# Optimal Fiscal and Monetary Policy

## Outline

(1) Background: Phelps-Friedman Debate

(2) Some Ideas from Public Finance - Ramsey Theory

– Policy

– Private Sector Equilibrium

– Private Sector Allocation Rule

– Ramsey Problem

– Ramsey Equilibrium

– Implementability Constraint

– Ramsey Allocation Problem

– Ramsey Allocations

(3) Simple One-Period Example

(4) Evaluating Phelps-Friedman Debate Using Lucas-Stokey Cash-Credit Good Model

  (a) General Remarks

  (b) Model

  (c) Ramsey Problem, Ramsey Allocation Problem

  (d) Surprising Result:
  Friedman is "Right" for Lots of Parameterizations (Used Homotheticity and Separability).

(5) Interpretation of Result

  (a) Homotheticity and Separability Corresponds to Unit Consumption Elasticity of Money Demand

  (b) Uniform Taxation Result in Public Finance for Non-Monetary Economies

  (c) What Happens When You Don't Have Unit Elasticity?

  (d) Who Is Right, Friedman or Phelps?

(6) What Happens When $g$, $z$ Are Random?
(Answer: Make $P$ Random)

(7) Financing a War: Barro versus Ramsey.

# Friedman-Phelps Debate

- Money Demand:

$$\frac{M}{P} = \exp[-\alpha R]$$

- Friedman:
  (a) Efforts to Economize Cash Balances when $R$ High is Socially Wasteful
  (b) Set $R$ as Low As Possible - $R = 1$.
  (c) Since $R = r + \pi$, Frieman Recommends $\pi = -r$.
    (i) $r \sim$ exogenous real interest rate rate
    (ii) $\pi \sim$ inflation rate, $\pi = (P - P_{-1})/P_{-1}$

- Phelps:

(a) Inflation Acts Like a Tax on Cash Balances -

$$\text{Seignorage} = \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}} \approx \frac{M}{P}\frac{\pi}{1+\pi}$$

(b) Use of Inflation Tax Permits Reducing Some Other Tax Rate

(c) Extra Distortion in Economizing Cash Balances Compensated by Reduced Distortion Elsewhere.

(d) With Distortions a Convex Function of Tax Rates, Would Always Want to Tax All Goods (Including Money) At Least A Little.

(e) Inflation Tax Particularly Attractive if Interest Elasticity of Money Demand Low.

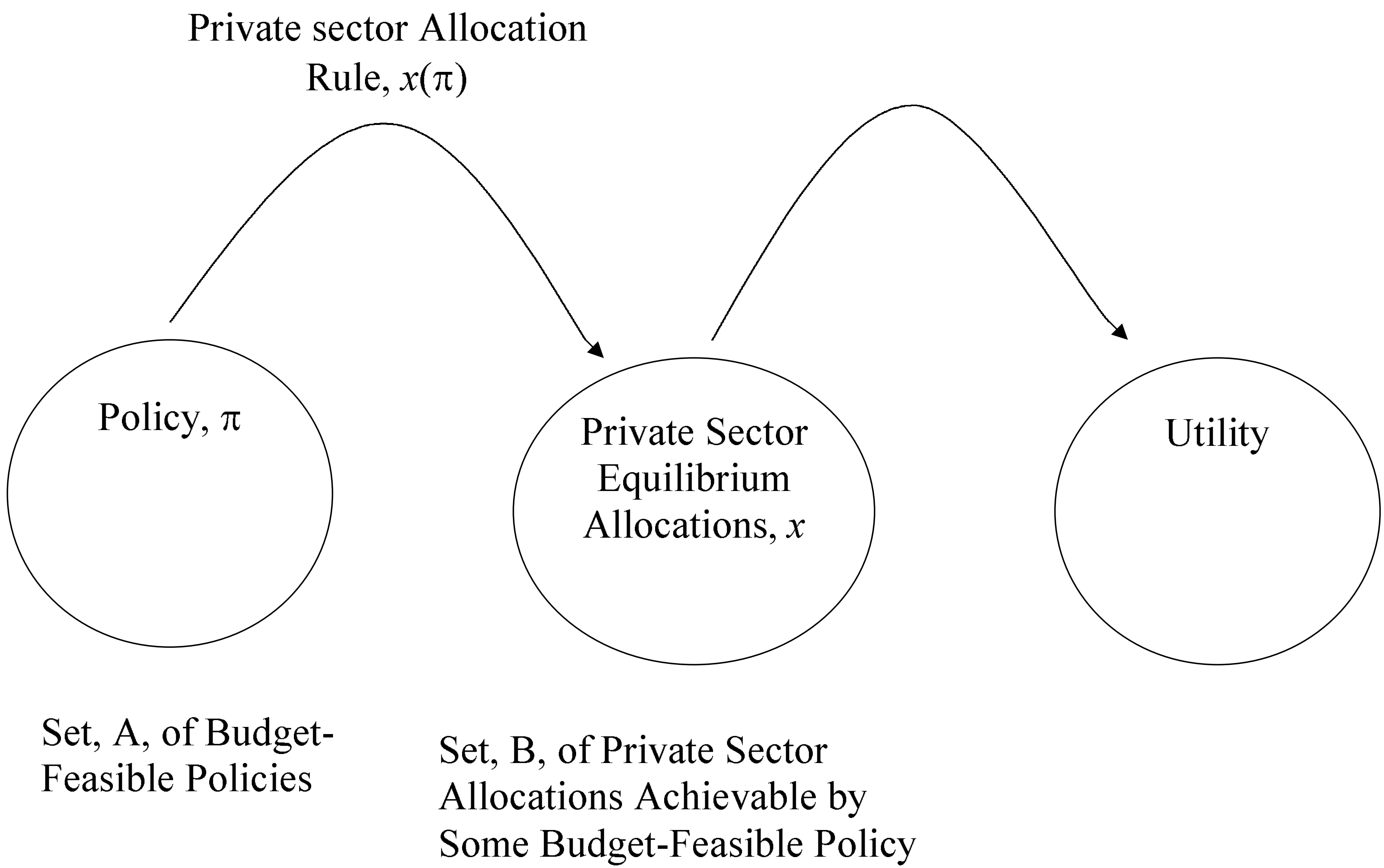
# Question: Who is Right, Friedman or Phelps?

- Answer: Friedman Right Surprisingly Often
- Depends on Income Elasticity of Demand for Money
- Will Address the Issue From a Straight Public Finance Perspective, In the Spirit of Phelps.
- Easy to Develop an Answer, Exploiting a Basic Insight From Public Finance.

# Some Basic Ideas from Ramsey Theory

- **Policy**, $\pi$, Belonging to the Set of 'Budget Feasible' Policies, $A$.
- **Private Sector Equilibrium Allocations**, Equilibrium Allocations, $x$, Associated with a Given $\pi$; $x \in B$.
- **Private Sector Allocation Rule**, mapping from $\pi$ to $x$ (i.e., $\pi : A \rightarrow B$).
- **Ramsey Problem**: Maximize, w.r.t. $\pi$, $U(x(\pi))$.
- **Ramsey Equilibrium**: $\pi^* \in A$ and $x^*$, such that $\pi^*$ solves Ramsey Problem and $x^* = x(\pi^*)$. 'Best Private Sector Equilibrium'.

- **Ramsey Allocation Problem**: Solve, $\tilde{x} = \arg max\, U(x)$ for $x \in B$
- Alternative Strategy for Solving the Ramsey Problem:
  (a) Solve Ramsey Allocation Problem, to Find $\tilde{x}$.
  (b) Execute the Inverse Mapping, $\tilde{\pi} = x^{-1}(\tilde{x})$.
  (c) $\tilde{\pi}$ and $\tilde{x}$ Represent a Ramsey Equilibrium.
- **Implementability Constraint**: Equations that Summarize Restrictions on Achievable Allocations, $B$, Due to Distortionary Tax System.


Private sector Allocation
Rule, $x(\pi)$
Policy, $\pi$
Private Sector
Equilibrium
Allocations, $x$
Utility
Set, A, of Budget-
Feasible Policies
Set, B, of Private Sector
Allocations Achievable by
Some Budget-Feasible Policy

# Example

- Households:

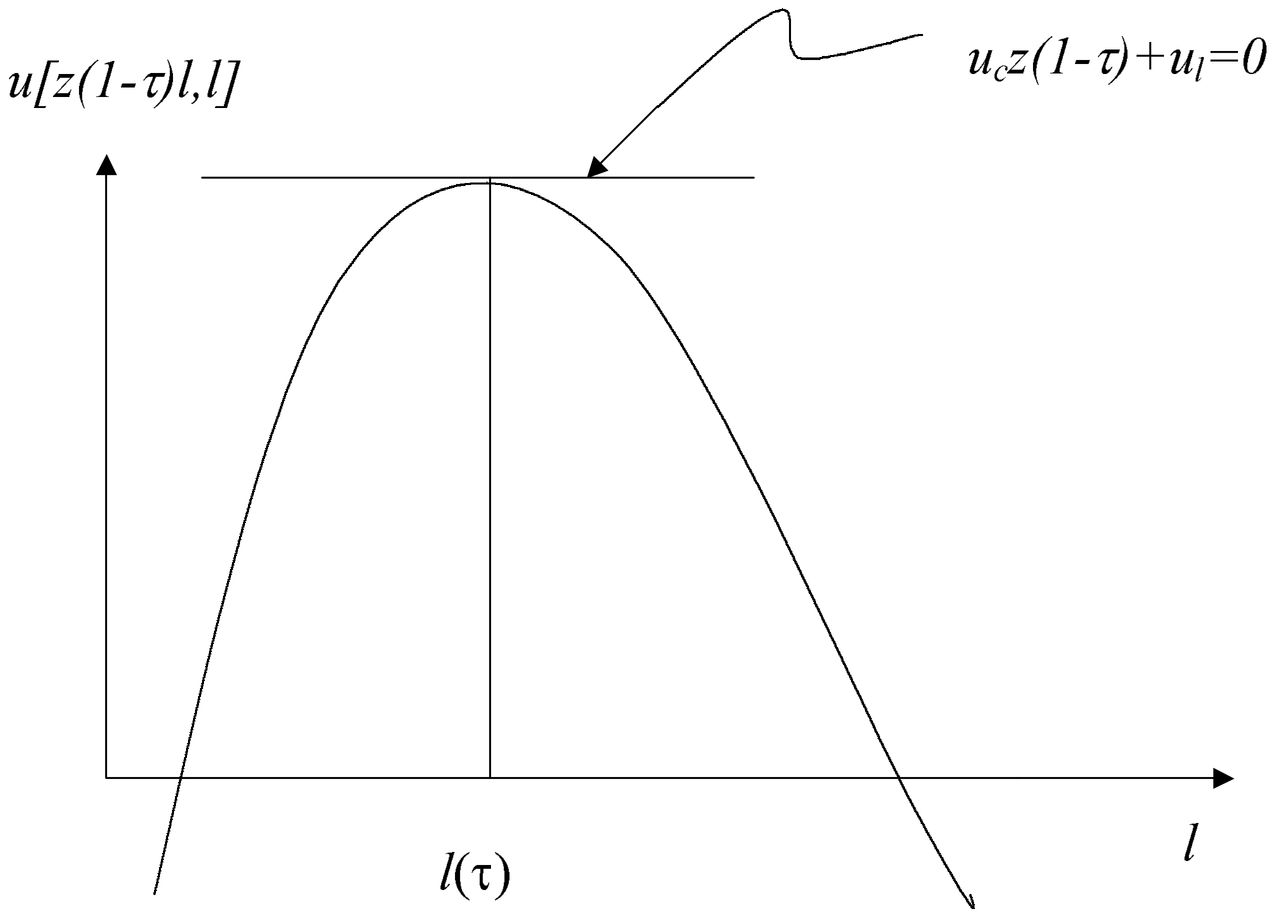
$$\max_{c,l} u(c,l)$$
$$\begin{aligned} c &\leq z(1-\tau)l, \\ z &\sim \text{wage rate} \\ \tau &\sim \text{labor tax rate} \end{aligned}$$

- Household Problem Implies Private Sector Allocation Rules:

$$l(\tau),\ c(\tau)$$

- Ramsey Problem:

$$\max_{\tau} u(c(\tau), l(\tau))$$
$$\text{subject to } g \leq zl(\tau)\tau$$




Private Sector Allocation Rules:

$l(\tau)$, $c(\tau) = z(1-\tau)l$

- Ramsey Equilibrium: $\tau^*$, $c^*$, $l^*$ such that

  (a) $c^* = c(\tau^*)$, $l^* = l(\tau^*)$
  ‘Private Sector Allocations are a Private Sector Equilibrium’

  (b) $\tau^*$ Solves Ramsey Problem
  ‘Best Private Sector Equilibrium’

# Analysis of Ramsey Equilibrium

- Simple Utility Specification:

$$u(c, l) = c - \frac{1}{2}l^2$$

- Two Ways to Compute the Ramsey Equilibrium
  (a) Direct Way: Solve Ramsey Problem (In Practice, Hard)
  (b) Indirect Way: Solve Ramsey Allocation Problem (Can Be Easy)

# Direct Approach

- Private Sector Allocation Rules:

$$u_c z(1-\tau) + u_l = 0, \; c \le (1-\tau) z l$$

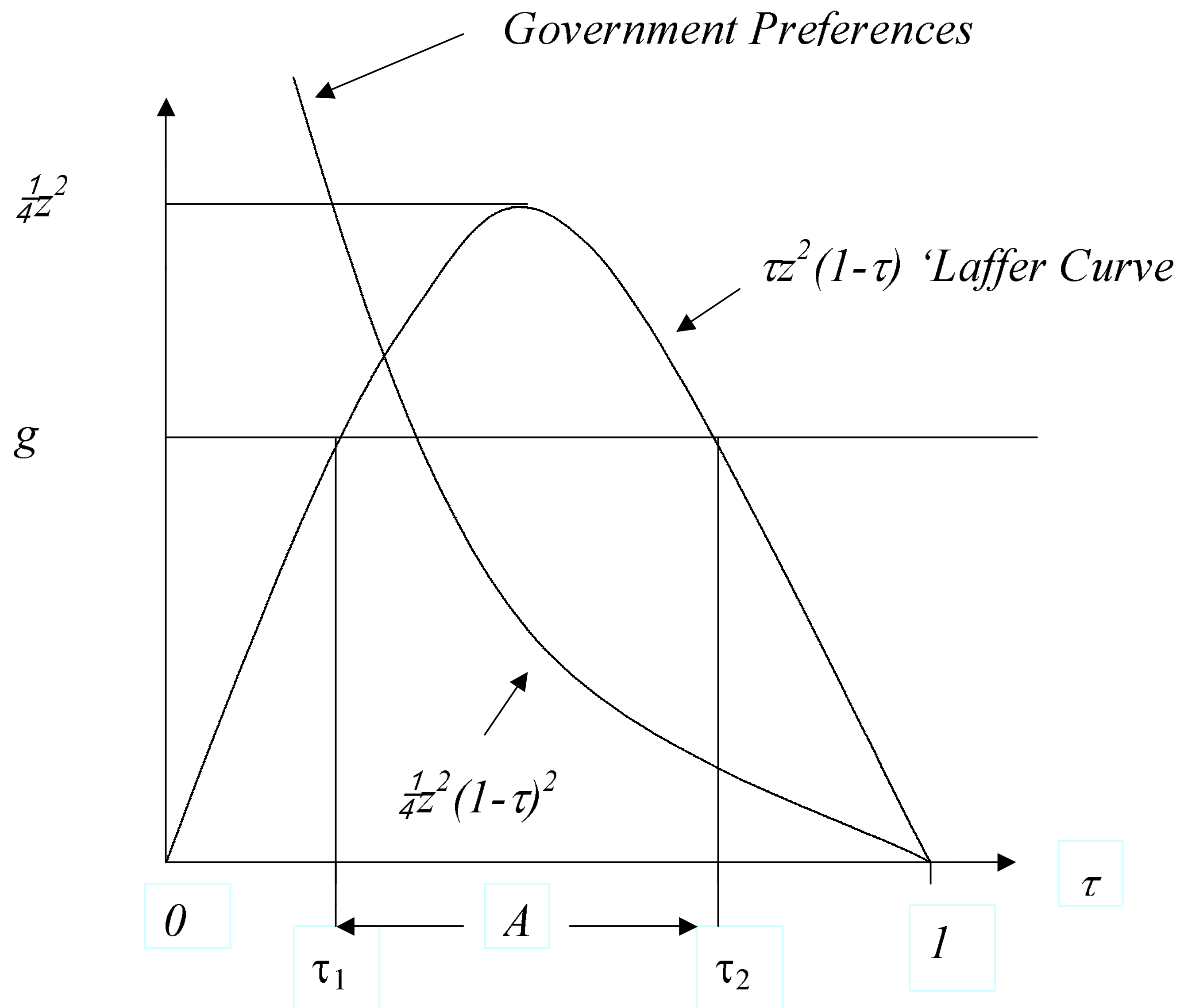
$$\Longrightarrow z(1-\tau) = l(\tau)$$

$$\Longrightarrow c(\tau) = z(1-\tau) l(\tau) = z^2(1-\tau)$$

- Ramsey Problem:

$$\max_{\tau} \frac{1}{2} z^2 (1-\tau)^2$$

$$\text{subject to}: \; g \le \tau z l(\tau) = \tau z^2 (1-\tau).$$




$$\tau^* = \tau_1 = \tfrac{1}{2} - \tfrac{1}{2}[\,1 - 4\,g/z^2\,]^{\frac{1}{2}} \qquad \tau_2 = \tfrac{1}{2} + \tfrac{1}{2}[\,1 - 4\,g/z^2\,]^{\frac{1}{2}}$$

$$l(\tau^*) = \tfrac{1}{2}\{\, z + [\, z - 4\,g\,]^{\frac{1}{2}} \}$$

# Indirect Approach

- Approach: Solve Ramsey Allocation Problem, Then 'Inverse Map' Back into Policies
- Problem: Need a Simpler Characterization of $B$

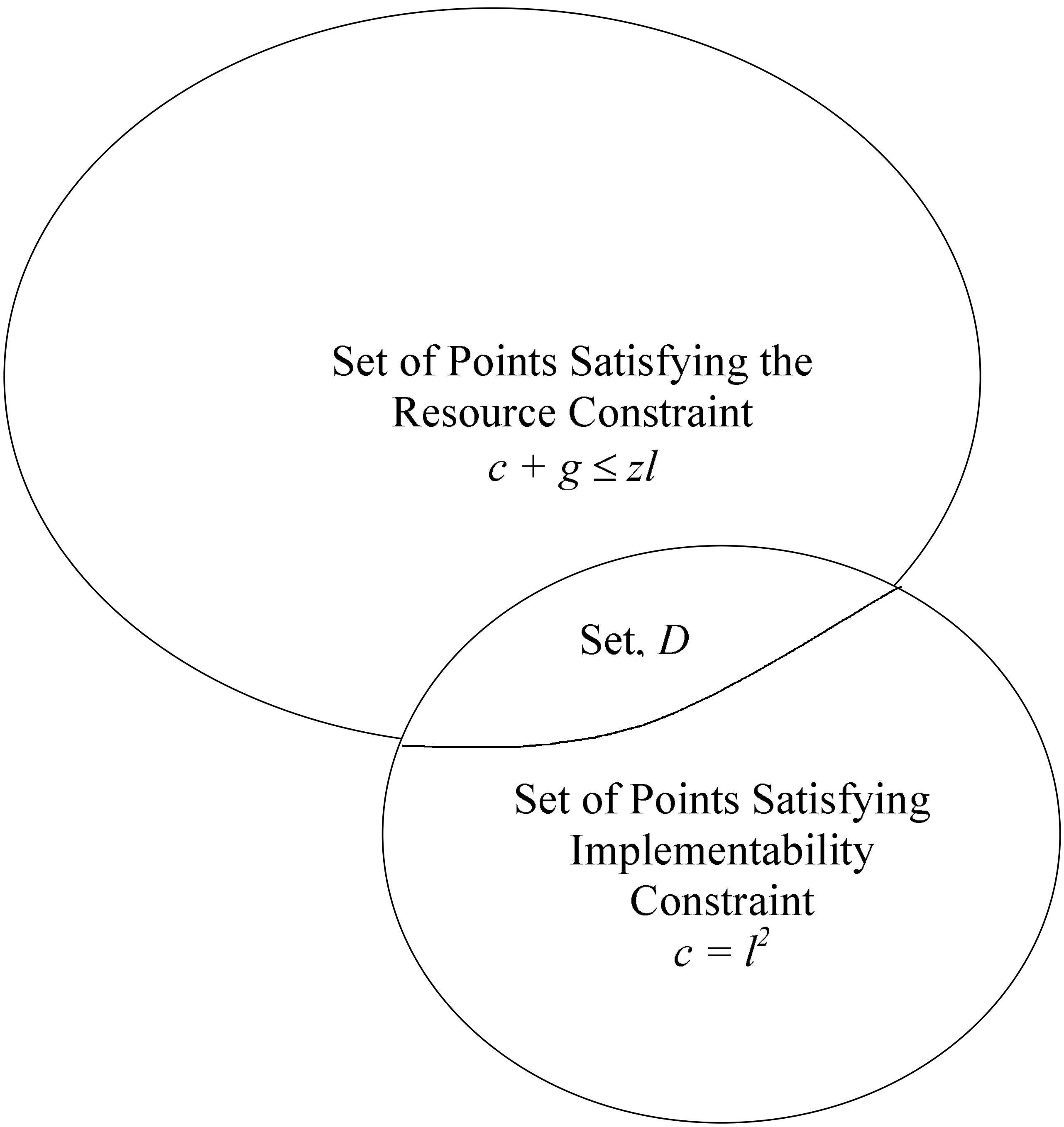
$$\begin{aligned} B &= \{(c,l) : \exists \tau \text{ s.t. } u_c(1-\tau)z + u_l l = 0, \\ c &= (1-\tau)zl,\ g \leq \tau z l\} \end{aligned}$$

- Consider the Following Set $D$ :

$$D = \left\{ (c,l) : \quad \underbrace{c + g \leq zl}_{\text{resource constraint}} , \quad \underbrace{u_c c + u_l l = 0}_{\text{implementability constraint}} \right\}$$

- Key Result: $D = B$

# Constraint Set, *D*, On Ramsey Allocation Problem



# Proof of Key Result, $D = B$

Show: $(c, l) \in D \Rightarrow (c, l) \in B$

- Suppose $(c, l) \in D$, i.e., $u_c c + u_l l = 0$, $c + g \leq zl$
- Need to show: $\exists \tau$ s.t. (i) $u_c(1 - \tau)z + u_l = 0$, (ii) $c = (1 - \tau)zl$, (iii) $g \leq \tau zl$
- Set $\tau$ so that

$$1 - \tau = \frac{-u_l}{u_c z}, \text{ so (i) holds.}$$

- Multiply Both Sides by $lz$ and rewrite:

$$(1 - \tau)\, lz = \frac{-u_l l}{u_c} = c, \text{ so (ii) holds.}$$

- (iii) follows (ii) and $c + g \leq zl$.

Show: $(c, l) \in B \Rightarrow (c, l) \in D$

- Suppose $(c, l) \in B$, i.e., $\exists \tau$ s.t. $u_c(1-\tau)z + u_l = 0$, $c = (1-\tau)zl$, $g \leq \tau zl$.
- Need to show: $(c, l) \in D$, i.e., (i) $u_c c + u_l l = 0$, (ii) $c + g \leq zl$
- Multiply by $l$:

$$u_c(1-\tau)zl + u_l l = 0, \text{ so (i) holds}$$

- Combine HH and Gov't Budget Constraints:

$$c + g \leq zl, \text{ so (ii) holds}$$

- Conclude:

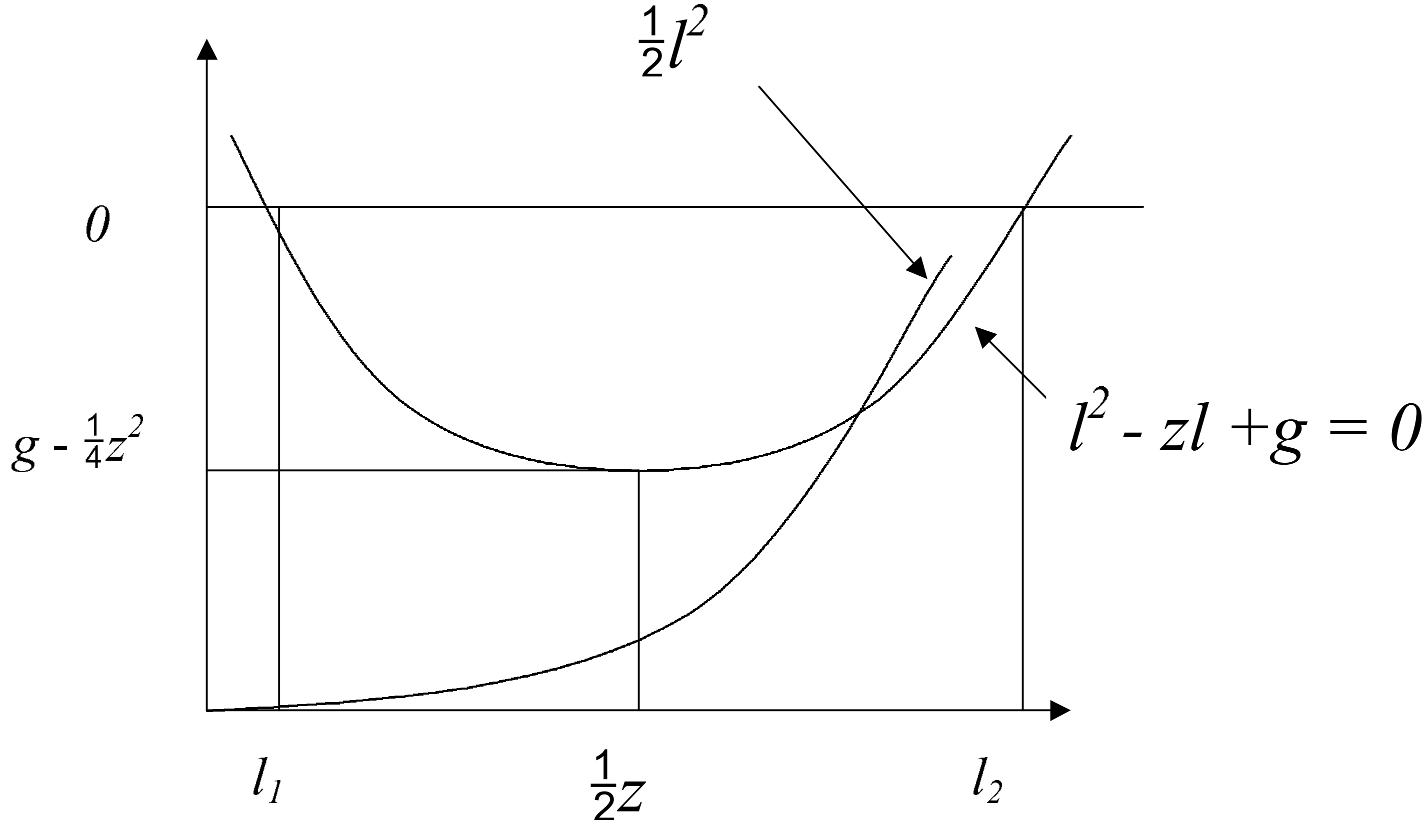
$$B = D$$

- Express Ramsey Allocation Problem:

$$\max_{c,l} u(c, l)$$
$$\text{s.t. } u_c c + u_l l \ = \ 0, \ c + g \leq zl$$

or

$$\max_{l} l^2$$
$$\text{s.t. } l^2 + g \ \leq \ zl$$




Ramsey Allocation Problem:

Max $\frac{1}{2}l^2$

Subject to $l^2 + g \leq zl$

Solution:

$l_2 = \frac{1}{2}\{ z + [ z^2 - 4g ]^{\frac{1}{2}} \}$

Same Result as Before!

# Lucas - Stokey Cash-Credit Good Model

## Households

$$\max \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, \ell_t)$$

s.t. $M_t^d + B_t^d \leq M_{t-1}^d - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1} + R_{t-1}B_{t-1}^d + (1-\tau_{t-1}) z \ell_{t-1}$

$$P_t c_{1t} \leq M_t^d$$

Euler equations:

$$\frac{U_{1t}}{U_{2t}} = R_t$$

$$U_{1t} = \beta U_{1,t+1} R_t \, P_t / P_{t+1}$$

$$U_{3t} + (1-\tau_t) z U_{2t} = 0$$

## Government

Budget constraint:

$$\underbrace{M_t^s - M_{t-1}^s + B_t^s}_{\text{inflow}} \geq \underbrace{R_{t-1}B_{t-1}^s + P_{t-1}g_{t-1} - P_{t-1}\tau_{t-1} z \ell_{t-1}}_{\text{outflow}}$$

Policy: $\pi = (M_0^s, M_1^s, \ldots\ldots$
$B_0^s, B_1^s, \ldots\ldots$
$\tau_0, \tau_1, \ldots\ldots )$

For each $\pi$ there is a private sector equilibrium:

$$x = (\{c_{1t}\}, \{c_{2t}\}, \{l_t\}, \{M_t\}, \{B_t\})$$

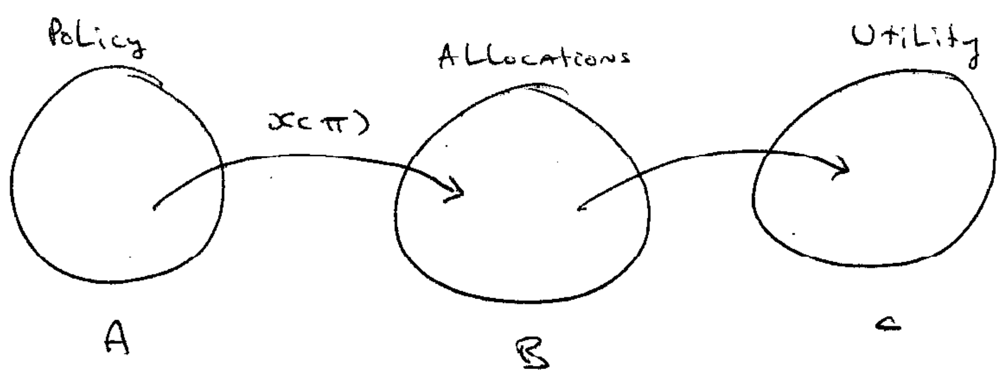
$$p = (\{P_t\}, \{R_t\})$$

$$M_t = M_t^s = M_t^d$$
$$B_t = B_t^s = B_t^d$$

$$R_t \geq 1 \quad (\text{i.e., } u_{1t}/u_{2t} \geq 1)$$

Ramsey problem:

$$\max_{\pi \in A} U(x(\pi))$$



# Ramsey Allocation Problem:

$$\max_{\{c_{1t}, c_{2t}, l_t\} \in D} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t),$$

where $D$ is the set of allocations, $c_{1t}, c_{2t}, l_t$, $t = 0, 1, 2, ...$, such that

$$\sum_{t=0}^{\infty} \beta^t [u_{1t}c_{1t} + u_{2t}c_{2t} + u_{3t}l_t] \; = \; u_{2,0}a_0,$$

$$c_{1t} + c_{2t} + g \; \leq \; zl_t, \; \frac{u_{1t}}{u_{2t}} \geq 1,$$

$$a_0 = \frac{R_{-1}B_{-1}}{P_0} \sim \text{ real value of initial government debt.}$$

Assumption:

$$B_{-1} = 0.$$

# Lagrangian Representation of Problem:

There is a $\lambda \geq 0$, Such that the Solution to the RA Problem and the Following Problem Coincide:

$$\max_{\{c_{1t}, c_{2t}, l_t\}} \sum_{t=0}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_t; \lambda)$$

subject to

$$c_{1t} + c_{2t} + g \leq z l_t, \; \frac{u_{1t}}{u_{2t}} \geq 1,$$

where

$$W(c_{1t}, c_{2t}, l_t; \lambda) = u(c_{1t}, c_{2t}, l_t) + \lambda[u_{1t} c_{1t} + u_{2t} c_{2t} + u_{3t} l_t].$$

# Restricting the Utility Function

- Utility Function:

$$
\begin{aligned}
u(c_1, c_2, l) &= h(c_1, c_2)v(l), \\
h &\sim \text{homogeneous of degree } k \\
v &\sim \text{strictly decreasing.}
\end{aligned}
$$

- Then, $u_1 c_1 + u_2 c_2 + u_3 l = h\left[kv + v'\right]$, so

$$
\begin{aligned}
W(c_1, c_2, l; \lambda) &= hv + \lambda h\left[kv + v'\right] \\
&= h(c_1, c_2)Q(l, \lambda).
\end{aligned}
$$

- Conclude - Homogeneity and Separability Imply:

$$
\frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = \frac{u_1(c_1, c_2, l)}{u_2(c_1, c_2, l)}.
$$

# Surprising Result: Friedman is Right More Often Than You Might Expect

- Equating 'Marginal Rate of Substitution' in $W$ with Associated Marginal Rate of Technical Transformation:

$$\frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = 1.$$

- Under Homogeneity and Separability:

$$\frac{u_1(c_1, c_2, l)}{u_2(c_1, c_2, l)} = 1.$$

- Conclude

$$R = 1.$$

- Friedman Is Right!

# Generality of the Result

- Result is True for the Following More General Class of Utility Functions:

$$u(c_1, c_2, l) = V(h(c_1, c_2), l),$$

  where $h$ is homothetic.

- Analogous Result Holds in 'Money in Utility Function' Models and 'Transactions Cost' Models (Chari-Christiano-Kehoe, *Journal of Monetary Economics*, 1996.)

- Actually, strict homotheticity and separability are not necessary.

# Consumption Elasticity of Demand

- Homotheticity and Separability Correspond to Unit Consumption Elasticity of Money Demand.
- Money Demand:

$$
\begin{aligned}
R = \frac{u_1}{u_2} = \frac{h_1}{h_2} &= f\left(\frac{c_2}{c_1}\right) \\
&= f\left(\frac{c - \frac{M}{P}}{\frac{M}{P}}\right) \\
&= \tilde{f}\left(\frac{c}{M/P}\right).
\end{aligned}
$$

- Note: Holding $R$ Fixed, Doubling $c$ Implies Doubling $M/P$

# Uniform Taxation Result from Public Finance For Non-Monetary Economies

- Households:

$$\max_{c_1, c_2, l} u(c_1, c_2, l)$$
$$\text{s.t. } zl \geq c_1(1+\tau_1) + c_2(1+\tau_2)$$
$$\Rightarrow c_1 = c_1(\tau_1, \tau_2),\ c_2 = c_2(\tau_1, \tau_2),\ l = l(\tau_1, \tau_2).$$

- Ramsey Problem:

$$\max_{\tau_1, \tau_2} u(c_1(\tau_1, \tau_2), c_2(\tau_1, \tau_2), l(\tau_1, \tau_2))$$
$$\text{s.t. } g \geq c_1(\tau_1, \tau_2)\tau_1 + c_2(\tau_1, \tau_2)\tau_2$$

- *Uniform Taxation Result*:

$$\text{if } u = V(h(c_1, c_2), l),\ h \sim \text{homothetic}$$
$$\text{then } \tau_1 = \tau_2.$$

Proof: trivial! (just study Ramsey Allocation Problem)

# Similarities to Monetary Economy

- Rewrite Budget Constraint:

$$\frac{zl}{1+\tau_2} \geq c_1 \frac{1+\tau_1}{1+\tau_2} + c_2.$$

- Similarities:

$$\frac{1}{1+\tau_2} \sim 1-\tau,\ \frac{1+\tau_1}{1+\tau_2} \sim R.$$

- Positive Interest Rate 'Looks' Like a Differential Tax Rate on Cash and Credit Goods.
- **Have the Same Ramsey Allocation Problem**, Except Monetary Economy Also Has:

$$\frac{u_1}{u_2} \geq 1.$$

# What Happens if You Don't Have Unit Elasticity?

- Utility Function:

$$u(c_1, c_2, l) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\delta}}{1-\delta} + v(l)$$

- Money Demand:

$$R = \frac{u_1}{u_2} = \frac{c_1^{-\sigma}}{c_2^{-\delta}} = \frac{\left(\frac{M}{P}\right)^{-\sigma}}{\left(c - \frac{M}{P}\right)^{-\delta}},$$

$$\varepsilon_M = \frac{d\log\left(\frac{M}{P}\right)}{d\log(c)}$$

- Can Verify:

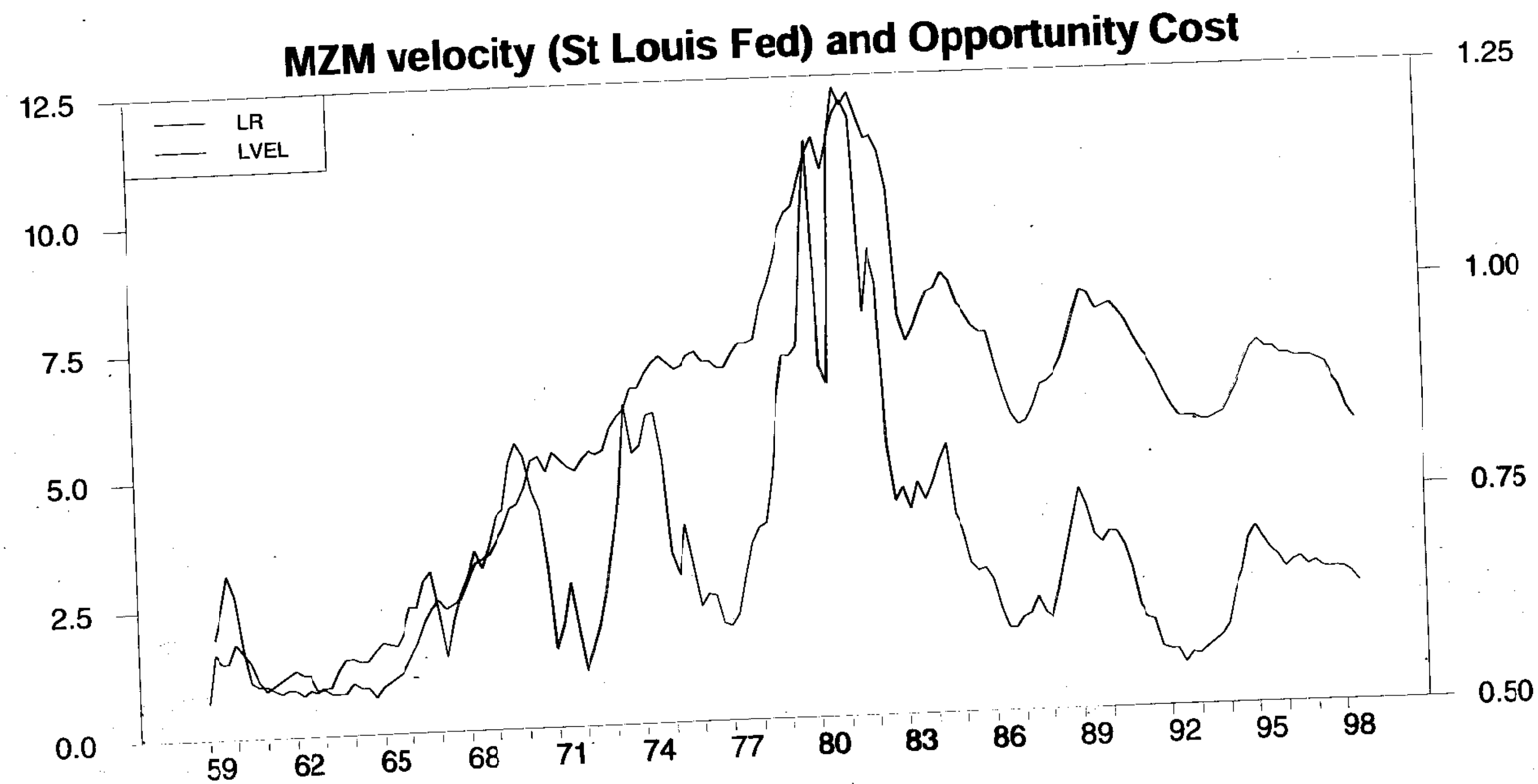
| Utility Function Parameters | $\varepsilon_M$ | Non-Monetary Economy | Monetary Economy |
|---|---|---|---|
| $\delta > \sigma$ | $\varepsilon_M > 1$ | $\tau_2 \geq \tau_1$ | $R = 1$ |
| $\delta < \sigma$ | $\varepsilon_M < 1$ | $\tau_2 < \tau_1$ | $R > 1$ |
| $\delta = \sigma$ | $\varepsilon_M = 1$ | $\tau_1 = \tau_2$ | $R = 1$ |

# Who is Right, Friedman or Phelps?

- Friedman is Right ($R = 1$) When Consumption Elasticity of Money Demand is Unity or Greater
- Close Connection to Uniform Taxation in Public Finance

  (But, $R = 1$ Holds More Generally Because of $R \geq 1$ Constraint in Monetary Economies)
- Basic Idea:

  Implicitly, High Interest Rates Tax Some Goods More Heavily that Others. Under Certain Conditions, Don't Want to Do That.
- What is Consumption Elasticity in the Data?


MZM velocity (St Louis Fed) and Opportunity Cost
LR
LVEL
12.5
10.0
7.5
5.0
2.5
0.0
1.25
1.00
0.75
0.50
59
62
65
68
71
74
77
80
83
86
89
92
95
98

# What To Do, When $g$, $z$ Are Random?

- Ramsey Principle: Minimize Tax Distortions
- If There is A Low Elasticity Item, Tax It
- If a Bad Shock Hits: Tax Capital
  (i.e., hit things that reflect *past* decisions like physical capital)
- Important ..... If a Good Shock Hits: Subsidize Capital
  (that minimizes *ex ante* distortions to capital accumulation)
- Movements in $P$ May Be Best Thing (see Simulations)
  This Conclusion Will Be Dependent on Degree of Price Stickiness

TABLE 3

PROPERTIES OF THE MONETARY MODELS

| Rates | Models | | |
|---|---|---|---|
| | Baseline | High Risk Aversion | I.I.D. |
| *Labor Tax* | | | |
| Mean | 20.05 | 20.18 | 20.05 |
| Standard Deviation | .11 | .06 | .11 |
| Autocorrelation | .89 | .89 | .00 |
| Correlation with | | | |
| Government Consumption | .93 | −.93 | .93 |
| Technology Shock | −.36 | .35 | −.36 |
| Output | .03 | −.06 | .02 |
| *Inflation* | | | |
| Mean | −.44 | 4.78 | −2.39 |
| Standard Deviation | 19.93 | 60.37 | 9.83 |
| Autocorrelation | .02 | .06 | −.41 |
| Correlation with | | | |
| Government Consumption | .37 | .26 | .43 |
| Technology Shock | −.21 | −.21 | −.70 |
| Output | −.05 | −.08 | −.48 |
| *Money Growth* | | | |
| Mean | −.70 | 4.03 | −2.78 |
| Standard Deviation | 18.00 | 54.43 | 3.74 |
| Autocorrelation | .04 | .07 | .00 |
| Correlation with | | | |
| Government Consumption | .40 | .28 | .92 |
| Technology Shock | −.17 | −.20 | −.36 |
| Output | .00 | −.07 | .02 |

# Financing War: Barro versus Ramsey

When War (or Other Large Financing Need) Suddenly Strikes:

- Barro:
  - Raise Labor and Other Tax Rates a Small Amount So That When Held Constant at That Level, Expected Value of War is Financed
  - This Minimizes Intertemporal Substitution Distortions
  - Involves a Big *Increase* in Debt in Short Run
  - Prediction for Labor Tax Rate: Random Walk.

- Ramsey:
  - Tax Existing Capital Assets (Human, Physical, etc) For Full Amount of Expected Value of War. Do This at the First Sign of War.
  - This Minimizes Intertemporal *and* Intratemporal Distortions (Don't Change Tax Rates on Income at all).
  - Example:
    - Suppose War is Expected to Last Two Periods, Cost: $1 Per Period
    - Suppose Gross Rate of Interest is $1.05$ (i.e., 5%)
    - Tax Capital $1 + 1/1.05 = 1.95$ Right Away.
    - Debt Falls $0.95 in Period When War Strikes.
  - Involves a *Reduction* of Outstanding Debt in Short Run.
  - Prediction for Labor Tax Rate: Roughly Constant.