Prior densities in DYNARE

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source: L. Bauwens, M. Lubrano, J.-F. Richard (1999) *Bayesian inference in dynamic econometric models*. Oxford University Press.

1 Gamma distribution

X follows a gamma distribution $G(\alpha, \beta)$ and

$$g(x|\alpha,\beta) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^{\alpha}}$$

$$E(X) = \mu = \alpha\beta \qquad\qquad Var(X) = \sigma^2 = \alpha\beta^2$$

and DYNARE computes α and β from

$$\beta = \frac{\sigma^2}{\mu} \qquad\qquad \alpha = \frac{\mu}{\beta}$$

If $Y = \frac{X}{\beta}$ then Y follows a distribution $G(\alpha, 1)$. This transformation is used to compute the gamma inverse distribution function with qgamma.m that uses only one parameter

2 Inverse gamma distribution of type II

$X \sim IG_2(s,\nu) \Leftrightarrow Z = X^{-1} \sim G(\frac{\nu}{2}, \frac{2}{s})$. The density is

$$ig_2(x|s,\nu) = \frac{x^{-\frac{1}{2}(\nu+2)}e^{-\frac{s}{2x}}}{\Gamma\left(\frac{\nu}{2}\right)\left(\frac{2}{s}\right)^{\frac{\nu}{2}}}$$

and

$$E(X) = \mu = \frac{s}{\nu-2} \qquad\qquad Var(X) = \sigma^2 = \frac{2}{\nu-4}\mu^2$$

Dynare computes s and ν from

$$\nu = 2\frac{\mu^2}{\sigma^2} + 4 \qquad\qquad s = \mu(\nu-2)$$

3 Inverse gamma distribution of type I

$Y \sim IG_1(s,\nu) \Leftrightarrow Y = \sqrt{X} \Leftrightarrow X \sim IG_2(s,\nu) \Leftrightarrow Z = Y^{-2} \sim G(\frac{\nu}{2}, \frac{2}{s})$. The density is

$$\begin{aligned} ig_1(y|s,\nu) &= 2\frac{y^{-(\nu+1)}e^{-\frac{s}{2y^2}}}{\Gamma\left(\frac{\nu}{2}\right)\left(\frac{2}{s}\right)^{\frac{\nu}{2}}} \\ \ln ig_1(y|s,\nu) &= \ln 2 - (\nu+1)\ln y - \frac{s}{2y^2} - \ln\Gamma\left(\frac{\nu}{2}\right) - \frac{\nu}{2}\left(\ln 2 - \ln s\right) \end{aligned}$$

and

$$E(Y) = \mu = \sqrt{\frac{s}{2}}\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \qquad Var(Y) = \sigma^2 = \frac{s}{\nu-2} - \mu^2$$

Dynare solves for ν in

$$2\mu^2\Gamma\left(\frac{\nu}{2}\right)^2 = (\nu-2)\left(\sigma^2+\mu^2\right)\Gamma\left(\frac{\nu-1}{2}\right)^2,$$

then computes

$$s = (\nu-2)\left(\sigma^2+\mu^2\right)$$