The Labor Market in the New Keynesian Model: Foundations of the Sticky Wage Approach and a Critical Commentary

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November 10, 2013

- Baseline NK model with no capital and with a competitive labor market.
  - private sector equilibrium conditions
  - Details: http://faculty.wcas.northwestern.edu/~lchrist/d16/d1613/Labor_market_handout.pdf
- Standard Labor Market Friction: Erceg-Henderson-Levin sticky wages.
  - we will consider an interpretation of EHL proposed by Gali, which deduces implications for unemployment.

New Keynesian Model with Competitive Labor Market: Households

- Problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \ \tau_t = \lambda \tau_{t-1} + \varepsilon_t^{\tau}$$

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t$$

- First order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5)$$

$$\exp(\tau_t) C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

New Keynesian Model with Competitive Labor Market: Goods

- Final good firms:
  - maximize profits:

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

  - Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \to \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

New Keynesian Model with Competitive Labor Market: Goods

- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$Y_{i,t} = \exp\left(a_t\right) N_{i,t},\ a_t = \rho a_{t-1} + \varepsilon_t^a$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

- Real marginal cost:

$$s_t = \frac{\frac{d\text{Cost}}{d\text{worker}}}{\frac{d\text{output}}{d\text{worker}}} = \frac{\overbrace{(1-\nu)}^{\text{minimize monopoly distortion by setting } = \frac{\varepsilon-1}{\varepsilon}} \frac{W_t}{P_t}}{\exp\left(a_t\right)}$$

Optimal Price Setting

- Let

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t},\ \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}}.$$

- Optimal price setting:

$$\tilde{p}_t = \frac{K_t}{F_t},$$

where

$$\begin{aligned} K_t &= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} (1) \\ F_t &= 1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}. (2) \end{aligned}$$

- Note:

$$\begin{aligned} K_t &= \frac{\varepsilon}{\varepsilon-1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+1} \\ &\quad + (\beta\theta)^2 E_t \bar{\pi}_{t+2}^{\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+2} + \ldots \end{aligned}$$

Goods and Price Equilibrium Conditions

- Cross-price restrictions imply, given the Calvo price-stickiness:

$$P_t = \left[(1-\theta)\, \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.$$

- Dividing latter by P_t and solving:

$$\tilde{p}_t = \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \rightarrow \frac{K_t}{F_t} = \left[\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3)$$

- Relationship between aggregate output and aggregate inputs:

$$C_t = p_t^* A_t N_t, \ (6)$$

$$\text{where } p_t^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad (4)$$

Linearizing around Efficient Steady State

- In steady state (assuming $\bar{\pi} = 1$, $1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$)

$$p^* = 1,\ K = F = \frac{1}{1 - \beta\theta},\ s = \frac{\varepsilon - 1}{\varepsilon},\ \Delta a = \tau = 0,\ N = 1$$

- Marginal cost:

$$\begin{aligned} s_t &= (1 - \nu)\frac{\bar{w}_t}{A_t}, \quad \overbrace{\bar{w}_t = \exp(\tau_t) N_t^{\varphi} C_t}^{\text{assumed competitive labor market}} \\ &\to \hat{\bar{w}}_t = \tau_t + a_t + (1 + \varphi)\hat{N}_t \end{aligned}$$

- Then,

$$\boxed{\hat{s}_t = \hat{\bar{w}}_t - a_t = (\varphi + 1)\left[\overbrace{\frac{\tau_t}{\varphi + 1} + \hat{N}_t}^{\text{output gap}}\right] = (\varphi + 1)\, x_t}$$

The Linearized Private Sector Equilibrium Conditions of the Competitive Labor Market Model

$x_t = x_{t+1} - \left[r_t - \pi_{t+1} - r_t^*\right]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$
$\hat{s}_t = (\varphi + 1)\, x_t$
$r_t^* = -\log(\beta) + E_t\left[a_{t+1} - a_t - \frac{\tau_{t+1}-\tau_t}{1+\varphi}\right]$

Reasons to consider frictions in the labor market:

- Play an essential role in accounting for response to a monetary policy shock.
  - With flexible wages, wage costs rise too fast in the wake of expansionary monetary policy shock.
  - High costs limit firms' incentive to expand employment.
  - High costs imply sharp rise in inflation.
  - But, the data suggest that after an expansionary monetary policy shock inflation hardly rises and output rises a lot!
  - Wage frictions play essential role in making possible an account of monetary non-neutrality (see CEE, 2005JPE).
- Important for understanding employment response to other shocks too.
- Introducing sticky wages and monopoly power in labor market provides a theory of unemployment.

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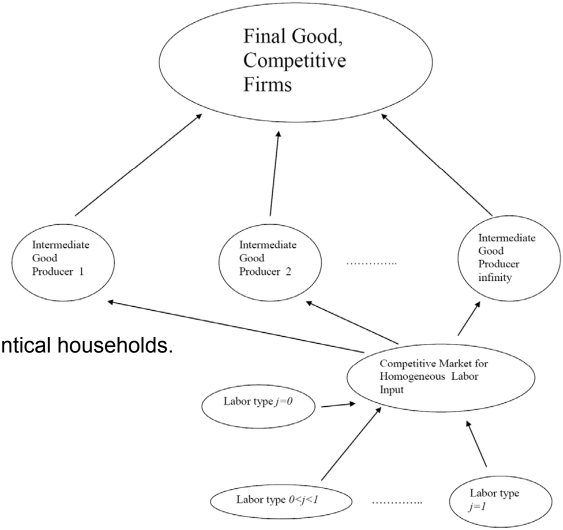
Sticky Wages

- Basic model is due to Erceg-Henderson-Levin.
  - We will follow the interpretation of EHL suggested by Gali, so that we have a theory of unemployment (see also Gali-Smets-Wouters).
- Worker heterogeneity is required:
  - Must have differences between workers if we're to have some unemployed and others employed.
  - Worker heterogeneity potentially introduces complications, which we will avoid through the (somewhat artificial) assumption that workers live in 'large families'.

Outline

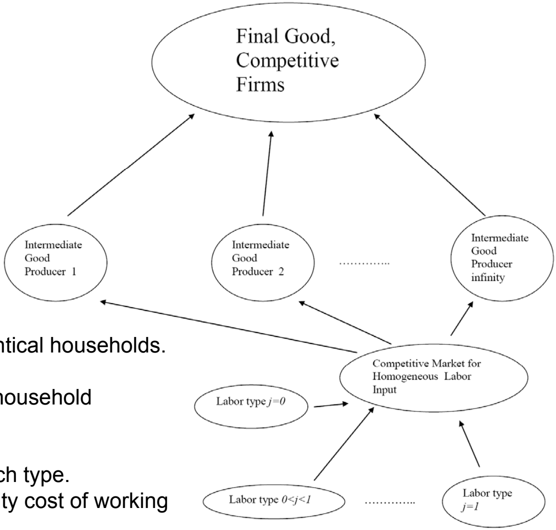
- Provide a broad sketch of the model.
- Discuss the equations of the model.
- Provide a critical assessment of the model.

Model



There are many identical households.

Model



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The representative household contains all workers:

Many workers of each type.
Differentiated by utility cost of working

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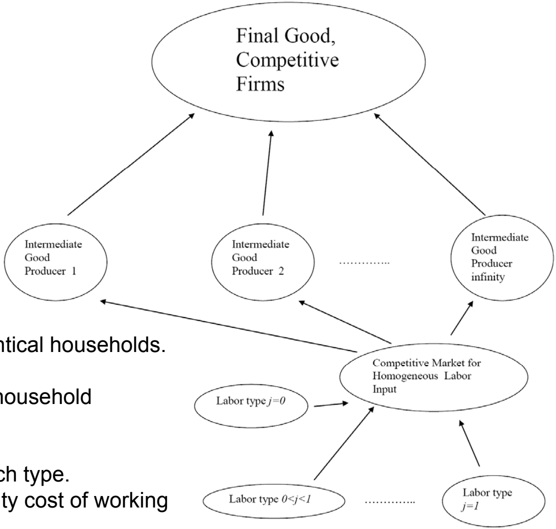

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The representative household contains all workers:

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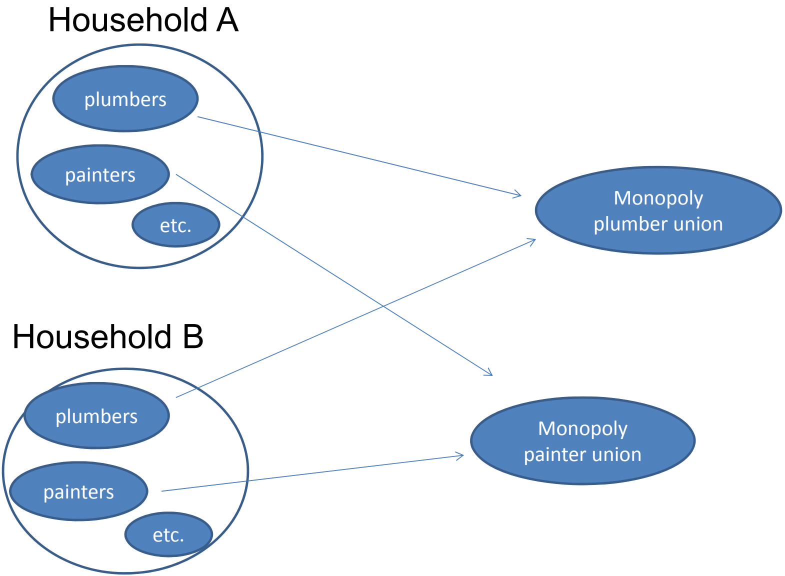
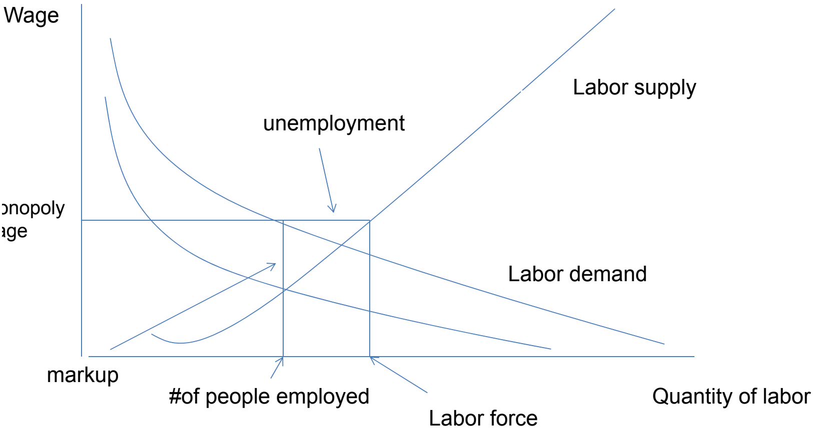
Every worker enjoys the same level of consumption.


Household A
plumbers
painters
etc.
Monopoly
plumber union
Household B
plumbers
painters
etc.
Monopoly
painter union

Type *j* Monopoly Union



The Equations of the Sticky Wage Model: Outline

- Describe relationship of workers and the households they live in.
- The source of heterogeneity that causes workers to experience different outcomes in the labor market.
  - definition of employment, unemployment, labor force.
- Household and worker utility functions.
- The nature of the labor market.
  - Driven by monopoly unions
  - Investigate the empirical implications of a theory of unemployment with monopoly unions.
- Explain Calvo-style wage stickiness.
- Explore the implications for unemployment of the model.
  - Some issues that come up.

Households and their Workers

- Economy has many identical households.

- Each household has a large number of workers.
  - All workers receive the same level of consumption, C_t, in exchange for obeying the rules.
  - Each worker is represented by a point in a unit-square box.
  - The vertical dimension of the box corresponds to $j \in (0,1)$ and j indexes the type of labor the worker does.
  - The horizontal dimension corresponds to $l \in (0,1)$ and l indexes the worker's degree of aversion to work.
    - Work aversion, l, is uniformly distributed among the workers, for each j.
    - That is, for each j, the 'number' of workers with work aversion, l, $f(l)$, has the property, $f(l) = 1$ for all $l \in (0,1)$.

Worker Utility

- For each j,
  - the utility of a worker that is not employed:
$$\log\left(C_t\right).$$
  - the utility of a worker that has aversion to work l and is employed:
$$\log\left(C_t\right) - l^{\varphi},\ \varphi > 0.$$

House Rules

- Wage taken as given by household and worker.
  - $W_{t,j}$ is determined by a union (more on this later).
  - Household does this in exchange for benefits of monopoly power.
- Type j employed workers send the wage, $W_{t,j}$, home.
  - They do this in exchange for consumption insurance.
- Household must supply all labor demanded.
  - workers with least work aversion are employed.
  - If labor demand is $h_{t,j}$ then workers with $0 \leq l \leq h_{t,j}$ must go to work and those with $l \geq h_{t,j}$ stay at home.
  - Under this rule, the household satisfies labor demand, since:

$$\int_0^{h_{t,j}} \overbrace{f(l)}^{\text{'number' (density) of workers with work aversion, } l} dl = \int_0^{h_{t,j}} dl = h_{t,j}.$$

Household Utility

- Equally-weighted sum of utility of all household workers.
- Utility of a household that supplies $h_{t,j}$ labor, $j \in (0,1)$:

$$\int_0^1 \left\{ \overbrace{\int_0^{h_{t,j}} f(l) \left[\log(C_t) - l^{\varphi}\right] dl}^{\text{utility of employed workers}} + \overbrace{\int_{h_{t,j}}^1 f(l) \log(C_t)\, dl}^{\text{utility of non-employed workers}} \right\} dj$$

$$= \int_0^1 \left\{ h_{t,j} \log(C_t) - \int_0^{h_{t,j}} l^{\varphi} dl + \left(1 - h_{t,j}\right) \log(C_t) \right\} dj$$

$$= \int_0^1 \left\{ \log(C_t) - \frac{h_{t,j}^{1+\varphi}}{1+\varphi} \right\} dj$$

$$= \log(C_t) - \int_0^1 \frac{h_{t,j}^{1+\varphi}}{1+\varphi} dj$$

Marginal Cost and Marginal Benefit of Labor

- The 'marginal cost of type j labor' is the cost experienced by the last employed worker:

$$h_{t,j}^{\varphi}.$$

- The marginal benefit (to household) of labor:
  - The utility value to the household of the wage, $W_{t,j}$, brought home by the marginal worker:

    $$W_{t,j}v_t.$$

    v_t ~household marginal utility of one unit of currency (Lagrange multiplier on budget constraint).
  - *private* benefit to worker of wage is zero.
- Labor supply: graph of $h_{t,j}^{\varphi}/v_t$ against $h_{t,j}$ for fixed v_t.

Labor Supply

- In principle, 'labor supply' is an ambiguous concept in a dynamic model.
  - Response to a temporary or permanent wage change?
  - In practice, assume temporary wage change.
    - So temporary (maybe even more temporary than just one whole period!), that there is no change in v_t.
    - Called the *Frisch elasticity* (after Ragnar Frisch) of labor supply.
  - Alternative interpretation of Frisch labor supply elasticity: pure substitution effect part of the response of labor to a wage change.
- Frisch labor supply elasticity:

$$\frac{d\log h_{t,j}}{d\log\left(h_{t,j}^{\varphi}/v_t\right)}|_{dv_t=0} = \frac{1}{\varphi}$$

Labor Force and Unemployment

- Gali/Gali-Smets-Wouters definition of 'type j labor force', $h_{t,j}^{s}$:

$$\frac{\left(h_{t,j}^{s}\right)^{\varphi}}{\upsilon_{t}} = W_{t,j}.$$

- For workers with $l \leq h_{t,j}^{s}$ the cost of working is less than the benefit (to the household).
  - These people are 'available for work'.
- For workers with $l \geq h_{t,j}^{s}$ it's not worth it to the household (which internalizes the worker's degree of work aversion) for them to go to work.
- Unemployment rate of type j workers:

$$\frac{h_{t,j}^{s} - h_{t,j}}{h_{t,j}^{s}}$$

Household Problem

- The household maximizes utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \int_0^1 \frac{h_{t,j}^{1+\varphi}}{1+\varphi} dj \right],$$

  w.r.t. C_t, B_{t+1}, subject to:
  - the budget constraint,

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + \int_0^1 W_{t,j} h_{t,j} dj + \overbrace{\pi_t}^{\text{profits net of lump sum government taxes}}, \text{ for all } t$$

  - the value of $W_{t,j}$, $j \in (0,1)$ chosen by the union
  - the value of $h_{t,j}$ implied by the demand curve for labor (see below).
- The household in this model only has a consumption/saving decision.

Labor Market

- In the simple New Keynesian model with competitive labor markets, labor is supplied directly by households.
- In model with sticky wages, N_t is constructed from the specialized labor supplied by households:

$$N_t = \left[\int_0^1 \left(h_{t,j} \right)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \varepsilon_w > 1.$$

- The above technology is operated by perfectly competitive labor ‘contractors’ or ‘aggregators’.
  - They are the analog of the final good producers.
  - They choose N_t, $h_{t,j}$, $j \in (0,1)$ to maximize profits:

$$W_t N_t - \int_0^1 W_{t,j} h_{t,j} dj$$

$$\rightarrow \quad h_{t,j} = N_t \left(\frac{W_t}{W_{t,j}} \right)^{\varepsilon_w}$$

Unions

- For each j, there is a monopoly union that sets $W_{t,j}$.
- The type j union sets the wage to promote the objectives of its members.
  - Union membership composed of 'coalitions' of people from each of the identical households.
  - Each identical household coalition is composed of all the $l \in (0,1)$ workers of type j from that household.
  - Integrating over the objectives of all $l \in (0,1)$ in a typical coalition:

$$\overbrace{v_t}^{\text{value, in utility terms, of wage } = 1/(P_t C_t)} \times \int_0^{h_{t,j}} f(l)\, W_{t,j} dl$$

$$- \overbrace{\int_0^{h_{t,j}} f(l)\, l^{\varphi} dl}^{\text{sum, across employed workers, of utility cost of working}}$$

$$= v_t W_{t,j} h_{t,j} - \frac{h_{t,j}^{1+\varphi}}{1+\varphi}$$

Union Wages when Wages are Set Flexibly

- The jth monopoly union would choose $W_{t,j}$ to maximize

$$\max\left\{v_t W_{t,j} h_{t,j} - \frac{h_{t,j}^{1+\varphi}}{1+\varphi}\right\} \text{ subject to } h_{t,j} = N_t \left(\frac{W_t}{W_{t,j}}\right)^{\varepsilon_w}.$$

$$\text{FONC: } W_{t,j} = \overbrace{\frac{\varepsilon_w}{\varepsilon_w - 1}}^{\text{markup}} \times \frac{\overbrace{h_{t,j}^{\varphi}}^{\text{nominal marginal cost of labor}}}{v_t}.$$

- Note that we have a 'theory of unemployment':

$$h_{t,j} = \left[v_t W_{t,j} \frac{\varepsilon_w - 1}{\varepsilon_w}\right]^{\frac{1}{\varphi}},\ h_{t,j}^s = \left[v_t W_{t,j}\right]^{\frac{1}{\varphi}},$$

So,

$$\text{unemployment} \equiv \frac{h^s - h}{h^s} = 1 - \left[\frac{\varepsilon_w - 1}{\varepsilon_w}\right]^{\frac{1}{\varphi}}.$$

Unemployment and Unionization in the Data

- According to the theory, the level of unemployment is a function of the labor markup and φ.
- How does the theory perform relative to the data?

$$
\begin{aligned}
u &= 1 - \left(\frac{1}{markup}\right)^{\frac{1}{\varphi}} \\
&= 4.8\%, \text{ when } markup = 1.05, \ \varphi = 1 \text{ (CEE calibration).}
\end{aligned}
$$

  – not bad!

- But, degree of unionization varies over time. Does the theory's prediction that with more unionization there should be higher unemployment hold up?

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- But, degree of unionization varies over time. Does the theory's prediction that with more unionization there should be higher unemployment hold up?
  – not really, though evidence is somewhat mixed.

Does the Degree of Union Power Affect the Unemployment Rate?

OECD Employment Outlook (2006, chap 7)

Norway and Denmark have unionization rates near 80 percent. Before the current crisis their unemployment rate was under 3.0 percent.

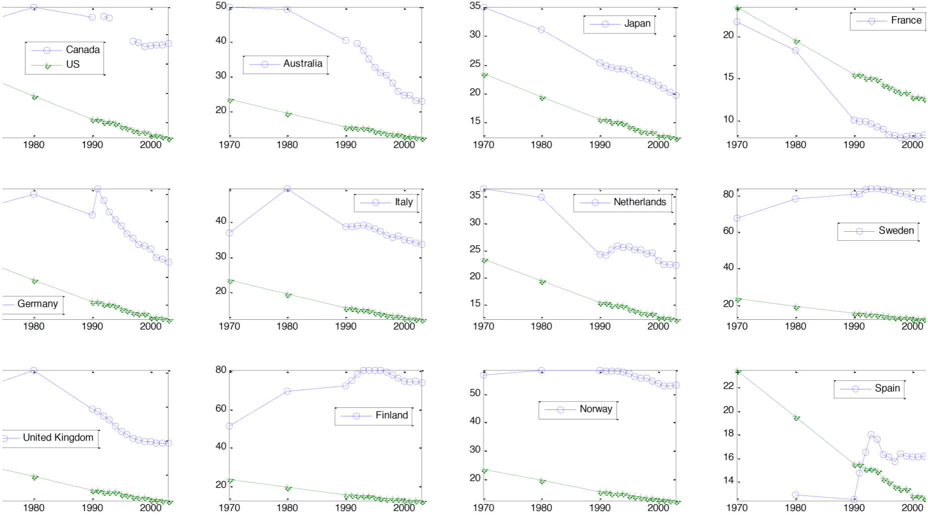
Union Density Rates

Jelle Visser, 2006 Monthly Labor Review

- Union density rates, 1970, 1980 and 1990–2003, adjusted for comparability.

- Definition: union membership as a proportion of wage and salary earners in employment.

Union Density Rates

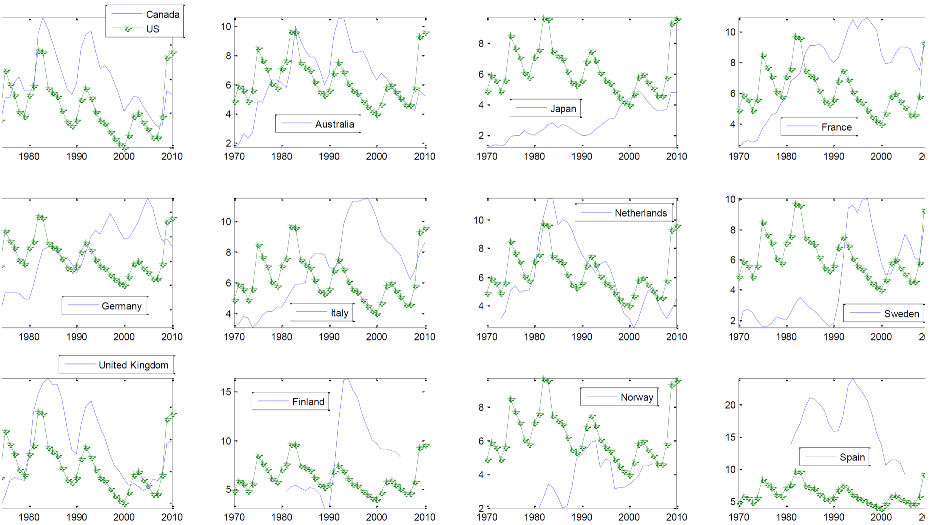


Unemployment Rates: Sources

BLS, "International Comparisons of Annual Labor Force Statistics," Adjusted to U.S. Concepts, 10 Countries, 1970-2010, Table 1-2.

Finland, Norway and Spain taken from ILO, "Comparable annual employment and unemployment estimates, adjusted averages"

Unemployment Rates



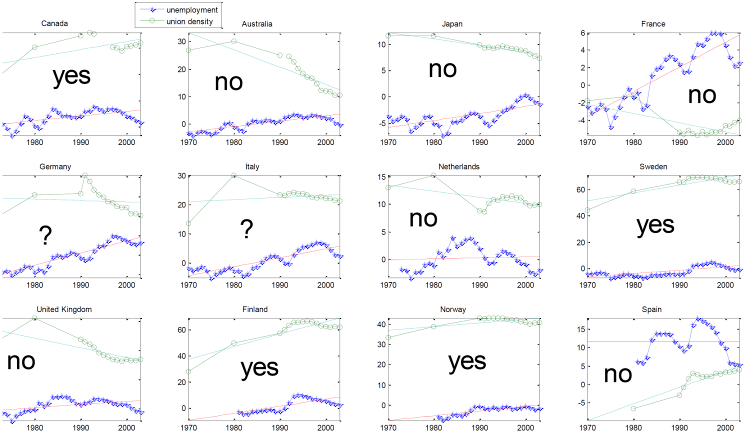
Monopoly Power Hypothesis

- If union density in country A grows faster than union density in US, then
  - Expect unemployment in country A to rise more than unemployment in US.

- Test is based on low frequency part of the data, not on the levels.

Data Consistent With Monopoly Power Hypothesis?

monopoly power ↑ → unemployment ↑



Wage Setting with Frictions

- When wages are flexible wage markup constant, so unemployment rate predicted to be *constant*.
- When wages are sticky, wage markup not constant, so unemployment rate fluctuates.
- To model wage stickiness, adopt Calvo-style frictions:
  - union optimizes wage with probability $1 - \theta_w$.
  - with probability θ_w,
$$W_{t,j} = W_{t-1,j}.$$

Wage Setting with Frictions, cnt'd

- In practice, assume 'indexation':

$$W_{t,j} = \pi_{w,t-1}\mu_{a,t-1}W_{t-1,j},$$

  where $\pi_{w,t-1}$ is lagged nominal wage inflation and $\mu_{a,t-1}$ is technology growth.

  - it appears that indexation is important for aggregate models to fit the data well (see Christiano-Eichenbaum-Trabandt (2013)).

  - indexation has been criticized as being inconsistent with micro data:

    - indexation implies all individual wages change in all periods, while in the data many wages remain unchanged for periods up to a year.

Wage Setting with Frictions, cnt'd

- The $1-\theta_w$ unions that reoptimize in t select $\tilde{W}_t$ to optimize

$$E_t \sum_{i=0}^{\infty} (\beta\theta_w)^i \left\{ v_{t+i}\tilde{W}_t h_{t+i}^t - \frac{\left(h_{t+i}^t\right)^{1+\varphi}}{1+\varphi} \right\}$$

$$\text{s. t. } h_{t+i}^t = N_{t+i} \left(\frac{W_{t+i}}{\tilde{W}_t} \right)^{\varepsilon_w} .$$

- Here,
  - h_{t+i}^t employment in $t+i$ of workers whose wage was set in t.
  - v_{t+i} household marginal utility of currency in $t+i$.
    - In general, v_{t+i} is multiplier on household budget constraint.
    - With our assumptions,

$$v_{t+i} = \frac{u_{c,t+i}}{P_{t+i}} = \frac{1}{P_{t+i}C_{t+i}}.$$

Wage Setting with Frictions, cnt'd

- The solution to this problem gives rise to a 'wage Phillips curve':

$$\hat{\pi}_{w,t} = \frac{\kappa_w}{1+\varphi\varepsilon_w}\left(\hat{C}+\varphi\hat{N}_t-\widehat{\bar{w}}_t\right)+\beta\hat{\pi}_{w,t+1}$$

$$\kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w},\ \pi_{w,t}\equiv\frac{W_t}{W_{t-1}},\ \bar{w}_t\equiv\frac{W_t}{P_t}.$$

- To understand this expression, note first:

$$\widehat{\left[\underbrace{\left(\frac{\varepsilon_w}{\varepsilon_w-1}\frac{N_t^{\varphi}}{v_t}\right)}_{\text{looks like markup of marginal cost of work}}/W_t\right]}$$

$$=\widehat{\frac{P_tC_tN_t^{\varphi}}{W_t}}=\widehat{\frac{C_tN_t^{\varphi}}{\bar{w}_t}}=\hat{C}+\varphi\hat{N}_t-\widehat{\bar{w}}_t.$$

Intuition for Wage Phillips Curve

- With flexible wages, all unions behave the same
  - so $h_{t,j} = h_{t,i} = h_t$, say, for all i, j and $N_t = h_t$:

$$N_t = \left[\int_0^1 (h_{t,j})^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} = h_t.$$

  - each union hits wage target each period:

$$W_{t,j} = \overbrace{\frac{\varepsilon_w}{\varepsilon_w - 1}}^{\text{markup}} \times \overbrace{\frac{N_t^{\varphi}}{\upsilon_t}}^{\text{nominal marginal cost of labor}}, \text{ for all } j.$$

- With sticky wages, unions aim to hit wage target on average:

$$W_t \text{ rises if } W_t < \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{N_t^{\varphi}}{\upsilon_t}$$

$$W_t \text{ falls if } W_t > \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{N_t^{\varphi}}{\upsilon_t}.$$

Collecting the Equations

- Variables to be determined:

$$x_t, r_t, \pi_t, \hat{r}_t^*, \hat{s}_t, \widehat{\bar{w}}_t, \hat{\pi}_{w,t}$$

- Equations:

$x_t = x_{t+1} - \left[r_t - \pi_{t+1} - r_t^*\right]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$
$\hat{s}_t = \widehat{\bar{w}}_t - a_t$
$\hat{\pi}_{w,t} = \frac{\kappa_w}{1+\varphi\varepsilon_w}\left(\hat{C}_t + \varphi\hat{N}_t - \widehat{\bar{w}}_t\right) + \beta\hat{\pi}_{w,t+1}$
$r_t^* = -\log(\beta) + E_t\left[a_{t+1} - a_t - \frac{\tau_{t+1}-\tau_t}{1+\varphi}\right]$

- Need more equations: relate $\hat{C}_t, \hat{N}_t$ to x_t and shocks, and connect $\widehat{\bar{w}}_t$ to $\hat{\pi}_{w,t}$ and π_t.

- Recall definition of level of output gap: X_t :

$$X_t \equiv \frac{C_t}{A_t \exp\left(-\frac{\tau_t}{1+\varphi}\right)} = p_t^* N_t \exp\left(\frac{\tau_t}{1+\varphi}\right).$$

- Then,

$$\begin{aligned} \hat{C}_t &= x_t + a_t - \frac{\tau_t}{1+\varphi},\ \hat{N}_t = x_t - \frac{\tau_t}{1+\varphi} \\ &\rightarrow \hat{C} + \varphi \hat{N}_t = (1+\varphi)\, x_t + a_t - \tau_t. \end{aligned}$$

- Also,

$$\begin{aligned} \pi_{w,t} &\equiv \frac{W_t}{W_{t-1}} = \frac{\frac{W_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}} \frac{P_t}{P_{t-1}} = \frac{\bar{w}_t}{\bar{w}_{t-1}} \bar{\pi}_t \\ &\rightarrow \hat{\pi}_{w,t} = \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \widehat{\bar{\pi}}_t \end{aligned}$$

Equations of the Sticky Wage Model

- Six private sector equations in seven variables:

$x_t = x_{t+1} - [r_t - \pi_{t+1} - r_t^*]$
$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{s}_t + \beta\pi_{t+1}$
$\hat{s}_t = \widehat{\bar{w}}_t - a_t$
$\hat{\pi}_{w,t} = \frac{\kappa_w}{1+\varphi\varepsilon_w}\left[(1+\varphi)\, x_t + a_t - \tau_t - \widehat{\bar{w}}_t\right] + \beta\hat{\pi}_{w,t+1}$
$\hat{\pi}_{w,t} = \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \hat{\pi}_t$
$r_t^* = \Delta a_{t+1} - \frac{\tau_{t+1}-\tau_t}{1+\varphi}$

where r_t and r_t^* now stand for their deviations from steady state, r.

- Monetary policy rule:

$$r_t = \alpha r_{t-1} + (1-\alpha)\left[\alpha_\pi \pi_t + \alpha_x x_t\right] + u_t,$$

where u_t denotes a monetary policy shock.

Conclusion

- Alternative interpretation of the labor supply parameter, φ:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \int_0^1 \frac{h_{t,j}^{1+\varphi}}{1+\varphi} dj \right].$$

  - previous interpretation - $1/\varphi$ is the labor supply elasticity, the willingness of individuals to adjust their hours in response to a change in the wage.
    - labor economists have estimated that the elasticity of labor supply on the intensive margin is small.
  - new interpretation - φ characterizes the nature of heterogeneity in the population, labor elasticity on extensive margin
    - large φ: population very heterogeneous in neighborhood of the working/not working margin, so few people jump in to the labor market when the wage rate rises (labor supply steep).
    - small φ : population relatively homogeneous with many people not working being close to the margin of jumping in to work in the response of a wage rise (labor supply flat).

Conclusion, cnt'd

- Sticky wages effective in getting wage not to rise much after a monetary policy shock, limiting the rise in inflation and amplifying the rise in output.
- But,
  - Underlying monopoly power theory of unemployment does not receive strong support in the data.
  - To fit the aggregate data, the model must incorporate wage indexation and this is not consistent with micro-data
  - To obtain a good model of unemployment and labor force requires removing income effects from utility.
    - Not clear if this makes sense from a micro perspective.
- Alternative approach to labor markets are being explored (see Christiano, Eichenbaum and Trabandt (2013)).