Advanced Macroeconomics,

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Econ 416

Homework #7

Due: November 21

1. Consider the linearized equilibrium conditions of the New Keynesian model, on the slide, 'The Equilibrium Conditions' in the handout, 'Simple New Keynesian Model: Foundations'. Consider the model of 'news' studied by Barsky-Sims, and Barsky-Basu-Lee (the latter is in this year's Macroeconomics Annual). This question will have to be done in MATLAB, and you'll need my model solving code, solveaa.m (to compute A) and solveb.m (to compute B). If you do not have it, let me know.

   Suppose the model of technology is as follows:

$$\Delta a_{t+1} = \varepsilon_{1,t+1} + g_t,\ \ g_t = 0.2 g_{t-1} + \varepsilon_{2,t}.$$

   Economic agents observe $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ at time t. Note that $\varepsilon_{2,t}$ is a news shock in that it only affects technology in period $t+1$ and has no impact on technology in period t. There are no other shocks in the system.

   (a) Explain why it is that in the simple NK model, it is first best for consumption and employment in period t to not respond to a period t news shock.

   (b) Suppose the econometrician observes a vector of data composed of TFP growth and employment:

$$Y_t = \begin{pmatrix} \Delta a_t \\ \log\left(\text{employment}_t\right) \end{pmatrix},$$

   $t = 0, 1, ..., T$. Display the state-space/observer representation of Y_t (i.e., the F, Q, H matrices in the lecture notes).

   (c) Suppose that T is large, so that issues of sampling uncertainty can be ignored.

      i. Can the news shock, $\varepsilon_{2,t}$, be recovered from current and past observations on Y_t? In particular, consider the vector autoregression representation of Y_t, in which the one-step-ahead forecast error in Y_t is denoted u_t. Can $\varepsilon_{2,t}$ be recovered from u_t?

      ii. How many lags are there in the VAR representation of Y_t? In the VAR representation, what is the 2×2 matrix lag on each of $Y_{t-1}, Y_{t-2}, Y_{t-3}$?

2. In the Eggertsson and Woodford (EW) approach to solving the NK model in the zero lower bound, linearized versions of the equilibrium conditions are used. There is no approximation theory to support this strategy, because the zero lower bound is discretely away from the steady state about which the approximation is taken. There, inflation is zero and the real and nominal rates of interest are positive, say 3 percent per year. So, there are serious questions about the accuracy of the approximation. The EW approach not only works with linearized equilibrium conditions, but the environment is further restricted so that there exists an 'EW equilibrium', i.e., an equilibrium characterized by a single inflation rate and level of output while the zlb is binding and then a jump to the positive interest rate steady state when the zlb ceases to bind. While analysis of an EW equilibrium is informative, the restrictions do rule out the investigation of some interesting questions. For example, one cannot investigate the impact on the zlb of a commitment to keep the interest rate low after the zlb ceases to bind ('forward guidance').

   This question adopts the assumption that agents have perfect foresight (we will relax this in the next question). Suppose that prior to $t = 0$, the economy was in a deterministic, zero inflation steady state (among other things, this implies $p^*_{-1} = 1$, $R_{-1} = 1/\beta$). Prior to $t = 0$, all agents in the economy expected to remain in steady state forever. But, in $t = 0$ everyone becomes aware that

$$\begin{aligned} r_1 &= r_2 = ... = r_{10} = r^l < 0 \\ 1 + r_t &= \frac{1}{\beta} \text{ for } t > 10. \end{aligned}$$

   Here, r_{t+1} is the rate at which utility in period $t + 1$ is discounted to period t. In the baseline of the model, let the parameters take on the following values:

$$\varepsilon = 6,\ \theta = 0.75,\ (1 - \nu)\frac{\varepsilon}{\varepsilon - 1} = \varphi = 1,\ \beta = 0.99,\ \phi_1 = 1.5,\ \phi_2 = 0,\ r^l = -0.01,\ \rho_R = 0.$$

   The model is quarterly, so the nominal rate of interest in the zero inflation steady state is $(1/\beta)^4 - 1$ or 4 percent, after rounding. The shock is that the discount rate drops from its steady state (annualize) value of 4 percent to minus 4 percent for 10 periods.

   The private sector equilibrium conditions are equations (1)-(6) in the handout, 'Simple New Keynesian Model: Foundations' (see the pages with the titles, 'Collecting Equilibrium Conditions' and 'Equilibrium Conditions'). The monetary policy rule is the one given in the handout that studies the zlb, which was discussed in Monday's class. (Careful: the dating convention for the interest rate in the zlb handout is different from the one in the NK handout.)

Solve the model using the extended path solution method described in the class handout (you could set $T^* = 150$). Do this directly in MATLAB. You can do the matrix inversion that is required by using the $inv(\cdot)$ command, though this is not very efficient. If you're so inclined, you could explore sparse matrix inversion methods in MATLAB, but this is not required for this homework. Or, you could avoid programming the gradient search method described in the handout and use a canned gradient search method that is in MATLAB (e.g., fsolve.m).

The objective of the algorithm is to compute a sequence,

$$C_t, F_t, K_t, \bar{\pi}_t, N_t, p_t^*, R_t, \; t = 0, 1, 2, ..., T^* - 1,$$

that solves the six private sector equilibrium conditions and the monetary policy rule on the assumption that the system is in steady state. Initiate the calculations at the steady state values of these variables.

One way to assess 'forward guidance' is to set $\rho_R > 0$. Relative to the case, $\rho_R = 0$, $\rho_R > 0$ represents a commitment to keep the interest rate low, longer. Graph output and inflation from $t = 0$ to the time it reaches steady state for both $\rho_R = 0.8$ and $\rho_R = 0$. You should find that the severity of the zlb is reduced with $\rho_R > 0$. Be sure to also graph p_t^*.

3. The assumption of perfect foresight in the previous question seems implausible. This is especially so, because we assume the drop in r_t (or, whatever shock made the zlb bind) was completely unanticipated. If agents had initially assigned zero probability to a binding zlb, then it seems a bit much to suppose that when it does bind, agents have perfect foresight over future values of the shock.

   For this reason we now explore a solution strategy that captures the idea that agents make decisions based on imperfect forecasts of the future. There is a second reason to introduce uncertainty. The Eggertsson-Woodford calculations (see the handout, 'The Zero Bound and Fiscal Policy', beginning with the section, 'Analysis of Case when the Non-negativity Constraint on the Nominal Interest Rate is Binding', discussed in Monday's class) adopt a particular stochastic environment and it would be interesting to know whether or not their linear approximation strategy is accurate.

One way to proceed applies a version of the projection method discussed earlier in the course. An advantage of this method is that it can be made arbitrarily accurate. A disadvantage is that in models of reasonable size the computational time needed to implement the projection method may be prohibitive. For this reason, we pursue the stochastic version of the extended path method in the handout.

The stochastic version of the extended path method adopts the approximation that agents have certainty equivalence. To implement the strategy requires applying the algorithm developed in the previous question many times, once for each date, $t = 0, 1, ...$ To determine the value taken on by the variables in $t = 0$, we solve for the deterministic equilibrium in which the exogenous shocks evolve according to their forecasts as of period $t = 0$. The period 0 value of the endogenous variables in this computation represents the period $t = 0$ realized values of the endogenous variables. In period $t = 1$ agents observe the period 1 realization of the exogenous shock(s) and they compute a new set of forecasts of the exogenous variables. We then use the solution strategy of the previous question again to solve for the sequence of variables that solve the equilibrium conditions for $t = 1, 2, ...$ conditional on the new forecasts. The $t = 1$ element of this deterministic equilibrium is the period 1 realization of the endogenous variables. Realizations of the endogenous variables in periods $t = 2, 3, ...$ are computed in the same way.

We adopt the law of motion for r_t that was posited by Eggertsson and Woodford. In particular, if in period t, $r_{t+1} = r$ is observed, then the discount rate is expected to remain in that state forever ($r = 1/\beta - 1$). If in period t, $r_{t+1} = r^l$ is observed, then with probability p the discount rate is expected to remain unchanged in the following period and with probability $1 - p$ it jumps to its absorbing state of r. Thus, the discount rate is a two-state, first order Markov chain. In particular, let the states be denoted $\mathbf{R}$, where

$$\mathbf{R} = \begin{pmatrix} r \\ r^l \end{pmatrix}.$$

The transition probabilities in the Markov chain are given by:

$$\mu_{i,j},$$

where $\mu_{i,j}$ denotes the probability that the system moves from state i to state j, for

$i, j = 1, 2$. Given that state 1 is r and state 2 is r^l, we have

$$\mu = \begin{bmatrix} 1 & 0 \\ 1-p & p \end{bmatrix}.$$

Then,

$$E_t\left[r_{t+1+j}|r_{t+1} = r^l\right] = \begin{bmatrix} 0 & 1 \end{bmatrix} \mu^j \mathbf{R}. \tag{1}$$

Suppose that the discount rate turns out to be r^l for 10 periods in a row. We simulate the response of the system to this realization of the discount rate shocks as follows. The forecasts of the future values of the discount rate are given by (1) in each of the periods when it is observed to be in its low state. The forecasts for the discount rate is r when it is observed to take on its steady state value. To compute the endogenous variables, we proceed as follows.

Let z^t_{t+j}, $j = 0, 1, 2, ..., T^* - 1$ denote the values of the endogenous variables that solve the deterministic private sector equilibrium conditions when y_{t+j} is replaced by y^t_{t+j}, the forecast of y_{t+j} as of date t, for $t = 1, ..., T^* - 1$. These variables are computed to satisfy the equilibrium conditions subject to going to non-stochastic steady state in period T^*. This method assumes certainty equivalence because future random variables are replaced by their forecasts. This is in fact a mistake, but perhaps not a very big one. Eggertsson and Woodford make two mistakes: they replace the equilibrium conditions by linear approximations and they assume certainty equivalence. In effect, we only make the second mistake.

(a) Solve the model for $t = 0, ..., T^* - 1$ using the stochastic extended path method and using the Eggertsson-Woodford procedure discussed in class. Set $p = 0.8$. Graph output, inflation and p^*_t given the realization that the discount rate remains low for 10 periods (unlike in the previous question, agents don't know this fact in advance). The solution based on Eggertsson and Woodford's procedure involves constant values of output, inflation and p^* while the discount rate is low and a jump to steady state as soon as the discount rate flips up to r. The extended path procedure procedure will generate (slightly, perhaps) different results because of the presence of the endogenous state variable, p^*_{t-1}, and the nonlinearities in the equilibrium conditions.

(b) Redo the computations using different degrees of price flexibility. Do you get the same extreme sensitivity that we find when linear approximations are used?

(c) Redo the computations using different values of p. Do you get the same extreme sensitivity that we find when linear approximations are used? (The intuition for why the zlb collapse gets worse with bigger p is that larger p induce a larger negative wealth effect on consumption.)

(d) Keep the situation as it is in (a) above, but increase ρ_R from 0 to 0.80. How does this change in the monetary policy rule affect the performance of the economy while in the zlb? How does this change affect the result that the drop in inflation and output are roughly constant while the economy is in the zlb?