \end{pmatrix}.$$

- Adjust β_i's:

$$\beta_0 = \begin{pmatrix} v_{K,6} & 0 \\ 0 & 0 \end{pmatrix}, \ \beta_1 = \begin{pmatrix} v_{K,7} & 0 \\ v_{N,4} & 0 \end{pmatrix},$$

- Again, Posit Following Solution:

$$z_t = Az_{t-1} + Bs_t.$$

- Substitute Into Canonical Form:$\alpha(A)z_{t-1}+$ $Fs_t + E_t\beta_0\varepsilon_{t+1} = \alpha(A)z_{t-1} + Fs_t = 0,$

$$\begin{aligned} &\mathcal{E}_t\left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] \\ =\ &\alpha(A)z_{t-1} + \mathcal{E}_t F s_t + \mathcal{E}_t\beta_0\varepsilon_{t+1} = \alpha(A)z_{t-1} + \mathcal{E}_t F s_t = 0, \end{aligned}$$

- Then,

$$\begin{aligned} \mathcal{E}_t F s_t \ &= \ \mathcal{E}_t \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} s_t = \mathcal{E}_t \begin{bmatrix} F_{11}\theta_t + F_{12}\theta_{t-1} \\ F_{21}\theta_t + F_{22}\theta_{t-1} \end{bmatrix} \\ &= \begin{bmatrix} 0 & F_{12} + \rho F_{11} \\ F_{21} & F_{22} \end{bmatrix} s_t = \tilde{F} s_t. \end{aligned}$$

- Equations to be solved:

$$\alpha(A) = 0, \ \tilde{F} = 0.$$

- $\tilde{F}$ Only Has *Three* Equations How Can We Solve for the Four Elements of B?
- Answer: Only *Three* Unknowns in B Because B Must Also Obey Information Structure:

$$B = \begin{bmatrix} 0 & B_{12} \\ B_{21} & B_{22} \end{bmatrix}.$$

- Bottom Line: With Exotic Information Structure, Lags of θ_t Must Appear in s_t.

Conclusion

- Solving Models Can Be Boiled Down to Finding A and B in:

$$(*)\ z_t = Az_{t-1} + Bs_t,$$

- To Set Up the Problem, Must First Write Equations of the Model and the Exogenous Variables in the following Form:

$$\mathcal{E}_t \left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right]$$

$$s_t = Ps_{t-1} + \epsilon_t,\ \epsilon_t \sim \text{ iid}$$

- The Matrices A and B Must Solve:

$$\begin{aligned} \alpha(A) &= 0, \\ \text{(standard information set) } F &= 0, \\ \text{(exotic information set) } \tilde{F} &= 0 \end{aligned}$$

Working capital loans for capital rental are:

$$\frac{\psi_{k,t} P_t r_t^k K_t}{M_t^b \left[1 - m + x + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)/m^b\right]}$$
$$= \frac{\psi_k r^k k/\mu_z}{m^b (1 - m + x) + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)}$$

Loans to entrepreneurs:

$$\frac{(k - n)/(\mu_z \pi)}{m^b (1 - m + x) + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)}$$

Total reserves are:

$$\frac{M_t^b - M_t + X_t}{M_t^b \left[1 - m + x + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)/m^b\right]}$$
$$= \frac{(1 - m + x)\, m^b}{m^b (1 - m + x) + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)}$$

Required reserves are:

$$\frac{\tau \left((1 - m + x)\, m^b + \left(\psi_l wl + \psi_k r^k k/\mu_z\right)\right)}{m^b (1 - m + x) + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)}$$

Excess reserves are:

$$\frac{(1 - m + x)\, m^b - \tau \left((1 - m + x)\, m^b + \left(\psi_l wl + \psi_k r^k k/\mu_z\right)\right)}{m^b (1 - m + x) + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)}$$

The ratio of firm demand deposits to total assets are:

$$\frac{\psi_{l,t} W_t l_t + \psi_{k,t} P_t r_t^k K_t}{M_t^b \left[1 - m + x + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)/m^b\right]}$$
$$= \frac{\psi_l wl + \psi_k r^k k/\mu_z}{m^b \left[1 - m + x + \left(\psi_l wl + \psi_k r^k k/\mu_z + (k - n)/(\pi \mu_z)\right)/m^b\right]}$$

3.2. Linearization

There are 24 endogenous variables whose values are determined at time t. We load them into a vector, z_t. The elements in this vector are reported in the following table. In addition, there is an indication about which shocks the variable depends on. If it depends on the realization of all period t shocks, then we indicate a, for 'all'. If it depends only on the realization of the current period non-financial shocks, then we indicate p, for 'partial'. The table also indicates the information associated with each of the 24 equations used to solve the model. These equations are collected below from the preceding discussion. Note that the number of equations and elements in z_t is the same. Note also, in each case, the third and fourth columns always have the same entry. In several cases, z_t contains variables dated $t+1$. In the case of $\widehat{\bar{k}}_t$, for example, the presence of a p in the third column indicates that $\widehat{\bar{k}}_t$ is a function of the realization of the period t non-financial shocks, and is not a function of the realization of period t financial shocks, or later period shocks. In the case of $\hat{R}^e_t$, the presence of an a indicates that this variable is a function of all period t shocks, but not of any period $t+1$ shocks.

	z_t	information, z	information, equation
1	$\hat{\pi}_t$	p	p
2	$\hat{s}_t$	a	a
3	$\hat{r}^k_t$	a	a
4	$\hat{\imath}_t$	p	p
5	$\hat{u}_t$	p	p
6	$\widehat{\bar{\omega}}_t$	a	a
7	$\hat{R}^k_t$	a	a
8	$\hat{n}_t$	a	a
9	$\hat{q}_t$	a	a
10	$\hat{\nu}^l_t$	a	a
11	$\hat{e}_{\nu,t}$	a	a
12	$\hat{m}^b_t$	a	a
13	$\hat{R}_t$	a	a
14	$\hat{u}^z_{c,t}$	a	a
15	$\hat{\lambda}_{z,t}$	a	a
16	$\hat{m}_t$	a	a
17	$\hat{R}_{a,t}$	a	a
18	$\hat{c}_t$	p	p
19	$\hat{w}_t$	p	p
20	$\hat{l}_t$	a	a
21	$\widehat{\bar{k}}_t$	p	p
22	$\hat{R}^e_t$	a	a
23	$\hat{x}_t$	a	a

(3.22)

The last of these variables is money growth, $\hat{x}_t$. As we show below, this is simply a trivial function of the underlying shocks. In addition, recall (2.35), in which the 10^{th} and 11^{th} variables are the same. A combination of the efficiency conditions for labor and capital in the firm sector, equations (1) and (2) below, are redundant with the efficiency conditions for labor and capital in the banking sector, (11) and (12). We deleted equation (11) below from our system.

In fact, we have 25 equations and unknowns in our model. The system we work with is one dimension less because we set $\Theta \quad 0$, so that $\hat{v}_t$ disappears from the system. When we want $\Theta > 0$, we can get our 25^{th} equation by linearizing (2.24), and $\hat{v}_t$ is then our 25^{th} variable.

3.2.1. Firms

The inflation equation, when there is indexing to lagged inflation, is:

$$(1)\ E\left[\hat{\pi}_t \quad \frac{1}{1+\beta}\hat{\pi}_t \quad\quad \frac{\beta}{1+\beta}\hat{\pi}_t \quad\quad \frac{(1 \quad \beta\xi_p)(1 \quad \xi_p)}{(1+\beta)\,\xi_p}\left(\hat{s}_t+\hat{\lambda}_{f,t}\right) \ \Omega_t\right]=0$$

The linearized expression for marginal cost is:

$$(2)\ \alpha\hat{r}_t^k+\frac{\alpha\psi_k R}{1+\psi_k R}\hat{\psi}_{k,t}+(1 \quad \alpha)\,\hat{w}_t+\frac{(1 \quad \alpha)\,\psi_l R}{1+\psi_l R}\hat{\psi}_{l,t}$$
$$+\left[\frac{\alpha\psi_k R}{1+\psi_k R}+\frac{(1 \quad \alpha)\,\psi_l R}{1+\psi_l R}\right]\hat{R}_t \quad \hat{\epsilon}_t \quad \hat{s}_t=0$$

Another condition that marginal cost must satisfy is that it is equal to the marginal cost of one unit of capital services, divided by the marginal product of one unit of services. After linearization, this implies:

$$(3)\ \hat{r}_t^k+\frac{\psi_k R\left(\hat{\psi}_{k,t}+\hat{R}_t\right)}{1+\psi_k R} \quad \hat{\epsilon}_t \quad (1 \quad \alpha)\left(\hat{\mu}_{z,t}+\hat{l}_t \quad \left[\widehat{\bar{k}}_t+\hat{u}_t\right]\right) \quad \hat{s}_t=0$$

3.2.2. Capital Producers

The 'Tobin's q' relation is:

$$(4)\ E\left\{\hat{q}_t \quad S \ \mu_z(1+\beta)\hat{\imath}_t \quad S \ \mu_z\hat{\mu}_{z,t}+S \ \mu_z\hat{\imath}_t \quad +\beta S \ \mu_z\hat{\imath}_t \quad +\beta S \ \mu_z\hat{\mu}_{z,t} \quad \Omega_t\right\}=0$$

The coefficients in the canonical form are:

$$\begin{aligned}
\alpha\ (4,9) &= 1 \\
\alpha\ (4,4) &= \quad S\ \mu_z(1+\beta) \\
\alpha\ (4,4) &= S\ \mu_z \\
\alpha\ (4,4) &= \beta S\ \mu_z \\
\beta\ (4,46) &= \beta S\ \mu_z \\
\beta\ (4,46) &= \quad S\ \mu_z
\end{aligned}$$

3.2.3. Entrepreneurs

The variable utilization equation is

$$(5)\ E\left[\hat{r}_t^k \quad \sigma_a \hat{u}_t\ \Omega_t\right] = 0,$$

where $\hat{r}_t^k$ denotes the rental rate on capital. The date t standard debt contract has two parameters, the amount borrowed and $\widehat{\bar{\omega}}_t$. The former is not a function of the period $t+1$ state of nature, and the latter is not. Two equations characterize the efficient contract. The first order condition associated with the quantity loaned by banks in period t in the optimal contract is:

$$(6)\ E\ \lambda\left(\frac{R^k \hat{R}_t^k}{1+R^k} \quad \frac{R^e \hat{R}_t^e}{1+R^e}\right)$$

$$[1 \quad \Gamma(\bar{\omega})]\frac{1+R^k}{1+R^e}\left[\frac{\Gamma\ (\bar{\omega})\bar{\omega}}{\Gamma\ (\bar{\omega})} \quad \frac{\lambda\left[\Gamma\ (\bar{\omega}) \quad \mu G\ (\bar{\omega})\right]\bar{\omega}}{\Gamma\ (\bar{\omega})}\right]\widehat{\bar{\omega}}_t \quad \Omega_t^\mu \quad = \quad 0.$$

Note that this is not a function of the period $t+1$ uncertainty. Also, note that when $\mu = 0$, so that $\lambda = 1$, then this equation simply reduces to $E\left[\hat{R}_t^k \quad \Omega_t^\mu\right] = \hat{R}_t^e$. The linearized zero profit condition is:

$$(7)\quad \left(\tfrac{k}{n} \quad 1\right)\frac{R^k}{R^k}\hat{R}_t^k \quad \left(\tfrac{k}{n} \quad 1\right)\frac{R^e}{R^e}\hat{R}_t^e + \left(\tfrac{k}{n} \quad 1\right)\frac{\omega \quad \mu G \quad \omega}{\omega \quad \mu G\ \omega}\bar{\omega}\widehat{\bar{\omega}}_t$$

$$\left(\hat{q}_t \quad +\widehat{\bar{k}}_t \quad \hat{n}_t\right) = \qquad 0.$$

The law of motion for net worth is:

$$(8)\quad \hat{n}_t \quad + a\ \hat{R}_t^k + a\ \hat{R}_t^e + a\ \widehat{\bar{k}}_t + a\ \hat{w}_t^e + a\ \hat{\gamma}_t + a\ \hat{\pi}_t + a\ \hat{\mu}_{z,t} + a\ \hat{q}_t \quad + a\ \widehat{\bar{\omega}}_t + a\ \hat{n}_t = 0$$

The definition of the rate of return on capital is:

$$(9)\ \hat{R}_t^k \qquad \frac{(1 \quad \tau^k)r^k + (1 \quad \delta)q}{R^k q}\pi\left[\frac{\left(1 \quad \tau^k\right) r^k \hat{r}_t^k \qquad \tau^k r^k \hat{\tau}_t^k + (1 \quad \delta)q\hat{q}_t}{(1 \quad \tau^k)r^k + (1 \quad \delta)q} + \hat{\pi}_t \qquad \hat{q}_t\right] \quad \frac{\delta\tau^k\hat{\tau}_t^k}{R^k}$$

These are the coefficients corresponding to this equation, in the canonical representation of the model:

$$\begin{aligned}
\alpha\ (9,7) &= 1 :\ \hat{R}_t^k,\ \alpha\ (9,3) = \quad \frac{\pi\left(1 \quad \tau^k\right) r^k}{R^k q} : \hat{r}_t^k \\
\beta\ (9,57) &= \left(r^k\pi \quad \delta\right)\frac{\tau^k}{R^k} : \hat{\tau}_t^k \\
\alpha\ (9,1) &= \quad \frac{(1 \quad \tau^k)r^k + (1 \quad \delta)}{R^k}\pi : \hat{\pi}_t \\
\alpha\ (9,9) &= \quad \frac{(1 \quad \delta)\pi}{R^k} : \hat{q}_t \\
\alpha\ (9,9) &= \frac{(1 \quad \tau^k)r^k + (1 \quad \delta)}{R^k}\pi :\ \hat{q}_t
\end{aligned}$$

3.2.4. Banking Sector

In the equations for the banking sector, it is capital services, k_t, which appears, not the physical stock of capital, $\bar{k}_t$. The link between them is:

$$\hat{k}_t = \hat{\bar{k}}_t + \hat{u}_t.$$

An expression for the ratio of excess reserves to value added in the banking sector is:

$$\begin{aligned}
&(10) \qquad \hat{e}_{v,t} + n_\tau\hat{\tau}_t + n_{m^b}\hat{m}_t^b + n_m\hat{m}_t + n_x\hat{x}_t + n_{\psi_l}\hat{\psi}_{l,t} \\
&+n_{\psi k}\hat{\psi}_{k,t} + (n_k \quad d_k)\left[\hat{\bar{k}}_t + \hat{u}_t\right] + n_{r^k}\hat{r}_t^k + n_w\hat{w}_t \\
&+(n_l \quad d_l)\,\hat{l}_t + (n_{\mu_z} \quad d_{\mu_z})\,\hat{\mu}_{z,t} \quad d_{\nu^k}\hat{\nu}_t^k \quad d_{\nu l}\hat{\nu}_t^l = 0
\end{aligned}$$

where m_t^b is the scaled monetary base, m_t is the currency-to-base ratio, x_t is the growth rate of the base

$$
\begin{aligned}
n_\tau &= \frac{-\tau m^b\left(1-m+x\right)-\tau\left(\psi_l wl+\frac{1}{\mu_z}\psi_k r^k k\right)-\tau\frac{1}{\mu_z}\psi_k r^k k}{n}, \\
n &= \left(1-\tau\right)m^b\left(1-m+x\right)-\tau\left(\psi_l wl+\frac{1}{\mu_z}\psi_k r^k k\right), \\
n_{m^b} &= \left(1-\tau\right)m^b\left(1-m+x\right)/n \\
n_m &= -\left(1-\tau\right)m^b m/n \\
n_x &= \left(1-\tau\right)m^b x/n \\
n_{\psi_l} &= n_w = n_l = -\tau\psi_l wl/n \\
n_{\psi_k} &= n_{r^k} = n_k = -\tau\frac{1}{\mu_z}\psi_k r^k k/n \\
n_{\mu_z} &= \tau\frac{1}{\mu_z}\psi_k r^k k/n
\end{aligned}
$$

and

$$
\begin{aligned}
d &= \left(\frac{1}{\mu_z}(1-\nu^k)k\right)^{\alpha}\left((1-\nu^l)l\right)^{1-\alpha} \\
d_{\mu_z} &= \frac{-\alpha\left(\frac{1}{\mu_z}(1-\nu^k)k\right)^{\alpha}\left((1-\nu^l)l\right)^{1-\alpha}}{\left(\frac{1}{\mu_z}(1-\nu^k)k\right)^{\alpha}\left((1-\nu^l)l\right)^{1-\alpha}} = -\alpha \\
d_k &= \alpha \\
d_{\nu^k} &= -\alpha\frac{\nu^k}{1-\nu^k} \\
d_l &= 1-\alpha \\
d_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l}
\end{aligned}
$$

The first order condition for capital in the banking sector is:

$$
\begin{aligned}
0 &= k_R\hat{R}_t + k_\xi\hat{\xi}_t - \hat{r}_t^k + k_x\hat{x}_t^b + k_e\hat{e}_{v,t} + k_\mu\hat{\mu}_{z,t} \\
&\quad + k_{\nu^l}\hat{\nu}_t^l + k_{\nu^k}\hat{\nu}_t^k + k_l\hat{l}_t + k_k\left[\widehat{\bar{k}}_t + \hat{u}_t\right] + k_\tau\hat{\tau}_t + k_{\psi_k}\hat{\psi}_{k,t}
\end{aligned}
$$

$$
\begin{aligned}
k_R &= \left[1 - \frac{\psi_k R}{1+\psi_k R}\right],\; k_\xi = 1 - \log\left(e_v\right)\xi + \frac{\tau h_{e^r}\left[\frac{}{\xi} + \log\left(e_v\right)\right]\xi}{1+\tau h_{e^r}} \\
k_x &= \frac{1}{1+\tau h_{e^r}},\; k_e = 1 - \xi + \frac{\tau h_{e^r}\xi}{1+\tau h_{e^r}},\; k_\mu = (1-\alpha) \\
k_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l},\; k_{\nu^k} = (1-\alpha)\frac{\nu^k}{1-\nu^k},\; k_l = (1-\alpha),\; k_k = -(1-\alpha) \\
k_\tau &= -\frac{\tau h_{e^r}}{1+\tau h_{e^r}},\; k_{\psi_k} = -\frac{\psi_k R}{1+\psi_k R}.
\end{aligned}
$$

The latter equation was deleted from our system, because it is redundant given the two firm Euler equations and the following equation.

The first order condition for labor in the banking sector is:

$$
\begin{aligned}
(11)\; 0 &= l_R\hat{R}_t + l_\xi\hat{\xi}_t - \hat{w}_t + l_x\hat{x}_t^b + l_e\hat{e}_{v,t} + l_\mu\hat{\mu}_{z,t} \\
&\quad + l_{\nu^l}\hat{\nu}_t^l + l_{\nu^k}\hat{\nu}_t^k + l_l\hat{l}_t + l_k\left[\hat{\bar{k}}_t + \hat{u}_t\right] + l_\tau\hat{\tau}_t + l_{\psi_l}\hat{\psi}_{l,t},
\end{aligned}
$$

where

$$
\begin{aligned}
l_i &= k_i \text{ for all } i,\text{ except} \\
l_R &= \left[1 - \frac{\psi_l R}{1+\psi_l R}\right],\; l_{\psi l} = -\frac{\psi_l R}{1+\psi_l R} \\
l_\mu &= k_\mu - 1,\; l_{\nu^l} = k_{\nu^l} + \frac{\nu^l}{1-\nu^l},\; l_l = k_l - 1, \\
l_{\nu^k} &= k_{\nu^k} - \frac{\nu^k}{1-\nu^k},\; l_k = k_k + 1.
\end{aligned}
$$

The production function for deposits is:

$$
\begin{aligned}
(12)\quad & \hat{x}_t^b - \xi\hat{e}_{v,t} - \log\left(e_{v,t}\right)\xi\hat{\xi}_t - \frac{\tau\left(m + m\right)}{(1-\tau)m - \tau m}\hat{\tau}_t \\
&= \left[\frac{m}{m + m} - \frac{(1-\tau)m}{(1-\tau)m - \tau m}\right]\left[\hat{m}_t^b + \frac{m\hat{m}_t + x\hat{x}_t}{1 - m + x}\right] \\
&\quad + \left[\frac{m}{m + m} + \frac{\tau m}{(1-\tau)m - \tau m}\right] \\
&\quad [\frac{\psi_l w l}{\psi_l w l + \psi_k r^k k/\mu_z}\left(\hat{\psi}_{l,t} + \hat{w}_t + \hat{l}_t\right) + \frac{\psi_k r^k k/\mu_z}{\psi_l w l + \psi_k r^k k/\mu_z}\left(\hat{\psi}_{k,t} + \hat{r}_t^k + \hat{k}_t - \hat{\mu}_{zt}\right)].
\end{aligned}
$$

The coefficients in the canonical form are:

$$
\begin{aligned}
\beta\ (12,16) &= \quad 1\text{: } \hat{x}_t^b, \\
\beta\ (12,4) &= \frac{\tau\,(m \quad + m \quad)}{m \qquad \tau\,(m \quad + m \quad)} : \ \hat{\tau}_t \text{ `ratio, required to excess reserves'} \\
\beta\ (12,13) &= \log\left(e_v\right)\xi : \ \hat{\xi}_t \\
\alpha\ (12,11) &= \xi : \ \hat{e}_{v,t}, \ \alpha\ (12,12) = m \quad ^{-} \ : \ \hat{m}_t^b \\
\alpha\ (12,19) &= \alpha\ (12,20) = \beta\ (12,7) = m \quad w_l : \ \hat{w}_t, \hat{l}_t, \hat{\psi}_{l,t} \\
\alpha\ (12,3) &= \alpha\ (12,5) = \alpha\ (12,21) = \beta\ (12,10) = \quad \beta\ (12,46) \\
&= m \quad w_k : \ \hat{r}_t^k, \ \hat{u}_t, \ \hat{\bar{k}}_t, \ \hat{\psi}_{k,t}, \ \hat{\mu}_{zt} \\
\alpha\ (12,23) &= m \quad ^{-} \frac{x}{1 \quad m + x} \ : \ \hat{x}_t \\
\alpha\ (12,16) &= \quad m \quad ^{-} \frac{m}{1 \quad m + x} \ : \hat{m}_t,
\end{aligned}
$$

where

$$
\begin{aligned}
^{-} \quad &= \frac{1}{m \quad + m} \qquad \frac{(1 \quad \tau)}{(1 \quad \tau)m \qquad \tau m} = \frac{(1 \quad \tau)m \qquad \tau m \qquad (1 \quad \tau)\,(m \quad + m \quad)}{(m \quad + m \quad)\,[m \qquad \tau(m \quad + m \quad)]} \\
&= \frac{m}{(m \quad + m \quad)\,[m \qquad \tau(m \quad + m \quad)]} \\
&= \frac{1}{m \quad + m} + \frac{\tau}{(1 \quad \tau)m \qquad \tau m} = \frac{(1 \quad \tau)m \qquad \tau m \quad + \tau\,(m \quad + m \quad)}{(m \quad + m \quad)\,[m \qquad \tau(m \quad + m \quad)]} \\
&= \frac{m}{(m \quad + m \quad)\,[m \qquad \tau(m \quad + m \quad)]}, \\
w_l &= \frac{\psi_l w l}{\psi_l w l + \psi_k r^k k/\mu_z}, \text{ labor component of working capital loans} \\
w_k &= \frac{\psi_k r^k k/\mu_z}{\psi_l w l + \psi_k r^k k/\mu_z}, \text{ capital component in working capital loans,}
\end{aligned}
$$

where $m \quad + m$ is total deposits and $m \qquad \tau(m \quad + m \quad)$ is excess reserves.

$$
\begin{aligned}
&(13)\ \hat{R}_{at} \quad \left[\frac{h_{e^r} \qquad \tau h_{e^r}}{(1 \quad \tau)\,h_{e^r} \quad 1} \qquad \frac{\tau h_{e^r}}{\tau h_{e^r} + 1}\right] \left[\quad \left(\frac{1}{1 \quad \xi} + \log\left(e_v\right)\right) \xi \hat{\xi}_t + \hat{x}_t^b \quad \xi \hat{e}_{v,t} \right] \\
&+ \left[\frac{\tau h_{e^r}}{(1 \quad \tau)\,h_{e^r} \quad 1} + \frac{\tau h_{e^r}}{\tau h_{e^r} + 1}\right] \hat{\tau}_t \quad \hat{R}_t = 0
\end{aligned}
$$

The coefficients in the canonical form are:

$$
\begin{aligned}
\alpha\ (13,17) &= 1\ :\ \hat{R}_{at} \\
\alpha\ (13,11) &= \xi h_{e^r}\left[\frac{1 \quad \tau}{(1 \quad \tau)\, h_{e^r} \quad 1} \qquad \frac{\tau}{\tau h_{e^r}+1}\right] : \hat{e}_{v,t} \\
\alpha\ (13,13) &= \quad 1 : \hat{R}_t \\
\beta\ (13,13) &= h_{e^r}\left[\frac{1 \quad \tau}{(1 \quad \tau)\, h_{e^r} \quad 1} \qquad \frac{\tau}{\tau h_{e^r}+1}\right]\left(\frac{1}{1 \quad \xi}+\log\left(e_v\right)\right)\xi :\ \hat{\xi}_t \\
\beta\ (13,16) &= \quad h_{e^r}\left[\frac{1 \quad \tau}{(1 \quad \tau)\, h_{e^r} \quad 1} \qquad \frac{\tau}{\tau h_{e^r}+1}\right] :\ \hat{x}_t^b \\
\beta\ (13,4) &= \tau h_{e^r}\left[\frac{1}{(1 \quad \tau)\, h_{e^r} \quad 1}+\frac{1}{\tau h_{e^r}+1}\right] \\
&= \frac{\tau h_{e^r}}{\left[(1 \quad \tau)\, h_{e^r} \quad 1\right]\left(\tau h_{e^r}+1\right)}\ :\ \hat{\tau}_t
\end{aligned}
$$

In the version of the model in which the banking sector is dropped, we must nevertheless have a loan market clearing condition:

$$\psi_{l,t} W_t l_t+\psi_{k,t} P_t r_t^k K_t=M_t^b \quad M_t+X_t .$$

The right side of this equation is the supply of base for the purpose of lending. The left hand side is the corresponding demand. Scale this by dividing by $P_t z_t$:

$$\psi_{l,t} w_t l_t+\psi_{k,t} r_t^k u_t \frac{\bar{k}_t}{\mu_{z,t}}=m_t^b\left(1 \quad m_t+x_t\right). \tag{3.23}$$

Linearizing this:

$$
\begin{aligned}
(25)\ \psi_l w l\left[\hat{\psi}_{l,t}+\hat{w}_t+\hat{l}_t\right]+\psi_k r^k \tfrac{k}{\mu_z}\left[\hat{\psi}_{k,t}+\hat{r}_t^k+\hat{u}_t+\widehat{\bar{k}}_t \quad \hat{\mu}_{z,t}\right] & \\
m^b(1 \quad m+x)\left[\hat{m}_t^b+\tfrac{m m_t \quad x x_t}{\quad m \quad x}\right]= & \qquad 0.
\end{aligned}
$$

The parameters of the reduced form are:

$$
\begin{aligned}
\alpha\ (25,19) &= \psi_l wl : \hat{w}_t,\ \alpha\ (25,20) = \psi_l wl\ :\ \hat{l}_t \\
\alpha\ (25,3) &= \psi_k r^k \frac{\bar{k}}{\mu_z}\ :\ \hat{r}_t^k,\ \alpha\ (25,5) = \psi_k r^k \frac{\bar{k}}{\mu_z}\ :\ \hat{u}_t \\
\alpha\ (25,12) &= \quad m^b (1 \quad m + x)\ :\ \hat{m}_t^b,\ \alpha\ (25,16) = m^b m\ :\ \hat{m}_t \\
\alpha\ (25,21) &= \psi_k r^k \frac{\bar{k}}{\mu_z}\ :\ \widehat{\bar{k}}_t,\ \alpha\ (25,23) = \quad m^b x\ :\ \hat{x}_t \\
\beta\ (25,7) &= \psi_l wl\ :\ \hat{\psi}_{l,t},\ \beta\ (25,10) = \psi_k r^k \frac{\bar{k}}{\mu_z}\ :\ \hat{\psi}_{k,t} \\
\beta\ (25,46) &= \quad \psi_k r^k \frac{\bar{k}}{\mu_z} :\ \hat{\mu}_{z,t}.
\end{aligned}
$$

3.2.5. Household Sector

The definition of u_c^z is:

$$
\begin{aligned}
(14)\ E\ \ u_c^z \hat{u}_{c,t}^z \quad & \left[\frac{\mu_z}{c\,(\mu_z \quad b)} \quad \frac{\mu_z c}{c\ \ (\mu_z \quad b)}\right] \hat{\mu}_{z,t} \quad b\beta \frac{\mu_z c}{c\ \ (\mu_z \quad b)} \hat{\mu}_{z,t} \\
+ & \frac{\mu_z + \beta b}{c\ \ (\mu_z \quad b)} c\hat{c}_t \quad \frac{b\beta\mu_z}{c\ \ (\mu_z \quad b)} c\hat{c}_t \quad \frac{b\mu_z}{c\ \ (\mu_z \quad b)} c\hat{c}_t \quad \Omega_t^\mu \quad = \quad 0.
\end{aligned}
$$

The coefficients in the canonical form are:

$$
\begin{aligned}
\alpha\ (14,18) &= \left(\frac{1}{c\,(\mu_z \quad b)}\right)\ \left[\mu_z + b\ \beta\right] c,:\ \hat{c}_t \\
\alpha\ (14,18) &= \quad b\beta \left(\frac{1}{c\,(\mu_z \quad b)}\right)\ \mu_z c :\ \hat{c}_t \\
\alpha\ (14,18) &= \quad \left(\frac{1}{c\,(\mu_z \quad b)}\right)\ b\mu_z c :\ \hat{c}_t \\
\beta\ (14,46) &= b\beta \left(\frac{1}{c\,(\mu_z \quad b)}\right)\ c\mu_z : \mu_{z,t} \\
\beta\ (14,46) &= \left[\quad \frac{\mu_z}{c\,(\mu_z \quad b)} + \left(\frac{1}{c\,(\mu_z \quad b)}\right)\ c\mu_z\right] : \mu_{z,t}
\end{aligned}
$$

The household's first order condition for time deposits is:

$$(15)\ E\left\{ \hat{\lambda}_{z,t} + \hat{\lambda}_{z,t} \quad \hat{\mu}_{z,t} \quad \hat{\pi}_t \quad \frac{R^e \tau^T}{1+(1 \quad \tau^T)\,R^e}\hat{\tau}_t^T \quad + \frac{R^e\left(1 \quad \tau^T\right)}{1+(1 \quad \tau^T)\,R^e}\hat{R}_t^e \quad \Omega_t^{\mu} \right\} = 0.$$

The household's first order condition for capital is:

$$(24)\ E\left\{ \hat{\lambda}_{zt} + \left[\frac{R^k}{1+R^k}\hat{R}_t^k \quad + \hat{\lambda}_{z,t} \quad \hat{\pi}_t \quad \hat{\mu}_{z,t} \quad \right] \Omega_t \right\}$$

The coefficients in the canonical form are:

$$\begin{aligned}
\alpha\ (24,15) &= 1:\ \hat{\lambda}_{z,t} \\
\alpha\ (24,7) &= \frac{R^k}{1+R^k}:\ \hat{R}_t^k \\
\alpha\ (24,1) &= \quad 1:\ \hat{\pi}_t \\
\alpha\ (24,15) &= \quad 1:\ \hat{\lambda}_{z,t} \\
\beta\ (24,46) &= \quad 1: \hat{\mu}_{z,t}
\end{aligned}$$

The first order condition for currency, M_t :

$$\begin{aligned}
(16)\ \hat{\upsilon}_t + (1 \quad \sigma_q)\,\hat{c}_t + \Bigg[\quad (1 \quad \sigma_q)\left(\theta \quad (1 \quad \theta)\frac{m}{1 \quad m+x}\right) \quad \frac{\frac{\theta}{m} + \frac{\theta}{m \quad x}{}^2 m}{\frac{\theta}{m} \quad \frac{\theta}{m \quad x}}\Bigg]\,\hat{m}_t \\
\left[\frac{(1 \quad \sigma_q)\,(1 \quad \theta)\,x}{1 \quad m+x} \quad \frac{\frac{\theta}{m \quad x}{}^2 x}{\frac{\theta}{m} \quad \frac{\theta}{m \quad x}}\right]\hat{x}_t \\
+\left[\quad (1 \quad \sigma_q)\,(\log(m) \quad \log(1 \quad m+x)) + \frac{1+x}{\theta\,(1+x) \quad m}\right]\theta\hat{\theta}_t \\
(2 \quad \sigma_q)\,\hat{m}_t^b \quad \left[\hat{\lambda}_{z,t} + \frac{\tau^D}{1 \quad \tau^D}\hat{\tau}_t^D + \hat{R}_{a,t}\right] \quad &= \quad 0
\end{aligned}$$

With ACEL preferences, set the coefficient on $\hat{c}_t$ to zero here. The household's first order condition for currency, M_t^b , is:

$$
\begin{aligned}
(17)\ E\ & \frac{\beta}{\pi\mu_z} \upsilon \left(1 \quad \theta\right) \left[c \left(\frac{1}{m}\right)^{\theta} \right]^{\quad \sigma_q} \left(\frac{1}{1 \quad m + x}\right)^{\quad \theta \quad \sigma_q} \quad \left(\frac{1}{m^b}\right)^{\quad \sigma_q} \\
& \hat{\upsilon}_t \quad \frac{\theta \hat{\theta}_t}{1 \quad \theta} + (1 \quad \sigma_q)\hat{c}_t \quad (1 \quad \sigma_q) \log(m)\, \theta \hat{\theta}_t \quad \theta(1 \quad \sigma_q)\hat{m}_t \\
& [(1 \quad \theta)(1 \quad \sigma_q) + 1] \left(\frac{1}{1 \quad m + x}\right) [x\hat{x}_t \quad m\hat{m}_t \quad] \\
& + (1 \quad \sigma_q) \log(1 \quad m + x)\, \theta\hat{\theta}_t \quad (2 \quad \sigma_q)\, \hat{m}_t^b \\
& + \frac{\beta}{\pi\mu_z} \lambda_z \left[1 + \left(1 \quad \tau^D\right) R_a\right] \hat{\lambda}_{z,t} \\
& + \frac{\beta}{\pi\mu_z} \lambda_z \left[\left(1 \quad \tau^D\right) R_a \hat{R}_{a,t} \quad \tau^D R_a \hat{\tau}_t^D \ \right] \quad \lambda_z \left[\hat{\lambda}_{zt} + \hat{\pi}_t \quad + \hat{\mu}_{z,t} \ \right] \Omega_t^{\mu} \\
& = 0.
\end{aligned}
$$

With ACEL preferences, replace c by unity and set $\hat{c}_t$ to zero. We now derive the coefficients in the canonical form are. Let

$$
\Upsilon = \beta \upsilon \left(1 \quad \theta\right) \left[c \left(\frac{1}{m}\right)^{\theta} \right]^{\quad \sigma_q} \left(\frac{1}{1 \quad m + x}\right)^{\quad \theta \quad \sigma_q} \quad \left(\frac{1}{m^b}\right)^{\quad \sigma_q}
$$

$$
\begin{aligned}
\alpha\ (17,18) &= \frac{\Upsilon}{\pi\mu_z}(1 \quad \sigma_q) \\
\alpha\ (17,16) &= \quad \frac{\Upsilon}{\pi\mu_z}\left[\theta(1 \quad \sigma_q) + [(1 \quad \theta)(1 \quad \sigma_q) + 1]\left(\frac{m}{1 \quad m + x}\right)\right] \\
\alpha\ (17,12) &= \quad \frac{\Upsilon}{\pi\mu_z}(2 \quad \sigma_q) \\
\alpha\ (17,15) &= \frac{\beta}{\pi\mu_z}\lambda_z\left[1 + \left(1 \quad \tau^D\right)R_a\right] \\
\alpha\ (17,17) &= \frac{\beta}{\pi\mu_z}\lambda_z\left(1 \quad \tau^D\right)R_a \\
\alpha\ (17,1) &= \lambda_z \\
\alpha\ (17,15) &= \quad \lambda_z : \hat{\lambda}_{z,t} \\
\beta\ (17,40) &= \frac{\Upsilon}{\pi\mu_z} : \hat{\upsilon}_t \\
\beta\ (17,22) &= \quad \frac{\Upsilon}{\pi\mu_z}\left[\frac{\theta}{1 \quad \theta} + (1 \quad \sigma_q)\log(m)\,\theta \quad (1 \quad \sigma_q)\log(1 \quad m + x)\,\theta\right] : \hat{\theta}_t \\
\alpha\ (17,23) &= \quad \frac{\Upsilon}{\pi\mu_z}[(1 \quad \theta)(1 \quad \sigma_q) + 1]\left(\frac{x}{1 \quad m + x}\right) : \hat{x}_t \\
\beta\ (17,25) &= \quad \frac{\beta}{\pi\mu_z}\lambda_z\tau^D R_a : \hat{\tau}_t^D \\
\beta\ (17,46) &= \quad \lambda_z : \hat{\mu}_{z,t}
\end{aligned}
$$

With ACEL preferences, replace c with unity in Υ. Also, $\alpha\ (17,18)$ should be zero. The first order condition for consumption is:

$$
\begin{aligned}
&(18)\ E\ \ u_c^z\hat{u}_{c,t}^z \quad \upsilon c^{\ \ \sigma_q}\left[\frac{1}{m^b}\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1 \quad m + x}\right)^{\quad \theta}\ \right]^{\quad \sigma_q} \\
&\quad [\hat{\upsilon}_t \quad \sigma_q\hat{c}_t + (1 \quad \sigma_q)\left(\quad \hat{m}_t^b \quad \theta_t\hat{m}_t \quad (1 \quad \theta_t)\left(\frac{m}{1 \quad m + x}\hat{m}_t + \frac{x}{1 \quad m + x}\hat{x}_t\right)\right) \\
&+ (1 \quad \sigma_q)\left[\log\left(\frac{1}{m}\right) \quad \log\left(\frac{1}{1 \quad m + x}\right)\right]\theta\hat{\theta}_t] \\
&\quad (1 + \tau^c)\,\lambda_z\left[\frac{\tau^c}{1 + \tau^c}\hat{\tau}_t^c + \hat{\lambda}_{z,t}\right]\ \Omega_t \quad = 0
\end{aligned}
$$

With ACEL preferences, the middle term is replaced by zero. The reduced form wage equation is:

$$(19)\ E\left\{\eta\ \hat{w}_{t} \quad + \eta\ \hat{w}_t + \eta\ \hat{w}_{t} \quad + \eta\ \ \hat{\pi}_{t} \quad + \eta\ \hat{\pi}_t + \eta\ \hat{\pi}_{t} \quad + \eta\ \hat{l}_t + \eta\ \left[\hat{\lambda}_{z,t} \quad \frac{\tau^l}{1 \quad \tau^l}\hat{\tau}^l_t\right] + \eta\ \hat{\zeta}_t\ \Omega_t\right\} = 0$$

where

$$\eta = \begin{pmatrix} b_w\xi_w \\ b_w\left(1+\beta\xi_w\right)+\sigma_L\lambda_w \\ \beta\xi_w b_w \\ b_w\xi_w \\ \xi_w b_w\left(1+\beta\right) \\ b_w\beta\xi_w \\ \sigma_L\left(1 \quad \lambda_w\right) \\ 1 \quad \lambda_w \\ \left(1 \quad \lambda_w\right) \end{pmatrix} = \begin{pmatrix} \eta \\ \eta \\ \eta \\ \eta \\ \eta \\ \eta \\ \eta \\ \eta \\ \eta \end{pmatrix}.$$

3.2.6. Aggregate Restrictions

The resource constraint is:

$$\begin{aligned}(20)\ 0 = \quad & d_y\left[\frac{G\left(\bar{\omega}\right)}{G(\bar{\omega})}\bar{\omega}\widehat{\bar{\omega}}_t + \frac{R^k}{1+R^k}\hat{R}^k_t + \hat{q}_{t} \quad + \hat{\bar{k}}_t \quad \hat{\mu}_{z,t} \quad \hat{\pi}_t\right] + u_y\hat{u}_t + g_y\hat{g}_t + c_y\hat{c}_t + \bar{k}_y\frac{i}{\bar{k}}\hat{\imath}_t \\ & +\Theta(1 \quad \gamma)v_y\hat{v}_t \quad \alpha\left(\hat{u}_t \quad \hat{\mu}_{z,t} + \hat{\bar{k}}_t + \hat{\nu}^k_t\right) \quad (1 \quad \alpha)\left(\hat{l}_t + \hat{\nu}^l_t\right) \quad \hat{\epsilon}_t\end{aligned}$$

$$
\begin{aligned}
\alpha\ (20,6) &= d_y \frac{G\left(\bar{\omega}\right)}{G(\bar{\omega})}\bar{\omega} :\ \widehat{\bar{\omega}}_t \\
\alpha\ (20,7) &= \frac{R^k}{1+R^k} d_y :\ \hat{R}^k_t \\
\alpha\ (20,1) &= \quad d_y :\ \hat{\pi}_t \\
\alpha\ (20,5) &= u_y \quad \alpha :\ \hat{u}_t \\
\alpha\ (20,18) &= c_y :\ \hat{c}_t \\
\alpha\ (20,4) &= \bar{k}_y \frac{i}{\bar{k}} :\ \hat{\imath}_t \\
\alpha\ (20,10) &= \quad \alpha \quad (1 \quad \alpha) :\ \hat{\nu}^l_t \\
\alpha\ (20,20) &= \quad (1 \quad \alpha) :\ \hat{l}_t \\
\alpha\ (20,21) &= d_y \quad \alpha :\ \widehat{\bar{k}}_t \\
\alpha\ (20,9) &= d_y :\ \hat{q}_t \\
\beta\ (20,46) &= \quad d_y + \alpha :\ \hat{\mu}_{z,t} \\
\beta\ (20,37) &= g_y :\ \hat{g}_t \\
\beta\ (20,52) &= \quad 1 :\ \hat{\epsilon}_t
\end{aligned}
$$

where

$$\bar{k}_y = \frac{\bar{k}}{y+\phi+d},$$

and the object in square brackets corresponds to the resources used up in monitoring.

$$(21)\ \widehat{\bar{k}}_t \qquad \frac{1 \quad \delta}{\mu_z}\left(\widehat{\bar{k}}_t \quad \hat{\mu}_{z,t}\right) \quad \frac{i}{\bar{k}}\hat{\imath}_t = 0.$$

Monetary policy is represented by:

$$(22)\ \hat{m}^b_t + \frac{x}{1+x}\hat{x}_t \quad \hat{\pi}_t \qquad \hat{\mu}_{z,t} \qquad \hat{m}^b_t \quad = 0$$

The parameters in the reduced form are:

$$
\begin{aligned}
\alpha\ (22,1) &= \quad 1 :\ \hat{\pi}_t \quad ,\ \alpha\ (22,12) = \quad 1 : \hat{m}^b_t \quad , \\
\alpha\ (22,12) &= 1 : \hat{m}^b_t,\ \alpha\ (22,23) = \frac{x}{1+x} :\ \hat{x}_t, \\
\beta\ (22,46) &= \quad 1 : \hat{\mu}_{z,t} \quad .
\end{aligned}
$$

The timing of this equation could be changed to:

$$(22)\ \hat{m}^b_t \quad + \frac{x}{1+x}\hat{x}_t \qquad \hat{\pi}_t \quad \hat{\mu}_{z,t} \quad \hat{m}^b_t = 0$$

3.2.7. Monetary Policy

Monetary policy has the following representation:

$$(23)\ \hat{x}_t = \sum_{i}^{p} x_{it},$$

where the x_{it}'s are functions of the underlying shocks.

3.2.8. Other Variables

The currency to deposit ratio is:

$$d_t^c = \frac{m_t}{1 \quad m_t + x_t + \psi_{l,t} \frac{w_t}{m_t^b} l_t + \psi_{k,t} \frac{r_t^k u_t k_t}{\mu_{z,t} m_t^b}},$$

with

$$d^c = \frac{m}{1 \quad m + x + \psi_l \frac{w}{m^b} l + \psi_k \frac{r^k k}{\mu_z m^b}}$$

$$\begin{aligned}
\hat{d}_t^c &= \hat{m}_t \quad \left[1 \quad m + x + \psi_l \frac{w}{m^b} l + \psi_k \frac{r^k k}{\mu_z m^b}\right] \\
&= \hat{m}_t \quad \frac{d\left[1 \quad m_t + x_t + \psi_{l,t} \frac{w_t}{m_t^b} l_t + \psi_{k,t} \frac{r_t^k u_t k_t}{\mu_{z,t} m_t^b}\right]}{1 \quad m + x + \psi_l \frac{w}{m^b} l + \psi_k \frac{r^k k}{\mu_z m^b}} \\
&= \hat{m}_t \quad \frac{d^c}{m} d \left[1 \quad m_t + x_t + \psi_{l,t} \frac{w_t}{m_t^b} l_t + \psi_{k,t} \frac{r_t^k u_t \bar{k}_t}{\mu_{z,t} m_t^b}\right]
\end{aligned}$$

so

$$\begin{aligned}
\hat{d}_t^c &= \hat{m}_t \\
&\quad + d^c \hat{m}_t \quad \frac{d^c x}{m} \hat{x}_t \quad \frac{d^c \psi_l}{m} \frac{w}{m^b} l \left[\hat{\psi}_{l,t} + \hat{w}_t \quad \hat{m}_t^b + \hat{l}_t\right] \\
&\quad \frac{d^c \psi_k}{m} \frac{r^k k}{\mu_z m^b} \left[\hat{\psi}_{k,t} + \hat{r}_t^k + \hat{u}_t + \widehat{\bar{k}}_t \quad \hat{\mu}_{z,t} \quad \hat{m}_t^b\right]
\end{aligned}$$