Christiano
FINC 520, Spring 2007
Final Exam.

Please do this exam on your own. It is due in Emiliano Pagnotta's box by noon, Thursday.

1. Consider the time series model you estimated in question 3, part (d) of homework #7. Do a bootstrap on the ARCH(2) model of the residuals, to evaluate how well the asymptotic sampling theory you used in the test in part (b) works in a small sample. By bootstrapping the ARCH residuals, I mean recover the fitted v_t's (see [21.1.9], page 659) and bootstrap those. Compute the bootstrap $p-$value of the test that the true number of lags is 2, against the alternative that the true number of lags is 5. Compare the bootstrap $p-$value against the asymptotic $p-$value. How well does the asymptotic theory fare?

2. Consider the VAR you estimated in question 2, homework #2. Recall that the actual long rate turned out to be higher than what the expectations hypothesis predicts during the post-1980 deflation. One possibility is that by working with a VAR estimated over the entire sample, we're giving the VAR a 'peek' into the future so that it can see that the drop in inflation after 1980 really did turn out to be 'permanent'. Perhaps this is why the VAR, coupled with the expectations hypothesis, expects a counterfactually low long term interest rate. An alternative strategy for doing the test in question 2 is to do 'rolling regressions'. In particular, run the VAR regression up to date t, and then compute the long rate for date t using the expectations hypothesis. Then, run the VAR regression up to date $t+1$ and compute the long rate for date $t+1$ using the expectations hypothesis. Continue like this to the end of the data set. Each regression has no idea what will happen to inflation in the future, and so perhaps it assigns some probability to the high inflation of the 1970s returning. In this case, perhaps it will predict higher long term interest rates, like the ones we actually saw. Graph the actual long rate, the long rate you computed in question 2, homework #2, and the long rate based on the rolling regressions. Is the predicted long rate based on the latter now closer to the actual long rate?

3. Exercise 11.5, page 349.

4. Consider a filter, $h(L)$, and let y_t be

$$y_t = h(L)\, x_t,$$

where x_t is a covariance stationary process with spectral density, $S_x(e^{-i\omega})$. We have discussed how $h(L)$ modifies the amplitude of fluctuation in y_t across frequencies:

$$S_y\left(e^{-i\omega}\right) = |h\left(e^{-i\omega}\right)| S_x\left(e^{-i\omega}\right).$$

A filter also alters the phase relationship between y_t and x_t. To see this, suppose x_t were simply a single cosine wave:

$$2\cos(\omega t),$$

for some specific frequency, $\omega \in (-\pi, \pi)$. Suppose that $h(e^{-i\omega})$ has the following polar form:

$$h\left(e^{-i\omega}\right) = s(\omega)\, e^{i\theta(\omega)}$$

   (a) Show that:

   $$y_t = s(\omega)\, 2\cos(\omega t + \theta(\omega)),$$

   so that x_t reaches its peak at $t = 0$, while y_t reaches its peak at $t = -\theta(\omega)/\omega$. Thus, if $\theta(\omega) > 0$ then x_t leads y_t, while if $\theta(\omega) < 0$ then x_t lags y_t. If $\theta(\omega) = 0$, then x_t and y_t are 'in phase at frequency ω' ($s(\omega)$ is the 'gain' of the filter). The function, $\theta(\omega)$, is said to be the phase angle of the filter because it determines whether y_t will lead or lag x_t at frequency ω. (Hint: recall that $2\cos(\omega t) = e^{-i\omega t} + e^{i\omega t}$.)

   (b) Prove that the phase angle of the HP filter is zero, so that it induces no phase shift.

   (c) Display the phase angle of the first difference filter, $1 - L$.

   (d) Prove that the centered moving average filter, $(1 - L)(1 - L)(1 - L^{-1})(1 - L^{-1})$, induces no phase shift.

5. Graph the gains of the first difference filter and the HP filter with $\lambda = 1600$. We discussed how the HP filter is roughly a 'high pass filter', letting through business cycle fluctuations and higher frequency fluctuations and setting to zero the other frequencies. What frequencies does the first difference filter permit to pass?

6. Problem 4.6, page 116.

7. Consider the log of US per capita GDP data provided in Angus Maddison's data set used for homework #7. Fit a second order autoregressive process around a deterministic time trend for these data (i.e., [16.3.1] with $p = 2$). Test the null hypothesis that $p = 2$ is correct, against the alternative of three lags. Compute the point estimate of the coefficient on time and display a 95 percent confidence interval based on the asymptotic theory.