Optimal Monetary Policy with Nominal Rigidities

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Abstract

This paper studies optimal monetary policy in a stochastic equilibrium model with monopolistically competitive firms and sticky goods prices. In an otherwise deterministic environment, a benevolent monetary authority can improve upon Friedman's rule by adding a purely random component to the money growth rate. This result is more likely when money is not important as a medium of exchange. Under parametric restrictions, we compute the welfare-maximizing stochastic process for money growth and calculate the welfare gains of a random monetary policy relative to Friedman's rule.

In a 1980 *Journal of Monetary Economics* paper "Banking in the Theory of Finance," Eugene Fama describes an economy with a sophisticated accounting system where transfers of wealth are conducted through bookkeeping entries. This economy does not require any physical medium, such as currency, to facilitate transactions. In the world Fama describes, an individual transfers wealth to a second individual by making a phone call or sending electronic mail to a "broker-banker" that signals the sale of securities in the first individual's portfolio. Another broker-banker, in turn, credits the portfolio of the second individual.[1]

Fama's paper was prophetic. Both technological innovations, in the form of high speed communication and powerful record-keeping computers, and financial deregulation have moved the financial systems of developed economies closer to

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[1] One interesting question, which is not the topic of our study, is how the price level is determined in the economy Fama describes. This issue is addressed in Fama (1983), Hall (1983) and Woodford (1995).

world Fama describes. Today, individuals in the U.S. can write checks against a portfolio of short-term Treasury securities using money market mutual funds and money market deposit accounts. Stock market mutual funds currently allow investors to write checks against a portfolio of equity shares.[2] Other changes to financial systems include the removal of reserve requirements on many types of assets, better cash management techniques and the introduction of sweep accounts. See Dotsey (1984, 1985) for case studies. The effect of these changes has been a reduced use of money for transactions. For example, Cole and Ohanian (1997) report that the ratio of M1 to output has fallen by a factor of three in the postwar U.S.

A number of researchers have begun to analyze the effect of financial innovation on the construction and evaluation of equilibrium monetary models. Cole and Ohanian (1997) analyze the implications of the reduced role of money on two classes of models with monetary non-neutrality: limited participation and sticky price models. They find that these real effects of a reduced role of money are very different across these two classes of models. Woodford (1998) and Rotemberg and Woodford (1997) argue that if money is sufficiently unimportant in facilitating transactions, then one may study the real effects of monetary policy without explicitly modelling money. In financially sophisticated economies, the authors argue that more important monetary phenomenon, such as nominal price dynamics, can be brought to the forefront by abstracting from the complications of modelling the transactions role of money.

This paper also addresses implications of the diminished role of money. We consider the effect on optimal monetary policy of the diminished role of money in a model with imperfect competition and sticky prices (ICSP). In an otherwise deterministic environment, we show a monetary authority can improve on Friedman's rule from the standpoint of households' ex ante expected utility by adding a purely random component to the money growth rate. This result holds if currency is sufficiently unimportant in facilitating transactions. We introduce a varying role for money in facilitating exchange by modelling both cash and credit goods as in Lucas and Stokey (1983, 1987).

We model imperfectly competitive product markets by assuming that firms engage in monopolistic competition. We model sticky product prices by assuming firms must set nominal prices one period prior to the realization of the money shock. There are two distinct mechanisms that make randomness welfare improving: (a) a monopolistically competitive firm that must set its future price under uncertainty about the future money supply chooses a price to be a linear function of the expected state-price-deflated wage. We derive this policy as a stochastic generalization of the Dixit-Stiglitz constant markup rule under certainty. Next, if the monetary authority adopts a random money rule, sticky prices induce positive comovement between aggregate consumption and the real wage. This positive comovement may lower the expected state-price-deflated wage, leading

[2] Innovations like these are not exactly the same as the system Fama describes. Transactions conducted with money market funds are conducted through banks that hold reserves.

firms to hire more workers, increase output and reduce the distortion associated with imperfect competition. (b) If the steady-state markup is sufficiently large, the indirect utility function in terms of the markup may be in a convex region. In the convex region, random monetary policy induces random variation in the markup that is welfare improving.

We study an ICSP model because it has become a standard model of monetary nonneutrality. Goodfriend and King (1997) lay out the 'new synthesis' in macroeconomics as requiring models with both imperfect competition and sticky prices. Additionally, ICSP models are useful because policy prescriptions of these models have become standard. Many analyses of optimal policy in ICSP economies conclude that the monetary authority should maintain a constant rate of inflation. Ireland (1997) reaches this conclusion by formally studying the decision of a benevolent monetary authority in an economy with imperfect competition and sticky prices. Our model differs from Ireland in that we assume money does not play an important role in facilitating transactions.

Another paper closely related to ours is Blanchard and Kiyotaki (BK, 1987). These authors study a environment with monopolistic competition and fixed prices. One main result of their paper is that holding nominal prices fixed, monetary injections can raise aggregate demand, thereby increasing output and welfare. Our model can be thought of as an extension of BK on two dimensions.[3] First, whereas BK place money in the utility function, we use the Lucas-Stokey cash-credit goods model as a primitive model of money demand. This is necessary for our purpose as we would like to think about an economy where the transactions role of money is diminishing. Second, we provide an explicitly dynamic model of staggered nominal price setting. BK, on the other hand, consider a static model. As we explained above, the negative comovement between the asset pricing kernel and wages can cause firms to reduce the average markup. Interest rate effects are not present in a static model.

This paper establishes that, in a standard imperfect competition-sticky price model with a diminished role of money, optimal monetary policy may involve a purely random component. This translates into a stochastic law of motion for the inflation rate. In the next section, we describe the model and its associated equilibria. In section 2, we prove the main theorems concerning the optimality of random monetary policy, as well as several extensions. Section 3 concludes.

1 An Expectational Phillips Curve Model

In this section, we present a dynamic equilibrium cash-in-advance (CIA) model with imperfect competition in product markets and sticky prices.[4] This model generates an expectational Phillips curve because unanticipated increases in the money supply generate unexpected inflation. Unexpected inflation reduces the real price of goods, whose nominal prices are set in advance. Lower real good

[3] Both of these potential extensions are mentioned but not explored in BK.

[4] The model presented here is exactly Ireland (1997) except that we allow for stochastic monetary policy and model both cash and credit goods.

prices generate a temporary increase in output.

The economy consists of a continuum of identical households, a continuum of firms that produce good 1, the cash good, a continuum of firms that produce good 2, the credit good, and a government. Each firm produces a distinct good, indexed by its producer (i, j), where i lies on the unit interval and $j = 1$ or 2. The government controls the nominal quantity of money via nominal lump-sum injections. The per-household money stock at the beginning of period t is M_t^s. The law of motion for the money supply is

$$M_{t+1}^s = x_t M_t^s \tag{1}$$

where x_t is an iid random variable with density f that has support $[\beta, +\infty)$.

At the beginning of period t, the representative household enters the period with nominal money M_t and firm (i, j) enters the period having chosen a nominal price $P_t(i)$ for its output in the previous period.[5] At the beginning of the period, each household receives a nominal transfer $(x_t - 1) M_t^s$.

The representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{\phi}{\alpha} (c_{1t})^{\alpha} + \frac{\psi}{\alpha} (c_{2t})^{\alpha} - n_t \right\} \tag{2}$$

where c_{1t}, c_{2t}, n_t denote period t consumption of the cash good composite, credit good composite and labor respectively, and $\alpha < 1$. Composite goods are defined by

$$c_{jt} = \left[\int_0^1 c_{jt}(i)^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)} \tag{3}$$

for $j = 1, 2$, $\theta > 1$ and

$$n_t = \int_0^1 n_t(i) \, di \tag{4}$$

Cash and credit goods are produced according to a linear technology

$$\int_0^1 [c_{1t}(i) + c_{2t}(i)] \, di = n_t$$

Consumers have rational expectations regarding and take as given the stochastic processes for $\{M_t^s, P_t(i), W_t\}$, where $P_t(i)$ denotes the dollar price of the cash and credit good i and W_t denotes the nominal wage. As in Lucas and Stokey (1987), cash and credit goods sell at the same price. Money from sales of cash goods collect in the register while trading occurs, while IOUs from the sale of credit goods accrue to the seller while trading occurs. For both goods, sales revenues become available for use only after goods trading ends. To the seller, there is no liquidity value of holding cash versus IOUs. Also, the two goods share the same linear production technology. Therefore, when both goods are in positive supply, they have identical prices.

[5] We explain below why the nominal price at time t does not depend on j.

Each household chooses cash and credit goods, labor supply and money demand $\{c_{1t}(i), c_{2t}(i), n_t, M_t\}$ to maximize (2) subject to (3),(4) and

$$\int_0^1 P_t(i)\, c_{1t}(i)\, di \leq M_t + (x_t - 1) M_t^s \tag{5}$$

$$c_{1t}(i), c_{2t}(i), M_t \geq 0 \tag{6}$$

$$\int_0^1 P_t(i)\left(c_{1t}(i) + c_{2t}(i)\right) di + M_{t+1} \leq M_t + (x_t - 1) M_t^s + \int_0^1 \left[D_{1t}(i) + D_{2t}(i)\right] di + W_t n_t \tag{7}$$

We follow Cole and Kocherlakota (1998) and Ireland (1997) in assuming that if the CIA constraint does not bind, then households carry just enough cash to make (5) hold with equality.

Next, consider the firms' problem. As explained above, producers of cash and credit goods charge identical prices. It suffices, therefore, to study the maximization problem of a typical producer of either the cash or credit good. Each cash good firm sells output on demand at a fixed price $P_t(i)$ during period t. At the end of period t, the firm makes wage payments, pays out profits as a dividend to the representative household, and chooses the next period nominal price. This price is set before the next period monetary shock is known. Firm $(i,1)$ chooses $P_{t+1}(i)$ to maximize $E_t\left[r_{t+1} D_{1t+1}(i)\right]$, where

$$D_{1t+1}(i) = \left[P_{t+1}(i) - W_{t+1}\right]\left[\frac{P_{t+1}(i)}{P_{t+1}}\right]^{-\theta} \frac{x_{t+1} M_{t+1}^s}{P_{t+1}} \tag{8}$$

and r_{t+1} is the asset pricing kernel. A step-by-step derivation of the dividend of a monopolistically competitive firm in a CIA economy, expressed in (8), is derived in the appendix to Ireland (1997).

The sequence of events and the timing of decisions that firms and households make is described in Diagram 1. Diagram 1 also includes a market for one-period state-contingent nominal debt that may be available to the household. This asset market is described completely in an appendix. Given our representative agent assumption, the existence of this market does not affect decisions of firms or households or equilibrium variables.

We are ready to define a rational expectations equilibrium for the model. We consider equilibria where currency is valued at all dates and in all states of the world.

Definition: A *rational expectations equilibrium* is a set of stochastic processes $\{c_{1t}(i), c_{2t}(i), n_t, M_t, M_t^s, D_{1t}(i), D_{2t}(i), P_t(i), W_t\}$ such that

(i) Households solve their problem.

(ii) Firms solve their problem.

(iii) The markets for cash goods, credit goods, labor and money clear.

(iv) The government specifies a random variable x_t, and the law of motion for the money supply obeys (1).

Consider a recursive formulation of the household's problem. First, let lower case variables denote the corresponding upper case variables scaled by the current per capita money stock, i.e., $p_t(i) = P_t(i)/M_t^s$. To study the recursive representation, we remove time subscripts from variables to denote this period's value of the variable and use a prime to denote next period's value. With this convention, the household Bellman equation is

$$v(m,p,x) = \max_{c_1(i),c_2(i),n(i)} \left\{ \frac{\phi}{\alpha}(c_1)^{\alpha} + \frac{\psi}{\alpha}(c_2)^{\alpha} - n + \beta E\left[v(m',p',x')\right] \right\}$$

subject to $m + x - 1 = \int_0^1 p(i)\, c_1(i)\, di$, where definitions of composite goods are given by (3) and (4), and

$$m' = \frac{1}{x}\left[m + x - 1 + wn - \int_0^1 p(i)\left(c_1(i) + c_2(i)\right) di + \int_0^1 \left(d_1(i) + d_2(i)\right) di\right]$$

We wish to study symmetric stationary monetary equilibria (SSME). By symmetric, we mean that in any period all firms choose identical nominal prices. By stationary, we wish to restrict attention away from sunspot, self-fulfilling inflationary and self-fulfilling deflationary equilibria. By monetary equilibria, we mean that the nominal price level is finite at all dates and under all contingencies.

The first order conditions for optimization in a symmetric equilibrium, where $p(i) = p$ and $c_j(i) = c_j$, are

$$\phi(c_1)^{\alpha-1} - \lambda p - \beta E\left[v_m(m',p',x')\, p/x\right] = 0 \qquad (9)$$

$$\psi(c_2)^{\alpha-1} = \beta E\left[v_m(m',p',x')\, p/x\right] \qquad (10)$$

$$1 = \beta E\left[v_m(m',p',x')\, w/x\right] \qquad (11)$$

where λ denotes the multiplier on the CIA constraint. The envelope condition requires

$$v_m(m,p,x) = \beta E\left[v_m(m',p',x')/x\right] + \lambda \qquad (12)$$

Throughout this paper, $E(\cdot)$ represents the expectation operator conditioned on the current money growth rate x.

In equilibrium, $m = m' = 1$ since the representative household holds the per capita quantity of money in every period and all states. Eliminating λ and substituting out the envelope condition, the necessary conditions for optimization by households are

$$pc_1 = x \qquad (13)$$

$$1 = \frac{\phi\beta w}{p'x} E\left(\left(c_1'\right)^{\alpha-1}\right) \qquad (14)$$

$$\psi(c_2)^{\alpha-1} = \frac{\phi\beta p}{xp'} E\left(\left(c_1'\right)^{\alpha-1}\right) \qquad (15)$$

Optimization by firms requires

$$p' = \frac{\theta}{\theta - 1} \frac{E(r'w')}{E(r')} \tag{16}$$

Equation (16) states that risk-neutral monopolistically competitive firms set price over expected state-price-deflated marginal cost equal to $\theta/(\theta-1)$. Equation (16) is a stochastic generalization of the deterministic Dixit-Stiglitz constant markup rule. The equilibrium asset pricing kernel is

$$r' = \frac{\beta}{x} \left(\frac{c_1'}{c_1} \right)^{\alpha - 1} \tag{17}$$

Before continuing on to solve the imperfect competition-sticky price model, the following lemma demonstrates that with flexible prices and perfect competition the model collapses to that of Lucas and Stokey (1987).

Lemma 1: If product markets are competitive and prices are flexible, the resource allocation does not depend on the realized money growth rate and is given by

$$c_1 = \left[\beta \phi E(1/x') \right]^{1/(1-\alpha)}$$
$$c_2 = \psi^{1/(1-\alpha)}$$

Proof: The first-order conditions for consumer optimization are (9), (10), (11), (12) and (13). With flexible prices and perfect competition, price equals marginal cost $p = w$.

Combining (10), (11) and $p = w$, we know that

$$c_2 = \psi^{1/(1-\alpha)}$$

Next, substituting λ out of (9) using the envelope condition, we have

$$v_m = \frac{\phi}{p} (c_1)^{\alpha - 1}$$

Using the CIA constraint and setting this equation one period forward,

$$v_m' = \frac{\phi}{x'} (c_1')^{\alpha} \tag{18}$$

Substituting (18) into (11) and using the CIA constraint again,

$$c_1 = \beta \phi E\left[(c_1')^{\alpha} / x' \right]$$

The single stationary solution to this equation for an iid x is

$$c_1 = \left[\beta \phi E(1/x') \right]^{1/(1-\alpha)}$$

■

Our first lemma states that with iid money growth, competitive product markets and flexible prices, the resource allocation does not depend upon the realization of the money growth rate, but only upon a single moment of the distribution of x. Recall that in the Lucas-Stokey model, only anticipated inflation affects households decisions. Furthermore, note that the pareto optimal allocation is achieved if $x = \beta$.

Now, let us return to the general model that admits imperfect competition and one-period ahead price setting. Summarizing, the recursive representation of a SSME consists of $\{c_1, c_2, w, p'\}$, a set of nonnegative functions of x, and r', a nonnegative function of (x, x'), that satisfy (13), (14), (15), (16), (17) as well as the restriction that $E(r') \leq 1$ for all x.[6]

Theorem 1: The law of motion for the scaled wage in a SSME is $w = Fx$, where

$$F = \left(\frac{\theta}{\theta - 1} E\left((x')^{\alpha}\right)\right)^{\alpha/(1-\alpha)} (\beta\phi)^{1/(\alpha-1)} E\left((x')^{\alpha-1}\right)^{-(1+\alpha)/(1-\alpha)} \tag{19}$$

Proof: We have five equations that characterize every SSME. First, we may ignore (15), the first-order condition for choice of c_2, since c_2 only appears in that equation. Of the remaining four equations, use the CIA constraint to substitute out c_1 and c_1'. This leaves us with three equations: (16) and

$$\frac{p'x}{\phi\beta w} = E\left[\left(\frac{x'}{p'}\right)^{\alpha-1}\right]$$

$$r' = \frac{\beta}{x}\left(\frac{x'/p'}{x/p}\right)^{\alpha-1}$$

in three unknown endogenous variables, r, p, w.

Next, substitute out r' using the last of these equations.

$$p' = \frac{\theta}{\theta - 1}\left[\frac{E\left((x')^{\alpha-1} w'\right)}{E\left((x')^{\alpha-1}\right)}\right]$$

$$p' = \left[\frac{\phi\beta w}{x} E\left((x')^{\alpha-1}\right)\right]^{1/\alpha}$$

Substituting out p' returns a single first-order nonlinear stochastic difference equation

$$\frac{\theta}{\theta - 1}\left[\frac{E\left((x')^{\alpha-1} w'\right)}{E\left((x')^{\alpha-1}\right)}\right] = \left[\frac{\phi\beta w}{x} E\left((x')^{\alpha-1}\right)\right]^{1/\alpha} \tag{20}$$

[6] The gross nominal interest rate $[E(r')]^{-1}$ must be greater than or equal to one in all states of the world. If not, agents could earn infinite profits in a monetary equilibrium by taking short positions in nominal bonds (not explicitly modelled here) and long positions in money.

Since x is iid, a solution to (20) corresponds to a law of motion for the scaled wage in a particular equilibrium. The single stationary solution to (20) is $w = Fx$. Using the method of undetermined coefficients, F is given by (19). ■

Not surprisingly, a high money growth rate increases the scaled wage, since $F > 0$. A larger money supply implies that the real price of goods, whose nominal values are pre-set, falls. This increases demand for goods. In order to meet demand at the fixed nominal prices, firms hire more workers. This bids up the real wage. Interpreting the coefficient F is difficult since it involves both preference parameters as well as several moments of the distribution of the money growth rate.

It is straightforward to find the remaining endogenous variables. Solving for equilibrium cash and credit good consumption, we have

$$c_1 = \eta_1 \left(\frac{E\left(\left(x'\right)^{\alpha-1} \right)^2}{E\left(\left(x'\right)^{\alpha}\right)} \right)^{1/(1-\alpha)} x \tag{21}$$

$$c_2 = \eta_2 \left(\frac{E\left(\left(x'\right)^{\alpha-1} \right)}{E\left(\left(x'\right)^{\alpha}\right)} \right)^{1/(1-\alpha)} x^{1/(1-\alpha)} \tag{22}$$

where $\eta_1 = (\phi\beta(\theta - 1)/\theta)^{1/(1-\alpha)}$ and $\eta_2 = (\psi(\theta - 1)/\theta)^{1/(1-\alpha)}$. The scaled nominal price p is given by the following expression.

$$p = p' = (\eta_1)^{-1} \left(\frac{E\left(\left(x'\right)^{\alpha-1} \right)^2}{E\left(\left(x'\right)^{\alpha}\right)} \right)^{1/(\alpha-1)} \tag{23}$$

Note that p is independent of the current realization of the money shock because we have assumed that x is iid. Using (17) and our assumption that the CIA constraint holds with equality, the asset pricing kernel is

$$r' = \frac{\beta}{x^{\alpha}} \left(x'\right)^{\alpha-1} \tag{24}$$

Equations (21) and (22) are instructive for understanding how and when randomizing monetary policy can be welfare improving.

Examining (22), note that c_2 is convex and increasing in the current money shock x. This convexity generates one of the benefits of adopting a random monetary policy. We call this the unanticipated inflation effect. Imperfect competition generates too little consumption of both the cash and credit goods in the model. In periods of high money growth, unanticipated inflation drives up the real wage. Since the scaled price of goods is fixed, the markup falls when money is injected and this reduces the distortion arising from imperfect competition. We explore the source of this convexity in the next section of the paper;

however, this is not the end of the story. In addition, we must also consider the effect of introducing randomness on the conditional moments of x' that appear in the policy function for c_2. We discuss this in the next section as well.

On the other hand, from (21) c_1 is linear in the current money shock. Since preferences are concave in c_1, by Jensen's inequality, adding randomness tends to reduce the expected utility contribution of the cash good. This works against the possibility that randomness is pareto improving. It is for this reason that our theorems hold only when the cash good is sufficiently unimportant in the utility function, that is, when ϕ is close to zero.

In this section, we constructed a closed-form expression for the endogenous variables in this stochastic ICSP model. These expressions are used repeatedly in the next section to develop several theorems concerning household welfare under alternative monetary policies.

2 Optimal Monetary Policy

This section develops a partial pareto ranking of alternative monetary policies in the ICSP model. Our ultimate goal is to demonstrate that a random policy for the money growth rate may dominate a deterministic one. As a prelude to the main result, we first characterize the optimal deterministic monetary policy.

Theorem 2: Restricting attention to deterministic monetary policies, the Ramsey outcome sets $x = \beta$.

Proof: Given that the next period money supply growth rate is known with certainty in this period, from (21), the quantity of the cash good is

$$c_1 = \left[\frac{\beta\phi\left(\theta-1\right)}{x\theta}\right]^{1/(1-\alpha)} \tag{25}$$

By equation (22), the quantity of the credit good is

$$c_2 = \left(\frac{\beta\phi}{x\psi}\right)c_1$$

Simplifying, c_2 does not depend upon the money growth rate

$$c_2 = \left[\psi\left(\theta-1\right)/\theta\right]^{1/(1-\alpha)} \tag{26}$$

It is clear from (26) that to compute the Ramsey policy it is sufficient to study the effect of changing x on the cash good.

Substituting (25) into the instantaneous utility function and suppressing the utility contribution of c_2 since it does not change as x varies:

$$U\left(x\right) = \frac{\phi}{\alpha}\left(\frac{\beta\phi\left(\theta-1\right)}{\theta x}\right)^{\alpha/(1-\alpha)} - \left(\frac{\beta\phi\left(\theta-1\right)}{\theta x}\right)^{1/(1-\alpha)}$$

The deterministic Ramsey problem becomes

$$\max_x \; U\left(x\right) \text{ subject to } x \geq \beta \tag{27}$$

The money growth rate must be no lower than the discount factor in order that $R \geq 1$, a condition required for existence of a monetary equilibrium. The solution to (27) is to choose $x^* = \beta$. ■

Deviating from Friedman's rule with a deterministic policy cannot alleviate the distortion generated by imperfect competition. The intuition for Theorem 2 is straightforward. With $x = \beta$ there is no monetary distortion; however, imperfect competition in product markets creates too little consumption and hence too much leisure relative to the social optimum. Increasing the gross money growth rate above β makes the cash good even more expensive. This leads households to substitute even further away from the cash good and into leisure. Both a high money growth rate and market power distort households' cash good-leisure margin in the same direction. Introducing a nonstochastic monetary distortion cannot undo the distortion generated by imperfect competition.

Next, we show that introducing a stochastic monetary policy can improve upon Friedman's rule. As we established in section 1, the unanticipated inflation effect increases the expected utility contribution of the credit good by driving down the markup. On the other hand, the potential for inflation leads to inflation expectation in every period, even if realized inflation only occurs occasionally. Expected inflation leads households to substitute out of cash goods and amplifies the distortion generated by imperfect competition, as demonstrated in Theorem 2. The size of this distortion is increasing in the gross nominal interest rate on a risk-free bond, which we denote R. This is because the private marginal rate of substitution between the cash and credit good is the opportunity cost of holding money in order to satisfy the CIA constraint. The following Lemma characterizes R.

Lemma 2: The gross one-period nominal interest rate is

$$R = x^{\alpha} / \left(\beta E\left[\left(x'\right)^{\alpha-1}\right]\right) \tag{28}$$

Proof: To compute the nominal interest rate, we price a risk-free nominal bond. The gross risk-free nominal rate of return is given by $R = \left[E\left(r'\right)\right]^{-1}$. Substituting out the asset pricing kernel using r' (17), we have

$$R = (x/\beta)\,\frac{\left(c_1\right)^{\alpha-1}}{E\left[\left(c_1'\right)^{\alpha-1}\right]} \tag{29}$$

Simplifying (29) by substituting in (13) and noting that $p = p'$ from (23), we have (28). ■

Under Friedman's rule, $x = \beta$ in all states. In this case, (13) demonstrates consumption is constant. From (28), $R = 1$. No monetary distortion occurs; however, there is also no possibility for surprise inflation.

From (28), we will show that any policy other than Friedman's rule, that is any policy with a density that does not concentrate all mass on β, will lead to a nominal interest rate that is greater than one in some states of the world. Since we would like to show that $R > 1$ in all states of the world, as long as Friedman's rule is not pursued, the lowest possible interest rate that can occur is when the money growth rate is β. By (28), $R > 1$ as long as

$$E\left[(x')^{\alpha-1}\right] > \beta^{1-\alpha}$$

To see this, note that as long as f is a density on $[\beta, \infty)$ without all mass on β, then

$$\int_{\beta}^{\infty} \left(x^{\alpha-1}\right)^2 \left[x^{1-\alpha} - \beta^{1-\alpha}\right] f(x)\, dx > 0$$

Simple algebra implies

$$\int_{\beta}^{\infty} x^{\alpha-1} f(x)\, dx > \beta^{1-\alpha}$$

If the government pursues a random monetary policy, the net nominal interest rate is positive in at least some states of the world; therefore, a monetary distortion arises from random monetary policy that is not present when the monetary authority follows Friedman's rule.

Anticipated inflation tends to reduce the quantity of the cash good held by households. One way to reduce the negative effects associated with random monetary policy is to reduce the preference for cash goods in the household utility function. As explained in the introduction, this assumption can be justified by developments in financially sophisticated economies that have reduced the reliance on money as a medium of exchange. For this reason, we assume that ϕ is close to zero.

The next lemma concerns the effect on the markup of introducing a small random component to the money growth rate.

Lemma 3: If $\alpha < 0$, there exists a random monetary policy with a lower expected markup than every deterministic policy.

Proof: By considering perturbations to the deterministic money growth rate, we can show that the expected markup falls upon the introduction of a small degree of uncertainty. Assume that the deterministic money growth rate is equal to $\bar{x} > \beta$. For this policy, the markup is given by $\mu = \mu' = \theta/(\theta - 1)$. For any iid money growth rate, the expected markup next period is:

$$E(\mu') = \frac{E\left((x')^{\alpha}\right) E\left((x')^{-1}\right)}{E\left((x')^{\alpha-1}\right)} \tag{30}$$

To simplify notation for the remainder of the proof, we suppress the prime on μ and x. We will consider perturbations of the random variable x indexed by δ. Let $x \equiv \bar{x} + \delta\epsilon$ where ϵ is a mean zero random variable with finite variance σ^2. Since the expected markup depends on moments of x that have a similar form, it is useful to define the following function.

$$E(x^m; \delta) = A_m(\delta) = \int \left[(\bar{x} + \delta\epsilon)^m\right] dF(\epsilon)$$

where F is a cdf with mean zero. In this case, it is straightforward to compute the first and second derivatives of A_m:

$$A'_m(\delta) = m \int \left[(\bar{x} + \delta\epsilon)^{m-1} \right] \epsilon dF(\epsilon)$$

$$A''_m(\delta) = m(m-1) \int \left[(\bar{x} + \delta\epsilon)^{m-2} \right] \epsilon^2 dF(\epsilon)$$

Evaluating these derivatives at $\delta = 0$, we have:

$$A_m(0) = \bar{x}^m$$

$$A'_m(0) = 0$$

$$A''_m(0) = m(m-1)(\bar{x})^{m-2} \sigma^2$$

From (30), the expected markup, indexed by δ, is

$$E(\mu;\delta) = \frac{\theta}{\theta-1} \frac{E(x^\alpha;\delta) E(x^{-1};\delta)}{E(x^{\alpha-1};\delta)}$$

Writing the RHS of this expression in terms of A, we have

$$E(\mu;\delta) = [\theta/(\theta-1)] A_\alpha(\delta) A_{-1}(\delta) [A_{\alpha-1}(\delta)]^{-1}$$

First of all, since $A'_m(0) = 0$, it is clear that $\partial E(\mu;0)/\partial\delta = 0$. In order to consider the effect on the markup of uncertainty, we must look at the second derivative. This also means that the formula for $\partial^2 E(\mu;0)/\partial\delta^2$ is relatively simple (every term with a first derivative in it equals zero):

$$\frac{\partial^2 E(\mu;0)}{\partial\delta^2} = \frac{\theta}{\theta-1} \left\{ \begin{array}{c} A''_\alpha(0) A_{-1}(0) [A_{\alpha-1}(0)]^{-1} + A_\alpha(0) A''_{-1}(0) [A_{\alpha-1}(0)]^{-1} \\ -A_\alpha(0) A_{-1}(0) [A_{\alpha-1}(0)]^{-2} A''_{\alpha-1}(0) \end{array} \right\}$$

Using the moments computed above, we know

$$\frac{\partial^2 E(\mu;0)}{\partial\delta^2} = \frac{\sigma^2\theta}{\theta-1} \left[\alpha(\alpha-1)(\bar{x})^{\alpha-2}(\bar{x})^{-\alpha} + 2(\bar{x})^{-3}(\bar{x})^1 - (\alpha-1)(\alpha-2)(\bar{x})^{\alpha-3}(\bar{x})^{1-\alpha} \right]$$

Simplifying, we have,

$$\frac{\partial^2 E(\mu;0)}{\partial\delta^2} = \frac{\sigma^2\theta}{\theta-1} [\alpha(\alpha-1) + 2 - (\alpha-1)(\alpha-2)] (\bar{x})^{-2}$$

Simplifying more, we have

$$\frac{\partial^2 E(\mu;0)}{\partial\delta^2} = \frac{\theta}{\theta-1} \left(\frac{2\alpha\sigma^2}{\bar{x}^2} \right)$$

Therefore, $\frac{\partial^2 E(\mu;0)}{\partial\delta^2} < 0$ if $\alpha < 0$. ■

Lemma 3 demonstrates that a mean preserving increase in the variance of the money growth rate can imply a decrease in the expected markup. A fall in the average markup reduces the distortion associated with imperfect competition. This occurs if there is sufficient negative comovement between the asset pricing kernel and the real wage. To see this, consider how the representative firm chooses the price. Recall that the firm chooses the next period price so that $p' = [\theta/(\theta-1)]\, E(r'w')/E(r')$. The firm sets a scaled price next period to be a linear function of the expected wage using the stochastic discount factor.

Substituting out p' using (16), we have

$$\mu' = \frac{\theta}{\theta-1}\frac{E(r'w')}{E(r')\,w'}$$

Next, multiply the numerator and denominator of the right-hand side of the above equation by $E(w')$.

$$\mu' = \frac{\theta}{\theta-1}\frac{E(w')}{w'}\left[\frac{E(r'w')}{E(r')\,E(w')}\right] \tag{31}$$

Using the definition of covariance, (31) becomes

$$\mu' = \frac{\theta}{\theta-1}\frac{E(w')}{w'}\left[\frac{Cov(r',w')}{E(r')\,E(w')}+1\right] \tag{32}$$

With a nonstochastic money growth rate, $E(w') = w'$ and $Cov(r', w') = 0$ since r' and w' are not stochastic. In this case, the markup is $\theta/(\theta-1)$.

Let us examine the effect on the markup of introducing a small random component to the money growth rate. First, consider the effect on the bracketed term in (32). We know that w' is increasing in x' because higher money growth decreases the real price of the credit good. In order to meet greater demand for goods, firms must hire more workers which drives up the wage. On the other hand, r' is decreasing in x' since goods are less expensive when they are more plentiful. This implies that the covariance between the pricing kernel and the real wage goes from being zero in the nonstochastic case to being negative upon the introduction of a small degree of randomness to the money growth rate. With randomness, when the wage is high the pricing kernel is low. This tends to drive down the state-price deflated expected wage and therefore lowers the price of consumption.

Examining (32), the markup is also a function of $E(w')/w'$. The effects in the numerator and denominator work in opposite directions. The term in the denominator states that the markup is decreasing in the realized wage. Since $w = Fx$, this is another way to express the unanticipated inflation effect discussed previously. The term in the numerator reflects the fact that the markup is increasing in the average wage. By Jensen's inequality, the term $E[E(w')/w']$ tends to increase upon the introduction of randomness. In order that the covariance term dominates the $E[E(w')/w']$ term, this requires that the elasticity of the pricing kernel with respect to the real wage is sufficiently negative. This occurs when $\alpha < 0$.

Although we have described the effect on the average markup of introducing a small amount of uncertainty, the benevolent monetary authority is interested in the effect on utility of the introduction of uncertainty. This will certainly depend upon the expected markup, but it will also depend upon the concavity or convexity of the utility function. The following theorem concerns the optimality of random monetary policy, under the maintained assumption that ϕ is close to zero.

Theorem 3: For a sufficiently small ϕ, there exists a random monetary policy that ex ante pareto dominates every deterministic policy if

$$\frac{\theta}{\theta-1} > \frac{2\left(1-\alpha\right)^2+\alpha}{1-2\alpha+2\alpha^2}$$

Proof: As in Lemma 3, we will consider small perturbations to a deterministic monetary policy. Each random monetary policy will be indexed by δ, and defined by $x = \bar{x} + \delta\epsilon$, where ϵ is mean zero with finite variance σ^2. Every expression in the utility function can be expressed using the following:

$$D_{m,p}\left(\delta\right) = \left[\int \left[\left(\bar{x}+\delta\epsilon\right)^m\right] dF\left(\epsilon\right)\right]^p = \left[A_m\left(\delta\right)\right]^p$$

We can compute the first two derivatives of D:

$$D'_{m,p}\left(\delta\right) = p\left[A_m\left(\delta\right)\right]^{p-1} A'_m\left(\delta\right)$$

$$D''_{m,p}\left(\delta\right) = p\left(p-1\right)\left[A_m\left(\delta\right)\right]^{p-2}\left(A'_m\left(\delta\right)\right)^2 + p\left[A_m\left(\delta\right)\right]^{p-1} A''_m\left(\delta\right)$$

We can evaluate these derivatives at $\delta = 0$, making use of expressions in the proof of Lemma 3:

$$D_{m,p}\left(0\right) = \left(\bar{x}\right)^{mp}$$

$$D'_{m,p}\left(0\right) = 0$$

$$D''_{m,p}\left(0\right) = mp\left(m-1\right)\left(\bar{x}\right)^{mp-2}\sigma^2$$

The expected utility of a representative agent, indexed by δ, is

$$EU\left(\delta\right) = \frac{\psi\left(\eta_2\right)^\alpha}{\alpha}\left(\frac{E\left(\left(x'\right)^{\alpha-1}\right)}{E\left(\left(x'\right)^\alpha\right)}\right)^{\frac{\alpha}{1-\alpha}} E\left(\left(x'\right)^{\frac{\alpha}{1-\alpha}}\right) - \eta_2\left(\frac{E\left(\left(x'\right)^{\alpha-1}\right)}{E\left(\left(x'\right)^\alpha\right)}\right)^{\frac{1}{1-\alpha}} E\left(\left(x'\right)^{\frac{1}{1-\alpha}}\right)$$

where the expectations on the RHS depend on the random variable x through δ. We will attack this in two parts. First, let EU_1 be defined by

$$EU_1\left(\delta\right) = \frac{\psi\left(\eta_2\right)^\alpha}{\alpha}\left(\frac{E\left(\left(x'\right)^{\alpha-1}\right)}{E\left(\left(x'\right)^\alpha\right)}\right)^{\frac{\alpha}{1-\alpha}} E\left(\left(x'\right)^{\frac{\alpha}{1-\alpha}}\right)$$

In our D notation, this is equal to

$$EU_1(\delta) = \frac{\psi(\eta_2)^\alpha}{\alpha} D_{\alpha-1,\frac{\alpha}{1-\alpha}}(\delta)\, D_{\alpha,\frac{\alpha}{\alpha-1}}(\delta)\, D_{\frac{\alpha}{1-\alpha},1}(\delta)$$

By the same argument as in the proof of Lemma 3, $\partial EU_1(\delta)/\partial\delta = 0$. Also, the expression for the second derivative is relatively simple:

$$\frac{\partial^2 EU_1(0)}{\partial\delta^2} = \left(\frac{\psi(\eta_2)^\alpha}{\alpha}\right)\left[\begin{array}{c} D''_{\alpha-1,\frac{\alpha}{1-\alpha}}(0)\, D_{\alpha,\frac{\alpha}{\alpha-1}}(0)\, D_{\frac{\alpha}{1-\alpha},1}(0) + D_{\alpha-1,\frac{\alpha}{1-\alpha}}(0)\, D''_{\alpha,\frac{\alpha}{\alpha-1}}(0) \times \\ D_{\frac{\alpha}{1-\alpha},1}(0) + D_{\alpha-1,\frac{\alpha}{1-\alpha}}(0)\, D_{\alpha,\frac{\alpha}{\alpha-1}}(0)\, D''_{\frac{\alpha}{1-\alpha},1}(0) \end{array}\right]$$

Using our expressions for D and D'' evaluated at zero, we have:

$$\frac{\partial^2 EU_1(0)}{\partial\delta^2} = \left(\psi(\eta_2)^\alpha/\alpha\right)\left[\frac{\alpha - 2\alpha^2 + 2\alpha^3}{(1-\alpha)^2}\right]\sigma^2(\bar{x})^{-2}$$

Next, the second term of $EU(\delta)$ is given by

$$EU_2(\delta) = -\eta_2 D_{\alpha-1,\frac{1}{1-\alpha}}(\delta)\, D_{\alpha,\frac{1}{\alpha-1}}(\delta)\, D_{\frac{1}{1-\alpha},1}(\delta)$$

Similarly,

$$\frac{\partial^2 EU_2(0)}{\partial\delta^2} = -\eta_2\left[\begin{array}{c} D''_{\alpha-1,\frac{1}{1-\alpha}}(0)\, D_{\alpha,\frac{1}{\alpha-1}}(0)\, D_{\frac{1}{1-\alpha},1}(0) + D_{\alpha-1,\frac{1}{1-\alpha}}(0)\, D''_{\alpha,\frac{1}{\alpha-1}}(0)\, D_{\frac{1}{1-\alpha},1}(0) + \\ D_{\alpha-1,\frac{1}{1-\alpha}}(0)\, D_{\alpha,\frac{1}{\alpha-1}}(0)\, D''_{\frac{1}{1-\alpha},1}(0) \end{array}\right]$$

Substituting in our expression for D and D'', and then simplifying,

$$\frac{\partial^2 EU_2(0)}{\partial\delta^2} = -\eta_2\left[2 + \frac{\alpha}{(1-\alpha)^2}\right]\sigma^2(\bar{x})^{-2}$$

Combining these two expressions, we see that

$$\frac{\partial^2 EU(0)}{\partial\delta^2} = \left(\frac{\sigma}{\bar{x}(1-\alpha)}\right)^2\left\{\psi(\eta_2)^\alpha\left(1 - 2\alpha + 2\alpha^2\right) - \eta_2\left[2(1-\alpha)^2 + \alpha\right]\right\}$$

Since we are solely interested in the sign of this function, we can ignore the first term on the RHS, which is always positive.

$$sgn\left[\frac{\partial^2 EU(0)}{\partial\delta^2}\right] = sgn\left[\psi(\eta_2)^\alpha\left(1 - 2\alpha + 2\alpha^2\right) - \eta_2\left(2(1-\alpha)^2 + \alpha\right)\right]$$

Then, for sufficiently small ϕ, a random monetary policy is preferred to every deterministic policy if

$$\psi(\eta_2)^{\alpha-1}\left(1 - 2\alpha + 2\alpha^2\right) > \left(2(1-\alpha)^2 + \alpha\right)$$

Since from the paper $\eta_2 = (\psi(\theta-1)/\theta)^{1/(1-\alpha)}$, the term ψ cancels from the above expression when we substitute out η_2. Therefore, a sufficiently small ϕ, a sufficient condition for randomness to be optimal is:

$$\frac{\theta}{\theta-1} > \frac{2(1-\alpha)^2+\alpha}{1-2\alpha+2\alpha^2}$$

There are two additional comments. First, we assumed that $\bar{x} > \beta$. This is necessary in order to rule out a negative net nominal interest rate upon the introduction of uncertainty. If we considered the contribution of the cash good to preferences, having an average money growth rate greater than β would be pareto dominated. We are assuming that ϕ is sufficiently small, that we can ignore the utility contribution of the cash good.

Second, ignoring the cash good requires that the derivative of the expected utility contribution of the cash good with respect to δ evaluated at 0 is finite. This is straightforward to verify using (2) and (21). ■

The above expression involves the interaction of several different influences. The first influence is that the average markup may fall upon the introduction of uncertainty into the monetary policy. In addition, there may be convex regions of the indirect utility function $V(\mu)$. It is straightforward to show that $V''(\mu) > 0$ as long as $\mu > 2-\alpha$. This effect implies that introducing random fluctuations in the money growth rate will tend to increase utility so long as households are in the convex region of the indirect utility function.

For the remainder of this section, we take up two extensions of our analysis: globally optimal random monetary policy and the quantitative significance of the cash good.

Globally Optimal Monetary Policy

Theorem 3 provided conditions under which a random money growth rate pareto dominates every deterministic policy. This leads one to ask what is the globally optimal random monetary policy. In this section, we solve a restricted version of this problem by placing two parametric forms on the density.[7]

Our first parameterization of a random monetary policy is

$$x_u = \beta + \gamma_u u$$

where u is a uniform random variable on the unit interval. We then maximize (2) by choice of a nonnegative γ_u. Note that this parameterization is equal to the deterministic policy (specifically, Friedman's rule) when $\gamma_u = 0$. For the second parameterization, we choose

[7] Instead of restrictively parameterizing the set of admissible distributions for x, we could use perturbation methods to characterize the general welfare-maximizing random money growth distribution. Bassetto (1998) applies these techniques in an optimal-taxation problem.

$$x_\epsilon = \beta + \gamma_\epsilon \epsilon$$

where ϵ has an exponential distribution.[8]

Next we must select parameters for the model. The model has five preference parameters and one policy parameter. The preference parameters consist of θ, governing the degree of imperfect competition, α, the utility function parameter determining the coefficient of relative risk aversion, β, the discount factor, ψ and ϕ, determining the utility weights on the cash and credit goods. We choose $\theta = 3$, $\psi = 1, \beta = 0.9$ and $\alpha = -5$. We maintain our assumption that ϕ is very close to zero. Figure 1 displays the density functions of the optimal policies under both parameterizations.

We also list the expectation, variance and expected utility associated with each monetary policy as well as Friedman's rule. As the figure indicates, the uniform money supply rule is the welfare maximizing policy among the three, and both random policies are preferred to Friedman's rule. Our second extension asks what is the quantitative significance of introducing a non-negligible cash good to the welfare implications of random monetary policy.

Introducing A Non-Negligible Cash Good

Next, we show that a significant fraction of cash goods may exist in an economy and still imply a random optimal monetary policy; therefore, Theorem 3 of the previous section is robust to allowing for a quantitatively significant preference for cash goods.

First, we must select policy parameters. The single government policy parameter is f, the density function for the money supply growth rate. Let $\theta = 3.0$, $\alpha = 0.8$, $\beta = 0.98$, $\phi = 0.2$, $\psi = 0.8$. The value of θ implies that in an equilibrium with a constant growth rate of money, the markup is 1.5, which lies within a range of empirical estimates. Setting $\phi = 0.2$ and $\psi = 0.8$ gives more weight to credit than cash goods in the utility function. In the future, we plan to impose more discipline on parameter choices. This example is intended to be illustrative.

Next, we parameterize two distinct monetary policies, f^1 and f^2. First, we let $f^1(\beta) = 1$, which corresponds to a constant decrease in the money supply at the rate of time preference. By Theorem 2, among the class of deterministic monetary policies, f^1 maximizes consumer welfare.

Our second rule money growth rate is a two-point discrete distribution given by

$$\pi = \Pr(x = \beta) \text{ and } 1 - \pi = \Pr(x = 1)$$

where $\pi = 0.5$. In most periods, the government decreases the money supply at the rate of time preference, however, occasionally the government does not change the money stock.

[8] The density of ϵ is $e^{-x}x$.

We can compute the unconditional expected utility $EU(f)$ under monetary policy f. Since among the class of deterministic monetary policies, f^1 is optimal, we know that if $EU(f^2) > EU(f^1)$ then a random policy is preferred to all deterministic monetary policies. Table 1 lists EU, expected interest rate and expected total consumption under both monetary regimes. Next, c_1^* denotes the percentage increase in consumption of the cash good that would make someone who lives under the deterministic policy f^1 be indifferent between f^1 and f^2. The two point distribution for f^2 was chosen arbitrarily. It is likely that other distributions for f correspond to higher utility. As we expect from the discussion in the previous section, $E(R) > 1$ under f^2. In addition, average total output of the cash and credit good is greater under f^2 because in the periods where the monetary injection occurs, aggregate production increases, becoming closer to the competitive equilibrium.

The above parameterization establishes that a random monetary policy can pareto dominate deterministic monetary policy in a dynamic equilibrium ICSP model for a plausible parameterization of the model. It is not true that for any parameterization of the model, the optimal monetary policy involves a random component. For example, if θ is sufficiently large, the product market becomes arbitrarily close to competitive. In this case, the potential benefits from surprise inflation go to zero while the distortions associated with $R > 1$ remain. Clearly, if θ is sufficiently high Friedman's rule is optimal.

3 Conclusion

This paper establishes that optimal monetary policy in an imperfect competition-sticky price model may contain a purely random component. This result was developed using a dynamic equilibrium monetary model with rational expectations where money played a minimal role in facilitating transactions. Two frictions caused a departure from the Lucas-Stokey cash-credit model—nominal price rigidity and imperfect competition. These frictions generate an expectational Phillips curve, where unexpected inflation can lead to a welfare-improving increases in physical output. Two distinct features generate a role for randomization. First, random monetary policy reduces the state-price-deflated expected wage that firms face. On average, this leads to a lower markup and higher output. Second, randomization allows the monetary authority to improve welfare in convex regions of the indirect utility function. Our results have two alternative interpretations: (a) the welfare benefits of inflation targeting and nominal price stabilization policies may be overstated, or (b) this version of the imperfect competition-sticky price model may have problems measuring the welfare costs of inflation.

Under interpretation (a), policies that attempt to minimize fluctuations in the inflation rate may hurt society. It has been observed for a long time that estimated policy functions of the Federal Reserve contain an idiosyncratic component. Cochrane (1994) asks, "Money VARs recognize that policy responds to the economy, and try to isolate the exogenous shocks as residuals to a policy rule. But why should a policy-maker deliberately introduce a random component to its decision?" Our results provide a resolution of Cochrane's puzzle using a standard imperfect competition-sticky price model.

Is the Federal Open Market Committee choosing monetary policy with a roulette wheel? Probably not; however, national economies are constantly being hit by small, random shocks to money demand. Whereas the model of this paper considered random money supply disturbances, alternatively random money demand shocks can generate variation in the markup. If a monetary authority chooses not to or cannot neutralize every shock, then these monetary demand shocks act as a natural randomization device. This may be one interpretation of the residual "exogenous" shock that is consistently identified in money VARs.

From the standpoint of policy, our results speak to an important operational question of monetary policy: how costly is low, variable inflation? Recently, a number of authors have attempted to characterize the welfare losses associated with low inflation in the U.S. (See Feldstein 1996, Lucas 1994 and Bullard and Russell, 1997).[9] Others advocate policies that, in principle, can insure price stability, such as targeting CPI futures (Cowen, 1997) and targeting inflation (Mishkin and Posen, 1997). This paper raises a new potential benefit of small, random movements in the money supply—small, random inflation allows a monetary authority to reduce the distortion of imperfect competition.

The research mentioned in the previous paragraph reflects a recent interest in understanding the potential welfare benefits of going from low to zero or negative inflation. Under interpretation (b) of the paper's results, an observer may ask what motivates prescriptions for a constant price level or inflation rate. If the primary reason for reducing the size and volatility of inflation stems from Milton Friedman's (1960, p. 73) concern that inflation increases "the discrepancy between social and private costs that lead individuals to hold smaller than optimal cash balances," then the reduced use of money as a medium of exchange in financially sophisticated economies may ease these concerns. If the primary reason for advocating price level or inflation rate stability stems from nominal price frictions, then the results of this paper may call this justification into question.

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[9] None of these studies model either sticky prices or imperfect competition.

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5 Appendix: A Contingent Claims Market in Nominal Debt

Here is a description of the model with a complete contingent claims market in nominal bonds. Each period is divided into two subperiods. Let Ω_t denote the nominal consumer wealth at the beginning of the first subperiod of t after the money injection is announced but before the injection occurs. When the household enters a period with nominal wealth, each must first allocate wealth between money and bonds:

$$M_t + E_t\left[r_{t+1}B_{t+1}\right] \leq \Omega_t \tag{33}$$

Let B_{t+1} be a random variable measurable with respect to I_{t+1} chosen by households at time t. If r_{t+1} is the asset pricing kernel, then $E_t\left[r_{t+1}B_{t+1}\right]$ denotes the time t dollar price of a bond portfolio that pays out at time $t+1$.

Second the law of motion for nominal wealth is given by

$$\Omega_{t+1} = M_t + (x_t - 1)\, M_t^s + W_t n_t - P_t\left(c_{1t} + c_{2t}\right) + B_{t+1} + D_{1t} + D_{2t} \tag{34}$$

Here we suppress i for simplicity. Note that Ω_{t+1} may depend upon x_{t+1} since households may receive payoffs of state contingent debt B_{t+1}. After the bond market closes, households receive a nominal sum transfer $(x_t - 1)\, M_t^s$.

The second subperiod then begins with the opening of cash goods, credit goods and labor markets. The CIA constraint requires

$$P_t c_{1t} \leq M_t + (x_t - 1)\, M_t^s \tag{35}$$

The nonnegativity constraints are identical to those presented in section 1. Also, the introduction of nominal debt requires a standard constraints to rule out Ponzi schemes. At the end of the second subperiod, firms pay dividends and choose next period's price. The sequence of events and the timing of decisions that firms and households make are represented in Diagram 1.

There are three things to note regarding the addition of a nominal asset market. First of all, the Euler equations for the choice of c_{1t}, c_{2t}, n_t, M_t are not changed. Second, setting $B_t = 0$ and combining (33) and (34), we get back the flow budget constraint without bonds (7) presented in section 1. Third, one may verify by taking the first order condition with respect to B_{t+1} of the asset-market augmented household problem that the asset pricing kernel is that posited in (17).

Alternative Firm Objective

Let us consider more closely the structure of trading in bonds and the delivery of dividends in this economy. Recall that firm i chooses $P_{t+1}(i)$ at time t. In the body of the paper, we assume firms maximize $E_t\left[r_{t+1}D_{t+1}(i)\right]$. It should be clear why firms discount one-period ahead dollar profits by the nominal pricing kernel. However, examining (34), one may note that dividends acquired at time t cannot be transformed into cash to purchase the cash good until time $t+1$. That is, although all uncertainty regarding the dollar value of time $t+1$ dividends in resolved at time $t+1$, the structure of trading requires these dividends to 'sit out' an additional period.

In this case, firms may discount dividends for an additional period. Under this assumption, the objective of the firm becomes

$$\max_{P_{t+1}(i)} E_t \left[r_{t+1} r_{t+2} D_{1t+1}(i) \right] \tag{36}$$

Our original motivation for the paper, that money is becoming less important in facilitating transactions, provides some justification for ignoring the term r_{t+2} in the firm's objective. Including r_{t+2} in the firms objective implies that even if cash goods are not important to households, cash frictions still impinge on the decisions of firms. It may be reasonable to assume that in financially sophisticated economies, both firms and households find ways to reduce the costs associated with money as medium of exchange.

In any case, we will now sketch out why, under (36), the optimal monetary policy still contains a purely random component. Using the law of iterated expectations, this expression become

$$\max_{P_{t+1}(i)} E_t \left[r_{t+1} D_{1t+1}(i) E_{t+1}(r_{t+2}) \right]$$

From our discussion in the paper, $E_{t+1}(r_{t+2})$ is the inverse of the gross one period nominal interest rate R_{t+1}. Let us define $\rho_{t+1} = r_{t+1} E_{t+1}(r_{t+2})$. If the normalized price is time independent $p = p'$, then (17) implies

$$r_{t+1} = \frac{\beta}{x_t} \left(\frac{x_{t+1}}{x_t} \right)^{\alpha - 1}$$

This implies

$$\rho_{t+1} = \frac{\beta^2 E_{t+1} \left[(x_{t+2})^{\alpha-1} \right]}{(x_t)^{\alpha}} (x_{t+1})^{-1}$$

Note that all the terms in the fraction are predetermined at time t. This means that they can be dropped without affecting the firm's first order condition. The argmax from (36) is equal to

$$\arg \max_{P_{t+1}(i)} E_t \left[(x_{t+1})^{-1} D_{1t+1}(i) \right]$$

The first-order condition for optimization is therefore

$$P_{t+1}(i) = \frac{\theta}{\theta - 1} \frac{E_t \left[(x_{t+1})^{-1} W_{t+1} \right]}{E_t \left[(x_{t+1})^{-1} \right]} \tag{37}$$

Writing (37) in the recursive, normalized form,

$$p' = \frac{\theta}{\theta - 1} \frac{E \left[(x')^{-1} w' \right]}{E \left[(x')^{-1} \right]} \tag{38}$$

Note that (38) is extremely similar to the first-order condition for firm optimization (16) presented in section 1. The only difference between the two is that r' is replaced here with $1/x'$. We can use (38) to replicate the argument made in section 2 that a small amount of randomness lowers the markup. Following the steps described in section 2,

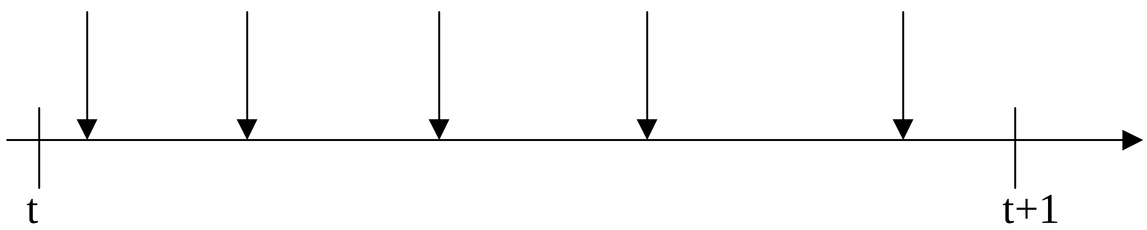
$$\mu' = \frac{\theta}{\theta - 1}\frac{E(w')}{w'}\left[\frac{Cov\left[(x')^{-1}, w'\right]}{E\left[(x')^{-1}\right]E(w')} + 1\right]$$

Under this firm objective, it is straightfoward to show that, to a second order, the expected markup does not change upon the introduction of randomness; however, there remains a convex region of households' indirect utility function. Therefore, randomness may be welfare improving even if the firms use (36) to select next period's nominal price.

Diagram 1: Sequence of Events

Government and Markets

- Money shock, x_t, announced
- (Asset market meets)
- Money injection occurs
- Goods & labor markets meet
- Settlement



Firms

- Enter period with preset price $P_t(i)$
- Choose $P_{t+1}(i)$, pay dividends

Households

- Enter period with Ω_t - B_t, where B_t is a random variable
- Collect $B_t(x_t)$ Allocate portolio between M_t and $E_t(r_{t+1} B_{t+1})$
- Receive $(x_t - 1)M^s_t$
- Buy goods, sell labor

Notes: 1. Recall $\Omega_{t+1} = M_t + (x_t - 1)M_t^s + W_t n_t - p_t(c_{1t} + c_{2t}) + B_{t+1} + D_{1t} + D_{2t}$

2. Availability of asset market and nominal bonds is optional.

Table 1: Simulation of Model under Alternative Parameterizations

	F1	*F2*
EU	1.88971	1.88976
E(R)	1.00000	1.00101
$E(c_1 + c_2)$	0.04319	0.04320
c_1*	1.46 %	