Topics in Quantitative Macroeconomics, Fall 1998
Homework #3.
Due: Wednesday, October 28.
Christiano

1. Consider the two period time-to-build model in the handout. (Beware of the possibility of typos in the handout!! I wrote it carefully, but there might still be errors.)

   (a) Set $\phi = 1/2$. Find γ such that there are multiple, non-explosive solutions to the state-space model, equation (7) in the handout. Display all the A matrices with non-explosive eigenvalues, which solve $\alpha(A) = 0$.

   (b) For each of the above A matrices, compute the corresponding B matrix and display it.

   (c) For each (A, B) pair that solves the linearized model, compute the impulse responses of c_t, I_t, n_t, and y_t to $\varepsilon_t = .01$. Report the responses in percent of steady state values.

   (d) Now set $\gamma = 0$.

      i. Repeat the impulse response function calculations you just did.

      ii. Choose σ_ε so that the standard deviation of log output is .02. Compute σ_I/σ_y, σ_n/σ_y, σ_c/σ_y using a long realization (i.e., 1,000 observations) of artificial data from this model. Here, σ_x is the standard deviation of the log of the variable, x_t (careful! if I_t is negative this won't work!) Also, compute $corr(I_t, y_{t-\tau})$ for $\tau = -3, -2, -1, 0, 1, 2, 3$. In the data, the values of this correlation are biggest for positive values of τ, when investment is measured as investment in business structures.

      iii. Let $\alpha_i(I)$ denote i^{th} impulse response computed above for investment (in percent of steady state), $i = 0, 1, 2, 3, ..., 50$. Define $\alpha_i(y)$ in the same way, for output. Compute and display

$$\sum_{i=0}^{50} \alpha_i(I)\alpha_i(y).$$

This should look a lot like $corr(I_t, y_t)$ computed above. Compute and display

$$\sum_{i=0}^{49} \alpha_{i+1}(I)\alpha_i(y).$$

This should look a lot like $corr(I_t, y_{t-1})$.

(e) Now set $\phi = .001$. Repeat the above impulse response, second moment calculations and cross-product calculations. Any differences? Any improvement with the correlation? The cross-product calculations should be helpful here.

2. This is a numerical integration exercise. We will need these types of calculations to compute the interest rate implications of dynamic, equilibrium models.

   (a) Let y have a normal distribution with mean $\mu = 2$ and standard deviation $\sigma = 2$, so that its density function is:

   $$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right].$$

   Consider the integrals:

   $$\int yf(y)dy, \quad \int E(y-\mu)^2 f(y)dy.$$

   You know the value of these integrals: μ and σ^2. Approximate these integrals using a Gaussian-Quadrature method. The one best suited to evaluating these integrals is Gauss-Hermite Quadrature. The $M-$point Gauss-Hermite quadrature formula is:

   $$\int_{\infty}^{\infty} l(x)e^{-x^2}dx \approx \sum_{i=1}^{M} w_i l(x_i).$$

   The weights and abscissas of the Gauss-Hermite integral formula are calculated in a Numerical Recipes program, *gauher.m*, that I have emailed to all of you. Approximate the above integrals using $M = 2, 3, 4, 5, 10$ point Gauss-Hermite Quadrature. How well do the approximations work?

(b) Now consider the problem,

$$E\exp(y),$$

where y has the same distribution as above. The exact value of this integral is $\exp(\mu+\sigma^2/2)$. How well does $M = 2,3,4,5,10$ Gauss-Hermite Quadrature do at approximating this integration problem?

3. Consider the following time series model:

$$y_t = \rho y_{t-1} + \varepsilon_t,$$

where ε_t is iid with variance σ_ε^2. Then, the covariance function for this process is:

$$E y_t y_{t-\tau} = c(\tau) = c(0)\rho^{|\tau|}, \text{ for integer } \tau,$$

where

$$c(0) = \sigma_\varepsilon^2/(1-\rho^2).$$

The spectral density of the process is:

$$g(\omega) = \sum_{\tau=-\infty}^{\infty} c(\tau) e^{-i\omega\tau} = \frac{\sigma_\varepsilon^2}{(1-\rho e^{-i\omega})(1-\rho e^{i\omega})},$$

for $\omega \in (-\pi,\pi)$. Compute $c(\tau)$ by:

$$c(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\omega) e^{i\omega\tau} d\omega.$$

Use the approximation to this sum suggested on page 35 of my manuscript ('Solving Dynamic Equilibrium Models....'). What value of N do you need to get a good approximation? A good reference for this material is Sargent, chapter XI.