Topics in Quantitative Macroeconomics, Fall 1998
Homework #1.
Due: Wednesday, October 30.
Christiano

You should do all the calculations for this homework in MATLAB.

Consider an economy in which household preferences have the following form:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \ \beta = .99,$$

and $u(c_t, n_t) = \log(c_t) + \psi \log(1 - n_t)$. The household budget constraint is:

$$c_t + k_{t+1} - (1-\delta)k_t = r_t k_t + w_t n_t + \pi_t, \ \delta = .028,$$

where r_t, w_t, π_t denote the rental rate on capital, the wage rate, and profits, respectively. Firms operate the following technology:

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha}, \ \alpha = 0.36.$$

Here,

$$A_t = Y_t^{\gamma},$$

where Y_t denotes the economy-wide level of output, in per capita terms. Firms are competitive, and maximize profits.

(a) Explain why profits, π_t, must be zero in equilibrium.

(b) Derive the household static and dynamic Euler equations. Derive the firm Euler equation.

(c) In the household Euler equations, substitute out the rental rate on capital and the wage rate for labor, using the firm first order conditions. You should have a static and dynamic Euler equation.

(d) You should focus on symmetric equilibria, in which the economy-wide average stock of capital and level of employment coincide with the firm's stock of capital and employment. After making this identification, the static and dynamic Euler equations should have the following arguments:

$$\begin{aligned} \text{`static'} &: \quad v_h(n_t, k_t, k_{t+1}) = 0 \\ \text{`dynamic'} &: \quad v_k(k_t, k_{t+1}, k_{t+2}, n_t, n_{t+1}) = 0. \end{aligned}$$

Linearize these equations about the steady state values of employment and and the firm's stock of capital. In computing the steady states, fix steady state employment at 1/3 and choose the value of ψ that rationalizes that. Initially, fix $\gamma = 0.01$.

(e) Substitute out for labor in the dynamic Euler equation, using the static Euler equation. This will give you one dynamic Euler equation in k_t, k_{t+1}, k_{t+2}.

(f) Set the Euler equation up as a first order difference equation:

$$AY_{t+1} + BY_t = 0.$$

Show the values of A, and B. What are the eigenvalues of $\Pi = -A^{-1}B$? Find a value of γ such that, for values larger than that, both eigenvalues of Π are less than unity in absolute value. Display the left eigenvectors of Π associated with each of these two eigenvalues. Hint: to find out how to compute eigenvalues and eigenvectors in MATLAB, type 'help eig', in MATLAB.

(g) Let $k_0 = .9k^*$. Generate 50 observations on n_t, y_t and k_t from the two minimal state solutions. (Hint: a minimal state solution is one in which Y_0 is selected so that $\tilde{P}_i Y_0 = 0$ for $i = 1$ or 2.) Can you make the case that these two sequences represent equilibria for the model economy?