

# Unemployment and Business Cycles

Lawrence J. Christiano  
Martin Eichenbaum  
Mathias Trabandt

# Background

- Key challenge for business cycle models.
  - How to account for observed volatility of labor market variables?
- Standard diagnosis
  - For plausibly parameterized models, in a boom, wages rise too rapidly, limiting expansion of employment.
  - Classic RBC models (Chetty), standard efficiency wage models (Alexopoulos), standard DMP models (Shimer).

# Sticky Wages...

- New Keynesian DSGE models successful in matching time series data, including hours worked, employment and real wages.
  - but, they assume the result by positing that wages are *exogenously* sticky.
    - model provides no rationale for wage stickiness.
  - approach criticized on micro data grounds
    - good macro fit requires wage indexation, so that all wages change all the time.
    - but, in micro data individual wages constant for lengthy spells.
  - underlying ‘monopoly power’ theory of unemployment
    - on questionable empirical grounds (Christiano (2010))
  - does not contribute to contemporary policy discussions (e.g., effects of extending unemployment benefits).

# Outline

- Describe variants of DMP labor market environments, define the problem, and several proposed solutions.
  - Standard DMP model: Shimer critique (AER)
  - Early solutions: Hagedorn and Manovskii (AER), Ljungqvist-Sargent (working paper).
- Current solutions:
  - Hiring versus search costs (see Pissarides (P), Christiano-Trabandt-Walentin (CTW), Christiano-Eichenbaum-Trabandt (CET)).
  - Hall-Milgrom (2008, HM).
- Steady State Properties.
- Integrate framework into New Keynesian model.

# Simple model

- Households

- have large numbers of workers, provide consumption insurance to each one.

- Consumption Euler equation:

$$1 = E_t m_{t+1} \tilde{R}_{t+1}$$

- workers

- Value function of unemployed worker ( $D$ –unemployment benefits,  $f_{t+1}$  job finding rate):

$$U_t = D + E_t m_{t+1} [f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1}]$$

- Employed worker ( $w_t$  wage rate)

$$V_t = w_t + E_t m_{t+1} [\rho V_{t+1} + (1 - \rho) (f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1})]$$

- Actually, only the difference,  $S_{w,t}$ , matters in the model:

$$S_{w,t} \equiv V_t - U_t = w_t - D + \rho E_t m_{t+1} (1 - f_{t+1}) S_{w,t+1}.$$

## Simple Model, cnt'd

- A firm that wants to meet worker must first post a vacancy, at cost  $c$ .
  - Probability that a vacancy is filled is denoted  $q_t$ .
- After meeting a worker, the firm must pay a fixed cost,  $\kappa$ , before bargaining.
- Free entry condition:

$$c = q_t (J_t - \kappa).$$

- Value of a worker to the firm,  $J_t$  :

$$J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}$$

## Simple Model, cnt'd

- Determination of job filling rate and vacancy filling rate
  - matching function:

$$\underbrace{\tilde{x}_t l_{t-1}}_{\text{new meetings}_t} = \sigma_m \left( \underbrace{v_t l_{t-1}}_{\text{vacancies}_t} \right)^{1-\sigma} \times \left( \underbrace{(1-\rho)l_{t-1} + 1 - l_{t-1} = 1 - \rho l_{t-1}}_{\text{people searching for jobs}_t} \right)^\sigma .$$

- Job finding rate:

$$f_t = \frac{\tilde{x}_t l_{t-1}}{1 - \rho l_{t-1}} = \sigma_m \left( \frac{\underbrace{v_t l_{t-1}}_{\text{labor market tightness} \equiv \Gamma_t}}{1 - \rho l_{t-1}} \right)^{1-\sigma} = \sigma_m \Gamma_t^{1-\sigma}$$

## Simple Model, cnt'd

- Vacancy filling rate:

$$q_t = \frac{\tilde{x}_t l_{t-1}}{v_t l_{t-1}} = \sigma_m \left( \frac{1 - \rho l_{t-1}}{v_t l_{t-1}} \right)^{-\sigma} = \sigma_m \Gamma_t^{-\sigma}$$

- Law of motion of labor force

$$l_t = (\rho + \tilde{x}_t) l_{t-1}$$

- Resource constraint:

$$C_t + x_t l_{t-1} \left( \kappa + \frac{c}{q_t} \right) = \vartheta_t l_t$$

- Nash sharing rule:

$$J_t = \frac{1 - \eta}{\eta} S_{w,t},$$

$\eta$  share of total surplus,  $J_t + S_{w,t}$ , given to workers



# Collecting Equilibrium Conditions

- 11 equations and 11 unknowns  $J_t, w_t, m_t, S_{w,t}, l_t, f_t, q_t, \tilde{x}_t, C_t, \tilde{R}_t, \Gamma_t$ :

$$J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}$$

$$J_t = \frac{c}{q_t} + \kappa$$

$$S_{w,t} = w_t - D + \rho E_t m_{t+1} (1 - f_{t+1}) S_{w,t+1}$$

$$l_t = (\rho + \tilde{x}_t) l_{t-1}, f_t = \frac{\tilde{x}_t l_{t-1}}{1 - \rho l_{t-1}}$$

$$f_t = \sigma_m \Gamma_t^{1-\sigma}, q_t = \sigma_m \Gamma_t^{-\sigma}$$

$$\vartheta_t l_t = C_t + x_t l_{t-1} \left( \kappa + \frac{c}{q_t} \right)$$

$$J_t = \frac{1 - \eta}{\eta} S_{w,t}$$

$$1 = E_t m_{t+1} \tilde{R}_{t+1}, m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

# Analysis

- Dynamics of the system hard to analyze analytically.
- Steady state much easier, and surprisingly (as we'll see later) informative about dynamics.
  - Can ignore some equations, (\*)

$$J = \frac{\vartheta - w}{1 - \rho\beta}, J = \frac{c}{q} + \kappa$$

$$S_w = \frac{w - D}{1 - \rho\beta(1 - f)}$$

$$1 = \rho + \tilde{x}, f = \frac{(1 - \rho)l}{1 - \rho l}$$

$$f = \sigma_m \Gamma^{1-\sigma}, q = \sigma_m \Gamma^{-\sigma}$$

$$\vartheta l = C + \tilde{x}l \left( \kappa + \frac{c}{q} \right) (*)$$

$$J = \frac{1 - \eta}{\eta} S_w, 1 = \beta \tilde{R} (*), m = \beta$$

# Core Steady State Equations

- Two equations in two unknowns,  $\Gamma$  and  $w$ 
  - understood, that  $f = \sigma_m \Gamma^{1-\sigma}$ ,  $q = \sigma_m \Gamma^{-\sigma}$

$$\text{('wage equation')} \quad \frac{c}{q} + \kappa = \frac{\vartheta - w}{1 - \rho\beta}$$

$$\rightarrow w = \vartheta - (1 - \rho\beta) \left( \frac{c}{q} + \kappa \right)$$

$$\text{('bargaining equation')} \quad \frac{\vartheta - w}{1 - \rho\beta} = \frac{1 - \eta}{\eta} \frac{w - D}{1 - \rho\beta(1 - f)}$$

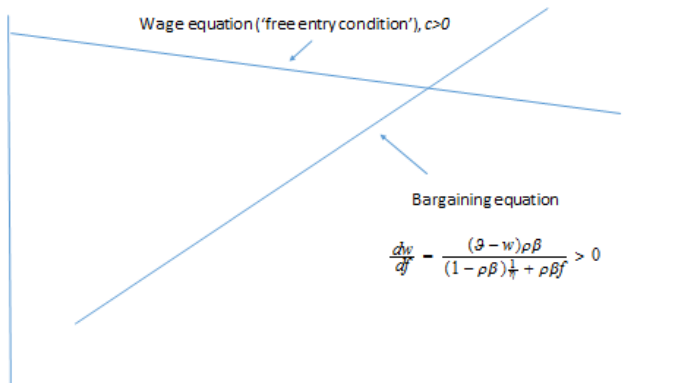
$$\rightarrow w = \frac{\vartheta (1 - \rho\beta(1 - f)) + (1 - \rho\beta) \frac{1 - \eta}{\eta} D}{(1 - \rho\beta) \frac{1 - \eta}{\eta} + 1 - \rho\beta(1 - f)}$$

- Alternatively, think of these as two equations in two unknowns,  $l$  and  $w$  :

$$f(l) = \frac{(1 - \rho)l}{1 - \rho l}, \quad \Gamma(l) = \left( \frac{f(l)}{\sigma_m} \right)^{\frac{1}{1-\alpha}}, \quad q(l) = \sigma_m \Gamma(l)^{-\sigma}$$

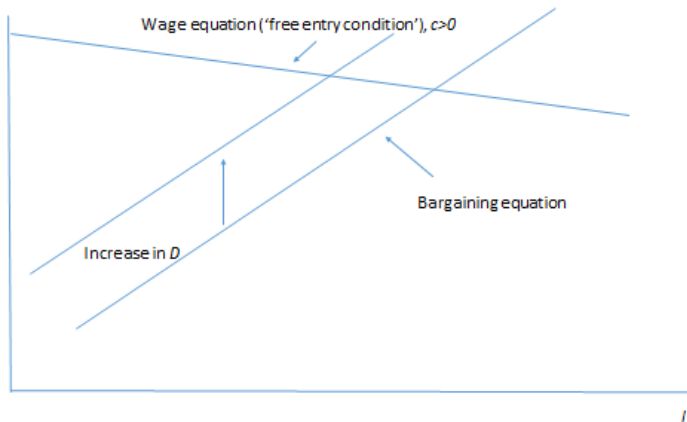
# Simple Characterization of Steady State Equilibrium

Steady State Equilibrium



# Comparative Static on Steady State

Steady State Effect of Increase in  $D$



# Steady State Properties of the Model

- Consider the long run (steady state) impact of a permanent shock to  $\vartheta$ .
  - Steady state effects (sort of) correspond, in dynamic setting, to medium run effects of persistent shock to  $\vartheta_t$ .
  - We are interested in the effects of all sorts of shocks.
    - In broader model, the primary avenue by which other shocks affect labor market is via their impact on  $\vartheta_t$ , which is endogenous in those models.
- Evaluate the effect of  $\vartheta$  on market tightness,  $\Gamma$  :

$$\epsilon_{\Gamma, \vartheta} \equiv \frac{d \log \Gamma}{d \log \vartheta}$$

- All other labor market variables are a monotone transform on  $\Gamma$ .

# Elasticity of Labor Market Tightness with Respect to Technology Shock

$$\epsilon_{\Gamma, \vartheta} = Y \frac{\vartheta}{\vartheta - D - \tau_{\kappa} \kappa'}$$

$$Y = \frac{\eta \rho \beta f + (1 - \rho \beta)}{\eta \rho \beta f + \sigma (1 - \rho \beta)} \geq 1$$

$$\tau_{\kappa} = \frac{Y \sigma}{1 - \eta} [\eta \rho \beta f + (1 - \rho \beta)]$$

# Shimer Puzzle

- Shimer (AER 2005) puzzle: "for any reasonable parameterization of the model, can't get  $\epsilon_{\Gamma,\theta}$  close to what it is in the data."
  - Shimer estimates  $\epsilon_{\Gamma,\theta}$  by ratio of standard deviation of market tightness (from JOLTS data) to standard deviation of labor productivity.
    - Later, we will display stochastic simulations of the model that provide some support for Shimer's approach to estimating  $\epsilon_{\Gamma,\theta}$ .
- Pissarides (2009, p. 1351), Mortensen-Nagypal (RED, 2007,p.333): argue that Shimer's estimate is too high.
  - Reports that in a regression of log tightness on log labor productivity, coefficient is 7.56
  - Argue that 7.56 is a better estimate of  $\epsilon_{\Gamma,\theta}$ .



## Hagedorn-Manovskii (AER 2008)

- HM examine elasticity in standard DMP model:

$$\epsilon_{\Gamma, \vartheta} = Y \frac{\vartheta}{\vartheta - D}, \quad Y = \frac{\eta \rho \beta f + (1 - \rho \beta)}{\eta \rho \beta f + \sigma (1 - \rho \beta)}.$$

- HM argue that to get elasticity up and solve Shimer puzzle, have two choices, given that in practice the data do not permit much playing around with  $\rho, \beta, f$ .
  - Reduce responsiveness of wage to  $\vartheta$  by increasing share of surplus going to labor,  $\eta$  ( $w \rightarrow D$  as  $\eta \rightarrow 0$ )
    - But,  $\eta$  has relatively small impact. When  $\eta = 0$ ,  $Y = 1/\sigma$ , and  $\sigma$  conventionally in neighborhood of 1/2. So,  $Y$  bounded between 1 (when  $\eta = \infty$ ) and 2 (when  $\eta = 0$ ).
  - Raising  $D$  has potentially *huge* impact via effect on  $\vartheta / (\vartheta - D)$ .
- Hagedorn and Manovskii conclude that only way to solve Shimer puzzle is to increase  $D$ .

# Costain-Reiter

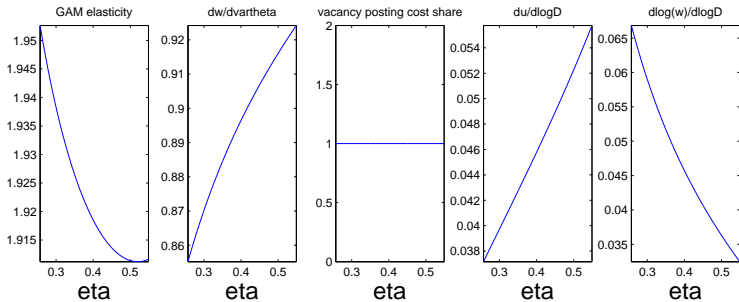
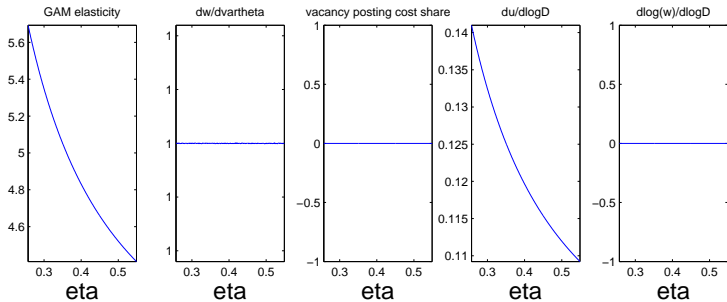
- Two objections to HM solution:
  - In practice,  $D$  is not very high.
  - Costain-Reiter: Solutions to Shimer puzzle tend to imply unreasonably high values of  $du/d \log D$ , where  $u = 1 - l$  is the unemployment rate -

$$\frac{du}{d \log D} = (1 - \rho l) l (1 - \sigma) \frac{D}{\vartheta} \times \epsilon_{\Gamma, \vartheta}$$

- Pissarides (2009, p. 1365):
  - Argues that in the data,  $du/d \log D \simeq 0.011$  : "A ten percent increase in unemployment benefits results in a 1.1 percentage point increase in unemployment."

# Numerical Properties of Elasticity Formula

- Following graph varies  $\eta$  from 0.26 to 0.55 (results are only reported for parameterizations that imply  $q, f \in (0, 1)$ )
  - first row:  $c = 0$ ,  $\kappa = 0.816$ ,  $\sigma_m = 0.664$ ,  $\sigma = 0.52$ ,  $\vartheta = 1$ ,  $\rho = 0.9$ ,  $\beta = 1.03^{-1/4}$ ,  $D = 0.365$ .
  - second row (standard DMP): same as first row, except  $c = 0.571$ ,  $\kappa = 0$ .
- Findings
  - Shifting meeting costs to fixed portion (first row) substantially boosts  $\epsilon_{\Gamma, \vartheta}$ .
  - Consistent with HM, increasing  $\eta$  has tiny impact on  $\epsilon_{\Gamma, \vartheta}$  and presumably achieves this via the (tiny or no, in case of first row) reduction in wage response.
  - With fixed costs of hiring labor,  $\epsilon_{\Gamma, \vartheta}$  is bigger. Increased wage inertia (reducing  $dw/d\vartheta$ ) plays no role in this.
- Consistent with Shimer, no parameterization comes close to solving the Shimer puzzle.



# Elasticity of Labor Market Tightness with Respect to Technology Shock

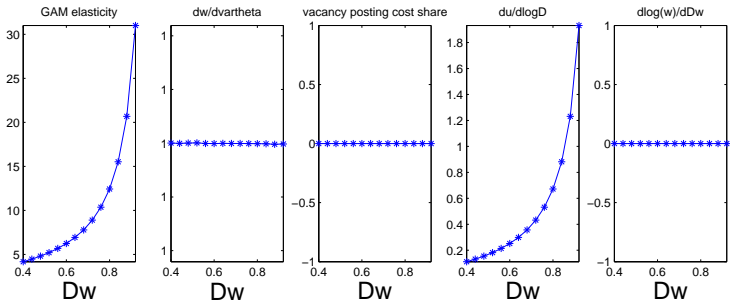
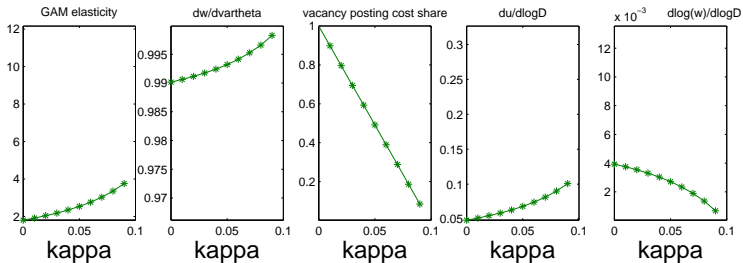
- Formula:

$$\epsilon_{\Gamma, \vartheta} = Y \frac{\vartheta}{\vartheta - D - \tau_{\kappa} \kappa}$$

- Increase in  $D$  raises elasticity, illustrates Hagedorn and Manovskii (2008, AER)
- Increase in  $\kappa$  raises elasticity, illustrates Pissarides (P, Econometrica, Vol. 77, No. 5, September, 2009, pp. 1339-1369), and Christiano, Trabandt and Walentin (CTW, Journal of Economic Dynamics & Control, vol. 35, 2011, pages 2038-2039).
- Formula similar to the one in Ljungqvist and Sargent ("The Fundamental Surplus in Matching Models", manuscript)

# More Numerical Properties of Elasticity Formula

- To establish baseline parameter values:
  - set:  $\beta = 1.03^{-0.25}$ ,  $\eta = 0.9$ ,  $\rho = 0.9$ ,  $\sigma = 0.52$ ,  $\vartheta = 1$ ,  $\kappa = 0.1$ .
  - select values of  $D$ ,  $\sigma_m$  and  $c$  so that  $l = 0.945$ ,  $q = 0.70$ ,  $D/w = 0.4$ .
- In following graph,
  - First row: vary value of  $\kappa$ , holding  $\beta$ ,  $\eta$ ,  $\rho$ ,  $\sigma$ ,  $\vartheta$ ,  $D$  fixed at baseline and setting  $\sigma_m$  and  $c$  so that  $l = 0.945$ ,  $q = 0.70$ .
  - Second row: vary  $D/w$ , holding  $\beta$ ,  $\eta$ ,  $\rho$ ,  $\sigma$ ,  $\vartheta$  fixed at baseline,  $c = 0$ , and setting  $\sigma_m$ ,  $\kappa$  and  $D$  so that  $l = 0.945$ ,  $q = 0.70$ ,  $D/w$  takes on specified value.
- Of course, the *elasticity* is the change in market tightness with respect to  $\vartheta$ , *holding all other structural parameters constant*.



# Messages from the Formula

- As  $\kappa$  increases and share of costs to meet worker due to vacancy costs declines,  $\epsilon_{\Gamma, \vartheta}$  increases (P,CTW,CET).
- As  $D/w$  increases,  $\epsilon_{\Gamma, \vartheta}$  increases.
- Response of wage to  $\vartheta$  shows little variation across experiments.
  - Response even moves in the ‘wrong’ direction in response to  $\kappa$
  - Hall-Shimer intuition, that key to resolving Shimer puzzle lies in reducing responsiveness of wage, is called into question by the results (see Hagedorn and Manovskii (2008, AER) and Ljungqvist and Sargent for further discussion).
- Perturbations which work towards resolution of Shimer puzzle, simultaneously imply a big response of unemployment to unemployment benefits,  $D$  (see Costain and Reiter, JEDC 2008).
- For intuition, see below.



# Alternating Offer Bargaining

- Hall-Milgrom intuition (later, called into question)
  - With alternating offer bargaining, wage rate in part reflects costs and benefits of bargaining, detaching wage a little from broader economic conditions.
  - Result: with reduced sensitivity of wage to general conditions, when a shock launches an expansion, firms enjoy a larger share of the rents from a match and therefore have a greater incentive to expand employment.
  - Hall-Milgrom suggested that this is how alternating offer bargaining may contribute to resolution of Shimer puzzle.
  - To resolve puzzle, still need to reduce  $c$  and raise  $\kappa$ .
- Baseline specification:
  - Each worker-firm pair bargains each period.
  - Bargain over current wage rate, taking outcome of future wage bargains given.
  - ‘Period-by-Period Bargaining’.

# Alternating Offers

- Each quarter is divided into  $M$  equal subperiods,  $m = 1, \dots, M$ .
  - Firm makes an opening wage offer in  $m = 1$ .
  - Worker may reject and make a counter offer in  $m = 2$ .
  - Firm may reject worker's wage offer and make a new offer in next sub-period,...
  - If there is a whole sequence of rejections, worker makes a take-it-or-leave-it offer in last subperiod  $M$ .
- If an offer is accepted in any sub period  $m$ , production begins immediately.
  - Value of production in any subperiod is  $\vartheta_t/M$ .
- Solution to the bargaining problem:

$$w_t^1 (\equiv w_t), w_t^2, \dots, w_t^M.$$

# Firm's Offer: round 1

- Firm offers  $w_t^1$  as low as possible subject to worker not rejecting it:

utility of worker who accepts  
firm offer and goes to work

$$\underbrace{V_t^1}$$

utility of worker who rejects  
firm offer and intends to make counteroffer

$$= \overbrace{\delta U_t^1 + (1 - \delta) \left( \frac{D}{M} + V_t^2 \right)}$$

where,

$$V_t^1 \equiv w_t^1 + E_t m_{t+1} [\rho V_{t+1} + (1 - \rho) (f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1})]$$

## Worker Offer: round 2

- Worker proposes highest possible wage  $w_t^2$  subject to firm not rejecting it:

$$\underbrace{\text{value of firm that accepts worker offer}}_{J_t^2} = \underbrace{\text{value of firm that rejects worker offer and intends to make counteroffer}}_{\delta \times 0 + (1 - \delta) [-\gamma + J_t^3]}$$

- The firm incurs cost  $\gamma$  to make a counter offer.
- Firm value:

$$J_t^2 \equiv \underbrace{\vartheta_t \frac{M-1}{M}}_{\text{value of worker output in subperiods 2 to } M} - w_t^2 + \rho E_t m_{t+1} J_{t+1}$$

# Alternating Offers, Final Round

- Each bargaining round requires the wage for the next round.
- If they go to last round with no agreement, the worker makes a final, take-it-or-leave-it offer:

$$\begin{array}{ccc} \text{value of firm that} & & \text{value of firm that rejects worker's} \\ \text{accepts worker offer in last round} & & \text{take-it-or-leave-it offer} \\ \underbrace{J_t^M} & = & \underbrace{0} \end{array}$$

or

$$J_t^M \equiv \frac{1}{M} \vartheta_t - w_t^M + \rho E_t m_{t+1} J_{t+1} = 0,$$

or

$$w_t^M = \frac{1}{M} \vartheta_t + \rho E_t m_{t+1} \overbrace{J_{t+1}}^{=\kappa}$$

# Calculations

- To determine  $w_t \equiv w_t^1$ , firm first solves  $w_t^M, w_t^{M-1}, w_t^{M-2}, \dots, w_t^2$ .
- $M$  equilibrium conditions for the  $M$  unknowns.
- Linearity of bargaining equilibrium conditions implies:
  - simple equation determines spot wage,  $w_t$ .

# Alternating Offer Sharing Rule

- Wage satisfies:

$$J_t = \beta_1 \overbrace{(V_t - U_t)}^{=S_{w,t}} - \beta_2 \gamma + \beta_3 (\vartheta_t - D),$$

where

$$\alpha_1 = 1 - \delta + (1 - \delta)^M, \quad \alpha_2 = 1 - (1 - \delta)^M,$$
$$\alpha_3 = \alpha_2 \frac{1 - \delta}{\delta} - \alpha_1, \quad \alpha_4 = \frac{1 - \delta}{2 - \delta} \frac{\alpha_2}{M} + 1 - \alpha_2,$$

and  $\beta_i = \alpha_{i+1} / \alpha_1$ ,  $i = 1, 2, 3$ .

# Alternative Bargaining Arrangements

- Alternative arrangement has workers and firms bargaining just once, when they first meet. Equilibrium allocations always the same.
  - negotiate over wage rates in each date and state of nature associated with the duration of their match.
  - they do not care about the precise pattern of wage payments, only the present discounted value (PV).
  - many patterns are possible, including the pattern in the period-by-period bargaining assumed in the paper.
    - one pattern: worker receives fixed nominal wage as long as she's with firm.
    - Wages of new hires more volatile than wages of incumbents.
- Key issue associated with PV bargaining: commitment.
  - no need to address these issues in period-by-period bargaining.



# Effect of Alternating Offer Bargaining on Equilibrium Conditions

- Replace Nash Sharing rule:

$$J_t = \frac{1 - \eta}{\eta} S_{w,t}$$

with alternating offer sharing rule:

$$J_t = \beta_1 S_{w,t} - \beta_2 \gamma + \beta_3 (\vartheta_t - D).$$

- With this sharing rule, the steady state elasticity,  $d \log \Gamma / d \log \vartheta$  is altered.

# Elasticity of Market Tightness with Respect to Productivity

- elasticity formula now has  $\gamma$  in there...higher  $\gamma$ , higher elasticity...

$$\epsilon_{\Gamma, \vartheta} = Y \frac{\vartheta}{\vartheta - D - \tau_{\kappa} \kappa - \tau_{\gamma} \gamma}$$

## Elasticity formula: the pieces

$$\psi \equiv \frac{\rho\beta f + \sigma(1 - \rho\beta)(1 + \beta_1)}{\rho\beta f + (1 - \beta\rho)(1 + \beta_1)},$$

$$Y = \frac{\beta_1 + \beta_3(1 - \rho\beta(1 - f))}{\psi a}$$

$$\tau_\kappa = \frac{(1 + \beta_1)(1 - \rho\beta) + \rho\beta f + \frac{\rho\beta f(\sigma - 1)}{\psi}}{a},$$

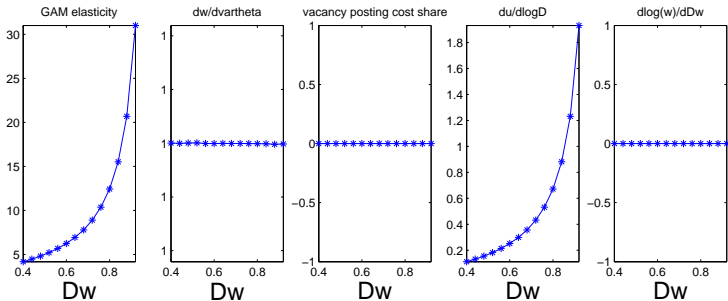
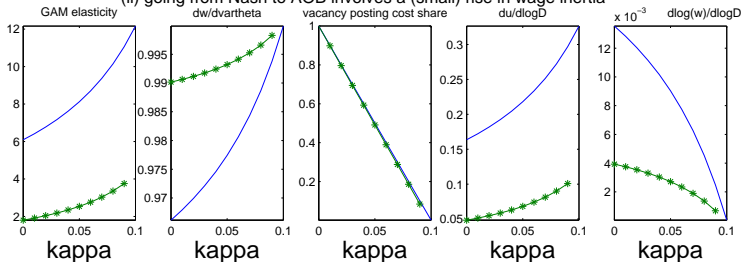
$$\tau_\gamma = \frac{\left(1 - \beta\rho(1 - f) + \frac{\rho\beta f(\sigma - 1)}{\psi}\right) \beta_2}{a}$$

$$a = \beta_1 + \beta_3 \left(1 - \beta\rho(1 - f) + \frac{\rho\beta f(\sigma - 1)}{\psi}\right)$$

## Following Figure

- Top and bottom rows constructed as in earlier figure which is reproduced here.

Two things in top row: (i) Nash (star) to AOB has big effect on market tightness elasticity  
(ii) Search ( $\kappa = 0$ ) to hiring ( $\kappa > 0$ ) has big effect, but has bigger impact on AOB than Nash  
Bottom row: (i) rise in  $D/w$  has huge effect on labor market volatility but also raises impact of benefits  
(ii) going from Nash to AOB involves a (small) rise in wage inertia



# Message of Formula

- Parameter values:

$$\beta = 1.03^{-0.25}, \sigma = 0.52, \rho = 0.9, \varepsilon = 6, \rho_a = 0.95,$$

$\underbrace{\eta = 0.9}_{\text{Nash}}, \underbrace{\gamma = 0.01, \delta = 0.005, M = 60}_{\text{Alternating offer bargaining}}$

- Going to alternating offer bargaining (AOB) increases elasticity of tightness with respect to  $\vartheta$ .
  - However, it simultaneously increases the impact of permanent changes in unemployment benefits.
- Wage inertia plays quantitatively little or no (second row) role in results (in first row, moves in ‘wrong’ direction with  $\kappa$ ).

# Some Intuition About Elasticity Formula

- The wage equation with  $q$  expressed in terms of  $\Gamma$  :

$$\frac{c}{\sigma_m} \Gamma^\sigma + \kappa = \frac{\vartheta - w}{1 - \rho\beta}.$$

- Differentiate:

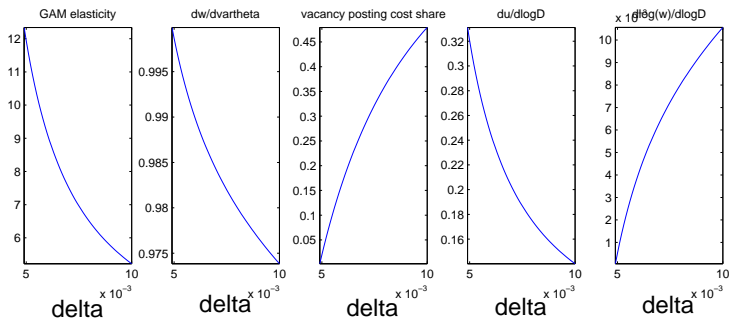
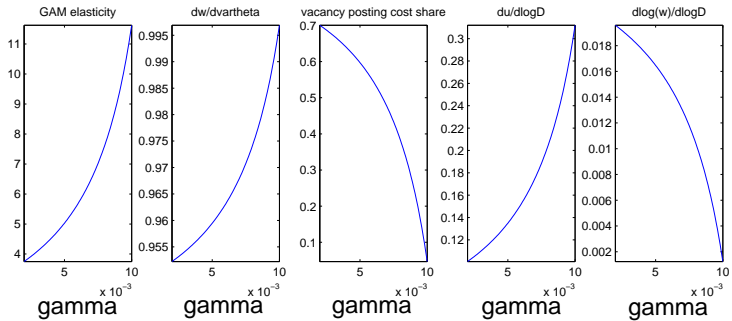
$$\epsilon_{\Gamma, \vartheta} = \frac{1}{\sigma} \frac{\vartheta (1 - dw/d\vartheta)}{\vartheta - w - \kappa (1 - \rho\beta)}.$$

- Two ways to raise  $\epsilon_{\Gamma, \vartheta}$  :
  - Numerator: wage inertia (reduce  $dw/d\vartheta$ ), but this exhibits too little variation to play an appreciable role
  - Denominator: reduce steady state profits, net of amortized fixed cost of meeting worker.
- Reducing profits:
  - Rise in  $\kappa$  directly reduces (relevant measure of) profits.
  - Rise in  $D$  or  $\gamma$  shifts bargaining equation (i.e., sharing rule) up and, given downward-sloped wage equation with  $c > 0$ ,  $w$  rises, reducing profits.

# Impact of Bargaining Parameters

- Baseline model parameters:
  - set:  $\beta = 1.03^{-0.25}$ ,  $\rho = 0.9$ ,  $\sigma = 0.52$ ,  $\vartheta = 1$ ,  $\kappa = 0.098$ ,  
 $\delta = 0.005$ ,  $\gamma = 0.01$ ,  $M = 60$ ,  $D = 0.396$ .
  - select values of  $\sigma_m$  and  $c$  so that  $l = 0.945$ ,  $q = 0.70$ .
- Change  $\gamma$  (top row) and  $\delta$  (bottom row)
  - always adjust  $\sigma_m$ ,  $c$  to fix  $q$ ,  $l$ , but keep all other parameters at baseline.
- Results:
  - Reducing  $\delta$  and/or raising  $\gamma$  raises  $\epsilon_{\Gamma, \vartheta}$ .
  - Change in wage inertia very small, but, significantly, moves in 'wrong' direction.
  - $du/d \log(D)$  is mirror image of  $\epsilon_{\Gamma, \vartheta}$  (Costain-Reiter).





# Why Wage Inertia Seems to Move in 'Wrong' Direction with Market Tightness

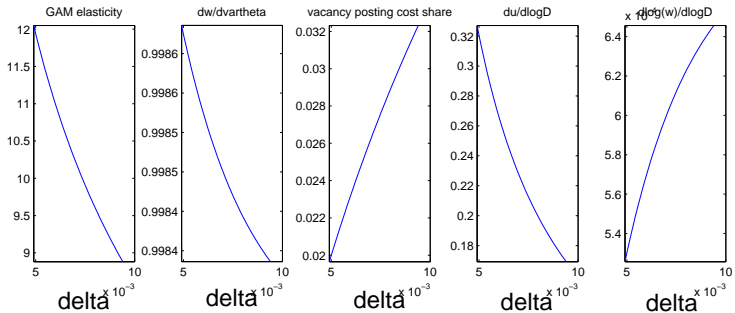
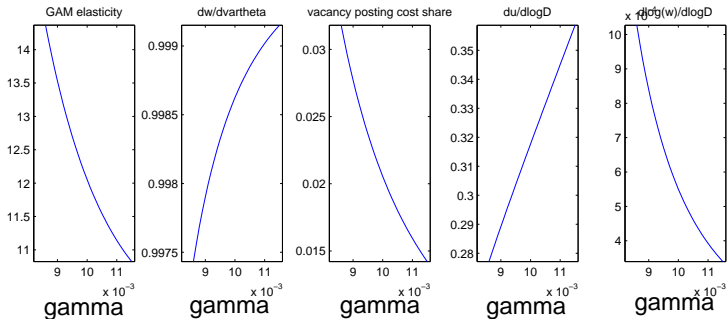
- Note from the previous slide that  $dw/d\vartheta$  appears to comove with  $\epsilon_{\Gamma,\vartheta}$ .
- This is a complete contradiction to the Hall-Shimer intuition that to get volatility up you need to have wage inertia.
- Following is the analytic formula for  $dw/d\vartheta$  :

$$\frac{dw}{d\vartheta} = \frac{\beta_1 \frac{(w-D)\beta\rho(1-\sigma)f}{\vartheta(1-\beta\rho+\beta\rho f)^2} \epsilon_{\Gamma,\vartheta} + \frac{1}{1-\rho\beta} - \beta_3}{\frac{\beta_1}{1-\beta\rho+\beta\rho f} + \frac{1}{1-\rho\beta}}$$

- Note that, for given  $f$  and  $w$ , this is an increasing function of  $\epsilon_{\Gamma,\vartheta}$ .
- It was verified numerically, that an increase in  $\gamma$  or decrease in  $\delta$  raises  $w$ , reinforcing positive association between  $dw/d\vartheta$  and  $\epsilon_{\Gamma,\vartheta}$ .

# Role of Wage Inertia and Effects of Alternating Offer Bargaining

- Conventional intuition:
  - rise in  $\gamma$  or reduction in  $\delta$  'insulates' wage from business cycle
  - with greater wage inertia, labor market variables respond more to shocks.
- Conventional wisdom contradicted in previous numerical results.
- Could the results reflect the calibration of  $c$  and  $\sigma_m$  in previous results?
- Next Figure: vary  $\gamma$  and  $\delta$  holding all other structural parameters fixed at baseline.
  - Bottom row: results unaffected; top row: impact of  $\gamma$  on  $\epsilon_{\Gamma, \theta}$  reversed.



# Understanding Previous Results

- Following graph repeats:

$$\epsilon_{\Gamma, \vartheta} = \frac{1}{\sigma} \frac{\vartheta (1 - dw/d\vartheta)}{\vartheta - w - \kappa (1 - \rho\beta)'} ,$$

from the previous graph.

- In addition, display

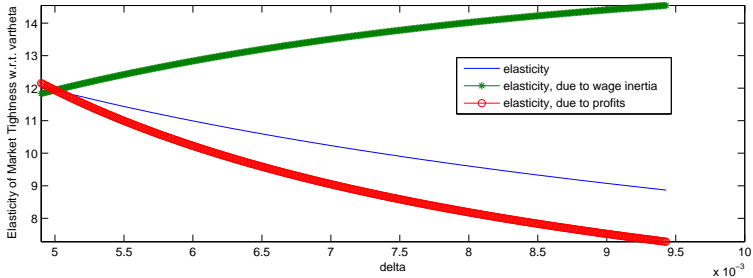
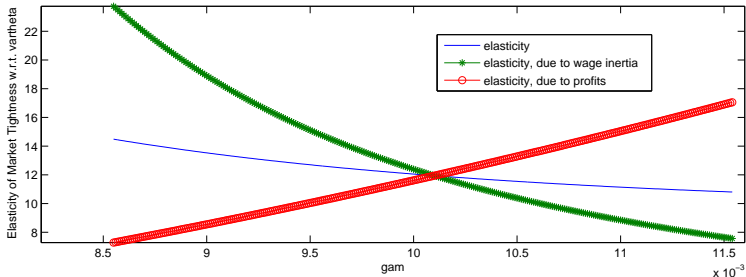
$$\text{"}\epsilon_{\Gamma, \vartheta} \text{ due to wage inertia" = } \frac{1}{\sigma} \frac{\vartheta (1 - dw/d\vartheta)}{[\vartheta - w - \kappa (1 - \rho\beta)]^{\text{baseline}}}$$

where  $[\vartheta - w - \kappa (1 - \rho\beta)]^{\text{baseline}}$  represents profits in the baseline parameterization.

- Also, display

$$\text{"}\epsilon_{\Gamma, \vartheta} \text{ due to profits" = } \frac{1}{\sigma} \frac{\vartheta [(1 - dw/d\vartheta)]^{\text{baseline}}}{\vartheta - w - \kappa (1 - \rho\beta)} .$$

- The figure shows how the numerator and denominator contribute to the previous results.



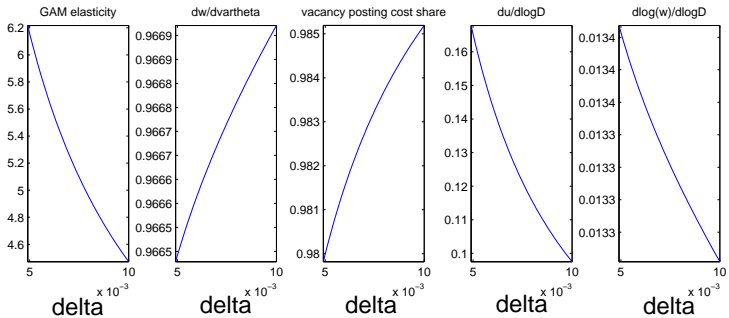
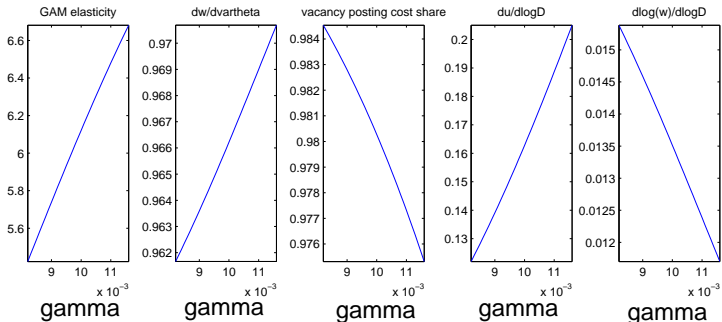
# Previous Results

- Results for increase in  $\gamma$  :
  - Rise in  $\gamma$  reduces  $\epsilon_{\Gamma,\vartheta}$ .
  - Consistent with earlier intuition, focusing on profits alone leads to implication that  $\epsilon_{\Gamma,\vartheta}$  rises.
  - Elasticity actually falls because rise in  $dw/d\vartheta$  dominates  $\epsilon_{\Gamma,\vartheta}$ .
  - Hall-Milgrom focus on wage inertia justified, but their intuition that an increase in  $\gamma$  reduces  $dw/d\vartheta$  is contradicted.
- Results for decrease in  $\delta$  :
  - Leads to increase in  $\epsilon_{\Gamma,\vartheta}$ .
  - Hagedorn-Manovskii intuition that focuses on level effect on profits is vindicated.
  - Hall-Milgrom intuition that focuses on wage inertia is contradicted.

# Role of Wage Inertia and Effects of Alternating Offer Bargaining

- Baseline parameter values in previous computations:
  - set:  $\beta = 1.03^{-0.25}$ ,  $\rho = 0.9$ ,  $\sigma = 0.52$ ,  $\vartheta = 1$ ,  $\kappa = 0.098$ ,  $\delta = 0.005$ ,  $\gamma = 0.0082$ ,  $M = 60$ ,  $D = 0.396$ .
  - select values of  $\sigma_m$  and  $c$  so that  $l = 0.945$ ,  $q = 0.70$  ( $c = 0.0014$ ,  $\sigma_m = 0.6638$ ).
  - $v^c = 0.02$ .
- Conventional wisdom about role of wage inertia in getting  $\epsilon_{\Gamma, \vartheta}$  up contradicted in previous numerical results.
- Is this due to low value of  $v^c$ ?
  - new baseline: set  $\kappa, l, q$  so that  $l = 0.945$ ,  $q = 0.70$ ,  $v^c = 0.98$  ( $\kappa = 0.002$ ,  $c = 0.069$ ,  $\sigma_m = 0.6638$ ).
- Next figure: vary  $\gamma$  and  $\delta$  holding all other structural parameters fixed at baseline.
  - Results unaffected.





# Analysis of Dynamics

- Must use computer to do model simulation (e.g., perturbation, projection, extended path)
- When  $\kappa$  large compared to  $c$ , steady state equilibrium is indeterminate.
  - Can be shown analytically for the case,  $m_{t+1} = \beta$ ,  $c = 0$  (see below).
- With price setting frictions (and  $\eta$  large enough, in the Nash sharing case), steady state equilibrium determinate.
- Do the analysis with sticky prices.
  - With  $c = 0$  and alternating offer bargaining, model performs well relative to data.
  - No Shimer puzzle.
  - This is so, despite the absence of exogenous stickiness in wages.

# Indeterminacy of Steady State with Hiring Costs

- Equilibrium conditions (sum of firm and worker surplus value functions; law of motion of labor; job finding rate; sharing rule):

$$S_{w,t} + \kappa = \vartheta_t - D + \rho\beta [(1 - f_{t+1}) S_{w,t+1} + \kappa]$$

$$l_t = (\rho + \tilde{x}_t) l_{t-1},$$

$$f_t = \frac{\tilde{x}_t l_{t-1}}{1 - \rho l_{t-1}},$$

$$\kappa = \frac{1 - \eta}{\eta} S_{w,t},$$

for  $t = 0, 1, 2, \dots$  ( $m_{t+1} = \beta$ , for analytical convenience).

- *Steady state equilibrium*: constant values of  $S_{w,t}, l_t, \tilde{x}_t, f_t$ , that satisfy equilibrium conditions for  $\vartheta_t = \vartheta$ , all  $t \geq 0$ .
- Steady state equilibrium is unique, and corresponds to  $S_{w,t} = S_w, l_t = l, \tilde{x}_t = \tilde{x}, f_t = f$  for all  $t \geq 0, l_{-1} = l$ .

# Indeterminacy of Steady State Equilibrium

- Let  $m_{t+1} = \beta$ ,  $c = 0$ .
- The steady state equilibrium is *indeterminate*: given  $l_{-1} = l$ , one can find another sequence of values of  $S_{w,t}, l_t, \tilde{x}_t, f_t$ ,  $t = 0, 1, 2, \dots$ , that satisfy the equilibrium conditions and which remain arbitrarily close to the steady state equilibrium.
  - Sharing rule implies  $S_{w,t} = S_w$  for all  $t \geq 0$ .
  - Total surplus equation implies  $f_{t+1} = f$  for  $t = 0, 1, 2, \dots$
  - Choose a value for  $\tilde{x}_0 \neq x$ , and set  $f_0 = \tilde{x}_0 l / (1 - \rho l) \neq f$ ,  
 $l_0 = f_0 + \rho(1 - f_0)l \neq l$ .
  - Set  $l_t = f + \rho(1 - f)l_{t-1}$ ,  $t \geq 1$ .
  - Note that all these  $\tilde{x}_t, f_t, l_t$  paths:
    - converge to steady state (because  $0 < \rho(1 - f) < 1$ )
    - satisfy the equilibrium conditions,
    - can be made arbitrarily close to steady state equilibrium by setting  $\tilde{x}_0$  arbitrarily close to  $\tilde{x}$ .

# Outline

- Integrate bargaining model into sticky price NK model.
- Evaluate the model in two ways:
  - Shimer style comparison to unconditional moments.
  - Impulse response matching.
- Conclude
  - Alternating offer bargaining and  $c = 0$  solve Shimer puzzle.

# Households and Final Good Firms

- Household intertemporal condition:

$$\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R_{t+1}^*} \frac{R_t}{\bar{\pi}_{t+1}},$$

$$R_{t+1}^* \equiv \frac{1}{\beta} \exp(a_{t+1} - a_t)$$

$$X_t \equiv \frac{C_t}{\exp(a_t)}, \quad \bar{\pi}_t = 1 + \pi_t.$$

- Final good firms solve

$$\max P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj, \quad \text{s.t. } Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Demand curve for  $i^{\text{th}}$  monopolist ('sticky price retailers')

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

# Intermediate Good Firms

- Production function:

$$Y_{i,t} = \exp(a_t) h_{i,t}, \quad a_t = \rho_a a_{t-1} + \varepsilon_t^a,$$

where  $h_{i,t}$  is a homogeneous input (not labor!) with after-subsidy nominal price,  $(1 - \nu) \vartheta_t P_t$ .

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases} .$$

# Price-setting equilibrium conditions

$$\tilde{p}_t = \frac{K_t}{F_t}, \tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}$$

$$K_t = \frac{\varepsilon}{\varepsilon-1} s_t \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1}$$

$$\tilde{p}_t = \left[ \frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \rightarrow \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}$$

$$s_t = \frac{(1-\nu)\vartheta_t}{\exp(a_t)}$$

$$p_t^* = \left[ (1-\theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}$$



# Resource Constraint and Monetary Policy

- Resource constraint:

$$C_t + \left( \frac{c}{q_t} + \kappa \right) \tilde{x}_t l_{t-1} = Y_t$$
$$p_t^* l_t \exp(a_t) = Y_t.$$

- Clearing in demand and supply, input good to intermediate good producers:

$$h_t = l_t.$$

- Taylor rule:

$$\frac{R_t}{R} = \exp \{ \phi_\pi (\bar{\pi}_t - 1) \}, \phi_\pi = 1.5.$$

# Labor Market Equations

- Value of worker to firm; firm free entry condition; recursive representation of household surplus,  $S_{w,t}$ ; law of motion of  $l_t$ ; labor market tightness,  $\Gamma_t$ ; finding rate; vacancy filling rate and sharing rule:

$$J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}$$

$$J_t = \frac{c}{q_t} + \kappa$$

$$S_{w,t} = w_t - D + \rho E_t m_{t+1} (1 - f_{t+1}) S_{w,t+1}$$

$$l_t = (\rho + \tilde{x}_t) l_{t-1}, \quad \Gamma_t = \frac{\tilde{x}_t l_{t-1}}{1 - \rho l_{t-1}}$$

$$f_t = \sigma_m \Gamma_t^{1-\sigma}, \quad q_t = \sigma_m \Gamma_t^{-\sigma}$$

$$J_t = \beta_1 S_{w,t} - \beta_2 \gamma + \beta_3 (\vartheta_t - D)$$

# Usefulness in Steady State Elasticity Formula for Understanding Dynamics

- Surprisingly good at predicting model dynamics ( $\theta = 0.75$ ).

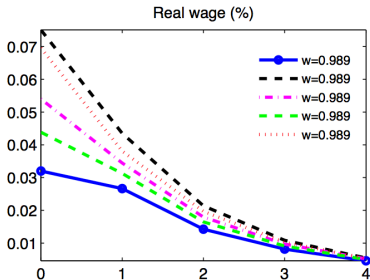
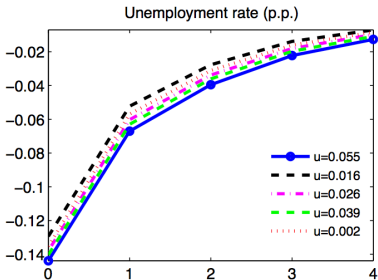
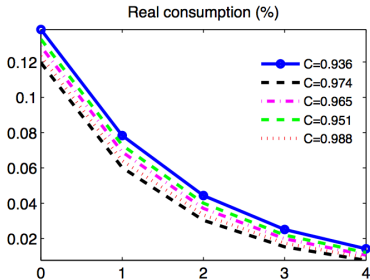
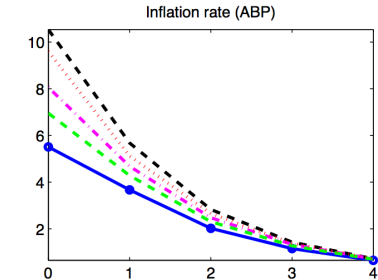
	$\frac{d \log \Gamma}{d \log \vartheta}$	$\frac{std(\log \Gamma)}{std(a)}$
Nash Sharing		
$\kappa = 0$ ('pure search costs')	1.8	1.8
$c = 0$ ('pure hiring costs')	4.0	4.3
Alternating Offer Bargaining		
$\kappa = 0$	6.0	6.1
$c = 0$	11.7	12.2
$c = 0, \delta = 0.001$	47.1	41.4

# Calibration/Parameterization

Parameter	Value	Description
Panel A: Parameters		
$\beta$	$1.03^{-0.25}$	Discount factor
$\xi$	0.66	Calvo price stickiness
$\lambda_f$	1.2	Price markup parameter
$\rho_R$	0.7	Taylor rule: interest rate smoothing
$r_\pi$	1.7	Taylor rule: inflation coefficient
$r_y$	0.1	Taylor rule: employment coefficient
$\rho$	0.9	Job survival probability
$\delta$	0.005	Prob. of bargaining session break-up
$M$	60	Max bargaining rounds per quarter
$\rho_a$	0.95	Roots for AR(1) technology
Panel B: Steady State Values		
$400(\pi - 1)$	0	Annual net inflation rate
$l$	0.945	Employment
$\kappa x l / Y$	0.01	Hiring cost to output ratio
$D/w$	0.4	Replacement ratio

Figure 1: Small Model Impulse Responses to a 25 ABP Monetary Policy Shock

—●— Baseline    - - - Higher  $\delta$     - · - · Lower  $\gamma$     - · - · Lower  $D$     ····· Lower  $M$



# Intuition

- Policy shock drives real interest rate down.
  - Induces increase in demand for output of final good producers and therefore output of sticky price retailers.
  - Latter must satisfy demand, so retailers purchase more of wholesale good driving up its relative price.
  - Marginal revenue product ( $\vartheta_t$ ) associated with worker rises.
  - Wholesalers hire more workers, raising probability that unemployed worker finds a job.
- Workers' disagreement payoffs rise.
  - Increase in workers' bargaining power generates rise in real wage.
- Alternating offer bargaining mutes rise in real wage.
  - Allows for large increase in employment, substantial decline in unemployment, small rise in inflation.

# Medium-Sized DSGE Model

- Standard empirical NK model (e.g., CEE, ACEL, SW).
  - Calvo price setting frictions, but no indexation
  - Habit persistence in preferences.
  - Variable capital utilization.
  - Investment adjustment costs.
- Our labor market structure

# Estimated Medium-Sized DSGE Model

- Estimate VAR impulse responses of aggregate variables to a monetary policy shock and two types of technology shocks.
- 11 variables considered:
  - Macro variables and real wage, hours worked, unemployment, job finding rate, vacancies.
- Estimate model using Bayesian variant of CEE (2005) strategy:
  - Minimizes distance between dynamic response to three shocks in model, analog objects in the data.
  - Particular Bayesian strategy developed in Christiano, Trabandt and Walentin (2011).



# Posterior Mode of Key Parameters

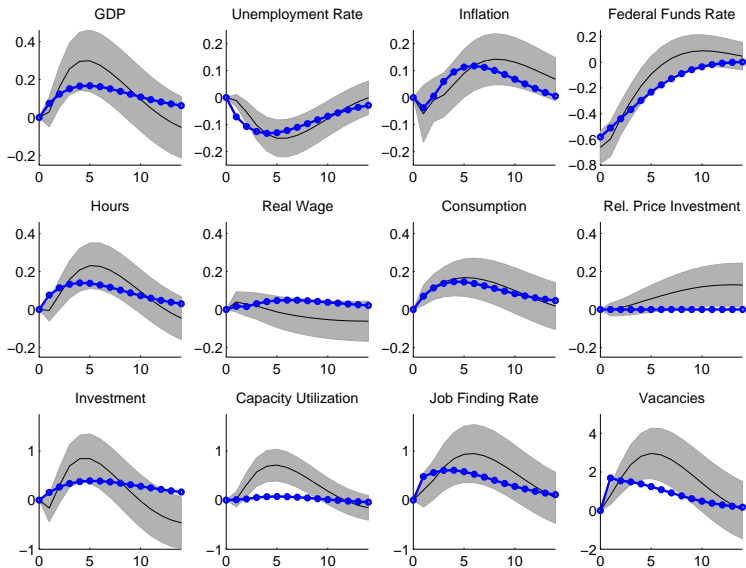
- Prices change on average every 2.5 quarters.
- $\delta$  : roughly 0.26% chance of a breakup after rejection.
- $\gamma$  : cost to firm of preparing counteroffer is 1/4 of a day's worth of production.
- Posterior mode of hiring cost as a percent of output (depends on  $\kappa$ ): 0.54% of GDP.

# Posterior Mode of Key Parameters

- Replacement ratio is 0.62.
  - Defensible based on micro data (Gertler-Sala-Trigari, Aguiar-Hurst-Karabarbounis).
- Gertler, Sala and Trigari (2008) : plausible range for replacement ratio is 0.4 to 0.7.
  - Lower bound based on studies of unemployment insurance benefits
  - Upper boundary takes into account informal sources of insurance.

# Medium-Sized Model Impulse Responses to a Monetary Policy Shock

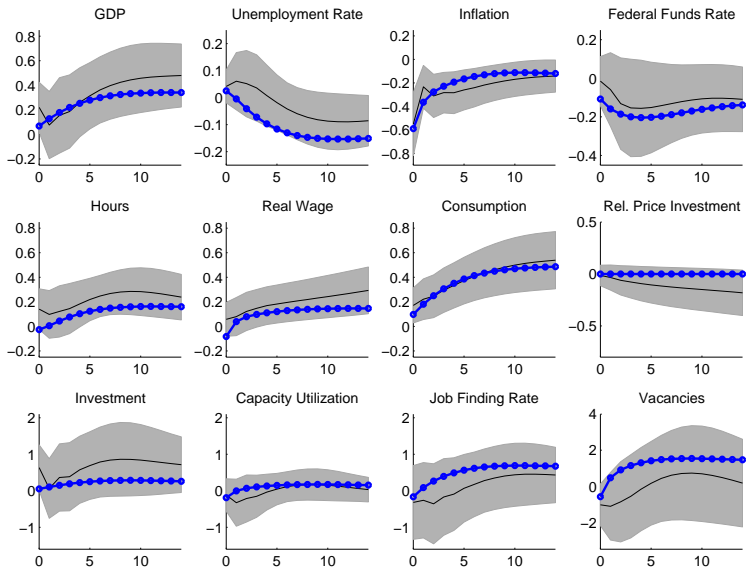
■ VAR 95% — VAR Mean —●— Alternating Offer Bargaining Model



Notes: x-axis: quarters, y-axis: percent

# Medium-Sized Model Impulse Responses to a Neutral Technology Shock

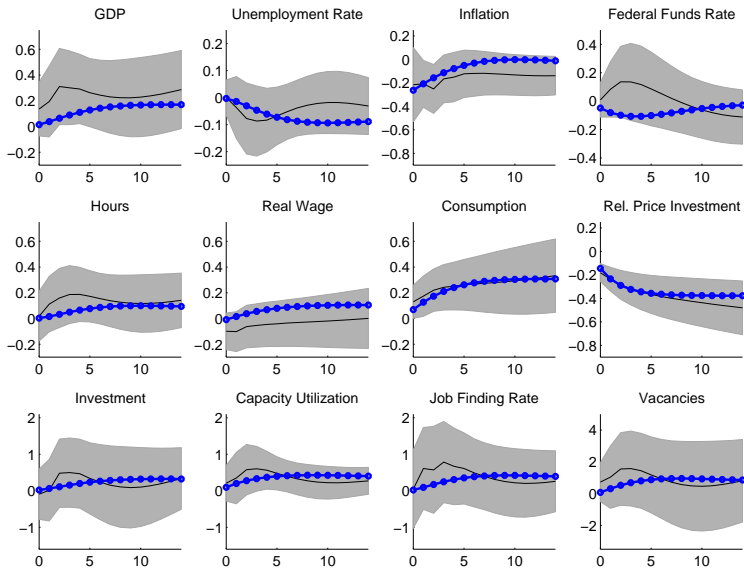
VAR 95% — VAR Mean Alternating Offer Bargaining Model



Notes: x-axis: quarters, y-axis: percent

# Medium-Sized Model Responses to an Investment-specific Technology Shock

■ VAR 95% — VAR Mean ● Alternating Offer Bargaining Model



Notes: x-axis: quarters, y-axis: percent

# Comparison With Two Other Models

- Standard DMP setup:
  - Firms post vacancies and meet workers probabilistically ( $c > 0, \kappa = 0$ ).
  - Workers and firms split surplus using a Nash-sharing rule.
- Standard New Keynesian sticky wage model following Erceg-Henderson-Levin (2000).
  - No wage indexation.
- Embed labor market models in CEE-style empirical model.
  - Calvo price rigidities, but no price indexation.

# Model Comparisons

- Marginal likelihood:
  - strongly prefers our model over standard DMP and NK sticky wage models by about 24 and 54 log points, respectively.
- Also, other models have relatively extreme parameter estimates.
  - For example, standard DMP formulation (Nash-sharing plus search), posterior mode of replacement ratio is 0.97.

# Cyclicality of Unemployment and Vacancies

- Similar to Shimer (2005), we simulate our model subject to a stationary neutral technology shock only.
  - Fixed parameter values.

---

---

## Standard Deviations of Data vs. Models

---

---

$$\frac{\sigma(\text{Labor market tightness})}{\sigma(\text{Labor productivity})}$$

---

Data	27.6
Standard DMP Model	13.6
Our Model	33.5

---

---

- Estimated DMP models also do well here.



# Conclusion

- We constructed a model that accounts for the economy's response to various business cycle shocks.
- Our model implies that nominal and real wages are inertial.
  - Allows to account for weak response of inflation and strong responses of quantity variables to business cycle shocks.
- Model outperforms sticky wage (no-indexation) NK in terms of statistical fit.
- Given limitations of sticky wage model, there's simply no need to work with it.