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ABSTRACT

Essays in Investment and Macroeconomics

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Although only a small component of national expenditure, aggregate investment is crucial in the development of nations and in the fluctuation of their economies. The three essays presented study the macroeconomic implications of the time firms need to invest, and of the liquidity of the collateral used to secure the loans through which firms finance their capital expenditure.

The second chapter presents a novel formulation of the time-to-build (TTB) assumption, whereby firms invest in multiple investment projects that have complementarities, and where the duration of the project is uncertain. The TTB model is shown to capture well the response of aggregate investment to shocks. Further the model is proposed as a possible explanation of the empirical failure of capital adjustment cost models. The model is shown to be equivalent, up to first order linearization, to investment adjustment cost models.

The third chapter compares properties of the business cycle of high and low income economies, and shows how the TTB model presented in the first chapter helps to understand the empirical difference in business fluctuations between the two income groups. In low income countries projects last more due to institutional constraints such as government regulation, import restrictions and financial underdevelopment.

The fourth chapter presents a model where the inter-sectoral allocation of capital and the liquidity of the assets pledged by borrowers as collateral are jointly determined. Due to feedback effects between the liquidity of the collateral and the levels of investment, two equilibria can coexist in each sector: in one investment and the liquidity are low, while in the other both are high. The paper provides conditions such that the aggregate equilibrium involves a misallocation of capital across sectors.

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CHAPTER 1

Introduction

This thesis consists of three essays on investment and macroeconomics.

Chapter 2 presents a novel specification of the time-to-build (TTB) assumption that has a number of new implications for the theory of investment. Recent studies at the macroeconomic and microeconomic level have downplayed the importance of TTB as a model of investment. At the macroeconomic level Rouwenhorst [1991] and Cogley and Nason [1995], show that the formulation of TTB, first introduced by Kydland and Prescott [1982] (KP), has only little effects on the response of the aggregate variables. Further, few empirical models of investment at the microeconomic level consider the TTB assumption, while Oliner, Rudebusch, and Sichel [1995a] argue that TTB falls short in explaining the sluggish response of investment.

The novel formulation presented in this paper departs from the one considered by KP in two ways: firms invest in many investment projects that have complementarities, and the duration of each investment project is uncertain. The investment decision of the firm is in part predetermined, because the firm cannot change the scale of the ongoing projects. Further, due to the complementarity of investment projects, the firm fully compensates the earlier commitments when adjusting the scale of the projects under its control. The resulting dynamic response of investment is gradual, due to the direct link between investment decisions at different dates.

The novel TTB formulation is compared to capital (e.g. Hayashi [1982]) and investment adjustment cost models (e.g. Christiano, Eichenbaum, and Evans [2005]). The TTB formulation is shown to be equivalent, up to first order linearization, to investment adjustment cost models. The response of the TTB model is instead different from the one of capital adjustment cost models, where investment swiftly responds to the realization of the shocks. A vast empirical literature has tested the investment model with (convex) capital adjustment costs, and the model is usually rejected in the data. In the chapter, I simulate artificial data using the TTB model calibrated on US sectoral data, and then estimate the empirical model with convex adjustment costs. Some of the empirical failures of the capital adjustment cost models are shown to be consistent with a misspecification of the empirical regression model when the TTB technology underlying the data-generating process is not taken into account in the empirical model.

Finally, the TTB specification is embedded in an RBC model and shown to capture well the response of aggregate investment to aggregate productivity shocks. Due to the equivalence result with the investment adjustment cost model, it is argued that the TTB model also helps explain the response of aggregate investment to fiscal and monetary policy shocks.

Chapter 3 presents new evidence that, relative to high income countries (2003 World Bank Classification), business cycles in low income economies are characterized by a higher volatility of aggregate consumption relative to GDP, a low correlation of investment and GDP, and a more procyclical trade balance. I argue that institutional constraints increase the time needed to build and plan for investment, so that investment cannot swiftly respond to innovations in exogenous shocks.

Ample evidence supports the view that firms in LDCs operate under severe institutional constraints (Tybout [2000]), because of financial underdevelopment (Levine [Forthcoming]), import barriers (Loayza, Oviedo, and Serven [2004]) and bureaucratic delays (Loayza, Oviedo, and Serven [2004]).

In order to analyze the role of longer project duration on the LDCs' business fluctuations, I augment a standard closed and small open economy (SOE) RBC model with the specification of TTB presented in Chapter 2. LDCs' economies differ from developed ones in terms of the duration of projects. Numerical simulations of both models show that investment cyclicality falls as a direct consequence of its *delayed* response to shocks. In the SOE model augmented with TTB, trade balance cyclicality increases with lower investment flexibility, due to the lower response of investment to the shocks. Households can perfectly smooth income fluctuations in the international financial markets; therefore the volatility of consumption is not affected by the lower investment flexibility. In a closed economy, on the other hand, as investment flexibility falls consumption adjusts to shocks relatively more, i.e. the volatility of consumption increases.

In Chapter 4 I present a model where the inter-sectoral allocation of capital and the liquidity of the assets that secure loans are jointly determined. Would-be entrepreneurs need to borrow in order to invest, and lenders prefer to lend in sectors where the collateral is more liquid in order to reduce the cost incurred in case of default. The liquidity of the collateral in turn depends on the level of investment, and thus on the amount of loans issued to a specific sector.

Due to the imperfect substitutability and a common production function, the levels of investment are equalized across sectors absent contracting frictions. As a result of the feedback effects between investment levels and collateral values, however, multiple (stable) equilibria may coexist in each sector. Entrepreneurs are constrained in the amount that they borrow in only one of these equilibria, and, for a given interest rate, investment is lower in this equilibrium. In general equilibrium, aggregate investment will be *necessarily* misallocated across sectors for a set of parameter values. Recent studies find evidence of capital misallocation across sectors, and financial frictions are usually considered as a potential source of this misallocation. But for the misallocation to occur, these frictions need to be coupled with heterogeneity in entrepreneurs' or technology's characteristics, such as entrepreneurs incomes. The mechanism presented in this model, instead, is sufficient for a misallocation to occur even when all agents and production technologies are similar. Thus frictions arising from imperfect contractibility are likely to affect real outcomes even when the empirical measures of heterogeneity are small.

CHAPTER 2

Resuscitating Time-To-Build

2.1. Introduction

Due to the time required to build (TTB) and plan (TTP) for investment, capital accumulation is an often lengthy process: should this feature be modelled when characterizing firms' investment decisions? In contrast with recent literature, this paper argues that it should. The departure from the literature is a novel specification of the TTB assumption that has a number of new implications for the theory of investment, and great analytical tractability.

The idea of time-to-build and plan is not new in economics as it can be traced back in time at least to the work of Kalecki [1935]. Although probably acknowledged by most economists, the assumption has not been often considered in both empirical and theoretical models of investment (see Nickell [1978]). In their seminal contribution to quantitative macroeconomic modelling, instead, Kydland and Prescott [1982] (KP hereafter) argue that TTB is key to understanding post-World War II business cycle fluctuations in the United States. In the specification of TTB considered by KP, and almost universally by the subsequent literature, firms invest in one type of investment that requires a fixed amount of resources for each period up to maturity, and the investment projects increase the capital stock only upon maturity.

More recent studies have, however, downplayed the role of TTB. Rouwenhorst [1991] shows that KP's formulation of TTB has little effect on the response of a real business cycle (RBC) model to productivity shocks, but for introducing unrealistic cyclicalities in the response of the main aggregates. Similarly, Cogley and Nason [1995] shows that KP's TTB formulation has little effects on the persistence of output growth. Following these works, few macroeconomic studies have explicitly considered the length of the investment process. The common approach is to assume a one period delay in the capital accumulation process no matter if the time unit of analysis is a quarter or a year.[1] Further, few empirical models of investment at the microeconomic level have considered the TTB assumption. An interesting exception is the work of Oliner, Rudebusch, and Sichel [1995a] who shows that TTB falls short in explaining the sluggish response of investment.

The novel formulation presented in this paper departs from the one considered by KP in two ways: firms invest in many investment projects that have complementarities, and, the duration of each investment project is uncertain. Because, as in KP, the scale of the ongoing projects is fixed, the investment decision of the firm is in part predetermined by earlier commitments. Further, due to the complementarity of the investment projects, the firm optimally decides not to fully compensate the earlier commitments when adjusting the scale of the projects under its control.

The optimal investment decision of the firm is first analyzed in partial equilibrium. The representative firm, calibrated on US manufacturing data, is subject to productivity and interest rate shocks. The dynamic responses of the model show that investment moves only gradually after the realization of the shocks, due to the direct link between

[1]One period delay means that current investment increases the capital stock only in the next period.

investment decisions at different dates. Two assumptions are crucial for the analytical tractability of the model. First, the duration of the investment project follows a Poisson process, so that the probability that a project matures is independent of when it started. Second, there is imperfect substitutability between the investment types but the capital stock of the firm is a homogeneous good that depreciates at a constant rate. Although these assumptions greatly simplify the analysis, they are not key for the hump-shaped response to exist. The key elements are a source of imperfect substitutability among the investment types, and the ex-post heterogeneity in the duration of the investment projects, not necessarily due to an uncertain duration. The paper makes these points by comparing the model with uncertainty with a deterministic formulations.

The novel TTB formulation is then compared to capital (e.g. Hayashi [1982]) and investment adjustment cost models (e.g. Christiano, Eichenbaum, and Evans [2005]). With capital adjustment costs, the firm pays costs that depend on the change in the capital stock, while with investment adjustment costs the cost of adjustment depends on the difference in the investment levels. The TTB formulation is shown to be equivalent, up to first order linearization, to investment adjustment cost models; thus the model presented in the paper directly links these models with the TTB assumption.

The response of the TTB model is instead different from the one of capital adjustment cost models, where investment swiftly responds to the realization of the shocks. A vast empirical literature has tested the investment model with (convex) capital adjustment costs (see e.g. Chirinko [1993]), and the model is usually rejected in the data. The level of Tobin's Q is not a sufficient statistic for the investment decision, as predicted by the model, and the estimated adjustment costs appear to be unreasonably large. Further, investment appears to be more sluggish than predicted by the model, as past realizations of Tobin's Q enter significantly in the regressions and error terms have a high autocorrelation. Finally the current level of cash flows appear to be an important determinant of investment decisions, a fact that is often interpreted as a sign of financial frictions (e.g. Hubbard [1998]). In the paper I generate artificial data using the TTB model calibrated on US sectoral data, and then estimate the empirical model with convex adjustment costs. The three empirical findings discussed above are shown to be consistent with a misspecification of the empirical regression model when the TTB technology underlying the data-generating process is not taken into account in the empirical model.

Finally the TTB model is embedded in an otherwise canonical RBC model. As for the partial equilibrium model, the dynamic response of investment to a technology shock is hump shaped, and thus describes well the empirical response of aggregate investment on US data (Christiano, Eichenbaum, and Vigfusson [2004]). Further, neither investment nor the other variables display the cyclicalities highlighted by Rouwenhorst [1991] for the KP formulation. Investment adjustment cost models have shown to help explain the response of aggregate investment and other macroeconomic variables to both fiscal (Basu and Kimball [2005], Burnside, Eichenbaum, and Fisher [2004]) and monetary policy shocks (Christiano, Eichenbaum, and Evans [2005]). Due to the equivalence result between the TTB and the investment adjustment cost model, the TTB formulation of this paper thus helps understand the response of aggregate economies also to fiscal and monetary policy shocks.

The remaining of the paper is organized as follows. The TTB model with uncertain duration is presented in the next Section, and then solved in Section 2.3.3. Section 2.4

compares the model with two alternative TTB formulations with deterministic duration. Section 2.5 relates the model to investment and capital adjustment cost models, while Section 2.6 discusses the implication of the model for empirical tests of capital adjustment cost models. Finally Section 2.7 embeds the TTB model in an otherwise canonical RBC model.

2.2. The Model

Consider a firm that produces the good, Y_t, using the stock of capital, K_{t-1}, and a vector of variable factor inputs $\mathbf{x}_t$ with the production function: $Y_t = f(A_t, K_{t-1}, \mathbf{x}_t)$, where A_t is the level of productivity. Let $p_{x,t}$ the vector price of variable factors. The date t flow of firms' revenues net of variable factor costs, after maximizing out the variable factors are $\pi_t(K_{t-1}) \equiv \max_{\mathbf{x}_t} f(K_{t-1}, \mathbf{x}_t) - p'_{x,t}\mathbf{x}_t$. The function $\pi_t(\cdot)$ is increasing and weakly concave.

To increase the stock of capital, the firm invests in a fixed but large number of perfectly symmetric investment goods indexed by their type $j \in [0, 1]$. It takes time to build and plan for the construction of each investment good. The firm chooses the desired quantity of type-j investment *good* when it starts a type-j investment *project*. Each firm can only run one project per-investment good at a time, and the scale of the project cannot be modified once initiated. If a project matures at t a new scale may be chosen at $t + 1$. Let $\iota_t(j)$ denote the date t scale of the type-j project, $M_t \subseteq [0, 1]$ be the set of investment projects maturing at t, and NM_t its complement in $[0, 1]$. The scale of a project that has not matured is fixed, thus $\iota_t(j) = \iota_{t-1}(j)$ for $j \in NM_{t-1}$. This is a key element of the model as it implies that, at each date, part of the investment decision is predetermined as some projects are still under way, and thus outside the control of the firm. It is also important to note that the notation maintains information on the scale of the investment for each type,$\iota_t(j)$, but omits an explicit indication of the date when the scale of each investment was chosen. In particular, the scale of all projects that are still under way where chosen in the past, and thus cannot include any information on the current realization of the shocks.

The duration of each investment project is uncertain: the maturity of each project follows a Poisson-process with arrival rate θ. A project started at date t has a probability θ of being completed at the same date t. Uncompleted projects mature with the constant probability θ at each of the subsequent dates, so that a firm expects projects to mature at date $t + (1/\theta - 1)$.

An investment project increases the capital stock only when it matures. Accordingly let the variable

$$\iota_t^m(j) = \begin{cases} \iota_t(j) & \text{if } j \in M_t, \\ 0 & \text{otherwise,} \end{cases} \tag{2.1}$$

denote the level of type j investment that will increase the capital stock. The variable $\iota_t^m(j)$ is equal to the scale of the project, $\iota_t(j)$, if it matures at date t ($j \in M_t$), and it is equal to zero otherwise. Investment goods are characterized by complementarities – the return to each good is increasing with the availability of the others – and each $\iota_t^m(j)$ enters symmetrically in the date t investment *basket*

$$I_t \equiv \left(\int_0^1 \iota_t^m(j)^{1-1/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}, \tag{2.2}$$

where $\varepsilon > 1$, so that none of the investment goods are essential in increasing the capital stock.

The level of the investment basket I_t increases the firm's capital stock, which depreciates at the constant rate δ :

$$K_t = (1 - \delta)K_{t-1} + I_t. \tag{2.3}$$

The installation of capital is costly: the firm pays adjustment costs $C(I_t, K_{t-1}) \equiv c(I_t/K_{t-1})\, K_{t-1}$, where $c(\cdot)$ is increasing and convex and such that $c'(\delta) = c(\delta) = 0$ and $c''(\delta) = \phi$.

Now consider the level of investment expenditure. An investment project started at t requires $\theta\iota_t(j)$ units of Y_s at all dates $s \geq t$ up to maturity. The date t investment expenditure is equal to the sum of all projects' expenditure

$$E_t \equiv \theta \int_0^1 \iota_t(j)\; dj. \tag{2.4}$$

The firm uses on average one unit of Y for each unit of investment. Indeed, the projects last on average $1/\theta$ periods and the firm uses $\theta\iota_t(j)$ units of Y_t per period. A time-to-plan formulation of the model, in which investment expenditure only occurs when the project matures, would yield the same expenditure function as in (3.2). Because the maturity of the project follows a Poisson-process, it follows that $E_t = \int_{j \in M_t} \iota_t(j)dj = \theta \int_0^1 \iota_t(j)dj$.

The firm acts in the interest of the shareholders and maximizes at each date t value of future dividends discounted using the gross rate R_{s+1} between each subsequent dates date s and $s+1$. Thus the firm solves

$$\max \ \mathbb{E}_t \left\{ \sum_{r=0}^{\infty} \left(\prod_{s=1}^{r} R_{t+r}^{-1} \right) D_{t+r} \right\}, \tag{2.5}$$

with respect to K_{t+r} and $\{\iota(j)\}_{j \in M_{t+r-1}}$,subject to law of motion of the capital stock (2.3) and the investment technology discussed above. The dividend at date $t+r$ is

$$D_{t+r} = \pi_{t+r}(K_{t-1+r}) - E_{t+r} - C(I_{t+r}, K_{t+r-1}). \tag{2.6}$$

At the beginning of each period t, the firm observes the level of productivity, A_t, the price of the variable factor inputs, the interest rate R_t and which projects that have matured in the previous period $j \in M_{t-1}$. Given the stock of K_{t-1}, the firm then decides the level production, Y_t, and the corresponding vector of variable inputs $\mathbf{x}_t$. The firm then decides on how much to invest. This decision is only in part under the control of the firm. Investment in the projects still under way, $j \in NM_{t-1}$, cannot be modified, while the firm decides how to invest for investments types that have matured in the previous period, $j \in M_{t-1}$. The uncertainty regarding which projects matures at date t is then resolved. The level of investment for the maturing projects increase the capital stock at t, and their scale is under control of the firm. The scale of all remaining projects remains fixed in the following period.

2.3. Solution of the Model

Due to the uncertain duration of the projects, when it picks a new scale of investment, the firm takes into account the implication on all future levels of the investment basket and expenditure. The optimal investment decision can be characterized using two different solution strategies.

A direct approach is to choose $\iota_t(j)$ for $j \in M_{t-1}$ by directly considering its effects on all future levels of the investment basket and expenditure. The first order condition that characterize the decision involves the future values of E_t and I_t at all future dates. Following the literature on staggered pricing decisions (e.g. Yun [1996]) the first order condition can be linearized, and after some algebra, it is possible to rewrite it only in terms of variables at two consecutive dates.

A simpler solution method makes use of the large number of investment projects in which the firm invests: despite the uncertain duration of each investment project, there is no uncertainty in the overall investment decision of the firm. Further it is possible to show that the levels of E_t and I_t depend on earlier investment decisions only through the lagged levels of E_{t-1} and I_{t-1}. As it will be shown in the next subsection this greatly simplifies the solution of the investment problem by separating it into an intratemporal allocation problem and an intertemporal decision. The remaining of the section specifies the functional forms left unmodelled in the previous section. Finally the model is calibrated using US manufacturing data, and then simulated and discussed.

2.3.1. Optimal Investment Decision

The optimal investment decision is solved by first characterizing the date t intratemporal decision of the firm of how to allocate investment among the different investment goods, whose scale is under control of the firm, $j \in M_{t-1}$. It is then possible to characterize the intertemporal investment decision only in terms of E_t, I_t and K_t, and not in terms of the scale of the projects $\iota_t(j)$'s.

Consider the intratemporal investment decision of the firm. Because the maturity of projects follows a Poisson process, all projects have an equal probability of maturing irrespective of their initial starting date. Then, the date t averages of the quantities $\iota_t(j)$ and $\iota_t(j)^{1-1/\varepsilon}$ among the projects that mature at the end of the period, $j \in M_t$, and that do not mature, $j \in NM_t$, are equal. Furthermore, because of the large number of projects, the total fraction of projects that matured is equal to θ, and the remaining fraction $1-\theta$ are projects that have not matured. It follows from this discussion that

(2.7)
$$\frac{\int_{j\in M_t} x_t(j)\ dj}{\theta} = \frac{\int_{j\in NM_t} x_t(j)\ dj}{1-\theta} = \int_0^1 x_t(j)\ dj \ \text{ for each } \ x_t(j) = \{\iota_t(j),\, \iota_t(j)^{1-1/\varepsilon}\}.$$

where the expression after the second equality is the average scale over all projects. Using (3.7), (3.4) becomes

$$I_t = \left(\theta \int_0^1 \iota_t(j)^{1-1/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}. \tag{2.8}$$

Now follows the crucial step of the intratemporal investment problem, which is to express investment expenditure, E_t, and the investment basket, I_t, in terms of their respective lagged values and the scale of the projects that matured at $t-1$. First note that I_t can be expressed as

$$
\begin{aligned}
I_t &= \left(\theta\left(\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon}dj + \int_{j\in NM_{t-1}} \iota_t(j)^{1-1/\varepsilon}dj\right)\right)^{\varepsilon/(\varepsilon-1)} = \\
&= \left(\theta\left(\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon}dj + (1-\theta)\left(\int_{j\in NM_{t-1}} \left(\iota_{t-1}(j)\right)^{1-1/\varepsilon} dj\right)/(1-\theta)\right)\right)^{\varepsilon/(\varepsilon-1)} = \\
&\left(\theta\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon}dj + (1-\theta)\left(I_{t-1}\right)^{1-1/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}. \qquad (2.9)
\end{aligned}
$$

The expression after the first equality follows simply from rewriting the integral in (3.8). The second equality makes use of the fact that for projects that did not mature in the last period, $j \in NM_{t-1}$, the scale of the investment project is fixed, or: $\iota_t(j) = \iota_{t-1}(j)$. The third equality follows from (3.8) lagged by one period and (3.7). Using analogous steps, (3.2) can be rewritten as

$$
E_t = \theta\int_{j\in M_{t-1}} \iota_t(j)dj + (1-\theta)\ E_{t-1}. \qquad (2.10)
$$

The firm only controls the current expenditure for a fraction θ of the investment projects that had matured, while the remaining fraction $1-\theta$ is predetermined.

The firm will choose the same scale for all maturing investment projects as long as $\varepsilon < \infty$, because they enter symmetrically into I_t and all have the same expected cost of one unit of Y. This result follows from a simple expenditure minimization of (3.10) subject to (3.9) with respect to all projects that had matured in the previous period $\{\iota_t(j)\}_{j\in M_{t-1}}$. Let ι_t be the optimal scale that is chosen by the firm, then $\iota_t(j) = \iota_t$ for all $j \in M_{t-1}$.[2] Using this result in (3.9) and (3.10), and then substituting ι_t from (3.9) in (3.10) yields

$$E_t = \Omega(I_t, I_{t-1}) + (1-\theta)E_{t-1}, \tag{2.11}$$

where

$$\Omega(I_t, I_{t-1}) \equiv \theta^{\frac{2}{1-\varepsilon}} \left(I_t^{1-\frac{1}{\varepsilon}} - (1-\theta)\ I_{t-1}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}.$$

The function $\Omega(I_t, I_{t-1})$ is increasing in its first argument and decreasing in the second. Indeed, a larger difference between the current and the lagged level of the investment basket corresponds to a larger scale of the investment project that are under the control of the firm. Condition (3.11) is the only additional condition in an otherwise standard intertemporal firm maximization problem.

The Bellman-Jacobi equation associated to the intertemporal problem is

(2.12)

$$V_t(K_{t-1}, E_{t-1}, I_{t-1}) = \max_{\{K_t, I_t, E_t\}} \ \pi_t(K_{t-1}) - E_t - C(I_t, K_{t-1}) + \mathbb{E}_t\ R_{t+1}^{-1}\ V_t(K_{t-1}, E_{t-1}, I_{t-1}),$$

subject to the constraint (3.11) and the law of motion of the capital stock (2.3). In the general case in which $\varepsilon < \infty$ and $\theta < 1$, the value function, as well as the optimal investment rules, depend directly on the lagged levels of E_t and I_t. As it will be discussed below, this is a crucial difference between the current model and one where investment does not require time to be built. The value function is also time dependent due to the temporal dependence of prices (p_{xt}, R_t) and productivity shock (A_t).

[2]Because the $\iota_t(j)$'s affect future quantities only through E_t and I_t, the optimal choice of $\iota_t(j)$ in the static minization problem is equivalent to that following from the intertemporal program.

Let q_t and $-\mu_t$ be the date t shadow values of K_t and E_t respectively. The first order conditions associated with (2.12) are

$$
\begin{aligned}
&(K_t) \quad q_t = \mathbb{E}_t R_{t+1}^{-1} V_{1,t+1}(K_t, I_t, E_t),\\
(2.13) \qquad &(E_t) \quad \mu_t = 1 - \mathbb{E}_t R_{t+1}^{-1} V_{2,t+1}(K_t, I_t, E_t),\\
&(I_t) \quad \mathbb{E}_t R_{t+1}^{-1} V_{3,t+1}(K_t, I_t, E_t) + q_t = C_{1,t}(I_t, K_{t-1}) + \mu_t \Omega_1(I_t, I_{t-1}),
\end{aligned}
$$

where a numeric subscript indicates the argument with respect to which a derivative is taken. The first order condition with respect to K_t equalizes the shadow value of capital, q_t to the discounted marginal partial change in the discounted value function (or marginal Q) which, using the envelope condition with respect to K_{t-1}, is:

$$V_{1t}(K_{t-1}, I_{t-1}, E_{t-1}) = \pi_{1,t} + (1-\delta)q_t - C_{2,t}(I_t, K_{t-1}).$$

The first order condition with respect to E_t, equates E_t's shadow *cost*, $\mu_t > 0$, to the sum the cost of one additional unit of E_t and the marginal increase of $(1-\theta)$ in E_{t+1} evaluated at E_{t+1}'s shadow *cost*. Indeed using the envelope condition for E_{t-1} one obtains that

$$V_{2t}(K_{t-1}, I_{t-1}, E_{t-1}) = -(1-\theta)\mu_t.$$

The first order condition with respect to I_t equates the marginal benefit to the cost of an additional unit of I_t. The marginal benefit is equal to the marginal increase in the capital stock evaluated at q_t and the discounted marginal reduction in E_{t+1} evaluated at μ_{t+1}. Indeed from the enveloped condition for E_{t-1} it follows that

$$V_{3t}(K_{t-1}, I_{t-1}, E_{t-1}) = -\mu_t \ \Omega_2(I_t, I_{t-1}).$$

The marginal cost of an additional unit of I_t, is equal to the sum of the marginal increase in the installation cost and in the investment expenditure evaluated at μ_t.

Substituting in the partial derivatives of the value function from the envelope conditions above into the (2.13) one obtains

$$
\begin{aligned}
&(K_t) \quad q_t = \mathbb{E}_t R_{t+1}^{-1}(\pi_{1,t+1} + (1-\delta)q_{t+1} - C_{2,t+1}(I_{t+1}, K_t)), \\
&(E_t) \quad \mu_t = 1 + (1-\theta)\mathbb{E}_t R_{t+1}^{-1}\mu_{t+1}, \\
&(I_t) \quad q_t - \mathbb{E}_t R_{t+1}^{-1}\mu_{t+1}\ \Omega_2(I_{t+1}, I_t) = C_{1,t}(I_t, K_{t-1}) + \mu_t\ \Omega_1(I_t, I_{t-1}),
\end{aligned}
\tag{2.14}
$$

that fully characterize firm's optimal investment decision.

2.3.2. Parametrization of $\pi_t(K_{t-1})$ and Steady State

The cash flow function $\pi_t(\cdot)$, which has been left unmodelled so far, is specified in the first part of this section. Using the functional form for $\pi_t(\cdot)$ and the first order conditions (2.14), the model is then solved in steady state.

A consistent portion of the investment literature considers the case of perfectly competitive firms that operate a constant return to scale production function. These assumptions simplify the empirical test of models of investment with adjustment costs, as marginal Q, a crucial determinant of the investment decisions but not observed by the researcher, is equal to average Q, which is easily measurable (Hayashi [1982]).

In this section I consider, instead, a monopolistically competitive firm that operates an increasing returns to scale production function, due to overhead costs. There are two advantages in considering this model over the competitive and constant return to scale one. The first, is that this model fits well with the empirical evidence at the sectoral level on US data, where firms tend to set prices above marginal costs and economic profits are close to zero (e.g. Hall [1988]).[3] The second, is that the resulting cash flow function $\pi_t(\cdot)$ is strictly concave, so that the scale of the firm, measured by its capital stock, is pinned down in the steady state of the model. The model will be solved in the next section by linearization around the non-stochastic steady state.[4]

The firm produces Y_t using the production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} - \phi, \tag{2.15}$$

with $0 < \alpha < 1$ and $\phi > 0$. The only variable factor of production is labor, L_t, and the firm takes as given the wage rate w_t. The parameter ϕ is an overhead fixed cost such that economic profits are equal to zero in steady state. The firm faces the isoelastic demand function

$$p_t = z_t Y_t^{-\eta},$$

where $0 < \eta < 1$ is the inverse of the elasticity of demand, and z_t is a demand shifter. Solving (2.15) for L_t, the cash flow function can be written as

$$\pi_t(K_{t-1}) = \max_{Y_t} \; z_t Y_t^{1-\eta} - w_t \left(\frac{Y_t + \phi}{A_t K_t^{a}} \right)^{\frac{1}{1-\alpha}} . \tag{2.16}$$

[3]Aside from the empirical evidence, the assumption of zero economic profits in steady state, implies that even in the presence of imperfect competition cost and revenue based factor shares are equal. This simplifies the calculation of the production function elasticity with respect to the production factors (see Hall [1988]).

[4]With perfect competition and constant returns to scale production function, the scale of the firm is indeterminate. Because linearization tecniques are local, the quality of the overall approximation might be poor when linearizing around other points of interest (e.g. sample averages).

The choice of Y_t yields that the firm sets its price as a constant markup over marginal cost;

$$p_t = \frac{1}{1-\eta} MC_t, \tag{2.17}$$

where the marginal cost is:

$$MC_t = w_t(1-\alpha)^{-1}((Y_t+\phi)^{\alpha}\ (A_t K_t)^{-1})^{1/(1-\alpha)}.$$

Using the envelope theorem, the derivative of the cash-flow function is then

$$\pi_t'(K_{t-1}) = \alpha\ MC_t \left(\frac{Y_t+\phi}{K_t}\right). \tag{2.18}$$

The next step is to compute the value of economic profits in steady state. To pin down the value of ϕ this value is then set to zero. I will further normalize the steady state values of the productivity shock, the wage rate and the demand shifter to one. Then $A_{ss}=1$, $w_{ss}=1$ and $z_{ss}=1$, where for all variables the subscript ss denote the steady state level of each corresponding variable. Note that the shadow rental rate on capital is given by (2.18) and that the total variable cost is $w_t L_t = (1-\alpha)(Y_t+\phi)MC_t$. Thus using (2.17) the value of economic profits in steady state is

$$\pi_{ss}(K_{ss}) - \pi_{ss}'(K_{ss})K_{ss} = p_{ss}(Y_{ss} - (1-\varepsilon)(Y_{ss}+\phi)). \tag{2.19}$$

Equating (2.19) to zero, the overhead cost is

$$\phi = \frac{\eta}{1-\eta} Y_{ss}. \tag{2.20}$$

Evaluating (E_t) of (2.14) at steady state, the shadow cost of investment expenditure is equal to the discounted value of expenditure costs

$$\mu_{ss} = \frac{1}{(1 - R^{-1}(1-\theta))}. \tag{2.21}$$

From (I_t) and (2.21) of (2.14) the shadow price of capital is

$$q_{ss} = \theta^{-\frac{1}{\varepsilon - 1}}. \tag{2.22}$$

The shadow value of capital in steady state is larger than one for the following reason. Investment projects increase the capital stock when they mature. For given E_{ss} the size of I_{ss} falls with θ, due to the lower fraction (θ) of projects mature in each period and the complementarity between the investment goods.[5] The higher shadow price of capital will reduce the the value of K_{ss} in steady state. Using (2.16), (2.20), (2.22) in (I_t) of (2.14) it follows that

$$K_{ss} = \Lambda \left(\frac{\alpha \theta^{\frac{1}{\varepsilon - 1}}}{R - (1 - \delta)} \right)^{\frac{\alpha + \eta(1-\alpha)}{\eta}},$$

where $\Lambda \equiv \left((1-\eta)^{1-\eta} (1-\alpha)^{(1-\alpha)(1-\eta)} \right)^{1/\eta}$. From (2.17) evaluated in steady state then

$$Y_{ss} = \left((1-\eta)^{\frac{1}{1-\alpha}} (1-\alpha) \right)^{\frac{1-\alpha}{\alpha + \eta(1-\alpha)}} K_{ss}^{\frac{\alpha}{\alpha + \eta(1-\alpha)}},$$

which used into (2.20) yields the value of ϕ.

[5]It is possible to eliminate the steady state inefficiency, by assuming that each investment project costs $\theta^{\frac{\varepsilon}{\varepsilon - 1}}$ per period rather than θ.

Name	Variable	Value
Gross Interest Rate in s.s.	R	1.08
Depreciation Rate	δ	.08
Demand Elasticity	η^{-1}	4
Capital Share in Production	α	.27
Capital Adjustment Cost Parameter	ψ	2
Persistency of log A_t	ρ_A	.78
Persistency of R_t	ρ_R	.31
Wage Rate	w_t	1
Demand Shifter Variable	z_t	1

Table 2.1. Partial Equilibrium Model: Calibration ofParameter Values

2.3.3. Calibration and Numerical Solution

The properties of the investment model discussed so far are described in this section by means of the impulse responses to unexpected innovations in the exogenous shocks. For brevity attention is restricted to the response to interest rate and productivity shocks, while the wage rate, w_t, and the demand shifter, z_t, are assumed constant. The productivity and interest rate shocks evolve according to first order autoregressive processes

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A \tag{2.23}$$

$$R_t = (1-\rho_R)R + \rho_R R_{t-1} + \varepsilon_t^R$$

The solution of the model is computed by loglinearizing the equilibrium conditions around the non stochastic steady state. The resulting system of expectational difference equations is solved using Anderson and Moore [1985] routines.

The parameters of the model are calibrated on post World War II data in the US at yearly frequencies, and are summarized in Table 2.1.

The gross interest rate R_t is the rate of return to firm's share and debt holders. The value of R_t is measured empirically as the weighted average of the ex-post real returns on the S&P 500 index and the Moody's Baa Corporate Bond Yield during the years 1950-2000.[6] The weight on the equity return is the median share of equity over total asset (3/4) of firms in the Compustat database over 1960-2000 (Welch [2004]). The series for A_t, adjusted to account for the monopolistically competitive setting with no economic profits, is from the NBER-CES Manufacturing Industry Database (Bartelsman, Becker, and Gray [2000]). The database reports data on the entire US manufacturing sector at the 4-digit SIC code over the years 1958-1996.[7] The capital share in production, α, is computed as the average share in the database, while the depreciation rate is set to 8%. The elasticity of labor demand, η^{-1} implies a markup over marginal cost of 33% (Woodford and Rotemberg [1999]). The adjustment cost parameter is from Cummins, Hasset, and Oliner [2003].[8]

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[6]The index and dividend of the S&P500 are from Shiller Robert [2000] , while the S&P 500 index and the Moody's Baa Corporate Bond Yield is from the Board of Governors. Inflation is computed from the Consumer Price Index.

[7]The (unadjusted) TFP series is the 5-factor TFP annual growth rate, computed as the difference between real sales' growth rate and the sum of production inputs' growth rate weighted by the respective production function elasticities. The elasticities are calculated from the share of each factor's expenditure over total revenues. With imperfect competition and overhead costs, the revenue based shares are unbiased estimates of the true elasticities so long as the economic profits are zero in steady state(Hall [1988]). In the TFP calculation, however, the weight on the sales growth depends on the markup. With labor being the only variable production factor, for example: $\dot{A}_t = (1-\eta)\dot{Y}_t - \alpha\dot{K}_t - (1-\alpha)\dot{L}_t$, where a dotted variable denotes the logarithmic growth rate. Starting with the TFP growth rate computed under perfect competition, $\dot{A}_t^{comp}$, the growth rate of TFP is then simply constructed as $\dot{A}_t = \dot{A}_t^{comp} - \eta\dot{Y}_t$. In the theoretical model TFP was assumed constant in the steady state. The TFP index is thereby detrended in each sector. The value of ρ^a and the volatility in the innovation (used in the simulations of the next sections) is calculated by estimating a linear AR(1) model for $\log A_t$ by pooling all the available data (over time and across sector).

[8]The value is taken from Table 3, second panel. Cummins, Hasset, and Oliner [2003] assume a quadratic adjustment cost function. Close to steady state ψ is approximately equal to the sensitivity of the investment to capital ratio with respect to marginal Q, when there is no TTB. The calibration below, uses the same value of ψ also used for the TTB model so that the models can be easily compared. As shown in Section 2.6, however, the empirically measured value of ψ would in general differ with TTB.

The impulse response functions to unexpected innovations of one percent in the interest rate and the productivity shocks are reported in Figure 2.2 to 2.3. The figures display the responses of investment, the scale of new investment projects ($j \in M_{t-1}$), the shadow value of capital , the capital stock and labor input.[9] All responses are expressed as percentage deviations from steady state, and the term investment refers to both E_t and I_t, as the two variables are equal in percentage deviations from steady state.

Empirical evidence on the investment process on US data finds an approximate duration of 2 years for projects involving structures, and lower values for investment in equipment.[10] The next section discusses how to it is possible to parametrize the elasticity of substitution, ε, between the investment goods in the basket I_t, from available literature at the aggregate level.

Consider, first, the response of the model to an interest rate shock. Due to the positive serial autocorrelation, a higher realization of R_t increases the expected rates in the future. Higher expected future rates, reduce firm's weight on future dividends, so that the shadow value of capital q_t falls. The incentives to invest are reduced and so the capital stock and future levels of production. The response of investment, however, depends on the values of ε and θ. Consider the comparative static with respect to θ shown in Figure 2.1, where ε is equal to 2. With a higher expected duration of the investment projects (lower θ),

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[9] The shadow value of capital is equal to the discounted value of marginal Q as shown in (2.13). The values of marginal and average Q differ in the model. Although most of the literature uses average Q as a proxy of marginal Q, other papers construct direct measures of marginal Q (Abel and Blanchard [1986], Gilchrist and Himmelberg [1998] and Cummins, Hasset, and Oliner [2003])

[10] Empirical work on TTB is almost exclusive to US data and is usually done at three different levels of aggregation: either at the project level (Mayer and Sonenblum [1955]), at the firm level (Koeva [2001]) or at the aggregate level (e.g. Altug [1989], Oliner, Rudebusch, and Sichel [1995a] and Christiano and Vigfusson [2003]).

investment responds less and only gradually (hump-shaped response) to the shock. Two separate channels are in action. First, loglinearizing (3.4) yields to

$$\hat{I}_t = \theta\ \hat{\iota}_t + (1-\theta)\ \hat{I}_{t-1}. \tag{2.24}$$

In each period, the fraction of projects under control of the firm falls with a longer project duration, and thus investment is increasingly inertial–depends on earlier choices of I_t, for given values of $\iota_t(j)$. Moreover as θ falls, the scale of the new investment projects, $\iota_t(j)$, respond less to the current realization of the shocks. To see why this is so, consider the first order condition for the optimal scale of the investment project. As shown in Appendix **??** this is

$$\mathbb{E}_t \sum_{r=0}^{\infty} \left(\prod_{s=1}^{r} R_{t+r}^{-1} (1-\theta) \right) \left\{ q_{t+r} \left(\frac{I_{t+r}}{\iota_t(j)} \right)^{1/\varepsilon} - 1 - C_1(I_{t+r}, K_{t+r-1}) \right\} = 0.$$

The firm equates the expected marginal increase in the basket, $(I_{t+r}/\iota_j(t))^{1/\varepsilon}$, weighted by shadow value of capital, q_{t+r}, to the expected marginal cost of the investment project including the installation cost of capital $(1 - C_1(I_{t+r}, K_{t+r-1}))$. As θ falls, the firm puts additional weight in the future trade off between costs and benefits. As the impact of the current innovation gets dissipated as time goes by, the firm optimally chooses to respond less to the current realization of the shock as it carries fewer information relevant to evaluate the trade off.

The degree of complementarity among investment projects, ε, also determines the response of investment. Figure 2.1 displays the responses for different values of ε, holding θ fixed at 2/3. From (2.24) the degree of investment inertia depends on how $\hat{\iota}_t$ adjusts to the shocks for given θ. The average scale of investment that had been chosen at earlier dates, which as shown in (3.4) and (3.2) is fully summarized in E_{t-1} and I_{t-1}, affects the marginal return to current investment due to the complementarity of the investment goods. With a high degree of complementarity, low ε, the firms does not have much incentive to cut the scale of the new projects, because, due to the high scale of the other projects, the return to investment is relatively high. On the contrary, the current choice of the investment projects is hardly affected by earlier investment choices with high level of ε. As shown for $\varepsilon = 50$, the firm will then fully rebalance I_t by over adjusting the scale of the new investment projects, and the inertial response of I_t disappears.

Now consider the response to a productivity shock. Due to the positive serial correlation, the expected marginal product of capital increases after an innovation, and thus firm invests increasing the stock of capital at future dates. The higher productivity also raises the demand for labor. Due to the higher factor inputs and productivity, output increases. As shown in Figure 2.2 and 2.4, response of investment depends on θ and ε. As for the interest rate shock, investment response is gradual and dampened with longer duration and higher complementarity. Once again with lower values of θ, for given $\iota_t(j)$, investment is increasingly inertial from (2.24). Further, $\iota_t(j)$ responds less to the realization of the shock, as it carries fewer information about future returns. The adjustment of $\iota_t(j)$ also crucially depends on the value of ε : as the degree of complementarity falls, the firm over adjusts the scale of the new investment projects, so as to offset the portion of investment that is predetermined, and thus investment responds swiftly to the shocks.

2.4. TTB with Deterministic Duration

This Section compares the TTB model with uncertain duration presented so far, with analogous models where the duration of the investment projects is deterministic. The comparison serves to highlight the elements that are key in the model. As discussed in Section 2.3.3, the gradual (or hump-shaped) response of investment to shocks follows from the pre-commitment of the firm to earlier investment decisions for the projects that are not completed, and from the imperfect substitutability of the investment goods that induces the firm not adjust the overall investment decision through the projects that are under its control.

In the model presented so far, some investment projects last longer than others ex-post, but, because the maturity of the project follows a Poisson process, all projects are perfectly homogeneous before it is known that which project matures at each date. This implies, for example, that projects started many periods in advance have the same probability to mature as the ones just started. Further, due to the uncertain maturity and the assumption that the investment projects increase the scale of the investment basket when the project mature, only a fraction θ of the investment goods increase the scale of the investment basket at each date. The Poisson assumption and the impossibility of the firm to store the investment goods to increase the investment basket at later dates, are not crucial for the gradual response of investment. The crucial element, instead, is the ex-post heterogeneity in the projects' duration and the imperfect substitutability.

In the first model presented in this Section, the firm invests in different investment projects, that are imperfect substitutes in the investment basket, as for the model presented in the previous sections. The duration of each investment project is certain, but the duration of the projects is different among the different investment goods. As shown, from the impulse responses the response of investment expenditure is also gradual in this model although the shape of the responses in the two models differ as it is discussed below.

Also in the second model presented, the duration of the investment projects is heterogeneous ex-ante. This model, however differs, in that the capital stock is composite of different capital types that are imperfect substitutes. The stock of each capital type is increased when the corresponding investment project matures. The response of investment expenditure in this model is also hump-shaped. This highlights that key element to deliver the gradual investment response is a mechanism that makes the firm reluctant to adjust the investment decision through the projects that are under his control. The specific mechanism, whether complementarity between investment goods or capital goods, for example, is not crucial.

I will now describe the model with complementarity among investment goods and deterministic duration of the investment projects. The model is a generalization of the model of Kydland and Prescott [1982], from which the notation is borrowed. The firm invests in $j = 1, ..., J$ investment goods, and each investment project lasts $N_j \geq 1$ periods. Let $S_{t,j,n}$ denote the scale of the project of type j at date t which is n stages from completion. The scale of the investment project is chosen once and for all at its initiation at N_j stages from completion. After then the scale of the projects is fixed

$$S_{t+1,j,n} = S_{t,j,n+1} \text{ for all } j = 1, ..J \text{ and } n = 1, ..., N_j. \tag{2.25}$$

At each point in time the expenditure on each type j of investment is equal to the j expenditures for all stages of completion

$$E_{t,j} = \sum_{n=1}^{N_j} \omega_{j,n} S_{t,j,n}. \tag{2.26}$$

Following Kydland and Prescott [1982] it is assumed that $\omega_{j,n} = \omega_j$ for all n. The value of the ω_j's is chosen below so that the firm's discounted investment expenditure on each project j is independent of N_j. The overall expenditure on investment, E_t, is then

$$E_t = \sum_{j=1}^{J} a_j \; E_{t,j}, \tag{2.27}$$

where the a_j's weight each expenditure's importance in steady state, as discussed below. The weights are such that $0 < a_j < 1$ and $\sum_{j=1}^{J} a_j = 1$. Only completed projects add onto the capital stock, and following the formulation of Section 2.2

$$I_t = \left(\sum_{j=1}^{J} a_j \iota_{t,j}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}, \tag{2.28}$$

where

$$\iota_{t,j} = S_{t,j,1}, \tag{2.29}$$

and the capital stock K_t depreciates at the constant rate δ as in (2.3). Having maximized out the variable factors of production, the intertemporal investment decision of the firm is to maximize (2.5), where D_t is given by (2.6), subject to the investment technology (2.25)-(2.29). Let q_t denote be the date t shadow value of capital. The first order conditions that characterize the optimal investment decisions are then

(2.30)

$$\mathbb{E}_t\left\{\phi_j\left(\sum_{n=0}^{N_j-1}\left(\prod_{s=1}^{n}R_{t+s}^{-1}\right)\right)\right\}=\mathbb{E}_t\left\{\left(\prod_{n=1}^{N_j}R_{t+n}^{-1}\right)\left(q_{t+N_j-1}-C_1\left(I_{t+N_j-1},K_{t+N_j-2}\right)\right)\left(\frac{I_{t+N_j-1}}{\iota_{t+N_j-1}}\right)^{\frac{1}{\varepsilon}}\right\},$$

for all $j=1,..,J$ and equation (K_t) of (2.14). The optimal choice of $S_{t,j,1}$ equates the discounted marginal investment expenditure, shown in the left-hand side of left-hand side of (2.30), to the marginal benefit which, is equal to the discounted marginal increase in the investment basket N_j-1 from when decision is taken, weighted by the shadow value of capital q_{t+N_j-1} net of the marginal increase in the installation cost $C_1(\cdot,\cdot)$. The interpretation of the first order condition with respect to capital is as in Section 2.3.3. The values of $\omega_j' s$ are such that the steady state is symmetric $\iota_{ss,j}=\iota_{ss,j}$ and the shadow cost of capital is equal to one. From (2.30) evaluated at steady state follows that

$$\omega_j=\frac{1-R^{1+N_j}}{1-R}. \tag{2.31}$$

The steady state level of the capital stock and of the other variables are then as in Section 2.3.3, evaluated at $\theta=1$. The model is solved by linearization around the non-stochastic steady state. The parameters of the model are the ones used in the calibration of the uncertain duration model listed in Table 2.1. The average duration of the investment project is one and a half year, thus $J=2$, $N_j=j$ and $a_j=1/2$. The elasticity of substitution in the investment basket, ε, is equal to 2. The response of the model to one-percent innovation in the interest rate shock is shown in Figure 2.5, which also shows the response of the uncertain duration model calibrated with the parameters (thus θ is equal to 2/3). The response of investment expenditure after an interest rate shock is gradual as for the model with uncertain duration. This demonstrates that the key element to obtain a gradual investment response is ex-post heterogeneity in the duration of the investment projects. Although not crucial for the gradual response, the uncertain duration of the project affects the response of investment. The response of the scale of the investment project in the deterministic model falls with its duration. The same is true for the model with uncertain duration. But due the uncertain duration, it is possible for some projects to last more than two periods. Thus the scale of the investment project with uncertain duration responds less than the projects in the deterministic models, also resulting into a lower response of investment expenditure. Further, total expenditure with uncertain duration displays a smooth hump-shape, while in the deterministic model, the response of expenditure is delayed only for the first two years, after which the model response is as in the model with no TTB. In the model with deterministic duration the expenditure averages investment decisions of the preceding two periods. In the uncertain duration model, instead, the investment expenditure summarizes the decisions at all previous dates due to the randomness in the project's duration, although with small weights to earlier decisions.

The alternative specification of the deterministic model differs from the one just presented in the source of imperfect substitutability among the different types of investment. The capital stock of the firm is made of J types of capital goods

$$K_t = \left(\sum_{j=1}^{J} a_j \ k_{t,j}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)},$$

where the weights a_j's as defined as before. Each type of capital good j depreciates at the constant rate δ and as before investment projects add onto the capital stock only when the project is completed, thus

$$k_{t,j} = (1-\delta)k_{t-1,j} + \iota_{t,j},$$

where $\iota_{j,t} = S_{t,j,1}$.It is assumed that the firm pays installation costs for each type of capital, thus the dividend flow is

$$D_t = \pi_t(K_{t-1}) - E_t - \sum_{j=1}^{J} a_j \; C(S_{t,j,1}, k_{t-1,j}),$$

where E_t is given by (2.27). Let $a_j \; q_{t,j}$ be the shadow value of the capital of type j then the first order conditions obtained by maximizing (2.5) subject to the investment technology are given by

$$\begin{array}{ll} (S_{t,j,1}) & \mathbb{E}_t \left\{ \phi_j \left(\sum_{n=0}^{N_j-1} \left(\prod_{s=1}^{n} R_{t+s}^{-1} \right) \right) \right\} = \mathbb{E}_t \left\{ \left(\prod_{n=1}^{N_j} R_{t+n}^{-1} \right) \left(q_{t+N_j-1} - C_1 \left(I_{t+N_j-1}, K_{t+N_j-2} \right) \right) \right\}, \\ (k_{jt}) & q_{t,j} = \mathbb{E}_t R_{t+1}^{-1} (\pi_{1,t+1} \left(\frac{K_t}{k_{t,j}} \right)^{\frac{1}{\varepsilon}} + (1-\delta) \; q_{t+1,j} - C_{2,t+1}(\iota_{t+1,j}, k_{t,j})), \end{array}$$

for $j = 1, ...J$. The interpretation of these first order conditions are analogous as the ones previously obtained in this Section. As in the previous model, the values of the ω_j's are equal to 2.31, so that the steady state is symmetric: $k_{ss,j} = k_{ss}$ and $\iota_{ss,j} = \iota_{ss}$. The model is calibrated as for the previous specification: $J = 2$, $N_j = j$ and $a_j = 1/2$. The remaining parameters are from Table 2.1. The impulse responses to a one percent innovation in the interest rate shock are reported in Figure 2.6. Once again the delayed response of investment expenditure lasts for the first two years after the interest rate shock. The important result, is that the overall response of investment expenditure is once again hump-shaped, although investment now peaks after the shock.[11] Once again, investment expenditure is in part predetermined by previous decisions and the firm does not adjust completely in the first periods after the shock due to the imperfect substitutability among the different types of investment.

An interesting alternative formulation of the model with uncertain duration is one, where the capital stock of the firm is made of heterogeneous capital types, as for the model just presented. Due to the random maturity of the investment project, however, the level of the single capital types is uncertain even in the steady state, which is not perturbed by shocks. Thus one cannot use standard linearization techniques to solve for the model.[12] The results just discussed that for the deterministic models, however, highlight that the response of investment expenditure in such model is also going to be hump-shaped. Indeed as long as investment in some projects is partial predetermined and there is imperfect substitutability among the different investment types, the firm will optimally choose to respond only gradually to realizations of the shocks.

The next section compares the investment model to models with costs of adjusting the capital stock and the flow of investment. The main result is an equivalence, up to first linearization, of the model with TTB and investment adjustment costs. These models have been recently estimated on aggregate. The shape of the estimated adjustment costs will be handy in assessing the empirical size of ε.

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[11] The parameters of the investment technology (ε and the a_j's) were kept constant to the ones of Figure 2.5 to facilitate the comparison. Note, however, that the parameters have a different meaning in the two formulations and the investment expenditure can peak in the second period for alternative choices of the parameter values.

[12] Further it is not possible to the express the intertemporal decision problem only in terms of "aggregated" quantities as done in Section 2.2 in the model with a homogeneous capital type.

2.5. Time-to-Build and Adjustment Cost Models

In this section, I compare the properties of the TTB presented so far with two classes of investment adjustment cost. In the first class of models, that will be referred as capital adjustment costs (Lucas Jr [1967], Treadway [1969], Uzawa [1969]), firms face costs that depend on the size of the adjustment in the capital stock relative to the initial capital stock., such as, for example, the cost function $C(I_t, K_{t-1}) = c(I_t/K_{t-1})K_{t-1}$, which is part of the model. More recently Christiano, Eichenbaum, and Evans [2005] have introduced an alternative specification of the adjustment cost function, referred to investment-flow adjustment cost. In this formulation the firm pays costs of the form $S(I_t, I_{t-1})$ that depend on the difference between current and lagged investment decisions.

The main result of this section is to show how both the TTB and the investment adjustment cost model display a delayed and dampened response of investment to shocks. In particular it is shown that, up to first order linearization around the steady state, the two models both in their dynamic response and in the steady state for an appropriate choice of the parameters in the two models. As it will be discussed in the next paragraph, the response of investment with capital adjustment costs is different due to the lack of inertia in the dynamic response of investment to shocks.

An extensive literature has studied the role of convex capital adjustment costs on firm's investment behavior. Consider the model with no TTB ($\theta = 1$). From (2.14), the shadow price of capital, q_t, is always equal to one absent capital adjustment costs. The firm adjusts the capital so as to fully compensate the impact of either productivity or interest rate shocks from (K_t) of (2.14). With capital adjustment costs, instead, the shadow price of capital diverges from unity after a shock, as shown in Figures 2.1 to 2.4. Due to the presence of the adjustment costs, the firms adjusts the capital stock after a shock only in part. Similarly to the TTB model, thus capital adjustment costs reduce the response of investment to shocks and thus its volatility. The dynamic response of investment with capital adjustment costs is different in that it won't be hump-shaped as for the TTB model. [...two forces...] In both models the firm has the highest incentives to invest right after a positive productivity or negative interest rate shock. In the TTB model, however, the adjustment of investment will be gradual due to the fraction of investment which is predetermined and the complementarity between the different projects. For a given level of investment, capital adjustment costs also fall after a shock as the capital stock changes. The important difference, however, is that around steady state investment is only a very small fraction of the capital stock, and so the movement in the capital stock will tend to be small. The reduction in the overall cost of adjustment in general, is always smaller than the higher incentives to invest after a shock, and thus investment will always be highest after the shock and will display a hump shaped response. The specification of the adjustment cost function is obviously crucial for this result, as it is discussed below.

Recent literature in macroeconomics finds that the dynamic response of aggregate investment to monetary and supply shocks is hump shaped. Christiano, Eichenbaum, and Evans [2005] have thus proposed a different formulation of the adjustment cost function whereby the cost of adjustment is a function of the change in the flow of investment rather than in stock of capital. Consider a firm that solves

$$\max_{\{K_{t+r},I_{t+r}\}} \mathbb{E}_t \left\{ \sum_{r=0}^{\infty} \left(\prod_{s=1}^{r} R_{t+r}^{-1} \right) \left(\left(\pi_{t+r}(K_{t+r-1}) - \tau \ \left(I_{t+r} - S(I_{t+r}, I_{t+r-1}) \right) \right) \right) \right\}, \tag{2.32}$$

subject to the law of motion for the capital stock (2.3).[13] The cost function is assumed to take to the form

$$(2.33) \qquad S(I_{t+r}, I_{t+r-1}) \equiv s\left(\frac{I_{t+r}}{I_{t+r-1}}\right) I_{t+r-1}, \text{ where } s(1) = s'(1) = 0 \text{ and } s''(1) = \chi$$

where $s(1) = s'(1) = 0$ and $s''(1) = \chi$, while τ is a fixed parameter that measures the price of the investment good.

Proposition 1 (Equivalence between Investment Adjustment Costs and TTB)**.** *Consider a firm that solves (2.32) subject to (2.33) and (2.3), and one that solves (2.12) with* $\psi = 0$ *subject to (3.11) and (2.3). If* $\tau = \theta^{\frac{1}{1-\varepsilon}}$ *and* $\chi = \frac{(1-\theta)}{\theta\varepsilon(1-R^{-1}(1-\theta))}$ *then the two models share the same steady state and local dynamics in a neighborhood of the steady state.*

2.6. TTB and Empirical Models with Capital Adjustment Costs

A large literature in recent years has empirically tested the investment model with convex capital adjustment costs. The literature finds only weak support for the model (see Chirinko [1993] for a review) as it will be discussed below. The objective of this section is to present implications of the TTB model presented so far for this empirical literature. In order to do so, I will first generate artificial data by simulating the model calibrated on US sectoral data. I will then run the same regression models considered in the literature, and show that some of the empirical failures discussed in the literature

[13]The formulation of (2.32) follows the investment literature by including the cost in the dividend, so that it is expressed in units of output. Christiano, Eichenbaum, and Evans [2005], instead, include the adjustment cost in the law of motion of the capital stock which is transformed into

$$K_t = (1 - \delta)K_{t-1} + I_t - S(I_t, I_{t-1}).$$

In this formulation the cost is paid in units of capital. The discussion that follows does not hinge upon the units of the adjustment cost function.

are consistent with a specification error of the model when TTB is not included in the empirical model.

Consider the empirical model tested in the literature. The model follows from the first order conditions presented in Section 2.3.1, when the capital adjustment cost function takes quadratic form

$$c\left(\frac{I_t}{K_{t-1}}\right) \equiv \frac{\psi}{2}\left(\frac{I_t}{K_{t-1}} - \delta - e_t\right)^2. \tag{2.34}$$

The variable e_t is a shock to the adjustment cost function assumed to be serially uncorrelated. Using (2.34) into (I_t) of (2.14) yields the linear regression model

$$\frac{I_t}{K_{t-1}} = \beta_0 + \beta_1 Q_t + e_t, \tag{2.35}$$

The model (2.35) implies that Q_t is a sufficient statistic for the investment decision, and that the capital adjustment cost parameter ψ is simply can be estimated as $\psi = 1/\beta_1$. This simple characterization of the investment decision has been rejected in the data along several dimensions. First, the estimated costs of capital adjustment are unreasonably large (e.g. Summers, Bosworth, Tobin, and White [1981]). Further, the value of Q_t is hardly a sufficient statistic for the investment decision. First, lagged values of Q_t also enter significantly in the regression model. This evidence, taken together with a high serial correlation of the error term, e_t, is interpreted (e.g. Oliner, Rudebusch, and Sichel [1995b]) as indicating that the investment decision is more inertial than what predicted by the model in (2.35). Second, the literature on financial frictions has also included as a right hand side variable of (2.34) the ratio of a firm's cash flow scaled by the capital stock, CF_t/K_{t-1} (for a review of this literature see Hubbard [1998]). The value of a firm's cash flow (revenues less taxes and expenses, excluding investment), is used to approximate to a firm's change in net worth. Firms with higher net worth have more internal funds, and thus, tend to invest when financial frictions are present, due to the lower costs of financing and more relaxed financing constraints. The empirical literature finds that that cash flows are highly significant when included in the regressions, and, in terms of economic magnitudes, they tend to be more important than Q_t in explaining investment.

As I show next, the evidence discussed above is consistent with investment decisions underlying the data that include TTB and an empirical model that does not. To show this, I first generate data on investment decisions using the model calibrated with the parameter values of Table 2.1, that match the evidence at the sectoral level in the US, and then regress the model (2.35) on the data. The adjustment cost shock e_t is assumed to be serially uncorrelated and has a standard deviation of 1 percent.[14] Each simulation is made of $5,000$ observations, which is comparable to empirical studies at the firm (e.g. Cummins, Hasset, and Oliner [2003]) and sectoral level (Bartelsman, Becker, and Gray [2000]). I consider two data generating processes. The value of ε is equal to 2 in both models. The value of θ is equal to one in the first model (no TTB) and to 2/3 in the second (average duration of one and a half years). While the true error term is serially uncorrelated in the empirical model allows the possibility of first order serial correlation: $e_t = \rho e_{t-1} + \varepsilon_t^e$. The parameter of the model and ρ are estimated using the Cochrane-Orcutt estimation procedure. An important issue in estimating the model is the empirical measure of marginal Q_t. The vast majority of the empirical literature approximates the value of marginal Q with average

[14]The standard deviation of the capital adjustment cost is not calibrated on empirical data. The value of the volatility, however, is not crucial in the results and discussion that follows. The main effect of a higher volatility in e_t is a lower fit of the regression in terms of R^2 and standard errors of the estimated coefficients.

Q, which can be easily measured for publicly traded companies. Indeed as shown by Hayashi [1982], average and marginal Q are equal when the firm is perfectly competitive and operates a constant return to scale technology. Both assumptions are not likely to hold empirically, and Abel and Blanchard [1986] and Cummins, Hasset, and Oliner [2003] among others, construct empirical measures for marginal Q. The model presented in the previous sections implies that average and marginal Q differ, and it can be shown that this is not only due to the monopolistically competitive framework but also because of the TTB technology. Because, the objective here is to show the implications of the TTB technology, the analysis abstracts from measurement errors on Q_t and assumes that the econometrician directly can directly observe its value.

The results of the simulations are presented in the Table 2.2, where all figures are averages over 100 simulations. The Table has six columns: the value of θ in the data generating process in the first three columns is 1 and in the last three is 2/3. For each of the two data generating processes, I consider three regression models. The first model only includes the contemporaneous value of Q_t. The second model augments (2.35) with four lags in the value of Q_t, while the third model also includes CF_t/K_{t-1}.

Consider the estimates of the regression model on the data generating process with TTB. As shown in the first three columns, the coefficient on Q_t is always statistically significant and is close to .5, so that ψ is roughly two as in the true data generating process. In model (2), only the second lab is statistically significant, but the magnitude is close to zero. The coefficients on all other lags are even smaller and none of them are statistically significant. In model (3), cash flows scaled by the capital stock is not statistically significant. As shown in the bottom part of the Table the estimate of ρ is roughly zero as in the true data generating process, and the Durbin Watson statistic indicates the lack of serial correlation in the model.

Now consider the regression models when the data generating process includes TTB of one and a half year. Although the coefficient on Q_t is always statistically significant, the magnitude of the estimated coefficient is less than half than its true value. As discussed in Section 2.3.3, investment expenditure responds less to the realization of the shocks as TTB increases, as a lower fraction of the investment projects in under the control of the firm. As shown in Figures 2.1 to 2.4, the value of Q is hardly affected by the longer duration of TTB, and thus the volatility of investment expenditure, E_t, with respect to Q_t falls. Because the empirical model (2.35) does not include TTB, however, the lower volatility of E_t is erroneously measured in the model as higher installation costs of capital. As discussed in Section 2.3.3, investment responds gradually to shocks with TTB, while the value of Q_t immediately responds to the productivity and interest rate shocks. The investment model without TTB cannot account for the inertial response in investment. As a result the residuals of the regression model are serially correlated over time. Indeed from the bottom panel of Table 2.2, the estimated values of ρ fall between .7 and .8, and, aside from model (5), the Durbin Watson statistic indicates that the residual display a persistence of order greater than one. Moreover, in the regression model (5), all lagged value of Q_t are statistically significant in the regression, and the magnitude of the regression coefficients are comparable to that of Q_t. Finally in the regression model (6), the cash-flows to capital stock ratio is statistically significant, and is economically more important than Q_t as an explanatory variable for investment. Indeed a one standard deviation increase in the cash flow ratio (equal to .12) raise the investment to capital ratio by almost twice the amount following a one standard deviation in the cash-flow to capital ratio (equal to .07). In the model presented in this paper no financial frictions are present, and cash flows help to explain the investment decision only because of the specification error.

How would it be possible to directly estimate the TTB model in the data? First note that, due to the unobservability of I_t, it is only possible to estimate a linear approximation of the TTB model. For example, by linearizing the first order conditions (2.14) around the steady state values of $\frac{E_t}{E_{t-1}}$ and $\frac{E_t}{K_{t-1}}$, one obtains that

$$\frac{E_t}{E_{t-1}} = \beta_0 + \beta_1 Q_t + \beta_2 \frac{E_{t+1}}{E_t} + \beta_3 \frac{E_t}{K_{t-1}} + \varepsilon_t, \tag{2.36}$$

where ε_t is the expectation error which is orthogonal to the information set available at date t, i.e. $\mathbb{E}_t\left(\varepsilon_t\right) = 0$. The value of the structural parameter can be obtained from the estimated β's by noting that $\beta_2 = R^{-1}$, $\beta_1 = \frac{\theta\varepsilon\left(1-R^{-1}(1-\theta)\right)}{(1-\theta)}\theta^{\frac{1}{\varepsilon-1}}$ and $\beta_3 = -\theta^{\frac{1}{\varepsilon-1}}\beta_1\psi$. Using the orthogonality condition of ε_t, the model in 2.36 can be estimated using a linear-GMM by instrumenting the right hand side variables with all variables that belong to the date t information set. It is unclear how well the linear model is capable of approximating the true model, especially at higher levels of aggregation (e.g. sectoral or aggregate investment) where the the investment series might not be stationary. Further the correct estimate of the structural parameters will depend on how goodness of the empirical approximation of the marginal Q_t.

Although an empirical test of the TTB is beyond the scope of this work, the results on simulated data presented in this section underscore how the TTB formulation presented

	Dependent Variable: E(t)/K(t-1)					
Theta in data generating process		1			2/3	
Model	(1)	(2)	(3)	(4)	(5)	(6)
Q(t)	.480 (.003)	.478 (.003)	.418 (.006)	.165 (.001)	.202 (.001)	.066 (.005)
Q(t-1)		.006 (.003)			.105 (.001)	
Q(t-2)		-.001 (.003)			.051 (.001)	
Q(t-3)		-.001 (.003)			.023 (.001)	
Q(t-4)		-.001 (.003)			.008 (.001)	
CF(t)/K(t-1)			.004 (.006)			.064 (.005)
Durbin Watson Statistic	2	2	2	1.36	2.01	1.37
Rho	.002	.001	.013	.745	.793	.773
Adj. R squared	.828	.83	.833	.732	.953	.758

Notes: Linear regression models estimated on simulated data from the models calibrated with parameters of Table 1 and sigma(e)=.01. There are 5,000 observations in each simulation, and the numbers reported are averages over 100 simulations. The value of epsilon= 2 and theta is equal to 1 or 2/3. All regression models are estimated using the Cochrane-Orcutt estimation procedure. The variable rho is the first order serial correlation in the error term. Standard errors of the estimated coefficients are reported in parenthesis below each coefficient.

Table 2.2. Regression Models on Artificially Generated Data

in this paper has the potential of capturing the empirical behavior of firms' investment decisions.

2.7. Real Business Cycle Model

This section embeds the time-to-build technology presented in Section 2.2 in an otherwise canonical real business cycle general equilibrium model. The response of the model to an unexpected innovation in the TFP shock is then compared with a model with no time-to-build and with a model that embeds the time-to-build technology considered by Kydland and Prescott [1982].

The representative household is infinitely lived. At each date t he decides how much to consume, C_t, work, L_t, and how many stocks, S_t, to hold so as to maximize the discounted value of future utility flows

$$\max_{\{C_t, L_t, S_t\}} \; \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r U\left(C_t, 1 - L_t\right),$$

where the utility function is

$$U\left(C_t, 1 - L_t\right) = \log C_t + \upsilon \; \log(1 - L_t),$$

and $0 < \beta < 1$ is a discount factor measuring household's rate of impatience. At every date, his decisions are subject to the flow budget constraint

$$C_t + S_t V_t \leq W_t L_t + S_{t-1}\left(V_t + D_t\right),$$

where $W_t L_t$ is his labor income, while V_t and D_t are the stock price and dividend of the representative firm. Finally S_t is the level of stock holdings.

The production sector of the economy is made of large number (measure one) of firms that produce Y_t, using the production function (2.15). Date t prices are expressed in terms of Y_t, and the prices of the investment good and of C_t are equal to one. For brevity, I consider the case in which the demand function is perfectly elastic, so that the representative firm's markup is equal to one and the overhead cost (2.20) is equal to zero.

The first order conditions of the household's maximization problem yield

$$\begin{aligned} &(L_t) \qquad U_{2,t} = w_t U_{1,t} \\ &(S_t) \quad U_{c,t} S_t = \mathbb{E}_t \beta U_{c,t+1}\left(S_{t+1} + D_{t+1}\right). \end{aligned} \tag{2.37}$$

Integrating (S_t) of (2.37) and using the transversality condition $\lim_{t\to\infty} \beta^r U_{c,t+r} S_{t+r}$ yields that the firm's ex-post realization of the discount factor is

$$R_{t+1}^{-1} = \beta \frac{U_{c,t+1}}{U_{c,t}}.$$

From (2.18) it also follows that in equilibrium

$$\pi_t'(K_{t-1}) = \alpha A_t K_{t-1}^{\alpha} L_t^{1-\alpha},$$

while the dividend of the representative firm equal to the constant fraction $(1-\alpha)$ of Y_t in equilibrium. The first order conditions that characterize the firm's optimal decisions are given by (2.14) along with the inverse demand for labor services, which, in equilibrium, is $w_t = (1-\alpha)Y_t/L_t$. Finally market clearing in the financial market yields the aggregate resource constraint $C_t + E_t = Y_t$.

Given the initial levels of K, E and I,and a sequence of the TFP shock A_t, an equilibrium is defined as a state-contingent sequence of prices and quantities such that a) firms and households solve their respective maximization problem, b) goods, labor and the financial markets clear.

Now consider the TTB model of Kydland and Prescott [1982]. They consider an investment technology with TTB but where the duration of the investment project is certain. Further, they assume that the representative firm makes one type of investment. Their model is easily obtained by considering the case of $J = 1$ in either of the two models with deterministic maturity that were presented in Section 2.4. I also follow their original analysis by assuming that the time unit is a quarter, that the investment projects last for four quarters, and that the investment expenditure is equally split along the duration

Name	Variable	Value
Gross Interest Rate in s.s.	R	(1.08)ˆ.25
Depreciation Rate	δ	.02
Demand Elasticity	η^{-1}	∞
Capital Share in Production	α	.36
Capital Adjustment Cost Parameter	ψ	0
Labor weight in utility	υ	such that $L_{ss} = .28$
Persistency of log A_t	ρ_A	.95

Table 2.3. RBC Model: Calibration ofParameter Values

of the projects, or $\omega_n = 1/4$ for all n. Consistently I also assume that the duration of the project in the random maturity TTB model is one year, or $\theta = 1.4$. The remaining parameter values common to the two models are reported in Table 2.3.

The parameter values common to the two models are calibrated on post-World War II US data (e.g. Prescott [1986]). The impulse response functions to a one percent innovation in the productivity shock are shown Figure 2.7. The Figure reports the response of the uncertain TTB (TTB) and the Kydland and Prescott [1982] (TTB KP) model, and of a canonical RBC model with no TTB (no TTB).

First consider the response of the canonical RBC model. With the exception of aggregate consumption, all variable peak when the productivity shock hits and then decay exponentially to the pre-shock level. The response of aggregate consumption is, instead, hump-shaped due to households's preferences to intertemporally smooth the temporary increase in income. The response of all variables in the Kydland and Prescott [1982] model, follow the overall response of the canonical RBC model. The main difference between the two model, is the existence of deterministic cycles in the responses of the Kydland and Prescott [1982] model, as first highlighted by Rouwenhorst [1991]. The representative firm increases the scale of the new projects when the technology shock hits the economy. In the three quarters after the shock, however, the firm reduces the scale of the new projects, because it has to live up to the previous commitments. In the fourth quarter after the technology shock the projects with a large scale mature increasing the capital stock and the incentives of the households to consume more and of the firms to choose a large scale of the new projects. The pattern then repeats itself again, thus creating the cyclicalities in the impulse response functions. As noted by Rouwenhorst [1991], however, the cyclicalities are small relative to the overall response of the economy, which is close to the canonical RBC model. Thus Rouwenhorst [1991] challenges the central role of TTB posed by Kydland and Prescott [1982].

Now consider the TTB model with uncertain duration. As for the partial equilibrium analysis, investment expenditure responds only gradually to the higher productivity levels. The representative firm only controls the scale of the investment projects that have just matured, and it optimally decides not to perfectly adjust the overall expenditure through the projects under control due to the complementarities. The gradual investment response is in stark contrast with the swift response in the canonical RBC model and in that of Kydland and Prescott [1982], and describes well the empirical response investment to shocks. Indeed the response of investment to productivity shocks (Christiano, Eichenbaum, and Vigfusson [2004]) and to monetary policy shocks (Christiano, Eichenbaum, and Evans [1998]) is hump-shaped. For monetary policy shocks, Christiano, Eichenbaum, and Evans [2005] find that firms with investment adjustment costs capture well the response of investment, thus from the results of Section 2.5, so will the TTB technology proposed in this paper.

Due to the lower response of investment expenditure, the response of aggregate consumption is amplified in a closed economy. Further due to the initial spike in aggregate consumption, households substitute labor for leisure and thus labor can fall after a positive productivity shock.[15] Lucca[2006] exploits the higher response of consumption due TTB to explain the higher volatility of aggregate consumption in low versus high income countries. For high income economies, it is possible to eliminate the initial spike in consumption and the fall in hours worked through preferences with habit-persistence (see Edge [2000] and Christiano, Eichenbaum, and Evans [2005]).

[15]For the empirical response of labor to technology shocks see the discussion in Gali [1999], Basu, Fernald, and Kimball [2004] and Christiano, Eichenbaum, and Vigfusson [2004].

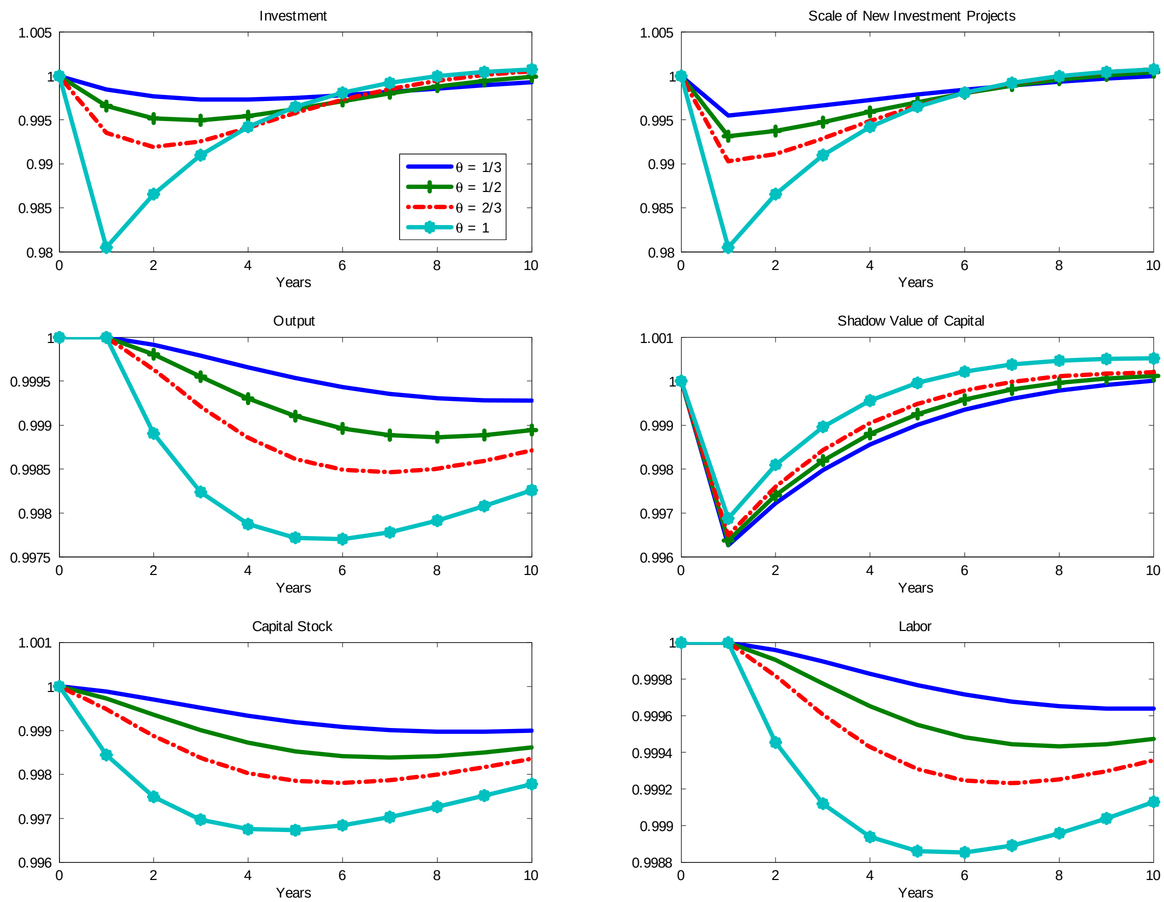

Figure 2.1. IRFs with $\varepsilon = 2$: Interest Rate Shock

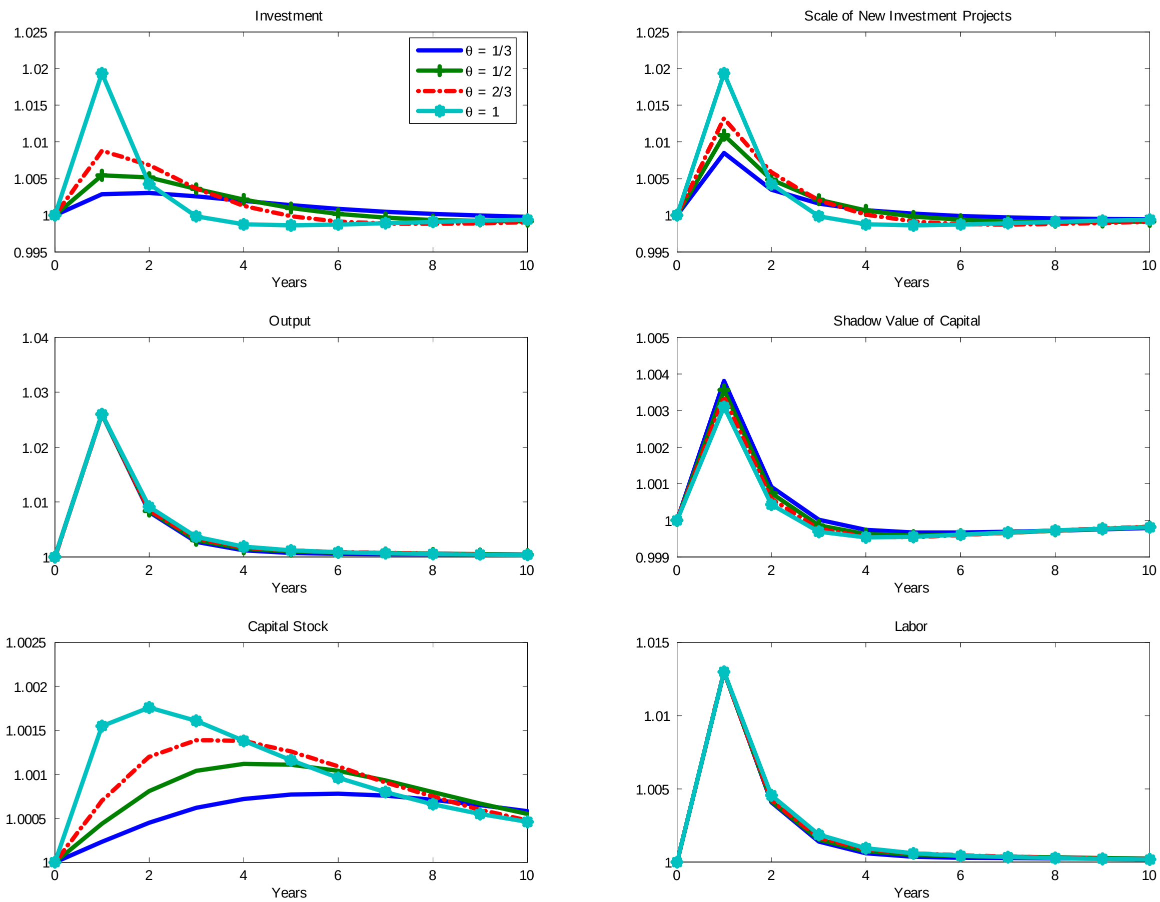

Figure 2.2. IRFs with $\varepsilon = 2$: Technology Shock

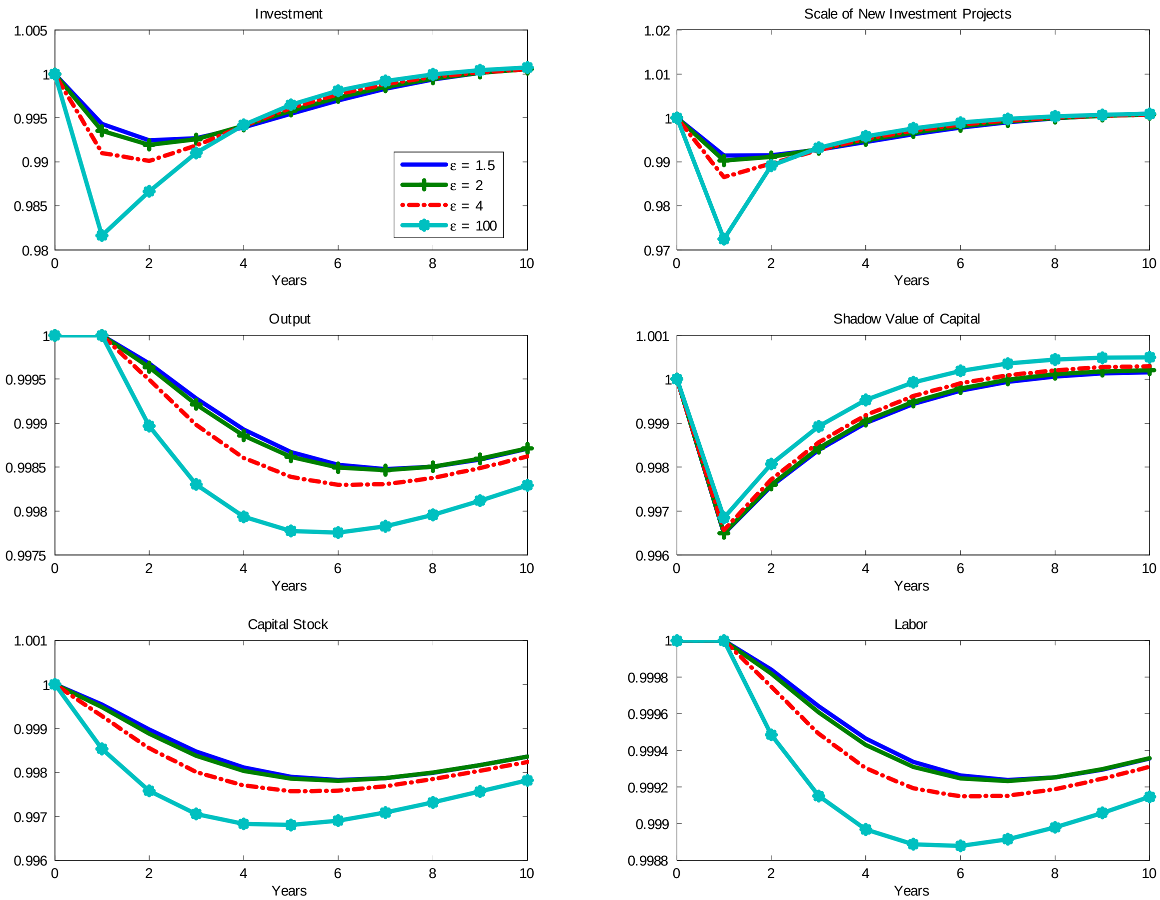

Figure 2.3. IRFs with $\theta = 2/3$: Interest Rate Shock

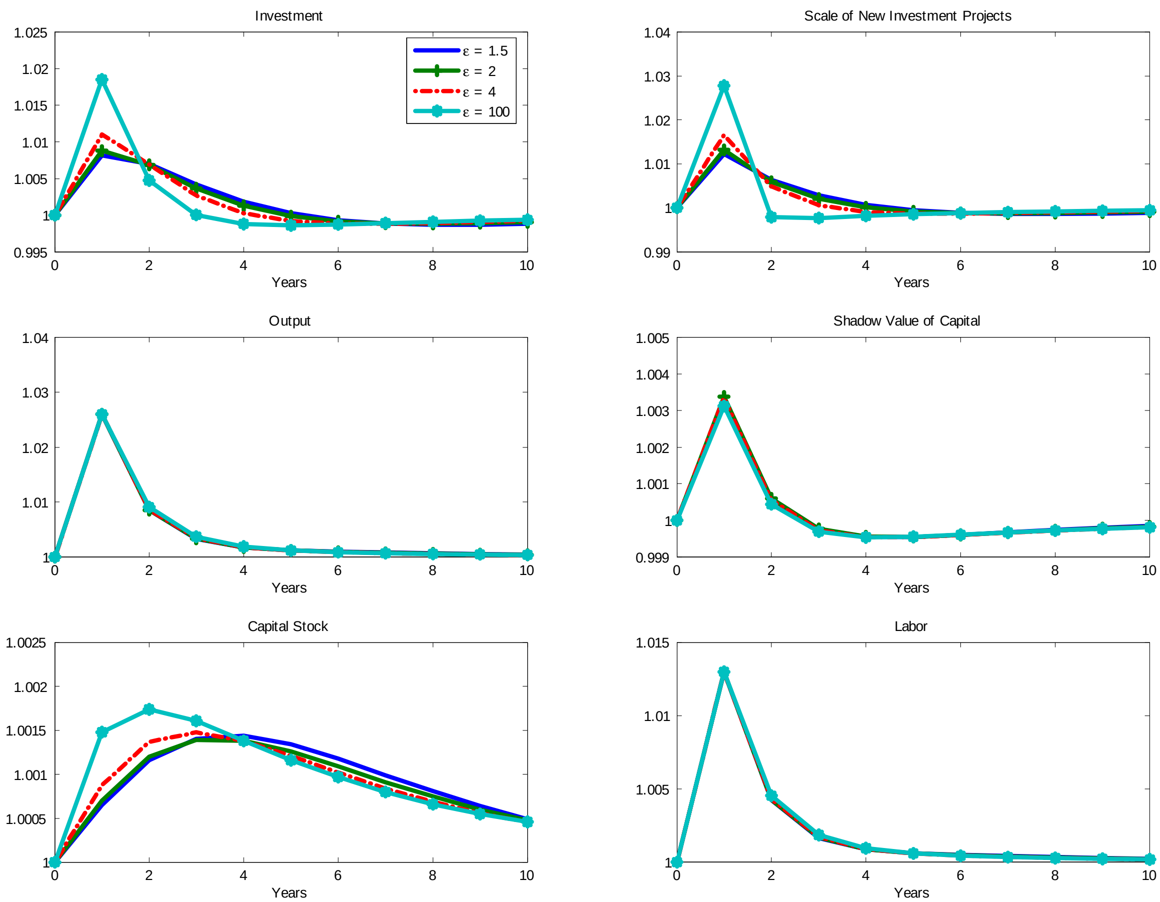

Figure 2.4. IRFs with $\theta = 2/3$: Technology Shock

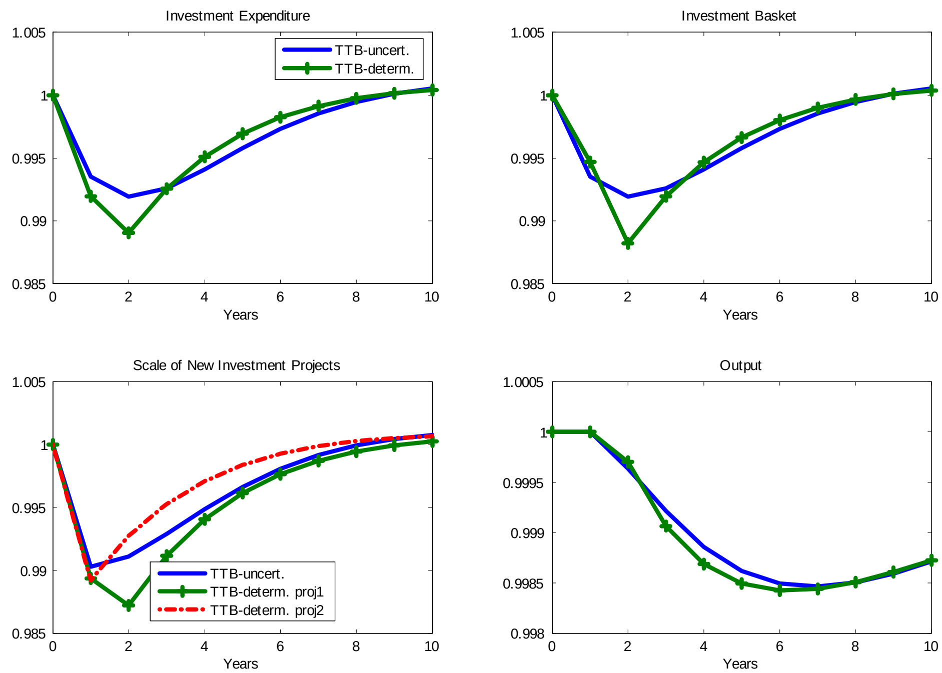

Figure 2.5. IRFs: Comparison with Deterministic TTB Model with Multiple Inv. Types; Interest Rate Shock

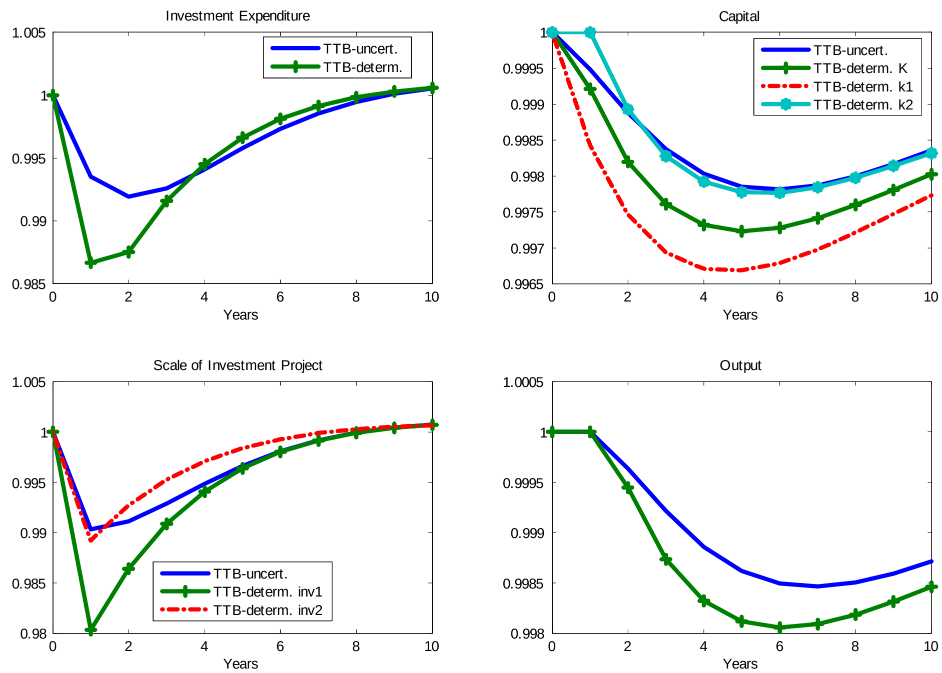

Figure 2.6. IRFs: Comparison with Deterministic TTB Model with Multiple Cap. Types; Interest Rate Shock

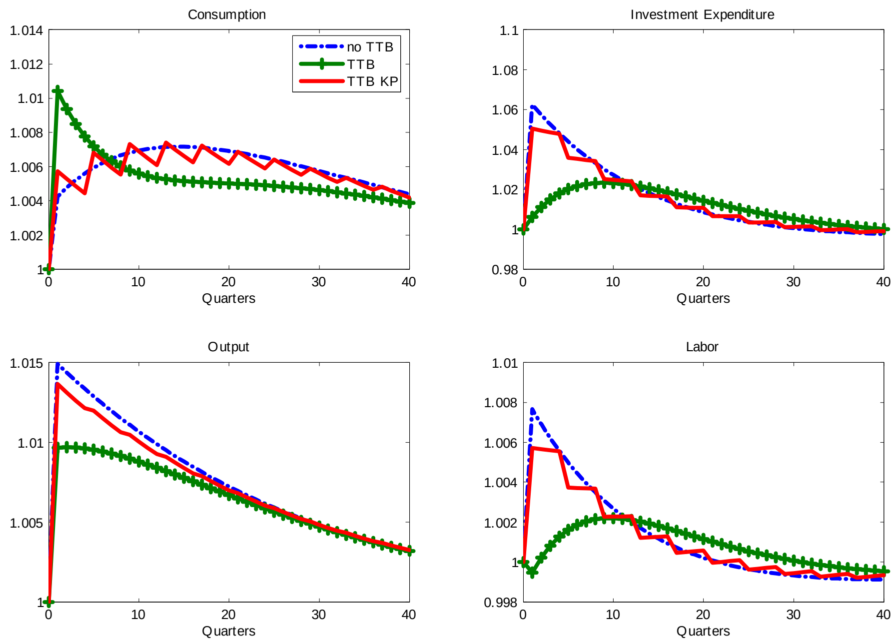

Figure 2.7. IRFs: RBC Model; Technology Shock

CHAPTER 3

Investment Flexibility and Aggregate Fluctuations in LDCs

3.1. Introduction

This paper shows that, relative to high income countries (2003 World Bank Classification), business cycles in low income economies are characterized by a higher volatility of aggregate consumption relative to GDP, a low correlation of investment and GDP, and a more procyclical trade balance. The goal of this paper is to show that accounting for a lower flexibility of investment in response to exogenous shocks helps understand these business cycle regularities.

The flexibility of investment depends on the time required to plan and build investment projects. If investment decisions must be made in advance, they can only incorporate expectations of future economic conditions rather than their realization. As the time horizon of investment lengthens, firms optimally choose a *smaller* adjustment, because the state of the economy carries less information about future returns. Moreover, if firms invest in multiple investment projects that have complementarities, they also partially *delay* the adjustment with longer horizons because many projects initiated in the past are still under way. Although a firm could invest in additional equipment immediately after a productivity shock, it prefers to wait for the new plant to be completed, because the two different types of investment will be jointly used in production. The delayed adjustment yields a hump-shaped response of investment to shocks. Dynamic investment complementarities, which formalize earlier ideas of von Hayek [1937], are the novel feature of the model and are crucial in understanding the business cycle regularities mentioned above.[1]

Economists such as Kalecki [1935], Nickell [1978] and Kydland and Prescott [1982] have thought about the time needed to plan for investment in purely technological terms. But constraints set by economic institutions are also important determinants of the lengthiness of the investment process. For instance, delays may arise from the government approval process to construct structures or use equipment and from the sluggishness of credit markets in channelling funds to firms. Ample empirical evidence supports the view that firms in LDCs operate under severe institutional constraints (Tybout [2000]). In these countries, project financing is a lengthy and complicated process because of financial markets' underdevelopment (Levine [Forthcoming]). High tariffs and hidden import barriers (Loayza, Oviedo, and Serven [2004]) translate into complex regulation and time-consuming customs clearance procedures; a particularly relevant issue since firms in LDCs import most of their equipment goods from abroad (Eaton and Kortum [2001]). In addition to a large number of official hurdles due to bureaucratic delays and regulation (Loayza, Oviedo, and Serven [2004]), firms also face unofficial hurdles, arising from the high level of corruption (de Soto [1989]). A permit application, for instance, may be stranded unless officials receive illicit payments. Given these constraints, firms plan investment projects well ahead of implementation. As a result, investment responds less and only gradually to current economic conditions.

[1] For a review of the role of strategic complementarities in macroeconomics see Cooper [1999]. For dynamic complementarities see Cooper and Johri [1997] and the references therein.

Based on a panel from the UNIDO INDSTAT database, which covers 28 manufacturing industries in over 100 countries, I provide evidence that, compared to high income countries, investment in LDCs responds less to current measures of sectoral productivity. This evidence at a more disaggregated level supports the idea that the low cyclicality of investment in aggregate data of LDCs stems from a different response of investment at the firm level.

In the novel specification of the time-to-build (TTB) (Kydland and Prescott [1982]) and time-to-plan (Christiano and Todd [1996]) hypothesis presented in the paper, firms invest in a large number of complementary investment projects.[2] The scale of new projects can only be modified when previous projects mature. A crucial mechanism in the model is that some investment projects take longer to complete than others, thus investment decisions at different points in time overlap. A natural interpretation of the heterogenous investment horizon is a technological one. A firm needs a plant and equipment to produce, but begins to invest in the plant well in advance because structures require more time to complete than equipment. Due to complementarities of the plant and the equipment, investment decisions at different dates are linked, and this intertemporal link results in a delayed adjustment.

In the model presented in the paper, investment projects are homogeneous ex-ante but differ ex-post in their TTB because the duration of a specific project is uncertain.

[2]Investment decisions are made in advance in both TTB and time-to-plan, however, resources are used only when capital is installed in time-to-plan, while also at intermediate dates in TTB. In the model of this paper the two specifications deliver identical outcomes. Studies following Kydland and Prescott [1982] downplay the role of time-to-build for RBC models. Rouwenhorst [1991] shows that TTB models display impulse responses to unexpected shocks that have deterministic cycles. Cogley and Nason [1995] show that TTB has little effect on output dynamics and persistency. The time-to-plan specification of this paper displays smooth impulse responses and leads to higher persistence. Both results follow from the imperfect substitutability among different types of investment.

Furthermore, the maturity of each project follows a Poisson process and each firm invests in a large number of projects. These assumptions greatly simplify the analysis, as the resulting investment decision only involves an additional recursive constraint to an otherwise standard problem. The optimal choice is also equivalent, up to a first order approximation, to that of firms facing investment adjustment costs of the form assumed in Christiano, Eichenbaum, and Evans [2005]. These types of adjustment costs depend on the *change* in capital adjustment rather than on the *level* (Hayashi [1982], Abel and Eberly [1994]). Although it is possible to interpret these costs literally, the model in this paper directly links the assumption of Christiano, Eichenbaum, and Evans [2005] to TTB.[3]

I augment a standard closed and small open economy (SOE) RBC model with this new specification of TTB to show how lower investment flexibility helps explain the key features of business cycles in LDCs. Numerical simulations of both models show that investment cyclicality falls as a direct consequence of its *delayed* response to shocks. The magnitude of the decrease in cyclicality is comparable to the empirical difference between the two income groups for reasonable parametrizations. Furthermore, the relative volatility of investment to GDP falls as TTB increases because of *smaller* adjustments. Volatile investment-specific shocks are needed in the model in order to maintain constant the volatility of investment, as it is observed in the data.

[3]Investment adjustment costs are a common assumption in recent large scale DSGE models, e.g. Smets and Wouters [2003]. Basu and Kimball [2005] use an investment planning adjustment cost function, which has implications that are very similar to the adjustment cost of Christiano, Eichenbaum, and Evans [2005]. They justify the assumption with time-to-plan, without however explicitly deriving an analytical link.

In the SOE model augmented with TTB, trade balance cyclicality increases with lower investment flexibility. In baseline SOE models, the trade balance is countercyclical because consumption and investment increase more than output after positive supply shocks (Mendoza [1991]). As investment flexibility falls, investment reacts less to the shock, and the trade balance becomes increasingly procyclical. However, since households can perfectly smooth income fluctuations in the international financial markets, consumption volatility is not affected by the lower investment flexibility. In a closed economy, on the other hand, as investment flexibility falls consumption adjusts to shocks relatively more, i.e. the volatility of consumption increases.

This paper is related to cross-country business cycle studies such as Neumeyer and Perri [2005] and Aguiar and Gopinath [2004]. These works are concerned with emerging market economies during the 'sudden-stop' phenomenon when consumption is volatile and, unlike in LDCs, the trade balance is strongly countercyclical and the cyclicality of investment is as large as in high income economies.

The business cycle evidence presented in this paper is related to the paper Agenor, McDermott, and Prasad [2000]. They also describe business cycle moments for LDCs, and find that in these countries the relative volatility of consumption is high and the trade balance tends to be acyclical. The authors use quarterly data and, due to its limited availability, the sample of countries in their work is significantly smaller than in this paper.[4]

[4]Agenor, McDermott, and Prasad [2000] do not report moments for investment. Also Kaminsky, Reinhart, and Vegh [2004] find that almost all countries with countercyclical capital inflows are in the low income group which implies that–up to changes in national reserves and interest payments– countries with procyclical trade balances are typically in the low income group.

The empirical results at the sectoral level are related to the work of Wurgler [2000], who finds a lower response of investment to value added for LDCs. Furthermore, Caballero, Cowan, Engel, and Micco [2004] find that the sectoral labor input's response to innovations in productivity is smaller in countries with a high level of labor market regulation. Their interpretation of a low flexibility of labor due to regulation is analogous to the one proposed here for investment.

The model in this paper is related to the ones of Edge [2000] and Gertler and Gilchrist [2000] who present a TTB and time-to-plan formulation with heterogenous capital goods. There are two important differences between this model and theirs, although in all three the response of investment to shocks is hump-shaped. The sluggish response of investment in their models lasts for a number of periods exactly equal to the number of capital types, while in this model there is a smooth transition to the long run when investment responds fully. Furthermore, the large number (needed for a lengthy delay) and ex-ante heterogeneity of investment types in their models complicates the analysis through a large number of first order conditions, each involving numerous leads and lags of the variables. In comparison, the model presented in this paper is very tractable due to ex-ante homogeneity of the projects. Compared to a model with no TTB, it only involves one additional equation, and has the same number of leads and lags.

The remainder of the paper is organized as follows. The next section summarizes the properties of low income countries' business cycles. Section 3.3 discusses the role of institutional constraints in increasing the lengthiness of the investment process in LDCs. The model economy is developed in Section 3.4, and numerical exercises are presented in Section 3.5. Finally, Section 3.6 reports evidence at the sectoral level. Section 3.7 concludes with some final remarks and directions for future research.

3.2. Aggregate Fluctuations in LDCs

This section describes how business cycles in LDCs differ from those in middle and high income economies in terms of unconditional business cycle moments. The main results of this section are that in low income countries, (a) the cyclicality of aggregate investment is low, (b) the relative volatility of consumption is high, and (c) the trade balance is acyclical compared to countercyclical trade balances of middle and high income economies.

The sample, which is described in Table 3.6, is composed of approximately 85 countries depending on the series being considered.[5] The data is annual and covers the years 1960-2000. Countries are divided in high, middle and low income according to the 2003 World Bank Income Classification. The low income group is composed of 29 countries, most of which are African economies. The middle income group is composed of 38 countries ranging from emerging market economies, such as Latin American countries, to the so-called middle low income economies (e.g. Morocco). The high income group is composed of 30 countries, most of which are OECD countries.

The series are filtered using three alternative methods: a Hodrick and Prescott [1997] filter with a smoothing parameter of 100, a Baxter and King [1999] band pass filter set to capture frequencies between two and eight years, and an annual first difference. I discuss the HP filtered series, while I mention the results for the other filters (Table 3.7)

[5]Some countries report series that cannot be compared with other countries. For example, Argentina reports government and private consumption in a single series until the beginning of the 1990s. Further the HP and band pass filters used to detrend the data require the series to be continuous. All series are in constant prices. The source of the data is either the International Financial Statistics (IFS) database or the World Development Indicators (WDI). The source for each country is listed in Table 3.6.

	Low	Middle	High
Corr(C,Y)	0.58	0.73	0.68
	[.45 ; .71]	[.65 ; .81]	[.63 ; .79]
Corr(I,Y)	0.46	0.73	0.79
	[.33 ; .58]	[.61 ; .85]	[.72 ; .88]
Corr(G,Y)	0.26	0.53	0.19
	[.05 ; .28]	[.44 ; .62]	[.41 ; .81]
Corr(nx,Y)	-0.02	-0.35	-0.30
	[-.22 ; .15]	[-.48 ; -.15]	[-.40 ; -.23]
Sd(Y)	4.39	4.12	2.29
	[3.9 ; 5.8]	[3.4 ; 4.6]	[2.0 3.2]
Sd(C)/Sd(Y)	1.46	1.19	1.06
	[1.2 ; 1.9]	[.90 ; 1.3]	[.87 ; 1.11]
Sd(G)/Sd(Y)	2.10	1.89	1.36
	[1.85; 2.72]	[1.44 ; 2.17]	[1.17 ; 1.86]
Sd(I)/Sd(Y)	4.38	3.83	3.94
	[3.6 ; 4.9]	[3.4 ; 4.2]	[3.4 ; 4.3]
Sd(nx)	4.90	2.89	1.28
	[2.8 ; 6.0]	[2.6 ; 3.5]	[1.0 ; 1.8]

Notes: Median of statistic by income group. 95% confidence intervals for the medians are reported in brackets. For each country, series are logged (with the expeption of nx which is the ratio of NX to GDP) and then HP-filtered with a smoothing parameter of 100. Standard deviations are reported in percentage terms.

Table 3.1. Median Statistics by Income Group of HP filtered national series over the sample 1960-2000

only when they are significantly different. Business cycle moments for each country are reported in Tables 3.8, 3.9 and 3.10. The standard errors of the moments are not reported in the tables. Because at most forty observations are available for each country, the vast majority of country by country differences are not statistically significant at conventional levels. On the other hand, because of relatively many countries per income group, it is possible to draw a meaningful comparison between the medians of the income groups.[6] Table 3.1 displays the median per income group of the business cycle moments. The 95% confidence intervals for the median of each moment are reported in square brackets below the corresponding moment.

[6]Because they are robust to outlier observations, group medians, rather than means, are used. The error bands of the business cycles moments at the country level can be computed by GMM. See Ogaki [1999] for a detailed implementation.

The relative volatility of consumption to GDP is higher in low income countries than for the other income groups, although only the difference with the high income group is statistically significant as seen from the confidence intervals reported in Table 3.1. In high income countries, consumption is as volatile as GDP, while in low income countries it is by 40% more volatile. Analogous results are also reported by Agenor, McDermott, and Prasad [2000].[7] The median cyclicality of consumption is slightly lower in low income countries. The difference, however, is only significant with the middle income group and, further, the result is sensitive to the filtering method as seen from Table 3.7. In summary, compared to high income countries, the relative volatility of consumption is higher while the cyclicality is similar in low income countries.

The median cyclicality of investment for low income countries is by 40% smaller than that of high and middle income economies. Only the difference with the high income countries is, however, statistically significant. Figure 3.1 reports a scatter plot of the correlation between investment and GDP against the initial level of the log of GDP per-capita in $PPP\$$ in 1960 obtained from the Penn World Tables (6.1).

The coefficient and T-statistic of the displayed linear regression line are reported at the bottom of Figure 1. The initial level of GDP per-capita and the cyclicality of investment have an economically and statistically strong linear relation[8]. The average correlation in the sample is 0.65 and a one standard deviation increase in the initial level of the log of

[7]Neumeyer and Perri [2005] and Aguiar and Gopinath [2004] discuss the higher volatility of consumption in emerging market economies

[8]The linear regression line is only meant to summarize the information in the scatter plot, and it is clearly not the optimal statistical tool since the correlation is a bounded measure. It is possible to transform the correlation domain into an unbouded one (e.g. using logistic transformations). The choice of any of these transformation would be subjective, and so the linear model is chosen instead.

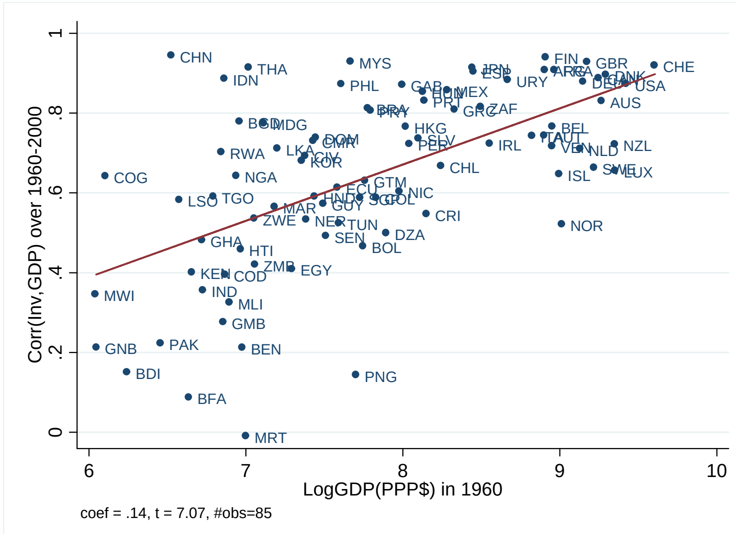

Figure 3.1. Correlation of Aggregate Investment and GDP against the 1960 GDP p.c. in PPP$

GDP per-capita in $PPP\$$ increases it by the amount 0.15, which is a 25% increase from the mean value.[9]

Low income countries' relative volatility of investment is larger than for the other income groups. However, the difference between the two income groups is barely significant, and not even so when the series are Band Pass filtered. In summary, compared to high income countries, the relative volatility of investment is constant while the cyclicality in LDCs is significantly lower. These empirical regularities are novel in the literature.

The trade balance is significantly more volatile in low and middle income countries compared to high income ones. Mendoza [1991] explains this phenomenon through a higher volatility of terms-of-trade shocks. The model presented in the paper does not

[9]The cross-sectional pattern of investment's cyclicality is very similar when investment is measured as gross *fixed* capital formation which excludes changes in inventories for the countries in which this measure is available. The pattern is also similar when excluding *central* government investment (source: WDI) from gross capital formation. A consolidated measure of investment–comprehensive of central, state and local investment– is only available on the IMF GFS database starting in 1990.

account for such types of shocks and so it will not be consistent with this empirical fact. The median trade balance in the low income group is acyclical, thus half of the low income economies display a procyclical balance, while it is countercyclical in middle and high income economies. As shown in Tables 3.8, 3.9 and 3.10, the trade balance is countercyclical for the majority of high income countries.[10] Although for almost 40% of the middle income economies the trade balance is procyclical, the group's median value is lower than for the high income group. Indeed, the middle income group is composed of two very different set of countries: the emerging market ones, many of which had experienced a strongly countercyclical trade balance during the sudden-stop phenomenon, and the low middle income ones (e.g. Algeria, Egypt) that are closer to the low income ones in terms of their experience of financial inflows and outflows. These empirical regularities for the cyclicality of the trade balance are consistent with results in the literature. Agenor, McDermott, and Prasad [2000] find that, in their sample of twelve low and low-middle income countries, the trade balance is often positively correlated with output. In Kaminsky, Reinhart, and Vegh [2004], almost all of the countries with countercyclical capital inflows are either low or middle-low income countries.[11]

The volatility of government consumption relative to that of GDP is roughly twice as large in low and middle income countries compared to high income countries. Further, the cyclicality of government consumption is highest for middle income countries. The model presented in the paper does not analyze the role of goverment-spending shocks. A more detailed analysis of fiscal policy, which also includes measures of taxation, can be

[10] The eight exeptions are: Austria, Cyprus, Israel, Kuwait, Luxembourg, Malta, Singapore, Sweden. For many of these countries, however, the trade balance is barely procyclical.

[11] Up to changes in official reserves and interest payments, countercyclical capital inflows are equivalent to procyclical trade balances.

found in Talvi and Vegh [2000] and Kaminsky, Reinhart, and Vegh [2004]. Finally, low and middle income countries display a volatility of GDP which is twice as large as that of high income countries on HP filtered data, and even more on annual growth rates as shown in Table 3.7. The higher output volatility of low and middle income countries, which has received a great deal of attention in the economic literature (e.g. Acemoglu and Zilibotti [1997]), cannot be accounted for by the model unless more volatile shocks are assumed in LDCs.

In the model presented in Section 3.4, low investment flexibility is the common thread that explains the different pattern of the trade balance, the relative volatility of consumption and the cyclicality of investment in LDCs compared to high income economies. The trade balance is increasingly procylical in the SOE model because a country demands fewer resources from abroad when investment does not respond. Consumption volatility is higher in a closed economy, because it absorbs a larger variation of output induced by supply shocks, due to the low investment response. I will argue in the next section that constraints set by several economic institutions are responsible for the low flexibility of investment that is observed in LDCs.

3.3. Institutional Constraints and Investment Flexibility

This section discusses how institutional constraints – such as credit market frictions, imperfect contract enforcement, excessive government regulation, and import restrictions – reduce a firm's ability to adjust investment decisions in response to changes in economic conditions. These constraints produce two distinct effects: lengthening the investment process, and increasing a firm's cost of adjusting the current level of investment relative to what previously planned and built.

For the sake of concreteness, consider the building process of a plant. Two phases are commonly distinguished (Krainer [1968]). In the first phase – the planning phase – the firm decides on the characteristics of the plant, enters into contracts with outside parties for the construction of the plant, and finances the investment through banks and other financial intermediaries. In the second phase – the construction phase – the investment project is physically implemented.

Studies such as that of Krainer [1968] on US data find that the planning phase can take up to a third of the entire investment process. The technological content of the construction involved, its novelty and complexity for example, is an important determinant of the length of the planning phase as well as of the ability of the firm to subsequently revise actual investment relative to the plan. But the role of economic institutions is also relevant. In the case of LDCs, poor contract enforcement, the lack of credit registries, and the general weakness of credit market institutions (WorldBank [2005]) extend the time necessary to finance projects. To counteract the high risks of lending stemming from inadequate institutional safeguards, lenders need to spend time and resources screening borrowers and assessing projects. The screening phase is particularly burdensome when establishing a new relationship, as lenders have little or no information on would-be borrowers due to the lack of credit registries.[12] Moreover, firms' ability to revise their investment plans once loans are granted is impaired. In order to reduce their exposure to defaults, lenders

[12]Relational contracting can partly offset the inefficiencies of anonymous contracting, although in practice they cannot substitute in full for the lack of jurisdicial protection (Johnson, McMillan, and Woodruff [2002]) Also see Fafchamps [2004] on the role of relational contracting in Sub-Saharan Africa.

need to closely monitor borrowers; since it is costly to do so, they will prefer to lend against the initial business plan (which borrowers would commit to and is known to both parties) rather than offer open and flexible lines of credit that would allow firms to revise their investment decisions.

In addition to bank-firm relationships, weak contract enforcement in LDCs also affects contractor-firm relationships. Due to the lack of enforcement, firms in LDCs choose contractual arrangements that minimize the moral hazard of contractors. They will thus favor fixed-cost contracts, where the payment to the contractors is set in advance, over cost-plus contracts, whereby firms compensate contractors for their costs plus a fee (Bajari and Tadelis [2001]). But at the same time as they seek to address moral hazard, these contractual provisions also reduce the flexibility of the investment process, as both parties are constrained by what was spelled out in the initial plan. Investment flexibility is also affected by the long bureaucratic delays, heavy government regulation and corruption (WorldBank [2005], Loayza, Oviedo, and Serven [2004], de Soto [1989]) that characterize LDCs' economies and contribute to extending the length of the planning phase. In addition, these institutions increase time and resource costs incurred by firms to modify the construction process relative to what was initially authorized.

Inefficient institutions also affect the length of the construction phase. Empirical evidence shows that firms in LDCs import most of their equipment goods (Eaton and Kortum [2001], Caselli and Wilson [2004]). It is thus particularly important in these countries that goods transit quickly through customs, to allow firms to react to current economic conditions. LDCs are, however, characterized by complex import regulations, including tariffs and hidden import barriers (Loayza, Oviedo, and Serven [2004]), which lead to lengthy customs clearance procedures (Gwartney and Lawson [2002]).

Quantifying the degree of inflexibility induced by these economic institutions, either through higher costs of plan readjustment or longer duration of investment projects, is not trivial. Adjustment costs are difficult to measure, and cross-country analyses of the length of the investment process are not available, particularly for LDCs. However, in order to gauge the difference in TTB between LDCs and more developed countries, it is interesting to compare existing empirical studies. A relatively large number of studies cover US data, either at the aggregate level (e.g. Altug [1989], Oliner, Rudebusch, and Sichel [1995a]), at the firm level (e.g. Koeva [2001]), or at the project level (e.g. Mayer and Sonenblum [1955], Koeva [2001]). Empirical estimates on US data find a TTB for structures of approximately two years.[13] Fewer studies are based on international data. Peeters [1996] cannot reject a TTB of more than two years on Dutch industry data, while del Boca, Galeotti, Himmelberg, and Rota [2005] find a project duration for structures on a panel of Italian firms of three years. Interestingly, they interpret the longer project duration on Italian data in terms of institutional constraints. This empirical finding is used as a rough guide for parametrizing the average duration of investment projects for LDCs in the numerical exercises of Section 3.5.

In the model presented in the next section, the effect of inefficient institutions will only be to increase the duration of investment projects, while firms cannot readjust the investment decision relative to the initial plan as in Kydland and Prescott [1982]. The

[13] An excellent review of the empirical estimates of TTB and time-to-plan on US data can be found in del Boca, Galeotti, Himmelberg, and Rota [2005].

choice of not explicitly modelling the readjustment decision follows from the their qualitative equivalence to longer project duration, and from the difficulty in assessing their empirical magnitudes.

3.4. The Model Economy

Consider a small open economy (SOE) with incomplete asset markets. The economy is populated by a large number of infinitely-lived households who can trade internationally in a single non-state contingent bond. The household side of the model economy is standard. At each date t, households make consumption decisions, C_t, supply labor services, L_t, at the wage rate W_t and choose their level of international borrowing B_t.

Following Mendoza and Uribe [2000] and others, the interest rate paid on international borrowing, r_t, is an increasing function of the domestic stock of debt $\bar{B}_t$, taken as given by each household. In particular, it is assumed that $r_t = r^* + p(\bar{B}_t/Z_t)$. The variable Z_t is the permanent component of the aggregate productivity shock, which will be defined below. The variable r^* is the international interest rate. The function $p(\cdot)$ is the interest premium function, which is increasing and convex and such that $p(\bar{b}_{ss}) = 0$ and $p'(\bar{b}_{ss}) = \pi$, where $\bar{b}_{ss}$ denotes the steady state level of the scaled domestic stock of debt, $\bar{b}_t \equiv \bar{B}_t/Z_t$. The role of the interest premium function is to induce stationarity in the series of consumption and foreign debt.[14] The closed economy version of the model corresponds to the case in which the economy's initial stock of debt is zero and the supply of foreign debt is perfectly inelastic ($\pi = \infty$).

Households receive income from labor services, W_tL_t, and dividends, D_t, and pay interest on their outstanding stock of debt. Every household owns the same fraction

[14]A detailed discussion can be found in Schmitt Grohe and Uribe [2003].

of each firm. Without loss of generality, stocks are not traded. At each date t, the representative household solves:

$$\max\ \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, 1 - L_{t+j}),$$

subject to the flow budget constraint:

$$C_t + B_{t-1}(1 + r_{t-1}) \leq W_t L_t + B_t + D_t.$$

The production sector of the economy is composed of a large number of firms producing a homogeneous output good, Y_t, with a Cobb-Douglas production function: $Y_t = S_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{1-\alpha}$, using capital, K_{t-1}, and labor. The level of total factor productivity S_t is common across all firms and is the product of a permanent (Z_t) and a transitory shock (A_t). The output good is either consumed or converted by firms into investment inputs X_t at the rate $\frac{\bar{E}}{\omega_t}$ per unit of output good, where ω_t is an investment specific shock, and $\bar{E}$ is a fixed parameter. Firms use investment inputs when investing.

The exogenous shocks evolve according to:

$$\begin{aligned} S_t &\equiv Z_t A_t, \\ \log A_t &= \rho_a \log A_{t-1} + \varepsilon_t^a, \\ Z_t &= \Gamma_t\ Z_{t-1}, \\ \log \Gamma_t &= (1 - \rho_\Gamma) \log \bar{\Gamma} + \rho_\Gamma \log \Gamma_{t-1} + \varepsilon_t^\Gamma, \\ \log \omega_t &= \rho_\omega \log \omega_{t-1} + \varepsilon_t^\omega. \end{aligned} \tag{3.1}$$

The innovations ε_t^a, ε_t^Γ and ε_t^ω are jointly independent, i.i.d over time, and distributed according to $\varepsilon_t^s \sim N(0, \sigma_s^2)$, for each $s = a, \Gamma, \omega$. The long run growth rate of TFP, S_t, is equal to $\bar{\Gamma}$.

Each firm invests in a fixed but large number of perfectly symmetric investment goods indexed by their type $j \in [0, 1]$. It takes time to build and plan for the construction of each investment good. The firm chooses the desired quantity of type-j investment *good* when it starts a type-j investment *project*. Each firm can only run one project per-investment good at a time, and the scale of the project cannot be modified once initiated. If a project matures at t a new scale may be chosen at $t + 1$. The date t scale of the type-j project will be denoted with $\iota_t(j)$. It is important to note that this notation omits an explicit indication of the date– and corresponding information set– at which the scale $\iota_t(j)$ has been chosen. An investment project started at t requires $\theta\iota_t(j)$ units of the investment input X_s, at all dates s up to maturity. Thus the date t investment expenditure in units of X_t is given by the sum of the expenditures for each project

$$E_t \equiv \theta \int_0^1 \iota_t(j)\ dj. \tag{3.2}$$

As in the canonical TTB model, investment projects add to the firm's capital stock at the date at which the project matures. The impossibility for a firm to delay the use of an investment good to a later date is implicit in this assumption.

Let $M_t \subseteq [0, 1]$ be the set of investment projects maturing at t, and NM_t its complement in $[0, 1]$. The scale of a project that has not matured is assumed to grow at the constant steady state growth rate of the economy $\bar{\Gamma}$. The growing scale of the project is a simple normalization which ensures that firm's steady state investment (I) is independent of the length of the project. This assumption has an analog in the automatic indexation in recent models of staggered price adjustment (Christiano, Eichenbaum, and Evans [2005]). Thus $\iota_t(j) = \bar{\Gamma}\iota_{t-1}(j)$ for $j \in NM_{t-1}$. The time needed for each project to mature is uncertain, so that investment in some of the projects will be predetermined by the earlier choices. The maturity of each project follows a Poisson-process with arrival rate θ. A project started at t has a probability θ of being completed at the same date t. Uncompleted projects mature with the constant probability θ at each of the subsequent dates, so that a firm expects projects to mature at $t + (1/\theta - 1)$. Define with

$$\iota_t^m(j) = \begin{cases} \iota_t(j) & \text{if } j \in M_t, \\ 0 & \text{otherwise,} \end{cases} \tag{3.3}$$

the level of type j investment good that will increase the capital stock at $t+1$. The level of $\iota_t^m(j)$ is equal to the scale of the project if the project matures at date t ($j \in M_t$), and it equals zero otherwise. The firm uses on average one unit of the investment input X_t per $\iota_t(j)$ project, since the projects last on average $1/\theta$ periods and the firm uses $\theta\iota_t(j)$ units of X_t per period. A time-to-plan formulation of the model, in which investment inputs are only used when the project matures, would yield the same expenditure function as in (3.2). Because the maturity of the project follows a Poisson-process, it follows that $E_t = \int_{j \in M_t} \iota_t(j)dj = \theta \int_0^1 \iota_t(j)dj$.[15]

Investment goods are characterized by complementarities – the return to each good is increasing with the availability of the others – and each $\iota_t^m(j)$ enters symmetrically in

[15]See equation (3.7) below.

the date t investment *basket*

$$I_t \equiv \left(\int_0^1 \iota_t^m(j)^{1-1/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}, \tag{3.4}$$

where $\varepsilon > 1$, so that none of the investment goods are essential in increasing the capital stock.

The level of the investment basket I_t increases the firm's capital stock, which depreciates at the constant rate δ. The model also includes capital adjustment costs, which are only used to compare the TTB model to the benchmark neoclassical SOE model in which projects mature instantaneously ($\theta = 1$). The law of motion of the capital stock then follows

$$K_t = (1-\delta)K_{t-1} + I_t - \Phi\left(\frac{K_t}{K_{t-1}}\right) K_{t-1}. \tag{3.5}$$

The function $\Phi(\cdot)$ is a standard capital adjustment cost function which satisfies $\Phi(\bar{\Gamma}) = \Phi'(\bar{\Gamma}) = 0$ and $\Phi''(\bar{\Gamma}) = \phi$. The parameter ϕ is equal to zero for most of the numerical experiments.

At each date t, the firm chooses labor services and scales for matured projects $\{\iota_t(j)\}_{j\in M_{t-1}}$, to maximize the discounted value of future profit flows

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left(Y_{t+j} - W_{t+j} L_{t+j} - \omega_{t+j}^{-1} \ \bar{E} \ E_{t+j}.\right), \tag{3.6}$$

subject to (3.1)-(3.5), and $Y_t = S_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{1-\alpha}$.

In (3.6), λ_{t+j} is the marginal utility of consumption of the representative household at $t+j$ which is taken parametrically by firms. Firms' cash flows are given by $Y_{t+j} - W_{t+j} L_{t+j}$, and E_{t+j} is equal to the total units of X_t that are used in producing the investment goods. The expression in parenthesis is equal to D_{t+j}, which is the dividend obtained by the representative household.[16]

3.4.1. Solution of the Model

The model is solved by first characterizing the date t intratemporal decision problem of the firm. The advantage of doing so is that it is then possible to express the intertemporal problem only in terms of E_t, I_t and K_t, and not in terms of the scale of the projects $\iota_t(j)$'s. I then solve the intertemporal problems of households and firms.

The firm demands labor services in the amount $L_t = ((1-\alpha)S_t/W_t)^{1/\alpha} K_{t-1}$, so that dividends may be rewritten as $D_t = R_t K_{t-1} - \omega_t^{-1}\, \bar{E}\, E_t$, where $R_t \equiv \alpha\left((1-\alpha)S_t/W_t\right)^{(1-\alpha)/\alpha}$ is the marginal product of capital, and $R_t K_{t-1}$ are the firm's cash flows.

Now consider the allocation of investment among the different investment projects. First note that because the maturity of projects follows a Poisson process, all projects have an equal probability of maturing irrespective of their initial starting date. Then, the date t averages of the quantities $\iota_t(j)$ and $\iota_t(j)^{1-1/\varepsilon}$ among the projects that mature, $j \in M_t$, and that do not mature, $j \in NM_t$, are equal. Furthermore, because of the large number of projects, the total fraction of projects that matured is equal to θ, and the remaining fraction $1-\theta$ are projects that have not matured. It follows from this

[16] Since only households borrow in the international markets, dividends can take negative values. It is straightforward to guarantee the non negativity of the dividend flow by allowing firms to borrow through bonds.

discussion that

$$\frac{\int_{j\in M_t} x_t(j)\ dj}{\theta} = \frac{\int_{j\in NM_t} x_t(j)\ dj}{1-\theta} = \int_0^1 x_t(j)\ dj \text{ for each } x_t(j) = \{\iota_t(j),\ \iota_t(j)^{1-1/\varepsilon}\}. \tag{3.7}$$

where the expression after the second equality is the average scale over all projects. Using (3.7), (3.4) becomes

$$I_t = \left(\theta \int_0^1 \iota_t(j)^{1-1/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}. \tag{3.8}$$

Now follows the crucial step of the intratemporal investment problem, which is to express investment expenditure, E_t, and the investment basket, I_t, in terms of their respective lagged values and the scale of the projects that matured at $t-1$. First note that I_t can be expressed as

$$\begin{aligned} I_t &= \left(\theta\left(\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + \int_{j\in NM_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj\right)\right)^{\varepsilon/(\varepsilon-1)} = \\ &= \left(\theta\left(\int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + (1-\theta)\left(\int_{j\in NM_{t-1}} \left(\bar{\Gamma}\iota_{t-1}(j)\right)^{1-1/\varepsilon} dj\right)/(1-\theta)\right)\right)^{\varepsilon/(\varepsilon-1)} = \\ &\left(\theta \int_{j\in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + (1-\theta)\left(\bar{\Gamma} I_{t-1}\right)^{1-1/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}. \end{aligned} \tag{3.9}$$

The expression after the first equality follows simply from rewriting the integral in (3.8). The second equality makes use of the law of motion of projects that did not mature in the last period, which are beyond the firm's control at date t. The third equality follows from (3.8) lagged by one period and (3.7). Using analogous steps, (3.2) can be rewritten as

$$E_t = \theta \int_{j \in M_{t-1}} \iota_t(j) dj + (1-\theta)\ \bar{\Gamma}\ E_{t-1}. \tag{3.10}$$

The firm only controls the current expenditure for a fraction θ of the investment projects that had matured, while the remaining fraction $1-\theta$ is predetermined.

The firm will choose the same scale for all maturing investment projects as long as $\varepsilon < \infty$, because they enter symmetrically into I_t and all have the same expected cost of one unit of investment input. This result follows from a simple expenditure minimization of (3.10) subject to (3.9) with respect to all projects that had matured in the previous period $\{\iota_t(j)\}_{j \in M_{t-1}}$. Let ι_t be the optimal scale that is chosen by the firm, then $\iota_t(j) = \iota_t$ for all $j \in M_{t-1}$. Using this result in (3.9) and (3.10), and then substituting ι_t from (3.9) in (3.10) yields

$$E_t = \Omega(I_t, \bar{\Gamma} I_{t-1}) + (1-\theta)\bar{\Gamma} E_{t-1}, \tag{3.11}$$

where

$$\Omega(u_1, u_2) \equiv \theta^{\frac{2}{1-\varepsilon}} \left(u_1^{1-\frac{1}{\varepsilon}} - (1-\theta)\ u_2^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}.$$

Condition (3.11) is the only additional condition in an otherwise standard intertemporal firm maximization problem.

In order to solve the intertemporal problem of firms and households it is convenient to express both problems in terms of variables scaled by the level of the permanent shock, so that all variables converge to a non-stochastic steady state level. For each variable $V_t = \{B_t, C_t, E_t, I_t, K_t, Y_t\}$ let the lower case letter be the rescaled variable $v_t \equiv V_t/Z_t$.

Households' utility function is assumed to be consistent with the existence of a balanced growth path. In particular let $U(C_t, 1-L_t) = Z_t^{1-\sigma} U(c_t, 1-L_t)$. Finally, define the scaled marginal utility of consumption as $\tilde{\lambda}_t \equiv \lambda_t \; Z_t^{\sigma}$, and note that, since Ω is constant returns to scale $\frac{\Omega(I_t, \bar{\Gamma} I_{t-1})}{z_t} = \Omega(i_t, \bar{\Gamma} i_{t-1}/\Gamma_t)$.

The date t intertemporal maximization problem of the firm in terms of scaled variables is

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} \tilde{\beta}_{t,j} \; \tilde{\lambda}_{t+j} \left(R_{t+j} \; \Gamma_{t+j}^{-1} \; k_{t+j-1} - \bar{E} \omega_{t+j}^{-1} e_{t+j} \right),$$

where $\tilde{\beta}_{t,j} \equiv z_t^{1-\sigma} \beta^j \prod_{\tau=1}^{j} \Gamma_{t+\tau}^{1-\sigma}$,[17] subject to the scaled condition (3.11)

$$e_t = \Omega(i_t, \bar{\Gamma} i_{t-1}/\Gamma_t) + (1-\theta) \; \bar{\Gamma} \; \Gamma_t^{-1} e_{t-1}, \tag{3.12}$$

and the scaled law of motion for the capital stock (3.5)

$$k_t = (1-\delta)\Gamma_t^{-1} k_{t-1} + i_t - \Phi\left(\frac{k_t \Gamma_t}{k_{t-1}}\right) \Gamma_t^{-1} k_{t-1}. \tag{3.13}$$

Let $\tilde{\beta}_{t,j}\mu_{t+j}$ and $\tilde{\beta}_{t,j}\gamma_{t+j}$ be the date $t+j$ Lagrange multipliers on (3.12) and (3.13) respectively. The associated first order conditions are

(3.14)

$$(k_t) \quad \gamma_t \;\; (1 + \Phi'(\kappa_t)) = \beta \; \mathbb{E}_t \; \Gamma_{t+1}^{-\sigma} \; \left\{ \tilde{\lambda}_{t+1} \; R_{t+1} + \gamma_{t+1} \;\; ((1-\delta) + \Phi'(\kappa_{t+1}) \; \kappa_{t+1} - \Phi(\kappa_{t+1})) \right\},$$

$$(i_t) \quad \gamma_t = \mu_t \; \Omega_1(i_t, \bar{\Gamma} i_{t-1}/\Gamma_t) + \beta \; \mathbb{E}_t \; \Gamma_{t+1}^{-\sigma} \; \mu_{t+1} \; \Omega_2(i_{t+1}, \bar{\Gamma} i_t/\Gamma_{t+1}) \; \bar{\Gamma},$$

$$(e_t) \quad \mu_t = \bar{E} \; \omega_t^{-1} \; \tilde{\lambda}_t + \beta(1-\theta) \; \mathbb{E}_t \; \mu_{t+1} \Gamma_{t+1}^{-\sigma} \; \bar{\Gamma},$$

where $\kappa_t \equiv k_t \Gamma_t / k_{t-1}$, and the numeric subscript denotes the argument with respect to which the derivative is taken.

[17] For $j = 0$, let $\tilde{\beta}_{t,0} \equiv z_t^{1-\sigma}$.

Now consider the date t household problem, which is

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} \tilde{\beta}_{t,j} U(c_{t+j}, 1 - L_{t+j}),$$

subject to the scaled flow budget constraint

$$c_t + (1 + r_{t-1}) b_{t-1} \Gamma_t^{-1} \leq b_t + w_t L_t + d_t, \tag{3.15}$$

and the no-Ponzi condition $\lim_{j \to \infty} \mathbb{E}_t \; b_{t+j} \; \Pi_{s=0}^{j} \left(1 + r_s\right)^{-1} \leq 0$. Households' first order conditions are

$$\begin{aligned} (c_t) \quad & U_1(c_t, 1 - L_t) = \tilde{\lambda}_t, \\ (b_t) \quad & \tilde{\lambda}_t = \beta \; \mathbb{E}_t \; \Gamma_{t+1}^{-\sigma} \; (1 + r_t) \; \tilde{\lambda}_{t+1}, \\ (L_t) \quad & U_2(c_t, 1 - L_t) = \tilde{\lambda}_t w_t. \end{aligned} \tag{3.16}$$

where $\tilde{\beta}_{t,j} \tilde{\lambda}_{t+j}$ is the date $t + j$ Lagrange multiplier on (3.15).

Given initial levels for k, e, i, and a sequence of exogenous shocks, an equilibrium is defined as a state-contingent sequence of prices and quantities such that a) firms and households solve their respective maximization problem, b) goods, labor and international financial markets clear. The equilibrium conditions are given by equations (3.12)-(3.15) along with firms' inverse labor demand $w_t = (1 - \alpha) y_t / L_t$, $R_t = \alpha \left((1 - \alpha) A_t / w_t\right)^{(1-\alpha)/\alpha}$, the equilibrium condition $b_t = \bar{b}_t$, and the inverse supply of foreign debt $r_t = r^* + p(\bar{b}_t)$.

In what follows the utility function is assumed Cobb-Douglas in consumption and leisure

$$U(C_t, L_t) \equiv \frac{\left(C_t (1 - L_t)^{\psi}\right)^{1-\sigma}}{1 - \sigma}. \tag{3.17}$$

Also assume that the international interest rate equals the domestic rate in the non-stochastic steady state $r^* = \beta^{-1}\bar{\Gamma}^{\sigma} - 1$. The scaled level of the capital stock in steady state is

$$k_{ss} = \left(\frac{\alpha \bar{\Gamma}^{1-\alpha} L_{ss}^{1-\alpha}}{(\beta^{-1}\bar{\Gamma}^{\sigma} - (1-\delta))\bar{E}\ \theta^{1/(1-\varepsilon)}} \right)^{\frac{1}{1-\alpha}},$$

and ψ in (3.17) is chosen to match a given level of L_{ss}. Note that the level of θ affects the steady state level of the economy. This might be an interesting channel through which a longer project duration might affect an economy's level of development. Because I want to focus on the dynamic effects, it is assumed that $\bar{E} = \theta^{1/(\varepsilon-1)}$, so that θ has no effects on k_{ss} and on the other variables.[18] Bencivenga, Smith, and Starr [2000] present an endogenous growth model in which investment gestation lags affect the growth rate of the economy. The numerical solution of the model is computed by loglinearizing the equilibrium conditions around the non stochastic steady state. The resulting system of expectational difference equations is solved using Anderson and Moore [1985] routines.

The next section goes deeper into the mechanism through which different project durations, or equivalently different values of θ, affect the dynamic response of the economy to exogenous shocks. A different value for θ, in Section 3.5, will be the main difference between high and low income economies. In low income economies, investment projects last longer–a lower θ– because of the delays stemming from institutional constraints.

[18]The independence of the other variables' steady state values from θ can be seen from the steady state aggregate resource constraint: $c_{ss} + \bar{E}\ \theta^{1/(1-\varepsilon)}\ k_{ss}\left(1 - (1-\delta)\bar{\Gamma}^{-1}\right) + (\beta\bar{\Gamma}^{\sigma-1} - 1)b_{ss} = \bar{\Gamma}^{-\alpha} k_{ss}^{\alpha} L_{ss}^{1-\alpha}$.

3.4.2. Investment Flexibility

In the model, the flexibility of investment depends on the average duration of TTB $(1/\theta)$ and on the degree of complementarity between the different types of investment (lower ε).

As TTB lengthens, firms optimally choose a smaller adjustment for each type of the projects, because the current realizations of the shocks carry less information about future returns. Furthermore, the firm adjusts a smaller fraction of the projects with longer TTB.

However, TTB is not in itself sufficient to make the choice of the investment *basket* inflexible. If the different investment projects were perfect substitutes, a firm could fully readjust the choice of the investment basket, since a fraction θ^2 of the projects at each date t are under the control of the firm *and* mature with certainty in the current period. Only with TTB and complementarity between the different investment projects, does the current choice of the investment basket not fully respond to the current realization of the shock and depend on its earlier choice leading to the delayed adjustment.

The first part of this section discusses in more detail how the dynamic response of aggregate investment to the shocks depends on the average duration of the project and on the complementarity of investment goods in the investment basket.

The second part of this section compares the current model to two alternative investment models with convex costs of adjusting either the capital stock (Hayashi [1982]) or the investment flow (Christiano, Eichenbaum, and Evans [2005]).

A large part of the discussion will be formulated in terms of exact analytical expressions, which require more stringent assumptions. It is assumed throughout this section that the supply of international bonds is perfectly elastic $(\upsilon = \infty)$, so that $r_t = r^*$, and that households' labor supply is perfectly inelastic ($\psi = 1$), so that $L_t = 1$. A perfectly elastic supply of international bonds implies that aggregate investment is not affected by households' savings decisions. Further, this section only studies the dynamic response of investment to the non-permanent component of total factor productivity. Thus the investment shock and the permanent shocks are normalized to one ($\omega_t = 1$ and $\Gamma_t = \bar{\Gamma} = 1$). Since $\bar{\Gamma} = 1$ all scaled and unscaled variables are equal, and the unscaled notation is used.

3.4.2.1. Time-to-build and Projects' Complementarities. Suppose that capital adjustment costs are zero, $\phi = 0$, and denote with a hat each variable's percentage deviation from its steady state level. As shown in the Appendix, the assumptions of this section imply that the local dynamics of aggregate investment around the non-stochastic steady state are approximately characterized by the system of equations

$$\begin{aligned}
&\hat{Q}_t = (1-\beta(1-\delta))\,(1-\alpha)\,(\mathbb{E}_t\hat{A}_{t+1} - \hat{K}_t) + \beta(1-\delta)\mathbb{E}_t\hat{Q}_{t+1},\\
&\eta(\hat{E}_t - \hat{E}_{t-1}) = \hat{Q}_t + \beta\eta\;\mathbb{E}_t\;(\hat{E}_{t+1} - \hat{E}_t),\\
&\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{E}_t,\\
&\hat{A}_{t+1} = \rho_a\hat{A}_t + \varepsilon_t^a.
\end{aligned} \tag{3.18}$$

where[19]

$$\eta \equiv \frac{(1-\theta)}{\varepsilon\,(1-\beta(1-\theta))\,\theta}. \tag{3.19}$$

The variable $Q_t \equiv \gamma_t/\lambda_t$ is the date t Tobin's Q, or the price of capital in units of consumption. As shown in the Appendix, percentage deviations from steady state of

[19] The parameter η is a decreasing and convex function of θ. It is equal to zero for $\theta = 1$ and to infinity for $\theta \to 0$.

investment expenditure and the investment basket are equal, $\hat{E}_t = \hat{I}_t$, and for simplicity the term investment in this section refers to both .[20] Using the method of undetermined coefficients, (3.18) can be solved when capital fully depreciates, $\delta = 1$, to yield

$$\hat{E}_t = \chi_0 \hat{A}_t + \chi_1 \hat{E}_{t-1}. \tag{3.20}$$

The exact analytical expressions for χ_0 and χ_1 are provided in Appendix.

When investment projects mature instantaneously, investment only responds to innovations in the current level of productivity, with a magnitude that depends on the autocorrelation of the shock. For instantaneous maturity, $\theta = 1$, the values of the parameters in (3.20) are $\chi_0 = \rho_a$ and $\chi_1 = 0$. The investment response to the current productivity level follows from the shock's serial correlation: current high levels of productivity imply high levels in the future and hence higher returns to the current level of investment. As the expected maturity of the project increases, a lower θ, the value of χ_1 increases, while χ_0 falls. In the limiting case in which investment projects last forever ($\theta \to 0$) investment is completely inelastic to innovations in productivity, $\chi_0 = 0$.

These results only hold when investment projects are complements $\varepsilon < \infty$. Indeed in the case of perfect substitutability firms perfectly rebalance the investment choice by adjusting the scale of the projects that just matured relatively more. In this case, the values of the parameters in (3.20) are $\chi_0 = \rho_a$ and $\chi_1 = 0$ as in the case of instantaneous maturity.

[20] The measure in levels that is empirically relevant is E_t, which is what would appear in national and firms' accounts.

Thus both θ and ε affect investment through the value of η in (3.19). Although the two parameters cannot be separately identified by the model's first order approximation,[21] they act on investment through two different channels. The value of θ controls the frequency of the firm's adjustment of each one of the investment projects, while ε measures how much flexibility given the TTB constraint is left to the firm to adjust the level of investment.

The dynamic link between investments due to projects' complementarity is similar to the one proposed by von Hayek [1937]. In this paper, he discusses the idea that the marginal return to investment might be increasing, rather than decreasing, with different types of capital that have complementarities. The intertemporal linkage between investment decisions follows from different types of investment being made at different dates: a plant is built first, and then the equipment is installed. Similarly, in the investment model presented in this paper, larger investment at earlier dates increase the productivity of later investment because of the complementarities among the different types of investment.

A more natural interpretation of the complementarity between investment goods is through the complementarity of capital types. In recent work (Lucca [2005]), I explore an alternative formulation of the model in which a firm operates a stock of capital – a plant –, which is made of several complementary capital goods. Each good depreciates over time and the firm can improve the quality of the plant by increasing the quality of each component. When TTB is assumed at the level of the capital goods, firms' investment behaves in a way which is analogous to the one in this paper in terms of the lack of response to current shocks and delay of the adjustment. The main advantage of the formulation presented in this paper is its simplicity. To derive the analytic equivalent of expression

[21] The parameters could be identified through the joint effect on the model dynamics and steady state if the value of $\bar{E}$ were normalized to a constant.

(3.11), one needs additional technical assumptions, and the analysis is complicated further by the fact that with heterogenous capital goods the firm's capital stock depreciates at a non linear rate.

3.4.2.2. Comparison with Adjustment Cost Models. This section compares the TTB formulation in this paper with capital and investment adjustment cost models. In this section it is assumed that investment projects mature instantaneously, i.e. $\theta = 1$.

First consider a model with standard capital adjustment costs, thus let ϕ be different from zero in (3.5). Also assume that $\delta < 1$ so that the investment flow and the future capital stock differ. The local dynamics of investment and capital around the non-stochastic steady state are then approximately described by a system analogous to (3.18). This system can be solved, similarly to what done in Appendix for (3.18), to obtain the linear policy function $\hat{K}_t = \chi_0^K \hat{A}_t + \chi_1^K \hat{K}_{t-1}$. By substituting this into the loglinearized capital accumulation equation, it follows that $\hat{E}_t = \delta^{-1}(\chi_0^K \hat{A}_t + (\chi_1^K + (1-\delta))\hat{K}_{t-1})$.

Larger costs of capital adjustment decrease the value of χ_0^K and increase χ_1^K, reducing investment's response to innovations in productivity. An analogous effect also occurs in the TTB model by increasing project's duration or the degree of investment complementarities. The main difference among the two models is that with capital adjustment costs, investment only depends on the lagged capital stock and not on its lagged level. Firm's response to a shock is then very different in the two models for the case in which δ is small as usually assumed.[22] In the short run, the capital stock is roughly constant because

[22]In the general case in which $\delta < 1$ in the TTB model, (3.20) becomes $\hat{E}_t = \chi_0 \hat{A}_t + \chi_1 \hat{E}_{t-1} + \chi_2 \hat{K}_{t-1}$. As I explain below, a positive value for χ_2 only affects the response of investment by reducing the response. The most important difference between the TTB and the capital adjustment cost model is that in the latter $\chi_1 = 0$ so that the adjustment is delayed. In other words, considering the case of full depreciation in the TTB model does not affect the main economic intuitions and the discussion of this section.

investment in steady state is only a small fraction of the capital stock. The dynamic response of investment with capital adjustment costs, then, only depends on the path of $\hat{A}_t$ and it is highest when the innovation occurs, as in a frictionless investment model. In the TTB model, investment depends on its lagged level, $\hat{E}_{t-1}$ and on the current shock $\hat{A}_t$. The effects of the two variables are countervailing after a temporary shock. In the periods immediately after the shock, the firm would like to increase investment the most because productivity is at its peak, but faces the largest constraints because a large fraction of the projects is fixed. As time passes, more and more projects are readjusted. Moreover the size of the adjustment involved is increasing because a larger fraction of projects had readjusted. Due to the temporary nature of the shock, over time the firm starts reducing investment because of the lower current and future productivity. The long run, and not just the short run, must be taken into account by the firm because current investment choices spill into future ones. The resulting response of investment will be hump-shaped with a peak of the response that occurs at a date that lags the realization of the shock.

The reader familiar with adjustment cost models on investment flows of the type assumed by Christiano, Eichenbaum, and Evans [2005], might have already noticed the close resemblance of these type of models with the TTB model presented in this paper. The two models are actually identical up to a first-order-approximation, for appropriate choices of the parameter values. Indeed for a given shape of the adjustment cost function of the Christiano, Eichenbaum, and Evans [2005] type, it is possible to find project durations and investment goods' substitution elasticities such that investment's local dynamics are identical in the two models. Christiano, Eichenbaum, and Evans [2005] directly assume that it is costly to change investment from its lagged level; the results of this paper interpret their adjustment costs in terms of TTB and investment complementarities. I explore further results of this TTB specification in work in progress (Lucca [2005]). I now turn to the numerical exercises and show how longer project durations help understand aggregate fluctuations in LDCs. The longer project duration is interpreted as following from tighter institutional constraints.

3.5. Numerical Experiments

This section shows how a lower flexibility of investment helps understand the empirical differences between business cycle moments of high and low income economies that were presented in Section 3.2. Section 3.3 discussed how institutional constraints of LDCs lengthen the investment process in these countries, and as shown in the previous Section, firms then optimally choose to delay and not to fully adjust investment in response to shocks.

The numerical experiments presented here show the aggregate consequences of longer project duration. The main results are that, in economies with longer TTB, the correlation between aggregate investment and GDP is lower, the relative volatility of aggregate consumption is higher, and the correlation between the trade balance and GDP is higher. As discussed in Section 3.2, the relative volatility of aggregate investment in LDCs is comparable to that of high income economies. I will show that more volatile investment-specific shocks are needed in the model in order to counteract the reduction in investment volatility stemming from the longer project duration.

This section presents numerical results for a closed and small open economy parametrization of the model. Longer project duration reduces consumption volatility only in the closed economy parametrization, whereas in the frictionless SOE households perfectly smooth consumption by borrowing and lending on the international bond market.

The numerical experiments in this section should ideally involve a calibration of the model for each of the countries under consideration, such that the properties of exogenous shocks and the other parameters of the model are representative of their empirical counterparts. This type of exercises is not feasible, however, because many of the necessary series are not available for low income economies.[23] Instead, I am currently developing a country-specific analysis of the model by performing a parameter estimation by GMM as in Christiano and Eichenbaum [1992] and Aguiar and Gopinath [2004]. At this stage I am performing numerical experiments to understand the implications of the model in terms of business cycle moments. In the numerical experiments, all economies share the parameters of a canonical high income economy except for the duration of investment projects and the volatility of the investment-specific shock. The results of this section should be, therefore, interpreted as a numerical comparative statics exercise along these two dimensions, rather than a calibration in the spirit of Kydland and Prescott [1982]. The values of the parameters used both in the closed and small open economy parametrization are summarized in Table 3.2.

The national account data of Section 3.2 have annual frequency, so a period in the model is set to be one year. The choice of the discount factor implies a 5% bond return along the balanced growth path. The values of the labor weight in the utility function, the capital share in production, and the depreciation rate are standard in RBC models of the

[23]First, measures of labor services are either poor or missing for LDCs and emerging economies. Second, although it is possible to compute a country's capital stock using perpetual inventory methods, this would imply dropping a large part of the small number of observations.

Name	Variable	Value
Discount Factor	β	$(1.01)^{-1}$
Curvature of Utility	σ	2
Labor Weight	ψ	such that $L_{ss} = .28$
Capital Share in Production	α	.36
Depreciation Rate	δ	.08
Elasticity of subst. of $\iota_t(j)$'s	ε	2.5
Average Growth Rate of Z_t	Γ	1.02
Persistency of $\log A_t$	ρ_a	.7
Persistency of $\log \Gamma_t$	ρ_Γ	.1
Persistency of $\log \omega_t$	ρ_ω	.7

Table 3.2. Parameter Values common to Closed Economy and SOE

US economy. The curvature of the utility function is in line with the SOE business cycle literature (e.g. Mendoza [1991] and Aguiar and Gopinath [2004]). The same curvature is assumed for the closed economy as it plays a limited role in this parametrization. The economy grows at a 2% annual rate on the balanced path, and the serial correlation of the transitory TFP shock matches the one on US post-war data at annual frequencies, assuming a constant growth rate for the economy (linear detrending). The SOE version of the model also considers the role of permanent TFP shocks. It is assumed that the serial correlation, ρ_Γ, of the innovation in the permanent shock equals 0.1. This number corresponds to the half life of the innovation estimated by Aguiar and Gopinath [2004] on Canadian quarterly data. The choice of ρ_Γ implies that a unit innovation in ε_t^Γ permanently increases TFP by a factor of 1.1.[24] The serial correlation of the investment-specific shock plays no important role in the exercises, and it is simply assumed to be 0.7 as for the transitory component of the TFP shock.

[24] The presence of a permanent component in the TFP shock for the SOE parametrization, implies that the persistency of the transitory component no longer corresponds to the empirical counterpart on US data, as such value was computed assuming a constant growth rate of TFP. Roberts [2001] presents alternative methods for estimating the properties of TFP in the presence of a permanent and temporary component. Such analysis is beyond the scope of the comparative statics exercise of this section.

As discussed in Section 3.3, empirical studies on the US economy find a length of TTB for structures of approximately two years. To account for the lower duration of TTB for equipment, it is thus assumed that projects in high income economies last on average 1.5 years, so that $\theta = 0.67$. Empirical estimates of TTB for LDCs are not available, as discussed in Section 3.3. However, del Boca, Galeotti, Himmelberg, and Rota [2005] find a TTB for structures on Italian data of three years. To account for even longer project durations in LDCs, I assume an average duration of three years for total investment, not only for structures.

The estimated investment adjustment cost function of Christiano, Eichenbaum, and Evans [2005] is used to calibrate the elasticity of substitution between investment projects. One can show that, in the log-linearized version of their model, the parameter η is equal to the steady state value of the second derivative of the adjustment cost function. For a given TTB duration and their estimate of η, one obtains the implied value of elasticity of substitution from (3.19). The elasticity of substitution in Table 3.2 corresponds to a $\eta = 0.3$, which is smaller than the 0.9 (Table 2 on page 17–no habit in the utility function) estimated by Christiano, Eichenbaum, and Evans [2005]. They use quarterly data while I use annual data. The lower value for η takes into account lower adjustment costs over longer horizons.

3.5.1. The Closed Economy

The first part of this section discusses the response of the closed economy to unexpected innovations in the transitory productivity shock and in the investment-specific shock. The second part of the section compares the theoretical moments of the model to their empirical counterparts. Permanent shocks are only considered in the SOE version of the model and will not be discussed for the closed economy.[25]

Table 3.3 reports levels of foreign debt and the derivative of the premium function in steady state, such that the model economy is closed.

Name	Variable	Value
Level of Foreign Debt in SS	$\bar{B}$	0
Derivative of the premium function in SS	π	∞
Capital Adjustment Cost	ϕ	0

Table 3.3. Parameter Values Specific to the Closed Economy

In the closed economy capital adjustment costs are set to zero. Figures 3.2 and 3.4 display the impulse responses of the model economy to a one percent innovation in the temporary and investment-specific shock, respectively. Each figure shows the response of consumption, investment, output and labor for three different parametrizations: the basic RBC, the high income and the low income. The basic RBC model is a benchmark economy in which investment projects mature instantaneously. As discussed above, projects take an average of 1.5 years in the high income and 3 years in the low income model to mature.

First, consider the response of the model economy to a transitory TFP shock. In the basic RBC model consumption, investment, output and labor all peak when the shock hits the economy, and then fall. The rate of decay of consumption's response is the smallest because of households' desire to intertemporally smooth. Conversely, investment in the high and low income parametrization does not fully respond to the shocks on impact. As discussed in Section 3.4.2, firms optimally choose not to fully adjust and

[25]The IRs of the closed economy to permanent shocks are reported in Figure 3.3. Closed economy RBC models do not usually account for permanent shocks (see Hansen [1997] for a discussion), and this is why I do not examine them here. The next section discusses their role in the SOE model.

delay investment because the scale of a large fraction of investment projects is initially fixed, and it is costly to tilt the composition of the investment basket by increasing the share of adjustable projects. The peak of investment's response in the low income economy lags the one in the high income one because of the longer project duration. Compared with the basic RBC model, investment is lower in the short run but also larger in the long run. The reason for this result is that short run investment decisions spill over into future decisions as discussed in Section 3.4.2. Consumption reacts more in low income economies because of a lack of domestic smoothing. Since the level of investment is low in the short run, consumption absorbs a large fraction of the output expansion induced by the shock. Firms take into account households' desire to smooth consumption through the stochastic discount factor λ_t in their objective function. However, returns on investments are too low and firms choose to invest a small amount. Due to the short-run increase in consumption, households choose to substitute labor services for leisure, and thus labor and output respond less in low income countries, as does output.

Now consider the response of the model to an unexpected innovation in the investment-specific shock. A positive shock reduces the cost of producing the investment input good X_t, makes investment relatively less expensive, and thus increases the incentives to invest in the short run. As shown in Figure 3.4, in the basic RBC model investment, labor and output increase when the shock hits while consumption falls. Investment's response in the high income economy is smaller and lags the realization of the shock because of a longer project duration. This is even more pronounced in the low income economy, where investment actually falls in the first period because $\hat{E}_t$ is smaller than $\hat{\omega}_t$. Indeed, the percentage deviation of investment expenditure in consumption units from steady state is $\hat{E}_t - \hat{\omega}_t$, so that a negative response of investment implies that $\hat{E}_t < \hat{\omega}_t$.[26]

The overall picture emerging from the IRs is in line with the intuition provided in Section 3.4.2: with longer project duration, investment's response is smaller and delayed compared to the basic RBC model.

I now compare the low income and high income theoretical moments with the two income groups' corresponding empirical median moments. In evaluating the model's overall performance, I compare each relative difference with its empirical counterpart without taking into account the precision of the empirical moments that were reported in Table 3.1. The moments for the two parametrizations are reported in the first two columns at the bottom of Table 3.11. The empirical moments of Table 3.1 are reported at the top of Table 3.11 for convenience. The complete set of parameters in the simulations are summarized in Table 3.2 and 3.3 and at the bottom of Table 3.11. The volatility of the temporary TFP shock is common and equal to 2% for the two economies. In addition to the different values of θ, the volatility of the investment-specific shock is equal to 3% in the low income economy and it is zero in the high income one. The theoretical moments of the basic RBC and of the low income economy without investment shocks will not be discussed, and are reported in Table 3.12.

[26]One can see from the IRs of the investment-specific shock, the cyclicality of consumption and investment both fall with more volatile investment shocks. By simulating the basic RBC model with volatile investment shocks, moreover, it appears that the reduction in cyclicality is much stronger for consumption than for investment. Because consumption cyclicality in low income countries is comparable to that of high income economies, a basic RBC model with *only* more volatile investment shocks cannot explain the business cycle regularities of LDCs.

I first begin discussing the correlations and then the volatility of the series. Compared with the high income model, in the low income model the correlation of consumption with GDP is smaller by a factor of 10% while that of investment by 22%. The median low income economy has a correlation of GDP and consumption which is 15% lower than the median high income economy, and of investment which is 40% lower than the high income economy. The reduction in investment cyclicality is exactly equal to the empirical counterpart in the SOE version of the model, as the next section shows.

Now consider the volatility of the series. The low income theoretical economy displays a relative volatility of consumption which is 60% larger than that of the high income economy. This number is significantly larger to what empirically observed: the median low income country has a relative volatility of consumption which is 37% higher than that of the high income country. Thus the model accounts for more than the actual difference in relative volatility between the two income groups. The model also predicts a lower volatility of GDP and a lower relative volatility of investment. As discussed above, households substitute leisure for labor, thus dampening the response of GDP to shocks. The model can account for the higher volatility of GDP which is observed in low income economies through a higher volatility of the shocks. The lower volatility of GDP due to tighter institutional constraints is an interesting theoretical result which contrasts with the empirical findings of Loayza, Oviedo, and Serven [2004]), who relate stronger regulation with a higher volatility of GDP. An important dimension along which the model does not perfectly match the data is the relative volatility of investment. The length of the investment project reduces the sensitivity of investment to innovations in the shocks. Increasing the volatility of the investment shock increases the relative volatility of investment, but also reduces consumption cyclicality.[27]. Even with a more careful calibration, it may be necessary to consider the role of additional shocks for the model to match the volatility of investment-specific shocks. As discussed in Section 3.2, for example, government expenditure in low income economies is much more volatile than in high income countries. A higher volatility of government shocks can increase the volatility of investment when households value government consumption significantly less than private consumption. In this case investment reacts strongly after a government spending shock, so that households maintain a relatively stable stream of total consumption. The scope of this work is limited to the role of supply shocks, but analyzing the role of government appears an interesting avenue for future research.

3.5.2. The Small Open Economy

As with the closed economy, the first part of this section analyzes the impulse responses of the model economy to unexpected innovations in the shocks. The second part compares the unconditional moments generated by the model with their empirical counterparts.

In addition to temporary TFP, A_t, and investment specific, ω_t, shocks, the model economy is also subject to permanent TFP shock, Z_t. As noted by Aguiar and Gopinath [2004], the permanent shock helps reproduce the empirically observed countercyclical trade balance for high income countries. I discuss this issue further in the remainder of the section.

Table 3.4 displays the values of the parameters specific to the small open economy parametrizations. The level of foreign debt along the balanced growth path is equal to

[27]This can be seen by comparing the low income specification of Table 3.12 with that of Table 3.11.

Name	Variable	Value
Level of Foreign Debt in SS	B	$Y_{ss}/10$
Derivative of the premium function in SS	π	.0001

Table 3.4. Parameter Values Specific to the SOE

10% of the GDP level.[28] The value assumed for π implies an almost perfectly elastic foreign debt supply.[29] As for the closed economy, three parametrizations of the model economy are considered: basic RBC, low, and high income. As noted in the literature, the basic RBC model needs capital adjustment costs for the theoretically predicted relative volatility of investment to be close to the empirically observed one. I set $\phi = 1$ in the basic RBC model; in the high and low income specifications there are no capital adjustment costs, as TTB itself reduces investment volatility as previously discussed. I will now turn to discuss the response of the model economy to the three types of shocks. The main result of the analysis is that longer project duration makes the trade balance more procyclical, as observed in low income economies, because investment's response to all shocks is diminished. Once again the reported results refer to a one percent unexpected innovation in the relevant shock.

Consider the response of the model economies to a temporary shock A_t, reported in Figure 3.5. Consumption, investment, and output increase in each of the three specifications. The response of consumption and output is very similar in the three specifications, while investment responds significantly less in the high and low income model and peaks after the realization of the shock. Also note that since households can smooth consumption through the international bond, the reduction in the response of investment is markedly

[28] The same ratio is also used by Aguiar and Gopinath [2004]; the ratio does not play an important role for the results of this section.

[29] The interest rate debt elasticity close to steady state is approximately $r/(\pi\bar{B})$.

larger than for what observed in the closed economy (see Figure 3.2). The trade balance responds less than output, so that the trade balance to GDP ratio increases after the technology shock. When productivity shocks are only transitory, the cyclicality implied by the three models would be counterfactually strongly positive. To obtain a countercyclical trade balance ratio, investment and consumption need to react by a sufficiently large amount to the shocks. The response of consumption with Cobb-Douglas preferences is not large enough, because of substitution between consumption and leisure. It is thus common in SOE model economies to assume preferences with no income effect on labor supply (Greenwood, Hercowitz, and Huffman [1988]).[30] These preferences, however, cannot generate countercyclical trade balances for the considered parametrizations of project duration, because investment's response does not peak when the shock hits the economy.[31] As noted by Aguiar and Gopinath [2004], the ratio of the relative trade balance turns strongly negative after a permanent TFP shock, as shown in Figure 3.6. The level of consumption, investment, and output permanently increase by 1.1% in the long run, and households' consumption level jumps up immediately because of the wealth effect. Given the magnitude of this effect, the trade balance ratio is negative even when investment's immediate response is dampened. An interesting property of the basic RBC model's response to a permanent productivity shock is that investment suddenly jumps, while output smoothly increases to the new balanced path. Thus, by increasing the importance of the permanent shock as compared to the temporary one, the cyclicality of

[30] Mendoza [1991] is an early example. An alternative route for the basic RBC model would be to reduce the capital adjustment cost parameter, but this would also imply that the relative volatility of investment becomes too large.

[31] Note that this is also empirically true. Christiano, Eichenbaum, and Vigfusson [2004] estimate a humped response of investment to a TFP shock innovation on US data.

investment falls in the SOE. However, modelling an LDC economy as having more volatile permanent shocks only, would lead to a strongly countercyclical trade balance which is counterfactual.[32] Finally note that, although the trade balance ratio turns negative after a permanent shock, the magnitude is smaller for the low income economy, thus its cyclically is higher. The same is true for the trade balance ratio after an investment-specific shock as shown in Figure 3.7.

As for the closed economy, I now compare the relative difference between the low income and high income theoretical moments, with the difference between the two income groups' corresponding empirical medians. Once again, I evaluate the performance of the model by comparing each relative difference with its empirical counterpart, without taking into account the precision of each estimate. The moments for the two parametrization are reported in the last two columns at the bottom of Table 3.11. The volatility of the innovations in the shocks are chosen as follows. I set the volatility of ε_t^A to 1% and then choose the standard deviation of ε_t^Z to roughly match the theoretical cyclicality of the high income economy's trade balance with the median value of the high income economies. I choose the volatility of ε_t^ω to be the same as that chosen in the closed economy.[33] I will now describe the theoretical predictions in decreasing order of their closeness to the empirical values. The model predicts that investment cyclicality is 40% lower and that the trade balance turns acyclical from countercyclical for the low income group. Thus

[32]Aguiar and Gopinath [2004] argue that a larger fraction of permanent versus temporary shocks help explain aggregate fluctuations in emerging market economies over the last 20 years. These economies have, however, experienced strongly countercyclical trade balances in the last twenty years during the so called sudden-stop phenomenon. It is also interesting to note that the initial jump in investment does not occur in the closed economy, due to the countervailing wealth effect on consumption (see Figure 3.3).
[33]The remaining parameters appear in Tables 3.2 and 3.4. Also see Table 3.12 for the theoretical moments of the basic RBC model and of the low income parametrization without investment-specific shocks.

from the upper portion of the Table 3.11, the model matches the two moments well. The model predicts that the low income group's cyclicality of consumption and the volatility of output is the same for the two income groups, and that the volatility of investment is smaller. These predictions do not match the data, but it is possible to improve the performance of the model by increasing the volatility of the investment-specific shock for low income economies.[34]

The main counterfactual prediction of the SOE economy is in terms of relative volatility of consumption, which is the same for the high and low income specifications. Because households perfectly smooth income fluctuations in the international financial markets, the response of investment no longer influences consumption volatility. Perfect access to financial markets is a rather strong assumption for the countries in the low income group, and the model could be modified to take this fact into account by reducing the elasticity of foreign debt supply (higher π). But the lower integration has opposite effects on the volatility of consumption depending on the type of the TFP shock. A steeper foreign debt supply reduces households' incentives to access financial markets as they pay high interest rates when they borrow and receive low interest rates when they lend. As previously discussed, consumption and investment suddenly increase after a permanent positive shock, and the economy imports goods from abroad. A reduction in international integration reduces the amount of imports, and thereby also consumption's initial increase. As a result, consumption volatility falls. After a transitory shock, on the other hand, consumption and investment increase relatively less and the trade balance turns positive. A reduction in international integration in this case reduces exports, and as a result the volatility of

[34]By doing so, the cyclicality of consumption no longer falls, as was the case with the closed economy. For example for $Sd(\varepsilon_t^{\omega}) = 5\% : Corr(C, Y) = .73$, $Sd(I)/Sd(Y) = 3.5$ and $Sd(Y) = 1.5$.

consumption increases because investment is inflexible. I am planning to analyze in detail the quantitative role of international integration when estimating the model as discussed at the beginning of this section.

Before concluding, I will examine in the next section the response of investment in LDCs on a panel of cross-country data of manufacturing sector.

3.6. Investment Flexibility at the Sectoral Level

This section shows that the response of investment to productivity within manufacturing sectors of low income countries is lower than that of middle and high income economies. This evidence on investment at a more disaggregated level supports the view that the lower cyclicality observed in the aggregate data stems from firms being constrained in their investment decisions.

Why should a firm respond to the level of productivity? Equation (3.20) in Section 3.4.2, described the optimal choice of investment for a *representative* firm as a function of its lagged level and the contemporaneous level of productivity. Investment responds positively to the level of productivity because higher productivity coupled with a positive serial correlation, raises the expected return on investment. Further, investment depends on its lagged level, because investment decisions taken at earlier dates affect the productivity of current investment through the complementarity of the types of investments.

In this section, sectoral panel regressions are estimated on cross-country data with specifications that are analogous to equation (3.20). Cross-country sectoral data are from the INDSTAT3 2002 ISIC Rev.2 database. The sample roughly covers 180 countries over the years 1963-2000, although many observations are missing. Data on economic activity are collected at the 3-digit level ISIC code revision 2 and comprise a total of 28 manufacturing sectors. The sample considered in the regressions[35] includes a total of 36,324 observations in 105 countries. The measure of investment is the sectoral gross fixed capital formation, while productivity is measured as value added per worker, a measure of labor productivity rather than a measure of TFP as in (3.20) since the latter is not available.[36]

I consider two alternative fixed effect panel regression models. The fixed effects in both models are defined at the joint country-sector level, and also include year dummies. The logarithm of investment is regressed on the logarithm of the marginal product of labor in the first regression model. The second regression model is a dynamic panel model, which in addition includes the lagged logarithm of investment as a right hand side variable. The OLS estimator is inconsistent in dynamic panel models and, as shown in Hsiao [1986], the inconsistency is of order $1/T$. To reduce the size of the inconsistency in the dynamic panel regression, I exclude sector-country groups with fewer than ten observations[37]. Each regression model is estimated for the three income groups. Investment should positively

[35]Gross fixed capital formation and value added are in constant US dollars, after being deflated using the GDP deflator from the WDI(2004). Value added is either measured at producer, factor prices or the valuation is not defined. For each unit of observation (country-ISIC) only data with the same valuation are used and data where the valuation is not defined are dropped from the sample. Further, the series for two or more sectors can be aggregated together, sometimes not consistently over the sample. In this case only observations that mantain a common definition are considered.

[36]Measures of the capital stock are needed to compute a sector's TFP. Although it is theoretically possible to compute the stock of capital using perpetual inventory methods, in practice such calculations are unfeasible becasue of the discontinutity in the series of gross capital formation

[37]The conclusions that will be drawn from these regressions hold with a higher cut-off level. Increasing the cutoff level reduces the size of the sample of low income countries–which is the smallest among the three income groups.

respond to the contemporaneous level of productivity, because of its positive serial correlation. Indeed, the average first order serial correlation of investment in the sample is 0.75.

The empirical estimates of the two models are reported in Table 3.5. The first three columns report the results of the static panel model. The point estimates of the elasticity of investment with respect to productivity are positive for all the three income groups, although the estimated coefficient is statistically different from zero only at the 10% level in the low income group. The magnitude of the estimated coefficient differs in the three income groups. The coefficient for the high and middle income countries is approximately four time larger than the one for low income countries, and the difference is statistically significant at conventional levels. The last three columns of Table 3.5 report the results for the dynamic panel model. The estimated first order serial correlation of the logarithm of investment is roughly 25% larger in middle and high income economies compared to low income economies. This result in not in line with the specification of equation (3.20). The relatively strong assumptions with which the relation was derived are a likely cause for its failure to hold in the data.[38] Once again, sectoral investment displays a lower response to contemporaneous productivity in low income countries than in middle and high income countries. The elasticity of investment is roughly three times larger in middle

[38]Equation (3.20) was a local approximation of the optimal investment decision derived under the assumptions of full capital depreciation, fixed interest rates and labor services that held when all firms were assumed to be identical. Fixed labor services at the sectoral level, implies that in addition to being inelastically supplied, labor is also specific to each sector. In the sample, however, labor is not constant (the standard deviation of labor's yearly growth is 18%). With imperfect capital depreciation, the lagged level of the capital stock appears as a right hand side variable in equation (3.20). Finally, the empirical measure of labor depends on the lagged level of investment, and in this case it would probably be more reasonable to use an empirical model (e.g. a panel VAR model) in which labor productivity and investment are jointly determined.

	Dependent Variable: Log(Inv(t))					
	Regression Model (1)			Regression Model (2)		
Log(MPL(t))	0.14	.678	.589	.114	.398	0.284
	(0.076)	(.042)	(.056)	(.063)	(.028)	(0.03)
Log(Inv(t-1))				.4275	.493	.548
				(.030)	(.016)	(.015)
Income Group	Low	Middle	High	Low	Middle	High
Number of Observations	3786	14312	17203	1708	10809	15289
Number of Groups	523	1008	837	138	587	705
R-squared	0.018	0.02	0.21	0.70	0.77	0.94

Notes: All regressions include a constant, year fixed effects and joint country-sector fixed effects. s.e. are reported in parenthesis below the coefficients and are clustered at the country-sector level.

Table 3.5. Sectoral Panel Regression Models

and high income economies relative to low income countries. Thereby, although the empirical results do not support the validity of the model presented in equation (3.20), they show that response of sectoral investment to contemporaneous measures of productivity is significantly smaller in low income countries. Because inefficient institutions act as a constraint on firms' choices, investment at different levels of aggregation, in each manufacturing sector or for the economy as a whole, displays a lower response to the state of the economy.

3.7. Conclusions

This paper provided evidence that business cycles in LDCs are characterized by high volatility of consumption, low correlation of investment and GDP, and acyclical trade balances.

These empirical regularities were interpreted within the countries' institutional context. Constraints arising from poor contract enforcement, low financial development, trade barriers, and regulation limit firms' response to changes in the current state of the economy, because of lengthy time to plan and build new investment.

The model described in the paper departs from the standard TTB specification (Kydland and Prescott [1982]) by considering distinct types of investment. Complementarities across types lead to intertemporal linkages between investment decisions and generate a hump-shaped response of investment to shocks.

Due to institutional constraints, firms in LDCs respond less and with delay to changes in economic conditions. Because the state of the economy carries less information about future than current returns, it is optimal not to fully adjust investment. Furthermore, since a large number of projects are still under way, a delayed and gradual adjustment is optimal because past choices influence current marginal returns to investment.

The staggered response of firms' adjustment in LDCs is confirmed by empirical evidence on cross-country data in the manufacturing sector, where investment responds less to current productivity than in middle and high income economies.

The paper shows that augmenting a standard closed and small open economy (SOE) RBC model driven by supply shocks with the new specification of TTB helps explain key features of the business cycle in LDCs. Investment cyclicality falls as a direct consequence of longer TTB, with a magnitude comparable to what is empirically observed for reasonable parametrizations. The relative volatility of investment to GDP also falls as TTB increases and more volatile investment-specific shocks are needed to maintain the constant volatility of investment observed in the data. In the SOE version of the model, the trade balance becomes increasingly procyclical; given that investment reacts to a smaller extent after a shock, a country imports relatively less in booms and more in recessions.

In a SOE, households perfectly smooth income fluctuations in the international financial markets, and thus consumption volatility is not affected by longer TTB. In a closed economy, on the other hand, consumption adjusts more to the shocks because of the lower investment response, and consumption volatility therefore increases. The effects of lengthy TTB on the cyclicality of the trade balance and on the volatility of consumption are quantitatively important and comparable to the empirical difference between high and low income countries.

In the numerical exercise presented in this paper, LDCs shared all parameters and shocks with high income countries except for the length of the TTB and the volatility of investment-specific shocks. An alternative country by country calibration is not feasible due to limited data availability for LDCs. Another alternative, which I am currently pursuing, is to estimate the parameters of the model using the generalized method of moments, by matching empirical moments of interest for each country with the ones predicted by the model. This type of analysis has the advantage of not only assessing whether the flexibility of investment can explain the business cycle regularities, but also testing the validity of the hypothesis against others, such as an alternative composition of the shocks.

This study takes a step toward understanding the impact of LDCs' institutional environment on their business cycles; however, much work is still ahead. LDCs' economies are more volatile compared to high income economies. Recent empirical studies (e.g. Loayza, Oviedo, and Serven [2004]) have associated institutional constraints with higher volatility. This empirical result does not find an explanation in the theory presented in this paper. Indeed, in the model higher regulation reduces investment flexibility, thus dampening the volatility of the economy. An interesting line of future research would be to explore additional channels through which institutional constraints affect business cycles in LDCs; for example, institutional constraints may be responsible for the fact that productive structures move from more volatile to less volatile sectors as a country develops (Koren and Tenreyro [2004]). A full understanding of the role of these institutions would allow to assess and possibly influence the outcome of the reforms currently under way in many LDCs (WorldBank [2005]).

LOW INCOME				MIDDLE INCOME				HIGH INCOME			
ISOCODE	COUNTRY	YEARS	SOURCE	ISOCODE	COUNTRY	YEARS	SOURCE	ISOCODE	COUNTRY	YEARS	SOURCE
BGD	Bangladesh	1965-2000	WDI	DZA	Algeria	1960-2000	WDI	AUS	Australia	1960-2000	IFS
BEN	Benin	1960-2000	WDI	ARG	Argentina	1960-2000	WDI	AUT	Austria	1964-2000	IFS
BFA	Burkina Faso	1965-2000	WDI	BOL	Bolivia	1970-2000	WDI	BEL	Belgium	1960-2000	IFS
BDI	Burundi	1960-2000	WDI	BRA	Brazil	1960-2000	WDI	CAN	Canada	1960-2000	IFS
CMR	Cameroon	1960-2000	WDI	CHL	Chile	1960-2000	IFS	DNK	Denmark	1966-2000	IFS
COD	Congo, Dr (Zaire)	1960-2000	WDI	CHN	China	1978-2000	WDI	FIN	Finland	1960-2000	IFS
COG	Congo, Pr	1960-2000	WDI	COL	Colombia	1960-2000	WDI	FRA	France	1960-2000	IFS
CIV	Cote D'Ivoire	1960-2000	WDI	CRI	Costa Rica	1960-2000	IFS	DEU	Germany	1971-2000	WDI
GMB	Gambia	1966-2000	WDI	DOM	Dominican Re	1960-2000	WDI	GRC	Greece	1960-2000	IFS
GHA	Ghana	1960-2000	WDI	ECU	Ecuador	1960-2000	WDI	HKG	Hong Kong	1961-2000	IFS
GNB	Guinea-Bissau	1970-2000	WDI	EGY	Egypt	1974-2000	WDI	ISL	Iceland	1960-2000	IFS
HTI	Haiti	1965-2000	WDI	SLV	El Salvador	1960-2000	IFS	IRL	Ireland	1971-2000	WDI
IND	India	1960-2000	IFS	GAB	Gabon	1960-2000	WDI	ITA	Italy	1960-2000	IFS
KEN	Kenya	1960-2000	WDI	GTM	Guatemala	1960-2000	IFS	JPN	Japan	1960-2000	IFS
LSO	Lesotho	1960-2000	WDI	GUY	Guyana	1960-2000	WDI	KOR	Korea	1960-2000	IFS
MDG	Madagascar	1960-2000	WDI	HND	Honduras	1960-2000	IFS	LUX	Luxembourg	1965-2000	WDI
MWI	Malawi	1960-2000	WDI	HUN	Hungary	1970-2000	IFS	NLD	Netherlands	1960-2000	IFS
MLI	Mali	1967-2000	WDI	IDN	Indonesia	1960-2000	WDI	NZL	New Zealand	1960-2000	IFS
MRT	Mauritania	1960-2000	WDI	MYS	Malaysia	1960-2000	WDI	NOR	Norway	1960-2000	WDI
NIC	Nicaragua	1960-2000	WDI	MEX	Mexico	1960-2000	WDI	PRT	Portugal	1971-2000	WDI
NER	Niger	1960-2000	WDI	MAR	Morocco	1964-2000	IFS	SGP	Singapore	1960-2000	IFS
NGA	Nigeria	1960-2000	WDI	PRY	Paraguay	1960-2000	WDI	ESP	Spain	1960-2000	IFS
PAK	Pakistan	1960-2000	WDI	PER	Peru	1960-2000	IFS	SWE	Sweden	1960-2000	IFS
PNG	Papua New Guin	1961-2000	WDI	PHL	Philippines	1960-2000	IFS	CHE	Switzerland	1965-2000	WDI
RWA	Rwanda	1960-2000	WDI	ZAF	South Africa	1960-2000	IFS	GBR	United Kingdon	1960-2000	IFS
SEN	Senegal	1960-2000	WDI	LKA	Sri Lanka	1960-2000	IFS	USA	United States	1960-2000	IFS
TGO	Togo	1960-2000	WDI	THA	Thailand	1960-2000	IFS				
ZMB	Zambia	1960-2000	WDI	TUN	Tunisia	1961-2000	IFS				
ZWE	Zimbabwe	1976-2000	WDI	URY	Uruguay	1960-2000	WDI				
				VEN	Venezuela	1960-2000	IFS				

Notes: WDI is the World Development Indicators 2004 database, while IFS is the International Financial Statistics Database of the IMF. World Bank Income Group classification in terms of 2003 World Bank Atlas GNI per capita. Low income, \$765 or less; middle income, \$766–9,385; and high income, \$9,386 or more. Years refer to the national account series with the least available data.

Table 3.6. Income Group Classification and Data Source for National Account Data

		HP	BP	FD		HP	BP	FD
Low Income					Sd(Y)	4.40	3.14	5.51
	Corr(C,Y)	0.58	0.63	0.58	Sd(C)/Sd(Y)	1.46	1.63	1.51
	Corr(I,Y)	0.46	0.37	0.40	Sd(I)/Sd(Y)	4.38	4.62	4.62
	Corr(G,Y)	0.26	0.23	0.20	Sd(G)/Sd(Y)	2.10	2.13	2.15
	Corr(nx,Y)	-0.02	-0.01	0.06	Sd(nx)	4.95	3.53	5.33
Middle Income					Sd(Y)	4.12	2.20	3.98
	Corr(C,Y)	0.73	0.61	0.64	Sd(C)/Sd(Y)	1.19	1.32	1.19
	Corr(I,Y)	0.73	0.73	0.71	Sd(I)/Sd(Y)	3.83	4.15	3.99
	Corr(G,Y)	0.53	0.35	0.42	Sd(G)/Sd(Y)	1.89	2.33	2.09
	Corr(nx,Y)	-0.35	-0.31	-0.25	Sd(nx)	2.89	2.26	3.46
High Income					Sd(Y)	2.29	1.48	2.65
	Corr(C,Y)	0.68	0.64	0.71	Sd(C)/Sd(Y)	1.06	0.97	0.99
	Corr(I,Y)	0.79	0.78	0.75	Sd(I)/Sd(Y)	3.94	4.50	3.72
	Corr(G,Y)	0.19	0.08	0.27	Sd(G)/Sd(Y)	1.36	1.30	1.45
	Corr(nx,Y)	-0.30	-0.27	-0.25	Sd(nx)	1.28	0.96	1.54

Notes: Median of statistic by income group. All series are logged (with the expeption of nx which is the ratio of NX to GDP) before filtering. Series are HP-filtered with a smoothing parameter of 100 in column HP, Band Passed filtered at frequencies between 2 and 8 years in column BP, and first differenced in column FD. Standard deviations are expressed in percentage terms.

Table 3.7. Medians within Income Groups using all Filters: Contemporaneous Correlations with GDP and Volatilites

Country	Corr(C,Y)	Corr(I,Y)	Corr(G,Y)	Corr(nx,Y)	Sd(Y)	Sd(C)/Sd(Y)	Sd(I)/Sd(Y)	Sd(G)/Sd(Y)	Sd(nx)
Bangladesh	.	0.78	.	-0.28	3.84	.	4.86	.	1.62
Benin	0.55	0.21	0.41	-0.07	3.08	1.46	6.82	3.11	3.17
Burkina Faso	0.73	0.09	0.20	-0.36	2.42	1.84	6.49	3.96	1.98
Burundi	0.70	0.15	0.50	0.01	5.09	1.51	4.38	2.75	3.03
Cameroon	0.80	0.73	0.46	-0.22	6.04	1.27	2.50	1.42	3.40
Congo, Dr (Zaire)	0.89	0.39	0.24	-0.40	5.69	1.31	4.83	3.02	2.56
Congo, Pr	0.41	0.64	0.44	-0.45	6.92	1.93	4.57	1.85	8.76
Cote D'Ivoire	0.68	0.69	0.84	-0.61	4.40	1.05	4.99	1.91	2.70
Gambia	0.10	0.27	-0.13	0.20	2.93	4.17	7.91	4.28	14.44
Ghana	0.77	0.48	0.04	0.01	4.27	1.33	4.15	2.41	2.87
Guinea-Bissau	0.49	0.21	0.39	0.17	6.46	2.11	3.22	1.87	5.01
Haiti	.	0.46	.	0.24	4.10	.	4.27	.	5.42
India	0.65	0.35	0.57	0.15	2.36	1.07	2.44	2.49	0.66
Kenya	0.72	0.40	0.06	-0.09	3.99	1.85	4.28	1.78	4.95
Lesotho	0.47	0.58	0.29	0.33	6.28	1.10	3.47	1.68	11.11
Madagascar	0.76	0.77	0.50	-0.35	3.25	1.13	5.46	1.85	2.41
Malawi	0.35	0.34	-0.01	0.01	4.13	2.20	5.52	2.29	9.68
Mali	0.77	0.32	0.20	0.50	4.76	1.06	2.48	2.10	2.60
Mauritania	0.33	-0.01	-0.08	0.25	3.98	5.25	7.57	4.13	10.72
Nicaragua	0.26	0.60	0.26	-0.18	6.46	1.39	3.27	2.34	5.93
Niger	0.65	0.53	0.26	-0.22	6.63	1.84	4.92	1.69	5.15
Nigeria	0.45	0.64	0.25	0.14	8.47	1.19	2.89	1.96	10.05
Pakistan	0.58	0.22	0.18	-0.21	2.02	2.44	4.41	3.37	3.29
Papua New Guinea	.	0.14	.	.	4.91	.	3.94	.	.
Rwanda	0.79	0.70	0.50	0.76	12.25	0.76	2.22	1.72	5.54
Senegal	0.52	0.49	0.40	0.43	3.25	1.00	3.68	1.77	2.46
Togo	0.43	0.59	0.09	-0.20	5.13	2.42	5.25	1.95	8.06
Zambia	0.16	0.42	0.10	-0.02	3.61	2.77	7.15	4.19	12.19
Zimbabwe	.	0.53	.	.	6.36	.	3.06	.	.

Notes: All series are logged (with the expeption of nx which is the ratio of NX to GDP) and HP filtered using a smoothing parameter of 100. Standard deviations are expressed in percentage terms. Missing statistics are due to either discountinuos or missing series.

Table 3.8. Low Income Countries: Contemporaneous Correlations with GDP and Volatilities

Country	Corr(C,Y)	Corr(I,Y)	Corr(G,Y)	Corr(nx,Y)	Sd(Y)	Sd(C)/Sd(Y)	Sd(I)/Sd(Y)	Sd(G)/Sd(Y)	Sd(nx)
Algeria	0.72	0.50	0.66	0.24	6.00	1.62	2.45	1.59	5.23
Argentina	.	0.91	.	-0.84	4.65	.	2.68	.	1.39
Bolivia	0.71	0.46	0.72	0.11	3.67	0.85	4.75	1.66	2.59
Brazil	0.46	0.81	0.53	-0.60	4.26	1.28	2.83	2.11	0.98
Chile	0.81	0.67	0.52	-0.42	5.67	1.25	3.92	1.45	28.06
China	.	0.94	0.84	.	7.31	.	3.03	1.15	.
Colombia	0.84	0.59	0.40	-0.43	2.20	1.25	5.71	3.00	2.28
Costa Rica	0.74	0.55	0.65	-0.47	3.30	1.70	2.93	2.36	2.69
Dominican Rep	0.81	0.74	0.11	-0.27	4.88	1.19	3.35	4.20	2.89
Ecuador	0.73	0.61	0.67	-0.18	3.20	0.90	3.49	2.95	3.67
Egypt	.	0.41	.	0.32	3.33	.	4.88	.	4.72
El Salvador	0.79	0.73	0.16	-0.05	4.49	0.90	3.96	1.32	2.45
Gabon	-0.26	0.87	0.54	-0.29	10.89	0.80	2.72	1.17	5.56
Guatemala	0.88	0.63	0.30	-0.36	2.77	0.99	5.44	2.87	1.81
Guyana	.	0.57	.	-0.18	5.41	.	4.14	.	10.99
Honduras	0.32	0.59	0.07	0.27	3.10	0.77	5.21	2.16	2.51
Hungary	0.38	0.85	0.44	0.25	4.27	0.68	3.05	1.36	2.62
Indonesia	0.40	0.88	0.53	-0.17	4.37	1.52	3.44	1.89	3.75
Malaysia	0.90	0.93	0.61	-0.86	3.68	1.52	4.56	1.59	5.50
Mexico	0.91	0.86	0.60	-0.66	3.39	1.18	3.73	0.83	2.43
Morocco	0.56	0.56	0.50	-0.05	3.34	0.95	4.32	3.32	2.86
Paraguay	0.64	0.80	0.52	-0.46	3.96	1.35	3.54	3.10	3.48
Peru	0.67	0.72	0.65	-0.34	5.28	1.19	3.59	2.03	3.38
Philippines	0.74	0.87	0.82	-0.55	3.56	0.75	4.36	2.15	2.54
South Africa	0.54	0.81	0.08	-0.51	1.98	1.38	4.92	2.14	2.69
Sri Lanka	0.98	0.71	0.87	0.11	11.68	0.97	1.48	1.17	3.48
Thailand	0.90	0.91	0.38	-0.78	4.45	0.89	3.38	1.48	3.09
Tunisia	0.72	0.52	-0.14	-0.09	2.93	1.25	3.96	1.37	2.52
Uruguay	0.85	0.88	0.52	-0.59	4.77	1.27	3.94	0.90	3.11
Venezuela	0.66	0.72	0.53	-0.52	3.98	1.74	5.88	2.61	6.48

Notes: All series are logged (with the expeption of nx which is the ratio of NX to GDP) and HP filtered using a smoothing parameter of 100. Standard deviations are expressed in percentage terms. Missing statistics are due to either discountinuos or missing series.

Table 3.9. Middle Income Countries: Contemporaneous Correlations with GDP and Volatilities

Country	Corr(C,Y)	Corr(I,Y)	Corr(G,Y)	Corr(nx,Y)	Sd(Y)	Sd(C)/Sd(Y)	Sd(I)/Sd(Y)	Sd(G)/Sd(Y)	Sd(nx)
Australia	0.64	0.83	-0.10	-0.14	1.81	0.52	4.05	1.44	1.14
Austria	0.55	0.74	0.03	0.01	1.70	0.79	3.58	1.15	0.81
Belgium	0.63	0.76	0.12	-0.14	1.92	1.13	3.74	2.28	0.94
Canada	0.84	0.89	-0.56	-0.31	2.03	0.93	4.00	1.78	1.01
Denmark	0.66	0.89	0.05	-0.47	1.96	1.17	4.82	1.06	1.26
Finland	0.83	0.94	0.06	-0.56	3.50	0.72	4.00	0.84	1.72
France	0.56	0.91	0.10	-0.23	1.51	0.82	4.31	1.98	0.74
Germany	0.92	0.88	0.28	-0.30	1.97	1.06	2.49	0.70	0.66
Greece	0.58	0.81	-0.19	-0.03	2.12	0.93	4.70	2.59	1.30
Hong Kong	0.52	0.76	0.15	-0.32	3.61	1.08	4.72	1.28	6.46
Iceland	0.86	0.65	0.78	-0.26	4.01	1.18	2.90	1.20	3.21
Ireland	0.69	0.72	0.42	-0.41	2.65	1.26	4.26	1.25	1.83
Italy	0.72	0.74	0.22	-0.40	1.94	1.94	3.77	2.39	1.37
Japan	0.74	0.91	0.15	-0.29	3.07	0.79	2.42	1.59	0.91
Korea	0.59	0.68	0.27	-0.26	3.30	1.08	3.87	2.28	11.36
Luxembourg	0.71	0.65	0.55	0.20	3.54	0.80	3.08	0.62	2.53
Netherlands	0.84	0.71	0.37	-0.03	3.95	1.07	1.93	2.05	1.19
New Zealand	0.63	0.72	0.23	-0.37	2.51	1.09	4.96	1.94	2.39
Norway	0.70	0.52	-0.03	-0.27	1.69	1.92	4.63	0.88	2.05
Portugal	0.58	0.83	0.62	-0.54	3.41	1.06	4.90	0.86	1.75
Singapore	0.66	0.59	0.24	.	4.28	0.92	2.85	1.74	.
Spain	0.63	0.90	0.24	-0.45	2.45	1.39	3.68	1.63	2.01
Sweden	0.61	0.66	-0.16	0.06	1.73	0.89	4.78	3.15	1.17
Switzerland	0.97	0.92	0.62	-0.40	2.84	0.84	2.79	0.68	0.85
United Kingdom	0.91	0.93	-0.13	-0.62	2.04	1.15	4.35	1.25	1.28
United States	0.93	0.87	0.40	-0.49	1.96	0.85	3.22	1.21	0.57

Notes: All series are logged (with the expeption of nx which is the ratio of NX to GDP) and HP filtered using a smoothing parameter of 100. Standard deviations are expressed in percentage terms. Missing statistics are due to either discountinuos or missing series.

Table 3.10. High Income Countries: Contemporaneous Correlations with GDP and Volatilities

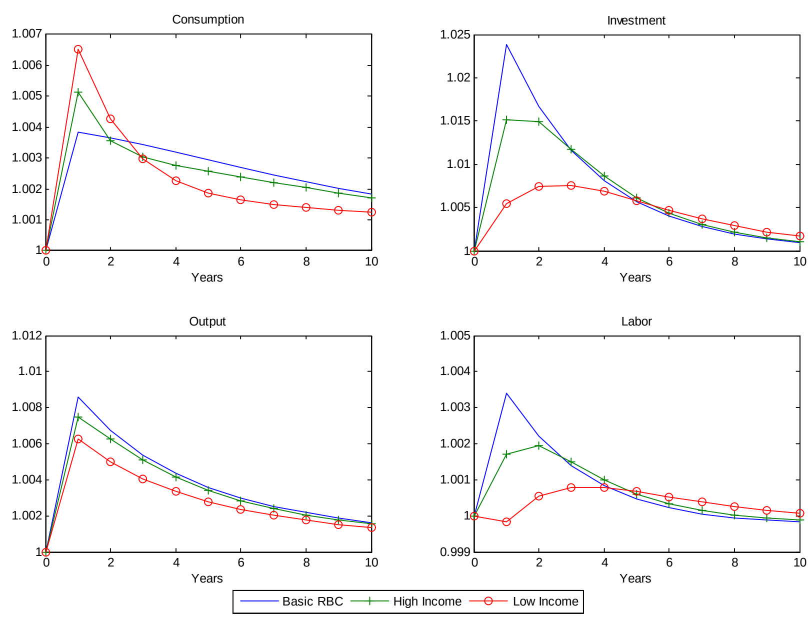

Figure 3.2. IRFs for the Closed Economy: Temporary TFP shock

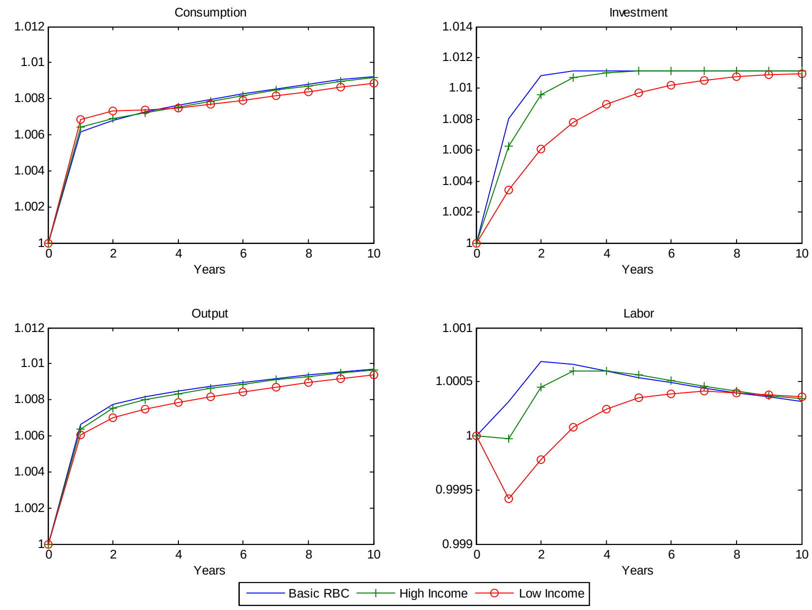

Figure 3.3. IRFs for the Closed Economy: Persistent TFP Shock

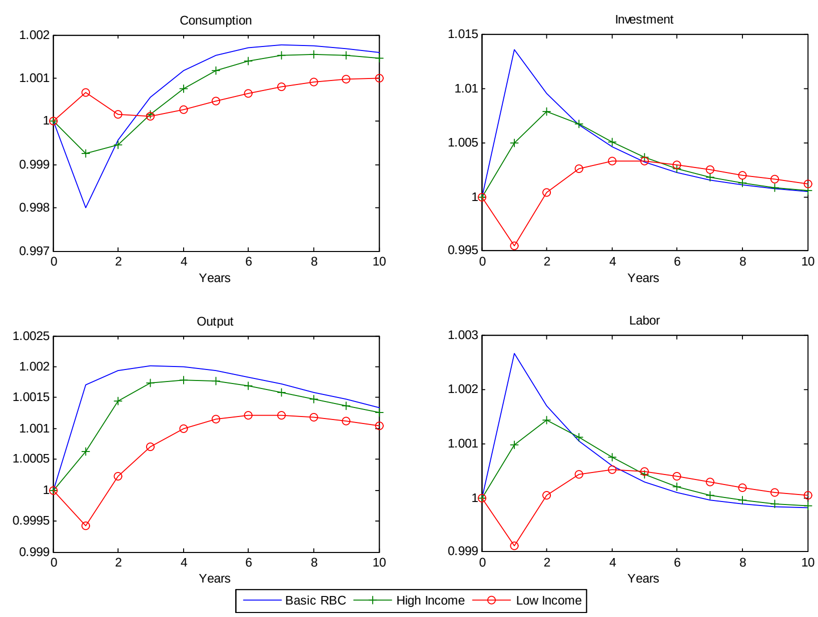

Figure 3.4. IRFs for the Closed Economy: Investment Specific Shock

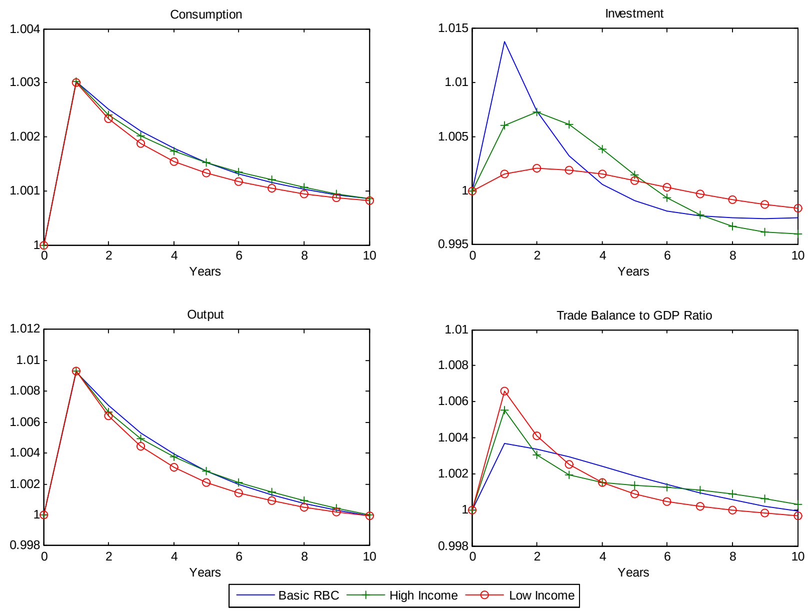

Figure 3.5. IRFs for the SOE: Temporary TFP shock

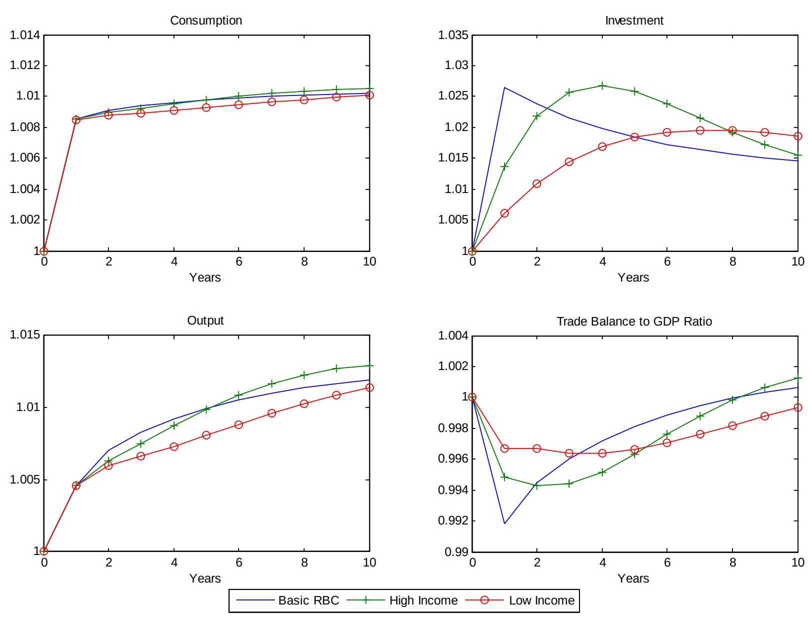

Figure 3.6. IRFs for the SOE: Permanent TFP shock

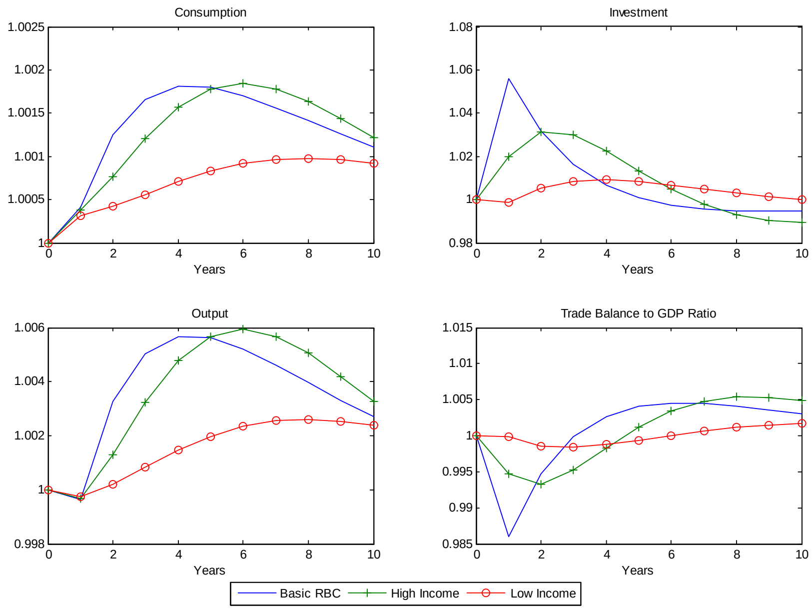

Figure 3.7. IRFs for the SOE: Investment Specific shock

Empirical Moments				
	Low Income	Middle Income	High Income	US
Corr(C,Y)	0.58	0.73	0.68	0.93
Corr(I,Y)	0.46	0.73	0.79	0.87
Corr(nx,Y)	-0.02	-0.35	-0.30	-0.49
Sd(Y)	4.40	4.12	2.29	1.96
Sd(C)/Sd(Y)	1.46	1.19	1.06	0.85
Sd(I)/Sd(Y)	4.38	3.83	3.94	3.21
Sd(nx)/Sd(Y)	4.90	2.89	1.28	0.57

Theoretical moments				
	Closed Economy		Small Open Economy	
	Low Income	High Income	Low Income	High Income
Corr(C,Y)	0.88	0.98	0.76	0.74
Corr(I,Y)	0.78	0.99	0.46	0.74
Corr(nx,Y)	.	.	0.01	-0.27
Sd(Y)	1.26	1.44	1.29	1.29
Sd(C)/Sd(Y)	0.94	0.58	1.22	1.22
Sd(I)/Sd(Y)	1.88	2.17	2.41	3.14
Sd(nx)/Sd(Y)	.	.	0.94	1.18
Parameter Values				
Phi	0.00	0.00	0.00	0.00
Theta	0.33	0.67	0.33	0.67
Sd(Eps(A))	2.00	2.00	1.00	1.00
Sd(Eps(Z))	0.00	0.00	2.00	2.00
Sd(Eps(om))	3.00	0.00	3.00	0.00

Note: Additional parameters can be found in the main text. In the theoretical moments, I refers to investment expenditure in units of consumption. All series are logged (with the expeption of nx which is the ratio of NX to GDP) and then HP-filtered with a smoothing parameter of 100. Standard deviations in absolute terms are expressed in percentage.

Table 3.11. Numerical Experiments: High and Low Income Economies

Theoretical moments				
	Basic RBC		Low Income w/out Inv shocks	
	Closed Economy	SOE	Closed Economy	SOE
Corr(C,Y)	0.97	0.79	0.97	0.78
Corr(I,Y)	0.99	0.74	0.88	0.66
Corr(nx,Y)	.	-0.30	.	-0.01
Sd(Y)	1.64	1.36	1.21	1.20
Sd(C)/Sd(Y)	0.43	1.17	0.95	1.29
Sd(I)/Sd(Y)	2.56	3.23	1.36	1.45
Sd(nx)/Sd(Y	.	1.24	.	0.82
Parameter Values				
Phi	0.00	1.00	0.00	0.00
Theta	0.00	0.00	0.33	0.33
Sd(Eps(A))	2.00	1.00	2.00	1.00
Sd(Eps(Z))	0.00	2.00	0.00	2.00
Sd(Eps(om))	0.00	0.00	0.00	0.00

Note: Additional parameters can be found in the main text. In the theoretical moments, I refers to investment expenditure in units of consumption. All series are logged (with the expeption of nx which is the ratio of NX to GDP) and then HP-filtered with a smoothing parameter of 100. Standard deviations in absolute terms are expressed in percentage.

Table 3.12. Numerical Experiments: Alternative Specifications

CHAPTER 4

Collateral Liquidity and Inter-Sectoral Capital Allocation

4.1. Introduction

Secured lending is a common financing practice for firms in more and less developed countries (LDCs). For instance, around 70% of loans issued to large and publicly traded US firms are secured by collateral (Bharath, Sunder, and Sunder [2004] on the Dealscan database). Although it is more difficult to collect analogous figures for LDCs, Braun [2003] provides evidence that collateral pledging is even more important in less developed economies, where the enforcement of financial contracts is weak (e.g. Levine [1997]).

An important characteristic of a secured loan is the value to the lender of the collateral pledged by the borrower. Indeed, as the lender values the collateral more, financing terms improve and he is willing to lend larger amount of funds. But what makes the collateral more valuable to a lender? Previous literature, such as Williamson [1988] and Shleifer and Vishny [1992], stress the importance of assets' redeployability and of the correlation of firms' returns within the same industry. According to Williamson [1988], lenders are more willing to finance sectors that make use of redeployable assets, such as land, as they can be easily converted to alternative use. Shleifer and Vishny [1992] point that the majority of assets are non-redeployable, and therefore the financial conditions of industry insiders, who have the highest valuation for the assets, are likely to affect the liquidity of the collateral.

But the value of the collateral also depends on the diffusion of the underlying physical assets. Because a lender might lack the skills to operate the asset pledged as collateral he is likely to resell it in case of default, and thus prefers to lend against assets that are more liquid. With liquid assets it is easier for the lender to find interested buyers. But the number of potential buyers itself depends on the number of loans being issued to a specific sector. The objective of this paper is to study the implications for the allocation of investment across sectors of these feedback effects between collateral liquidity and sectoral investment.

I present a simple general equilibrium model where investment projects are only financed through secured lending because projects' returns are non-verifiable by courts as in Hart and Moore [1994]. The production sector of the model economy is composed of a large number of perfectly symmetric sectors and a final good sector. The goods produced by the intermediate sector are used as inputs in the final good sector and the intermediate goods are imperfect substitutes in the production function of the final good sector.

Due to the imperfect substitutability and common production function, the levels of investment in the intermediate good sector are equalized in equilibrium absent any contracting friction. When returns are non-verifiable, instead, entrepreneurs can invest only if the liquidity of their productive assets is high. Higher investment rates increase the diffusion of the assets and the thickness of the markets in which they are exchanged, namely, the ease with which buyers and sellers meet in the collateral's secondary market. As a result of the endogenous collateral values, multiple (stable) equilibria may coexist in each sector: investment is determined by its profitability in one equilibrium, and by the liquidity of its collateral in the other, and for a given interest rate, sectoral investment is lower in the financially constrained equilibrium. In general equilibrium, depending on parameter values, investment is *necessarily* asymmetric so that capital is misallocated. In a positive fraction of sectors the *collateral constraint* necessarily binds and these sectors underinvest as compared to frictionless economy. Since the amount of resources is fixed in the economy, the remaining unconstrained sectors overinvest. In the model I show that the asymmetric allocation of capital necessarily occurs in middle income countries and if industry insiders are very productive relative to lenders in operating the projects' assets.

Several studies find evidence of capital misallocation within the same economy. Banerjee and Duflo [2004] summarize empirical micro-studies conducted in developing countries and argue that very high and very low rates of returns often coexist in the same economy for the same factors of production suggesting the presence of misallocation. Many economists (e.g. Banerjee and Duflo [2004]) believe that financial frictions are the likely cause of capital misallocation. But standard financial contracting frictions, resulting for instance from moral hazard or asymmetric information, must be coupled with heterogeneity in entrepreneurs' or technology's characteristics to deliver misallocation of fixed amount of resources. Among the relevant sources of agents' heterogeneity are differences in investors' income levels or the ease with which they can reach lenders through local networks. Relevant sources of technological heterogeneity are the informational content of the production process, which make some investment projects easier to monitor. The financial channel analyzed in this paper, namely the endogeneity of the collateral values, is sufficient for misallocation to occur even when all agents and the production technologies are perfectly symmetric. Thereby frictions arising from imperfect contractibility are likely to affect real outcomes even when the broadly considered measures of heterogeneity are small.

This paper is closely related to ideas in Shleifer and Vishny [1992]. My model differs from theirs in several dimensions. First, here potential buyers always have enough cash flows to purchase the collateral[1], however industry conditions matter as sellers and buyers might not meet when market thickness is low.[2] In Shleifer and Vishny [1992], instead, the within-industry correlation of shocks to cash-flows play a central role. When the correlation is large, industry insiders are themselves in financial trouble when competitors' assets are being sold, so that the collateral ends up being sold to deep pockets who fetch a lower valuation for the asset. Further, the model of this paper makes the more realistic assumption that firms in the same industry not only share the same technology but also compete in the product market. Finally, their model, which is in partial equilibrium, does not address inter-sectoral interaction, while a central result of this paper is to provide condition sufficient for capital to be asymmetrically allocated across sectors. In this respect, the model is closely related to the work of Matsuyama [2004] who presents a model of the world economy, composed of a large number of identical countries, and provides conditions under which the world is polarized in two sets of countries some of which are rich while others remain poor. In Matsuyama [2004]'s model financial frictions do not constrain investment in the richer countries because in equilibrium entrepreneurs are wealthier, while in this model, frictions do not constrain investment levels in some sectors because the collateral is liquid in equilibrium. Finally Kiyotaki and Moore [1997]

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[1]

[2] McLaren [2000] presents a model where contracting externalities affect the firm's decision to vertically integrate or produce at arm's length through assets' market thickness.

also study endogenous determinants of collateral values. The endogeneity of the collateral value is used in their model as an amplification mechanism of supply shocks, but has no implications for sectoral allocations as the asset (land) which secures the loan is perfectly redeployable.

The remaining of the paper is organized as follows. Section 4.2 describes the model economy, while in Section 4.3 I describe the form of the optimal financial contract. Section 4.4 characterizes the properties of the sectoral equilibrium in partial equilibrium. Finally Section 4.5, solves the general equilibrium of the model and provides conditions for the existence of the symmetric and asymmetric general equilibrium.

4.2. The Model Economy

The economy lasts for three dates: $t = 0, 1, 2$. Agents are born at date 0 with $e < 1$ units of endowment, E. In the remaining of the paper the initial endowment e is assumed to be higher in more developed countries. The endowment E must be transformed in the final good, Y, in order to be consumed. The final good sector operates at date 2 and is competitive. The final good is produced with the constant elasticity of substitution production function

$$Y = \left(\int_0^1 K_i^{1-\frac{1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

The variables K_i's denote outputs from the measure one of intermediate sectors indexed by $i \in [0, 1]$. The elasticity of substitution is assumed to be $\sigma > 1$, so that although the K_i's are imperfect substitutes for one another, no single K_i is indispensable in production.

Agents only consume at date 2, are risk neutral, don't value leisure, and differ in their entrepreneurial skill, which is indexed by the sector $i \in [0, 1]$ where their skill is of use.

An entrepreneur of skill $\hat{\imath}$ may only start an entrepreneurial project in sector $\hat{\imath}$ and there is exactly a measure one of agents with a specific entrepreneurial skill, so that the total measure of the population is also one. Firms in the intermediate sector require one unit of E in order to operate.

At date 0, each agent decides to become an entrepreneur in the intermediate sector by borrowing $(1-e)$ and investing, or to lend his endowment e to the other entrepreneurs. A project in sector $\hat{\imath}$ converts one unit of E into R_1 units of sectoral output $\hat{\imath}$ at date 1 and $\tilde{R}_2$ units at date 2. The return R_1 at date 1 is certain. [3] The date 2 return is uncertain due to the possibility of failure of the physical asset: the project produces $\tilde{R}_2$ units of the output good when the asset is non-defective, which occurs with probability λ. If the machine is defective, no output is produced at date 2. Following Hart and Moore [1994], the levels of R_1 and $\tilde{R}_2$ cannot be observed by courts.[4] Outside parties and courts can only observe transfers between contracting parties and asset ownership.

Because the asset does not produce any return after date 2, it has no value by then. Thus, trading in the secondary market only occurs at date 1. The lender may decide to run himself the project at date 2, and produce $\underline{\tilde{R}}_2 < \tilde{R}_2$ units of the intermediate output $\hat{\imath}$. However, industry insiders can produce $\tilde{R}_2$ units of capital.[5] Gains from trade in the secondary market occur when lenders who own a projects' asset after a borrower's default, meet an industry insider, namely, an entrepreneur with a skill of type of the asset. But since entrepreneurs run at most one project, they only value the asset at date 1, if their

[3]The return R_1 can be stored without cost up to date 2 when production of the final good occurs.

[4]The return R_1 is certain, yet its value is assumed non verifiable. Hart and Moore [1998] discuss the case in which the return is uncertain, and show that in this case the optimal contract is no longer given by debt contract.

[5]Agents can tell defective from non defective projects, once the uncertainty is resolved.

own asset had been defective. The secondary market for the projects is characterized by one sided search frictions; a lender costlessly finds only one agent to whom to sell the project. The type of the agent is common knowledge[6], but it is unknown who among the agents of a given type is an entrepreneur. Thus the probability for a lender to find an interested buyer for the asset $\hat{\imath}$ increases when more agents of type $\hat{\imath}$ are entrepreneurs. If the match in the secondary market is profitable, the lender pockets the entire quasi-rents of the match.

fThe detailed unfolding of the events is the following. At date 0, agents choose whether to borrow or to lend. At date 1, a) R_1 is produced, b) borrowers default or repay, and renegotiation between borrowers and lenders occurs, c) the uncertainty over $\tilde{R}_2$ is resolved, d) trading in the secondary market for the physical assets occurs. At date 2, a) R_2 is produced, b) the final output is produced, and agents consume.

4.3. Optimal Contract and Liquidation Value

In this section I first briefly characterize the optimal financial contract following Hart and Moore [1994], and then determine the liquidation value expected by a lender who owns a project's asset. Without loss of generality, I restrict attention to renegotiation-proof contracts where the borrower never has incentives to declare default in order to renegotiate down the payment.

It is also assumed that the returns of the project are such that

Assumption 1 $R_1 > R_2 \equiv (1-\lambda)\tilde{R}_2$,

so that liquidation never occurs on the equilibrium path because the borrower is insolvent. When Assumption 1 fails to hold, *involuntarily* default might occur when R_1 is

[6]This assumption is needed because there is a continuum of types.

low relative to R_2, r, the interest rate on the loan, is high, and the cost of liquidation is not too high. In this situation, it could be profitable for the entrepreneur to start the project, however he might fail to have enough cash flows at date 1 in order to repay the loan, thus default will occur in equilibrium. The non-verifiability of a project's return is crucial in shaping the optimal contract. Indeed a borrower in this environment can always divert away from the borrower the project's cash flow by simply declaring to the court that the asset produced a zero return at date 1 and at date 2. Thus no lending would ever occur absent any mechanism that induces a borrower to repay. As discussed in Hart and Moore [1994], the optimal mechanism to allow financial trade is asset ownership: loans are secured by the physical asset: when a borrower does not repay, the lender obtains the ownership of the asset (transfers are verifiable by courts). But this threat is not always sufficient to induce the borrower to repay in full.

No payment from the borrower occurs at date 2 since the asset is worthless at this date. To determine the maximum payment that a borrower is willing to make at date 1, first note that the asset's expected value as of date 1 is $(1-\lambda)\tilde{R}_2$ and that the joint borrower-lender surplus is maximized when the borrower owns the asset. Thus the two parties find it optimal to renegotiate the contract upon a borrower's default as in such case the lender owns the asset. Following Hart and Moore [1994], the borrower has all the bargaining power in the renegotiation process, so that the borrower pays the lender a sum which makes him indifferent between holding the asset and selling it back to the borrower.

Thus the exact amount depends on the lender's outside option, which I know turn to determine. The lender who owns the asset will first try to sell the asset in the secondary market and then run the project himself if he cannot find an interested buyer.[7] Let p_i denote the price of the capital good of type i in units of the final good. Agents can tell defective from non defective assets, thus an insider whose project failed, is willing to pay $p_i\tilde{R}_2$ for a non defective asset, and, from Assumption 1, he has sufficient cash flows at date 1 to buy the asset after repaying his initial loan[8] Let n_i be the probability that a lender meets an industry insider of type i. Then the probability for a lender to find an interested buyer is $n_i\lambda$. If the search effort is unsuccessful, the lender runs the project and produces $\underline{\tilde{R}}_2$ units of type i output So the expected liquidation value of the project is: $p_i(\lambda n_i(R_2 - \underline{R}_2) + \underline{R}_2) \equiv p_i(1-\lambda)(n_i\lambda\tilde{R}_2 + (1-n_i\lambda)\underline{\tilde{R}}_2)$, where $(1-\lambda)$ is the probability that the asset is not defective. Let r be the price of e in terms of Y, or real interest rate between date 0 and date 2. Then a borrower chooses not to default only if the size of the loan gross of interest is smaller than the expected liquidation value to the lender.

$$\text{(CC)} \qquad r(1-e) \leq p_i(\lambda n_i(R_2 - \underline{R}_2) + \underline{R}_2).$$

Equation (CC) is the collateral constraint for entrepreneurs in sector i. Having outlined the form of the optimal contract and determined the liquidation value of the asset, the next section characterizes the properties of the sectoral equilibrium.

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[7]The lender cannot resell the project to the borrower. If this were the case, the lender would obtain all the bargaining power in the renegotiation process and the the liquidity of the asset would no longer affect the terms of the contract.

[8]Shleifer and Vishny [1992] consider an economy where there are no search frictions, assumption **??** does not hold and liquidation occurs on the equilibrium path. They show that sectoral illiquidity emerges when the shocks to the industry insiders' cash flows are highly correlated, so that all entrepreneurs are insolvent at the same time.

4.4. The Sectoral Equilibrium

Profit maximization in the perfectly competitive final good sector determines the inverse demand for each type of capital good

$$p_i = Y^{\frac{1}{\sigma}} K_i^{-\frac{1}{\sigma}}.$$

An agent of type i finds it profitable to become an entrepreneur if:

$$\text{(PC)} \qquad (R_1 + R_2) p_i \geq r,$$

and has enough funds to fund the project if (CC) is satisfied. Thereby he becomes an entrepreneur if and only if:

$$\text{(4.1)} \qquad p_i \geq \hat{p}_i \equiv \max \left\{ \frac{r}{(R_1 + R_2)}, \frac{r(1-e)}{n\lambda(R_2 - \underline{R}_2) + \underline{R}_2} \right\}.$$

Note that the fraction of type i agents who become entrepreneurs in each sectors, k_i is simply equal to $\frac{K_i}{(R_1+R_2)}$. Because the demand for the capital goods satisfies the Inada conditions, either a positive fraction of agents become entrepreneurs, $p_i = \hat{p}_i$ and the fraction of entrepreneurs is $0 < k_i < 1$, or all agents do and $k_i = 1$. Define the function

$$\text{(4.2)} \qquad g(n_i) \equiv \left(\frac{Y}{r^\sigma} \right) \frac{(\lambda n_i (R_2 - \underline{R}_2) + \underline{R}_2)^\sigma}{(1-e)^\sigma (R_1 + R_2)}.$$

The from (4.1), the fraction of type i agents that become entrepreneurs is equal to

$$\text{(KK)} \qquad k_i = \phi(n_i) \equiv \min \left\{ \left(\frac{Y}{r^\sigma} \right) (R_1 + R_2)^{\sigma - 1}, g(n_i), 1 \right\},$$

A sectoral equilibrium is defined as a k_i and n_i such that n_i equals k_i and solves (KK). The subscript i is dropped in the remaining of this section. The equilibrium is stable if the map $\phi(\cdot)$ cuts the 45-degree line from above. The value of n such that both (PC) and (CC) hold with equality if from (KK)

$$n_{cc} \equiv \frac{(R_1 + R_2)(1-e) - \underline{R}_2}{\lambda(R_2 - \underline{R}_2)}.$$

Also let $\underline{n}_{cc} \equiv \min\{1, n_{cc}\}$ The full characterization of the sectoral equilibrium is provided in the following Proposition

Proposition 2 (Properties of the Sectoral Equilibrium)**.** *Let* $\hat{n} \equiv \frac{\underline{R}_2}{(\sigma-1)\ \lambda\ (R_2 - \underline{R}_2)}$.

(1) *There is at least one equilibrium.*

(2) *There is at most one equilibrium where the collateral constraint does not bind. The equilibrium is stable and the fraction of entrepreneurs is* $k_h = \min\left\{\left(\frac{Y}{r^\sigma}\right)(R_1 + R_2)^{\sigma-1}, 1\right\} \geq \underline{n}_{cc}$.

(3) *There are at most two equilibria where the collateral constraint binds. If there's one then the fraction of entrepreneurs is* $k_l < \hat{n}$, *and the equilibrium is stable. If there are two then* $k_l < \hat{n} < k_m < n_{cc}$, k_l *is stable while* k_m *is unstable.*

Proof. The existence of the sectoral equilibrium directly follows from the Brouwer fixed point and the continuity of $\phi(\cdot)$ over $[0,1]$. If in equilibrium, (CC) does not bind then $k_h > n_{cc}$. Uniqueness and stability of k_h follows from the fact that $\phi(n)$ is constant where strictly less than $g(n)$. If in equilibrium (CC) binds, the derivative of the map $\phi(\cdot)$ in equilibrium is $\frac{\partial \phi}{\partial n}\Big|_{k=n} = \frac{\sigma\lambda(R_2 - \underline{R}_2)n}{(\lambda n(R_2 - \underline{R}_2) + \underline{R}_2)}$, which is less than one only if $n < \hat{n}$. The map

$\phi(\cdot)$ is convex and thus there are at most two equilibria where the collateral constraint can bind. □

The three configurations of proposition 2 are shown in figure 1 for the non-generic case in which $k_h < 1$ in equilibrium. I discuss each of the three classes of equilibria after the next Proposition that provides conditions for each of the three equilibria to occur

Proposition 3 (Characterization of Sectoral Equilibrium)**.** *Let* $f(t) \equiv t\ (t\lambda(R_2 - \underline{R}_2) + \underline{R}_2)^{-\sigma}$, $y_1 \equiv (1-e)^{\sigma}(R_1 + R_2)f(\underline{n}_{cc})$ *and* $y_3 \equiv (1-e)^{\sigma}(R_1 + R_2)f(\hat{n})$.

A. *If* $\left(\frac{Y}{r^{\sigma}}\right) < y_1$ *there is a unique equilibrium where (CC) binds.*

B. *If either* $\frac{Y}{r^{\sigma}} > y_3$ *or* $\frac{Y}{r^{\sigma}} > y_1$ *and* $\hat{n} > \underline{n}_{cc}$ *there is one equilibrium where (CC) does not bind.*

C. *If* $\left(\frac{Y}{r^{\sigma}}\right) \in (y_1, y_3)$ *and* $\hat{n} < \underline{n}_{cc}$ *there are three equilibria* k_l, k_m *and* k_h *such that* $k_l < \hat{n} < k_m < \underline{n}_{cc} \leq k_h$ *and* $k_h = \min\left\{\left(\frac{Y}{r^{\sigma}}\right)(R_1 + R_2)^{\sigma-1}, 1\right\}$

Proof. Consider the case in which $n_{cc} < 1$. If $\frac{Y}{r^{\sigma}} = y_1$ then $n_{cc} = \phi(n_{cc})$, so that n_{cc} is the unique equilibrium. Consider an increase of $\frac{Y}{r^{\sigma}}$ starting at y_1. The map $\phi(n)$ shifts up, and always intersects the 45 degree line above the value of n_{cc}. Intersections below n_{cc} only occur if for $\frac{Y}{r^{\sigma}} = y_1$, the derivative of $\phi(n)$ at n_{cc} is greater than one, that is, from proposition 3 when $\hat{n} < n_{cc}$. If there is a unique equilibrium below n_{cc}, the map $\phi(n)$ meets the 45 degree line tangentially, and from Proposition 2, the intersection occurs exactly at $\hat{n}$. From $\hat{n} = \phi(\hat{n})$ then it follows that $\frac{Y}{r^{\sigma}}$ is equal to y_3. The function $f(t)$ reaches its maximum at $\hat{n}$, thus, $y_3 > y_1$. The map $\phi(n)$ is convex where (CC) binds, thus below n_{cc} there are no equilibria if $\frac{Y}{r^{\sigma}} > y_3$, while, there are two equilibria if $\frac{Y}{r^{\sigma}} \in (y_1, y_3)$. Thus the claim follows when $n_{cc} < 1$. The case of $n_{cc} > 1$ is proved analogously. □

In case $A.$, the collateral constraint binds in the unique stable equilibrium, while in case $B.$ the profitability constraint binds. In case $B.$ two stable equilibria coexist: in one the collateral constraint binds and the liquidity of the collateral is low; in the other it is the profitability constraint that binds and the collateral is liquid. The conditions for case $C.$ in proposition 3, are sufficient for multiple equilibria to exist in each of the intermediate sector in isolation. These conditions are, however, not sufficient for an aggregate asymmetry in the allocation of capital to occur. First, the level of the aggregate demand Y and the interest rate r are determined in equilibrium, and thus, one first needs to solve for their levels to determine whether, in equilibrium, case $C.$ occurs or not. Further, although case $C.$ shows the possibility for capital to be asymmetrically allocated across sectors, it does not imply that the asymmetry is inevitable, in other words, a configuration in which the collateral constraint binds in all sectors or in none is as likely as one in which the constraint only binds in a portion of sectors so that the allocation of capital will be asymmetric. In the next sections I will solve for the values of Y and r, and provide conditions on the economy's primitives such that the asymmetric capital allocation will be inevitable and not just a possibility.

4.5. The General Equilibrium

In this section I solve for the general equilibrium of the model economy. I restrict attention to stable general equilibria, in which all sectors lie in a stable sectoral equilibrium and r clears the funds' market. Since from proposition 3, each sector has at most two stable equilibria, the general equilibrium can be either symmetric or asymmetric, depending on whether sectors produce an equal or unequal amount of output. When the equilibrium is symmetric, each sectors' overall output is $(R_1 + R_2)e$, and the collateral constraint might either bind or not. In an asymmetric equilibrium, on the other hand, the collateral constraint binds in some sectors, but not in others. From proposition 3, investment is lower in the sectors where the collateral constraint binds, thereby it immediately follows from the aggregate resource constraint for e, that in the asymmetric general equilibrium overinvestment occurs in some sectors while under-investment in others relative to the investment levels in symmetric equilibria.

The remaining of this section characterizes the general equilibrium in two benchmark economies where collateral liquidity does not matter; this serves to highlight the role of endogenous collateral values in generating the asymmetric allocation of investment.

First suppose that a project's returns are verifiable. In this case, enforceable contracts that specify how the returns are divided between borrowers and lenders are feasible. Then projects are always funded if they are profitable, thus, only (PC) determines the investment level, or $k_i = \min\left\{\left(\frac{Y}{r^\sigma}\right)(R_1 + R_2)^{\sigma-1}, 1\right\}$. It then follows that in all sectors $k_i = k$, and market clearing in the market for funds, implies that $k = \frac{Y}{r^\sigma}(R_1 + R_2)^{\sigma-1} = e < 1$, so that $Y = (R_1 + R_2)k = (R_1 + R_2)e$, and $r = (R_1 + R_2)$. Because the production technology for the intermediate good has decreasing returns to scale, all sectors produce an equal amount of output.

Now consider the case in which asset returns are non-verifiable but $\underline{R}_2 = R_2$, so that lenders are as productive as the industry insiders in running the second project's return. The liquidity of the collateral is irrelevant in this parametrization, but entrepreneurs may be subject to borrowing constraints as they may only pledge the return R_2 to outside investors. From (KK), $k_i = \phi(n_i) \equiv \min\left\{\left(\frac{Y}{r^\sigma}\right)(R_1 + R_2)^{\sigma-1},\ \left(\frac{Y}{r^\sigma}\right)R_2^\sigma(R_1 + R_2)^{-1}\,(1-e)^{-\sigma}, 1\right\}$, and thus $k_i = k = e$ and $Y = (R_1 + R_2)e$, that is all sectors produce an equal amount of output. From the aggregate resource constraint, $r = \min\left\{(R_1 + R_2), \frac{R_2}{(1-e)}\right\}$. If e is low enough, the borrowing constraint binds in all sectors and the equilibrium interest rate is lower than without the assumption of limited pledgeability. However, limited pledgeability does not affect the aggregate allocation of capital and all sector produce the same amount of intermediate good[9].

Depending on the primitives of the economy, aggregate investment might be asymmetric when $\underline{R}_2 < R_2$, on the other hand, as the next sections show.

4.5.1. The Symmetric Equilibrium

In the symmetric equilibrium, an equal amount of capital is produced by each sector and an equal proportion of agents become entrepreneurs within each agents' type. Thereby $k_i = k = e$, $Y = (R_1 + R_2)e$ and, since $e < 1$, either (PC) or (CC) bind in all sectors. From proposition 2, if (PC) binds then $k = e > n_{cc}$, which can be rewritten in terms of endowment levels as $e > e^{**}$, where

$$e^{**} \equiv 1 - \frac{\lambda(R_2 - \underline{R}_2) + \underline{R}_2}{(R_1 + R_2) + \lambda(R_2 - \underline{R}_2)}.$$

Using Y into the resource constraint it follows that $r = (R_1 + R_2)$, so that in equilibrium $\frac{Y}{r^\sigma} > y_1$ which, from proposition 3, is the necessary condition for the existence of a sectoral equilibrium in which the collateral constraint does not bind. On the contrary, if (CC) binds, then $e < e^{**}$ and from proposition 2 the equilibrium is stable only if it is also the

[9]When the borrowing constraint binds, equilibrium credit rationing occurs in that the agents who lend would rather borrow to start a project at the equilibrium level of r. However they cannot credibly promise to pay higher interest rates and thus attract funds away from current borrowers.

case that $e < \hat{n}$. From the resource constraint $r = \frac{\lambda e(R_2 - \underline{R}_2) + \underline{R}_2}{(1-e)}$ so that now $\frac{Y}{r^\sigma} < y_3$ in equilibrium; the latter condition is necessary for a sectoral equilibrium to exist in which the collateral constraint binds. From the previous discussion it immediately follows that if the primitives are such that $\hat{n} < e < e^{**}$ there is no stable general equilibrium in which all sectors produce an equal amount of intermediate good. The previous results are summarized in the following proposition:

Proposition 4 (Symmetric General Equilibrium)**.** *When* $e \in (\hat{n}, e^{**})$ *there is no symmetric stable general equilibrium, while when:*

A. $e > e^{**}$, *a symmetric stable general equilibrium exists where (PC) binds in all sectors,*

B. $e < \min\{\hat{n}, e^{**}\}$, *a symmetric stable general equilibrium exists where (CC) binds in all sectors*

Proof. Follows from the discussion above. □

Note[10] that $\hat{n} < e^{**}$ only if $\underline{R}_2 < \underline{R}_2^{**}$, where:

$$\underline{R}_2^{**} \equiv R_2 - \frac{\sqrt{\Phi} - (R_1 + R_2)(1-\lambda) + R_1 \lambda \sigma}{2\lambda\sigma},$$

and $\Phi \equiv ((R_1 + R_2)(1-\lambda) + \lambda\sigma(R_1 + 2R_2))^2 - 4\lambda^2 \sigma R_2 (R_1 + R_2)(\sigma - 1)$. The conditions of proposition 4 are shown in figure 2. The value[11] of e^{**} separates A from C and B from BC, while $\hat{n}$ separates B from $\tilde{A}B$ and AB. The symmetric equilibrium where (PC) binds exists in the areas A, AB and $A\tilde{B}$, while the equilibrium in which the (CC) binds exists

[10] The function $h(\underline{R}_2) \equiv \hat{n} - e^{**}$ is decreasing over the relevant parameter space. Futher $g(0) = -\frac{R_1 + R_2}{R_1 + R_2(1+\lambda)}$ and $g(\underline{R}_2^{**}) = 0$.

[11] From straighforward differentiation, e^{**} is decreasing and concave in $\underline{R}_2$.

in BC and C. There is no symmetric equilibrium in area B, so this is the area where the equilibrium will be necessarily asymmetric.

4.5.2. The Asymmetric equilibrium

This section provides conditions for the existence of an asymmetric general equilibrium. The investment level differs across sectors: the collateral constraint binds in some sectors while either the profitability constraint or the availability of entrepreneurial skills determines the investment levels in the remaining sectors. Suppose that the collateral constraint binds in a portion of sectors $(1 - x)$, then in these sectors the number of entrepreneurs is given by:

$$k_l = g(k_l) < \underline{n}_{cc}. \tag{4.3}$$

From proposition 3 it must also be the case that

$$k_l < \hat{n}, \tag{4.4}$$

for the sectoral equilibrium to be stable. In sectors where the collateral constraint does not bind, the number of entrepreneurs is given by

$$k_h = \min\left\{\frac{Y}{r^\sigma}(R_1 + R_2)^{\sigma-1}, 1\right\} < g(k_h). \tag{4.5}$$

Finally, for the market for the endowment to clear, the following condition must be satisfied:

$$(1 - x)\ k_l + x\ k_h = e. \tag{4.6}$$

Then an asymmetric stable general equilibrium exists if it exists a solution to the set of conditions (4.3)-(4.6). The parameter values for which the conditions may be satisfied are provided in the following proposition

Proposition 5 (Asymmetric General Equilibrium). *Let* e_3 *be defined by* $e_3\,(1-e_3) \equiv (R_1+R_2)^{\sigma}\;f(\hat{n})$, e_1 *by* $f(n_{cc}) \equiv f(1)$ *and* $e_1 < \hat{n}$, $\underline{e} \equiv 1 - \frac{\underline{R}_2+(R_2-\underline{R}_2)\;\lambda}{R_1+R_2}$, *and* $e^* \equiv \frac{(R_1+R_2)(\sigma-1)-\sigma\underline{R}_2}{(\sigma-1)(r_1+R_2)}$ *There exist an continuum of asymmetric general equilibria, where* k_h *agents become entrepreneurs in a portion* $x \in [0,1]$ *of intermediate sectors while* k_l *in the remaining sectors if* e *is such that either* $e \in (e_1, \underline{e})$, *or* $e \in (\max\{\hat{n}, \underline{e}\}, e_3)$ *or* $e \in (\underline{e}, \min\{e^*, \hat{n}\})$ *and* $f(e) > f(n_{cc})$ *hold. If further* $e \in (\hat{n}, e^{**})$ *then* $x \in (0,1)$.

Proof. See Appendix. □

The parameter space of proposition 5 is shown in figure 2. The value of $\underline{e}$ is plotted as the dashed line going through area B and the line that separates AB from $\tilde{A}B$, while[12] e_1 is the curve that separates AB and the lower portion of $\tilde{A}B$ from A. Thereby $e \in (e_1, \underline{e})$ holds in AB and in the lower portion of B that is delimited from above by the dashed line. The[13] value of e_3 separates BC from C. Thus $e \in (\min\{\hat{n}, \underline{e}\}, e_3)$ holds in area BC and in the portion of B bounded from below by the dashed line. Finally[14] e^* separates the upper portion of $\tilde{A}B$ from A. The shape of the area where $f(e) > f(n_{cc})$ holds depends on the values of the parameters, however the condition is never satisfied where $e \in (\underline{e}, \min\{e^*, e_1\})$. Indeed, when $e < e_1$ then $f(e) < f(1)$ while $f(n_{cc}) > f(1)$ for

[12]By its defition e_1 is always smaller than $\hat{n}$ unless $\hat{n} = 1$. Further $\frac{\partial e_1}{\partial \underline{R}_2} = \frac{(1-e)eR_2\lambda\sigma}{(\underline{R}_2(1-\lambda)+\underline{R}_2)(\underline{R}_2(1+(\sigma-1)e\lambda)} > 0$.
[13]Note that $sign\left(\frac{\partial e_3}{\partial \underline{R}_2}\right) = sign\left(\underline{R}_2 - \frac{R_2(\sigma-1)}{\sigma}\right)$ and $R_2^{**} < \frac{R_2(\sigma-1)}{\sigma}$ so that e_3 is decreasing when $\underline{R}_2 < R_2^{**}$. Further if $\underline{R}_2 \neq R_2^{**}$ then $e_3 > e^{**}$. To see this this note that $e_3 > e^{**}$ can be rewritten as $e^{**}(1-e^{**})^{-\sigma} < f(\hat{n})\;(R_1+R_2)^{-\sigma}$, or: $f(e^{**}) < f(\hat{n})$ which is always true for $e^{**} \neq \hat{n}$ or $\underline{R}_2 < \underline{R}_2^{**}$.
[14]It can be shown with some algebra that $\hat{n} > e^{**} > e^*$ for $\underline{R}_2 > R_2^{**}$ while the three values are equal when $\underline{R}_2 = R_2^{**}$.

$e \in (\underline{e}, \max\{e^*, e_1\})$. Thus $e \in (\underline{e}, \min\{e^*, \hat{n}\})$ and $f(e) > f(n_{cc})$ hold in a subset of the area $\tilde{A}B$.

As it can be seen from figure 2, no asymmetric equilibrium exists when $\underline{R}_2 > R_2^*$, where $R_2^* \in (R_2^{**}, R_2)$ is defined by $f(e^*) = f(1)$ and $e^* < 1$. In other words, sectoral market thickness does not matter for aggregate allocations when lenders' productivity is close to that of industry insiders'. The collateral constraint might still bind when $\underline{R}_2 > R_2^*$ since only the return R_2 can be pledged to outside investors, but as $\underline{R}_2$ increases market thickness has an increasingly smaller role, and in the extreme case in which $\underline{R}_2 = R_2$, (KK) no longer depends on the value of n. When $\underline{R}_2 < R_2^*$, the asymmetric equilibrium is likely to occur for intermediate levels of e. To understand this result, note that when e is very low, and $\underline{R}_2$ is positive, entrepreneurs are always constrained by the value of R_2 as their financial leverage is very high. On the contrary, when e is high, entrepreneurs only need few outside funds invest, so the collateral constraint never binds no matter what is the thickness level. It is for intermediate levels of e that the collateral constraint might bind or not, and this is when multiple equilibria will coexist. In area B, the only possible equilibrium is asymmetric. Indeed in this area $e < n_{cc}$ so that there is no symmetric equilibrium in which the collateral constraint does not bind, while $e > \hat{n}$ so that the symmetric equilibrium in which the collateral constraint binds in all sectors is unstable.

What are the aggregate consequences of endogenous differences in collateral's liquidity? It directly follows from the Jensen inequality and the decreasing returns of the final good production function, that the productivity of the endowment good is lower whenever capital allocation is asymmetric. The degree of inefficiency is easily computed in the special case in which $\underline{R}_2 = 0$. With $\underline{R}_2 = 0$, then $\hat{n} = 0$ so that k_l is equal to zero, thus, the only stable equilibrium where the collateral constraint binds is one in which the sector is completely inactive. Then from (4.6), $k_h = e/x$, and

$$Y = x^{\frac{1}{\sigma-1}}(R_1 + R_2)e. \tag{4.7}$$

From the requirement that $k_h \in (n_{cc}, 1)$ and that the fraction of active sectors is less than one, then:

$$x = \begin{cases} \left[e, \min\left\{\frac{e\ \lambda\ R_2}{(1-e)\ (R_1+R_2)}, 1\right\}\right] & \text{if } e \geq \underline{e} \\ e & \text{otherwise} \end{cases} \tag{4.8}$$

From (4.7), the aggregate final good output level relative to the case in which the equilibrium is symmetric is simply: $x^{\frac{1}{\sigma-1}}$. Whenever $e < e^{**}$ the equilibrium is asymmetric. As seen in proposition 5 the model generates a continuum of equilibria for given parameter values, each being characterized by a different fraction of sectors x where the collateral constraint does not bind[15]. Each of these equilibria has a different level of aggregate productivity, which decreases with the number of inactive sectors. Further, for given x, the inefficiency increases as σ tends to one.

4.6. Conclusion

Lenders accept collateral against loans, to induce a would-be defaulter to repay a loan through the threat of expropriation. The credibility of this threat diminishes as the collateral is increasingly illiquid, because lenders face larger costs in reselling the asset. On the other hand, the collateral is increasingly liquid with a higher number of industry insiders, who are the natural buyers for a competitor's asset as they have the highest

[15]When $\underline{R}_2 > 0$, differently from (4.8) the continuum of equilibria exists also where $e < \underline{e}$.

valuation for it. But the number of insiders depends itself on the amount of loans issued to a specific sector, and it is thus possible that, in some sectors, investment and the liquidity of the assets used in production are jointly low. The amount of internal funds that entrepreneurs start with, given the scale of the investment projects, determine whether or not the endogeneity of the collateral values translate into asymmetric investment at the aggregate level. Entrepreneurs in LDCs need to borrow large amounts so that in general equilibrium they are financially constrained in all sectors, and the equilibrium is symmetric. In high income economies, entrepreneurs need to borrow small amounts and thus are financially unconstrained in all sectors, and again the general equilibrium is symmetric. It is in middle income economies that financially constrained and unconstrained entrepreneurs coexist in different sectors. For these economies, investment in sectors where the collateral is illiquid is low, because would-be entrepreneurs cannot credibly commit to repay their loans, further reinforcing the illiquidity of the collateral. In the same sectors, active entrepreneurs repay their loans but can only do so at a relatively low interest rate. In sectors where the collateral is liquid, on the other hand, would be entrepreneurs decide to enter a crowded market only because they can borrow at the rates that are kept artificially low by the constrained sectors. In the paper I show that the asymmetry in the aggregate investment level *necessarily* occurs in equilibrium for middle income countries. In these economies resources will be misallocated generating losses of efficiency at the aggregate level. Although establishing the size of these inefficiencies still remains an open question, the endogenous liquidity of the collateral is an important channel through which contracting frictions at the microeconomic level can have macroeconomic implications.

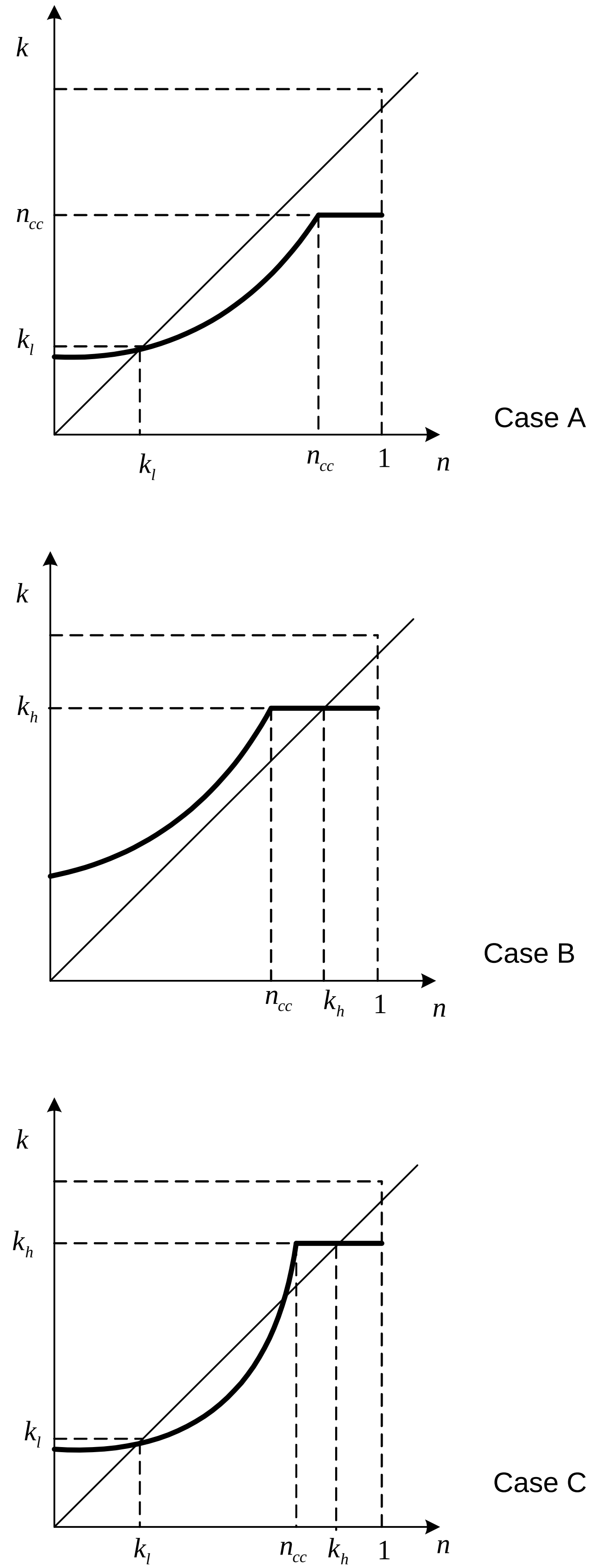

Figure 4.1. Sectoral Equilibrium when $k_h < 1$.

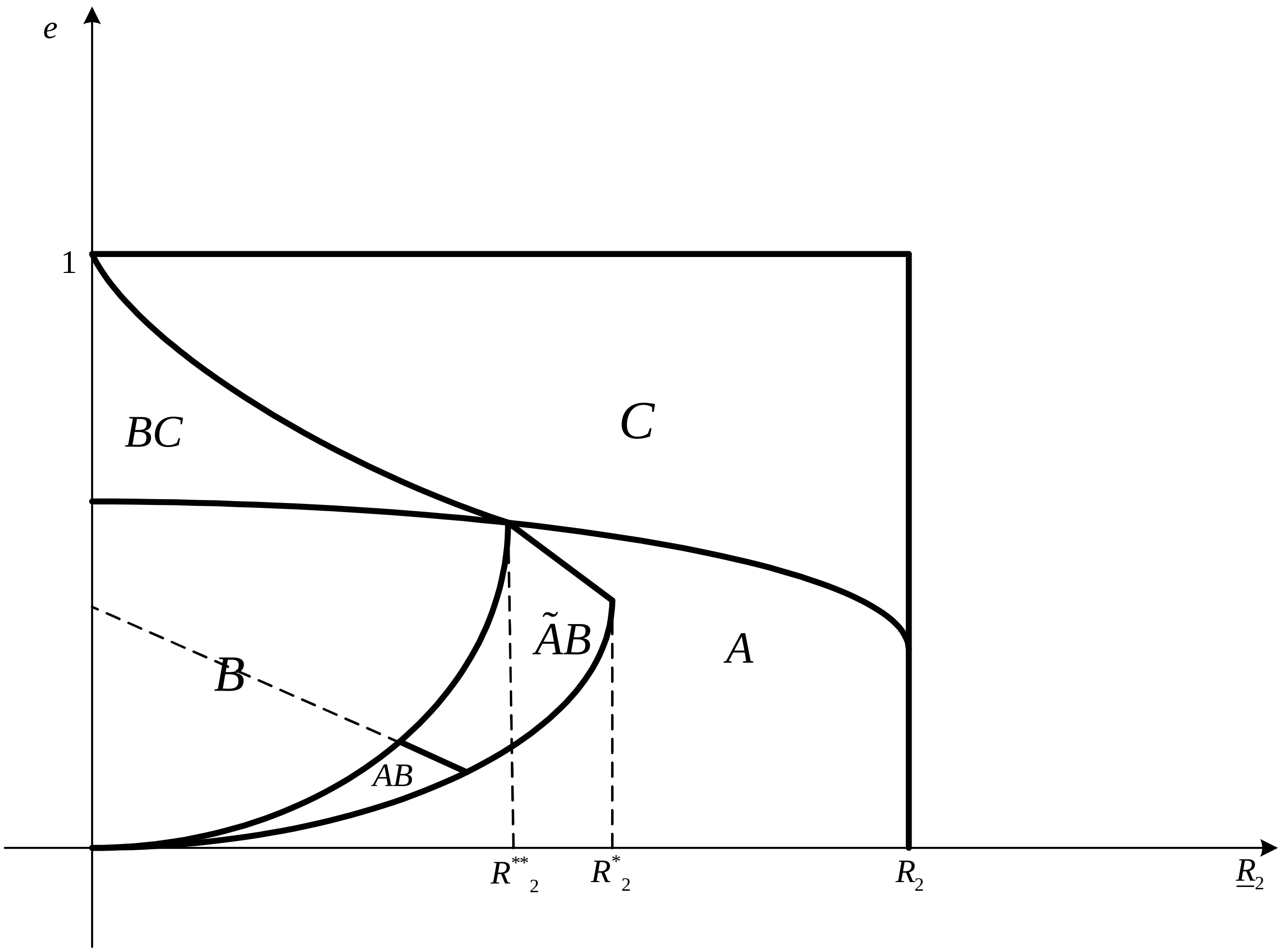

Figure 4.2. Parameter Configuration for General Equilibrium

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APPENDIX A

Appendix

A.1. Proof of Proposition 1

First consider the investment problem of a firm that faces investment adjustment costs. The firm solves (2.32) subject to (2.33) and (2.3). The first order conditions of the maximization problem are

$$\text{(A.1)} \qquad \begin{aligned} &(K_t) \quad q_t = \mathbb{E}_t R_{t+1}^{-1}(\pi_{1,t+1} + (1-\delta)q_{t+1}), \\ &(I_t) \quad q_t = \tau(1 + S_1(I_t, I_{t-1}) + \mathbb{E}_t R_{t+1}^{-1}\mu_{t+1}\ S_2(I_{t+1}, I_t)). \end{aligned}$$

From (A.1) the steady state level of capital solves

$$\text{(A.2)} \qquad \pi'(K_{ss}) = \tau(R - (1-\delta)).$$

For each variable X_t, let the corresponding hatted variable denote a percentage deviation from steady state, $\hat{X}_t = (X_t - X_{ss})/X_{ss}$. Equation (I_t) of (A.1) can be log-linearized in

$$\text{(A.3)} \qquad \tau\hat{q}_t = \chi\left(\Delta\hat{I}_t - \mathbb{E}_t R_{t+1}^{-1}\Delta\hat{I}_{t+1}\right),$$

where $\Delta\hat{I}_t = \hat{I}_t - \hat{I}_{t-1}$.

Now consider the TTB model. The first order conditions that characterize the investment decision are (2.14), from which the capital stock in steady state solves

(A.4) $$\pi'(K_{ss}) = \theta^{\frac{1}{1-\varepsilon}}(R - (1-\delta))$$

Loglinearizing (E) of (2.14) it follows that

(A.5) $$\hat{\mu}_t = R^{-1}(1-\theta)\mathbb{E}_t\left(\hat{\mu}_{t+1} - \hat{R}_{t+1}\right).$$

Loglinearizing (I) of (2.14) and substituting $\hat{\mu}_t$ from (A.5) it follows that

(A.6) $$\hat{q}_t = \frac{(1-\theta)}{\theta\varepsilon\ \left(1 - R^{-1}(1-\theta)\right)}\left(\Delta\hat{I}_t - \mathbb{E}_t R^{-1}_{t+1}\Delta\hat{I}_{t+1}\right).$$

From (A.2) and (A.4), the capital stock in steady state is the same in the two models when

(A.7) $$\tau = \theta^{\frac{1}{1-\varepsilon}},$$

holds. It then easily follows that all other quantities are also equal in steady state. When (A.7) and $\chi = \frac{(1-\theta)}{\theta\varepsilon(1-R^{-1}(1-\theta))}$ hold, the loglinearized first order condition (A.3) and (A.6) are also equal. Further note that the first order condition with respect to (K) is the same in both models, absent capital adjustment costs in the TTB model ($\phi = 0$). In the investment adjustment cost model, investment expenditure is equal to τI_t, so that the log-deviations of the basket and expenditure are equal. This is also the case in the TTB model. By loglinearizing (3.4) and (3.11) it follows that

$$\hat{E}_t = \int_0^1 \hat{\imath}_t(j)\ dj = \hat{I}_t.$$

Thus the claim of the Proposition follows

A.2. Derivation of Equation (3.18)

From $Z_t = 1$ it follows that all scaled quantities are equal to the corresponding levels. To obtain the second line of (3.18), first loglinearize (e_t) of (3.14) to obtain

(A.8)
$$\hat{\mu}_t = (1 - \beta(1-\theta))\ \hat{\lambda}_t + \beta(1-\theta)\mathbb{E}_t\hat{\mu}_{t+1},$$

and then (i_t) in the following expression

(A.9)
$$\gamma_{ss}\hat{\gamma}_t - \mu_{ss}\hat{\mu}_t\Omega_{1ss} - \mu_{ss}\Omega_{11ss}I_{ss}\hat{I}_t - \mu_{ss}\Omega_{12ss}I_{ss}\hat{I}_{t-1} = \beta\mathbb{E}_t\mu_{ss}\left(\Omega_{2ss}\hat{\mu}_{t+1} + \Omega_{21ss}I_{ss}\hat{I}_{t+1} + \Omega_{22ss}I_{ss}\hat{I}_{t+1}\right).$$

Using the steady state derivatives: $\Omega_{1ss} = \theta^{\frac{1}{1-\varepsilon}}$, $\Omega_{2ss} = -(1-\theta)\theta^{\frac{1}{1-\varepsilon}}$, $\Omega_{11ss} = \Omega_{22ss} = -\Omega_{12ss} = \frac{(1-\theta)\theta^{\frac{\varepsilon}{1-\varepsilon}}}{I_{ss}\varepsilon}$, and substituting $\mathbb{E}_t\hat{\mu}_{t+1}$ from (A.8) into (A.9), yields

(A.10)
$$\eta\Delta\hat{I}_t = \hat{\gamma}_t - \hat{\lambda}_t + \beta\eta\mathbb{E}_t\Delta\hat{I}_{t+1},$$

where η is defined in (3.19). By rewriting (k_t) of (3.14) it follows that the ratio $\frac{\gamma_t}{\lambda_t}$ is equal to the price of capital in terms of consumption, or Tobin's marginal Q_t, so that

(A.11)
$$\hat{Q}_t = \hat{\gamma}_t - \hat{\lambda}_t.$$

Since $\iota_{j,ss} = \iota_{ss}$ for all $j \in [0,1]$, from (3.2) and (3.4) it follows that $\hat{E}_t = \hat{I}_t = \int_0^1 \hat{\iota}_t(j)\, dj$. Using this equality and (A.11) into (A.10) yields the second line of (3.18).

Since the supply of international bonds is perfectly elastic, line (b_t) of (3.16) can be linearized in

$$\hat{\lambda}_t = \mathbb{E}_t \hat{\lambda}_{t+1}. \tag{A.12}$$

Linearization of line (k_t) of (3.14) yields

$$\hat{\gamma}_t = (1-\beta(1-\delta))\ \mathbb{E}_t \Big((1-\alpha)(\hat{A}_{t+1} - \hat{K}_t) + \hat{\lambda}_{t+1} \Big) + \beta(1-\delta)\ \mathbb{E}_t\ \hat{\gamma}_{t+1}. \tag{A.13}$$

The second equation of (3.18) follows from (A.11), (A.12) and (A.13). The third equation (3.18) is the loglinearized capital accumulation equation (3.5).

A.3. Values of χ_0 and χ_1, and Derivation of Equation (3.20)

The method of undetermined coefficients is used to solve for $\hat{E}_t$ in (3.18). First from $\delta = 1$, it follows that $\hat{K}_t = \hat{E}_t$. Using this equality and substituting the value of $\hat{Q}_t$ from the first line into the second line of (3.18) one obtains

$$\eta \Delta \hat{E}_t = (1-\alpha)(\rho_a \hat{A}_t - \hat{E}_t) + \beta \eta \mathbb{E}_t \Delta \hat{E}_{t+1}. \tag{A.14}$$

Substitute the guess $\hat{E}_t = \chi_0 \hat{A}_t + \chi_1 \hat{E}_{t-1}$ and use the fourth line of (3.18) into (A.14), to yield $\pi_0(\chi_1)\hat{E}_{t-1} + \pi_1(\chi_0, \chi_1)\hat{A}_t = 0$, where $\pi_0(\cdot)$ and $\pi_1(\cdot,\cdot)$ are polynomials in χ_0 and χ_1. Since (A.14) holds for all $\hat{A}_t$ and $\hat{E}_{t-1}$, it must also be true that: $\pi_0(\cdot) = 0$ and

$\pi_1(\cdot,\cdot) = 0$. The equation $\pi_0(\chi_1) = 0$ has two zeros, of which only the smallest

$$\chi_1^* = \frac{((1-\alpha+\eta(1+\beta)) - \sqrt{\varphi})}{2\beta\eta},$$

where $\varphi \equiv (1-\alpha+\eta(1+\beta))^2 - 4\beta\eta^2$, is less than one and thus corresponds to the non explosive solution of the system (3.18). The equation $\pi_1(\cdot, \chi_1^*) = 0$ has the unique solution

$$\chi_0^* = \frac{(1-\alpha)\rho_a}{1-\alpha+\eta(1+\beta(1-\chi_1^*-\rho_a))}.$$

The parameters of equation (3.20) are equal to χ_0^* and χ_1^*.

A.4. Proof of Proposition 5

From proposition 3 $k_h > k_l$, thus a solution $x \in [0,1]$ to (4.6) exists only if

$$k_h > e > k_l. \tag{A.15}$$

We first consider the case of $n_{cc} > 1$, or $e < \underline{e}$, and then of $n_{cc} < 1$.

If $n_{cc} > 1$, then $g(k) < \left(\frac{Y}{r^\sigma}\right)\ (R_1+R_2)^{\sigma-1}\ \forall k \leq 1$ thereby $k_h = 1$ and (4.5) holds only if $g(1) < 1$. By substitution of $\frac{Y}{r^\sigma}$ from (4.3), the latter condition may be rewritten as:

$$f(k_l) > f(1). \tag{A.16}$$

Since $e < 1 < n_{cc}$, k_l satisfies (4.3), (4.4) and (A.15) if:

$$k_l < \min\{e, \hat{n}\}. \tag{A.17}$$

The function $f(\cdot)$, defined in proposition 3, is increasing up to $\hat{n}$, $f(0) = 0$ and $f(1) > 0$. Thus the set of k_l's that solves (A.16) and (A.17) is non-empty only if $\hat{n} < 1$, or $\underline{R}_2 < \hat{R}_2 = \frac{R_2 \lambda(\sigma-1)}{1+\lambda(\sigma-1)}$, and $e > e_1$ where e_1 is the unique solution to $f(e_1) = f(1)$ such that $e_1 < \hat{n}$. Note that when $\underline{R}_2 = \hat{R}_2$ then $e_1 = \hat{n} > \underline{e}$, so $\underline{R}_2 < \underline{R}_2^*$ always holds when

$$e \in (e_1, \underline{e}). \tag{A.18}$$

Now let $n_{cc} < 1$ so that $\underline{n}_{cc} = n_{cc}$.Substituting $\frac{Y}{r^\sigma}$ from (4.3) into the definition of k_h:

$$k_h = \min\left\{\frac{n_{cc}\ f(k_l)}{f(n_{cc})}, 1\right\}. \tag{A.19}$$

The inequality in (4.5) holds if and only[1] $k_h > n_{cc}$, thus k_h satisfies (4.5) and (A.15) when:

$$k_h > \max\{e, n_{cc}\} \tag{A.20}$$

Further since $k_l < n_{cc}$, for $k_h > n_{cc}$ to hold it must always be the case that $\hat{n} < n_{cc}$, or $e < e^* \equiv \frac{(R_1+R_2)(\sigma-1)-\sigma \underline{R}_2}{(\sigma-1)(r_1+R_2)}$ If the latter condition were not satisfied, $k_l < n_{cc}$ would yield from (A.19) that $\frac{n_{cc}\ f(k_l)}{f(n_{cc})} < n_{cc} < 1$ contradicting[2] (4.5). Thereby (4.3), (4.4) and (A.15) hold if (A.17) is satisfied.

Thus when $e > \underline{e}$, the asymmetric equilibrium exists if there are a k_l's that satisfy (A.17), (A.19) and (A.20), or equivalently if the set $A \equiv \left\{k_l : \frac{n_{cc}\ f(k_l)}{f(n_{cc})} > \max\{e, n_{cc}\} \text{ and } k_l < \min\{e, \hat{n}\}\right\}$ is non-empty, since $\max\{e, n_{cc}\} < 1$. When $e < \hat{n}$, A is non-empty if $f(e) > f(n_{cc})$ and when $e > \hat{n}$, only if $e < e_3$, where e_3 is the unique solution to $e_3\ (1 - e_3)^{-\sigma} \equiv$

[1]From $n_{cc} < 1$ (4.5) is always satisfied when $k_h = 1$, while it holds only if $k_h > n_{cc}$ when $k_h < 1$.
[2]Tecall that $f(t)$ is increasing for $t < \hat{n}$.

$f(\hat{n})\ (R_1+R_2)^{\sigma}$. Hence A is non-empty if either

(A.21) $$e \in \left(\max\left\{\hat{n}, \underline{e}\right\}, \min\left\{e_3, e^*\right\}\right)$$

or

(A.22) $$e \in \left(\underline{e}, \min\left\{e^*, \hat{n}\right\}\right) \text{ and } f(e) > f(n_{cc}).$$

The value[3] of $\hat{n}$ is smaller than e^* only if $\underline{R}_2 < R_2^{**}$ and over the same range[4] $e_3 < e^*$, thus (A.21) may be rewritten as:

(A.23) $$e \in \left(\left(\max\left\{\hat{n}, \underline{e}\right\}, e_3\right).$$

Thus the asymmetric equilibrium exists if either (A.18) or (A.22) or (A.23) is satisfied. To see that $x \in (0,1)$ when $e \in (\hat{n}, e^{**})$, note that if on the contrary $x = 1$, then $k_h = e < n_{cc}$ from $e < e^{**}$ which contradicts (A.20) while if $x = 0$ then $k_l = e > \hat{n}$ which contradicts (A.17).

[3]The function $o(\underline{R}_2) = \hat{n} - e^*$ is decreasing, $o(0) = 1$ and $o(R_2^{**}) = 0$.

[4]From the definition of e_3, $e_3 < e^*$ is equivalent to $f(\hat{n})\ (R_1+R_2)^{-\sigma} < e^*(1-e^*)^{-\sigma}$, or $f(\hat{n}) < \frac{e^*}{\hat{n}} f(\hat{n})$. This last condition always holds for $\underline{R}_2 < \underline{R}_2^{**}$.