

Advanced Macroeconomics,

Econ 416

Homework #6

Due: November 13

1. Use first order perturbation to solve the simple six equation New Keynesian model discussed in class (you may use Dynare if you wish, but if you've figured out how to do symbolic arithmetic in MATLAB, then using Dynare doesn't offer huge advantages). Or, you could use the linearized conditions derived in class. Consider the case where the technology shock is difference stationary, so that you have to scale consumption in the model to ensure that there exists a steady state independent of initial conditions. You might as well scale 'natural consumption', so that scaled consumption,  $X_t$ , has an interpretation as the output gap. Set the monopoly subsidy so that the effects of monopoly power are extinguished in steady state. Let the interest rate rule be given by:

$$R_t = \frac{1}{\beta} \bar{\pi}_t^{\phi_\pi},$$

where  $\bar{\pi}_t$  denotes the gross inflation rate and  $R_t$  denotes the gross nominal rate of interest on a nominally risk free loan taken out in period  $t$  and repaid in  $t + 1$ . Consider the following parameterization:

$$\begin{aligned}\beta &= 0.97, \phi_\pi = 1.5, \rho = 0.2, \lambda = 0.5, \\ \varphi &= 1, \theta = 0.75, \sigma_{\varepsilon^a} = \sigma_{\varepsilon^\tau} = 0.02.\end{aligned}$$

- (a) In the case of the technology and preference shocks, compute the impulse response functions of the variables to each shock.
  - i. Consider the response of the economy to a technology shock and a preference shock. In each case, indicate whether the economy over- or under- responds to the shock, relative to their 'natural' responses. What is the economic intuition in each case? Display the output gap, employment, inflation, output and the shock in graphs. Display the output gap as the log-deviation of output from natural output and similarly for employment. Do you reproduce the results reported in class?
  - ii. Replace the time series representation of  $a_t$  with the trend stationary represen-

tation:

$$a_t = \rho a_{t-1} + \varepsilon_t^a.$$

How does the response of the economy to  $\varepsilon_t^a$  with this representation compare to the response to  $\varepsilon_t^a$  with the unit root representation?

- (b) Do the calculations with  $\phi_\pi = 0.99$ , and note that you get indeterminacy (you saw this before). Can you provide economic intuition for this result?
- (c) Return to the parameterization,  $\phi_\pi = 1.5$ . Now, insert  $r_t$  into the Calvo pricing equation. Redo the calculations and note how you get indeterminacy again. Provide economic intuition for this result.
- (d) Replace the monetary policy rule by  $R_t = E_t R_{t+1}^*$ , and try to solve the model. Explain why the Ramsey equilibrium is not the only solution to the equilibrium conditions (i.e., the indicated policy rule does not support the natural equilibrium uniquely). This shows that implementation of the Ramsey allocations is not a trivial exercise. That is why this is called the ‘implementation problem’.
- (e) Now replace the monetary policy rule with

$$R_t = \bar{\pi}_t^{\phi_\pi} \times E_t R_{t+1}^*.$$

Verify computationally that this policy rule uniquely supports the natural equilibrium (in the sense of satisfying determinacy), as long as  $\phi_\pi$  is large enough. Provide intuition. Conclude that the Taylor rule uniquely supports the natural equilibrium if the natural rate of interest is included in the rule.

- (f) Consider the following alternative representation for the technology shock:

$$a_t = \rho a_{t-1} + \xi_t^0 + \xi_{t-1}^1,$$

where both shocks are iid, so that the sum is iid too. Here, we assume agents see  $\xi_t^0$  at time  $t$  and they see  $\xi_{t-1}^1$  at  $t - 1$ . Thus, agents have advance information (or, ‘news’) about the future realization of a shock. Introduce this change into the code and set  $\rho = 0.2$ . Verify that when there is a shock to  $\xi_t^1$ , inflation falls contemporaneously and the output gap jumps. Provide intuition for this apparently contradictory result. What happens when the natural rate of interest is introduced in the policy rule? (This example is the subject of my 2010 Jackson Hole paper, <http://faculty.wcas.northwestern.edu/~lchrist/research/ECB/JacksonHole/paper.pdf>)

2. We now consider a modification to the New Keynesian model above that resembles the modification of the RBC model in the take home exam. The problem of the household is to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t),$$

subject to

$$C_t \leq w_t N_t + (1 - N_t) D - T_t + \pi_t,$$

where  $\pi_t$  denotes firm profits (discussed in the take-home) and  $T_t$  denotes taxes raised to finance government unemployment payments. The government budget constraint is:

$$(1 - N_t) D = T_t.$$

Instead of hiring labor in competitive markets, intermediate good firms in the NK model purchase an input,  $h_t$ , in competitive markets at an equilibrium price of  $\vartheta_t$ . The  $h_t$  goods are produced by the *bargaining firms* in the take-home. The value functions of the bargaining firms and workers are as they are in the take home. The resource constraint in this economy is:

$$C_t + \kappa x_t N_{t-1} \leq Y_t,$$

where  $Y_t$  denotes the output of the final good firms. Now the labor preference shock plays no role in the analysis, but the technology shock does.

- (a) Parameters whose values are not specified in the first question above should correspond to the values you used in the take home. Recompute the impulse response function to a difference and trend stationary shocks that you did in the previous question. What are the differences? Can you provide intuition?
- (b) Set the replacement ratio to 0.40. Note that you get a (locally) unique solution for the model. This is different from what you got in the take home. Can you provide intuition (hint: see what happens when  $\theta = 0$ , i.e., there are no sticky prices)? To me, this seems like a hard question, and I don't know the answer.
- (c) How do the impulse response functions in the case of a replacement ratio of 0.40 compare to what you got in question 1?
- (d) Modify the model so that the bargaining firm sector corresponds to the version of the model in the previous homework in which  $H = 0$ . In this version of the model, the only cost the firm incurs is a vacancy posting cost. I'll call this specification

the standard DMP specification. Recompute the impulse response functions for this case. Note that the response of employment is relatively modest. This small response has been called the *Shimer puzzle* (Shimer, AER 2005) because Shimer showed that the standard DMP specification implies little volatility in labor market variables. A problem with the standard DMP specification (as stressed by Pissarides) is that the increase in labor market tightness in a boom produces a higher wage, and this robs firms of the incentive to expand employment.