Topics in Quantitative Macroeconomics, Fall 1998
Homework #2.
Due: Monday, October 12.
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1. Consider the model economy of homework #1. Solve the model again, but this time do it *without* substituting out for labor in the dynamic Euler equation. Compute the set of minimal state solutions that have the form, $\tilde{k}_{t+1} = d\tilde{k}_t$, $\tilde{n}_t = q\tilde{k}_t$. How many are there? How do they compare with what you got in homework #1? You can do this just for the case $\gamma = 0$.

   To answer this question, you will need to apply the QZ decomposition in MATLAB. As I indicated in class, MATLAB's version of this decomposition does not order H_0 so that the zeros on the diagonal are located along the bottom right part of the diagonal. My software for doing this implements Chris Sims' programs, QZDIV.M and QZSWITCH.M, which is called by QZDIV.M. Here is the way I implement the decomposition: (the $m \times m$ matrices a and b below correspond to A and B, respectively, in class)

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   %we are looking for q and z, where
   % q*a*z = H0, q*b*z = H1
   % and q, z are orthonormal. H0 and H1 are upper triangular.
   [H0,H1,q,z,v]=qz(a,b);
   % H0*gam(t+1) + H1*gam(t) = 0
   % gam(t) = z'*y(t)
   %re-order matrices:
   %stake1 compares H0(i,i) to H1(i,i) and 0; stake2 compares H0(i,i) to 0
   %Success of QZ decomp can be very sensitive to these values
   stake1 = 1e+08;
   stake2 = 1e-05;
   [H0,H1,q,z] = qzdiv(stake1,H0,H1,q,z);
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%Make sure H0(i,i) ~= 0 and H1(i,i) ~= 0
root = abs([diag(H0) diag(H1)]);
chord = sqrt(root(:,1).*root(:,1)+root(:,2).*root(:,2));
if (min(chord) < (1/stake1))
error('fatal (solvea) element of (H0(i,i),H1(i,i)) is close to (0,0)');
end;
%Count zero elements of H0(i,i)
root(:,1) = root(:,1)-(root(:,1)<stake2).*(root(:,1)+root(:,2));
root(:,2) = root(:,2)./root(:,1);
zct = length(find((root(:,2) > stake1 | root(:,2) < -0.1)));
K = m-zct;
%L2*y(t) = 0 by construction
zz = z';
L1 = zz([1:K],:);
L2 = zz([K+1:m],:);
g0 = H0([1:K],[1:K]);
g1 = H1([1:K],[1:K]);
gg = H1([K+1:m],[K+1:m]);
PIE = -g0\g1;
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2. Consider again the model of homework #1, with a value of γ which is large enough that both eigenvalues are smaller than unity in absolute value. Replace the utility function by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t),$$

so that there is now an expectation in front of it. The model has no shocks inside the utility function or in the production function. Still, it is possible to generate solutions with bounded, but positive, variance by letting ξ_{t+1}, the Euler error, be random. Generate a long data series

(500 observations) on output, y_t, from this model and compute the standard deviation of $log(y_t)$ using the command, *std*, in MATLAB. Choose the variance for ξ_{t+1} so that the standard deviation of output is about 0.02 (i.e., 2 percent).

(a) In the US data, the standard deviation of investment (in percent terms) is 2-3 times what it is for output. How does this model do on this statistic?

(b) In the US data, hours worked is about as volatile (again, measured as the standard deviation of logged hours) as output. How does the model look?

(c) In the data, productivity is procylical (i.e., log output and log output/labor effort are positively correlated). How does the model look on this?

(d) Think about how to interpret this equilibrium that you have studied. In the competitive decentralization, household are observing prices and rates of return that are random processes. That this randomness is not fundamental is, of course, of no concern or even interest to them. All they care about is maximizing utility subject to their budget constraint.

3. Consider again the model economy of homework #1, but with

$$A_t = z_t^{(1-\alpha)}, \ z_t = z_{t-1} \exp(\theta_t),$$

and θ_t is a serially uncorrelated process with standard deviation 0.001. Scale this economy by z_t. For example, let $\bar{k}_{t+1} \equiv k_{t+1}/z_t$. Note how the depreciation rate in the scaled economy is now a random variable. The Euler equations are:

$$\begin{aligned} v_k(\bar{k}_t, \bar{k}_{t+1}, \bar{k}_{t+2}, n_t, n_{t+1}, \theta_t, \theta_{t+1}) &= \xi_{t+1} \\ v_n(n_t, \bar{k}_t, \bar{k}_{t+1}, \theta_t) &= 0 \end{aligned}$$

The purpose of this question is to study the unique non-explosive solution for the scaled economy obtained by linearizing the Euler equations in two ways.

(a) Consider first a log-linearization. Rewrite the equation for capital as

$$\tilde{v}_k(K_t, K_{t+1}, K_{t+2}, N_t, N_{t+1}, \theta_t, \theta_{t+1}) = \xi_{t+1},$$

where $K_t = \log(\bar{k}_t)$, $N_t = \log(n_t)$, and

$$\begin{aligned} &\tilde{v}_k(K_t, K_{t+1}, K_{t+2}, N_t, N_{t+1}, \theta_t, \theta_{t+1}) \\ \equiv\ & v_k(\exp(K_t), \exp(K_{t+1}), \exp(K_{t+2}), \exp(N_t), \exp(N_{t+1}), \theta_t, \theta_{t+1}). \end{aligned}$$

Define

$$\tilde{v}_n(N_t, K_t, K_{t+1}, \theta_t) = 0$$

in a similar way.

i. Linearly approximate $\tilde{v}_k$ and $\tilde{v}_n$, and find the unique non-explosive solution in which K_{t+1} is a linear function of K_t, N_t, θ_t. Display the parameter values. Display the parameters of the linear function relating the Euler errors to the fundamental shock, θ_t.

ii. Write out the solution in terms of the fundamental variable of interest, namely, k_{t+1}. (Thus, k_{t+1} will be a multiplicative function involving k_t, θ_t, z_{t-1}.) Compute consumption as follows:

$$c_t = y_t - [k_{t+1} - (1-\delta)k_t].$$

Given the approximations to the policy rules that you computed for n_t and k_{t+1}, c_t is a function of θ_t and other variables that are given at time t. Graph the response of c_t to θ_t for values of θ_t from zero to 5.

(b) Now linearize v_k and v_n directly in terms of the scaled variables, without logging.

i. Display the unique non-explosive policy rules relating $\bar{k}_{t+1}$ to $\bar{k}_t$ and θ_t and n_t to $\bar{k}_t$ and θ_t. Is there a simple connection between the policy rule parameters computed under the log-linear approximation and the straight linear approximation?

ii. Compute the implied policy rules for the variables of interest, k_{t+1} and n_t. Plot the response of consumption to θ_t as in the previous part of this question. Do these responses look very different? They should, and the difference should cast doubt on the straight linear approximation.

4. Consider the following process:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \upsilon_t + \theta_1 \upsilon_{t-1} + \theta_2 \upsilon_{t-2},$$

where υ_t is a white noise process. Show how to write this in the form:

$$\theta_t = \rho \theta_{t-1} + \varepsilon_t.$$