Advanced Macroeconomics,

Econ 416

Homework #5

Consider the simple labor market model in which a representative household maximizes, over C and B_{t+1},

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \ \beta \in (0,1),$$

subject to

$$B_{t+1} + C_t \leq w_t N_t + (1 - N_t) D + R_t B_t + \pi_t,$$

where B_{t+1}, C_t, D, w_t denote bonds, consumption, government unemployment payments and the wage rate, respectively. Also, π_t denotes lump sum profits, net of taxes levied by the government to finance unemployment benefits (the government runs a balanced-budget policy). The number of potential workers in the household is unity and a fraction, N_t, is employed while the complementary fraction is unemployed. The first period is $t = 0$, when variables dated $t = -1$ and earlier are given.

The labor market is setup as follows. The value to a firm of an employed worker is denoted J_t, where

$$J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1},$$

and ϑ_t denotes an exogenous shock to worker productivity. Also, ρ is the exogenous probability that a firm-worker match persists into the next period and m_{t+1} denotes the stochastic discount factor:

$$m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)},$$

which the firm treats as exogenous. A firm that is not matched at the start of period t, but wishes to be matched with a worker must undertake two costly activities at the start of period t. First, to find a worker the firm must post a vacancy, whereupon it finds a worker with probability, q_t. To post a vacancy, the firm must purchase a quantity of goods, κ. If the firm finds a worker, it must then purchase a fixed quantity of goods, H, before it can begin bargaining with the worker over the wage rate (see Pissarides, Econometrica 2009 for an explanation of this setup). At the time of bargaining, both costs of finding a worker are sunk, so that they are not taken into account in the bargaining. If period t bargaining is concluded successfully, then work begins immediately.

The present discounted value of an employed worker in period t is V_t. In recursive form, this is

$$V_t = w_t + \beta E_t m_{t+1} \left[\rho V_{t+1} + (1-\rho) \left(f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right].$$

Here, f_{t+1} denotes the probability, at the start of period $t+1$, that a worker who is not then matched, finds a firm. Also, U_t denotes the present discounted value of a worker that is not employed by a firm in period t. In recursive form,

$$U_t = D + \rho E_t m_{t+1} \left[f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right].$$

At time t, the worker treats m_{t+1}, V_{t+1} and U_{t+1} as exogenous.

Bargaining is undertaken at the start of period t, after search by firms and workers is finished. At that time, bargaining occurs between the $f_t (1 - \rho N_{t-1})$ firms and workers that have just met for the first time and between the ρN_{t-1} firms and workers that met in the past and remain attached. Here, $1 - \rho N_{t-1}$ is the sum of the unemployed workers in period $t-1$ and the $(1-\rho) N_{t-1}$ workers who became separated at the end of $t-1$ from their employers.

Firms and workers do Nash bargaining. That is, they take the sum of the surplus associated with a match

$$J_t + V_t - U_t,$$

and give a fraction, η, to workers and the complementary fraction, $1-\eta$, to firms. Thus,

$$J_t = \frac{1-\eta}{\eta} (V_t - U_t).$$

Since firms and workers take $J_{t+1}, V_{t+1}, U_{t+1}$ as given, Nash bargaining can be thought of as determining the wage rate, w_t.

We assume that firms make zero profits, so that

$$\kappa = q_t (J_t - H),$$

or,

$$\tilde{\kappa}_t = J_t,$$

where

$$\tilde{\kappa}_t = \frac{\kappa}{q_t} + H.$$

We must still describe the determination of the probabilities, q_t and f_t. These are treated as exogenous by firms and workers. Vacancies meet workers randomly and workers meet firms randomly, so that

$$f_t = \frac{x_t N_{t-1}}{1 - \rho N_{t-1}}, q_t = \frac{x_t N_{t-1}}{v_t N_{t-1}},$$

where x_t is the aggregate meeting rate, so that $x_t N_{t-1}$ is the number of new meetings between firms and workers. Also, v_t is the time t vacancy rate, so that $v_t N_{t-1}$ is the number of vacancies posted at the start of period t. We assume that the total number of new meetings, $x_t N_{t-1}$, are the result of the total number of vacancies and of the number of job searchers according to the matching function:

$$x_t N_{t-1} = \sigma_m \left(1 - \rho N_{t-1}\right)^{\sigma} \left(v_t N_{t-1}\right)^{1-\sigma},$$

where $\sigma_m > 0$ and $\sigma \in (0, 1)$. The variable, Γ_t, is referred to as 'labor market tightness' and is defined as follows

$$\Gamma_t \equiv \frac{v_t N_{t-1}}{1 - \rho N_{t-1}}.$$

1. (a) Show that

$$\begin{aligned} q_t &= \frac{\sigma_m}{\Gamma_t^{\sigma}},\ \Gamma_t = \left(\frac{f_t}{\sigma_m}\right)^{\frac{1}{1-\sigma}},\ \tilde{\kappa}_t = \kappa^* \left(f_t\right)^{\frac{\sigma}{1-\sigma}} + H, \\ \kappa^* &\equiv \kappa \sigma_m^{\frac{-1}{1-\sigma}}, \end{aligned}$$

so that $q_t, \tilde{\kappa}_t$ and f_t can be expressed as functions of x_t, N_{t-1} alone. Derive and display the resource constraint for the economy from the household and government budget constraints and from the definition for profits.

(b) You may assume that utility is linear in C_t, so that $m_{t+1} = \beta$ and suppose that $\vartheta_t = 1$ for all t. By appropriate substitution among the equilibrium conditions, derive a dynamic equation in f_t and f_{t+1} alone that must be satisfied in equilibrium and let it be denoted, $F(f_t, f_{t+1}) = 0$, $t = 0, 1, 2, ...$. Suppose that you have found a (deterministic) sequence, $f_0, f_1, f_2, ...$ that satisfy $F(f_t, f_{t+1}) = 0$ and $f_t \in (0, 1)$ for all t. Does such a *candidate equilibrium* sequence correspond to an actual equilibrium, in the sense that you can find $C_t \geq 0$, $N_t \in [0, 1]$, $w_t \geq 0$ for $t = 0, 1, 2, ...$ that constitute an equilibrium for the system as a whole? Or, are further restrictions on $\{f_t\}$ required, in addition to satisfying F and $f_t \in (0, 1)$?

(c) Let $\sigma = 1/2$, $\rho = 0.9$, $\beta = 1.03^{-1/4}$, $\vartheta_t = 1$, $\eta = 1/2$ (these are all reasonable parameter values relative to the literature). Choose values for H and D such that, conditional on the other parameter values and a value for κ^*, $l = 0.945$ and $D/w = 0.40$.

i. Graph the mapping from f_t to f_{t+1} implied by $F = 0$ (f_t on the horizontal axis and f_{t+1} on the vertical axis) for $\kappa^* = 0,\ 0.1,\ 0.2,\ 0.3$. In each case, compute the slope of the mapping at the point where it crosses the horizontal axis (i.e., $-F_1/F_2$).

ii. Explain why it is that when the slope referred to above is less than unity in absolute value, then for f_0 sufficiently close to the steady state finding rate, f, there is another equilibrium that corresponds to the sequence, $f_1, f_2, ...$ generated by F starting from f_0. Explain why it may nevertheless not be true that the sequence of f_t's starting from f_0 far from f corresponds to an equilibrium.

iii. Describe the possibility that the mapping from f_t to f_{t+1} is consistent with the following situation: the steady state is determinate (as for large values of κ^*), but there is a two-period cycle (I don't believe that the model in this question has this property).

(d) Consider a value of κ^* in part (c) where the steady state is indeterminate. Construct a sunspot equilibrium by replacing $F(f_t, f_{t+1}) = 0$ with $E_t F(f_t, f_{t+1}) = 0$.

i. Explain why the sunspot shocks must not be too big (that is, indeterminacy of the steady state implies he existence of a sunspot equilibrium *with sufficiently small sized shocks*).

ii. Generate a long sequence of realizations of C_t, N_t, f_t, w_t from a sunspot equilibrium. (Don't do any linearization for this! The model is analytically tractable). Is this a decent business cycle model? That is, what is the ratio of the standard deviation of HP filtered log N_t to the standard deviation of HP filtered log C_t? Note that this is not a minimal state variable solution: f_t depends on the current realization of a non-fundamental shock and also on a variable, f_{t-1}, that is payoff irrelevant at time t.