The household's Lagrangian problem is:

$$E_t^j \sum_{l=0}^{\infty} \beta^l \{ u(c_{t+l} - bc_{t+l-1}) - \psi_L \frac{l_t^{1+\sigma_L}}{1+\sigma_L} - \upsilon \frac{\left(\frac{P_{t+l}c_{t+l}}{M_{t+l+1}^d}\right)^{1-\sigma_q}}{1-\sigma_q}$$
$$+\lambda_t \left[W_t l_{t,j} + P_t r_t^k k_t + (1+R_t) T_t - \left(P_t (c_t + i_t) + M_{t+1}^d - M_t^d + T_{t+1}\right)\right]$$
$$+\mu_t \left[(1-\delta) k_t + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t - k_{t+1} \right] \}$$

Note that we have introduced money in the utility function, and investment adjustment costs habit persistence in consumption have also been introduced.

The first order conditions for consumption is:

$$P_t \lambda_t = u'(c_t - bc_{t-1}) - \beta b u'(c_{t+1} - bc_t) - \upsilon \left(\frac{P_t}{M_t^d}\right)^{1-\sigma_q} c_t^{-\sigma_q},$$

or,

$$\lambda_{z,t} = \frac{1}{c_t - bc_{t-1}} - \beta b \frac{1}{c_{t+1} - bc_t} - \upsilon \left(\frac{\pi_t}{m_t}\right)^{1-\sigma_q} c_t^{-\sigma_q},$$

where $m_t = M_t / P_{t-1}$.

The first order condition for l_t is:

$$\psi_L l_t^{\sigma_L} = \lambda_t W_t = \lambda_{z,t} w_t,$$

where $w_t = W_t / P_t$.

The first order condition for T_{t+1} is:

$$\lambda_t = \beta \lambda_{t+1} (1 + R_t),$$

or,

$$\lambda_{z,t} = \beta \lambda_{z,t+1} \frac{1+R_t}{\pi_{t+1}}, \ \ \pi_{t+1} = \frac{P_{t+1}}{P_t}.$$

The first order condition for i_t is:

$$\lambda_t P_t = \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + \beta \mu_{t+1} S'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2.$$

Define

$$P_{k',t} = \frac{\mu_t}{\lambda_t P_t},$$

so that

$$1 = P_{k',t} \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} P_{k',t+1} S'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2,$$

The first order condition M_{t+1}^d :

$$\lambda_t = \beta \lambda_{t+1} + \beta \upsilon \left(P_{t+} c_{t+1}\right)^{1-\sigma_q} \left(M_{t+1}^d\right)^{\sigma_q - 2},$$

or, after multiplying by P_t :

$$\lambda_{z,t} = \beta \frac{\lambda_{z,t+1}}{\pi_{t+1}} + \pi_{t+1}^{1-\sigma_q} \beta \upsilon c_{t+1}^{1-\sigma_q} m_{t+1}^{\sigma_q - 2}.$$

The first order condition associated with k_{t+1} is:

$$\mu_t = \beta \left[\lambda_{t+1} P_{t+1} r_{t+1}^k + \mu_{t+1} (1 - \delta) \right],$$

or, after dividing by $\lambda_t P_t$:

$$P_{k',t} = \frac{\lambda_{z,t+1}}{\lambda_{z,t}} \beta \left[r_{t+1}^k + P_{k',t+1} (1 - \delta) \right].$$

The monetary policy rule is:

$$R_t = \rho_i R_{t-1} + (1 - \rho_i) \left[\frac{\bar{\pi}}{\beta} - 1 + \alpha_\pi \left[E_t (\pi_{t+1}) - \bar{\pi} \right] + \alpha_y \log \left(\frac{c_t + i_t}{Y^+} \right) \right] + \varepsilon_t.$$

Putting all the equations together, and including the equations related to sticky prices:

$$
\begin{aligned}
(1)\frac{1}{c_t - bc_{t-1}} - \beta b \frac{1}{c_{t+1} - bc_t} - \upsilon \left(\frac{\pi_t}{m_t}\right)^{1-\sigma_q} c_t^{-\sigma_q} &= \lambda_{z,t} \\
(2) P_{k',t}\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} P_{k',t+1} S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2 &= 1 \\
(3)\frac{\lambda_{z,t+1}}{\lambda_{z,t}}\beta\left[r_{t+1}^k + P_{k',t+1}(1-\delta)\right] &= P_{k',t} \\
(4)\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}} + \upsilon\beta\left(\pi_{t+1}c_{t+1}\right)^{1-\sigma_q} m_{t+1}^{\sigma_q - 2} &= \lambda_{z,t} \\
(5)\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}}(1+R_t) &= \lambda_{z,t} \\
(6)\,(1-\delta)k_t + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t &= k_{t+1} \\
(7)\ \ (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}}\left[\epsilon_t (K_t)^\alpha l_t^{1-\alpha} - \Phi\right] &= c_t + i_t \\
(8)\ \lambda_{z,t} s_t \epsilon_t (1-\alpha)\left(\frac{k_t}{l_t}\right)^\alpha &= \psi_L l_t^{\sigma_L} \\
(9)\ \ s_t \epsilon_t \alpha \left(\frac{l_t}{k_t}\right)^{1-\alpha} &= r_t^k \\
(10)\quad \left[(1-\xi_p)\left(\frac{1 - \xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_t}{\pi_t} p_{t-1}^*\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}} &= p_t^* \\
(11)\ E_t\left[\lambda_{z,t} Y_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} \beta\xi_p F_{p,t+1} - F_{p,t}\right] &= 0 \\
(12)\ E_t\left[\lambda_f \lambda_{z,t} Y_t s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t}\right] &= 0 \\
(13)\ \frac{1 - \xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{(1-\xi_p)} &= \left(\frac{K_{p,t}}{F_{p,t}}\right)^{\frac{1}{1-\lambda_f}} \\
(14)\ \rho_i R_{t-1} + (1-\rho_i)\left[\frac{\bar{\pi}}{\beta} - 1 + \alpha_\pi\left[E_t(\pi_{t+1}) - \bar{\pi}\right] + \alpha_y \log\left(\frac{c_t + i_t}{Y^+}\right)\right] + \varepsilon_t &= R_t
\end{aligned}
$$

The 14 variables to be solved using these 14 equations are $\lambda_{z,t}$, c_t, m_t, π_t, i_t, $P_{k',t}$, r_t^k, R_t, k_t, p_t^*, l_t, s_t, $F_{p,t}$, $K_{p,t}$.

4.2. Steady State

The equations associated with Calvo sticky prices imply:

$$\begin{aligned} s &= \frac{1}{\lambda_f}, \\ F_p &= \frac{\lambda_z Y}{1-\beta\xi_p} \end{aligned}$$

This is as expected. When there are no price distortions in the steady state, then the firm markup is $1/s = \lambda_f$ in the present case of a constant elasticity demand curve. The resource constraint is

$$c + \delta k = k^{\alpha} l^{1-\alpha} - \Phi.$$

We use the supposition that profits are zero in the steady state to determine a value for Φ. If we think of the Cobb-Douglas part of the production function as the firm's 'production function', then with fixed costs, Φ, the firm which sells Y_{jt} must actually produce $Y_{jt} + \Phi$. Given fixed marginal costs, the firms total cost associated with selling $Y_{jt} + \Phi$ is $s_t\left(Y_{jt} + \Phi\right)$, in units of the final good (i.e., scaling by P_t). The firms' revenues are $P_{jt}Y_{jt}$, so its profits in units of currency are

$$\frac{P_{jt}}{P_t} Y_{jt} - s_t\left(Y_{jt} + \Phi\right).$$

In steady state, $P_{jt} = P_t$, $Y_{jt} = Y_t$ for all j because our assumptions guarantee that prices and resources are not distorted in a steady state. So, the zero profit condition in steady state is

$$Y = \frac{1}{\lambda_f}\left(Y + \Phi\right),$$

or,

$$\left(\lambda_f - 1\right) Y = \Phi.$$

In the steady state, profits are $100\left(\lambda_f - 1\right)$ of output sold, and fixed costs must be equal to this amount if profits are to be zero. Substituting in the production function,

$$\left(\lambda_f - 1\right)\left(k^{\alpha} l^{1-\alpha} - \Phi\right) = \Phi,$$

so that

$$\left(\lambda_f - 1\right) k^{\alpha} l^{1-\alpha} = \lambda_f \Phi$$

Combining this with the resource constraint, we obtain:

$$c + \delta k = k^{\alpha} l^{1-\alpha} - \frac{\lambda_f - 1}{\lambda_f} k^{\alpha} l^{1-\alpha} = \frac{1}{\lambda_f} k^{\alpha} l^{1-\alpha}.$$

Collecting our results, we have that the steady state equations are:

$$
\begin{aligned}
P_{k',t} &= 1, \\
\lambda_z &= (1-\beta b)\, u'(c-bc) - \upsilon \left(\frac{\pi}{m}\right)^{1-\sigma_q} c^{-\sigma_q} \\
\frac{1}{\beta} &= r^k + 1 - \delta \\
\frac{\pi}{\beta} &= 1 + R \\
\lambda_z &= \beta \frac{\lambda_z}{\pi} + \upsilon\beta \left(\frac{\pi}{m}\right)^{1-\sigma_q} c^{-\sigma_q} \left(\frac{c}{m}\right) \\
\frac{\psi_L l^{\sigma_L}}{\lambda_z} &= w \\
c + \delta k &= \frac{1}{\lambda_f} k^{\alpha} l^{1-\alpha} \\
s &= \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r^k\right)^{\alpha} w^{1-\alpha} \\
s &= \frac{r^k}{\alpha \left(\frac{l}{k}\right)^{1-\alpha}} \\
s &= \frac{1}{\lambda_f},
\end{aligned}
$$

which represents 10 equations in 11 unknowns:

$$
c,\ s,\ l,\ \lambda_z,\ w,\ r^k,\ \pi,\ R,\ m,\ k,\ P_{k'}.
$$

Another equation is provided by the assumption that transfers are made to the household to provide them with money:

$$
M_{t+1} - M_t = X_t,
$$

and dividing this by M_t :

$$
\frac{M_{t+1} - M_t}{M_t} = x_t,
$$

so that in steady state,

$$
\frac{\pi m - m}{m} = \pi - 1 = x.
$$

We will just treat π as an exogenous variable, with the understanding that it is actually x that is exogenous. So, by deleting π from the list of 11 unknowns, we have 10 equations in 10 unknowns.

We solve these equations as follows. The variables, R, s, $P_{k'}$, r^k, l/k, c/k and w are virtually immediate. Thus, $P_{k'} = 1$, $s = 1/\lambda_f$, $R = \pi/\beta - 1$ and

$$
r^k = \frac{1}{\beta} - (1-\delta),
$$

and

$$r^k = \frac{1}{\lambda_f}\alpha\left(\frac{l}{k}\right)^{1-\alpha},$$

so that

$$l_k \equiv \frac{l}{k} = \left(\frac{r^k\lambda_f}{\alpha}\right)^{\frac{1}{1-\alpha}}.$$

The resource constraint can be written:

$$c_k \equiv \frac{c}{k} = \frac{1}{\lambda_f}l_k^{1-\alpha} - \delta,$$

and the wage rate can be solved using the fact that r^k is known and

$$w = \left[\frac{1}{\lambda_f\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(r^k\right)^{\alpha}}\right]^{\frac{1}{1-\alpha}}.$$

We still require k, λ_z, m. The equations that remain available to us are the following three:

$$\begin{aligned}
\lambda_z &= (1-\beta b)\frac{1}{c_k k(1-b)} - \upsilon\left(\frac{\pi}{m}\right)^{1-\sigma_q}(c_k k)^{-\sigma_q} \\
\lambda_z &= \beta\frac{\lambda_z}{\pi} + \upsilon\beta\left(\frac{\pi}{m}\right)^{1-\sigma_q}(c_k k)^{-\sigma_q}\frac{c_k k}{m} \\
\frac{\psi_L (l_k k)^{\sigma_L}}{\lambda_z} &= w
\end{aligned}$$

which now reduces to two equations in two unknowns, k and m. Multiply the first equation by $c_k k$ and use the expression for λ_z from the household's first order condition for labor:

$$c_k k\frac{\psi_L (l_k k)^{\sigma_L}}{w} = \frac{1-\beta b}{(1-b)} - \upsilon\left(\frac{\pi}{m}\right)^{1-\sigma_q}(c_k k)^{1-\sigma_q}$$

SSubstitute from the labor euler equation into the second of the previous two equations:

$$\frac{\psi_L (l_k k)^{\sigma_L}}{w}\left[1-\frac{\beta}{\pi}\right] = \upsilon\beta\left(\frac{\pi}{m}\right)^{1-\sigma_q}\frac{(c_k k)^{1-\sigma_q}}{m}.$$

or,

$$\begin{aligned}
a_0 k^{1+\sigma_L} + \upsilon a_1\left(\frac{k}{m}\right)^{1-\sigma_q} &= a_2 \\
b_0 k^{1+\sigma_L} &= \upsilon b_1\left(\frac{k}{m}\right)^{2-\sigma_q}.
\end{aligned}$$

for known constants, a_0, a_1, a_2, b_0, b_1 and the parameter, υ :

$$\begin{aligned}
a_0 &= c_k\frac{\psi_L {l_k}^{\sigma_L}}{w},\ a_1 = \pi^{1-\sigma_q}{c_k}^{1-\sigma_q},\ a_2 = \frac{1-\beta b}{1-b}, \\
b_0 &= \frac{\psi_L {l_k}^{\sigma_L}}{w}\left[1-\frac{\beta}{\pi}\right],\ b_1 = \beta\pi^{1-\sigma_q}{c_k}^{1-\sigma_q}.
\end{aligned}$$

From the second expression

$$\frac{m}{k} = \left[\frac{\upsilon b_1}{b_0 k^{1+\sigma_L}}\right]^{\frac{1}{2-\sigma_q}},$$

so that

$$k^{1+\sigma_L} + \upsilon^{\frac{1}{2-\sigma_q}} \frac{a_1}{a_0} \left[\frac{b_0}{b_1} k^{1+\sigma_L}\right]^{\frac{1-\sigma_q}{2-\sigma_q}} = \frac{a_2}{a_0} = \frac{w\frac{1-\beta b}{1-b}}{c_k \psi_L l_k^{\sigma_L}} > 0.$$

Note that the function on the left of the equality is zero at $k = 0$ and strictly increasing and convex. As a result, there is a unique value of k that solves this equation. In the case, $\upsilon = 0$, this value can be found analytically:

$$k = \left(\frac{w\frac{1-\beta b}{1-b}}{c_k \psi_L l_k^{\sigma_L}}\right)^{\frac{1}{1+\sigma_L}}.$$

With k in hand, k/m may be found from the previous expression. Note that when $\upsilon = 0$ then $k/m = \infty$ so that $m = 0$. We would expect that the price level would be infinite when $\upsilon = 0$ because in this case there is no use for money.

4.3. Numerical Analysis of the Model

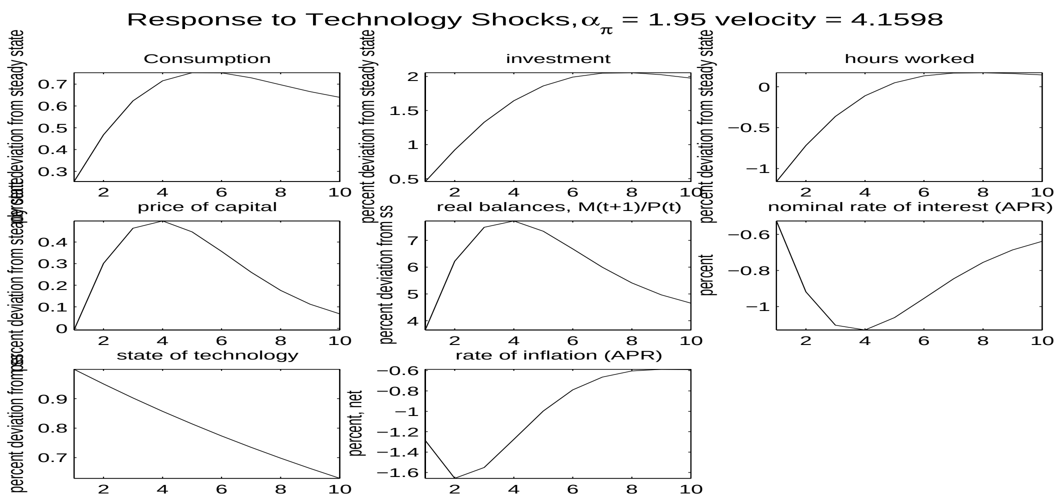
We considered the following parameter values.

$$\beta = 1.03^{-0.25},\ b = 0.63,\ \alpha = 0.36,\ \delta = 0.025,\ \rho = 0.95,$$

and

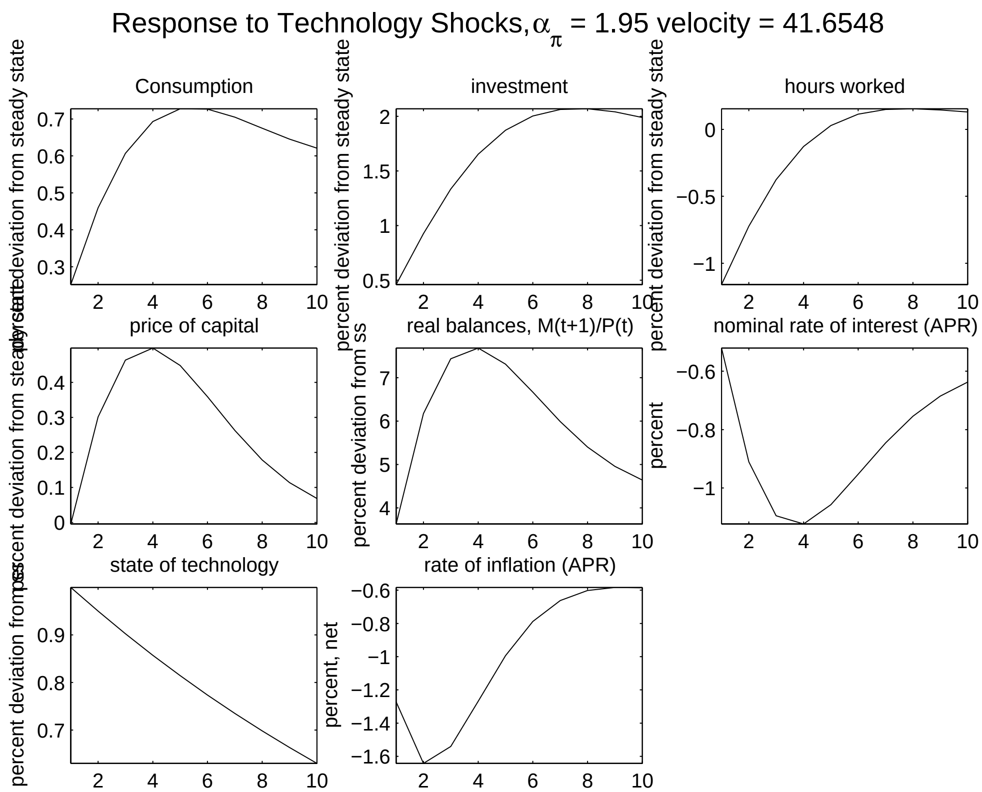
$$\begin{aligned} \lambda_f &= 1.20,\ \xi_p = 0.75,\ \iota = 0.84,\ \bar{\pi} = 1 + 0.025/4,\ \upsilon = 0.0005, \\ \rho_i &= 0.81,\ \alpha_\pi = 1.95,\ \alpha_y = 0.18,\ \sigma_L = 1,\ \sigma_q = -1,\ a = 5. \end{aligned}$$

With this parameterization, we have $l = 0.996$, $py/m = 4.16$, $k/y = 11.11$. The impulse response function to



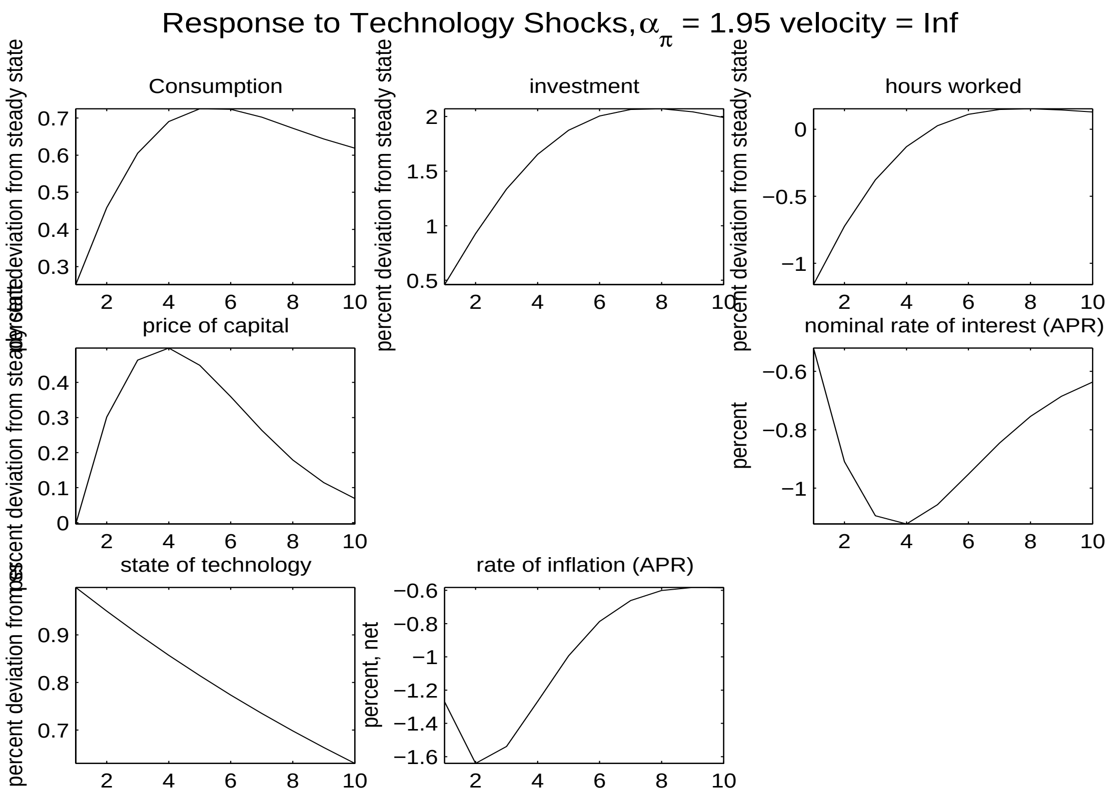
Next, we consider the effect of reducing the importance of money, by reducing v to

0.0000005. The resulting impulse responses are as follows:



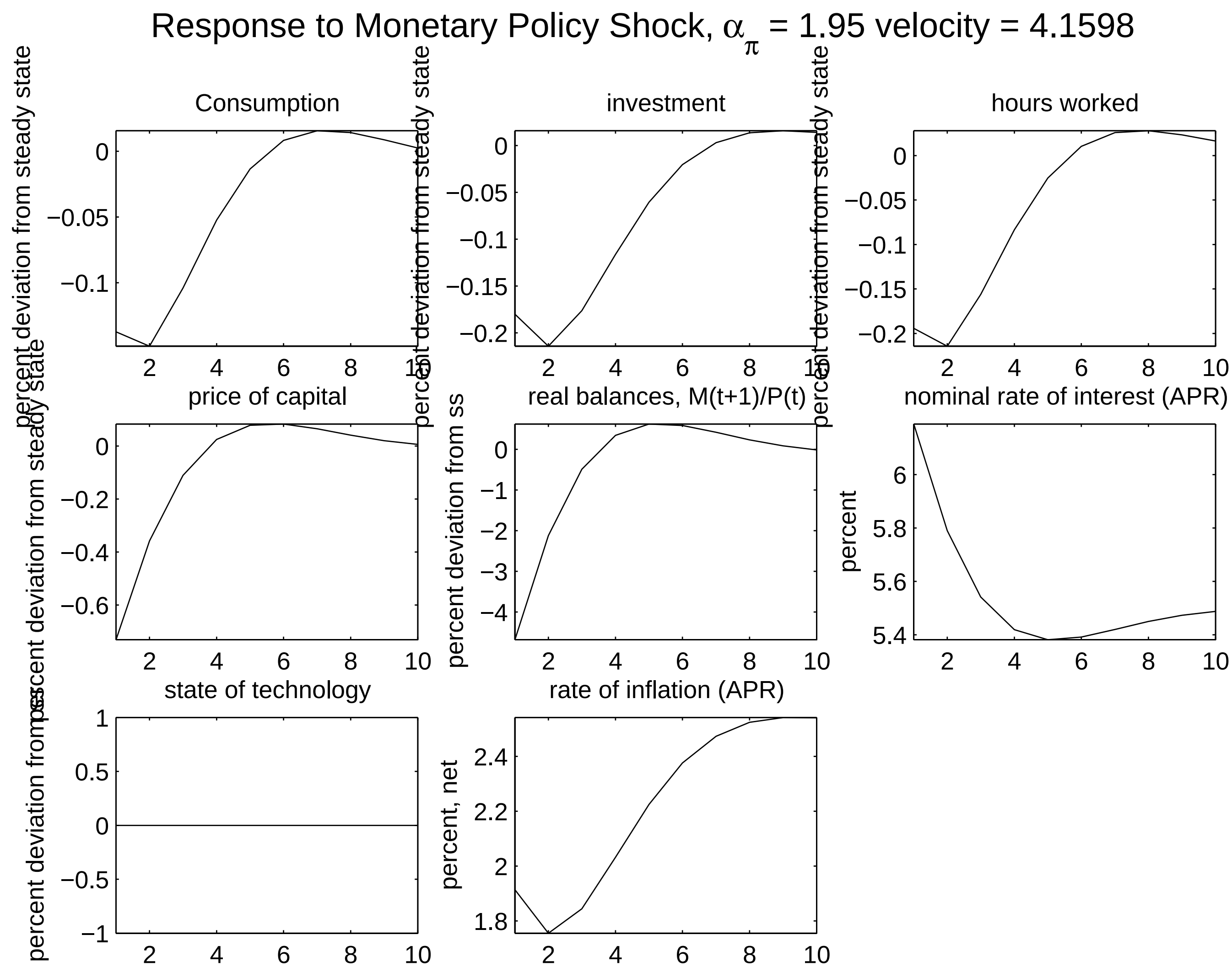
Note that with this ten-fold increase in velocity, the impulse responses have hardly change.

Next, we set $v = 0$. The steady state algorithm described above works for this case, and produces $m = 0$. In the dynamic equations, we set $v = 0$ in (1) and drop equation (4) and the variable, m_t. With $\alpha_\pi = 1.95$, the following impulse response function was obtained:



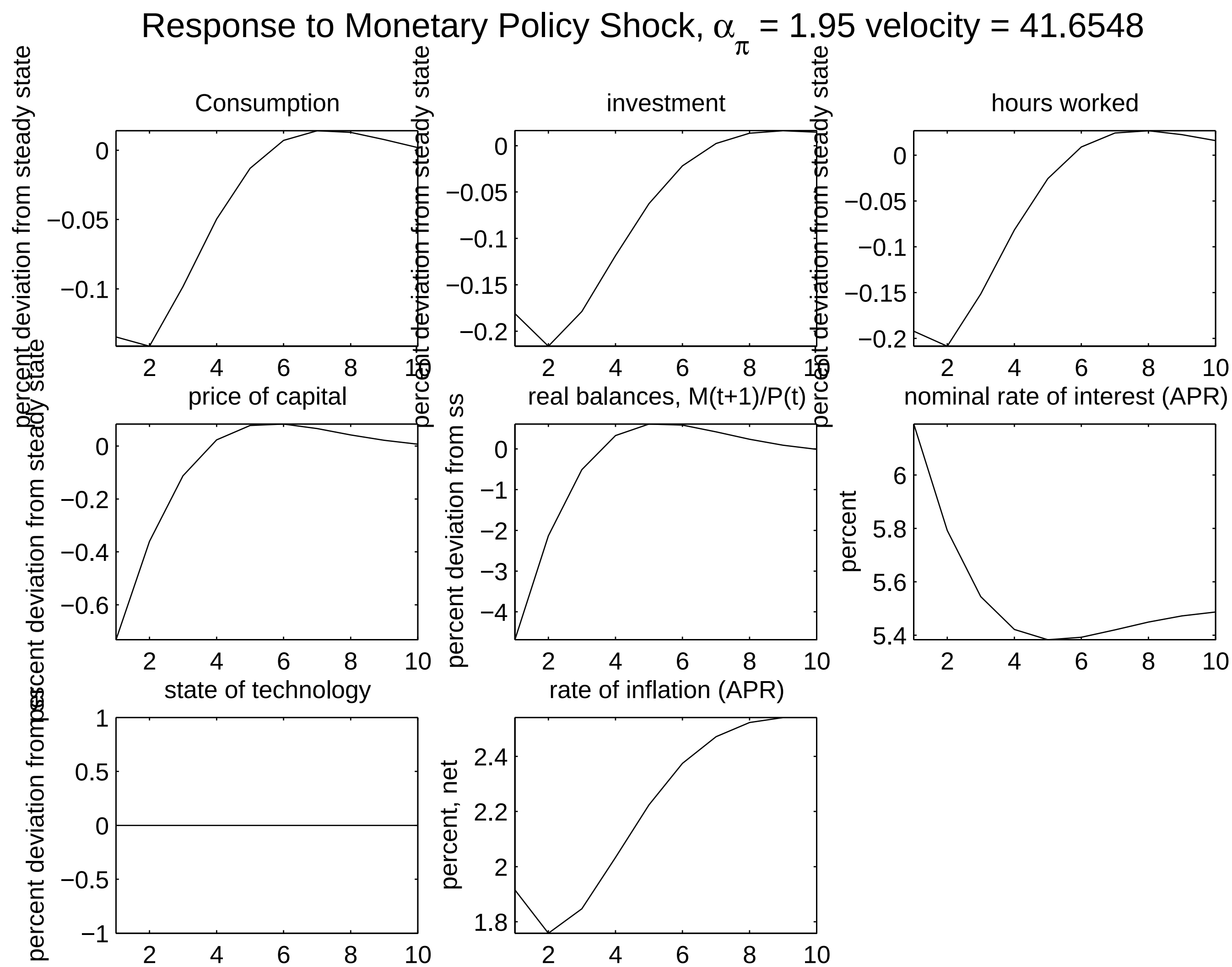
Note that the results are essentially the same. In addition, the range of determinacy is unchanged from before. From this we have to conclude that there must be a mistake somewhere because this model is essentially the standard model with no money.

Next, we consider the response of the system to a monetary policy shock. Here is the response to a 0.0025 shock to monetary policy. This corresponds to a 100 basis point rise in the interest rate, at an annual rate.



Note that to get the interest rate up, they need to reduce the money supply. Also, the interest rate does not rise by the full 100 basis points immediately, because the fall in prospective inflation exerts a countervailing force on the interest rate.

Now v is reduced, and the impulse response function becomes:



This result is the same as before.

Evidently, the size of v does not matter for the responses of the model economy to monetary policy and technology shocks.