

## Tutorial on Analysis of an Equilibrium Monetary Model

November 11, 2004

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This is a tutorial that takes you through the estimation and analysis of the general equilibrium model in Altig, Christiano, Eichenbaum and Linde ('Firm-Specific Capital, Nominal Rigidities and the Business Cycle') (ACEL). The estimation strategy chooses model parameters so that the model impulse response functions match the VAR-based estimated impulse response functions as closely as possible. Another tutorial reviewed the estimation of the VAR-based impulse response functions and this will be taken as a given here. The questions in this tutorial will take you through a limited range of experiments with the equilibrium model. As in the previous tutorial, this will be done by executing `main.m` with various different settings of the parameters at the start of that program. Once familiar with the experiments considered here, the interested reader can explore a wider range of experiments by changing the settings of the parameters in `main.m` in other ways. Of course, the more ambitious reader could expand the range of experiments even further by changing the computational code itself. In any case, you should peek inside `solveandsimulate.m` and notice how the various steps discussed in lecture are followed: (i) find the steady state, (ii) log-linearize about the steady state, (iii) solve the linearized system. Then, simulate.

Many of the experiments involve comparing the impulse response functions of the model to the impulse response functions from the VAR. In order for the latter to be available, it is necessary to start by estimating the VAR ( $estvar = 1$ ) and storing its impulse response functions and the associated confidence intervals ( $estvar = 1$ ,  $ndraws = 200$  - a larger value of  $ndraws$  would ensure a more reliable estimate of the sampling uncertainty in the impulse response functions, but would also take a lot of time). The program automatically stores the results of these calculations in several files<sup>1</sup>, which pass the necessary information on to the program that analyzes the equilibrium model. These calculations need not be repeated, unless experiments are done which require impulse response functions from a different VAR.

1. Compute and graph the impulse response functions from the benchmark equilibrium model (set  $mimp = 1$ , the model parameters are stored in `getparam.m`.) This question explores how different features of the model contribute to the shape of the impulse response functions. In practice, this is

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<sup>1</sup>The files are `var20pi_yunits.mat`, `compare20pi_yunits.mat`, and `VAR_IRFsAndSEs.mat`.

done by changing the value of a model parameter. To see the effect of this change it is convenient to have, on the same graph, both the response in the benchmark version of the model and the version of the model with the changed parameter value. This can be accomplished in `main.m`. To see this, note that the part of `main.m` which graphs the model impulse responses has two calls to `solveandsimulate.m`. The first call reads parameters from `getparams.m` and uses the setting of `taux` provided by the user. The second call uses the same parameter values, except for any that might be reset in the lines between the calls. So, to do an experiment simply introduce lines between the calls to `solveandsimulate.m` to set the parameter values you want to change.

- a. For this part of the question, we explore the role of sticky prices and wages in determining the shape of the impulse response functions. Consider first eliminating sticky prices. In fact,  $\xi_p$  does not appear as a parameter in the model. It enters via  $\gamma$  in the reduced form inflation equation. So, how do we incorporate a low value of  $\xi_p$ ? It turns out that whether we adopt the firm-specific or homogeneous capital versions of the model,  $\gamma$  is decreasing in  $\xi_p$ . So, we capture flexible prices by setting  $\gamma$  to a high number, say  $\gamma = 10,000$ . We capture flexible wages in the model by setting  $\xi_w$  to a low number, say 0.01. Note too, that in the benchmark version of the model, prices and wages (as well as consumption, investment and capital utilization) are set before the realization of the current period monetary policy shock. For prices and wages to be fully flexible, they should be set after the current realization of the monetary policy shock. Unfortunately, at the moment the software is not set up to easily implement this.
  - (i) Consider the effect of making wages and prices flexible (i.e., set  $\gamma = 10,000$ ,  $\xi_w = 0.01$ ). Note that monetary policy shocks are now essentially neutral.
  - (ii) Investigate the role of the monetary policy feedback rule when wages and prices are flexible. To do this, set  $\gamma = 10,000$ ,  $\xi_w = 0.01$  right before the first call to `solveandsimulate.m`, so that prices and wages are flexible in both calls. Shut down the monetary policy feedback rule before the second call to `solveandsimulate.m`. To do

this, recall that the policy rule is:

$$\begin{aligned}\hat{x}_{M,t} &= \rho_M \hat{x}_{M,t-1} + \varepsilon_{M,t} \\ \hat{x}_{z,t} &= \rho_{xz} \hat{x}_{z,t-1} + c_z \varepsilon_{z,t} + c_z^p \varepsilon_{z,t-1} \\ \hat{x}_{\Upsilon,t} &= \rho_{x\Upsilon} \hat{x}_{\Upsilon,t-1} + c_{\Upsilon} \varepsilon_{\Upsilon,t} + c_{\Upsilon}^p \varepsilon_{\Upsilon,t-1}\end{aligned}$$

So, to shut down the endogenous part of the policy rule, set  $c\_z=0$ ,  $cp\_z=0$ ,  $c\_ups=0$ ,  $cp\_ups=0$ . Do this between the two calls to `solveandsimulate.m`. Does the endogenous part of the monetary policy rule have any impact on the response of real quantities (e.g., consumption, employment, capital utilization, etc.) to shocks?

- (iii) Return  $\xi_w$  and the monetary policy rule to its benchmark specification, but keep  $\gamma = 1,000$ . The only difference from the benchmark specification now is that prices are flexible (apart from the fact that they are set before the monetary policy shock). Compare the responses of real variables to shocks between the benchmark and flexible price specification. Do these responses look very different?
- b. Note that inflation hardly falls after a positive, neutral technology shock. At the same time, money growth responds quite strongly.
- (i) Investigate whether the response of money growth to the neutral response is responsible for the counterfactual inflation response. To do this, cut the link between monetary policy and the neutral technology shock, by setting  $c\_z = cp\_z = 0$  in between the two calls to `solveandsimulate.m`. Can you see why the estimation program ‘chose’ to introduce a strong monetary policy response to a neutral technology shock (hint: look at the response of hours worked, investment and output)? Evidently, the advantages, in terms of model fit, produced by a strong monetary response outweigh the disadvantage of the counterfactual inflation response.
  - (ii) Setting  $c\_z = cp\_z = 0$  only makes the price level fall a little in response to a neutral technology shock. It does not fall as much as it does in the VAR. Could it be the sticky prices and wages that prevent the fall in the price level? To find out, set  $c\_z = cp\_z = 0$  and  $\xi_w = 0.01$ ,  $\gamma = 10,000$ . Note that inflation now drops a little relative to the benchmark, but not much. We can conclude that the strong response of money growth to a neutral technology shock

is part, but not all, of the reason for the model's counterfactual implication for the response of inflation to a neutral technology shock. This counterfactual implication draws attention to a weak point in the analysis. Perhaps there is a misspecification in the model. Perhaps in the VAR.

- c. The conclusions of the ACEL analysis depend strongly on  $\sigma_a$  being a large number. To understand why the estimation strategy settles on a large value for this parameter, set it to a very small value (say, 0.00001) and evaluate the impact of this change on the impulse response functions. How does the change affect the response of investment to a neutral technology shock? Provide an economic interpretation.
  - d. The conclusions of the ACEL analysis depend strongly on  $\lambda_f$  being close to unity. However, there appears to be little information in the aggregate data about this parameter. To see this, set  $\lambda_f$  to a larger number (try  $\lambda_f = 1.90$ , a number far greater than any reasonable estimate in the literature) and verify the relatively small impact on impulse response functions.
  - e. It is of interest to use the program to explore the impact of adjustment costs in investment and habit persistence in consumption.
    - (i) Consider investment adjustment costs nearly equal to zero,  $S'' = 0.1$  (actually,  $S''$  is  $\kappa$ , or *kappa*, in `main.m`). What does this change do to the response of investment to the three shocks? Does the change amplify the response of hours worked to shocks? Why?
    - (ii) Consider habit persistence nearly zero,  $b = 0.01$ . How does this change affect the response of consumption to a monetary policy shock? Explain the economic reason for the new response. Does the change have much of an effect on the transmission of the two technology shocks?
2. In the ACEL analysis, the model is estimated without taking a stand on whether capital is firm-specific or homogeneous. After estimation, the value of  $\xi_p$  is inferred from the parameters. Under the homogeneous capital interpretation, only  $\gamma$  is needed to infer  $\xi_p$ . Under the firm-specific capital model, the other parameters of the model play a role, most especially  $\lambda_f$  and  $\sigma_a$ . Inference about  $\xi_p$  is in fact very sensitive to the value taken on by these parameters. Under the benchmark parameter values, with  $\sigma_a = 2.0136$ ,

$\lambda_f = 1.01$ , the assumption that capital is firm-specific implies  $\xi_p = 0.36$ , which implies a duration of  $1/(1 - \xi_p) = 1.6$  quarters. The assumption that capital is homogeneous implies  $\xi_p = 0.83$  and a duration of 6 quarters. Thus, the implied duration is very sensitive to the assumption of firm-specificity of capital.

Determine what happens to  $\xi_p$  and duration when  $\lambda_f = 1.10$  and 1.20. (To do this, set `lambda_f` right before the call to `findksip.m`). These are values of  $\lambda_f$  that have been defended in the literature. However, a recent paper by David Bowman (see the course web site) argues that very low levels of  $\lambda_f$  are quite plausible.

Next, return  $\lambda_f$  to its benchmark value and consider a much lower value of  $\sigma_a$ , say  $\sigma_a = .01$ .

Can you provide intuition for the fact that  $\xi_p$  becomes is relatively insensitive to the degree of firm-specificity of capital when  $\lambda_f$  is close to unity and/or  $\sigma_a$  is close to zero?

3. Program `main.m` can be used to reestimate the model under alternative settings for the parameters. Additional instructions are presented on the website for the ACEL project.
  - a. One exercise that is worth doing is to reestimate the VAR using business sector labor productivity for labor productivity and business sector hours worked in the computation of per capita hours worked (set `product = 2`). Then, reestimate the model, allowing  $\lambda_f$  to be free. (To implement this, you must enter `setup.m` and set `II(26,1) = 1`. Read the information at the top of this program for an explanation. Then, run `main.m` with `estimatemodel = 1`, `estvar = 1`, `imp = 1`. This will take some time to recompute the VAR and the impulse response confidence intervals, as well as to reestimate the equilibrium model.) Does the econometrics still favor a low value of  $\lambda_f$  and high value of  $\sigma_a$ ?
  - b. Reestimate the equilibrium model setting the number of lags in the VAR to 6.