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Foreign Direct Investment and the Risk of Expropriation

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When an investor, for example a transnational corporation, invests abroad it runs the risk that its investment will be expropriated for the simple reason that international contracts are practically impossible to enforce Any agreements or contracts then undertaken by the transnational company and the host country must be designed to be self-enforcing It could be possible for the host country and the transnational corporation to find such self-enforcing agreements if there are future gains from trade Thus although the host country might have a short-term incentive to expropriate, it has a long-term incentive to foster good relations with potential investors to attract more investment in the future This conflict between short-term and long-term incentives determines the type of investment contracts agreed This paper extends previous work on the general underprovision of investment when contracts are incomplete or only partially enforceable (see e g Grout (1984)) to a dynamic context It is likewise shown that investment is initially underprovided but it increases over time and for certain parameter values it tends to the efficient level The expected future discounted returns to the transnational company decline over time, extending Vernon's observation of the obsolescing bargain (Vernon (1971)) The model is also extended to allow for capital accumulation and consideration is given to renegotiation-proof contracts

1 INTRODUCTION

An investor, for example a transnational corporation, that invests directly abroad[1] risks that its investment will be expropriated[2] for the simple reason that international contracts are practically impossible to enforce [3] Any agreement or contract undertaken by the transnational company and the host country must therefore be designed to be self-enforcing Providing there are always mutually advantageous trades to be made in the future such

1 In 1986 the flow of foreign direct investment from developed market economies to developing countries was $12 5 billion or roughly one-half of all private capital flows from the developed to the developing nations The total flow of all foreign direct investment was around $50 billion

2 Between 1960 and 1976 at least 1535 firms were forced to divest (either by direct expropriation, forced sale, forced renegotiation of contract resulting in ownership transfer or through extra-legal acts) in 76 different less-developed countries (see Kobrin (1980, p 73)) The book value of these firms represented 4 4% (3 3% if agriculture is excluded) of the total stock, including expropriated assets, of partially or wholly foreign-owned firms in the expropriating countries at the end of 1976 The period 1956 72 was even more striking with nearly 20% of all assets being expropriated without compensation (see Williams (1975 p 265))

3 A referee has pointed out to us that the legality of various forms of expropriation in international law is unclear It appears that no precise distinction between "lawful" and "unlawful" acts has been established in international law Neither is there agreement on the acceptable standard of compensation in the case of unlawful acts (see Weston (1981) and Norton (1991) on this issue) What seems clear is that no court or tribunal can ultimately extract more from a host country than it is willing to pay For a more detailed discussion see Eaton and Gersovitz (1983)

self-enforcing agreements may exist Thus although the host country might have a short-term incentive to expropriate it has a long-term incentive to foster good relations with the investor to attract more investment in the future This conflict between short-term and long-term incentives determines the type of self-enforcing investment contracts agreed and is the focus of this paper

Traditionally most studies of the relations between the host country and the transnational corporation use a bilateral monopoly framework This is, according to Kobrin (1987), " *the* currently accepted paradigm of HC-MNC relations in international political economy " A clear exposition is given by Kindlberger (1969, lecture 5) The host country has an investment opportunity that it is unable to exploit itself, either because it does not have the technical know-how or because its access to capital markets is restricted but which the transnational corporation can exploit as it has the necessary capital, technology, marketing and managerial skills The host country has control over access and conditions of operation It is then argued (see e g Penrose (1959)) that the actual outcome will depend on the relative bargaining strengths of the host country and the transnational corporation The lower bound on the return to the transnational corporation is just sufficient to cover the supply price of capital where the transnational corporation makes no economic profit and the upper bound is that level where the host country would just prefer to leave the opportunity unexploited

The one-period model is, however, inappropriate for studying expropriation risk since if there were just one period, or indeed a final period, the host country would certainly expropriate, and knowing this the transnational corporation would not be prepared to invest We therefore set up an infinitely repeated version of the bilateral monopoly model in Section 2 and suppose the transnational corporation and the host country negotiate a long-run contract specifying how much is to be invested in each period and how much of output is to be transferred to the host country at each date-state Thus there are no informational asymmetries the host country can observe how much the transnational corporation invests and both can observe the state of nature The crucial issue is that the host country can, because of its sovereign status, renege on the contract and expropriate output or capital The transnational corporation can choose to withdraw and not to invest in the future The only feasible contracts then are self enforcing in which the long-term benefits from adhering to the contract exceed any short-term gains to be had by reneging Such self-enforcing or implicit contracts are enforced by the threat to return to autarky following any infringement Section 3 presents the main results of the paper In it we assume that the host country is risk neutral and use a dynamic programming approach to find the set of Pareto-efficient self-enforcing contracts We show that the optimum contract evolves "ratchet-like" over time Investment is initially below the efficient level[4] and state-contingent transfers are zero at the start of the contract (a non-trivial contract always exists provided the production function satisfies an Inada condition) but investment rises over time until a stationary state is reached Depending on the discount factor the stationary state either has the efficient level of investment or investment is below the efficient level and all expected profits are taken by the host country Transfers to the host country are always zero until the period before the maximal level of investment is attained

4 It should be noted that this underinvestment result is related to results in the transactions cost literature that specific assets tend to be underinvested when their quasi-rents can be appropriated (see e g Williamson (1975) and Klein, Crawford and Alchian (1978)) Grout (1984) and Hart and Moore (1988) provide a formal analysis Our model has many similar features, investment is not contractible and risk aversion does not play a key role It has the added advantage that investment is repeated (capital accumulated in Section 4), so the time-structure of investment can be considered

Essentially this policy of delaying payments and investments makes the threat to return to autarky more effective by increasing the cost of any deviation It is a manifestation of "back-loading" the contract and a similar effect was at work in Thomas and Worrall (1988) and Holmstrom (1983) Another feature of the optimum contract is that investment moves pro-cyclically (lagged one period) and transfers to the host country are positively serially correlated When output is high there is a greater temptation for the host country to confiscate output To offset this more must be offered by the contract in the future [5]

Our back-loading result offers some support for the use of "fade-in" clauses and tax holidays [6] This result was foreshadowed by Kindeleberger (1969, lecture 5) who suggests that the bargaining strength of the host country might tend to increase over time, a theme which was taken up by Vernon (1971, p 46-59) under the title of "the obsolescing bargain" [7] Doyle and Van Wijnbergen (1984) also provide a similar rationale for tax holidays [8] They examine a repeated bargaining model where the investor pays a fixed entry cost and invests a single unit thereafter to remain productive They obtain a similar result that the transfer to the host country rises over time, but cannot say anything about investment level choice Another closely related paper is Eaton and Gersovitz (1983) where, as in our model, it is the threat of withdrawal of future capital which is used to forestall expropriation, but in their model each foreign investor makes only a negligible contribution to output so that the strategic interactions between the transnational corporation and the host country which we examine here are excluded by assumption

Since the underlying structure of the model is stationary (the production function is independent of time and states are i i d), it may be thought that the optimum contract ought itself to be stationary But this is wrong There is certainly a cost to having investment change over time as the production function is concave, but there are also benefits To see this suppose that the contract is stationary, that investment is constant and that, following the bargaining strength model, the host country gets a certain constant percentage share of profits each period Ideally investment should be at the efficient level where expected marginal revenue equals marginal cost This would be sustainable if the share going to the host country and the discount factor were high enough so that future benefits were attractive enough Otherwise the host country will have an incentive to confiscate current output rather than wait for the future returns, in which case the contract is not self-enforcing The contract can be made self-enforcing by reducing the investment level, it never pays to increase the investment level since this increases the temptation to renege and at the same time reduces the level of profit to be shared out This then is a stationary self-enforcing contract with underinvestment It is however, possible to do better As the host country is risk neutral it is only interested in the value of discounted payments and

5 A contrasting argument is made by King (1988) He argues that governments best maintain office by managing a steady growth in income In that case a positively serially correlated pattern of tax revenues is likely to lead to unstable governments This is of course an empirical question King finds some support for his hypothesis that tax revenues are counter-cyclical from data on bauxite mining in Jamaica

6 Fade-in clauses were used by the Andean Pact countries in the 1970s They required new foreign firms to sell 51% of ownership over a 15–20 year period (see Sigmund (1980, p 289 90)) Tax holidays which exempt investors from tax obligations for a prespecified period along with direct subsidies, duty-free imports and import tariffs and quotas have been used by host countries to encourage inward investment (see Reuber (1973 p 126 30) and Guisinger (1985 p 19–33))

7 Their model, however, was initially applied only to the resource sector and doubt has been cast on its applicability to the manufacturing sector (see Kobrin (1987)), whereas our model is more appropriate to the latter Bennett and Sharpe (1979) even suggest that bargaining power may shift in favour of the transnational corporation citing evidence from the Mexican car industry that local capital becomes increasingly dependent on the transnational corporation and provides a powerful lobby for the transnational corporation's cause

8 Other rationales for tax holidays are provided by Bond and Samuelson (1986) and Gersovitz (1987)

not when they actually occur Then delaying payments does not affect the current investment level but, as the time approaches when the payments are due the temptation to renege is diminished since, seen from that date, expected future payments are higher and so future investment can be raised without causing the host country to renege Since investment in the future is higher so too are the transnational corporation's profits and thus a better contract has been found Of course investment cannot rise without limit and eventually either the efficient investment level will be attained or the host country will end up taking all the expected profits

The dynamic programming approach we adopt is similar to that used by Abreu, Pearce and Stachetti (1990) to find pure-strategy sequential equilibria of repeated discounted games with imperfect monitoring and we show in Section 2 that a self-enforcing contract is a subgame-perfect equilibrium of a repeated discounted game between the host country and transnational corporation This was also the procedure adopted by Thomas and Worrall (1988) on self-enforcing wage contracts But in that paper the back-loading result applied only to a price variable whereas here it applies to a real variable namely investment [9]

The remainder of the paper deals with extensions Section 4 allows for capital accumulation Section 5 considers what happens when renegotiation of contracts is allowed Even though the contracts described in Section 3 correspond to subgame-perfect equilibria of a repeated game, the threat to return to autarky, if carried out, may be subject to renegotiation Nevertheless it is shown that renegotiation-proof contracts have essentially the same dynamic structure and that they can be computed using a modification of the dynamic programming technique Finally in Section 6 we show that although the host country cares about the timing of the transfers when it is risk averse most of our results go through provided the host country is not too risk averse

A closely related paper, though not concerned with foreign direct investment, is Allen (1983) Formally his model may be encompassed as a special case of Section 4 of this paper Allen studies a model of lending for production where capital is infinitely lived there is no uncertainty and borrowers can default only on the interest but not the principal of the loan without legal sanction, whereas we allow for any constant rate of capital depreciation between zero and one, uncertainty and a general default penalty Moreover Allen considers only the distribution in which the host country extracts all the surplus so that in his model the steady state is achieved immediately, whereas the interesting dynamic properties of the efficient contract arise only when the transnational corporation extracts at least some of the surplus

While it might be tempting to reinterpret our results in the context of sovereign lending there are certain differences that should be stressed First the host country might wish to save out of output so as to be able itself to finance projects in the future, or indeed it may choose to consume part of the current loan, these possibilities do not exist when the transnational corporation is providing not only capital but also technology and expertise not otherwise available to the host country Second the bilateral monopoly model is probably less appropriate in the lending context Third the lender may have to monitor to ensure that the host country is actually devoting the money lent to the specified project and finally, as shown in Bulow and Rogoff (1989), if the expected future value of debt were positive the host country could do better by reneging and investing in cash-in-advance contracts Sovereign-debt contracts have been studied using the dynamic programming

9 It should be noted that the dynamic programming approach has also been successfully used to study repeated principal-agent models, see e g Phelan and Townsend (1990) and Spear and Srivastava (1987)

approach by Atkeson (1991) who considers both repudiation and moral hazard and by Kletzer and Wright (1990) who address the issue of renegotiation in a consumption-smoothing model

2 MODEL

In this section we set up the basic repeated bilateral monopoly model of foreign direct investment between a transnational corporation and a host country In the basic model we assume that capital depreciates completely within a single period (capital accumulation is considered in Section 4) It is assumed throughout that the transnational corporation is risk neutral, in Sections 2-5 it is assumed that the host country is also risk neutral Section 6 considers the case of a risk-averse host country [10] At each date $\tau = 1, 2, \ldots, \infty$ there is a state of nature $s = 1, 2, \ldots, N$ The state of nature is i i d over time and the probability of state s is p_s, independent of time There are two goods a capital and a consumption good We take the capital good as numeraire Since the relative price of the two goods is assumed constant over time units are chosen so that the price of the consumption good is also unity (One possible interpretation of the state of nature is however as a variable consumption good price) The consumption good can only be produced in the host country with the help of foreign capital The transnational corporation provides this capital to the host country when it invests Together with the investment, I, provided by the transnational corporation, the state of nature determines output at each date through the production/restricted profit function $r(I, s)$ This does not mean that the two parties have no other investment opportunities, but only that the particular investment opportunity is specific to these two parties, and whatever happens here does not affect any other activities being undertaken Investment is chosen before the state of nature is known It is assumed that $r(I, s)$ is twice-continuously differentiable in I, increasing and concave in I, $r(0, s) \leq 0$, and by convention increasing in s Further it is assumed that $E[r(I, s)]$, where E is the expectations operator, is strictly concave, and that there is a positive I^* satisfying $E[r'(I^*, s)] = 1$ with $E[r(I^*, s)] - I^*$ positive given our assumptions this is unique and maximises expected returns

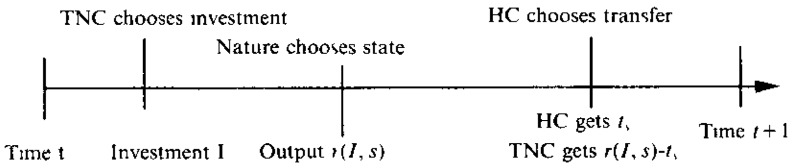


FIGURE 1

Time sequence of decisions

Let s^τ be the state that occurs at date τ, and let $h^\tau = (s^1, s^2, \ldots, s^\tau)$ denote the history of states up to and including date τ, with h^0 the empty history A contract, δ, between the host country and the transnational corporation is a pair $(I^\tau(h^{\tau-1}), t^\tau(h^\tau))_{\tau=1}^{\infty}$ of sequences of functions specifying the investment level I^τ, at the start of date τ, as a function of past history, and the transfer to the host country t^τ, at the end of date τ, as a function of the past history and the current state The actual sequence of events at any date τ is illustrated in Figure 1

10 Part A of the Appendix provides proofs for the case of a risk-neutral host country, and Part B for a risk-averse host country

It is assumed that the transfer to the host country cannot exceed current output and cannot be negative a contract is *feasible* if

$$r(I^\tau(h^{\tau-1}), s^\tau) - t^\tau(h^\tau) \geqq 0, \tag{2 1}$$

$$t^\tau(h^\tau) \geqq 0, \tag{2 2}$$

for any $h^{\tau-1}$ and s^τ, $t \geqq 1$ It is shown in the next section that (2 1) never binds in the optimum contract if the host country is risk neutral, whereas the constraint (2 2) plays a crucial role it implicitly means that the host country is credit constrained, and without a constraint of this type the first best could be achieved by having the host country pay a lump sum transfer to the transnational corporation at the beginning of the first period or by posting a bond which would be forfeited if it abrogated the contract

We assume that the host country and transnational corporation discount at the same constant discount factor $\alpha \in (0, 1)$, so the future payoff, discounted to time τ, to the transnational corporation from some contract δ at time τ after the history $h^{\tau-1}$ must satisfy the recursive relation

$$U(\delta, h^{\tau-1}) = -I^\tau(h^{\tau-1}) + E[r(I^\tau(h^{\tau-1}), s^\tau) - t^\tau(h^\tau) + \alpha U(\delta, h^\tau)],$$

and likewise the payoff to the host country satisfies

$$V(\delta, h^{\tau-1}) = E[t^\tau(h^\tau) + \alpha V(\delta, h^\tau)]$$

For the contract to be self-enforcing neither the host country nor the transnational corporation should ever have an incentive to violate it It is assumed that the host country has first call on any output produced within its territory so may choose any transfer satisfying (2 1) and (2 2) If, however, it chooses a transfer different from that specified by the contract it is assumed that the transnational corporation will make no further investment Given this assumption, if the host country ever violates the contract, it will clearly choose to expropriate the entire output as this is the most profitable deviation As capital depreciates within one period and the operation of the project requires the expertise of the transnational corporation, the host country will not renege on the contract at τ provided

$$t^\tau(h^\tau) + \alpha V(\delta, h^\tau) \geqq r(I^\tau(h^{\tau-1}), s^\tau), \tag{2 3}$$

for every history h^τ [11] Likewise it is assumed that if the transnational corporation does not invest the contracted amount the host country will always expropriate any output Clearly then if the transnational corporation chooses to default, it will do so by never investing again, thus it will not renege at τ provided

$$U(\delta, h^\tau) \geqq 0 \tag{2 4}$$

We define a contract to be *self-enforcing* if (2 3) and (2 4) hold for every history h^τ, $\tau \geqq 1$

Let $\Delta(h^{\tau-1})$ be the set of contracts satisfying (2 1)–(2 4) after the history $h^{\tau-1}$ (i e feasible and self-enforcing for all histories of length at least $\tau - 1$ coinciding with $h^{\tau-1}$ for the first $\tau - 1$ periods) We are interested in finding an optimum contract belonging to $\Delta(h^0)$ which maximizes the transnational corporation's ex ante payoff $U(\delta, h^0)$ subject to

11 If the host country could operate a less efficient technology than the transnational corporation it would be necessary to add further terms to the R H S of (2 3), but this would not qualitatively affect the analysis A more general default payoff is specified in Section 4

a participation constraint for the host country, $V(\delta, h^0) \geqq 0$ [12] Such an optimum contract cannot be Pareto-dominated after any history if it were it would be possible to replace that part of the contract which was dominated, raising the continuation payoffs of both parties and simultaneously relaxing all previous self-enforcing constraints We define the Pareto-frontier of the set of payoffs arising from feasible self-enforcing contracts as follows for any history $h^{\tau-1}$ and for any feasible value of the host country's payoff, V^τ, from the start of period τ onward (feasible in the sense that there exists a $\delta \in \Delta(h^{\tau-1})$ which yields at least this payoff to the host country),

$$U(V^\tau) = \sup_{\delta \in \Delta(h^{\tau-1})} \{U(\delta, h^{\tau-1}) | V(\delta, h^{\tau-1}) \geqq V\}$$

Because the constraint set is stationary, $U()$ does not depend upon $h^{\tau-1}$ To simplify notation write V_s^τ for $V(\delta, (h^{\tau-1}, s^\tau))$, t_s^τ for $t^\tau(h^{\tau-1}, s^\tau)$ and I^τ for $I^\tau(h^{\tau-1})$ Then using the recursive definitions given above, the Pareto-frontier is a fixed point[13] of the dynamic programming problem of choosing $(I^\tau, (t_s^\tau, V_s^\tau)_{s=1}{}^N) \in R_+{}^{2N+1}$ at date τ to maximize $\{-I^\tau + E[r(I^\tau, s^\tau) - t_s^\tau + \alpha U(V_s^\tau)]\}$ subject to (2 1) – (2 4) and

$$E[t_s^\tau + \alpha V_s^\tau] \geqq V^\tau \tag{2 5}$$

Thus in the dynamic programme V can be considered as the state variable of the contract and current values of I and t_s are the solutions of the programming problem given V The state equation itself is trivial the future value of V equals the value currently promised dependent on which state actually occurs Given then a starting value for the state variable V^1 (such that the host country gets at least its outside utility ex ante, in other words $V^1 \geqq 0$), the programme at date $\tau = 1$ determines the first-period investment and state-contingent transfers and the future utilities $V_s{}^1$ promised to the host country from date 2 onward, again contingent on the first-period state Then $V^2 = V_s{}^1$ if state s actually occurs, and the programme then determines investment and transfers in the second period, and so on

There are a number of points that should be made about this dynamic programming problem First, given our assumption about the production function, the constraint set is closed and an optimum contract exists although of course it may be the trivial contract which has no investment and no transfers Second, (2 5) should be treated as an inequality constraint because the transnational corporation can offer the host country more than it promised in the past without violating previous self-enforcing constraints (on the downward sloping part of the function it will bind however) Third, the presence of the optimum function itself in the constraint set ((2 4)) means that standard contraction mapping arguments cannot be used to show that a unique fixed point exists which corresponds to the true value function Nevertheless it can be shown that an iterative mapping starting from the first-best frontier does converge to the optimum value function and this is proved in Lemma 1 of the Appendix Fourth, the set of self-enforcing contracts $\Delta(h^{\tau-1})$ is not a convex set because of the term $r(I, s)$ on the R H S of (2 3) Fifth, the value function itself is not everywhere differentiable

12 That is to say that we are concentrating on the equilibrium which is the best from the point of view of the corporation This is for expositional purposes only other equilibria are picked out by requiring that the host country receive higher payoffs The contract characterisation given remains essentially the same, as will become clear

13 The argument is the same as that given in Thomas and Worrall (1988), see also van Damme (1991) and Abreu, Pearce and Stachetti (1990)

It will be helpful to describe how the solution method corresponds to game-theoretic concepts although it would be unnecessarily cumbersome to introduce all the machinery and notation of game theory. The problem is simply a repeated discounted (extensive form) two-person game. In the stage game the transnational corporation moves first by choosing the investment level. Nature then chooses a state. Having observed both the investment choice of the transnational corporation and the state of nature the host country chooses how much output to retain. The *optimum punishment* of this game is autarky: it is clearly a subgame-perfect equilibrium of any continuation game (strategies are to never invest and to always expropriate) and minimax payoffs (0, 0) are attained. So our assumption about what happens after either party reneges amounts to assuming that optimum punishments are imposed, and consequently what we have called a self-enforcing contract is just a subgame-perfect equilibrium outcome, and because the punishments are optimum, any subgame-perfect equilibrium outcome is a self-enforcing contract (Abreu (1988)). We are only interested in that equilibrium which is constrained Pareto-optimal and maximizes the payoff to the transnational corporation. The dynamic programming approach is used by Abreu, Pearce and Stachetti (1990) to characterise the equilibrium payoff set in a game with imperfect monitoring. Attention can be restricted here to the Pareto-frontier payoffs as the continuation payoffs must also be Pareto efficient and because in this case the optimum punishments do not depend on knowledge of the entire payoff set, so this part of the dynamic programme can be considered independently.

3. THE DYNAMICS OF AN EFFICIENT CONTRACT

This section solves the dynamic programming problem outlined in the last section and shows that the optimum contract evolves "ratchet-like" with investment increasing over time and the transfers to the host country being zero in the early periods.

It will be helpful first as a benchmark case to consider the first-best situation where the self-enforcing constraints are ignored. Investment each period will maximise $E[r(I, s)] - I$, and hence satisfy $E[r'(I^*, s)] = 1$. By assumption there is a unique solution I^*, where $\Pi^* = E[(I^*, s)] - I^* > 0$ so that per-period profits are positive, we shall call I^* the *efficient level of investment*. The first-best Pareto frontier, that is, taking into account only the feasibility constraints (2.1) and (2.2), is given by the equation $P^*(V) = (\Pi^*/(1-\alpha)) - V$ for $\in[0, \Pi^*/(1-\alpha)]$. Clearly the efficient first-best contract which maximises the transnational corporation's payoff involves an investment level I^* each period and no transfers. The host country gets its reservation payoff of zero. For any other point on the frontier the corresponding contract still involves an investment of I^* each period and although transfers are positive the actual time path of transfers is not uniquely determined (though it must satisfy $0 \leqq t_s \leqq r(I^*, s)$) as the host country is risk neutral and both parties discount at the same rate.

As we show below, if the self-enforcing constraints (2.3) and (2.4) are included, the second-best Pareto frontier—the solution of the dynamic programming problem—lies on or below the first-best frontier: $U(V) \leqq P^*(V)$ with strict inequality for some lower values of V. It may be that no non-trivial contract exists in which case the frontier is just the single point (0, 0). On the other hand if there is an optimum non-trivial contract it must offer the host country a positive rent, as the only way the transnational corporation can offer the host country a zero payoff is by making no transfers, but then any positive level of investment will yield a positive value of output which will be expropriated. Define this minimum rent as $V_{min} = \sup\{V | V \in \operatorname{argmax} U(V)\}$. Then given our assumptions it is obvious that $U(V)$ is non-increasing and there will be no distributional conflict between the

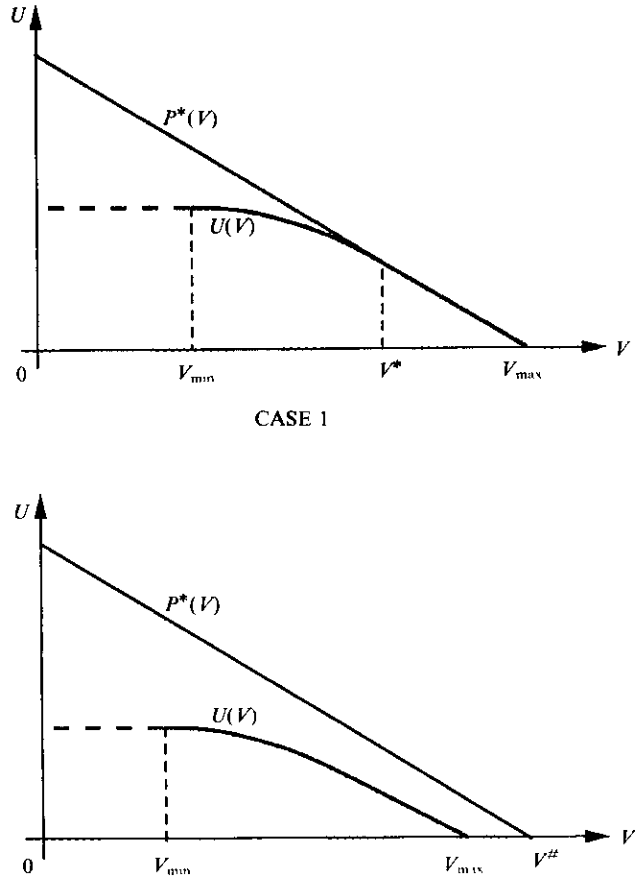


FIGURE 2
The value function

transnational corporation and the host country in the range $[0, V_{min}]$ $U(V)$ is horizontal in this range and equation (2 5) is a strict inequality Let V_{max} be the maximum feasible value of V, where $U(V)$ cuts the V-axis and the host country gets the maximum rent from the project, i e $U(V_{max})=0$ Lemmas 2 and 3 in an Appendix show that $U(V)$ is decreasing and concave on (V_{min}, V_{max}) with an absolute slope less than or equal to one (Concavity follows since a convex combination of any two self-enforcing contracts can be made self-enforcing by transferring any extra output directly to the host country and it will offer the host country and the transnational corporation at least the average from the original contracts) Thus if there exists a non-trivial contract there are just two possibilities either $V_{max}=\Pi^*/(1-\alpha)$ or $0<V_{max}<\Pi^*/(1-\alpha)$ These are drawn as CASE 1 and CASE 2 in Figure 2

In CASE 1 of Figure 2, the value function is strictly concave on (V_{min}, V^*) but coincides with the first-best frontier on $(V^*, \Pi^*/(1-\alpha))$ In CASE 2, $U(V)$ is strictly concave on (V_{min}, V_{max}) but $V_{max}<\Pi^*/(1-\alpha)$ (see Lemma 4) In CASE 1 for values of V in $(V^*, \Pi^*/(1-\alpha))$ the efficient level of investment I^* can be sustained as the host country is promised enough transfers now and in the future so that it has no incentive to expropriate even the efficient level of investment As V is reduced below V^* the utility that the host country gets falls below the utility obtained by confiscating output, $r(I^*, s)$ In order to avoid expropriation I must be reduced below I^*, implying that a one unit reduction in V will lead to a less than one unit gain in U, so the absolute slope of $U(V)$ is less than one In CASE 2 it is not possible to sustain I^* even for large values of V, because the transnational corporation's self-enforcing constraint (2 4) will bind Proposition 2 below examines the circumstances under which each case obtains, but we first turn to the dynamics of the optimum contract

Consider the dynamic programming problem and let $p_s\theta_s, p_s\pi_s, p_s\mu_s, \alpha p_s\phi_s$, and σ be the multipliers for the constraints (2 1)–(2 5) The first step is to notice that (2 1) never binds at the optimum (see Lemma 3), so the first-order conditions are

$$E[r'(I, s)(1-\mu_s)] = 1, \tag{3 1}$$

$$\mu_s + \sigma = 1 - \pi_s, \qquad s = 1, 2, \quad , N, \tag{3 2}$$

$$-(\mu_s + \sigma)/(1 + \phi_s) \in \partial U(V_s), \qquad s = 1, 2, \quad , N, \tag{3 3}$$

together with the relevant complementary slackness condition [14] The notation $\partial U(V_s)$ represents the set of super differentials of the value function at V_s There is also an "envelope condition"

$$-\sigma \in \partial U(V), \tag{3 3}$$

which implies that if there is a unique value of σ for which (3 2)–(3 3) hold, then $U(V)$ is differentiable at V

The key to understanding the first-order conditions and solving for the optimum contract is to remember that V is the state variable of the dynamic programme and that σ, which is (minus) the superdifferential of $U(V)$ is weakly monotone increasing in V since $U(V)$ is concave The analysis is slightly more complicated if $U(V)$ is not differentiable (it is differentiable almost everywhere since $U(V)$ is concave) so for the purposes of discussion we assume that $U(V)$ is differentiable and deal with the more general case in the Appendix (that $U(V)$ can have points of non-differentiability is illustrated by an example given below) Then for any given value of V, the corresponding value of σ can be used in the first-order conditions to determine I, t_s and V_s Next the actual state and the chosen value of V_s will determine the next period's state variable and the process can be repeated We will show that V increases "ratchet-like" over time and that each choice variable is continuously and monotonically related to V

First let $\sigma_s = -U'(V_s)$ and $\sigma_{\max} = -U'(V_{\max})$ (In CASE 1, $\sigma_{\max} = 1$ and in CASE 2, $\sigma_{\max} < 1$) Then equation (3 3) can be written as $\sigma_s(1+\phi_s) = (\mu_s + \sigma)$ and equation (3 4) becomes $\sigma = -U'(V)$ Starting with any value of V where $\sigma < 1$ it can be seen straightaway from (3 2) that if $\mu_s + \sigma < 1$, then $\pi_s > 0$ so that $t_s = 0$ This is clearly true if the host country's self-enforcing constraint is not binding, $\mu_s = 0$ But from (3 3) it is also true if $\mu_s > 0$ but $\sigma_s(1+\phi_s) < 1$ Thus unless $\sigma_s = 1$ or $\sigma_s = \sigma_{\max}$ and $\sigma_{\max}(1+\phi_s) \geqq 1$ no transfer from the transnational corporation to the host country will be made Next consider how V_s is related to V From (3 3), $\sigma_s \geqq \sigma$ since $\mu_s \geqq 0$ and if $\phi_s > 0$ then σ_s is at its maximum value, $\sigma_{\max}$, and by definition $\sigma_{\max} \geqq \sigma$ Thus if a state occurs where the self-enforcing

14 The Kuhn–Tucker constraint qualification holds everywhere apart from $V_{\max}$

constraint does not bind, $\mu_s = 0$, σ does not change, and because when $\sigma < 1$ U is strictly concave (Lemma 4), promised future transfers remain the same, $V_s = V$ If on the other hand a state occurs in which it does bind, $\mu_s > 0$, so $\sigma_s > \sigma$ and therefore $V_s > V$ Finally consider how I is determined when $\sigma < 1$ For a given value of $I, V_s = V$ and $t_s = 0$, unless this violates the host country's self-enforcing constraint $(r(I, s) > \alpha V)$ in which case utility must be increased to just the confiscation utility $(t_s + \alpha V_s = r(I, s)$ t_s will be kept at zero unless V_s reaches $V_{\max}$ or $V^*)$ Overall expected utility must equal V, so there is only one value of I consistent with this rule This can be easily calculated and it can be seen from (3 1) that it is less than the efficient level of investment since at least one μ_s must be positive

The optimum contract which maximises the transnational corporation's ex ante utility can be found by starting with an initial value $\sigma = 0$ which corresponds to $V^1 = V_{\min}$ (see Figure 2) As we have just seen $V_s^1 \geqq V^1$ with a strict inequality with positive probability, so $V^\tau > V^{\tau-1}$ with positive probability and is never less than $V^{\tau-1}$ The technical reason that V stays constant when the country's constraint is not binding is that because V_s is not constrained, were it chosen differently from the previous period's V the slope of the value function would differ at these two dates, but clearly these slopes cannot differ since one way of achieving an exchange of utility in the previous period is to move along U in an unconstrained state this period More intuitively, because of the concavity of the problem, V should be kept constant when it is possible to do so Reducing V_s in a non-binding state allows higher V_s's to be offered in binding states and current I could be increased, the cost is that I would be lower in the future, as I is positively related to V, if the non-binding state occurred, and the extra variability in the contract would mean that overall payoffs are reduced Thus over time V increases, moving around the Pareto-frontier In CASE 1 this proceeds until $V = V^*$ after which $\mu_s = 0$ in all states and from (3 1) the investment level is efficient, and V remains constant In CASE 2 V rises to $V_{\max}$ where it remains and the transnational corporation's payoff is zero each period

Since investment increases with V, investment rises to the efficient level in CASE 1 and to some constant less than the efficient level in CASE 2 Moreover investment is procyclical (with a one-period lag) This follows directly since a high value of s will produce a large temptation to renege leading to a larger increase in V_s and hence a large increase in I next period Transfers are always zero until either $V_s = V^*$ (in CASE 1) or $V_s = V_{\max}$ (in CASE 2) Intuitively because both parties are risk neutral and discount at the same rate it does not matter for discounted utilities when the host country receives the transfers But the presence of the host country's self-enforcement constraint means that it pays to delay the transfer to offer a "carrot" to prevent reneging Once $V^*(V_{\max})$ is attained any further postponement of the transfer would make it worthwhile for the transnational corporation to renege at some future date Thus we have the fundamental observation that the entire contract evolves according to a ratchet effect, sometimes increasing, sometimes staying the same, but never falling[15] and eventually tending to a stationary state This is summarized by

Proposition 1. *Investment is non-decreasing over time, attaining a maximum value in the steady state with probability one which may be less than the efficient level The discounted utility of the host country is also non-decreasing and transfers are zero until the period before the maximum value of investment is attained*

15 This is different from the ratchet effect identified by Laffont and Tirole (1988) There an agent who reveals too much good information in the first period faces a stiffer incentive scheme in the second period

As discussed once V_{max} or V^* is reached the contract is stationary, V remains constant with the transfer chosen appropriately to satisfy the feasibility and self-enforcing constraints. Therefore if a non-trivial contract exists at all, there must be a non-trivial stationary contract and if it is possible to attain the efficient level of investment, there must exist an efficient stationary contract. So the questions of existence and efficiency can be answered by looking for a non-trivial and an efficient stationary contract. Proposition 2 shows that if an Inada condition on the production function holds there is always a non-trivial contract. If on the other hand $E[(r(I, s)/I)]$ is bounded above in I then there will be a critical value of the discount factor below which no non-trivial contract exists and above which one always exists. Likewise the efficient level of investment will be sustainable if and only if the discount factor is above some critical value.

Proposition 2. (i) *There exists an* α^*, $0<\alpha^*<1$, *such that a stationary contract at* I^* *exists if and only if* $1>\alpha\geqq\alpha^*$. (ii) *If* $r(0, s)=0$ *and* $r'(I, s)\to\infty$ *as* $I\to 0$ *for all* s *then there exists a non-trivial stationary contract for all* $\alpha\in(0, 1)$. (iii) *If* $E[(r(I, s)/I)]$ *is bounded above then there exists an* α', $0<\alpha'<1$, *such that a non-trivial stationary contract exists if and only if* $1>\alpha>\alpha'$.

At this stage an example may help. Until now we have proceeded as if $U(V)$ was known, but of course it has to be calculated as part of the solution. It would be possible to calculate $U(V)$ by starting with the first-best frontier and repeatedly applying the mapping defined in Secion 2. Since V never decreases over time there is, however, a simpler procedure. $U(V)$ depends only on its properties above V, so it is possible to calculate it by working backward from V_{max}.

Consider a simple example with two equi-probable states in which $r(I, 1)=0$ and $r(I, 2)=4\sqrt{I}$, so $I^*=1$. The assumption that output is always zero in state one makes everything much simpler since $t_1=0$ and the host country's constraint does not bind, implying $V_1=V$. Further it can be shown that it is optimum to set $I=\min(1, V^2(2-\alpha)^2/16)$, $V_2=\min(V^*, \beta^{-1}V)$ and $t_2=\max(0, \alpha(\beta^{-1}V-V^*))$, where $\beta=\alpha/(2-\alpha)$ and that the value function is

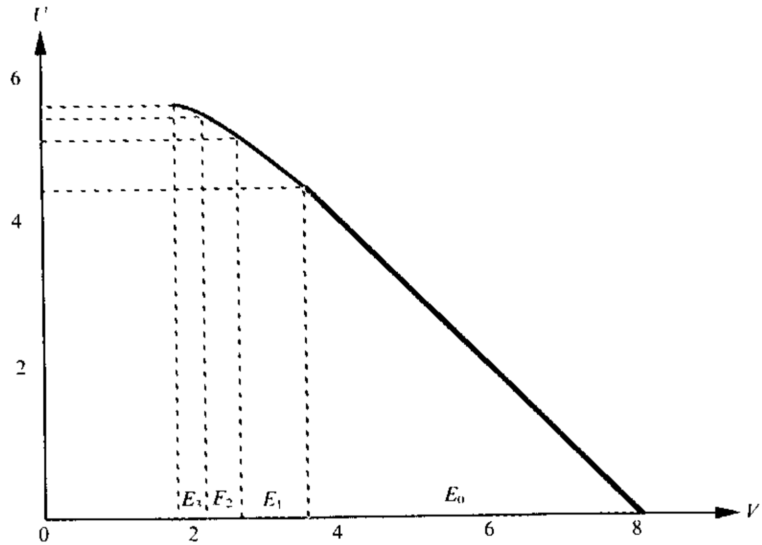
$$U(V)=V_{max}-V, \qquad V\in[V^*, V_{max}],$$

$$U(V)=\beta^n V_{max}+(n-1)V-\alpha(1/8)\left(\textstyle\sum_{i=1}^{n}\beta^{-i}\right)V^2, \qquad V\in[\beta^n V^*, \beta^{n-1}V^*]$$

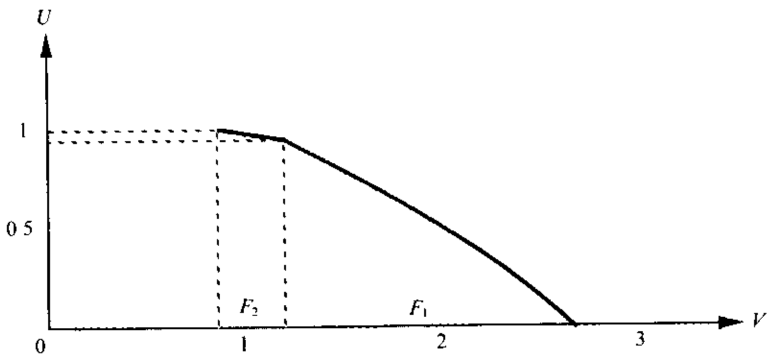
for $n=1, 2, \ldots, m$, where m is the number such that $V_{min}\in[\beta^m V^*, \beta^{m-1}V^*]$. It can be checked that this function is continuous and concave.

The example does not quite meet the conditions of Proposition 2(ii), nevertheless it is easy to show that a non-trivial contract exists for all α. First consider the value of V_{max} and suppose $I=1$. Since $U(V_{max})=0$, $t_2=2$ so that $V_{max}=1/(1-\alpha)$. For this to be feasible requires $t_2+\alpha V_{max}\geqq 4$, therefore CASE 1 applies if $\alpha\geqq 2/3$. Further the constraint will not bind provided $t_2+\alpha V_2\geqq 4$. But since $t_1=0$ and $V_1=V$, $t_2+\alpha V_2=V(2-\alpha)$ so that $I=1$ for any $V\geqq 4/(2-\alpha)$. Therefore for $\alpha\geqq 2/3$, $U(V)$ is linear in the range $E_0=[V^*, V_{max}]$ where $V^*=4/(2-\alpha)$. This means for $V\in E_0$ the choice of V_2 and t_2 is not uniquely defined. Further from Lemma 7, $U(V)$ is everywhere differentiable as can be easily checked. An example for the discount factor $\alpha=7/8$ is drawn in Figure 3. In this case $m=3$, $V_{min}=1.81$ and the regions of V are $E_0=[3.55, 8]$, $E_1=[2.77, 3.55)$, $E_2=[2.15, 2.77)$, $E_3=[1.81, 2.15)$.

If $\alpha<2/3$ CASE 2 applies and it is not possible to attain the efficient level of investment and the host country's self-enforcing constraint binds at $V=V_{max}$. Then $V_{max}=8\alpha/(2-\alpha)^2$ and $V(2-\alpha)=t_2+\alpha V_2=4\sqrt{I}$ or $I=V^2(2-\alpha)^2/16$. $U(V)$ is not differentiable at



(*a*) CASE 1 Discount factor 0.875



(*b*) CASE 2 Discount factor 0.625

FIGURE 3
Example of value functions

$\beta^n V_{\max}$ but is still concave. The value function for a discount factor of $\alpha = 5/8$ is drawn in Figure 3b. Here $m = 2$, $V_{\min} = 0.91$ and $F_1 = [1.2, 2.65]$ and $F_2 = [0.91, 1.2)$.

4 CAPITAL ACCUMULATION

Capital accumulation can be introduced in a simple manner by assuming that start of period investment adds to current capital stock and that a constant fraction $\delta > 0$ of the inherited capital stock depreciates. Capital stock at time τ is then $K^\tau = (1-\delta)K^{\tau-1} + I^\tau$ and the model of Section 3 is covered by the special case of $\delta = 1$. To choose the optimum capital stock the transnational corporation must take into account its user cost, $c =$

$\alpha(r+\delta)$, where $r=(1-\alpha)/\alpha$ is the interest carrying cost and δ is the depreciation cost, and the sum is multiplied by α to convert it into current period dollars It is assumed that there is a unique positive K^* maximizing $Er(K,s)-cK$ such that the expected value of output covers investment costs, $I^*=\delta K^*$, that is $Er(K^*,s)-\delta K^*>0$

The host country when it expropriates inherits the capital stock We shall let $D(K,s)$ be the benefit to the host country when it expropriates if the capital stock is K and the state is s The self-enforcing constraint for the host country in states is $t_s+\alpha V_s \geqq D(K,s)$ If, for example, the host country is unable to use the capital without the transnational's expertise, $D(K,s)=r(K,s)+(1-\delta)K$, the value of current output plus the scrap value of future capital in a perfect market It is assumed that $D(0,s)=0$, $D'>0$ and $D''\leqq 0$, with $D''(K,s)\geqq r''(K,s)$ for each K and s, and $D(K,s)\geqq D(K,s')$ for $s>s'$

Formally the value function will now depend also upon the capital stock so let $U(V^\tau,(1-\delta)K^{\tau-1})$ be the payoff to the corporation discounted to τ when the country gets V^τ and the inherited capital from the previous period is $(1-\delta)K^{\tau-1}$ As $D''(K,s)\geqq r''(K,s)$, Lemma 2 can be used mutatis mutandis to prove that $U(V^\tau,(1-\delta)K^{\tau-1})$ is concave in V^τ For the moment assume that negative investment is possible we shall see that the solution to this relaxed problem is also the solution to the problem with the non-negativity constraint $I\geqq 0$ Define $U(V)=U(V,0)$, that is, the value function at the beginning of the period if the initial capital stock is zero Then it follows that $U(V^\tau,(1-\delta)K^{\tau-1})=U(V^\tau)+(1-\delta)K^{\tau-1}$, since any contract which is self-enforcing and delivers V^τ starting from no capital will also do the same starting from $(1-\delta)K^{\tau-1}$ if it is changed so that I^τ is reduced by $(1-\delta)K^{\tau-1}$, *and vice versa* [16] hence the optimum contract must be the same except for this difference in initial investment which translates directly into the corporation's payoff The optimality equation is

$$U(V,(1-\delta)K^{-1}) = \max_{K(t_s\ V_s)\in R_+^{2N+1}} \{-(K-(1-\delta)K^{-1})+E[r(K,s)-t_s+\alpha U(V_s,(1-\delta)K)]\},$$

where the participation constraints are now $t_s+\alpha V_s\geqq D(K,s)$ and $U(V_s,(1-\delta)K)\geqq 0$, and K^{-1} denotes last period's capital stock The first order conditions (A 8)–(A 10) in the Appendix are unchanged (the superdifferentials being with respect to V), and Lemma 3(i) holds as before, so $\sigma\leqq 1$ Because σ depends only upon V, and not on K^{-1}, the updating dating rule for V can be derived from these conditions as argued in Section 3, and likewise K can be found by looking for the value which, using the updating rule for V_s, gives exactly the required V to the country This is as before except for two complications Firstly if $D(K,s)>r(K,s)$ we are faced with a new possibility, that the self-enforcing constraint in s may bind even when $t_s=r(K,s)$, and secondly $V_{\max}$ depends upon K we shall write it as $V_{\max}(K)$ in what follows

Consider first the problem of finding the largest feasible steady state level of K less than K^* choose K and (t_s) to maximize K subject to $K\leqq K^*$, $\delta K\leqq E[r(K,s)-t_s]$, $t_s+E[t_s]/(1-\alpha)\geqq D(K,s)$ and $0\leq t_s\leq r(K,s)$ all s Let $K_{\max}$ be the solution There are three possibilities (i) K^* is feasible, (ii) $K_{\max}<K^*$ and $t_s=0$ except where the country's self-enforcing constraint binds—this corresponds exactly to the earlier analysis, or (iii) $K_{\max}<K^*$ and at least one self-enforcing constraint is binding with $t_s=r(K,s)$, and $t_s>0$ in some states where the constraint is not binding—this is the new possibility just referred to,[17] these

16 The value function is defined so that the corporation's participation constraint holds from the second period on, but its overall payoff from the first period may be negative

17 If the constraint $t_s\leqq r(K,s)$ was dispensed with then (iii) could not arise

latter payments cannot be reduced without violating the constraint in the binding state(s) through their effect on V In this last case consider the slope of $U(V)$ just to the left of $V_{\max}(K_{\max})$ so long as the future value of V is set to $V_{\max}(K_{\max})$ in the states where the constraint is binding, then K can be set equal to $K_{\max}$, with payments in the non-binding states reduced, unlike case (ii) where K must be reduced Thus the slope will be -1 here, despite the fact that efficiency is not attainable Unravelling the first-order conditions as before leads to the following updating rule given a choice of K, in state s if $|U'(V)| < 1$ and $V \leqq V_{\max}(K)$ then $V_s = V$ and $t_s = 0$ unless at these values the country's self-enforcing constraint is violated, in which case assign the country a utility of $D(K, s)$ by setting $V_s = V_{\max}(K)$ and choosing t appropriately Now choose K so that following this rule delivers exactly V If $|U'(V)| = 1$ then set $K = K_{\max}$ and choose $V_s = V_{\max}(K_{\max})$ and $t_s = r(K, s)$ for any states where this would generate exactly $D(K, s)$, in other states set $V_s = V$ and choose t_s so that the self-enforcing constraint is satisfied and overall utility is V Notice that K must be non-decreasing in V, and as before, V is non-decreasing, provided $V \leqq V_{\max}(K)$

If $V_{\max}(K) < V$, the first-order conditions would dictate that each V_s is reduced to $V_{\max}(K)$ To see that this cannot arise, make the induction assumption that $V^{\tau} \geqq V^{\tau-1}$ The updating rule implies then that $K^{\tau} \geqq K^{\tau-1}$, so $V_{\max}(K^{\tau}) \geqq V_{\max}(K^{\tau-1})$ and since $V^{\tau} \leqq V_{\max}(K^{\tau-1})$, $V_{\max}(K^{\tau}) \geqq V^{\tau}$ So the problem would not arise, and V would indeed be non-decreasing as before The induction assumption holds in the initial period as K^1 cannot be negative and $V^1 \leqq V_{\max}(0)$ for the corporation to be prepared to participate, so $V^1 \leqq V_{\max}(K^1)$, and each $V_s^2 \geqq V^1$

To summarise the evolution of V will again follow a ratchet-like rule which says that V remains constant whenever the country's self-enforcing constraint does not bind, and rises to just satisfy the constraint otherwise K will follow the same ratchet-like path, except when $U'(V) = -1$, when it will remain constant even if V increases Consequently gross investment is always strictly positive, and so the model which incorporates the constraint $I \geqq 0$ will have the same solution as the unconstrained model

As before, questions concerning the existence of a non-trivial contract and its long-run properties can be answered simply by looking at the largest feasible steady-state capital stock ($\leqq K^*$), $K_{\max}$

Proposition 3. *K is non-decreasing over time, converging to $K_{\max}$ with probability one*

Provided $D(K, s)$ is bounded above the efficient capital stock, K^*, will be sustainable for a high enough discount factor, although it should be noticed that in this case K^* is itself increasing in α since an increase in α decreases the interest carrying costs of capital In the long-run a steady-state is attained with $I = \delta K_{\max}$

That the efficient capital stock is not attained instantaneously is often attributed to adjustment costs (see e g Gould (1968)) The slow adjustment here is caused by the absence of a legally binding contract The model of Allen (1983) can be viewed as a special case of this model in which there is no uncertainty, $D(K) = r(K)$ ("borrowers can only default on interest") and $\delta = 0$ Moreover he considers only the distribution in which the country extracts all the surplus $V^1 = V_{\max}(0)$ Our results then confirm Allen's that the steady state is achieved immediately, and this may involve K^*, $K < K^*$ (which he refers to as credit rationing) or no existing equilibrium depending upon the parameters

5 RENEGOTIATION-PROOFNESS

The solution identified in Section 3 is not renegotiation-proof despite being confined to the Pareto frontier of the set of all equilibrium payoffs The reason is simple the punishment meted out to the host country when it reneges, to be cut off from all future investment, is Pareto-dominated by points on the second-best frontier,[18] and would therefore be subject to renegotiation. The most severe punishment which can be imposed is $V_{\min}$, anything lower by definition also gives the transnational corporation a lower payoff.

It is nevertheless possible to find a renegotiation-proof set of equilibria by adding $V_{\min}$ to the R.H.S. of (2.3), and finding a fixed point of the mapping. (It is easy to see that the corporation's constraint is not a problem assuming we consider only renegotiations occurring at the end of each period.) We shall consider only the case of certainty.

Consider a set of points represented by the decreasing non-negative function $U(V)$ defined on some compact domain, dom $U(V)$, with $V_{\min}$ defined as the minimum V in dom $U(V)$. The constraint is

$$V \geqq r(I) + \alpha V_{\min} \tag{5.1}$$

Suppose that $LU(V)$ is the maximum value function of the problem to maximize $-I+r(I)-t+\alpha U(V^{+1})$ by choosing $V^{+1} \in \text{dom } U(\ )$, t, I, subject to (5.1), $V=t+\alpha V^{+1}$, and $0 \leqq t \leqq r(I)$, where V^{+1} is the continuation payoff to the host country. LU is defined whenever the constraint set is non-empty and the maximum value is non-negative. Let $L^*U(\ )$ correspond to the weak Pareto-efficient part of the graph of $LU(\ )$ (i.e. $\text{dom } L^*U(\ ) = \{V \in \text{dom } L^*U(\ )$ if $V' \in \text{dom } L^*U(\ )$ then either $V' \leqq V$ or $L^*U(V') \leqq L^*U(V)\}$). We say that $U(\ )$ is *renegotiation-proof* if dom $U(\ ) =$ dom $L^*U(\ )$ and $U(\ ) = L^*U(\ )$. Hence any fixed point of the mapping L^* corresponds to a set of payoffs which is "renegotiation-proof" (van Damme (1991)), since no payoff Pareto dominates any other and each payoff corresponds to an equilibrium in which all continuation payoffs also belong to the set.[19] The punishment strategies that support points in this set give the host $V_{\min}$ and the transnational corporation $U(V_{\min})$. We can show that contracts corresponding to points in this renegotiation-proof set have the same qualitative properties as before.

Proposition 4. *If there is no uncertainty then any renegotiation-proof solution has the same characterization as in Proposition 1.*

As an example we solve the case with $r(I)=\sqrt{I}$. Then $I^*=1/4$, and attention will be restricted to the case where efficiency is sustainable at some point in the set. Working backwards as in the example of Section 3 for a given value of $V_{\min}$, and computing the new value of $V_{\min}$, it is straightforward to show that there is a unique fixed point which, using the same notation as before, falls in the interval E_2. The value function is $U_{RP}(V)=((1/4(1-\alpha))-V)$ for $V \in E_0=(0.5+\alpha V_{\min}, 1/4(1-\alpha)]$ and $U_{RP}(V)=\sum_{i=1}^{n} (-V^2\alpha^{-i+1}-V_{\min}^2\alpha^{i+1}-V_{\min}\alpha^i)+V(2nV_{\min}+n-1)+\alpha^n/4(1-\alpha)$ for $V \in E_n=(\max\{V_{\min}, \alpha^n(0.5+\alpha V_{\min})\}, \alpha^{n-1}(0.5+\alpha V_{\min})]$ and for $i=1, 2$, where $V_{\min}=\alpha/2(1+\alpha-2\alpha^2)$. Notice that this solution is only valid (E_0 is an interval) if $\alpha \geqq \sqrt{0.5}$, whereas without imposing renegotiation-proofness the efficient level of investment is sustainable for $\alpha \geqq 0.5$. Since the set of payoffs (the graph of $U_{RP}(V)$ for $V \in [V_{\min}, 1/4(1-\alpha)]$) includes part of the

18. This is not true of the wage-contracts model in Thomas and Worrall (1988). Asheim and Strand (1991) have identified strategies which show that the solution identified there does satisfy renegotiation-proofness. The concept of renegotiation-proofness has been applied to problems of country risk by Kletzer and Wright (1990) who study sovereign debt in a consumption-smoothing model.

19. We require that domains are equal; some definitions require only weak inclusion of the graph of $U(V)$ in the set of all (V, U) pairs that can be sustained using continuation values drawn from $U(V)$. Our definition is the one given in van Damme (1991) for renegotiation proof sets (see also Ray (1989)), and is in contrast for example to Farrell and Maskin (1989) who use weak inclusion in their definition of 'weak renegotiation-proofness'. Our definition allows us to show that the domain must be an interval.

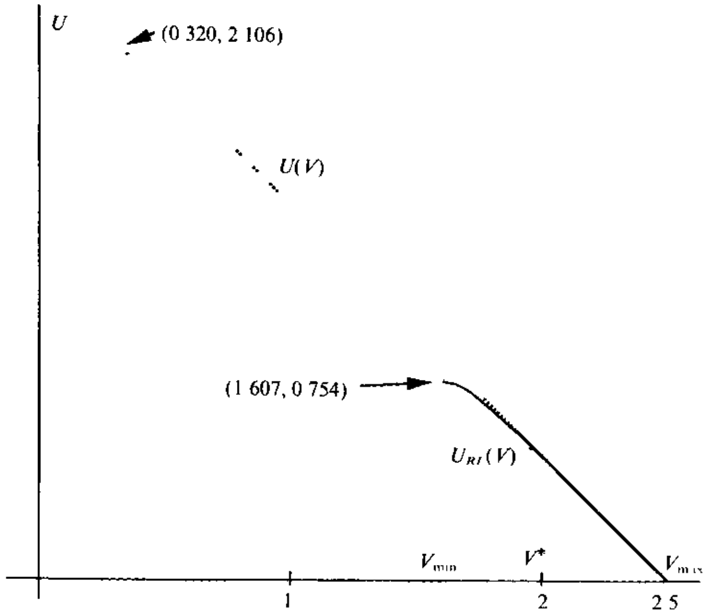


FIGURE 4
A renegotiation-proof set

unconstrained first-best frontier it must be true that no point in any other weakly renegotiation-proof set Pareto-dominates the whole set

The impact of renegotiation-proofness on utilities is illustrated for $\alpha = 0\ 9$ in Figure 4[20] It is also interesting to consider what happens for high discount factors First multiply payoffs by $(1-\alpha)$ to normalize Then $V_{min} = \alpha/2(1+2\alpha)$ which tends to 1/6 as α tends to one, while $V_{max} = 1/4$ for all $\alpha \geqq \sqrt{0\ 5}$, so U_{RP} converges to that part of the first-best frontier on [1/16, 1/4] whereas U converges to the entire frontier by the folk theorem

6 RISK-AVERSE HOST COUNTRY

Until now it has been assumed that the host country is risk neutral However, in some circumstances it may be pertinent to assume that the host country is risk averse, for example if the project is large, or if its returns are dependent upon one or two unstable markets, or if the host country has limited access to international capital markets to smooth out fluctuations in returns The latter is most obviously appropriate for many less developed countries In the risk-neutral case the host country is unconcerned about the timing of receipts and is only interested in the value of discounted payments As a consequence the optimum contract "back-loads" all transfers into the future This is clearly undesirable if the host country is risk averse and it is important to know to what extent our back-loading results are preserved in this case

The formal analysis is left to the Appendix In summary all the qualitative results of Section 3 carry over provided that the host country is not too risk averse What exactly is meant by "not too risk averse" is spelt out in Assumption B of the Appendix but roughly a condition on the relative curvatures of the utility function of the host country

20 To find $U(V)$, set $V_{min} = 0$ in the definition of $U_{RP}(V)$

and the production function needs to be satisfied, in the neighbourhood of unconstrained efficient investment the coefficient of absolute risk aversion of the host country should be smaller than $|r''/r'|$.

One of our main results of Section 3 was that investment was initially lower than the efficient level. But when the host country is risk averse it is less clear what is meant by this comparison since, even in the absence of the self-enforcing constraints, investment in a first-best contract will vary with the level of utility given to the host country because of the non-negativity constraints (2.1)–(2.2). A comparison will therefore be made between the optimum self-enforcing contract which gives the host country a net future utility of V and the first-best contract which gives the host country V in the absence of the self-enforcing constraints. Let $I^*(V)$ denote the optimum investment level in the first-best contract which gives the host country V, and $I(V)$ be the investment level in the first period of an optimum self-enforcing contract (Lemma 12 in the Appendix shows that $(I(V)$ is a continuous function). In the risk-neutral case $I^*(V) = I^*$ but with risk aversion $I^*(V) \geqq I^*$.

As in Section 3 if a non-trivial contract exists there are two possible cases: either $V_{\max} = V^{\#}$ (CASE 1) or $V_{\max} < V^{\#}$ (CASE 2) where $P^*(V^{\#}) = 0$ and $P^*(V)$ is the first-best frontier, i.e. either the first-best contract is self-enforcing for V high enough or not. Letting V^* be the smallest V in CASE 1 such that the first-best contract is self-enforcing we have the following result that if some self-enforcing constraint binds then there is underinvestment.

Proposition 5. *(i) in* CASE 1 $I(V) < I^*(V)$ *for* $V < V^*$ *and* $I(V) = I^*(V)$ *for* $V^* < V < V^{\#}$, *(ii) in* CASE 2 $I(V) < I^*(V)$.

The dynamics of the optimum self-enforcing contract are derived in the Appendix. The optimum contract evolves as in the risk-neutral case "ratchet-like" with V non-declining over time and increasing if the relevant self-enforcing constraint binds. Unlike the risk-neutral case however, the transfer is not necessarily zero in the initial periods and although the expected value of the transfer is non-decreasing over time the actual transfer may fall if a bad state occurs. As before investment is non-decreasing over time. In CASE 1 it can be shown that investment increases with positive probability in each period so that the efficient level of investment $I(V^*)$ is approached but, in contrast to the risk-neutral case, never quite reached. In CASE 2 $I(V_{\max})$ is attained with probability one.

Proposition 6. *(i) in* CASE 1 $V^{\tau} \to V^*$ *and* $I^{\tau} \to I^*(V^*)$ *with probability one with each increasing with positive probability in each period (so* $V^{\tau} < V^*$ *and* $I^{\tau} < I^*(V^*)$ *for all* τ*)* *(ii) If* CASE 2 *applies then* $V^{\tau} = V_{\max}$ *and* $I^{\tau} = I(V_{\max})$ *eventually with probability one.*

APPENDIX

Define $V^{\#}$ to be the largest discounted utility the host country can receive in the first-best (unconstrained) problem, subject to giving the investor zero utility ($V^{\#} = \Pi^*/(1-\alpha)$ under risk neutrality). In the space of bounded functions on $[0, V^{\#}]$ consider some decreasing not necessarily differentiable function P and define the mapping L as follows

$$L(P)(V) = \sup_{I,(t_s, V_s) \in R^{S+1}} \{-I + E[r(I,s) - t_s + \alpha P(V_s)]\}, \tag{A.1}$$

subject to

$$\sigma \quad E[v(t_s)+\alpha V_s] \geqq V, \tag{A 2}$$

$$p_s\mu_s \quad v(t_s)-v(r(I,s))+\alpha V_s \geqq 0, \qquad s=1,2, \quad ,N, \tag{A 3}$$

$$\alpha p_s\phi_s \quad P(V_s) \geqq 0, \qquad s=1,2, \quad ,N, \tag{A 4}$$

$$p_s\theta_s \quad r(I,s)-t_s \geqq 0, \qquad s=1,2, \quad ,N, \tag{A 5}$$

$$p_s\pi_s \quad t_s \geqq 0, \qquad s=1,2, \quad ,N \tag{A 6}$$

Notice that the constraint set is always non-empty provided $P(V) \geqq 0$ for some V—simply choose I large enough with $t_s = r(I,s)$ $LP(V)$ may be negative, but (A 4) ensures that only non-negative values are relevant (As U was defined in the text, its domain may be smaller than $[0, V^{\#}]$, it is only necessary to extend it by defining arbitrary negative values for V outside of the original domain and then to consider U to be L of this extended function this U is a fixed point of L) This is not a standard concave programming problem even when P is concave because the self-enforcing constraint (A 4) means that the constraint set is not convex Neither, unfortunately, is L a contraction mapping in the supremum metric, despite the presence of strict discounting, and L has more than one fixed point when a non-trivial contract exists, the zero function being one, and U itself another Technically, the reason for this is the presence of the value function itself in the constraints (in (A 4)) Nevertheless the following can be proved

Lemma 1 *Define P^* as the unconstrained first-best Pareto frontier for the problem without constraints* (A 3) *and* (A 4) *Then $L^n(P^*)$ converges pointwise to U as $n \to \infty$*

Proof (i) Notice that when P^* is the first-best frontier $L(P^*) \leqq P^*$

(ii) Make the induction assumption that $L^n(P^*) \leqq L^{n-1}(P^*)$ Compare $L(L^n(P^*))$ and $L(L^{n-1}(P^*))$ The constraint set in the latter case is at least as large as in the former, so $L(L^{n-1}(P^*)) \geqq L(L^n(P^*))$, i e $L^n(P^*) \geqq L^{n+1}(P^*)$, thus completing the induction assumption

(iii) Hence $L^n(P^*)$ is a decreasing sequence, and must therefore converge pointwise to some limit function say U^0

(iv) U^0 is a fixed point of L To see this, consider for any fixed V, the sequence of variables chosen at each application of L $(I^n, (t_s^N, V_s^n))$, which are a solution to $L(L^{n-1}(P^*))(V)$ Because $L^{n+1}(P^*) \leqq L^n(P^*)$ the constraint (A 3) does not relax as n increases Hence the sequence belongs to a compact set and has a convergent subsequence, converging to, say, $(I^*, (t_s^*, V_s^*))$ We have $L^{n-1}(P^*)(V_s^n) \geqq 0$, for each n in the subsequence, so in the limit $U^0(V_s^*) \geqq 0$, for each s, and the limit contract clearly satisfies all other constraints in the problem $L(U^0)(V)$, and gives the transnational corporation a utility of $U^0(V)$ Consequently $LU^0(V) \geqq U^0(V)$ However since $L^{n-1}(P^*) \geqq L^n(P^*) \geqq \quad \geqq U^0$, we have $L^n(P^*) \geqq L(U^0)$, and taking the limit as $n \to \infty$, $U^0 \geqq L(U^0)$ So $U^0 = L(U^0)$

(v) Every fixed point U^1 of L corresponds to a family of self-enforcing contracts in the sense that there is a self-enforcing contract which gives the host country a discounted utility of V, and the transnational corporation, $U^1(V)$, for any V satisfying $V \geqq 0$, $U^1(V) \geqq 0$ Consider the contract formed by the repeated application of L, starting from utility V, so that the variables in the first period of the contract are the I and $t_s(1)$'s that solve Problem A from V, the second period contract, contingent upon $s(1)$ occurring in the first period, is then the solution to Problem A from $V_s(1)$, and so on As in any discounted programming problem, this contract must deliver V and $U^1(V)$ respectively to the two parties, and this same argument guarantees that because constraints (A 3) and (A 4) are satisfied at each point in the future, the self-enforcing constraints proper are satisfied All other constraints are clearly also met, so this contract is as required

(vi) Since $P^* \geqq U$, $L^n(P^*) \geqq L^n(U) = U$, and in the limit $U^0 \geqq U$ From (v) and by definition of U, therefore, $U^0 = U$ ||

Now suppose that $P(V)$ is a concave function We can now state first-order conditions for the programming problem in the definition of the mapping L, using the fact that U is a fixed point

$$E[r'(I,s)(1-\mu_s v'(r(I,s))+\theta_s)] = 1, \tag{A 7}$$

$$\sigma+\mu_s+(\pi_s-\theta_s)/v'(t_s), \qquad s=1,2, \quad ,N, \tag{A 8}$$

$$-((\sigma+\mu_s)/(1+\phi_s)) \in \partial P(V_s), \qquad s=1,2, \quad ,N, \tag{A 9}$$

together with an envelope condition

$$-\sigma\in\partial L(P)(V), \tag{A 10}$$

where $\partial P(V)$ denotes the set of the superdifferentials of P at V. Under the assumption that P is concave, this set is an interval which is a single point at almost every value of V in the domain of P.

Part A. Risk-Neutral Case

Assumption A. The host country is risk neutral.

The next lemma shows that if the host country is risk neutral, the fixed point of the mapping U, is concave so that the first-order conditions (A 7)-(A 10) can be used to characterise the optimum contract.

Lemma 2. *$U(V)$ is strictly decreasing and concave on $(V_{\min}, V_{\max}]$.*

Proof. Assume that $L^{n-1}P(V)$ is concave. For a given V, V' and corresponding contracts $(I, (t_s, V_s))$, $(I', (t'_s, V'_s))$, consider the contract $(I^\lambda, (t^\lambda_s, V^\lambda_s))$ where $I^\lambda=\lambda I+(1-\lambda)I'$, $V^\lambda_s=\lambda V_s+(1-\lambda)V'_s$ and $t^\lambda_s=\lambda t_s+(1-\lambda)t'_s+r(I^\lambda, s)-(\lambda r(I, s)+(1-\lambda)r(I', s))$. This new contract is feasible, it satisfies (A 3), and offers neither the host country nor transnational corporation less overall utility. Then $L^nP(V)$ is concave and since P^* is concave and U is the pointwise limit of L^nP^* from Lemma 1, $U(V)$ is itself concave. ||

The next five lemmas establish the basic characteristics of the optimum contract which are described in Proposition 1. Lemma 3 shows that the slope of the value function is never greater than minus one and that the upward constraints on t_s, (A 5), never bind. Lemmas 4 and 5 show that t_s is positive only if next period the maximum or efficient level of investment is attained. Lemma 6 shows that I is increasing in V so that I is non decreasing over time, as are V_s and t_s. Lemma 7 is a technical lemma.

Lemma 3. (i) *$\sigma\leqq 1$ for $V\in[V_{\min}, V_{\max}]$ and* (ii) *$\theta_s=0$ for all s.*

Proof. (i) Suppose that $\sigma>1$. From (A 8) $\theta_s=(\sigma-1)+\mu_s+\pi_s>0$ for all s so that $t_s=r(I, s)$. By concavity if $\sigma>1$ anywhere then certainly at $V_{\max}$. If $\phi_s>0$ then $V_s=V_{\max}$ which implies $U(V_s)=0$. If $\phi_s=0$, then from (A 9) $-(\sigma+\mu_s)\in\partial U(V_s)$ which implies $-\inf\partial U(V_s)>1$, then in the next period $\theta>0$ and again $t_s=r(I, s)$ for all s and so on. This means the transnational corporation is making negative profits which is a contradiction. Thus $\sigma\leqq 1$.

(ii) If $\theta_s>0$ then as $\sigma\leqq 1$, from (A 8) either μ_s or π_s is strictly positive but by complementary slackness $\theta_s>0$ implies $\pi_s=0$. So $\mu_s>0$ and (A 3) binds and as (A 5) binds too $V_s=0$ which is inefficient and hence impossible. ||

Lemma 4. *If* CASE 1 *applies then there is by assumption some (t_s, V_s) for any $V\in[V^*, V_{\max}]$ such that I^* is sustainable and U is linear on $(V, V_{\max}]$ with slope of -1. On $(V_{\min}, V^*)$ U is strictly concave and $I<I^*$. If* CASE 2 *applies then U is strictly concave on $(V_{\min}, V_{\max})$ and $I<I^*$ for all $V\in[V_{\min}, V_{\max}]$.*

Proof. Suppose U is linear on some interval (V, V') and consider the contracts $(I, (t_s, V_s))$ and $(I', (t'_s, V'_s))$. First suppose $I<I^*$. Then from (A 7) $\mu_s>0$ for some s and $\sigma<1$. Then from linearity $\sigma'<1$ implying $I'<I^*$. Moreover since U is strictly decreasing the contract $(I^\lambda, (t^\lambda_s, V^\lambda_s))$ defined in Lemma 2 satisfies $U(V^\lambda)\geqq U(V^\lambda+E[r(I^\lambda)-(\lambda r(I, s)+(I', s))])\geqq\lambda U(V)+(1-\lambda)U(V')$ which can only hold with equality if $I=I'$. From (A 7) and using the implicit function theorem μ_s is a continuous function of I and hence $\mu_s=\mu'_s$. If $\mu_s>0$, $t_s+\alpha V_s=r(I, s)=r(I', s)=t'_s+\alpha V'_s$. If $\mu_s=0$, then $\pi_s=\pi'_s>0$ since $\sigma=\sigma'<1$ from (A 8). Then $t_s=t'_s=0$ and $V_s\in(V, V')$ from (A 9) and (A 10). But then $E[|(t_s+\alpha V_s)-(t'_s+\alpha V'_s)|]\leqq|V'-V|$ as $\alpha<1$ which contradicts (A 2). Thus $U(V)$ is strictly concave unless it is possible to sustain I^*. This occurs only in CASE 1 on $[V^*, V_{\max}]$. By definition $\sigma(V^*)=1$, so as $\sigma\leqq 1$ from Lemma 3 and since U is concave from Lemma 2 it follows that for any $V\in[V^*, V_{\max}]$ $I=I^*$ and $\sigma=1$. ||

If the slope is greater than -1 then it will always pay to have $t_s=0$ since this allows a higher V which generates a loss in future profits smaller than the gain due to reducing t_s.

Lemma 5 *$t_s > 0$ only if $-1 \in \partial U(V_s)$*

Proof Suppose $t_s > 0$, then $\pi_s = 0$ and $\mu_s = 1 - \sigma$ from (A 8) If $\phi_s > 0$ then $V_s = V_{max}$ and $\partial U(V_s) \in \partial U(V_{max})$ where $-1 \in \partial U(V_{max})$ If $\phi_s = 0$ then straightaway from (A 9) $-1 \in \partial U(V_s)$ ||

Higher levels of I up to I^* are desirable, the problem is they increase the temptation for the host country to renege, as V increases the benefit from adhering to the contract also increases so higher I is possible

Lemma 6 *In* CASE 1 *I is strictly increasing in V for $V \in (V_{min}, V^*)$ and in* CASE 2 *I is strictly increasing in V for $V \in (V_{min}, V_{max})$ In both cases V_s and t_s are non-decreasing in V for $V \in [V_{min}, V_{max})$*

Proof Consider $V_{max} \geqq V' > V$ First suppose $V'_s < V_s \leq V_{max}$ Then $\phi'_s = 0$ and since by assumption $\sigma' > \sigma$ we have from (A 9) $\mu_s > \mu'_s \geqq 0$ Then from (A 7), $I' > I$ Since $V'_s < V_{max}$, $t'_s = 0$ Therefore $t_s + \alpha V_s = r(I, s) < r(I', s) \leqq \alpha V'_s$ This implies $t_s < \alpha(V'_s - V_s) < 0$, a contradiction Therefore $V'_s \geqq V_s$

Suppose $\mu'_s > \mu_s \geqq 0$, then from (A 7), $I' < I$ From the first part of the proof $V_{max} \geqq V'_s \geqq V_s$, so that $\sigma_{max} \geqq \sigma'_s \geqq \sigma_s$ First consider $\sigma_s < \sigma_{max}$ Then $t_s = 0$, so $t'_s + \alpha V'_s = r(I', s) < r(I, s) \leqq \alpha V_s$ *But since $V'_s \geqq V_s$ this implies $t'_s < 0$* which is impossible If $\sigma_s = \sigma_{max}$ then $t'_s - t_s < 0$ But from (A 8) $\pi_s > \pi'_s \geqq 0$ since $\sigma' > \sigma$ again implying $t'_s < 0$ Thus $\mu_s \geqq \mu'_s \geqq 0$ and $I' \geqq I$, and if $\sigma < 1$, $I < I' \leqq I^*$

Now suppose $t_s > t'_s \geqq 0$ From Lemma 5, $-1 \in \partial U(V_s)$, but $V'_s \geqq V_s$ from above If $V'_s > V_s$ then the intersection of $\partial U(V'_s)$ and $\partial U(V_s)$ is empty since U is strictly concave But $\sigma \leqq 1$, so $-1 \in \partial U(V'_s)$, a contradiction If $V'_s = V_s$ on the other hand, from (A 8) $\mu_s > \mu'_s \geqq 0$ as $\sigma' > \sigma$ Then $t_s + \alpha V_s = r(I, s) < r(I', s) \leqq t'_s + \alpha V'_s$ or $t_s < t'_s$, a contradiction ||

Lemma 7 *In* CASE 1 *$U(V)$ is differentiable on $V \in (V_{min}, V_{max})$*

Proof Since V_s is non-decreasing over time in the interval $V \in [V_{min}, V^*)$, it attains some $V \in [V^*, V_{max}]$ Here $\partial U(V)$ is unique and equal to -1 and $\phi_s = 0$ Hence σ is also unique and $U(V)$ is differentiable ||

Proposition 1 *Investment is non-decreasing over time, attaining a maximum value in the steady state with probability one which may be less than the efficient level The discounted utility of the host country is also non-decreasing and transfers are zero until the period before the maximum value of investment is attained*

Proof It only remains to show that the steady-state is attained with probability one Consider the path in which state N occurs at each date, and recall that $V_N \geqq V_s$, all s, and from the updating rule, if $V' > V$ then $V'_N \geqq V_N$ If t_N is always zero on this path, then it must be on all other paths, which is impossible Therefore eventually $t_N > 0$, which can only be the case if V_{max} or V^* has been reached, let period T be the first date at which this is true Then on any other path in which state N occurs at least T times V_{max} or V^* will be attained, thus this happens with probability one ||

Proposition 2 (i) *There exists an α^*, $0 < \alpha^* < 1$, such that a stationary contract at I^* exists if and only if $1 > \alpha \geqq \alpha^*$* (ii) *If $r(0, s) = 0$ and $r'(I, s) \to \infty$ as $I \to 0$ for all s, then there exists a non-trivial stationary contract for all $\alpha \in (0, 1)$* (iii) *If $E[(r(I, s)/I)]$ is bounded above then there exists an α', $0 < \alpha' < 1$, such that a non-trivial stationary contract exists if and only if $1 > \alpha > \alpha'$*

Proof For a stationary contract $(I, (t_s))$ to be feasible it must satisfy

$$(t_s/I) - (r(I, s)/I) + ((\alpha/(1-\alpha))E[(t_s/I)] \geqq 0, \qquad s = 1, 2, \quad , N, \tag{A 11}$$

$$-1 + E[((r(I, s)/I) - (t_s/I)] \geqq 0, \tag{A 12}$$

$$(r(I, s)/I) \geqq (t_s/I) \geqq 0, \qquad s = 1, 2, \quad , N \tag{A 13}$$

(i) The first thing to notice about the constraints is that as at least one $t_s > 0$ they are strictly relaxed as α increases At I^*, $-1 + Er(I^*, s)/I^* > 0$ So consider (t_s) such that (A 12) and (A 13) are satisfied where at least one $t_s > 0$ since $V > 0$ Therefore for α near enough one (A 11) will be satisfied On the other hand for α near enough zero (A 11)–(A 13) cannot hold simultaneously from (A 11) $(r(I^*, s)/I^*) - (t_s/I^*) \leqq ((\alpha/(1-\alpha))E[t_s/I^*]$ and from (A 12) $((\alpha/(1-\alpha))E[t_s/I^*] \leqq ((\alpha/(1-\alpha))\, E[(r(I^*, s)/I^*) - 1]$ which can be made less than one for α small enough so $r(I^*, s)/I^*) - (t_s/I^*) < 1$ which contradicts (A 12)

(ii) Set $t_s = r(I, s)$, $s < N$ and $t_N = r(I, N) - I/p_N$. Then (A 12) holds with equality, (A 11) holds for $s < N$ and if $s = N$, then $-(1/p_N) + ((\alpha/(1-\alpha))E[(r(I, s)/I) - 1] \geq 0$ which is satisfied if I is small enough since $E[r(I, s)/I] \to \infty$ as $I \to 0$. Likewise (A 13) holds for $s < N$ and for $s = N$ it becomes $r(I, N)/I \geq (r(I, N)/)I - (1/p_N) \geq 0$, which again holds for I small enough. Thus for any $\alpha \in (0, 1)$ there is a feasible stationary contract with $I > 0$.

(iii) By assumption there is some $(I, (t_s))$ satisfying (A 12) and (A 13) with at least one $t_s > 0$. The proof proceeds along the lines of part (i) given that $E[(r(I, s)/I) - 1]$ is bounded above. ||

Proposition 3 *K is non-decreasing over time, converging to K_{max} with probability one.*

Proof. Suppose that state N recurs. Because V is non-decreasing and bounded it converges to some limit V_{lim} with corresponding K_{lim}. By continuity of the updating rule in V, at V_{lim} we have $V_N = V_{lim}$; this can only hold if either the first-best K^* has been attained, or $U(V_{lim}, (1-\delta)K_{lim}) = 0$ and the self-enforcing constraints are just binding in the sense that a tightening in any binding constraint would imply there is no feasible solution. If $K_{lim} \neq K_{max}$, then the limit contract, which is stationary, must be a convex combination (as in the concavity proof) of the stationary contracts $K = 0$ and $K = K_{max}$, with weight K_{lim}/K_{max} on the latter; this is feasible, offers both parties the same utilities as the limit contract, but because of the strict concavity of the production function, cannot have binding self-enforcing constraints, contrary to assumption, so $K_{lim} = K_{max}$. Since state N occurs infinitely often with probability one, the proposition follows. ||

Proposition 4 *If there is no uncertainty then any weakly renegotiation-proof solution has the same characterization as in Proposition 1.*

Proof. First we establish that any renegotiation-proof set is the graph of a continuous function $U(\ )$ with slope at least equal to -1 and which meets the $U = 0$ axis. Consider an arbitrary point $V \in \mathrm{dom}\, U(\ )$. By definition there exists I, t, and $V^{+1} \in \mathrm{dom}\, U(\ )$—we use $^{+1}$ to denote continuation payoffs throughout—satisfying $V = t + \alpha V^{+1}$, and $0 \leq t \leq r(I)$ with $U(V) = -I + r(I) - t + \alpha U(V^{+1})$. Consider a small increase in I to I' and a corresponding increase in t to t' with $t' - t = r(I') - r(I)$, leaving V^{+1} unchanged. The host country's utility is increased above V, to say V', while U falls. Provided $U(V) > 0$ initially this is feasible, since the new value of U, say U', is positive for a small enough increase and incentive compatibility is satisfied for the host, hence this shows that the constraint set is non-empty just to the right of V, and moreover $U(V') = (LU(V')) \geq U'$. We sketch the argument to show there cannot be any discontinuities: by the argument just given there cannot be any "downward jumps" in $U(V)$ (i.e. a value of U which does not belong to the graph but such that lower and higher values do). The only remaining possibility is a "horizontal jump" in the graph, that is, values V', $V \in \mathrm{dom}\, U(\ )$, such that $V' < V$, $U(V') = U(V)$ and there exists no V'', $V' < V'' < V$ with $V'' \in \mathrm{dom}\, U(\ )$. Assume that such a discontinuity exists. At V we must have $t = 0$ since otherwise the construction just used, but now with $I' < I$, also generates points to the north-west of $U(V)$ which would Pareto-dominate the payoff at V'. Hence $V^{+1} = V/\alpha$. It follows that at least above αV_{max} there can be no discontinuity, as this would imply an infeasible value for V^{+1}, and moreover there is a discontinuity such that the continuation V^{+1} corresponding to the right end of the discontinuity belongs to the interior of this continuous part (the most rightward one in the case of a finite number of discontinuities). A convex combination of the contracts corresponding to either end of this discontinuity, as in the proof of Lemma 2, with the weight on the right-hand contract sufficiently large to guarantee that V^{λ} belongs also to the continuous part generates a point in $L(U)$ with the same value for U as the end-points, but with $t > 0$, and the above construction can again be used to generate a Pareto-dominating point contrary to assumption. Consequently $U(V)$ must be defined on some interval, continuous in V and meet the $U = 0$ axis.

Next define $V^* = r(I^*) + \alpha V_{min}$; notice that for $V < V^*$, we have $I < I^*$ for the self-enforcing constraint (5 1) to hold. In such a range the slope (the right-hand derivative) is greater than -1 since using once more the above construction, an increase in I to I' is achieved together with an increase in V (to V') of $r(I') - r(I)$, so U falls to U' by the increase in I while V rises by the increase in $r(I)$ which is greater for a small increase in I because $I < I^*$; as $U(V')$ must be at least U' the result follows. Now for any $V \geq V^*$ investment can be set equal to I^* without violating (5 1), and indeed must be since choosing a different value of I only reduces profits. Suppose that $V \geq V^*$ (i.e. suppose the domain of $U(V)$ extends past V^*). We claim $U(V)$ can always be attained with a continuation payoff $V^{+1} \geq V^*$; to see this suppose $V^{+1} < V^*$. Then consider changing the contract so that V^{+1} is changed to $V^{+1\prime} = V^*$ itself, and t is changed to $t' = t - \alpha(V^* - V^{+1})$. This is feasible since if it had been the case that $t < \alpha(V^* - V^{+1})$, then $V = t + \alpha V^{+1} < \alpha V^*$ which is impossible as $V \geq V^* > 0$. Hence $t' \geq 0$ and all the constraints are satisfied. The change in payoff for the transnational corporation is $\alpha(V^* - V^{+1}) - \alpha(U(V^{+1}) - U(V^*)) > 0$ by the slope being greater than -1 between V^{+1} and V^*. Thus the change increases profits, establishing the claim We have shown that once above V^*, V will remain permanently above V^* This implies that I remains at I^* But $V+U(V)$, which is just the sum of discounted net revenue, equals $(r(I^*)-I^*)/(1-\alpha)$, a constant This establishes that the slope equals -1 for all $V \geq V^*$ Finally we show that $U(V)$ must be defined below V^* start with $V=V^*$ and notice that $V^{+1}=V^*$ attains the maximum We must have $t>0$ since otherwise $V=\alpha V$ which is impossible for $V>0$ By reducing I as in the first construction of our proof, and reducing t by the reduction in output (which must be feasible for small reductions), values of $U>U(V^*)$ are derived, hence $U(V)$ must be defined below V^* Summarizing whether or not the function $U(V)$ is defined to the right of V^*, it will have slope less than -1 initially

To see that the contract characterization of Proposition 1 is optimum, consider first V such that $V/\alpha < V^*$ Then t must be set to zero, and $V^{+1}=V/\alpha$, with I such that $r(I)+\alpha V_{\min}=V$ since if $t>0$, reducing t to zero and increasing V^{+1} by t/α leads to an increase in U of $t-(\alpha U(V^{+1})-\alpha U(V^{+1}+t/\alpha))>0$ by virtue of slope being greater than -1 (if this implies $V^{+1}>V_{\max}$ then increase V^{+1} only to $V_{\max}$, and if $V=V_{\max}$ then $V^{+1}=V_{\max}$ and $t>0$) For higher values of V it is only necessary to have $V^{+1} \geq V^*$, so t need not be zero Provided $V<V^*$, I satisfies $r(I)+\alpha V_{\min}=V$, so I is increasing The characterization then follows ||

Part B Risk-Averse Case

The method of argument to prove that the value function is concave in Part A no longer works when the host country is risk averse In order to prove concavity in the risk-averse case we introduce a new assumption

Assumption B The host country has a C^2, strictly concave, per-period, utility function $v(t)$ defined over the transfer t, $v(0)=0$, uncertainty is multiplicative, $r(I,s)=g(s)r(I)$ with $r(I)$ also C^2 and strictly concave, and $\{((r(I')-r(I))/r'(I)-(I'-I)\}-E[((v(g(s)r(I'))-v(g(s)r(I)))/v'(g(s)r(I))-g(s)(r(I')-r(I))]$ is negative for all $I \neq I'$

Before proceeding it will be useful to consider two benchmark problems the first-best problem in which the self-enforcing constraints (A 3) and (A 4) are not included and the sub-problem which solves the optimum way of giving the host country the minimum gain when output is known to be y

Consider first the problem of finding a stationary contract to maximize $-I+E[r(I,s)-t_s]$ subject to $Ev(t_s)=(1-\alpha)V$ and the constraints (A 5) and (A 6) Given Assumption B that $v(t)$ and $r(I)$ are strictly concave this is a standard concave programming problem Denote the solutions as $I^*(V)$ and $t_s(V)$ and the optimum value function as $P^*(V)$ Letting $\sigma^*=-P^{*\prime}(V)$ and $t(\sigma^*)$ satisfy $\sigma^*=1/v'(t)$ we have the following lemma

Lemma 8 (i) *t_s and I^* are continuous, differentiable and non-decreasing functions of V,* (ii) *$I^*(V) \geq I^*$ and $t_s^*(V)=\min\{t(\sigma^*), r(I^*(V),s)\}$,* (iii) *$P^*(V)$ is strictly concave and differentiable on $(0, V^{\#})$ where $-I^*(V^{\#})+E[r(I^*(V^{\#}),s)-t_s^*(V^{\#})]=0$*

Proof Straightforward ||

Since the value function $P^*(V)$ is strictly concave the optimum first-best contract is indeed stationary and the solution is described by Lemma 8 for any given value of V

Now consider the following sub-problem of choosing (t, V) to maximize $y-t+\alpha U(V)$ subject to $U(V) \geq 0$, $v(t)+\alpha V-v(y) \geq 0$, $t \leq y$ and $t \geq 0$ with multipliers $\alpha\underline{\phi}$, $\underline{\mu}$, $\underline{\theta}$ and $\underline{\pi}$ where a bar beneath the multiplier denotes that it refers to this sub-problem The solution $(\underline{t}, \underline{V})$ corresponds to the optimum way of giving the host country the minimum gain when output is known to be y If we assume for the moment that the undominated part of $U(V)$ is concave on $(V_{\min}, V_{\max})$ where $U(V_{\max})=0$ then this sub-problem is a standard concave programming problem and the solution $(\underline{t}, \underline{V})$ will be unique as $v(t)$ is strictly concave For any $\underline{V}>V_{\min}$, $\underline{\theta}=0$ and $\underline{\mu}>0$ We shall let $Q(y)$ which is continuous and strictly concave in y denote the maximum value function for this sub-problem

Lemma 9 (i) *$\underline{t}$ is a continuous non-decreasing (strictly increasing for $\underline{t}>0$) function of y and $\underline{V}$ is a continuous non-decreasing (strictly increasing for $\underline{V} \in (V_{\min}, V_{\max})$) function of y,* (ii) *If $\underline{\pi}>0$, $\underline{\phi}=0$ then $\underline{V}=v(y)/\alpha$, if $\underline{\pi}=\underline{\phi}=0$, then $\underline{V}$ satisfies $v^{-1}(v(y)-\alpha\underline{V})=t(\underline{\mu})$ and $-\underline{\mu} \in \partial U(\underline{V})$, if $\underline{\phi}>0$ then $\underline{V}=V_{\max}$,* (iii) *$\underline{t}=v^{-1}(v(y)-\alpha\underline{V})$,* (iv) *$\underline{\mu}$ is a possibly multi-valued non-decreasing function of y,* (v) *$1-\underline{\mu}v'(y) \in \partial Q(y)$*

Proof Straightforward ||

We now turn to the analysis of the optimum self-enforcing contract and show that the limit function of the mapping defined by (A 1) is concave under Assumption B

Lemma 10 *If $L^{n-1}P(V)$ is concave then so too is $L^nP(V)$.*

Proof. Consider any two values V and V' and the associated contracts $(I, (t_s, V_s))$ and $(I', (t'_s, V'_s))$.

$$L^nP(V')-L^nP(V)=E[(r(I',s)-r(I,s))-(I'-I)]-E[t'_s-t_s]+\alpha E[L^{n-1}P(V'_s)-L^{n-1}P(V_s)] \quad \text{(A.14)}$$

Consider each of the three terms on the R.H.S. in turn.

(i) Let $\Phi_s(I'-I)=(r(I',s)-r(I,s))-r'(I,s)(I'-I)$. $\Phi_s(I'-I)\leqq 0$ with equality iff $I'=I$, since $r(I,s)$ is strictly concave and differentiable in I. Similarly let $\Omega_s(I'-I)=(v(r(I',s))-v(r(I,s)))-v'(r(I,s))(r(I',s)-r(I,s))$. Again $\Omega_s(I'-I)\leqq 0$ with equality iff $I'=I$, since v is differentiable. Multiplying both sides of (A.7) by $(I'-I)$ gives

$$(I'-I)=(I'-I)E[(1+\theta_s-\mu_s v'(r(I,s)))r'(I,s)]=E[(1+\theta_s-\mu_s v'(r(I,s)))((r(I',s)-r(I,s))-\Phi_s(I'-I))]$$

Then rearranging terms and using the definition for $\Omega_s(I'-I)$ gives

$$\begin{aligned} E[(r(I',s)-r(I,s))-(I'-I)]&=E[\mu_s(v(r(I',s))-v(r(I,s)))]-E[\theta_s(r(I',s)-r(I,s))]\\ &\quad+E[(1+\theta_s-\mu_s v'(r(I,s)))\Phi_s(I'-I)]-E[\mu_s\Omega_s(I'-I)] \end{aligned} \quad \text{(A.15)}$$

(ii) From (A.5) and (A.6), $\theta_s(r(I,s)-t_s)=0\leqq\theta_s(r(I',s)-t'_s)$ and $\pi_s t_s=0\leqq\pi_s t'_s$ by complementary slackness. Therefore $-(t'_s-t_s)\leqq-(1-\pi_s+\theta_s)(t'_s-t_s)+\theta_s(r(I',s)-r(I,s))$. But from (A.8), $(1-\pi_s+\theta_s)=-(\sigma+\mu_s)V'(t_s)$. So $-(1-\pi_s+\theta_s)(t'_s-t_s)=-(\sigma+\mu_s)v'(t_s)(t'_s-t_s)\leqq-(\sigma+\mu_s)(v(t'_s)-v(t_s))$ since v is concave. Therefore combining terms and taking expectations

$$-E[t'_s-t_s]\leqq-E[(\sigma+\mu_s)(v(t')-v(t_s))]+E[\theta_s(r(I',s))] \quad \text{(A.16)}$$

(iii) From (A.4) $\phi_s L^{n-1}P(V_s)=0\leqq\phi_s L^{n-1}P(V'_s)$. So $L^{n-1}P(V'_s)-L^{n-1}P(V_s)\leqq(1+\phi_s)(L^{n-1}P(V'_s)-L^{n-1}P(V_s))$. But from (A.9) $-(\sigma+\mu_s)/(1+\phi_s)\in\partial L^{n-1}P(V_s)$ and since $L^{n-1}P(V_s)$ is concave $L^{n-1}P(V'_s)-L^{n-1}P(V_s)\leqq\partial L^{n-1}P(V_s)(V'_s-V_s)$. So

$$\alpha E[L^{n-1}P(V'_s)-L^{n-1}P(V_s)]\leqq-\alpha E[(\sigma+\mu_s)(V'_s-V_s)] \quad \text{(A.17)}$$

Then substituting (A.15), (A.16), (A.17) into (A.14) gives

$$\begin{aligned} L^nP(V')-L^nP(V)&\leqq E[\mu_s(v(t_s)+\alpha V_s-v(r(I,s)))]\\ &\quad-E[\mu_s(v(t'_s)+\alpha V'_s-v(r(I',s)))]\\ &\quad+E[(1+\theta_s-\mu_s v'(r(I,s)))\Phi_s(I'-I)]-E[\mu_s\Omega_s(I'-I)] \end{aligned}$$

But from (A.2), $E[v(t_s)+\alpha V_s]=V$ and $E[v(t'_s)+\alpha V'_s]=V'$ and from (A.4) $\mu_s(v(t_s)+\alpha V_s-v(r(I,s)))=0\leqq\mu_s(v(t'_s)+\alpha V'_s-v(r(I',s)))$ and from (A.10) $\sigma=\partial L^nP(V)$. So

$$L^nP(V')-L^nP(V)\leqq\partial L^nP(V)(V'-V)+E[(1+\theta_s-\mu_s v'(r(I,s)))(\Phi_s(I'-I)]-E[\mu_s\Omega_s(I'-I)] \quad \text{(A.18)}$$

With the assumption $r(I,s)=g(s)r(I)$, $\Phi_s(I'-I)=g(s)\Phi(I'-I)$, where $\Phi(I'-I)=[(r(I')-r(I))-r'(I)(I'-I)]$. Then

$$E[(1+\theta_s-\mu_s v'(r(I,s)))\Phi_s(I'-I)]=E[(1+\theta_s-\mu_s v'(g(s)r(I)))g(s)]\Phi(I'-I)=\Phi(I'-I)/r'(I)$$

using (A.7). From (A.8) $\mu_s v'(t_s)=1+\theta_s-\pi_s-\sigma v'(t_s)\leqq 1+\theta_s$. As $\theta_s\mu_s=0$, since otherwise $V_s=0$ which is obviously inefficient, and $t_s\leqq r(I,s)$ from (A.4), $\mu_s v'(r(I,s))\leqq\mu_s v'(t_s)\leqq 1$. Then as $\Omega_s(I'-I)\leqq 0$, $-\mu_s\Omega_s(I'-I)\leqq\Omega_s(I'-I)/v'(r(I,s))$. So

$$E[(1+\theta_s-\mu_s v'(r(I,s)))\Phi_s(I'-I)]-E[\mu_s\Omega_s(I'-I)]\leqq(\Phi(I'-I)/r'(I))-E[\Omega_s(I'-I)/v'(r(I,s))]\leqq 0$$

where the final inequality follows directly by assumption B with equality if and only if $I'=I$. ||

Lemma 11 *The limit function $U(V)$ is concave.*

Proof. From Lemma 8 $P^*(V)$ is concave and from Lemma 10 it follows that $L^nP^*(V)$ is concave. But from Lemma 1 the pointwise limit of $L^nP^*(V)$ is $U(V)$ so it too is concave.

Having proved that $U(V)$ is concave it is now possible to use the first-order conditions (A.7)–(A.10), replacing P and $L(P)$ by U to characterize the optimum solution. In fact it can be shown that $U(V)$ is strictly concave, but first we prove a useful technical result.

Lemma 12 (i) *There is a unique value of I which solves the dynamic programme and hence I is a continuous function of V, (ii) each μ_s and q_s are continuous functions of V*

Proof Using Lemma 10 and setting $V=V'$ in equation (A 18) applied to the limit function it can be seen that the inequality can only be satisfied if $I=I'$, i e there is a unique solution for I and so from the maximum theorem I is continuous in V (ii) Using the implicit function theorem for equation (A 7) shows that μ_s and θ_s are continuous functions of I given $E[r''(I, s)]<0$ But from part (i) of this lemma I is a continuous function of V ||

Lemma 12 is extremely useful it shows that $y_s(V)=r(I(V), s)$ is a continuous function of V and so from Lemma 9 the solution to the sub-problem for each possible state s are continuous functions of V, which we can write as $\underline{t}_s(V)$ and $\underline{V}_s(V)$

Lemma 13 *The limit function $U(V)$ is strictly concave*

Proof Suppose U is linear over some interval $[V, V']$ Then $U(V')-U(V)=U'(V)(V'-V)$ But the inequality (A 18) of Lemma 10 shows that this can only hold if $I=I'$ Consider then the contracts $(I, (t_s, V_s))$ and $(I', (t_s', V_s'))$ starting from V and V' From Lemma 9 and 12 it follows that $\underline{V}_s(V)=\underline{V}_s(V')$ and $\underline{t}_s(V)=\underline{t}_s(V')$ since I is unchanged Thus we refer simply to $\underline{t}_s$ and $\underline{V}_s$ There are three cases to consider (i) $\underline{V}_s \leqq V$, (ii) $V'<\underline{V}_s$, (iii) $V<\underline{V}_s \leqq V'$

(i) For $\underline{V}_s \leqq V$ it is shown that $V_s, V_s' \in [V, V']$ and that $t_s=t_s'=\max\{0, \min\{t(\sigma), r(I, s)\}\}$ where $\sigma=-U'(V)$ From the solution to the sub-problem $-\underline{\mu}_s \in \partial U(\underline{V}_s)$, remembering that $\underline{\phi}_s=0$ as $\underline{V}_s \leqq V<V' \leqq V_{\max}$, and from (A 9) $-(\sigma+\mu_s)/(1+\phi_s) \in \partial U(V_s)$ By assumption $\underline{V}_s \leqq V$ so using Lemma 11 that U is concave $\underline{\mu}_s \leqq \sigma$ First it is shown that $\mu_s=0$ So suppose otherwise $\mu_s>0$ If $\phi_s>0$ then $V_s=V_{\max}>\underline{V}_s$ and if $\phi_s=0$ $V_{\max} \geqq V_s \geqq V'>V$ where the middle inequality follows from concavity of U So in either case $V_s>\underline{V}_s$ Also from (A 3) and Lemma 9 $v(t_s)-v(t_s)=\alpha(\underline{V}_s-V_s)<0$ But from Lemma 9 $\underline{t}_s=\max\{0, t(\underline{\mu}_s)\}$ and from (A 8) $t_s=\max\{0, \min t(\sigma+\mu_s), r(I, s)\}\}$ Hence if $\mu_s>0$, then $t_s>\underline{t}_s$ as $\sigma+\mu_s>\sigma \geqq \underline{\mu}_s$, which implies $v(t_s)-v(\underline{t}_s)>0$ a contradiction So $\mu_s=0$ Then if $\phi_s>0$, $\sigma/(1+\phi_s)<\sigma$, which implies $V_s=V_{\max}$ by definition and $V_s<V<V_{\max}$ by concavity, a contradiction So $\phi_s=\mu_s=0$ which from (A 8) and (A 9) gives $t_s=\max\{0, \min t(\sigma), r(I, s)\}\}$ and $V_s \in [V, V']$ Now the same argument can be used to show $\phi_s'=\mu_s'=0$ so $t_s'=t_s$ and $V_s' \in [V, V']$

(ii) For $V'<\underline{V}_s$ it is shown $v(t_s)+aV_s=v(t_s')+\alpha V_s'=v(\underline{t}_s)+\alpha \underline{V}_s$ We first show $\mu_s>0$ Suppose $\mu_s=0$ From (A 9) $-\sigma/(1+\phi_s) \in \partial U(V_s)$ and from (A 10) $-\sigma \in \partial U(V)$ If $\phi_s>0$ then $V_s=V_{\max} \geqq V'$ But $\sigma/(1+\phi_s)<\sigma$ implies $V_s \leqq V<V_{\max}$ so $\phi_s=0$ Then as $-\sigma \in \partial U(V_s)$, $V_s \in [V, V']$ implying $V_s<\underline{V}_s$ But then $\underline{\mu}_s \geqq \underline{\mu}_s/(1+\phi_s) \geqq \sigma$ since $-\underline{\mu}_s/(1+\underline{\phi}_s) \in \partial U(\underline{V}_s)$ and $\underline{t}_s \geqq t_s$ since $t_s=\max\{0, \min\{t(\sigma), r(I, s)\}\}$ and $\underline{t}_s=\max\{0, t(\underline{\mu}_s)\}$ Using (A 3) then shows $v(t_s)+\alpha V_s \geqq V(\underline{t}_s)+\alpha \underline{V}_s$ or $\alpha(V_s-\underline{V}_s) \geqq v(\underline{t}_s)-v(t_s) \geqq 0$ a contradiction So $\mu_s>0$ and $v(t_s)+\alpha V_s=V(\underline{t}_s)+\alpha \underline{V}_s$ A similar argument shows $v(t_s')+\alpha V_s'=V(\underline{t}_s)+\alpha \underline{V}_s$

(iii) For $V<\underline{V}_s \leqq V'$ there are two sub-cases to consider (a) $\underline{\phi}_s>0$, then $\underline{V}_s=V_{\max} \geqq V_s$ and if $\mu_s=0$, $\underline{t}_s>t_s$ since $\sigma=\underline{\mu}_s$ which contradicts (A 3) So μ_s and μ_s' are strictly positive and $v(t_s)+\alpha V_s=v(t_s')+\alpha V_s'$ (*b*) $\underline{\phi}_s=0$, then by part (i) $\mu_s'=\phi_s'=0$ so that $t_s'=\underline{t}_s$ and $V_s' \in [\underline{V}_s, V']$ If $\mu_s>0$ then $v(t_s)+\alpha V_s=v(\underline{t}_s)+\alpha \underline{V}_s$ and $|v(t_s)+\alpha V_s-(v(t_s')+\alpha V_s')| \leqq \alpha|V'-\underline{V}_s|$ If $\mu_s=0$ then $t_s=\underline{t}_s$ and $V_s \in [\underline{V}_s, V']$ so again $|v(t_s)+\alpha V_s-(v(t_s')+\alpha V_s')| \leqq \alpha|V'-\underline{V}_s|$

In all three cases $|v(t_s)+\alpha V_s-(v(t_s')+\alpha V_s')|<|V-V'|$ for all s since $\alpha<1$ But given $\sigma>0$ we have a contradiction since (A 2) must hold with equality i e $E[v(t_s)+\alpha V_s-(v(t_s')+\alpha V_s')]=V-V'$ ||

Lemma 14 *For a given value of V and hence a given value of I* (i) *if $V<\underline{V}_s$ then $V_s=\underline{V}_s$ and $\mu_s>0$,* (ii) *if $V \geqq \underline{V}_s$ then $V_s=V$ and $\mu_s=0$*

Proof Follows directly from the proof of Lemma 13 ||

We refer to Lemma 14 as the updating lemma as it shows how V changes from one period to the next Of course from Lemma 9 and value $\underline{V}_s$ depends upon I So in order to know how the contract evolves over time it is necessary to know how I changes with V

Lemma 15 (i) *I is non-decreasing in V and strictly increasing unless $I=I^*$*, (ii) *t_s is non-decreasing in V*

Proof (i) Suppose $I'<I$ and $V<V'$ We show first that $\theta_s'\geqq\theta_s\geqq 0$ Suppose $\theta_s'<\theta_s$ Then $\mu_s=\pi_s=0$ and from (A 8) $\sigma'\leqq\sigma'+\mu_s'=(1-\pi_s'+\theta_s')$ and $(1+\theta_s)/v'(t_s)=\sigma$ But by definition $t_s'\leqq r(I',s)\leqq r(I,s)=t_s$ so $1/v'(t_s')\leqq 1/v'(t_s)$ and $\sigma'\leqq(1-\pi_s'+\theta_s')/v'(t_s')<(1+\theta_s)/v'(t)=\sigma$ But from the concavity of U, $\sigma'>\sigma$ a contradiction Moreover if $\theta_s>0$ then $\theta_s'>\theta_s$ since from (A 8) $(1+\theta_s)=\sigma v'(t_s)<\sigma' v'(t_s')=(1+\theta_s')$

Next consider the sign of $(1-\mu_s' v'(r(I',s))+\theta_s')-(1-\mu_s v'(r(I,s))+\theta_s)$ This is clearly positive if $\theta_s'>0$ since then $\mu_s'=0$ and $\theta_s'\geqq\theta_s\geqq 0$ from above This is equally true if $\mu_s'=0$ but $\mu_s>0$, since in that case $\theta_s=0$ If $\mu_s'>0$ then it follows directly from the updating rule (Lemma 14) and from Lemma 9 that $\underline{V}(y)$ is non-decreasing that $\mu_s>0$ With $\mu_s>0$ and $\mu_s'>0$ Lemma 14 and (A 3) show that $t_s=\underline{t}(y)$ and $t_s'=\underline{t}(y')$ so that $\underline{\mu}_s=\sigma+\mu_s$ and $\underline{\mu}_s'=\sigma'+\mu_s'$ Now from Lemma 9, $\underline{\mu}_s'<\underline{\mu}_s$ which implies $\mu_s'<\mu_s$ since $\sigma<\sigma'$ But in the solution to the sub-problem the maximum value function $Q(y)$ is concave so it can be inferred from Lemma 9 that $(1-\underline{\mu}_s' v'(r(I',s)))-(1-\underline{\mu}_s v'(r(I,s)))\geqq 0$ Hence $(1-\mu_s' v'(r(I',s)))-(1-\mu_s v'(r(I,s)))=(1-\underline{\mu}' v'(r(I',s)))-(1-\underline{\mu}_s v'(r(I,s)))+\sigma' v'(r(I',s))-\sigma v'(r(I,s))>0$ since $\sigma'>\sigma$ and $v'(r(I',s))\geqq v'(r(I,s))$ Then since $r'(I',s)\geqq r'(I,s)$ we have $E[r'(I',s)(1-\mu_s' v'(r(I',s)+\theta_s')-r'(I,s)(1-\mu_s v'(r(I,s)+\theta_s)]>0$ which contradicts equation (A 7) that the difference should be zero unless $\mu_s'=\mu_s=\theta_s'=\theta_s=0$ for all s in which case $I=I'=I^*$

(ii) Suppose $V'>V$ and $t_s'<t_s$ Since $I(V')\geqq I(V)$ from part (i) it follows from (A 8) that $\sigma+\mu_s>\sigma'+\mu_s'$ Now from part (i) and Lemma 9 it follows that $\underline{V}_s(V')\geqq\underline{V}_s(V)$ so that the updating lemma shows $V_s'\geqq V_s$ But if $V_s<V_{\max}$ from (A 9) $\sigma+\mu_s\leqq\sigma'+\mu_s'$ from concavity, a contradiction, and if $V_s=V_{\max}$ from (A 3) $v(t_s')+\alpha V_{\max}\geqq v(y')\geqq v(y)=v(t_s)+\alpha V_{\max}$ where the last equality holds as $\sigma+\mu_s>\sigma'+\mu_s'$ implies $\mu_s>0$ given $\sigma'>\sigma$ But then $t_s'\geqq t_s$ also a contradiction ‖

Since I and t_s are non-decreasing in V the updating lemma shows that the contract evolves "ratchet-like" with investment and the expected transfer non-decreasing over time Next we prove the main propositions of the text Proposition 5 shows that investment is below the first-best level if any self-enforcing constraint binds Proposition 6 deals with the long-run properties of the optimum contract, in CASE 1 investment increases with positive probability each period and approaches but never quite reaches the efficient level, in CASE 2 investment increases to $I(V_{\max})$ where it stays thereafter

Proposition 5 (i) *In* CASE 1 *$I(V)<I^*(V)$ for $V<V^*$ and $I(V)=I^*(V)$ for $V^*<V<V^{\#}$*, (ii) *in* CASE 2 *$I(V)<I^*(V)$*

Proof Suppose $I(V)\geqq I^*(V)$ and μ_s for some s If $\sigma\geqq\sigma^*$, then since $r(I(V),s)\geqq r(I^*(V),s)$ and $\sigma+\mu_s\geqq\sigma^*$ it follows from (A 8) and Lemma 8 that $t_s\geqq t_s'$ for all s, and since $\mu_s>0$ for some s, $V_s>V$, so that the host country's utility is higher in the constrained case—contrary to assumption Hence $\sigma<\sigma^*$ Then $\theta_s\leqq\theta_s^*$, since if $\mu_s>0$, $\theta_s=0$ and when $\mu_s=0$ and $\theta_s>0$, from (A 5) $t_s=r(I(V),s)\geqq r(I^*(V),s)\geqq t_s$ so that $v'(t_s^*)\geqq v'(t_s)$ which implies $\theta_s\leqq\theta_s^*$ from (A 8) given $\sigma<\sigma^*$ But $\theta_s\leqq\theta_s^*$ for all s and $\mu_s>0$ for some s implies from (A 7) that $I(V)<I^*(V)$, a contradiction If $\mu_s=0$ for all s then by definition $I(V)=I^*(V)$ and hence this applies for $V\geqq V^*$ in CASE 1 ‖

Proposition 6 (i) *In* CASE 1 *$V^\tau\to V^*$ and $I^\tau\to I^*(V^*)$ with probability one with each increasing with positive probability in each period (so $V^\tau<V^*$ and $I^\tau<I^*(V^*)$ for all τ)* (ii) *If* CASE 2 *applies then $V^\tau=V_{\max}$ and $I^\tau=I(V_{\max})$ eventually with probability one*

Proof (i) (a) At V^*, $\underline{V}_s(V^*)\leqq V^*$ for all s by the updating rule of Lemma 14, since otherwise V^* could not be a steady state But $\underline{V}(y)$ is strictly increasing in y up to the point where $\underline{V}(y)=V_{\max}$ from Lemma 9 and I is strictly increasing in V for $V<V^*$ from Lemma 15 so $\underline{V}_s(V)<\underline{V}_s(V^*)$ for $V<V^*$ But by the updating rule $V_s(V)\leqq\underline{V}_s(V)$ and hence $V_s(V)\leqq\underline{V}_s(V)<\underline{V}_s(V^*)\leqq V^*$

(b) Next consider V as a stochastic process $\{V^\tau\}_{\tau=0}$ Note that $\phi_s=0$ as $V^\tau<V^*$ So from (A 11) $\mu_s(V^\tau)=\sigma(V^{\tau+1})-\sigma(V^\tau)$ and as $\sigma\leqq\sigma_{\max}$ we have $\mu_s(V^\tau)\to 0$ along all paths Moreover state N occurs infinitely often with probability one, so on almost all paths by choosing those dates when state N occurs, there is a convergent subsequence of V^τ s such that $\mu_N(V^\tau)\to 0$ For any such path $\mu_N(\lim V^\tau)=0$ by the continuity of μ_N in V, Lemma 11 This implies $\mu_s(\lim V^\tau)=0$ for all s since $\mu_N\geqq\mu_s$ for all s from (A 8) and the updating rule Hence efficiency is attained at $\lim V^\tau$ which in view of part (a) must be V^* So $V^\tau\to V^*$ and $I^\tau\to I^*(V^*)$ with probability one

(ii) Given that V^τ is non-decreasing, $V^\tau=V_{\max}$ eventually with probability one unless Prob $\{V^\tau<V_{\max}$ all $\tau\}>0$ The latter implies Prob $\{\lim\mu_s(V^\tau)=0\}>0$, but by the argument of part (b) this means V^τ converges to a value with $\mu_N=0$ with positive probability This is impossible since such a point would be efficient, contrary to assumption ‖

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