

DSGE Models for Monetary Policy*

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Abstract

We begin with a detailed derivation of the equilibrium conditions of a simple New Keynesian DSGE model, and explore some of its implications for the analysis of monetary policy. We then review the consensus New Keynesian model that is rich enough to address actual data. We show how that model accounts for the slow response of inflation to a monetary policy shock and the strong response of real variables, without appealing to implausible degrees of price and wage frictions. We show that that model is simultaneously able to capture the dynamic response of aggregate variables to two technology shocks. We provide a detailed discussion of the model features that account for these findings.

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1. Introduction

There has been enormous progress in recent years in the development of dynamic, stochastic general equilibrium (DSGE) models for the purpose of monetary policy analysis. These models have been shown to fit aggregate data well by conventional econometric measures. For example, they have been shown to do as well or better than simple atheoretical statistical models at forecasting outside the sample of data on which they were estimated. In part because of these successes, a consensus has formed around a particular model structure, the New Keynesian model.

We begin by presenting a detailed derivation of the simple New Keynesian model with price setting frictions and no capital or investment. Although the model is too simple to address data directly, it is nevertheless useful for analyzing some key questions in monetary policy. For example, we discuss the role of the Taylor principle in achieving stable inflation and output. According to the Taylor principle, it is best to counteract evidence of a rise in inflation with a vigorous interest rate increase. We review the basis for this view. However, we also note that there are reasons to be concerned about this policy. In particular, a policy-induced rise in the interest rate may perversely destabilize the economy by providing a direct boost to inflation. This can happen if interest rates are an important cost of production, as when firms must borrow to finance their variable inputs. We also discuss other implications of this working capital channel. Another policy topic addressed is how the New Keynesian framework provides a basis for price level targeting, as opposed to inflation targeting. Finally, we address the much debated topic of the interaction of monetary policy and the asset price and other volatility of recent years. Here, we show how vigorous application of the Taylor principle could inadvertently trigger a boom/bust episode. Our policy analysis discussion is not meant to be definitive. Indeed, it will be evident that to make it definitive, further econometric work is necessary to assign values to key model parameters. Our objective is only to illustrate how the New Keynesian model provides a useful framework for analyzing monetary policy questions and for evaluating alternative points of view on those questions.

The new monetary DSGE models are of interest not just because they represent laboratories for the analysis of important monetary policy questions. They are also of interest because they appear to address key empirical puzzles about monetary policy. It has long been thought that it is virtually impossible to explain the very slow response of inflation to a monetary disturbance without appealing to completely implausible assumptions about price frictions (see, e.g., Mankiw (2000)). However, it turns out that modern DSGE models do provide an account of the inertia in inflation and the strong response of real variables to monetary policy disturbances, without appealing to seemingly implausible parameter values. Moreover, the models simultaneously explain the dynamic response of the economy to

other shocks. We review these important findings. We explain in detail the contribution of each feature of the consensus New Keynesian model in achieving this result. This discussion follows closely the analysis in Christiano, Eichenbaum and Evans (2005) (CEE) and Altig, Christiano, Eichenbaum and Evans (2005) (ACEL).

The econometric technique that is particularly suited to the shock-based analysis described in the previous paragraph, is the one that matches impulse response functions estimated by vector autoregressions (VARs) with the corresponding objects in a model. Using US macroeconomic data, we show how the parameters of the consensus DSGE model are estimated by this impulse-response matching procedure. The advantage of this econometric approach is transparency and focus. The transparency reflects that the estimation strategy has a simple graphical representation, involving objects - impulse response functions - about which economists have strong intuition. The advantage of focus comes from the possibility of studying the empirical properties of a model without having to specify a full set of shocks. An important methodological development of recent years is the adoption of Bayesian methods of econometric inference. We show how to implement the impulse response matching strategy using Bayesian methods. As a result, all the machinery of priors and posteriors, as well as the marginal likelihood as a measure of model fit, is available to researchers doing inference about DSGE models based on matching model and VAR-based impulse response functions.

The paper is organized as follows. Section 2 describes the simple New Keynesian model without capital. The following section reviews some policy implications of that model. Section 4 describes the larger-sized version of the model, designed to econometrically address macroeconomic data. Section 5 reviews the econometric aspects of the impulse-response estimator. Section 6 reviews the results and conclusions are offered in Section 7. Many algebraic derivations are relegated to an appendix.

2. Simple Model Without Capital

This section analyzes a version of the Calvo-sticky price New Keynesian model without capital. In practice, the analysis of this model often begins with the familiar three equations: the linearized ‘Phillips curve’, ‘IS curve’ and monetary policy rule. However, for our purposes it is necessary to derive the equilibrium conditions from their foundations. This is in part because we explore variations on the standard model and in part because we study the Ramsey-optimal equilibrium of our model. Our strategy for computing Ramsey equilibria requires the actual non-linear equilibrium conditions, not their linearized representation.¹

¹We follow the approach suggested by Kydland and Prescott (1980) and also implemented by Levin, Onatski, Williams and Williams (2005) and others.

The version of the New Keynesian model studied in this section is a modification of the classic model considered in Clarida, Gali and Gertler (1999) and Woodford (2003), modified in two ways. First, we accommodate the working capital channel emphasized by CEE and Barth and Ramey (2002).² The working capital channel results from the assumption that firms' variable inputs must be financed by short term loans. With this assumption, changes in the interest rate affect the economy by changing firms' variable production costs, in addition to operating through the usual spending mechanism. There are several reasons to take the working capital channel seriously. Using US Flow of Funds data, Barth and Ramey (2002) argue that a substantial fraction of firms' variable input costs are borrowed in advance. Ravenna and Walsh (2006) provide evidence supporting the working capital channel, based on instrumental variables estimates of a suitably modified Phillips curve. Finally, section 4 below shows that incorporating the working capital channel helps to explain the 'price puzzle' and provides an answer to Ball (1994)'s 'dis-inflationary boom' critique of sticky price models.

We explore a second modification to the classic New Keynesian model by incorporating the assumption about materials inputs proposed in Basu (1995). Basu argues that a large part (as much as half) of a firm's output is used as materials inputs by other firms. The working capital channel introduces the interest rate into costs while the materials assumption makes those costs big. In the next section of this paper we show that these two factors potentially have far-reaching consequences for monetary policy.

This section is organized as follows. The following subsection describes the private sector of the economy, and derives equilibrium conditions associated with optimization and market clearing. After that we specify the monetary policy rule and define the Taylor rule equilibrium. The last subsection below discusses the interpretation of a key parameter in our utility function. The parameter controls the elasticity with which the labor input in our model economy adjusts in response to a change in the real wage. Traditionally, this parameter has been viewed as being restricted by microeconomic evidence on the Frisch labor supply elasticity. We summarize recent thinking, according to which this parameter is in fact not restricted by observations about the Frisch elasticity. This discussion influences our interpretation of the analysis of monetary policy in the next section, as well as our interpretation of the empirical results in section 6 below.

²The first monetary DSGE model we are aware of that incorporates a working capital channel is Fuerst (1992). Other early examples include Christiano (1991) and Christiano and Eichenbaum (1992a).

2.1. Private Economy

2.1.1. Households

We suppose there is a large number of identical households. The representative household solves the following problem:

$$\max_{\{C_t, H_t, B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{H_t^{1+\phi}}{1+\phi} \right), \quad 0 \leq \beta \leq 1, \quad \phi \geq 0, \quad (2.1)$$

subject to

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + W_t H_t + \text{Transfers and profits}_t. \quad (2.2)$$

Here, C_t , H_t denote household consumption and market work, respectively. In (2.2), B_{t+1} denotes the quantity of a nominal bond purchased by the household in period t and R_t denotes the one-period gross nominal rate of interest on a bond purchased in period t . Finally, W_t denotes the competitively determined nominal wage rate. The parameter, ϕ , is discussed in section 2.3 below.

Optimization of H_t leads the representative household to equate the marginal cost of working, in consumption units, with the marginal benefit, the real wage:

$$C_t H_t^\phi = \frac{W_t}{P_t}. \quad (2.3)$$

Optimization of B_{t+1} leads the household to equate the utility cost of the consumption foregone in acquiring a bond with the corresponding benefit:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}}. \quad (2.4)$$

Here, π_{t+1} is the gross rate of inflation from t to $t+1$.

2.1.2. Firms

A key feature of the New Keynesian model is its assumption that there are price-setting frictions. These frictions are introduced in order to accommodate the evidence of inertia in aggregate inflation. Obviously, the presence of price-setting frictions requires that firms have the power to set prices, and this in turn requires the presence of monopoly power. A challenge is to create an environment in which there is monopoly power, without contradicting the obvious fact that actual economies have a very large number of firms. The Dixit-Stiglitz framework of production handles this challenge very nicely, because it has a very large number of price-setting monopolist firms. In particular, gross output is produced using a representative, competitive firm using the following technology:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right)^{\lambda_f}, \quad \lambda_f \geq 1. \quad (2.5)$$

The representative firm takes the price of gross output, P_t , and the price of intermediate inputs, P_{it} , as given. Profit maximization leads to the following first order condition:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\lambda_f}{\lambda_f-1}}. \quad (2.6)$$

Substituting (2.6) into (2.5) yields the following relation between the aggregate price level and the prices of intermediate goods:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\lambda_f-1}} di \right)^{-(\lambda_f-1)} \quad (2.7)$$

The i^{th} intermediate good is produced by a single monopolist, who takes (2.6) as its demand curve. The value of λ_f determines how much monopoly power the i^{th} producer has. If λ_f is large, then intermediate goods are poor substitutes for each other, and the monopoly supplier of good i has a lot of market power. Consistent with this, note that if λ_f is large, then the demand for $Y_{i,t}$ is relatively price inelastic (see (2.6)). If λ_f is close to unity, so that each $Y_{i,t}$ is almost a perfect substitute for $Y_{j,t}$, $j \neq i$, then i^{th} firm faces a demand curve that is almost perfectly elastic. In this case, the firm has virtually no market power.

The production function of the i^{th} monopolist is:

$$Y_{i,t} = z_t H_{i,t}^\gamma I_{i,t}^{1-\gamma}, \quad 0 \leq \gamma \leq 1, \quad (2.8)$$

where z_t is a technology shock whose stochastic properties are specified below. Here, H_{it} denotes the level of employment by the i^{th} monopolist. We follow Basu (1995) in supposing that the i^{th} monopolist uses the quantity of materials, I_{it} , as inputs to production. The materials, I_{it} , are converted one-for-one from Y_t in (2.5). For $\gamma < 1$, each intermediate good producer in effect uses the output of all the other intermediate produces as input. When $\gamma = 1$, then our model reduces to the classic New Keynesian model without capital.

The nominal marginal cost of the intermediate good producer is the following Cobb-Douglas function of the price of its two inputs:

$$\text{marginal cost}_t = \left(\frac{\bar{P}_t}{1-\gamma} \right)^{1-\gamma} \left(\frac{\bar{W}_t}{\gamma} \right)^\gamma \frac{1}{z_t}.$$

Here, $\bar{W}_t$ and $\bar{P}_t$ are the effective prices of H_{it} and I_{it} , respectively:

$$\begin{aligned} \bar{W}_t &= (1 - \nu_t) (1 - \psi + \psi R_t) W_t \\ \bar{P}_t &= (1 - \nu_t) (1 - \psi + \psi R_t) P_t. \end{aligned}$$

In this expression, ν_t denotes a subsidy to intermediate good firms and the term involving the interest rate reflects the presence of a ‘working capital channel’. For example, $\psi = 1$

corresponds to the case where the full amount of the cost of labor and materials must be financed at the beginning of the period. When $\psi = 0$, no advanced financing is required. A key variable in the model is the ratio of nominal marginal cost to the price of gross output, P_t :

$$s_t = (1 - \nu_t) \left(\frac{1}{1 - \gamma} \right)^{1-\gamma} \left(\frac{W_t/P_t}{\gamma} \right)^\gamma \frac{1}{z_t} (1 - \psi + \psi R_t). \quad (2.9)$$

If the intermediate good firms faced no price-setting frictions, they would all set their price as a fixed markup over nominal marginal cost:

$$\lambda_f P_t s_t. \quad (2.10)$$

In fact, we assume there are Calvo-style price setting frictions. An intermediate firm can set its price optimally with probability $1 - \xi_p$, and with probability ξ_p it must keep its price unchanged relative to what it was in the previous period:

$$P_{i,t} = P_{i,t-1}.$$

Consider the $1 - \xi_p$ intermediate good firms that are able to set price optimally in period t . There are no state variables in the intermediate good firm problem and all the firms face the same demand curve. As a result, all firms able to optimize price in period t choose the same price, which we denote by $\tilde{P}_t$. It is clear that optimizing firms do not set $\tilde{P}_t$ equal to (2.10). Setting $\tilde{P}_t$ to (2.10) would be optimal from the perspective of the current period, but it does not take into account the possibility that the firm may be stuck with $\tilde{P}_t$ for several periods into the future. Instead, the intermediate good firms that have an opportunity reoptimize their price in the current period, do so to solve:

$$\max_{\tilde{P}_t} E_t \sum_{j=0}^{\infty} (\xi_p \beta)^j v_{t+j} \left(\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right), \quad (2.11)$$

subject to (2.8) and the requirement that it satisfy demand, (2.6), in each period. In (2.11), $\beta^j v_{t+j}$ is the multiplier on the household's nominal period $t + j$ budget constraint. Because they are the owners of the intermediate good firms, households are the recipients of firm profits. In this way, it is natural that the firm should weigh profits in different dates and states of nature using $\beta^j v_{t+j}$. Intermediate good firms take v_{t+j} as given. The nature of the family's preferences, (2.1), implies:

$$v_{t+j} = \frac{1}{P_{t+j} C_{t+j}}.$$

In (2.11) the presence of ξ_p reflects that intermediate good firms are only concerned with future scenarios in which they are not able to reoptimize the price chosen in period t .

The first order condition associated with (2.11) is:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j (X_{t,j})^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j (X_{t,j})^{-\frac{1}{\lambda_f-1}}} = \frac{K_t^f}{F_t^f}, \quad (2.12)$$

where K_t^f and F_t^f denote the numerator and denominator of the ratio after the first equality, respectively. Also,

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, \quad X_{t,j} \equiv \begin{cases} \frac{1}{\pi_{t+j} \cdots \pi_{t+1}} & j > 0 \\ 1 & j = 0 \end{cases}.$$

Not surprisingly, (2.12) implies $\tilde{P}_t$ is set to (2.10) when $\xi_p = 0$. When $\xi_p > 0$, optimizing firms set price so that it is the desired markup, λ_f , over marginal cost on average. It is useful to write the numerator and denominator in (2.12) in recursive form. Thus,

$$K_t^f = \lambda_f s_t + \beta \xi_p E_t \pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}} K_{t+1}^f, \quad (2.13)$$

$$F_t^f = 1 + \beta \xi_p E_t \pi_{t+1}^{\frac{1}{\lambda_f-1}} F_{t+1}^f. \quad (2.14)$$

Expression (2.7) simplifies when we take into account that (i) the $1 - \xi_p$ intermediate good firms that set their price optimally all set it to $\tilde{P}_t$ and (ii) the ξ_p firms that cannot reset their price are selected at random from the set of all firms. Doing so,

$$\tilde{p}_t = \left[\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p} \right]^{-(\lambda_f-1)}.$$

It is convenient to use the last expression to eliminate $\tilde{p}_t$ in (2.12):

$$K_t^f = F_t^f \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p} \right)^{-(\lambda_f-1)}. \quad (2.15)$$

Cost minimization by the i^{th} intermediate good producer leads it to equate the relative price of its labor and materials inputs to the corresponding relative marginal productivities:

$$\frac{\bar{W}_t}{\bar{P}_t} = \frac{W_t}{P_t} = \frac{\gamma}{1-\gamma} \frac{I_{it}}{H_{it}} = \frac{\gamma}{1-\gamma} \frac{I_t}{H_t}. \quad (2.16)$$

Evidently, each firm uses the same ratio of inputs, regardless of its output price, P_{it} .

2.1.3. Aggregate Resources and the Private Sector Equilibrium Conditions

A notable feature of the New Keynesian model is the absence of an aggregate production function. That is, given information about aggregate inputs and technology, it is not possible

to say what aggregate output, Y_t , is. This is because Y_t also depends on how inputs are distributed among the various intermediate good producers. For a given amount of aggregate inputs, Y_t is maximized by distributing the inputs equally across producers. If the distribution is not uniform a lower level of Y_t occurs. In the New Keynesian model with Calvo price frictions, resources are unequally allocated across intermediate good firms if, and only if, P_{it} differs across i . Price dispersion in the model is caused by the interaction of inflation with price-setting frictions. With price dispersion, the price mechanism ceases to allocate resources efficiently, as too much production is done in firms with low prices and too little in the firms with high prices. Yun (1996) derived a very simple formula that characterizes the loss of output due to price dispersion. We derive that formula here.

Let Y_t^* denote the unweighted integral of gross output across intermediate good producers:

$$Y_t^* \equiv \int_0^1 Y_{i,t} di = \int_0^1 z_t \left(\frac{H_{it}}{I_{it}} \right)^\gamma I_{it} di = z_t \left(\frac{H_t}{I_t} \right)^\gamma I_t = z_t H_t^\gamma I_t^{1-\gamma}.$$

Here, we have used linear homogeneity of the production function, as well as the result in (2.16), that all intermediate good producers use the same labor to materials ratio. An alternative representation of Y_t^* makes use of the demand curve, (2.6):

$$Y_t^* = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\lambda_f}{\lambda_f-1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f-1}} \int_0^1 (P_{i,t})^{-\frac{\lambda_f}{\lambda_f-1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f-1}} (P_t^*)^{-\frac{\lambda_f}{\lambda_f-1}}. \quad (2.17)$$

Thus,

$$Y_t = p_t^* z_t H_t^\gamma I_t^{1-\gamma},$$

where

$$p_t^* \equiv \left(\frac{P_t^*}{P_t} \right)^{\frac{\lambda_f}{\lambda_f-1}}. \quad (2.18)$$

Here, $p_t^* \leq 1$ denotes Yun (1996)'s measure of the output lost due to price dispersion. From (2.17),

$$P_t^* = \left[\int_0^1 (P_{i,t})^{-\frac{\lambda_f}{\lambda_f-1}} di \right]^{-\frac{\lambda_f-1}{\lambda_f}}. \quad (2.19)$$

According to (2.18), p_t^* is a monotone transform on the ratio of two different weighted averages of intermediate good prices. The ratio of these two weighted averages can only be at its maximum of unity if all prices are the same.³

³The distortion, p_t^* , is of interest in its own right. It is a sort of ‘endogenous Solow residual’ of the sort called for by Prescott (1998). Whether the magnitude of fluctuations in p_t^* are quantitatively important given the actual price dispersion in data is something that deserves exploration. A difficulty that must be overcome, in such an exploration, is determining what the benchmark efficient dispersion of prices is in the data. In the model it is efficient for all prices to be exactly the same, but that is obviously only a convenient normalization.

Taking into account observations (i) and (ii) after (2.14), (2.19) reduces (after dividing by P_t and taking into account (2.18)) to:

$$p_t^* = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \xi_p \frac{\pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right]^{-1}. \quad (2.20)$$

According to (2.20), there is price dispersion in the current period if there was dispersion in the previous period and/or if there is a current shock to dispersion. Such a shock must operate through the aggregate rate of inflation.

We conclude that the relation between the aggregate inputs used to produce gross output and the uses of gross output is given by:

$$C_t + I_t = p_t^* z_t H_t^\gamma I_t^{1-\gamma}. \quad (2.21)$$

Here, $C_t + I_t$ represents total gross output, while C_t represents value added.

The private sector equilibrium conditions of the model are (2.3), (2.4), (2.9), (2.13), (2.14), (2.15), (2.16), (2.20) and (2.21). This represents 9 equations in the following 11 unknowns:

$$C_t, H_t, I_t, R_t, \pi_t, p_t^*, K_t^f, F_t^f, \frac{W_t}{P_t}, s_t, \nu_t. \quad (2.22)$$

As it stands, the system is underdetermined. This is not surprising, since we have said nothing about monetary policy or ν_t . We turn to this in the following section.

2.2. Log-linearized Equilibrium with Taylor Rule

We suppose that monetary policy follows a Taylor rule that is specified below. Because the Taylor rule responds to the deviation between actual inflation and a target zero inflation rate, it follows that inflation must be zero in steady state. In addition, we suppose that the intermediate good subsidy, ν , is the constant that enforces (3.21) in steady state. This treatment of policy implies that the steady state allocations of our model economy are efficient in the sense that they coincide with the solution to the following planning problem:

$$\max_{\{c_t, H_t, i_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \frac{H_t^{1+\phi}}{1+\phi} \right] \quad (2.23)$$

subject to

$$c_t + i_t = H_t^\gamma i_t^{1-\gamma},$$

where we have adopted the following scaling:

$$c_t \equiv \frac{C_t}{z_t^{1/\gamma}}, \quad i_t \equiv \frac{I_t}{z_t^{1/\gamma}}. \quad (2.24)$$

The problem, (2.23), is that of a planner who allocates resources efficiently across intermediate goods and who does not permit distortions due to monopoly power.

Because inflation, π_t , fluctuates in equilibrium, (2.20) suggests that p_t^* fluctuates too. It turns out, however, that p_t^* is constant to a first order approximation. To see this, note that the absence of inflation in the steady state also guarantees there is no price dispersion in steady state in the sense that p_t^* is at its maximal value of unity (see (2.20)). With p_t^* at its maximum in steady state, small perturbations have a zero first-order impact on p_t^* . This can be seen by noting that π_t is absent from the log-linear expansion of (2.20) about $p_t^* = 1$:

$$\hat{p}_t^* = \xi_p \hat{p}_{t-1}^*. \quad (2.25)$$

Here, a hat over a variable indicates:

$$\hat{\varrho}_t = \frac{d\varrho_t}{\varrho},$$

where ϱ denotes the steady state of the variable, ϱ_t , and $d\varrho_t = \varrho_t - \varrho$ denotes a small perturbation in ϱ_t from steady state. We suppose that in the initial period, $\hat{p}_{t-1}^* = 0$, so that, to a first order approximation, $\hat{p}_t^* = 0$ for all t .

We log-linearize the equilibrium conditions of our model economy about steady state. Log-linearizing (2.13), (2.14) and (2.15) we obtain the usual representation of the Phillips curve:

$$\hat{\pi}_t = \frac{(1 - \beta\xi_p)(1 - \xi_p)}{\xi_p} \hat{s}_t + \beta E_t \hat{\pi}_{t+1}. \quad (2.26)$$

Combining (2.3) with (2.9), taking into account (2.24) and setting the subsidy as just described, real marginal cost is:

$$s_t = \frac{1}{\lambda_f} \frac{1 - \psi + \psi R_t}{1 - \psi + \psi R} \left(\frac{1}{1 - \gamma} \right)^{1-\gamma} \left(\frac{c_t H_t^\phi}{\gamma} \right)^\gamma.$$

Then,

$$\hat{s}_t = \gamma \left(\phi \hat{H}_t + \hat{c}_t \right) + \frac{\psi}{(1 - \psi)\beta + \psi} \hat{R}_t. \quad (2.27)$$

Substituting out for the real wage in (2.16) using (2.3) and applying (2.24),

$$H_t^{\phi+1} c_t = \frac{\gamma}{1 - \gamma} i_t. \quad (2.28)$$

Similarly, scaling (2.21):

$$c_t + i_t = H_t^\gamma i_t^{1-\gamma}.$$

Using (2.28) to substitute out for i_t in the above expression, we obtain:

$$c_t + \frac{1 - \gamma}{\gamma} H_t^{\phi+1} c_t = H_t^\gamma \left[\frac{1 - \gamma}{\gamma} \exp(\tau_t) H_t^{\phi+1} c_t \right]^{1-\gamma}.$$

Log-linear approximation of this expression about steady state implies, after some algebra,

$$\hat{c}_t = \hat{H}_t.$$

Substituting the latter into (2.27), we obtain:

$$\hat{s}_t = \gamma (1 + \phi) \hat{c}_t + \frac{\psi}{(1 - \psi) \beta + \psi} \hat{R}_t. \quad (2.29)$$

Substituting out for $\hat{s}_t$ into the Phillips curve, we obtain:

$$\hat{\pi}_t = \frac{(1 - \beta \xi_p) (1 - \xi_p)}{\xi_p} \left[\gamma (1 + \phi) \hat{c}_t + \frac{\psi}{(1 - \psi) \beta + \psi} \hat{R}_t \right] + \beta \hat{\pi}_{t+1}. \quad (2.30)$$

In (2.30), $\hat{c}_t$ is the percent deviation of $C_t/z_t^{1/\gamma}$ from its steady state value. Since the value of $C_t/z_t^{1/\gamma}$ in steady state coincides with the first-best solution for $C_t/z_t^{1/\gamma}$, (2.23), we can refer to $\hat{c}_t$ as the output gap. When $\gamma = 1$ and $\psi = 0$, (2.30) reduces to the ‘Phillips curve’ in the classic New Keynesian model. When materials are an important factor of production, so that γ is small, then a given jump in the output gap has a smaller impact on inflation. The reason is that in this case the aggregate price index is part of the input cost for intermediate good producers. So, a small rise in prices in response to a given gap is an equilibrium because individual intermediate good firms have less of an incentive to raise their prices in this case. With $\psi > 0$, (2.30) indicates that a jump in the interest rate drives up prices. This is because with an active working capital channel a rise in the interest rate drives up marginal cost.⁴

Now consider the intertemporal Euler equation. Expressing (2.4) in terms of scaled variables,

$$1 = E_t \frac{\beta c_t}{c_{t+1} \mu_{z,t+1}^{\frac{1}{\gamma}}} \frac{R_t}{\pi_{t+1}}, \quad \mu_{z,t+1} \equiv \frac{z_{t+1}}{z_t}.$$

Log-linearly expanding about steady state:

$$0 = E_t \left[\hat{c}_t - \hat{c}_{t+1} - \frac{1}{\gamma} \hat{\mu}_{z,t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right],$$

or,

$$\hat{c}_t = E_t \left[\hat{c}_{t+1} - \left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^* \right) \right], \quad (2.31)$$

where

$$\hat{R}_t^* \equiv \frac{1}{\gamma} E_t \hat{\mu}_{z,t+1}.$$

We suppose that monetary policy, when linearized, is characterized by the following Taylor rule:

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) [r_\pi \hat{\pi}_{t+1} + r_c \hat{c}_t]. \quad (2.32)$$

The equilibrium of the log-linearly expanded economy is given by (??), (2.30), (2.31) and (2.32).

⁴Equation (2.30) resembles equation (13) in Ravenna and Walsh (2006), except that we allow for materials inputs, i.e., $\gamma < 1$.

2.3. Frisch Labor Supply Elasticity

The magnitude of the parameter, ϕ , in the household utility function plays an important role in the analysis in later sections. Indeed, this parameter has been the focus of much discussion in macroeconomics generally. Note from (2.3) that the elasticity of H_t with respect to the real wage, holding C_t constant, is $1/\phi$. The condition, “holding C_t constant”, could mean that the elasticity refers to the response of H_t to a change in the real wage that is of very short duration, so short that the household’s wealth - and, hence, consumption - is left unaffected. Alternatively, the elasticity could refer to the response of H_t to a change in the real wage that is associated with a lump sum transfer payment that keeps wealth unchanged. The debate about ϕ centers on the interpretation of H_t . Under one interpretation, H_t represents the amount of hours worked by a typical person in the labor force. With this interpretation, $1/\phi$ is the Frisch labor supply elasticity. This is perhaps the most straightforward interpretation of $1/\phi$ given our assumption that the economy is populated by identical households, in which H_t is the labor effort of the typical household. An alternative interpretation of H_t is that it represents a number of people, and that $1/\phi$ measures the elasticity with which marginal people substitute in and out of employment with a change in the wage. Under this interpretation, $1/\phi$ need not correspond to the labor supply elasticity of any particular person. The two different interpretations of H_t give rise to very different views about the appropriate value of ϕ .

There is an influential labor literature that estimates the Frisch labor supply elasticity using household level data. The general finding is that, although the Frisch elasticity varies somewhat across different types of people, on the whole the elasticities are very small. Some have interpreted this to mean that only large values of ϕ (say, larger than unity) are consistent with the data. Many macroeconomists accepted this interpretation initially, but this presented them with a puzzle. Over the business cycle, one observes that employment fluctuates a great deal more than real wages. It seemed puzzling that aggregate data appeared to suggest high labor supply elasticities, while the micro data appeared to suggest low labor supply elasticities. A consensus is now emerging according to which this ‘puzzle’ is in fact a false puzzle. The idea is that the Frisch elasticity in the micro data and the labor supply elasticity in the macro data represent different objects.

It is well known that much of the business cycle variation in employment reflects changes in the quantity of people working, not in the number of hours worked by the average household. Beginning at least with the work of Rogerson (1988) and Hansen (1985), it has been argued that even if the individual’s labor supply elasticity is *zero* over most values of the wage, aggregate employment could nevertheless respond highly elastically to small changes in the real wage. This can occur if there are household members who are just on the margin

between working in the market and devoting their time to other activities. An example is a spouse who is doing productive work in the home, and yet who might be tempted by a small rise in the market wage to substitute into the market. Another example is teenagers who may be close to the margin between pursuing additional education and working, who could be induced to switch to working by a small rise in the wage. Finally, there is the elderly person who might be induced by a small rise in the market wage to delay retirement. These examples suggest that aggregate employment might fluctuate substantially in response to small changes in the real wage, even if the individual household's Frisch elasticity of labor supply is zero over all values of the wage, except the one value that induces them to shift in or out of the market.

The ideas in the previous paragraphs can be illustrated in our model. Such an illustration obviously requires that households have several members. The realistic assumption is to suppose that 'several' means 3 or 4, but this would embroil us in technical complications which would take us away from the main idea. Instead, we adopt the technically convenient assumption that the household has a large number of members, one for each of the points on the line bounded by 0 and 1.⁵ In addition, we assume that a household member only has the option to work full time or not at all. Their Frisch labor supply elasticity is zero for most values of the wage. Let $l \in [0, 1]$ index a particular member in the family. Suppose this member enjoys the following utility if employed:

$$\log(C_t) - l^\phi, \quad \phi > 0,$$

and the following utility if not employed:

$$\log(C_t).$$

Household members are ordered according to their degree of aversion to work. Those with high values of l have a high aversion (for example, small children, and elderly or chronically ill people) to work, and those with l near zero have very little aversion. We suppose that household decisions are made on a utilitarian basis, in a way that maximizes the equally weighted integral of utility across all household members. Under these circumstances, efficiency dictates that all members receive the same level of consumption, whether employed or not. In addition, if H_t members are to be employed, then those with $0 \leq l \leq H_t$ should work and those with $l > H_t$ should not. For a household with consumption, C_t , and employment, H_t , utility is, after integrating over all $l \in [0, 1]$:

$$\log(C_t) - \frac{H_t^{1+\phi}}{1+\phi},$$

⁵Our approach is most similar to the approach of Gali (2010), though it also resembles the recent work of Mulligan (2001) and Krusell, Mukoyama, Rogerson and Sahin (2008).

which coincides with the period utility function in (2.1). Under this interpretation of the utility function, (2.3) remains the relevant first order condition for labor. In this case, given the wage, W_t/P_t , the household sends out a number of members, H_t , to work until the utility cost of work for the marginal worker, H_t^ϕ , is equated to the corresponding utility benefit to the household, $(W_t/P_t)/C_t$.

Note that under this interpretation of the utility function, H_t denotes a quantity of workers and ϕ dictates the elasticity with which different members of the households enter or leave employment in response to shocks. The case in which ϕ is large corresponds to the case where household members differ relatively sharply in terms of their aversion to work. In this case there are not many members with disutility of work close to that of the marginal worker. As a result, a given change in the wage induces only a small change in employment. If ϕ is very small, then there is a large number of household members close to indifferent between working and not working, and so a small change in the real wage elicits a large labor supply response.

Given that most of the business cycle variation in the labor input is in the form of numbers of people employed, we think the most sensible interpretation of H_t is that it measures numbers of people. As a result, ϕ is not to be interpreted as a Frisch elasticity, which is in fact zero.

3. Monetary Policy Analysis With the Simple Model Without Capital

This section uses three examples to illustrate how monetary DSGE are used to think about monetary policy. In the first two examples, we discuss the Taylor principle both in ‘normal times’ and at times when optimism about the future triggers economic volatility. In each case, we describe the rationale for the Taylor principle, as well as potential problems. The last subsection examines the Ramsey optimal policy for our model. The first and third examples show how the working capital channel can have important consequences for monetary policy.

3.1. Taylor Principle

A key objective of monetary policy is the maintenance of low and stable inflation. The classic New Keynesian model defined by $\gamma = 1$ and $\psi = 0$ can be used to articulate the risk that inflation expectations might become self-fulfilling unless the monetary authorities adopt the appropriate monetary policy. The classic model can also be used to explain the widespread consensus that ‘appropriate monetary’ policy means a monetary policy that embeds the Taylor principle: a 1% rise in inflation should be met by a greater than 1% rise in the nominal interest rate. This subsection explains how the classic New Keynesian model

provides support for the Taylor principle. However, when we incorporate the assumption of a working capital channel - particularly when the share of materials in gross output is as high as it is in the data - the Taylor principle to become a *source* of instability. This is perhaps not surprising. When the working capital channel is strong, if the monetary authority raises the interest rate in response to rising inflation expectations, they will simply produce the higher inflation that people expect.

It is convenient to summarize the linearized equations of our model here:

$$\hat{R}_t^* = E_t \frac{1}{\gamma} \hat{\mu}_{z,t+1} \quad (3.1)$$

$$\hat{\pi}_t = \kappa \left[\gamma (1 + \phi) \hat{c}_t + \alpha_\psi \hat{R}_t \right] + \beta E_t \hat{\pi}_{t+1} \quad (3.2)$$

$$\hat{c}_t = E_t \left[\hat{c}_{t+1} - \left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^* \right) \right] \quad (3.3)$$

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) [r_\pi \hat{\pi}_{t+1} + r_c \hat{c}_t], \quad (3.4)$$

where

$$\kappa = \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p}, \quad \alpha_\psi = \frac{\psi}{(1 - \psi)\beta + \psi}.$$

The specification of the model is complete when we specify the law of motion for the exogenous shocks. We do this in the following subsections as needed.

We begin by reviewing the case of the Taylor principle using the classic New Keynesian model, with $\gamma = 1$, $\psi = 0$. We get to the heart of the argument using the deterministic version of the model, in which $\hat{R}_t^* \equiv 0$. In addition, it is convenient to suppose that monetary policy is characterized by $\rho = r_c = 0$. Throughout, we adopt the presumption that the only valid equilibria are paths for $\hat{\pi}_t$, $\hat{R}_t$ and $\hat{c}_t$ that converge to steady state, i.e., 0.⁶ Under these circumstances, (3.2) and (3.3) can be solved forward as follows:

$$\hat{\pi}_t = \kappa \gamma (1 + \phi) \hat{c}_t + \beta \kappa \gamma (1 + \phi) \hat{c}_{t+1} + \beta^2 \kappa \gamma (1 + \phi) \hat{c}_{t+2} + \dots \quad (3.5)$$

and

$$\hat{c}_t = - \left(\hat{R}_t - \hat{\pi}_{t+1} \right) - \left(\hat{R}_{t+1} - \hat{\pi}_{t+2} \right) - \left(\hat{R}_{t+2} - \hat{\pi}_{t+3} \right) - \dots \quad (3.6)$$

⁶ Although our presumption is standard, justifying it is harder than one might have thought. For example, Benhabib, Schmitt-Grohe and Uribe (2002) have presented examples in which some explosive paths for the linearized equilibrium conditions are symptomatic of perfectly sensible equilibria for the actual economy underlying the linear approximations. In these cases, focusing on the non-explosive paths of the linearized economy may be valid after all if we imagine that monetary policy is a Taylor rule with a particular escape clause. The escape clause specifies that in the event the economy threatens to follow an explosive path, the monetary authority commits to switch to a monetary policy of targetting the money growth rate. There are examples of monetary models in which the escape clause monetary policy justifies the type of equilibrium selection we adopt in the text (see Benhabib, Schmitt-Grohe and Uribe (2002), and Christiano and Rostagno (2001) for further discussion). For a more recent debate about the validity of the equilibrium selection adopted in the text, see McCallum (2009) and Cochrane (2009) and the references they cite.

In (3.6) we have used the fact that in our setting a path converges to zero if, and only if, it converges fast enough so that a sum like the one in (3.6) is well defined.⁷ Equation (3.5) shows that inflation is a function of the present and future output gap. Equation (3.6) shows that the current output gap is a function of the long term real interest rate (i.e., the sum on the right of (3.6)) in the model.

Under the Taylor principle, the classic New Keynesian model implies that a rise in inflation expectations launches a sequence of events which ultimately leads to a moderation in actual inflation. Seeing this moderation in actual inflation, people's higher inflation expectations would quickly dissipate before they can be a source of economic instability. The way this works is that the rise in the real rate of interest slows down spending, causing the output gap shrink (see (3.6)). The fall in actual inflation occurs as the reduction in output reduces pressure on resources and drives down the marginal cost of production (see (3.2)). Strictly speaking, we have just described a rationale for the Taylor principle that is based on learning (for a formal discussion, see McCallum (2009)). Under rational expectations, the posited rise in inflation expectations would not occur in the first place if policy obeys the Taylor principle.

A similar argument shows that if the monetary authority does not obey the Taylor principle, i.e., $r_\pi < 1$, then a rise in inflation expectations can be self-fulfilling. This is not surprising, since in this case the rise in expected inflation is associated with a fall in the real interest rate. According to (3.6) this produces a rise in the output gap. By raising marginal cost, the Phillips curve, (3.5), implies that actual inflation rises. Seeing higher actual inflation, people's higher inflation expectations are confirmed. In this way, with $r_\pi < 1$ a rise in inflation expectations becomes self-fulfilling by triggering a boom in output and actual inflation. It is easy to see that with $r_\pi < 1$ many equilibria are possible. A drop in inflation expectations can cause a fall in output and inflation. Inflation expectations could be random, causing random fluctuations between booms and recessions.⁸

In this way, the classic New Keynesian model has been used to articulate the idea that the

⁷The reason for this can be seen below, where we show that the solution to this equation is a linear combination of terms like $a\lambda^t$. Such an expression converges to zero if, and only if, it is also summable.

⁸Clarida, Gali and Gertler (1999) argue that the high inflation of the 1970s in many countries can be explained as reflecting the failure to respect the Taylor principle in the early 1970s. Christiano and Gust (2000) criticize this argument on the ground that one did not observe the boom in employment in the 1970s that the CGG analysis predicts. Christiano and Gust argue that even if one thought of the 1970s as also a time of bad technology shocks (fuel costs and commodity prices soared then), the CGG analysis predicts that employment should have boomed in the 1970s. Christiano and Gust present an alternative model, a 'limited participation' model, which has the same implications for the Taylor principle that the CGG model has. However, the Christiano and Gust model has a very different implication for what happens to real allocations in a self-fulfilling inflation episode. Because of the presence of an important working capital channel in the Christiano and Gust model, the self-fulfilling inflation episode is associated with a recession in output and employment. Thus, Christiano and Gust conclude that the 1970s might well reflect the failure to implement the Taylor principle, but only if the analysis is done in a model different from the CGG model.

Taylor principle promotes stability, while absence of the Taylor principle makes the economy vulnerable to fluctuations in self-fulfilling expectations.

The preceding results are particularly easy to establish formally under the assumption of rational expectations. We continue to maintain the simplifying assumption, $\rho = r_x = 0$. We reduce the model to a single second order difference equation in inflation. Substitute out for $\hat{R}_t$ in (3.2) and (3.3) using (3.4). Then, solve (3.2) for $\hat{c}_t$ and use this to substitute out for $\hat{c}_t$ in (3.3). These operations result in the following second order difference equation in $\hat{\pi}_t$:

$$\hat{\pi}_t + [\kappa\gamma(1 + \phi)(r_\pi - 1) - (\kappa\alpha_\psi r_\pi + \beta) - 1] \hat{\pi}_{t+1} + (\kappa\alpha_\psi r_\pi + \beta) \hat{\pi}_{t+2} = 0,$$

The general set of solutions to this difference equation can be written as follows:

$$\hat{\pi}_t = a_0 \lambda_1^t + a_1 \lambda_2^t,$$

for arbitrary a_0, a_1 . Here, $\lambda_i, i = 1, 2$, are the roots of the following second order polynomial:

$$1 + [\kappa\gamma(1 + \phi)(r_\pi - 1) - (\kappa\alpha_\psi r_\pi + \beta + 1)] \lambda + (\kappa\alpha_\psi r_\pi + \beta) \lambda^2 = 0.$$

Thus, there is a two dimensional space of solutions to the equilibrium conditions (i.e., one for each possible value of a_0 and a_1). We continue to apply our presumption that among these solutions, only the ones in which the variables converge to zero (i.e., to steady state) correspond to equilibria. Thus, uniqueness of equilibrium requires that both λ_1 and λ_2 be larger than unity in absolute value. In this case, the unique equilibrium is the solution associated with $a_0 = a_1 = 0$. If one or both of $\lambda_i, i = 1, 2$ are less than unity in absolute value, then there are many solutions to the equilibrium conditions that are equilibria. We can think of these equilibria as corresponding to different, self-fulfilling, expectations.

The following result can be established for the classic New Keynesian model, with $\gamma = 1$ and $\alpha_\psi = 0$. The model economy has a unique equilibrium if, and only if $r_\pi > 1$ (see, e.g., Bullard and Mitra (2002)). This is consistent with the intuition about the Taylor principle discussed above.

We now re-examine the case for the Taylor principle when there is a working capital channel. The reason the Taylor principle works in the classic New Keynesian model is that a rise in the interest rate leads to a fall in inflation by curtailing aggregate spending. But, with a working capital channel, $\alpha_\psi > 0$, an increase in the interest rate has a second effect. By raising marginal cost (see (3.2)), a rise in the interest rate places upward pressure on inflation by raising marginal cost. If the working capital channel is strong enough, then monetary policy with $r_\pi > 1$ may add fuel to the fire when inflation expectations rise. The sharp rise in the nominal rate of interest in response to a rise in inflation expectations may actually cause the inflation that people expected. In this way the Taylor principle could

actually be destabilizing. Of course, for this to be true requires that the working capital channel be strong enough. For a small enough working capital channel (i.e., small α_ψ) the Taylor principle would still have the effect of inoculating the economy from destabilizing fluctuations in inflation expectations.

Whether the presence of the working capital channel in fact overturns the Taylor principle is a numerical question. We must assign values to the model parameters and investigate whether one or both of λ_1 and λ_2 are less than unity in absolute value. If this is the case, then the Taylor principle does not stabilize inflation expectations. Throughout, we set

$$\beta = 0.99, \quad \xi_p = 0.75, \quad r_\pi = 1.5.$$

The discount rate is 4 percent, at an annual rate and the value of ξ_p implies an average time between price reoptimization of one year. In addition, monetary policy is characterized by a strong commitment to the Taylor principle. Throughout, we set $\rho = 0$ and we consider $r_c = 0$ and $r_c = 0.1$. We consider the latter value because it lies slightly above the upper bound of the 95 percent probability interval associated with the posterior distribution for this parameter in our estimation below. For robustness, we also consider a version of (3.4) in which the monetary authority reacts to current inflation.

We do not have a strong prior about the value of ϕ (see section 2.3 above), so we consider two values, $\phi = 1$ and $\phi = 0.1$. To have a sense of the appropriate value of γ , we follow Basu (1995). He argues, using manufacturing data, that the share of materials in gross output is roughly 1/2. Recall that the steady state of our model coincides with the solution to (2.23), so that

$$\frac{i}{c+i} = 1 - \gamma.$$

Thus, Basu's empirical finding suggests a value for γ in a neighborhood of 1/2.⁹ The instrumental variables results in Ravenna and Walsh (2006) suggests that a value of ψ in a neighborhood of unity is consistent with the data.

Figure 1 displays our results. The upper row of figures provides results for the case in (3.4), in which the policy authority reacts to the one-quarter-ahead expectation of inflation, $E_t \hat{\pi}_{t+1}$. The lower row of figures corresponds to the case where the policy maker responds instead to current inflation, $\hat{\pi}_t$. The horizontal and vertical axes indicate a range of values for γ and ψ , respectively. The grey areas correspond to the parameter values where one or both of λ_i , $i = 1, 2$ are less than unity in absolute value. Technically, the steady state equilibrium of the economy is said to be 'indeterminate' for parameterizations in the grey area. Intuitively, the grey area corresponds to parameterizations of our economy in which the Taylor

⁹Actually, this is a conservative estimate of γ . Had we not selected ν to extinguish monopoly power in the steady state, our estimate of γ would have been lower. See Basu (1995) for more discussion of this point.

principle does not stabilize inflation expectations. The white areas in the figures correspond to parameterizations where the Taylor principle successfully stabilizes the economy.

Consider the upper two set of charts in Figure 1 first. Note that in each case, $\psi = 0$ and $\gamma = 1$ are points in the white area, consistent with the discussion above. However, for very small positive values of ψ , we see that we are into the grey area. Moreover, this is true regardless of the value of γ . For these parameterizations the aggressive response of the interest rate to higher inflation expectations only produces the higher inflation that people anticipate. We can see in the right two figures of the first row, that $r_c > 0$ greatly reduces the extent of the grey area. Still, for $\gamma = 0.5$ and ψ near unity we are still in the grey area and the Taylor principle does not work.

Now consider the bottom row of graphs. Note that in all cases, if $\gamma = 1$ then we are always in the white area. That is, for the economy to be vulnerable to self-fulfilling expectations, it must not only be that there is a substantial working capital channel, but it must also be that materials are a substantial fraction of gross output. The 2,2 graph shows that with $\gamma = 0.5$, $\phi = 0.1$ and ψ above roughly 0.6, we are in the grey area. When ϕ is substantially higher, the 2,1 graph indicates the grey area is smaller. Note that with $r_c > 0$, the grey area has almost shrunk to zero, according to the 2,3 and 2,4 graphs.

We conclude from this analysis that in the presence of a working capital channel, sharply raising the interest rate in response to higher inflation could actually be counterproductive. This is more likely to be the case when the share of materials inputs in gross output is high. When this is so, one cannot rely exclusively on the Taylor principle to ensure stable inflation and output performance. In the example, responding strongly to the output gap could restore stability. However, in practice the output gap is hard to measure and so presumably responding to other variables that are correlated with the output gap is desirable. Of course, this requires a more extended model than the one we have here. Still, we hope that the discussion illustrates how DSGE models can help place structure on thinking about the design of monetary policy.

3.2. Monetary Policy and Boom-Busts

In recent years, there has been extensive discussion about the interaction of monetary policy and economic volatility, in particular, asset price volatility. Prior to the recent financial turmoil, a consensus had developed that monetary policy should not actively seek to stabilize asset prices. The view was that in any case, a serious commitment to inflation targeting, one that implements the Taylor principle, would stabilize asset markets automatically.¹⁰ The idea is that an asset price boom is basically a demand boom, the presumption being

¹⁰See Bernanke and Gertler (2000).

that the boom is driven by optimism about the future, and not primarily by current actual developments. A boom that is driven by demand should - according to the conventional wisdom - raise production costs and, hence, inflation. The monetary authority that reacts vigorously to inflation then automatically raises interest rates and helps to stabilize asset prices.

When this scenario is evaluated in the classic New Keynesian model, we find that the boom is not necessarily associated with a rise in prices. In fact, if the optimism about the future concerns the expectations about cost saving new technologies, forward-looking price setters may actually reduce their prices. This is the finding of Christiano, Ilut, Motto and Rostagno (2007), which we briefly summarize here.

To capture the notion of optimism about the future, suppose that the time series representation of the log-level of technology is as follows:

$$\log z_t = \rho_z \log z_{t-1} + u_t, \quad u_t = \varepsilon_t + \xi_{t-1}, \quad (3.7)$$

so that the steady state of z_t is unity. In (3.7), u_t is an iid shock, uncorrelated with past $\log z_t$. The innovation in technology growth, u_t , is the sum of two orthogonal processes, ε_t and ξ_{t-1} . The time subscript on these two variables represents the date when they are known to private agents. Thus, at time $t - 1$ agents come to know of a piece of u_t , namely ξ_{t-1} . At time t they learn the rest, ε_t . For example, the initial ‘news’ about u_t , ξ_{t-1} , could in principle be entirely false, as would be the case when $\varepsilon_t = -\xi_{t-1}$.

Substituting (3.7) into (3.1):

$$\hat{R}_t^* = E_t [\log z_{t+1} - \log z_t] = (\rho_z - 1) \log z_t + \xi_t, \quad (3.8)$$

where $\gamma = 1$ since we now consider the classic New Keynesian model.¹¹ Our system of equilibrium conditions is (3.8) with (3.2), (3.3) and (3.4). We set $\alpha_\psi = 0$ (i.e., no working capital channel) and $r_c = \rho = 0$. We adopt the following parameter values:

$$\beta = 0.99, \quad \phi = 1, \quad r_\pi = 1.5, \quad \rho_z = 0.9, \quad \xi_p = 0.75.$$

We perform a simulation in which news arrives in period t that technology will jump one percent in period $t + 1$, i.e., $\xi_t = 0.01$. The value of ε_t is set to zero. The period t response

¹¹To see why we replaced $\hat{\mu}_{z,t+1}$ in (3.1) $\log z_{t+1} - \log z_t$, note first

$$\hat{\mu}_{z,t} = \frac{\mu_{z,t} - \mu_z}{\mu_z} = \mu_{z,t} - 1,$$

because in steady state $\mu_z \equiv z_t/z_{t-1} = 1/1 = 1$. Then,

$$1 + \hat{\mu}_{z,t} = \mu_{z,t}$$

Take the log of both sides and note, $\log \mu_{z,t} = \log (1 + \hat{\mu}_{z,t}) \simeq \hat{\mu}_{z,t}$. But, $\log \mu_{z,t} = \log z_t - \log z_{t-1}$.

of hours worked is positive 1 percent. This rise is entirely inefficient because in the first best equilibrium hours does not respond at all to a technology shock, whether it occurs in the present or it is expected to occur in the future (see (2.23)). Interestingly, inflation falls in period t by 10 basis points, at an annual rate.¹² Current marginal cost does rise (see (2.29)), but the expected future fall in marginal cost is why current inflation drops.

The efficient monetary policy sets $\hat{R}_t = \hat{R}_t^*$ which, according to (??), means the interest rate should rise when a positive signal about the economy occurs. A policy that applies the Taylor principle in this example moves policy in exactly the wrong direction in response to ξ_t . By responding to the fall in inflation, policy not only does not raise the interest rate - as it should - but it actually reduces the interest rate in response to the fall in inflation. By reducing the interest rate in the period of a positive signal about the future, policy over stimulates the economy and thereby creates excessive volatility.

So, the classic New Keynesian model can be used to challenge the conventional wisdom that an inflation-fighting central bank automatically moderates economic volatility. But, is this just an abstract example without any relevance? In fact, the typical boom-bust episode is characterized by low or falling inflation (see Adalid and Detken (2007)). For example, during the US booms of the 1920s and the 1980s and 1990s, inflation was low. This fact turns the conventional wisdom on its head and leads to a conclusion that matches that of our numerical example: an inflation-fighting central bank amplifies boom/bust episodes.

A full evaluation of the ideas in this subsection requires a more elaborate model, preferably one with financial variables such as the stock market. In this way, one could assess the impact on a broader set of variables in boom/bust episodes. In addition, one could evaluate what other variables the monetary authority might look at in order to avoid contributing to the type of volatility described in this example. We presume that it is not helpful to simply say that the monetary authority should set $\hat{R}_t = \hat{R}_t^*$, because in practice this may require more information than is actually available. Instead, the objective should be to find variables that are correlated with $\hat{R}_t^*$, so that these may be included in the monetary policy rule. For further discussion of these issues, see Christiano, Ilut, Motto and Rostagno (2007).

3.3. Ramsey Equilibrium

The previous two subsections studied the Taylor principle, once with and once without, the working capital channel. Here we consider the working capital channel in a Ramsey optimal equilibrium. This equilibrium is defined as the best equilibrium that is possible, given private sector optimization and market clearing. We study the Ramsey equilibrium for three reasons. First, we develop the result of Ravenna and Walsh (2006), that the working capital channel

¹²Because inflation is zero in steady state, $\hat{\pi}_t = \pi_t - 1$. This was converted to annualized basis points by multiplying by 40,000.

introduces an interesting trade-off between inflation and output stabilization. By establishing the existence of this conflict in the context of the Ramsey optimal policy, it is clear that the conflict is not an artifact of the details of some particular policy rule. The trade-off that the existence of the working capital channel creates is interesting because the classic New Keynesian model has difficulty articulating such a trade-off. In practice, researchers wanting to study a trade-off between inflation and output stabilization in the classic New Keynesian model are forced to appeal to disturbances in λ_f , the degree of monopoly power. These disturbances show up as an additive term in the Phillips curve, but are difficult to motivate. With the working capital channel one in effect also obtains an additive term in the Phillips curve (see (3.2)), but perhaps this way of introducing such a term has a more appealing motivation.

A second reason for considering the Ramsey equilibrium is that it represents an interesting benchmark for evaluating monetary policy rules. A third reason is that a version of the Ramsey equilibrium corresponds to ‘flexible inflation targeting’, a policy applied at least implicitly in many central banks and explicitly in some, such as the Swedish central bank. Although it is beyond the scope of this paper to provide a full discussion of the use of Ramsey equilibrium in the implementation of monetary policy, some key issues are addressed here. For example, we provide intuition about the nature of temptations to deviate from the Ramsey optimal policy that exist according to the New Keynesian model (the ‘time inconsistency problem’). The model is capable of articulating the classic time inconsistency problem in monetary economics, the one emphasized by Kydland and Prescott (1977) and Barro and Gordon (1983a, 1983b). In addition, there is a time inconsistency problem that specifically arises because of the assumed frictions in setting prices in the New Keynesian model.

Typically, a tax subsidy is included among the policymaker’s instruments in a Ramsey equilibrium. We show that the treatment of this subsidy is very important for the properties of the Ramsey equilibrium. For example, the Ramsey equilibrium that obtains when there are no restrictions on the way the tax subsidy is set is not only first best, but it is also time consistent. If restrictions are placed on what the policy maker can do with the tax subsidy, then the Ramsey equilibrium outcomes are no longer first best or time consistent. We discuss these issues with the use of numerical examples. We also use the example to illustrate why the New Keynesian model motivates price level targeting, as opposed to inflation targeting, as a desirable objective for monetary policy.

3.3.1. Definition of the Ramsey Equilibrium

Recall that the private sector optimization and market clearing conditions in section 2.1 supply us with 11 equations in the 9 unknowns listed in (2.22). Thus, the system at this

point is under-determined. There are many configurations of the variables in (2.22) which satisfy the private sector equilibrium conditions. The Ramsey-optimal equilibrium is the configuration of those variables that maximizes the utility of the representative household. In Lagrangian form, the Ramsey equilibrium allocations solve:

$$\begin{aligned}
& \max_{c_t, H_t, i_t, R_t, \pi_t, p_t^*, K_t^f, F_t^f, \nu_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \frac{H_t^{1+\phi}}{1+\phi} \right. \\
& + \lambda_{1t} \left[1 - \beta E_t \frac{c_t}{c_{t+1} \mu_{z,t+1}^{\frac{1}{\gamma}}} \frac{R_t}{\pi_{t+1}} \right] \\
& + \lambda_{2t} \left[-K_t^f + \lambda_f (1 - \psi + \psi R_t) (1 - \nu_t) \left(\frac{1}{1-\gamma} \right)^{1-\gamma} \left(\frac{c_t H_t^\phi}{\gamma} \right)^\gamma + \beta \xi_p E_t \pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}} K_{t+1}^f \right] \\
& + \lambda_{3t} \left[-F_t^f + 1 + \beta \xi_p E_t \pi_{t+1}^{\frac{1}{\lambda_f-1}} F_{t+1}^f \right] \\
& + \lambda_{4t} \left[K_t^f - F_t^f \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p} \right)^{-(\lambda_f-1)} \right] \\
& + \lambda_{5,t} \left[c_t H_t^\phi - \frac{\gamma}{1-\gamma} \frac{i_t}{H_t} \right] \\
& + \lambda_{6,t} \left[-\frac{1}{p_t^*} + (1 - \xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p} \right)^{\lambda_f} + \xi_p \frac{\pi_t^{\frac{\lambda_f}{\lambda_f-1}}}{p_{t-1}^*} \right] \\
& \left. + \lambda_{7,t} [c_t + i_t - p_t^* H_t^\gamma i_t^{1-\gamma}] \right\}.
\end{aligned} \tag{3.9}$$

In (3.9) we have dropped two private sector equilibrium conditions by using (2.3) to substitute out for W_t/P_t and (2.9) to substitute out for s_t . Also, we have written consumption and materials input in scaled form using (2.24).

3.3.2. A Simplification of the Ramsey Equilibrium

At first sight, the Ramsey problem, (3.9), may appear formidable. However, on closer inspection it turns out to be quite simple. This is because we can conjecture that the first four restrictions are non-binding. To see this, note that we can think of the first, third and fourth restrictions as simply defining R_t , F_t^f and K_t^f , respectively. Finally, we can think of the second restriction as defining ν_t . Note that the block of variables, R_t , F_t^f , K_t^f and ν_t do not enter the system anywhere else, and so our setting them does not restrict our ability to optimize the welfare criterion. Formally, the conjecture that the first four equations are non-binding on the problem can easily be verified ex post. Rewriting (3.9) using our conjecture,

$$\lambda_{1,t} = \dots = \lambda_{4,t} = 0 :$$

$$\begin{aligned} & \max_{c_t, H_t, i_t, \pi_t, p_t^*} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - \frac{H_t^{1+\phi}}{1+\phi} \right. \\ & + \lambda_{5,t} \left[c_t H_t^\phi - \frac{\gamma}{1-\gamma} \frac{i_t}{H_t} \right] \\ & + \lambda_{6,t} \left[-\frac{1}{p_t^*} + (1-\xi_p) \left(\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p} \right)^{\lambda_f} + \xi_p \frac{\pi_t^{\frac{\lambda_f}{\lambda_f-1}}}{p_{t-1}^*} \right] \\ & \left. + \lambda_{7,t} [-c_t + p_t^* H_t^\gamma i_t^{1-\gamma} - i_t] \right\}. \end{aligned} \quad (3.10)$$

This problem is considerably less daunting than (3.9). Two observations deserve to be made about this problem. First, note that the law of motion of the price distortions remains a restriction on the system. This should not be surprising. In effect, the Ramsey equilibrium problem has two degrees of freedom (i.e., two more variables than equilibrium conditions) and this is not enough to make all intermediate good firm prices identical in period t , if there was dispersion in the previous period, p_{t-1}^* . The second observation is that under our conjecture, all the equilibrium conditions with forward-looking variables disappear from the system. As a result, we can expect that the Ramsey-optimal equilibrium is time-consistent. That is, the solution to the problem resolved at some future date, $t+j$, is just the continuation of the solution to the problem at date t , for all $j > 0$.

The first order conditions with respect to c_t , H_t and i_t , respectively, are:

$$\lambda_{7,t} = \frac{1}{c_t} + \lambda_{5,t} H_t^\phi \quad (3.11)$$

$$0 = -H_t^\phi + \lambda_{5,t} \left[c_t \phi H_t^{\phi-1} + \frac{\gamma}{1-\gamma} \frac{i_t}{H_t^2} \right] + \lambda_{7,t} \gamma p_t^* H_t^{\gamma-1} i_t^{1-\gamma} \quad (3.12)$$

$$0 = -\lambda_{5,t} \frac{\gamma}{1-\gamma} \frac{1}{H_t} + \lambda_{7,t} [(1-\gamma) p_t^* H_t^\gamma i_t^{1-\gamma} - 1] \quad (3.13)$$

The first order conditions with respect to π_t and p_t^* , respectively, are, after some rearranging,

$$\frac{\pi_t}{p_{t-1}^*} = \left(\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p} \right)^{\lambda_f-1} \quad (3.14)$$

$$\frac{\lambda_{6,t}}{(p_t^*)^2} + \lambda_{7,t} H_t^\gamma i_t^{1-\gamma} = \beta \lambda_{6,t+1} \frac{\lambda_f}{\lambda_f-1} \xi_p \frac{\pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}}}{(p_t^*)^2}. \quad (3.15)$$

The law of motion for p_t^* , the equilibrium condition associated with $\lambda_{6,t}$, is simplified by substituting out for inflation using (3.14). Doing so we obtain, after some algebra,

$$p_t^* = \left[1 - \xi_p + \xi_p (p_{t-1}^*)^{\frac{1}{\lambda_f-1}} \right]^{\lambda_f-1}. \quad (3.16)$$

From (3.16), we see that in a Ramsey equilibrium price distortions are eliminated gradually over time. Interestingly, during this convergence to steady state, price distortions are not affected by the shocks included in our analysis. Also, equation (3.16) has a unique steady state at unity, where it has a slope equal to $\xi_p < 1$. It follows that (3.16) is a (locally) stable difference equation and that p_t^* converges to unity from below. To see the implications for inflation, substitute (3.16) into (3.14) to obtain:

$$\pi_t = \frac{p_{t-1}^*}{p_t^*}. \quad (3.17)$$

Thus, as the price distortions converge to unity from below, inflation converges to zero from below too.

Given the solution for p_t^* , it remains to solve for $\lambda_{5,t}$, $\lambda_{7,t}$, c_t , H_t , i_t using (3.11), (3.12), (3.13) and the restrictions associated with $\lambda_{5,t}$ and $\lambda_{7,t}$:

$$\begin{aligned} \lambda_{5,t} &= 0, \quad \lambda_{7,t} = \frac{1}{c_t}, \\ H_t &= 1, \quad i_t = [(1 - \gamma) p_t^*]^{\frac{1}{\gamma}}, \quad c_t = \frac{\gamma}{1 - \gamma} [(1 - \gamma) p_t^*]^{\frac{1}{\gamma}}. \end{aligned} \quad (3.18)$$

Note that yet one more multiplier is zero. To verify that the restriction associated with $\lambda_{5,t}$ is non-binding, simply verify that the restriction is satisfied by the indicated solutions for c_t , i_t and H_t .¹³

3.3.3. Properties of the Ramsey Equilibrium

There are several things worth noting about the Ramsey equilibrium. First, the allocations in Ramsey equilibrium coincide with the solution to the following problem:

$$\max_{\{c_t, H_t, i_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \frac{H_t^{1+\phi}}{1+\phi} \right]$$

subject to

$$c_t + i_t = p_t^* H_t^\gamma i_t^{1-\gamma}.$$

That is, the allocations in the Ramsey equilibrium solve the planning problem for an economy in which p_t^* is treated as an exogenous shock to technology which evolves according to (3.16). The effects of monopoly power and the working capital channel are extinguished in each period of the Ramsey equilibrium, and the misallocation of resources due to price dispersion disappears asymptotically. Second, a closely related observation is that the markup

¹³It is easily verified that there exist stochastic processes for R_t , K_t^f , F_t^f , ν_t such that the equilibrium conditions associated with $\lambda_{i,t}$, $i = 1, \dots, 4$ are satisfied.

of marginal cost over the aggregate price level is unity when evaluated at the allocations in the Ramsey equilibrium:

$$\left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{c_t H_t^\phi}{\gamma}\right)^\gamma \frac{1}{p_t^*} = 1. \quad (3.19)$$

Third, it is instructive to examine the subsidy, ν_t , in a Ramsey equilibrium. Consider the equilibrium condition associated with $\lambda_{2,t}$:

$$K_t^f = \lambda_f (1 - \psi + \psi R_t) (1 - \nu_t) \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{c_t H_t^\phi}{\gamma}\right)^\gamma + \beta \xi_p E_t \pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}} K_{t+1}^f \quad (3.20)$$

Combining (3.14) with the equilibrium condition on $\lambda_{6,t}$, we obtain $K_t^f = F_t^f p_t^*$. Use this to substitute out for K_t^f and K_{t+1}^f in (3.20), divide both sides by p_t^* and use (3.17), (3.19) to obtain:

$$F_t^f = \lambda_f (1 - \psi + \psi R_t) (1 - \nu_t) + \beta \xi_p E_t \pi_{t+1}^{\frac{1}{\lambda_f-1}} F_{t+1}^f.$$

This expression and the fact that the condition on $\lambda_{3,t}$ must be satisfied in a Ramsey equilibrium implies:

$$\lambda_f (1 - \psi + \psi R_t) (1 - \nu_t) = 1. \quad (3.21)$$

Thus, in the Ramsey equilibrium, the subsidy extinguishes the working capital distortion as well as the monopoly distortion.

A third observation about the Ramsey equilibrium is that its basic properties are very sensitive to the treatment of the subsidy rate, ν_t , and to the presence of the working capital channel. For example, if the subsidy rate is a constant that extinguishes monopoly power and there is no working capital channel (i.e., $\psi = 0$), then our conjectures about the multipliers remain valid. In this case, the Ramsey equilibrium is time consistent and our analytic representation of the solution given in (3.16), (3.17) and (3.18) is correct. Suppose we maintain the assumption that the subsidy rate is constant, but we introduce the working capital channel by setting $\psi > 0$. Now our conjecture, $\lambda_{2,t} = 0$, ceases to be valid and our the analytic solution is not relevant. Moreover, the introduction of the working capital channel causes the Ramsey equilibrium to be time inconsistent. This is because with the multipliers non-zero, forward-looking variables enter the problem in a non-trivial way (see (3.9)). This case is discussed in the next subsection.

3.3.4. Ramsey Equilibrium and Time Inconsistency

The previous section showed that when the subsidy rate is allowed to vary with time and is chosen optimally, then the Ramsey equilibrium is time consistent. This feature of the Ramsey equilibrium is convenient for expository purposes, but it is not robust. In practice,

one expects the Ramsey equilibrium to be time inconsistent. In this subsection we briefly discuss the sources of time inconsistency in the New Keynesian model.

We consider two examples. The first example illustrates a form of time inconsistency that does not exist in the steady state of the Ramsey equilibrium, but which does exist along a dynamic path. This form of time inconsistency is novel to the sticky price model. This example also illustrates the sense in which the presence of the working capital channel creates a trade off between output and inflation stabilization. The second example shows how the classic time inconsistency problem in monetary economics emerges in the New Keynesian model. Both examples are derived by placing different restrictions on the subsidy rate, ν_t .

Consider the case where the subsidy rate is restricted to be the constant value that enforces (3.21) in steady state. That is, the subsidy rate is chosen to extinguish the effects of monopoly power and working capital in the steady state of the Ramsey equilibrium. For the example to be interesting we must suppose there is a nontrivial working capital channel, $\psi > 0$, for otherwise the discussion of the previous section shows the Ramsey equilibrium is time consistent. Because the steady state is efficient, there is nothing to be gained from deviating and so the steady state allocations are time consistent. However, the equilibrium out of steady state is not time consistent. In our numerical example, we compute the Ramsey equilibrium at time $t = 0$, and we recompute it at date $t = 1$. As a benchmark, we include the calculations for the case in which ν_t is variable and chosen optimally (i.e., (3.21) is satisfied at each t). In all cases, we suppose that $p_{-1}^* = 1$ and we specify the law of motion of $\mu_{z,t}$ as follows:

$$\log(\mu_{z,t}) = 0.5 \log(\mu_{z,t-1}) + \varepsilon_t^z,$$

where ε_t^z is a mean-zero, iid process. We set $\log(\mu_{z,-1}) = 0$, $\varepsilon_0^z = 0.01$, $\varepsilon_t^z = 0$ for $t > 0$. Thus, technology, z_t , jumps in period $t = 0$ by 1 percent, and continues to rise until it asymptotes at a level permanently higher by 2 percent. After the initial jump in technology, the state of technology, $\log z_t$, follows a deterministic path up to its new, permanently higher value. The other model parameters are set as follows:

$$\psi = 0.7, \gamma = 0.5, \phi = 1, \beta = 0.99.$$

According to these parameter values, intermediate good firms must finance 70% of their inputs in advance, a substantial fraction of their gross output is used by other firms as materials inputs and the Frisch labor supply elasticity is unity.

Figure 2 displays our results. The upper left panel displays the response of $\log z_t$ to the shock, expressed in deviation from the constant path it would have followed in the absence of the shock (the deviation is multiplied by 100 so that the results are in percent terms). The other figures display the response of inflation, the nominal rate of interest and employment. Inflation and the nominal rate of interest are expressed in net terms and multiplied by 40,000

so that they are in annual, basis point terms. Employment is expressed as 100 times the log deviation of employment from its constant steady state.

The line with solid circles displays the results based on the benchmark experiment, when the subsidy rate is variable and chosen optimally.¹⁴ Inflation and hours worked do not deviate from their steady state values, as our analytic solution implies. Note that the interest rate rises substantially in response to the shock because this is necessary to reconcile households with the prospective rise in consumption in Ramsey equilibrium. We then re-solved the Ramsey equilibrium at $t = 1$ and found that hours worked, inflation and the interest rate are the continuation of the equilibrium computed in period $t = 0$. This is a manifestation of our finding that the Ramsey equilibrium with ν_t unrestricted is time consistent.

Now consider the case in which the subsidy rate is set to the constant value that extinguishes the monopoly and working capital distortions in steady state.¹⁵ The continuous line in Figure 2 indicates the resulting restricted Ramsey equilibrium as of date $t = 0$. The thin line with circles corresponds to the Ramsey equilibrium when that equilibrium is recomputed at the start of $t = 1$. Consider the $t = 0$ equilibrium first. In this case, the interest rate rises by a smaller amount than in the benchmark case. This is because the higher interest rate from the benchmark Ramsey equilibrium now introduces a distortion through the working capital channel. This distortion is the reason for the drop in employment.¹⁶ In addition, inflation rises in the $t = 0$ equilibrium because the rise in the rate of interest drives up marginal cost. Interestingly, inflation returns to steady state from below in the $t = 0$ equilibrium. In effect, inflation overshoots its steady state target of zero.

Consider the Ramsey equilibrium computed at time $t = 1$. Note that inflation is not as

¹⁴The computations were done using Dynare, version 4. We entered the equilibrium conditions associated with $\lambda_{i,t}$, $i = 1, \dots, 7$ in (3.9), and the utility function. The Dynare command, `Ramsey_policy`, launches the calculations. We set the subsidy rate to satisfy (3.21) for each t . This required replacing the relevant terms in the constraint on $\lambda_{2,t}$ with unity. Dynare then computes the 8 first order necessary conditions for optimality of the Ramsey problem. This gives a total of 15 equations, including the 7 constraints. These equations are used to solve for the 15 endogenous variables, $c_t, N_t, i_t, R_t, \pi_t, p_t^*, K_{p,t}, F_{p,t}$, plus the 7 multipliers. Dynare first computes the steady state values of these 15 variables, and then linearizes the 15 equations about steady state. The results reported in Figure 1 are the solution to the linearized system of 15 equations. This linearized system makes use of the second order derivatives of our model, because it involves linearization of the Ramsey first order conditions. This is why, for example, our solution takes into account the cost of price dispersion, even though the discussion below shows that to a first order approximation, p_t^* is a constant.

¹⁵To implement this, we replace the expression in (3.9),

$$\frac{\varepsilon}{\varepsilon - 1} (1 - \psi + \psi R_t) (1 - \nu_t),$$

with

$$\frac{1 - \psi + \psi R_t}{1 - \psi + \psi R},$$

where R denotes the steady state value of R_t .

¹⁶When $\psi = 1$, so that 100% of input costs must be financed in advance, then the impact effect of the technology shock on employment is zero. Thereafter, employment falls.

close to zero as it is projected to be in the $t = 0$ equilibrium. The reason for this inconsistency in the Ramsey plan is simple. At time $t = 0$ the Ramsey equilibrium takes into account the impact on $t = 0$ prices of the announcement about $t = 1$ inflation. At $t = 0$ it is desirable to create the expectation that inflation will be close to zero in $t = 1$ in order to minimize the welfare-reducing rise in the price level in period $t = 0$. This reasoning also explains why inflation is projected overshoot its steady state. The commitment to eventually restore the price level to roughly its pre-shock level helps to reduce the incentive of intermediate good producers to change their price in period $t = 0$. At time $t = 1$ prices set at $t = 0$ represent ‘water under the bridge’, and they are no longer have any impact on the Ramsey planner’s choice of inflation at time $t = 1$. This is why the plan as of period $t = 1$ is not the continuation of the $t = 0$ plan.

Note that in the preceding discussion we have explained the sense in which the sticky price model makes a case for price level targeting. The commitment to bring the price level roughly back to its pre-shock level has the effect of reducing the impact of a shock on the price level.¹⁷

Although the simple example in section 3.3 has a Ramsey equilibrium characterized by time consistency, the previous discussion shows that this property is not robust. The example can be used to illustrate additional reasons why the Ramsey equilibrium might be time inconsistent. For example, suppose we drop the subsidy rate altogether, by setting $\nu = 0$. In this case, output and employment are below their first best levels in the steady state of the Ramsey equilibrium. Since the nominal rate of interest now represents a distortion in the steady state, the Ramsey equilibrium is characterized by a nominal rate of interest that is somewhat less than $1/\beta$ in steady state. Because the household’s intertemporal Euler equation must hold at all dates (see the condition associated with $\lambda_{1,t}$ in (3.9)), this implies that inflation is negative in the Ramsey steady state. Because the Ramsey planner now has only one degree of freedom (i.e., an excess of only one variable over the number of equilibrium conditions) and several barriers to first best (i.e., monopoly power, working capital channel and inflation) the Ramsey equilibrium necessarily represents a compromise. The Ramsey planner who restarts the Ramsey plan in effect acquires another degree of freedom for getting close to the first best allocations. In the case where output is below first best in steady state, the Ramsey planner has an incentive to restart the plan with an interest rate cut and a jump in inflation. Relative to the previously announced Ramsey plan, the interest rate cut and inflation jump represent surprises. The Ramsey planner’s incentive to implement the surprise is to move the economy closer to first best. This particular source of

¹⁷In the $t = 0$ Ramsey equilibrium, $\pi_0 = P_0/P_{-1} = 1.0123$, where P_{-1} denotes the price level in period $t = -1$, the period before the shock. The cumulative product of the inflation rates, $\pi_0\pi_1 \times \dots \times \pi_t = P_t/P_{-1}$. In the experiment, P_t/P_{-1} reaches a maximum of 1.0168 in $t = 2$ and then declines monotonically thereafter. By period 40 P_t/P_{-1} has roughly converged, at 0.9986.

time inconsistency is the classic one emphasized by Kydland and Prescott (1977) and Barro and Gordon (1983a, 1983b) and many others.

Thus, in general the Ramsey equilibrium is time inconsistent. For the Ramsey equilibrium to actually occur in practice, policy makers must resist the temptation to deviate by restarting the Ramsey plan and ignoring commitments made in previous periods.¹⁸ Deviations from past promises would in practice soon be detected, in which case Ramsey would cease to be the relevant equilibrium concept. The equilibrium concepts that are relevant to the case where future commitments are not expected to be honored imply the possibility of highly suboptimal outcomes.¹⁹

3.3.5. Working Capital and the Trade Off Between Output and Price Stabilization

The previous discussion used the simulations displayed in Figure 2 to discuss the time inconsistency problems that arise in New Keynesian models. We now use the two $t = 0$ simulations to describe Ravenna and Walsh (2006)'s point that the introduction of a working capital channel creates a trade-off between output and price stabilization. We are free to think that in both simulations the subsidy rate, ν , is constant and extinguishes monopoly power and any working capital channel in steady state. We can think of the line with solid circles in Figure 2 as indicating the Ramsey equilibrium when there is no working capital channel. In this case, the Ramsey optimal policy stabilizes both employment and inflation in the wake of a shock to technology. There is evidently no trade-off in this case.

Now consider the continuous line, the one corresponding to the economy with the working capital channel. Now, the rise in the interest rate that is meant to stop a surge in consumption from expanding employment, drives inflation up. In addition, the rise in the interest rate drives employment down because of the working capital channel.

4. Medium-Sized DSGE Model

We develop a version of the model in Christiano, Eichenbaum and Evans (2005) (CEE). We describe the objectives and constraints of the agents in the model, and leave the derivation of the equilibrium conditions to the appendix. Each feature of the model is motivated by our desire to match inertial response of inflation to a monetary policy shock. However, when we investigate the ability of that model to quantitatively match our estimate of that response, we also investigate its ability to match estimated dynamic responses to two technology shocks.

¹⁸See Woodford (2003) for a discussion of the ‘timeless perspective’ in which a Ramsey equilibrium is implemented in a way that honors past promises.

¹⁹See Barro and Gordon (1983a, 1983b), Chari, Christiano and Eichenbaum (1998), and Albanesi, Chari and Christiano (2003).

4.1. Goods Production

An aggregate homogeneous good is produced using the technology, (2.5). The first order condition of the representative, competitive producer of the homogeneous good is given by (2.6). Substituting this first order condition back into (2.5) yields the restriction across prices, (2.7). Each intermediate good, $i \in (0, 1)$, is produced by a monopolist who treats (2.6) as its demand curve. The intermediate good producer treats the aggregate quantities, P_t and Y_t as exogenous.

We use a production function for intermediate good producers that is standard in the literature. It does not use materials inputs, but it does use the services of capital, $K_{i,t}$:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} K_{i,t}^\alpha - z_t^+ \varphi. \quad (4.1)$$

Here, z_t is a technology shock whose logarithmic first difference has a positive mean and φ denotes a fixed production cost. The economy has two sources of growth: the positive drift in $\log(z_t)$ and a positive drift in $\log(\Psi_t)$, where Ψ_t is the state of an investment specific technology shock discussed below. The object, z_t^+ , in (4.1) is defined as follows:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Along a non-stochastic steady state growth path, Y_t/z_t^+ and $Y_{i,t}/z_t^+$ converge to constants.

The two shocks, z_t and Ψ_t , are specified to be unit root processes in order to be consistent with the assumptions we use in our VAR analysis to identify the dynamic response of the economy to neutral and investment specific technology shocks. We adopt the following time series representations for the shocks:

$$\Delta \log z_t = \mu_z + \varepsilon_t^z, \quad E(\varepsilon_t^z)^2 = (\sigma_z)^2 \quad (4.2)$$

$$\Delta \log \Psi_t = \mu_\psi + \rho_\psi \Delta \log \Psi_{t-1} + \varepsilon_t^\psi, \quad E(\varepsilon_t^\psi)^2 = (\sigma_\psi)^2. \quad (4.3)$$

Our assumption that the neutral technology shock follows a random walk with drift matches closely the finding in Smets and Wouters (2007) who estimate $\log z_t$ to be highly autocorrelated. The direct empirical analysis of Prescott (1986) also supports the notion that $\log z_t$ is a random walk with drift. Finally, Fernald (2009) constructs a direct estimate of total factor productivity growth for the business sector. The first order autocorrelation of quarterly observations covering the period 1947Q2 to 2009Q3 is 0.0034, consistent with the idea of a random walk.

We assume that there is no entry or exit by intermediate good producers. The no entry assumption would be implausible if firms enjoyed large and persistent profits. The fixed cost in (4.1) is introduced to minimize the incentive to enter. We set φ so that intermediate good producer profits are zero in steady state. This requires that the fixed cost grows at the same

rate as the growth rate of economic output, and this is why φ is multiplied by z_t^+ in (4.1). A potential empirical advantage of including fixed costs of production is that, by introducing some increasing returns to scale, the model can in principle account for evidence that labor productivity rises in the wake of a positive monetary policy shock.

In (4.1), $H_{i,t}$ denotes homogeneous labor services hired by the i^{th} intermediate good producer. Firms must borrow the wage bill, so that one unit of labor costs is given by

$$W_t R_t. \tag{4.4}$$

Here, W_t denotes the aggregate wage rate and R_t denotes the gross nominal interest rate on working capital loans. The assumption that firms require working capital was introduced by CEE as a way to help dampen the rise in inflation after an expansionary shock to monetary policy. An expansionary shock to monetary policy drives R_t down and - other things the same - this reduces firm marginal cost. Inflation is dampened because marginal cost is the key input into firms' price-setting decision. Indirect evidence consistent with the working capital assumption includes the frequently-found VAR-based results, suggesting that inflation drops for a little while after a positive monetary policy shock. It is hard to think of an alternative to the working capital assumption to explain this evidence, apart from the possibility that the estimated response reflects some kind of econometric specification error.²⁰

Another motivation for treating interest rates as part of the cost of production has to do with the 'dis-inflationary boom' critique made by Ball (1994) of models that do not include interest rates in costs. Ball's critique focuses on the Phillips curve in (2.26), which we reproduce here for convenience:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{s}_t,$$

where $\hat{\pi}_t$ and $\hat{s}_t$ denote inflation and marginal cost, respectively. Also, $\kappa > 0$ is a reduced form parameter and β is slightly less than unity. According to the Phillips curve, if the monetary authority announces it will fight inflation by strategies which (plausibly) bring down future inflation more than present inflation, then $\hat{s}_t$ must jump. In simple models $\hat{s}_t$ is directly related to the volume of output (see, e.g., (2.29)). High output requires more intense utilization of scarce resources, their price goes up, driving up marginal cost, $\hat{s}_t$. Ball criticized theories that do not include the interest rate in marginal cost on the grounds that we do not observe booms during disinflations. Including the interest rate in marginal cost potentially avoids the Ball critique because the high $\hat{s}_t$ may simply reflect the high interest rate that corresponds to the disinflationary policy, and not higher output.

²⁰This possibility was suggested by Sims (1992) and explored further in Christiano, Eichenbaum and Evans (1999).

We adopt the Calvo model of price frictions. With probability ξ_p , the intermediate good firm cannot reoptimize its price, in which case it is assumed to set its price according to the following rule:²¹

$$P_{i,t} = \pi P_{i,t-1}. \quad (4.5)$$

Note that in steady state, firms that do not optimize price raise prices at the general rate of inflation. Firms that do optimize price in a steady state growth path choose to also raise their price at the steady state rate of inflation. This is a key reason why all firms' prices are the same in the steady state of the model. According to the discussion near (2.25), the fact that we analyze the first order approximation of DSGE model in a neighborhood of steady state means that we can impose the analog of $p_t^* = 1$.

With probability $1 - \xi_p$ the intermediate good firm can reoptimize its price. Apart from the fixed cost, the i^{th} intermediate good producer's profits are the analog of (2.11):

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} [P_{i,t+j} Y_{i,t+j} - s_{t+j} P_{t+j} Y_{i,t+j}],$$

where s_t denotes the marginal cost of production, denominated in units of the homogeneous good. The object, s_t , is a function only of the costs of capital and labor, and is described in Appendix C. Marginal cost is independent of the level of $Y_{i,t}$ because of the linear homogeneity of the first expression on the right of (4.1). The first order necessary conditions associated with this optimization problem are reported in Appendix E.

Goods market clearing dictates that the homogeneous output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t + \tilde{I}_t. \quad (4.6)$$

Here, C_t denotes household consumption, G_t denotes exogenous government consumption and $\tilde{I}_t$ is a homogenous investment good which is defined as follows:

$$\tilde{I}_t = \frac{1}{\Psi_t} (I_t + a(u_t) \bar{K}_t). \quad (4.7)$$

The investment goods, I_t , are used by households to add to the physical stock of capital, $\bar{K}_t$.²² The remaining investment goods are used to cover maintenance costs, $a(u_t) \bar{K}_t$, arising from capital utilization, u_t . The cost function, $a(\cdot)$, is increasing and convex, and has the property that in steady state, $u_t = 1$ and $a(1) = 0$. The relationship between the utilization of capital, u_t , capital services, K_t , and the physical stock of capital, $\bar{K}_t$, is as follows:

$$K_t = u_t \bar{K}_t.$$

²¹Equation (4.5) excludes the possibility that firms index to past inflation. We discuss the reason for this specification in section 6.2.2 below.

²²The notation, I_t , used here should not be confused with materials inputs in section . Our medium-sized DSGE model does not include materials inputs.

The investment and capital utilization decisions are discussed in section 4.2. See section 4.4 below for the functional form of the capital utilization cost function. Finally, Ψ_t in (4.7) denotes the unit root investment specific technology shock in (4.3).

4.2. Households

4.2.1. Preferences

There is a continuum of households indexed by $j \in (0, 1)$. The j^{th} household has the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - bC_{t-1}) - A_L \frac{(h_{j,t+i})^{1+\phi}}{1+\phi} \right], \quad A_L, \phi > 0, \quad \beta \in (0, 1). \quad (4.8)$$

Here, $h_{j,t}$ denotes the quantity of the j^{th} type of labor service supplied. The j^{th} household is the sole supplier of this type of labor service, which is imperfectly substitutable with the i^{th} household's labor service, for $i \neq j$. We discuss the household's participation in the labor market in the next subsection. In (4.8), C_t and C_{t-1} denote the j^{th} household's consumption at dates t and $t-1$, respectively. As explained below, it is the presence of the appropriate insurance markets which guarantees that individual household consumption is independent of $j \in (0, 1)$.

The presence of $b > 0$ in (4.8) is motivated by VAR-based evidence like that displayed below, which suggests that a positive monetary policy triggers (i) a hump-shape response in consumption and (ii) a persistent reduction in the real rate of interest.²³ With $b = 0$ and a utility function separable in labor and consumption like the one above, (i) and (ii) are difficult to reconcile. A positive monetary policy shock that triggers an increase in expected future consumption would be associated with rise in the real rate of interest, not a fall. Alternatively, a fall in the real interest rate would cause people to rearrange consumption intertemporally, so that consumption is relatively high right after the monetary shock and low later. Intuitively, one can reconcile (i) and (ii) by supposing the marginal utility of consumption is inversely proportional not to the level of consumption, but to its derivative. To see this, it is useful to recall the familiar intertemporal Euler equation implied by household optimization (see, e.g., (2.4)):

$$E_t \beta \frac{u_{c,t+1}}{u_{c,t}} \frac{R_t}{\pi_{t+1}} = 1.$$

Here, $u_{c,t}$ denotes the marginal utility of consumption at time t . From this expression, we see that a low R_t/π_{t+1} tends to produce a high $u_{c,t+1}/u_{c,t}$, i.e., a rising trajectory for the marginal utility of consumption. This illustrates the problematic implication of the model when $u_{c,t}$ is inversely proportional to C_t as in (4.8) with $b = 0$. To fix this implication we

²³The intuition in this paragraph was originates with Fuhrer (2000).

need a model change which has the property that a rising $u_{c,t}$ path implies hump-shape consumption. A hump-shaped consumption path corresponds to a scenario in which the slope of the consumption path is falling, suggesting that (i) and (ii) can be reconciled if $u_{c,t}$ is proportional to the slope of consumption. The notion that marginal utility is inversely proportional to the slope of consumption corresponds loosely to $b > 0$.²⁴ The fact that (i) and (ii) can be reconciled with the assumption of habit persistence is of special interest, because there is evidence from other places that also favors the assumption of habit persistence, for example in asset pricing (see, for example, Constantinedes (1990) and Boldrin, Christiano and Fisher (2001)) and growth (see Carroll et al. (1997, 2000)). In addition, there may be a solid foundation in psychology for this specification of preferences.²⁵

The logic associated with the intertemporal Euler equation above suggests that there are other ways to reconcile (i) and (ii). For example, Guerron-Quintana (2008) shows that non-separability between consumption and labor in (4.8) can help reconcile (i) and (ii). He points out that if the marginal utility of consumption is an increasing function of labor and the model predicts that employment rises with a hump shape after a positive monetary injection, then it is possible that consumption itself rises with a hump-shape.

4.2.2. Wage Setting by Households

The model incorporates Calvo-style wage setting frictions along the lines spelled out in Erceg, Henderson and Levin (2000). Because wages are an important component of costs, wage setting frictions help slow the response of inflation to a monetary policy shock. As in the case of prices, wage setting frictions require that there be market power. To make sure there is not too much market power, we follow Erceg, Henderson and Levin (2000) in adopting the Dixit-Stiglitz type framework used in the context of price-setting. The many households with specialized labor inputs, $h_{j,t}$ in (4.8) correspond to the many intermediate good firms producing specialized inputs.

²⁴In particular, suppose first that lagged consumption in (4.8) represents aggregate, economy wide consumption and $b > 0$. This corresponds to the so-called ‘external habit’ case, where it is the lagged consumption of others that enters utility. In that case, the marginal utility of household C_t is $1/(C_t - bC_{t-1})$, which corresponds to the inverse of the slope of the consumption path, at least if b is large enough. In our model we think of C_{t-1} as corresponding to the household’s own lagged consumption (that’s why we use the same notation for current and lagged consumption), the so-called ‘internal habit’ case. In this case, the marginal utility of C_t also involves future terms, in addition to the inverse of the slope of consumption from $t = 1$ to t . The intuition described in the text, which implicitly assumed external habit, also applies roughly to the external habit case that we consider.

²⁵Anyone who has gone swimming has experienced the psychological aspect of habit persistence. It is usually very hard at first to jump into the water because it seems so cold. The swimmer who jumps (or is pushed!) into the water after much procrastination initially experiences a tremendous shock with the sudden drop in temperature. However, after only a few minutes the new, lower temperature is perfectly comfortable. In this way, the lagged temperature seems to influence one’s experience of current temperature, as in habit persistence.

We suppose that, with probability $1 - \xi_w$, the j^{th} worker is able to reoptimize its wage and with probability ξ_w that worker sets $W_{j,t}$ according to the following rule:

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \quad (4.9)$$

$$\tilde{\pi}_{w,t+1} = \pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z+}, \quad (4.10)$$

where $\kappa_w \in (0, 1)$. Note that in steady state, non-optimizing workers raise their real wage at the rate of growth of the economy. Because optimizing workers also do this in steady state, it follows that in the steady state, the wage of each type of worker is the same.

To understand the problem of the $1 - \xi_w$ households which have the opportunity to reoptimize their wage, it is useful to understand the source of labor demand. We suppose that the labor power hired by intermediate good firms is homogeneous labor that is ‘produced’ by competitive labor contractors. Labor contractors produce homogeneous labor by aggregating the different types of specialized labor, $j \in (0, 1)$, as follows:

$$H_t = \left[\int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \quad 1 \leq \lambda_w < \infty. \quad (4.11)$$

Labor contractors take the wage rate of H_t and $h_{t,j}$ as given and equal to W_t and $W_{t,j}$, respectively. Profit maximization by labor contractors leads to the following first order necessary condition:

$$W_{j,t} = W_t \left(\frac{H_t}{h_{t,j}} \right)^{\frac{\lambda_w - 1}{\lambda_w}}. \quad (4.12)$$

Equation (4.12) is the demand curve for the j^{th} household’s type of labor. We assume that this demand curve must be satisfied at each point in time, whether or not the household has the opportunity to reoptimize its wage. In considering (4.12), the j^{th} household correctly treats H_t and W_t as given and beyond its control.²⁶

In principle, the idiosyncratic experiences of individual households will, over time, cause them to have different wealth holdings and therefore also different levels of consumption. Under these circumstances, aggregate economic outcomes may be dependent on the distribution of wealth across households. If so, then the distribution of wealth and the law of motion of that distribution must be solved for as part of the solution of the model. In practice, it is probably the case that solving a model of the size considered in this paper is infeasible when there is non-trivial heterogeneity among households. For this reason, we follow Erceg, Henderson and Levin (2000) in adopting the assumption that there are insurance markets on the realization of the Calvo uncertainty determining whether the household can or cannot adjust its wage.

²⁶Substituting out for $h_{t,j}$ in (4.11) using (4.12) we obtain an expression relating W_t to $W_{j,t}$ for $j \in (0, 1)$ that is analogous to (2.7).

It is possible, however, that the heterogeneity induced by idiosyncratic uncertainty in the setting of wages may only have a negligible impact on aggregate outcomes. This possibility is suggested by the recent literature on versions of our model in which intermediate good firms have a state variable, such as the capital stock or a stock of employment.²⁷ In these models, idiosyncratic uncertainty about the timing of price reoptimization gives rise to a distribution across firms of their state variable. In principle, this matters for aggregate outcomes in the same way that idiosyncratic uncertainty about the timing of wage reoptimization on the part of households might matter. In the context of intermediate good firms, Woodford (2004) has shown that as long as (i) the variables in a stochastic equilibrium are not too far from their value in non-stochastic steady state and (ii) agents in the non-stochastic steady state are identical, then standard linearization methods can be applied and the details of the distribution of firms by their state variable do not matter for economic aggregates.²⁸ The intuition is simple. Condition (i) guarantees that individual firm decision rules are well approximated by linearizing about the steady state and condition (ii) guarantees that all those decision rules have the same intercept and slope coefficients. To understand how this guarantees that the details of the microeconomic distribution of state variables does not matter, consider the old-fashioned Keynesian consumption function:

$$C = \alpha + \beta Y,$$

where C denotes consumption and Y denotes household income. In reality, different households have different levels of income and in principle poor households have different β 's than rich ones. If this were the case, then a simple relationship like the one above relating aggregate consumption to aggregate income would not exist: to predict C one would have to know not only aggregate Y , but also how it is distributed among rich and poor people. However, if everyone - poor and rich alike - all had consumption functions with the same slope and intercept terms, then aggregate consumption would be determined from aggregate income as in the old-fashioned Keynesian consumption function. This insight has been applied with success to models in which firms have idiosyncratic state variables, and it may also work in models like the present one in which households have different wealth levels because of the effects of idiosyncratic realizations of the ability to set wages. A challenge for the approach would be to ensure condition (i). This requires that the model incorporate forces that pull the distribution of wealth across households back together after a disturbance has pulled it apart.

²⁷See ACEL and Thomas (2009).

²⁸A write-up of the method in a simplified model appears in Christiano (2004). An application in a medium-scaled model with size approximating the size of our model appears in ACEL. See also Sveen and Weinke (2005) for an alternative strategy for solving a model with firm-specific factors. For another study that uses the assumption of firm-specific capital, see de Walque, Smets and Wouters (2006).

4.2.3. Accumulation of Capital

The household owns the economy's physical stock of capital, sets the utilization rate of capital and rents the services of capital in a competitive market. The household accumulates capital using the following technology:

$$\bar{K}_{t+1} = (1 - \delta) \bar{K}_t + F(I_t, I_{t-1}) + \Delta_t, \quad (4.13)$$

where Δ_t denotes physical capital purchased in a market with other households. Since all households are the same in terms of capital accumulation decisions, $\Delta_t = 0$ in equilibrium. We nevertheless include Δ_t so that we can assign a price to installed capital. In (4.13), $\delta \in (0, 1)$ and we use the specification suggested in CEE:

$$F(I_t, I_{t-1}) = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t, \quad (4.14)$$

where the functional form, S , that we use is described in section 4.4. In (4.14), $S = S' = 0$ and $S'' > 0$ along a nonstochastic steady state growth path.

Let $P_t P_{k',t}$ denote the nominal market price of Δ_t . For each unit of $\bar{K}_{t+1}$ acquired in period t , the household receives X_{t+1}^k in net cash payments in period $t + 1$:

$$X_{t+1}^k = u_{t+1} P_{t+1} r_{t+1}^k - \frac{P_{t+1}}{\Psi_{t+1}} a(u_{t+1}). \quad (4.15)$$

The first term is the gross nominal period $t + 1$ rental income from a unit of $\bar{K}_{t+1}$. The second term represents the cost of capital utilization, $a(u_{t+1}) P_{t+1} / \Psi_{t+1}$. Here, P_{t+1} / Ψ_{t+1} is the nominal price of the investment goods absorbed by capital utilization. That P_{t+1} / Ψ_{t+1} is the equilibrium market price of investment goods follows from the technology specified in (4.6) and (4.7), and the assumption that investment goods are produced from homogeneous output goods by competitive firms.

The introduction of variable capital utilization is motivated by a desire to explain the slow response of inflation to a monetary policy shock. In the baseline model, prices are heavily influenced by costs. These in turn are influenced by the elasticity of the factors of production. If factors can be rapidly expanded with a small rise in cost, then inflation will not rise much after a monetary policy shock. Allowing for variable capital utilization is a way to make the services of capital elastic. If there is very little curvature in the a function, then households are able to expand capital services without much increase in cost.

The form of the investment adjustment costs in (4.13) is motivated by a desire to reproduce VAR-based evidence that investment has a hump-shaped response to a monetary policy shock. Alternative specifications include $F \equiv I_t$ and

$$F = I_t - \frac{S''}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t. \quad (4.16)$$

Specification (4.16) has a long history in macroeconomics, and has been in use since at least Lucas and Prescott (1971). To understand why DSGE models generally use the adjustment cost specification in (4.14) rather than (4.16), it is useful to define the rate of return on investment:

$$R_{t+1}^k = \frac{x_{t+1}^k + \left[1 - \delta + S'' \left(\frac{I_{t+1}}{\bar{K}_{t+1}} - \delta \right) \frac{I_{t+1}}{\bar{K}_{t+1}} - \frac{S''}{2} \left(\frac{I_{t+1}}{\bar{K}_{t+1}} - \delta \right)^2 \right] P_{k',t+1}}{P_{k',t}}. \quad (4.17)$$

The numerator is the one-period payoff from an extra unit of $\bar{K}_{t+1}$, and the denominator is the corresponding cost, both in consumption units. In (4.17), $x_{t+1}^k \equiv X_{t+1}^k/P_{t+1}$ denotes the earnings net of costs. The term in square brackets is the quantity of additional $\bar{K}_{t+2}$ made possible by the additional unit of $\bar{K}_{t+1}$. This is composed of the undepreciated part of $\bar{K}_{t+1}$ left over after production in period $t+1$, plus the impact of $\bar{K}_{t+1}$ on $\bar{K}_{t+2}$ via the adjustment costs. The object in square brackets is converted to consumption units using $P_{k',t+1}$, which is the market price of $\bar{K}_{t+2}$ denominated in consumption goods. Finally, the denominator is the price of the extra unit of $\bar{K}_{t+1}$.

The price of extra capital, in competitive markets corresponds to the marginal cost of production. Thus,

$$\begin{aligned} P_{k',t} &= -\frac{dC_t}{d\bar{K}_{t+1}} = -\frac{dC_t}{dI_t} \times \frac{dI_t}{d\bar{K}_{t+1}} \\ &= \frac{1}{\Psi_t} \left[\frac{1}{\frac{d\bar{K}_{t+1}}{dI_t}} \right] = \frac{1}{\Psi_t} \left\{ \frac{1}{1 - S'' \times \left(\frac{I_t}{\bar{K}_t} - \delta \right)} \quad \begin{array}{l} F = I \\ F \text{ in (4.16)} \end{array} \right\}. \end{aligned} \quad (4.18)$$

The derivatives in the first line correspond to marginal rates of technical transformation. The marginal rate of technical transformation between consumption and investment is implicit in (4.6) and (4.7). The marginal rate of technical transformation between I_t and $\bar{K}_{t+1}$ is given by the capital accumulation equation. The relation in the second line of (4.18) is referred to as ‘Tobin’s q ’ relation, where Tobin’s q here corresponds to $\Psi_t P_{k',t}$. This is the market value of capital divided by the price of investment goods. Here, q can differ from unity due to the investment adjustment costs.

We are now in a position to convey the intuition about why DSGE models have generally abandoned the specification in (4.16) in favor of (4.13). The key reason has to do with VAR-based evidence that suggests the real interest rate falls persistently after a positive monetary policy shock, while investment responds in a hump-shaped pattern. Any model that is capable of producing this type of response will have the property that the real return on capital, (4.17) - for arbitrage reasons - also falls after an expansionary monetary policy shock. Suppose, to begin, that $S'' = 0$, so that there are no adjustment costs at all and $P_{k',t} = 1$. In this case, the only component in R_t^k that can fall is x_{t+1}^k , which is dominated

by the marginal product of capital. That is, approximately, the rate of return on capital is:

$$K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} + 1 - \delta.$$

In steady state this object is $1/\beta$ (ignoring growth), which is roughly 1.03 in annual terms. At the same time, the object, $1 - \delta$, is roughly 0.9 in annual terms, so that the endogenous part of the rate of return of capital is a very small part of that rate of return. As a result, any given drop in the return on capital requires a very large percentage drop in the endogenous part, $K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}$. An expansion in investment can accomplish this, but it has to be a very substantial surge. To see this, note that the endogenous part of the rate of return is not only small, but the capital stock receives a weight substantially less than unity in that expression. Moreover, a model that successfully reproduces the VAR-based evidence that employment rises after a positive monetary policy implies that hours worked rises. This pushes the endogenous component up, increasing the burden on the capital stock to bring down the rate of return on investment. For these reasons, models without adjustment costs generally imply a counterfactually strong surge in investment in the wake of a positive shock to monetary policy.

With $S'' > 0$ the endogenous component of the rate of return on capital is much larger. However, in practice models that adopt the adjustment cost specification, (4.16), generally imply that the biggest investment response occurs in the period of the shock, and not later. To gain intuition into why this is so, suppose the contrary: that investment does exhibit a hump-shape response in investment. Equation (4.18) implies a similar hump-shape pattern in the price of capital, $P_{k',t}$.²⁹ This is because that $P_{k',t}$ is primarily determined by the contemporaneous flow of investment. So, under our supposition about the investment response, a positive the monetary policy shock generates a rise in $P_{k',t+1}/P_{k',t}$ over at least several periods in the future. According to (4.17), this creates the expectation of future capital gains, $P_{k',t+1}/P_{k',t} > 1$ and increases the immediate response of the rate of return on capital. Thus, households would be induced to substitute away from a hump-shaped response, towards one in which the immediate response is much stronger. In practice, this means that in equilibrium, the biggest response of investment occurs in the period of the shock, with later responses converging to zero.

The adjustment costs in (4.14) do have the implication that investment responds in a hump-shaped manner. The reason is (4.14)'s implication that a quick rise in investment from previous levels is expensive.

²⁹Note from (4.18) that the price of capital increases as investment rises above its level in steady state, which is the level required to just meet the depreciation in the capital stock. Our assertion that the price of capital follows the same hump-shaped pattern as investment after a positive monetary policy shock reflects our implicit assumption that the shock occurs when the economy is in a steady state. This will be true on average, but not at each date.

There are other reasons to take the specification in (4.14) seriously. Lucca (2006) and Matsuyama (1984) have described interesting theoretical foundations which produce (4.14) as a reduced form. For example, in Matsuyama, shifting production between goods and capital involves a learning by doing process, which makes quick movements in either direction expensive. Also, Matsuyama explains how the abundance of empirical evidence that appears to reject (4.16) may be consistent with (4.14). Consistent with (4.14), Topel and Rosen (1988) argues that data on housing construction cannot be understood without using a cost function that involves the change in the flow of housing construction.

4.2.4. Household Optimization Problem

The j^{th} household's period t budget constraint is as follows:

$$P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) + B_{t+1} + P_t P_{k',t} \Delta_t \leq W_{t,j} h_{t,j} dj + X_t^k \bar{K}_t + R_{t-1} B_t + a_{jt} \quad (4.19)$$

where $W_{t,j}$ represents the wage earned by the j^{th} household, B_{t+1} denotes the quantity of risk-free bonds purchased by the household, R_t denotes the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t , and a_{jt} denotes the payments and receipts associated with the insurance on the timing of wage reoptimization. The household's problem is to select sequences, $\{C_t, I_t, \Delta_t, W_{t,j}, B_{t+1}, \bar{K}_{t+1}\}$, to maximize (4.8) subject to (4.12), (4.9), (4.10), (4.13), (4.15), (4.19) and the mechanism determining when wages can be reoptimized.

4.3. Fiscal and Monetary Authorities, and Equilibrium

We suppose that monetary policy follows a Taylor rule of the following form:

$$\log \left(\frac{R_t}{R} \right) = \rho_R \log \left(\frac{R_{t-1}}{R} \right) + (1 - \rho_R) \left[r_\pi \log \left(\frac{\pi_{t+1}}{\pi} \right) + r_y \log \left(\frac{gdp_t}{gdp} \right) \right] + \varepsilon_{R,t}, \quad (4.20)$$

where $\varepsilon_{R,t}$ denotes an iid shock to monetary policy. As in CEE and ACEL, we assume that the period t realization of $\varepsilon_{R,t}$ is not included in the period t information set of the agents in our model. This ensures that our model satisfies the restrictions used in the VAR analysis to identify a monetary policy shock. In (4.20), gdp_t denotes scaled real GDP defined as follows:

$$gdp_t = \frac{G_t + C_t + I_t}{z_t^+}. \quad (4.21)$$

In (4.21), gdp denotes the nonstochastic steady state value of gdp_t and G_t denotes government consumption. We adopt the model of government consumption suggested in Christiano and Eichenbaum (1992):

$$G_t = g z_t^+.$$

In principle, g could be a random variable, though our focus in this paper is just on monetary policy and technology shocks. So, we set g to a constant. Lump-sum transfers are assumed to balance the government budget.

An equilibrium is a stochastic process for the prices and quantities which has the property that the household and firm problems are satisfied, and goods and labor markets clear.

4.4. Adjustment Cost Functions

We adopt the following functional forms. The capacity utilization cost function is:

$$a(u) = 0.5b\sigma_a u^2 + b(1 - \sigma_a)u + b((\sigma_a/2) - 1), \quad (4.22)$$

where b is selected so that $a(1) = a'(1) = 0$ in steady state and σ_a is a parameter that controls the curvature of the cost function. The closer σ_a is to zero, the less curvature there is and the easier it is to change utilization. The investment adjustment cost function takes the following form:

$$\begin{aligned} S(x_t) &= \frac{1}{2} \left\{ \exp \left[\sqrt{S''} (x_t - \mu_z + \mu_\Psi) \right] + \exp \left[-\sqrt{S''} (x_t - \mu_z + \mu_\Psi) \right] - 2 \right\}, \\ &= 0, \quad x = \mu_z + \mu_\Psi. \end{aligned} \quad (4.23)$$

where $x_t = I_t/I_{t-1}$ and $\mu_z + \mu_\Psi$ is the growth rate of investment in steady state. With this adjustment cost function, $S(\mu_z + \mu_\Psi) = S'(\mu_z + \mu_\Psi) = 0$. Also, $S'' > 0$ is a parameter having the property that it is the second derivative of $S(x_t)$ evaluated at $x_t = \mu_z + \mu_\Psi$. Because of the nature of the above adjustment cost functions, the curvature parameters have no impact on the model's steady state.

5. Estimation Strategy

Our estimation strategy is a Bayesian version of the two-step impulse response matching approach applied by Rotemberg and Woodford (1997) and CEE. We begin with a discussion of the two steps. After that, we discuss the computation of a particular weighting matrix used in the analysis.

5.1. VAR Step

We estimate the dynamic responses of a set of aggregate variables to three shocks, using standard vector autoregression methods. The three shocks are the monetary policy shock, the innovation to the permanent technology shock, z_t , and the innovation to the investment specific technology shock, Ψ_t . The contemporaneous and 14 lagged responses to each of $N = 9$ macroeconomic variables to the three shocks are stacked in a vector, $\hat{\psi}$. These

macroeconomic variables are a subset of the variables that appear in the VAR. The additional variables in our VAR pertain to the labor market. We use this augmented VAR in order to facilitate comparison between the analysis in this manuscript and in other research of ours which integrates labor market frictions into the monetary DSGE model.³⁰ The Y_t vector of variables in the VAR is:

$$\underbrace{Y_t}_{14 \times 1} = \begin{pmatrix} \Delta \ln(\text{relative price of investment}_t) \\ \Delta \ln(\text{real } GDP_t/\text{hours}_t) \\ \Delta \ln(GDP \text{ deflator}_t) \\ \text{unemployment rate}_t \\ \text{capacity utilization}_t \\ \ln(\text{hours}_t) \\ \ln(\text{real } GDP_t/\text{hours}_t) - \ln(W_t/P_t) \\ \ln(\text{nominal } C_t/\text{nominal } GDP_t) \\ \ln(\text{nominal } I_t/\text{nominal } GDP_t) \\ \text{vacancies}_t \\ \text{job separation rate}_t \\ \text{job finding rate}_t \\ \log(\text{hours}_t/\text{labor force}_t) \\ \text{Federal Funds Rate}_t \end{pmatrix} \quad (5.1)$$

An extensive general review of identification in VAR's appears in Christiano, Eichenbaum and Evans (1999). The specific technical details of how we simultaneously identify the responses to all three structural shocks in our model appear in ACEL. We estimate a two-lag VAR using data that are quarterly and seasonally adjusted and cover the period 1951Q1 to 2008Q4. Our identification assumptions are as follows. The only variable that the monetary policy shock affects contemporaneously is the Federal Funds Rate. We make two assumptions to identify the dynamic response to the technology shocks: (i) the only shocks that affect labor productivity in the long run are the two technology shocks and (ii) the only shock that affects the price of investment relative to consumption is the innovation to the investment specific shock. All these identification assumptions are satisfied in our model. Details of our strategy for computing impulse response functions imposing the shock identification are reported in ACEL.³¹

Our data set extends over a long range, while we estimate a single set of impulse response functions and model parameters. In effect, we suppose that there has been no parameter

³⁰See Christiano, Trabandt and Walentin (2010a, 2010b).

³¹The identification assumption for the monetary policy shock by itself imposes no restriction on the VAR parameters. Similarly, Fisher (2006) showed that the identification assumptions for the technology shocks when applied without simultaneously applying the monetary shock identification, also imposes no restriction on the VAR parameters. However, ACEL showed that when all the identification assumptions are imposed at the same time, then there are restrictions on the VAR parameters. We found that the test of the overidentifying restrictions on the VAR fails to reject the null hypothesis that the restrictions are satisfied at 5 percent critical level.

break over this long period. Whether or not there has been a break is a question that has been debated. For example, it has been argued that the parameters of the monetary policy rule have not been constant over this period. We do not review this debate here. Implicitly, our analysis sides with the conclusions of those that argue that the evidence of parameter breaks is not strong. For example, Sims and Zha (2006) argue that the evidence is consistent with the idea that monetary policy rule parameters have been unchanged over the sample. Christiano, Eichenbaum and Evans (1999) argue that the evidence is consistent with the proposition that the dynamic effects of a monetary policy shock have not changed during this sample. Standard lag-length selection criteria led us to work with a VAR with 2 lags.³²

The number of elements in $\hat{\psi}$ corresponds to the number of impulses estimated. Since we consider $n = 15$ lags in the impulses, there are in principle 3 (i.e., the number of shocks) times 9 (number of variables) times 15 (number of lags) = 405 elements in $\hat{\psi}$. However, we do not include in $\hat{\psi}$ the 8 contemporaneous responses to the monetary policy shock that are required to be zero by our monetary policy identifying assumption. Taking the latter into account, the vector $\hat{\psi}$ has 387 elements.

According to standard classical asymptotic sampling theory, when the number of observations, T , is large, we have

$$\sqrt{T} \left(\hat{\psi} - \psi(\theta_0) \right) \overset{a}{\sim} N(0, W(\theta_0, \zeta_0)),$$

where θ_0 represents the true values of the parameters that we estimate. The vector, ζ_0 , denotes the true values of the parameters of the shocks that are in the model, but that we do not formally include in the analysis. We find it convenient to express the asymptotic distribution of $\hat{\psi}$ in the following form:

$$\hat{\psi} \overset{a}{\sim} N(\psi(\theta_0), V(\theta_0, \zeta_0, T)), \quad (5.2)$$

where

$$V(\theta_0, \zeta_0, T) \equiv \frac{W(\theta_0, \zeta_0)}{T}.$$

5.2. Impulse Response Matching Step

In the second step of our analysis, we treat $\hat{\psi}$ as ‘data’ and we choose a value of θ to make $\psi(\theta)$ as close as possible to $\hat{\psi}$. We give our strategy an approximate Bayesian interpretation.³³ This interpretation uses (5.2) to define an approximate likelihood of the data, $\hat{\psi}$, as a function

³²We considered VAR specifications with lag length 1, 2, ..., 12. The Schwartz and Hannan-Quinn criteria indicate that a single lag in the VAR is sufficient. The Akaike criterion indicates 12 lags, though we discounted that result. Later, we investigate the sensitivity of our results to lag length.

³³Our approach follows in the spirit of Chernozhukov and Hong (2003).

of θ :

$$f\left(\hat{\psi}|\theta, V\left(\theta_0, \zeta_0, T\right)\right) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V\left(\theta_0, \zeta_0, T\right)|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}\left(\hat{\psi}-\psi\left(\theta\right)\right)' V\left(\theta_0, \zeta_0, T\right)^{-1}\left(\hat{\psi}-\psi\left(\theta\right)\right)\right]. \quad (5.3)$$

As we explain below, we treat the true value of $V\left(\theta_0, \zeta_0, T\right)$ as a known object. Under these circumstances, the value of θ that maximizes the above function represents an approximate maximum likelihood estimator of θ . It is approximate for two reasons: (i) the central limit theorem underlying (5.2) only holds exactly as $T \rightarrow \infty$ and (ii) the value of $V\left(\theta_0, \zeta_0, T\right)$ that we use is guaranteed to be correct only for $T \rightarrow \infty$. Interestingly, our approximation does not require (as in standard Bayesian analysis, which works with a Normal likelihood) that the data underlying the VAR, Y_t , be Normal. This is an advantage of the method, because the Normality assumption is not a good one for macroeconomic variables (see Christiano (2007)).

Treating the function, f , as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior of θ conditional on $\hat{\psi}$ and $V\left(\theta_0, \zeta_0, T\right)$ is:

$$f\left(\theta|\hat{\psi}, V\left(\theta_0, \zeta_0, T\right)\right) = \frac{f\left(\hat{\psi}|\theta, V\left(\theta_0, \zeta_0, T\right)\right) p\left(\theta\right)}{f\left(\hat{\psi}|V\left(\theta_0, \zeta_0, T\right)\right)}, \quad (5.4)$$

where $p\left(\theta\right)$ denotes the priors on θ and $f\left(\hat{\psi}|V\left(\theta_0, \zeta_0, T\right)\right)$ denotes the marginal density of $\hat{\psi}$:

$$f\left(\hat{\psi}|V\left(\theta_0, \zeta_0, T\right)\right) = \int f\left(\hat{\psi}|\theta, V\left(\theta_0, \zeta_0, T\right)\right) p\left(\theta\right) d\theta.$$

As usual, the mode of the posterior distribution of θ can be computed by simply maximizing the value of the numerator in (5.4), since the denominator is not a function of θ . The marginal density of $\hat{\psi}$ is required when we want an overall measure of the fit of our model and when we want to report the shape of the posterior marginal distribution of individual elements in θ . To compute the marginal likelihood, we can use a standard random walk metropolis algorithm or a Laplace approximation. We explain the latter in section 5.4 below. The results that we report are based on a single Monte Carlo Markov Chain (MCMC) of length 600,000. The first 100,000 draws were dropped and the average acceptance rate in the chain is 27 percent. We confirmed that the chain is long enough so that all the statistics reported in the paper have converged.

5.3. Computation of $V\left(\theta_0, \zeta_0, T\right)$

A crucial ingredient in our empirical methodology is the matrix, $V\left(\theta_0, \zeta_0, T\right)$. The logic of our approach requires that we have an at least approximately consistent estimator of

$V(\theta_0, \zeta_0, T)$. A variety of approaches is possible here. We use a bootstrap approach. Using our estimated VAR and its fitted disturbances, we generate a set of M bootstrap realizations for the impulse responses. We denote these by ψ_i , $i = 1, \dots, M$, where ψ_i denotes the i^{th} realization of the 397×1 vector of impulse responses.³⁴ Consider

$$\bar{V} = \frac{1}{M} \sum_{i=1}^M (\psi_i - \bar{\psi}) (\psi_i - \bar{\psi})', \quad (5.5)$$

where $\bar{\psi}$ is the mean of ψ_i , $i = 1, \dots, M$. We set $M = 10,000$. The object, $\bar{V}$, is a 397 by 397 matrix, and we assume that the small sample (in the sense of T) properties of this way (or any other way) of estimating $V(\theta_0, \zeta_0, T)$ are poor. To improve small sample efficiency, we proceed in a way that is analogous to the strategy taken in the estimation of frequency-zero spectral densities (see Newey and West (1987)). In particular, rather than working with the raw variance-covariance matrix, $\bar{V}$, we instead work with $\hat{\bar{V}}$:

$$\hat{\bar{V}} = f(\bar{V}, T).$$

The transformation, f , has the property that it converges to the identity transform, as $T \rightarrow \infty$. In particular, $\hat{\bar{V}}$ damps some elements in $\bar{V}$, and the damping factor is removed as the sample grows large. The matrix, $\hat{\bar{V}}$, has on its diagonal, the diagonal elements of $\bar{V}$. The entries in $\hat{\bar{V}}$ that correspond to the correlation between the l^{th} lagged response and the j^{th} lagged response in a given variable to a given shock equals the corresponding entry in $\bar{V}$, multiplied by

$$\left[1 - \frac{|l-j|}{n} \right]^{\theta_{1,T}}, \quad l, j = 1, \dots, n.$$

Now consider the components of $\bar{V}$ that correspond to the correlations between components of different impulse response functions, either because a different variable is involved or because a different shock is involved, or both. We damp these entries in a way that damps more the greater is τ , the separation in time of the two impulses. In particular, we adopt the following damping factors for these entries:

$$\beta_T \left[1 - \frac{|\tau|}{n} \right]^{\theta_{2,T}}, \quad \tau = 0, 1, \dots, n.$$

We suppose that

$$\beta_T \rightarrow 1, \quad \theta_{i,T} \rightarrow 0, \quad T \rightarrow \infty, \quad i = 1, 2,$$

³⁴To compute a given bootstrap realization, ψ_i , we first simulate an artificial data set, $Y_1, \dots, Y_T$. We do this by simulating the response of our estimated VAR to an iid sequence of 14×1 shock vectors that are drawn randomly with replacement from the set of fitted shocks. We then fit a 2-lag VAR to the artificial data set using the same procedure used on the actual data. The resulting estimated VAR is then used to compute the impulse responses, which we stack into the 397×1 vector, ψ_i .

where the rate of convergence is whatever is required to ensure consistency of $\widehat{\bar{V}}$. These conditions leave completely open what values of β_T , $\theta_{1,T}$, $\theta_{2,T}$ we use in our sample. At one extreme, we have

$$\beta_T = 0, \theta_{1,T} = \infty,$$

and $\theta_{2,T}$ unrestricted. This corresponds to the approach in CEE and ACEL, in which $\widehat{\bar{V}}$ is simply a diagonal matrix composed of the diagonal components of $\bar{V}$. At the other extreme, we could set β_T , $\theta_{1,T}$, $\theta_{2,T}$ at their $T \rightarrow \infty$ values, in which $\widehat{\bar{V}} = \bar{V}$. Here, we work with the approach taken in CEE and ACEL. This has the important advantage of making our estimator particularly transparent. It corresponds to selecting θ so that the model implied impulse responses lie inside a confidence tunnel around the estimated impulses. When non-diagonal terms in $\bar{V}$ are also used, then the estimator aims not just to put the model impulses inside a confidence tunnel about the point estimates, but it is also concerned about the pattern of ‘misses’ across different impulse responses. Precisely how the off-diagonal components of $\bar{V}$ give rise to concerns about cross-impulse response patterns of misses is virtually impossible to understand intuitively. This is both because $\bar{V}$ is an enormous matrix and because it is not $\bar{V}$ itself that enters our criterion but its inverse.

5.4. Laplace Approximation

Because the likelihood we work with is only approximate, it is perhaps appropriate that we also work with an approximation to the posterior distribution. This is not essential, however, since Monte Carlo algorithms apply perfectly well in our setting, for computing marginal posteriors or θ and the marginal likelihood of $\hat{\psi}$.

To derive the Laplace approximation to $f(\theta|\hat{\psi}, V(\theta_0, \zeta_0, T))$, define

$$g(\theta) \equiv \log f(\hat{\psi}|\theta, V(\theta_0, \zeta_0, T)) + \log p(\theta).$$

Let θ^* denote the mode of the posterior distribution and define the following Hessian matrix:

$$g_{\theta\theta} = -\frac{\partial^2 g(\theta)}{\partial\theta\partial\theta'}|_{\theta=\theta^*}.$$

Note that the matrix, $g_{\theta\theta}$, is an automatic by-product of standard gradient methods for computing the mode, θ^* . The second order Taylor series expansion of g about $\theta = \theta^*$ is:

$$g(\theta) = g(\theta^*) - \frac{1}{2}(\theta - \theta^*)' g_{\theta\theta} (\theta - \theta^*),$$

where the slope term is zero if θ^* is an interior optimum, which we assume. Then,

$$f(\hat{\psi}|\theta, V(\theta_0, \zeta_0, T)) p(\theta) \approx f(\hat{\psi}|\theta^*, V(\theta_0, \zeta_0, T)) p(\theta^*) \exp \left[-\frac{1}{2}(\theta - \theta^*)' g_{\theta\theta} (\theta - \theta^*) \right].$$

Note:

$$\frac{1}{(2\pi)^{\frac{m}{2}}} |g_{\theta\theta}|^{\frac{1}{2}} \exp \left[-\frac{1}{2} (\theta - \theta^*)' g_{\theta\theta} (\theta - \theta^*) \right]$$

is the m -variable Normal distribution for the m random variables, θ , with mean θ^* and variance-covariance matrix, $g_{\theta\theta}^{-1}$. By the standard property of a density function,

$$\int \frac{1}{(2\pi)^{\frac{n}{2}}} |g_{\theta\theta}|^{\frac{1}{2}} \exp \left[-\frac{1}{2} (\theta - \theta^*)' g_{\theta\theta} (\theta - \theta^*) \right] d\theta = 1. \quad (5.6)$$

Bringing together the previous results, we obtain:

$$\begin{aligned} f(\hat{\psi}|V(\theta_0, \zeta_0, T)) &= \int f(\hat{\psi}|\theta, V(\theta_0, \zeta_0, T)) p(\theta) d\theta \\ &\approx \int f(\hat{\psi}|\theta^*, V(\theta_0, \zeta_0, T)) p(\theta^*) \exp \left[-\frac{1}{2} (\theta - \theta^*)' g_{\theta\theta} (\theta - \theta^*) \right] d\theta \\ &= (2\pi)^{\frac{n}{2}} |g_{\theta\theta}|^{-\frac{1}{2}} f(\hat{\psi}|\theta^*, V(\theta_0, \zeta_0, T)) p(\theta^*), \end{aligned}$$

by (5.6). We now have the marginal distribution for $\hat{\psi}$. We can use this to compare the fit of different models for $\hat{\psi}$. In addition, we have an approximation to the marginal posterior distribution for an arbitrary element of θ , say θ_i :

$$\theta_i \sim N(\theta_i^*, [g_{\theta\theta}^{-1}]_{ii}),$$

where $[A]_{ii}$ denotes the i^{th} diagonal element of the matrix, A .

6. Results

We first describe our VAR results. We then turn to the estimation of the DSGE model.

6.1. VAR Results

We briefly describe the impulse response functions implied by the VAR. The solid line in Figures 3-5 indicate the point estimates of the impulse response functions, while the gray area displays the corresponding two standard error probability bands. Inflation and the interest rate are in annualized percent terms, while the other variables are measured in percent. The solid lines with squares and the dashed lines will be discussed when we review the DSGE model estimation results.

6.1.1. Monetary Policy Shocks

We make five observations about the estimated dynamic responses to a 50 basis point shock to monetary policy, displayed in Figure 3. Consider first the response of inflation. Two

important things to note here are the price puzzle and the delayed and gradual response of inflation.³⁵ In the very short run the point estimates indicate that inflation moves in a seemingly perverse direction in response to the expansionary monetary policy shock. This transitory drop in inflation in the immediate aftermath of a monetary policy shock has been widely commented on, and has been dubbed the ‘price puzzle’. Christiano, Eichenbaum and Evans (1999) review the argument that the puzzle may be the outcome of the sort of econometric specification error suggested by Sims (1992), and find evidence that is consistent with that view. Here, we follow ACEL and CEE in taking the position that there is no econometric specification error. Although price puzzle is not statistically significant, it nevertheless deserves comment because it has potentially great economic significance. For example, the presence of a price puzzle in the data complicates the political problem associated with using high interest rates as a strategy to fight inflation. High interest rates and the consequent slowdown in economic growth is politically painful and if the public sees it producing higher inflation in the short run, support for the policy may evaporate unless the price puzzle has been explained.³⁶ Regarding the slow response of inflation, note how inflation reaches a peak after two years. Of course, the exact timing of the peak is not very well pinned down (note the wide confidence intervals). However, the evidence does suggest a sluggish response of inflation. This is consistent with the views of others, arrived at by other methods, about the slow response of inflation to a monetary policy shock.³⁷ It has been argued that this is a major puzzle for macroeconomics. For example, Mankiw (2000) argues that with price frictions of the type used here, the only way to explain the delayed and gradual response of inflation to a monetary policy shock is to introduce a degree of stickiness in prices that exceeds by far what can be justified based on the micro evidence. For this reason, when we study the ability of our models to match the estimated impulse response functions, we must be wary of the possibility that this is done only by making prices and wages very sticky. In

³⁵Here, we have borrowed Mankiw’s (2000) language, ‘delayed and gradual’, to characterize the nature of the response of inflation to a monetary policy shock. Though Mankiw wrote 10 years ago and he cites a wide range of evidence, Mankiw’s conclusion about how inflation responds to a monetary policy shock resembles our VAR evidence very closely. Mankiw argues that the response of inflation to a monetary policy shock is gradual in the sense that it does not peak for 9 quarters.

³⁶There is an important historical example of this political problem. In the early 1970s, at the start of the Great Inflation in the US, Arthur Burns was chairman of the US Federal Reserve and Wrigth Patman was chairman of the United States House Committee on Banking and Currency. Patman had the opinion that, by raising costs of production, high interest rates increase inflation. Patman’s belief had enormous significance because he was influential in writing the wage and price control legislation at the time. He threatened Burns that if Burns tried to raise interest rates to fight inflation, Patman would see to it that interest rates were brought under the control of the wage-price control board (see “The Lasting, Multiple Hassles of Topic A”, Time Magazine, Monday, April 9, 1973.).

³⁷For example, Mankiw (2000) cites Hume’s 1752 essay ‘Of Money’, in which Hume says that an increase in the money supply ‘..must first quicken the diligence of every individual, before it increases the price of labour.’

addition, we must be wary of the possibility that the econometrics leans too hard on other features (such as variable capital utilization) to explain the gradual and delayed response of inflation to a monetary policy shock.

The third observation is that output, consumption, investment and hours worked all display a slow, hump-shape response to a monetary policy shock, peaking a little over one year after the shock. As emphasized in section 4, these hump-shape observations are the reason that researchers introduce habit persistence and costs of adjustment in the flow of investment into the baseline model. In addition, note that the effect of the monetary shock on the interest rate is roughly gone after one year, yet the economy continues to respond well after that. This suggests that to understand the dynamic effects of a monetary policy shock, one must have a model that displays considerable sources of internal propagation.

A fourth observation concerns the response of capacity utilization. Recall from the discussion of section 4 that the magnitude of the empirical response of this variable represents an important discipline on the analysis. In effect, those data constrain how heavily we can lean on variable capital utilization to explain the slow response of inflation to a monetary policy shock. The evidence in Figure 3 suggests that capacity utilization responds very sharply to a positive monetary policy shock. For example, it rises three times as much as employment. In interpreting this finding, we must bear in mind that the capital utilization numbers we have are for the manufacturing sector. To the extent that the data are influenced by the durable part of manufacturing, they may overstate the volatility of capacity utilization generally in the economy.

Our fifth observation concerns the price of investment. In our model, this price is unaffected by shocks other than those to the technology for converting homogeneous output into investment goods. Figure 3 indicates that the price of investment rises in response to a positive shock to investment, contrary to our model. This suggests that it would be worth exploring modifications to the technology for producing investment goods so that the trade-off between consumption and investment is nonlinear.³⁸ Under these conditions, the rise in the investment to consumption ratio that appears to occur in response to an expansionary monetary policy shock would be associated with a rise in the price of investment.

³⁸For example, instead of specifying a resource constraint in which $C_t + I_t$ appears, we could adopt one in which C_t and I_t appear in a CES function, i.e.,

$$\left[a_1 C_t^{1/\rho} + a_2 I_t^{1/\rho} \right]^\rho.$$

The standard linear specification is a special case of this one, with $a_1 = a_2 = \rho = 1$.

6.1.2. Technology Shocks

Figures 4 and 5 display the responses to neutral and investment specific technology shocks. Overall, the confidence intervals are wide. The width of these confidence intervals should be no surprise in view of the nature of the question being addressed. The VAR is informed that there are two shocks in the data which have a long run effect on labor productivity, and it is being asked to determine the dynamic effects of these shocks on the data. To understand the challenge that such a question poses, imagine staring at a data plot and thinking how the technology shocks might be detected visually. It is no wonder that in many cases, the VAR response is, ‘I don’t know how this variable responds’. This is what the wide confidence intervals tell us. For example, nothing much can be said about the response of capacity utilization to a neutral technology shock.

Though confidence intervals are often wide there are some responses that are significant. For example, there is a significant rise in consumption, output, and hours worked in response to a neutral shock, according to these results. A particularly striking result in Figure 4 is the immediate drop in inflation in the wake of a positive shock to neutral technology. This drop has led some researchers to conjecture that the rapid response of inflation to a technology shock spells trouble for sticky price/sticky wage models. We investigate this conjecture in the next section.

6.2. Model Results

6.2.1. Parameters

Parameters whose values are set a priori are listed in Table 1. We found that when we estimated the parameters, κ_w and λ_w , the estimator drove them to their boundaries. This is why we simply set λ_w to a value near unity and we set $\kappa_w = 1$. The steady state value of inflation (a parameter in the monetary policy rule and the price and wage updating equations), the steady state government consumption to output ratio, and the growth rate of the investment specific technology were chosen to coincide with their corresponding sample means in our data set.³⁹ The growth rate of neutral technology was chosen so that, conditional on the growth rate of investment specific technology, the steady state growth rate of output in the model coincides with the corresponding sample average in the data. We set $\xi_w = 0.75$, so that the model implies wages are reoptimized once a year on average. We did not estimate this parameter because we found that it is difficult to separately identify the value of ξ_w and the curvature parameter of household labor disutility, ϕ .

The parameters for which we report priors and posteriors are listed in Table 2. Note

³⁹In our model, the relative price of investment goods represents a direct observation the technology shock for producing investment goods.

first that the degree of price stickiness, ξ_p , is modest. The time between price reoptimizations implied by the posterior mean of this parameter is a little less than 3 quarters. The amount of information in the likelihood, (5.3), about the value of ξ_p is reasonably large. The posterior standard deviation is roughly an order of magnitude smaller than the prior standard deviation and the posterior 95 percent probability interval is half the length of the prior probability interval. Generally, the amount of information in the likelihood about all the parameters is large in this sense. An exception to this pattern is the coefficient on inflation in the Taylor rule, r_π . There appears to be relatively little information about this parameter in the likelihood. Note that ϕ is estimated to be quite small, implying a consumption-compensated labor supply elasticity for the family of around 8. Such a high elasticity would be regarded as empirically implausible if it were interpreted as the elasticity of supply of hours by a representative agent. However, as discussed in section 2.3 above, this is not our interpretation.

Table 3 reports steady state properties of the model, evaluated at the posterior mean of the parameters. According to the results, the capital output ratio is lower than the empirical value of 12 typically reported in the real business cycle literature.

6.2.2. Impulse Responses

Figures 3-5 display the response of the indicated macroeconomic variables to our three shocks. In each case, the solid black line is the point estimate of the dynamic response generated by our estimated VAR. The grey area is an estimate of the corresponding 95% probability interval.⁴⁰ Our estimation strategy in effect selects a model parameterization that places the model-implied impulse response functions as close as possible to the center of the grey area, while not suffering too much of a penalty from the priors. The estimation criterion is less concerned about reproducing VAR-based impulse response functions where the grey areas are the widest.

The line with solid squares in the figures display the impulse responses of our model, at the posterior mean of the parameters. The dashed lines display the 95 percent probability interval for the impulse responses implied by the posterior distribution of the parameters. These intervals are in all cases reasonably tight, reflecting the tight posterior distribution on the parameters as well as the natural restrictions of the model itself.

Consider Figure 3, which displays the response of standard macroeconomic variables to a monetary policy shock. Note how well the model captures the delayed and gradual response of inflation. In the model it takes two years for inflation to reach its peak response after the monetary policy shock. Note that the model even captures the ‘price puzzle’ phenomenon,

⁴⁰The probability interval is defined by the point estimate of the impulse response, plus and minus 1.96 times the square root of the relevant term on the diagonal of $\bar{V}$ reported in (5.5).

according to which inflation moves in the ‘wrong’ direction initially. This apparently perverse initial response of inflation is interpreted by the model as reflecting the reduction in labor costs associated with the cut in the nominal rate of interest. The notable result here is that the slow response of inflation to a monetary policy shock is explained with a modest degree of wage and price-setting frictions. In addition, the gradual and delayed response of inflation is not due to an excessive or counterfactual increase in capital utilization. Indeed, the model substantially understates the rise in capital utilization. While on its own this is a failure of the model, it does draw attention to the apparent ease with which the model is able to capture the inertial response of inflation to a monetary shock.

The model also captures the response of output and consumption to a monetary policy shock reasonably well. However, the model apparently does not have the flexibility to capture the relatively sharp rise and fall in the investment response, although the model responses lie inside the grey area. The relatively large estimate of the curvature in the investment adjustment cost function, S'' , suggests that to allow a greater response of investment to a monetary policy shock would cause the model’s prediction of investment to lie outside the grey area in the initial and later quarters. These findings for monetary policy shocks are broadly similar to those reported in CEE and ACEL.

Figure 4 displays the response of standard macroeconomic variables to a neutral technology shock. Note that the model is reasonably successful at reproducing the empirically estimated responses. The dynamic response of inflation is particularly notable, particularly in light of the estimation results reported in ACEL. Those results suggest that the sharp and precisely estimated drop in inflation in response to a neutral technology shock is difficult to reproduce in a model like ours. In describing this problem for their model, ACEL express a concern that the failure reflects a deeper problem with sticky price models.⁴¹ They suggest that perhaps the emphasis on price and wage setting frictions, largely motivated by the inertial response of inflation to a monetary shock, is shown to be misguided by the evidence that inflation responds rapidly to technology shocks.⁴² Our results suggest a far more mundane possibility. There are two key differences between our model and the one in ACEL which allow it to reproduce the response of inflation to a technology shock more or less exactly without hampering its ability to account for the slow response of inflation to a monetary policy shock. First, in our model there is no indexation of prices to lagged inflation (see (4.5)). ACEL follows CEE in supposing that when firms cannot optimize their

⁴¹See Paciello (2009) for another discussion of this point.

⁴²The concern is reinforced by the fact that an alternative approach, one based on information imperfections and minimal price/wage setting frictions, seems like a natural one for explaining the puzzle of the slow response of inflation to monetary policy shocks and the quick response to technology shocks (see Maćkowiak and Wiederholt (2009), Mendes (2009), and Paciello (2009)). Dopor, Han and Tsai (2009) suggest more modest changes in the model structure to accommodate the inflation puzzle.

price, they index it fully to lagged aggregate inflation. The position of our model on price indexation is a key reason why we can account for the rapid fall in inflation after a neutral technical shock while ACEL cannot. We suspect that our way of treating indexation is a step in the right direction from the point of view of the microeconomic data. Micro observations suggest that individual prices do not change for extended periods of time. A second distinction between our model and the one in ACEL is that we specify the neutral technology shock to be a random walk (see (4.2)), while in ACEL the growth rate of the estimated technology shock is highly autocorrelated. In ACEL, a technology shock triggers a strong wealth effect which stimulates a surge in demand that places upward pressure on marginal cost and thus inflation.

Figure 5 displays dynamic responses of macroeconomic variables to an investment specific shock. The evidence indicates that the two models, parameterized at their posterior modes, do well in accounting for these responses.

6.3. Assessing VAR Robustness and Accuracy of the Laplace Approximation

It is well known that when the start date or number of lags for a VAR are changed, the estimated impulse response functions change. In practice, one hopes that the width of probability intervals reported in the analysis is a reasonable rule-of-thumb guide to the degree of non-robustness. In Figures 7 and 8 we display all the estimated impulse response functions from our VAR when we apply a range of different start dates and lag lengths. The VAR point estimates are displayed in Figures 7 and 8 in the form of the solid line with solid squares. The 95% probability intervals are indicated by the dashed lines. According to the figures, the degree of variation across different samples and lag lengths corresponds roughly to the width of probability intervals. Although results do change across the perturbed VARs, the magnitude of the changes are roughly what is predicted by the rule of thumb. In this sense, the degree of non-robustness in the VAR is not great.

Figure 9 displays the priors and posteriors of the model parameters. The posteriors are computed by two methods: the MCMC method, and the Laplace approximation described in section 5.4. It is interesting that the Laplace approximation and the results of the MCMC algorithm are very similar. These results suggest that one can save substantial amounts of time by computing the Laplace approximation during the early and intermediate phases of a research project. At the end of the project, when it is time to produce the final draft of the manuscript, one can then perform the time-intensive MCMC calculations.

7. Conclusion

The literature on DSGE models for monetary policy is too large to review in all its detail here. Necessarily, we have been forced to focus on only a part. In the introduction we argued that a major success of that literature is that monetary DSGE models perform well in forecasting, yet we have not discussed that topic at all. In our discussion about the estimation of DSGE models we have focused on a partial information method that allowed us to accomplish two things. First, we were able to explain some of the basic choices that were made in the construction of monetary DSGE models. This includes the decision to include habit persistence in preferences, a particular type of adjustment costs in investment, etc. Second, we were able to highlight a basic success of monetary DSGE models, that they provide an account for the slow response of inflation to a monetary policy shock and the substantial response of real variables. One interpretation of the field of monetary economics is that it is all about the identification of frictions that allow a model to explain these monetary non-neutralities. Although we think that our discussion of results using a limited information econometric technique is useful for the reasons given, the literature has moved on to use full information methods. In effect, these methods allow one to build in the additional model details required to provide a full account of the data. We have not been able to review the important results and themes in this literature.

Another important topic concerns the limitations of monetary DSGE models. A central puzzle concerns the famous statistical rejections of the intertemporal Euler equation that lies at the heart of DSGE models (see, e.g., Hansen and Singleton (1983)). These rejections of what is in effect the “IS equation” in the New Keynesian model, pose a challenge for that model’s account of the way shocks propagate through the economy. At the same time, the limited information econometric technique that we apply suggests that the New Keynesian model is able to capture the basic features of the transmission of three important shocks.⁴³ An outstanding question is how to resolve these apparently conflicting pieces of information.

Finally, we have not been able to review the new frontiers for monetary DSGE models. The recent financial turmoil has accelerated the introduction of a richer financial sector into the New Keynesian model. With these additions, the model is able to address important policy questions that cannot be addressed by the models described here: how should monetary policy respond to an increase in interest rate spreads?, how do we modify the framework to allow it to place structure on the recent forays into ‘unconventional monetary policy’ in which the monetary authority purchases privately issued liabilities such as mortgages and commercial paper? The model described here is silent on these questions. However, an ex-

⁴³In our empirical analysis we have not reported our VAR’s implications for the importance of the three shocks that we analyzed. However, ACEL documents that these shocks together account for well over 50 percent of the variation of macroeconomic time series like output, investment and employment.

ploding literature too large to review here has begun to introduce the modifications necessary to address them.⁴⁴

Another frontier for new model development concerns the labor market. The basic model developed here does not allow one to think about labor market variables such as unemployment. Yet, this is a variable of key interest to policy makers. To some extent, analysis of unemployment necessarily also involves the analysis of financial frictions. Limited access to credit markets is part of the reason unemployment is socially costly and of concern to policy makers. Extensive research on this frontier is now also well under way.⁴⁵

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⁴⁴For a small sampling, see, for example, Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2003,2009), Cúrdia and Woodford (2009) and Gertler and Kiyotaki (2010).

⁴⁵A recent analysis of unemployment from the point of view of credit market limitations appears in Christiano, Trabandt and Walentin (2010a). A small open economy model with financial and labor market frictions, estimated by full information Bayesian methods, appears in Christiano, Trabandt and Walentin (2010c). Important other papers on the integration of unemployment and other frictions into monetar DSGE models include Gertler, Sala and Trigari (2008) and Thomas (2009).

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A. Data Appendix

A.1. Data Sources

FRED2: Database of the Federal Reserve Bank of St. Louis available at:

<http://research.stlouisfed.org/fred2/>.

BLS: Database of the Bureau of Labor Statistics available at: <http://www.bls.gov/>.

FR: Fujita and Ramey (2006) data on job separations and job findings available at:

http://www.philadelphiafed.org/research-and-data/economists/fujita/transition_rates.xls.

SH: Data on job separations and job findings available at Robert Shimer's Homepage:

<http://robert.shimer.googlepages.com/>.

NIPA: Database of the National Income And Product Accounts available at:

<http://www.bea.gov/national/nipaweb/index.asp>

BGOV: Database of the Board of Governors of the Federal Reserve System available at:

<http://www.federalreserve.gov/econresdata/default.htm>.

CONFB: Database of the Conference Board available at:

<http://www.conference-board.org/economics/HelpWanted.cfm>

A.2. Raw Data

Nominal GDP (GDP): nominal gross domestic product, billions of dollars, seasonally adjusted at annual rates, NIPA.

GDP Deflator (P) : price index of nominal gross domestic product, index numbers, 2005=100, seasonally adjusted, NIPA.

Nominal nondurable consumption ($C_{nondurables}^{nom}$) : nominal personal consumption expenditures: nondurable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal durable consumption ($C_{durables}^{nom}$) : nominal personal consumption expenditures: durable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal consumption services ($C_{services}^{nom}$) : nominal personal consumption expenditures: services, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal investment (I^{nom}) : nominal gross private domestic investment, billions of dollars, seasonally adjusted at annual rates, NIPA.

Price index: nominal durable consumption ($PC_{durables}^{nom}$) : price index of durable goods, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.

Price index: nominal investment (PI^{nom}) : price index of nominal gross private domestic investment, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.

Employment (E): civilian employment, CE16OV, seasonally adjusted, monthly, thousands, persons 16 years of age and older, FRED2.

Federal Funds Rate (FF): effective federal funds rate, H.15 selected interest rates, monthly, percent, averages of daily figures, FRED2.

Treasury bill rate ($TBill$): 3-month treasury bill: secondary market rate, H.15 selected interest rates, monthly, percent, averages of business days, discount basis, FRED2.

Population (POP): civilian noninstitutional population, not seasonally adjusted, monthly, thousands, FRED2.

Capacity utilization (CAP): capacity utilization, G.17 - industrial production and capacity utilization, UTL: manufacturing (SIC) G17/CAPUTL/CAPUTL.B00004.S.Q. , seasonally adjusted, percentage, BGOV.

Job separation rate (S): separation rate: E to U, seasonally adjusted, monthly, 1976M1-2008M12, FR. Spliced with corresponding data from Robert Shimer for the sample before 1976, quarterly. SH.

Job finding rate (F): Job finding rate: U to E, seasonally adjusted, monthly, 1976M1-2008M12, FR. Spliced with corresponding data from Robert Shimer for the sample before 1976, quarterly. SH.

Vacancies (V): index of help wanted advertising in newspapers, HELPWANT, The Conference Board, seasonally adjusted, monthly, index 1987=100,

Unemployment rate (U): unemployment rate labor force status: unemployment rate, LNS14000000, seasonally adjusted, percent, 16 years and over, monthly frequency, BLS.

Nominal wage (W): nominal hourly compensation, PRS85006103, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.

Average hours (H^{avg}): average weekly hours, PRS85006023, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.

Participation rate ($LabForce$) : civilian participation rate, CIVPART, the employment situation, seasonally adjusted, monthly, percent, BLS.

A.3. Data Transformations

Raw data are transformed as follows. POP is seasonally adjusted using the X12 (multiplicative) method. The indices for W and H^{avg} are normalized such that 2005=100. E , FF , $TBill$, POP , V , U and $LabForce$ are converted to quarterly frequencies by averaging monthly observations. For the job finding rate F , we compute the quarterly measure from monthly data as follows:

$$F_{q1} = F_{m1} + (1 - F_{m1})F_{m2} + (1 - (1 - F_{m1})F_{m2})F_{m3},$$

where F_{q1} denotes the finding rate of quarter 1 and F_{m1} , F_{m2} , F_{m3} are the corresponding monthly finding rates. The case for the separation rate, S , follows accordingly.

Due to missing data we use *TBill* as a proxy for the *FF* prior 1954Q3. All data are available from 1948Q1 except for vacancies for which the first observation is 1951Q1.

We calculate the following time series which, among others, is used in the VAR:

$$\begin{aligned} \text{real } GDP &= \frac{GDP}{P * POP} \\ \text{hours} &= \frac{H^{avg.} * E}{Pop} \\ \text{nominal consumption} &= C_{nondurables}^{nom} + C_{services}^{nom} \\ \text{nominal investment} &= I^{nom} + C_{durables}^{nom} \end{aligned}$$

The price of investment is calculated as a Torn price index using PI^{nom} , $PC_{durables}^{nom}$, I^{nom} and $C_{durables}^{nom}$. The resulting price index PIT and quantity index QIT are used to calculate the relative price of investment as follows:

$$\text{relative price of investment} = \frac{PIT * I^{nom}}{P * QIT}.$$

B. Scaling of Variables in Medium-sized Model

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t , is an investment specific technology shock and it is convenient to define the following combination of our two technology shocks:

$$\begin{aligned} z_t^+ &\equiv \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &\equiv \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{B.1}$$

Capital, $\bar{K}_t$, and investment, I_t , are scaled by $z_t^+ \Psi_t$. Consumption goods C_t , government consumption G_t and the real wage, W_t/P_t are scaled by z_t^+ . Also, v_t is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, v_t is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $v_t P_t$. The latter must be multiplied by z_t^+ to induce stationarity. Output, Y_t , is scaled by z_t^+ . Optimal prices, $\tilde{P}_t$, chosen by intermediate good firms which are subject to Calvo price setting frictions are scaled by the price, P_t , of the homogeneous output good. Similarly, optimal wages, $\tilde{W}_t$, chosen by monopoly unions which are subject to Calvo wage setting frictions are scaled by the wage, W_t , of the homogenous labour input. Thus our

scaled variables are:

$$\begin{aligned}
k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t}, \quad \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t}, \quad i_t = \frac{I_t}{z_t^+ \Psi_t}, \quad c_t = \frac{C_t}{z_t^+}, \\
g_t &= \frac{G_t}{z_t^+}, \quad \psi_{z^+,t} = v_t P_t z_t^+, \quad \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \quad \tilde{y}_t = \frac{Y_t}{z_t^+}, \\
\tilde{p}_t &= \frac{\tilde{P}_t}{P_t}, \quad w_t = \frac{\tilde{W}_t}{W_t}.
\end{aligned} \tag{B.2}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t}, \quad \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the homogeneous good. The inflation rate is defined as:

$$\pi_t = \frac{P_t}{P_{t-1}}.$$

C. Equilibrium Conditions for the Medium-sized Model

C.1. Firms

We let s_t denote the firm's marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

$$s_t = \frac{\left(\frac{r_t^k P_t}{\alpha}\right)^\alpha \left(\frac{W_t R_t}{1-\alpha}\right)^{1-\alpha}}{P_t z_t^{1-\alpha}}.$$

When expressed in terms of scaled variables, this reduces to:

$$s_t = \left(\frac{\bar{r}_t^k}{\alpha}\right)^\alpha \left(\frac{\bar{w}_t R_t}{1-\alpha}\right)^{1-\alpha}. \tag{C.1}$$

Productive efficiency dictates that s_t is also equal to the ratio of the real cost of labor to the marginal product of labor:

$$s_t = \frac{(\mu_{\Psi,t})^\alpha \bar{w}_t R_t}{(1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t}} / H_{i,t}\right)^\alpha}. \tag{C.2}$$

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. The first order necessary conditions associated

with price optimization are, after scaling:⁴⁶

$$E_t \left[\psi_{z^+,t} y_t + \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{t+1}^f - F_t^f \right] = 0, \quad (\text{C.3})$$

$$E_t \left[\lambda_f \psi_{z^+,t} y_t s_t + \beta \xi_p \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{t+1}^f - K_t^f \right] = 0, \quad (\text{C.4})$$

$$\dot{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{f,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right)^{\lambda_f} + \xi_p \left(\frac{\tilde{\pi}_{f,t}}{\pi_t} \dot{p}_{t-1} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}}, \quad (\text{C.5})$$

$$\left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{f,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{(1-\lambda_f)} = \frac{K_t^f}{F_t^f}, \quad (\text{C.6})$$

$$\tilde{\pi}_{f,t} \equiv \pi. \quad (\text{C.7})$$

C.2. Households

We now derive the equilibrium conditions associated with the household. We first consider the household's consumption saving decision. We then turn to its wage decision. The Lagrangian representation of the household's problem is:

$$\begin{aligned} & E_0^j \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t - bC_{t-1}) - A_L \frac{h_{j,t}^{1+\phi}}{1+\phi} dj \right\} \\ & v_t \left[W_{t,j} h_{t,j} dj + X_t^k \bar{K}_t + R_{t-1} B_t + a_{t,j} - P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) - B_{t+1} - P_t P_{k',t} \Delta_t \right] \\ & + \omega_t \left[\Delta_t + (1 - \delta) \bar{K}_t + \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) I_t - \bar{K}_{t+1} \right] \end{aligned}$$

The first order condition with respect to C_t is:

$$\frac{1}{C_t - bC_{t-1}} - E_t \frac{b\beta}{C_{t+1} - bC_t} = v_t P_t,$$

or, after expressing this in scaled terms and multiplying by z_t^+ :

$$\psi_{z^+,t} = \frac{1}{c_t - b \frac{c_{t-1}}{\mu_{z^+,t}}} - \beta b E_t \frac{1}{c_{t+1} \mu_{z^+,t+1} - b c_t}. \quad (\text{C.8})$$

⁴⁶When we log-linearize about the steady state, we obtain,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} \hat{s}_t,$$

where a hat indicates log-deviation from steady state.

The first order condition with respect to Δ_t is, after rearranging:

$$P_t P_{k',t} = \frac{\omega_t}{v_t}. \quad (\text{C.9})$$

The first order condition with respect to I_t is:

$$\omega_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) - S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + E_t \beta \omega_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 = \frac{P_t v_t}{\Psi_t}.$$

Making use of (C.9), multiplying by $\Psi_t z_t^+$, rearranging and using the scaled variables,

$$\begin{aligned} \psi_{z^+,t} p_{k',t} \left[1 - S \left(\frac{\mu_{z^+,t} \mu_{\Psi,t} i_t}{i_{t-1}} \right) - S' \left(\frac{\mu_{z^+,t} \mu_{\Psi,t} i_t}{i_{t-1}} \right) \frac{\mu_{z^+,t} \mu_{\Psi,t} i_t}{i_{t-1}} \right] \\ + \beta \psi_{z^+,t+1} p_{k',t+1} S' \left(\frac{\mu_{z^+,t+1} \mu_{\Psi,t+1} i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \mu_{z^+,t+1} \mu_{\Psi,t+1} = \psi_{z^+,t}, \end{aligned} \quad (\text{C.10})$$

Optimality of the choice of $\bar{K}_{t+1}$ implies the following first order condition:

$$\omega_t = \beta E_t v_{t+1} X_{t+1}^k + \beta E_t \omega_{t+1} (1 - \delta) = \beta E_t v_{t+1} [X_{t+1}^k + P_{t+1} P_{k',t+1} (1 - \delta)].$$

Using (C.9) again,

$$v_t = E_t \beta v_{t+1} \left[\frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1 - \delta)}{P_t P_{k',t}} \right] = E_t \beta v_{t+1} R_{t+1}^k, \quad (\text{C.11})$$

where R_{t+1}^k denotes the rate of return on capital:

$$R_{t+1}^k \equiv \frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1 - \delta)}{P_t P_{k',t}}$$

Multiply (C.11) by $P_t z_t^+$ and express the results in scaled terms:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1} \mu_{z^+,t+1}}. \quad (\text{C.12})$$

Expressing the rate of return on capital, (4.15), in terms of scaled variables:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{u_{t+1} \bar{r}_{t+1}^k - a(u_{t+1}) + (1 - \delta) p_{k',t+1}}{p_{k',t}}. \quad (\text{C.13})$$

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'(u_t),$$

or, in scaled terms,

$$\bar{r}_t^k = a'(u_t). \quad (\text{C.14})$$

The first order condition with respect to B_{t+1} is:

$$v_t = \beta v_{t+1} R_t.$$

Multiply by $z_t^+ P_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1} \pi_{t+1}} R_t. \quad (\text{C.15})$$

Finally, the law of motion for the capital stock, in terms of scaled variables is as follows:

$$\bar{k}_{t+1} = \frac{1 - \delta}{\mu_{z^+,t} \mu_{\Psi,t}} \bar{k}_t + \left(1 - \tilde{S} \left(\frac{\mu_{z^+,t} \mu_{\Psi,t} i_t}{i_{t-1}} \right) \right) i_t. \quad (\text{C.16})$$

C.3. Resource Constraint

The resource constraint after scaling by z_t^+ is given by:

$$y_t = g_t + c_t + i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}}. \quad (\text{C.17})$$

In appendix D we derive a relationship between total output of the homogeneous good, Y_t , and aggregate factors of production which in scaled form looks as follows:

$$y_t = (\hat{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha H_t^{1-\alpha} - \varphi \right], \quad (\text{C.18})$$

where

$$k_t = \bar{k}_t u_t. \quad (\text{C.19})$$

Finally, GDP is given by:

$$gdp_t = g_t + c_t + i_t. \quad (\text{C.20})$$

C.4. Wage Setting by the Monopoly Union

We turn now to the equilibrium conditions associated with the household wage-setting decision. Consider the j^{th} household that has an opportunity to reoptimize its wage at time t . We denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{(h_{j,t+i})^{1+\phi}}{1+\phi} dj + v_{t+i} W_{j,t+i} h_{j,t+i} dj \right],$$

where v_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$. Substituting this into the objective function and optimizing (see appendix F for details) yields the following equilibrium equations associated with wage setting:

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1} \mu_{z^+,t+1} \pi_{t+1}}{\bar{w}_t}, \quad (\text{C.21})$$

$$h_t = \hat{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \quad (\text{C.22})$$

$$\hat{w}_t = \left[(1 - \xi_w) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \hat{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \quad (\text{C.23})$$

In addition to (C.23), we have following equilibrium conditions associated with sticky wages⁴⁷:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \hat{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \quad (\text{C.24})$$

$$K_{w,t} = \left(\hat{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1+\phi} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} K_{w,t+1} \quad (\text{C.25})$$

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(1+\phi)} \bar{w}_t F_{w,t} = K_{w,t} \quad (\text{C.26})$$

$$\tilde{\pi}_{w,t+1} = \pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}. \quad (\text{C.27})$$

⁴⁷Log-linearizing these equations about the nonstochastic steady state we obtain,

$$E_t \left[\begin{array}{c} \eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_3 \hat{\pi}_t + \eta_4 \hat{\pi}_{t+1} + \eta_5 \hat{\pi}_{t-1} \\ + \eta_6 \hat{\psi}_{z^+,t} + \eta_7 \hat{H}_t + \eta_8 \hat{\mu}_{z^+,t} + \eta_9 \hat{\mu}_{z^+,t+1} \end{array} \right] = 0,$$

where

$$\begin{aligned} b_w &= \frac{[\lambda_w \sigma_L - (1 - \lambda_w)]}{[(1 - \beta \xi_w)(1 - \xi_w)]}, \eta_0 = b_w \xi_w, \eta_1 = \sigma_L \lambda_w - b_w (1 + \beta \xi_w^2), \eta_2 = b_w \beta \xi_w, \\ \eta_3 &= -b_w \xi_w (1 + \beta \kappa_w), \eta_4 = b_w \beta \xi_w, \eta_5 = b_w \xi_w \kappa_w, \eta_6 = (1 - \lambda_w), \\ \eta_7 &= -(1 - \lambda_w) \sigma_L, \eta_8 = -b_w \xi_w, \eta_9 = b_w \beta \xi_w. \end{aligned}$$

C.5. Equilibrium Equations

The equilibrium conditions of the model correspond to the following 28 equations,

$$(C.1), (C.2), (C.3), (C.4), (C.5), (C.6), (C.7), (C.16), (C.8), (C.10), (C.14), \\ (C.15), (C.22), (4.20), (C.17), (C.18), (4.22), (C.19), (4.23), (C.13), \\ (C.21), (C.24), (C.25), (C.26), (C.23), (C.27), (C.20), (C.12),$$

which can be used to solve for the following 28 unknowns:

$$\bar{r}_t^k, \bar{w}_t, R_t, s_t, \pi_t, p_{k',t}, k_{t+1}, \bar{k}_{t+1}, u_t, h_t, H_t, i_t, c_t, \psi_{z^+,t}, y_t, \\ K_t^f, F_t^f, \tilde{\pi}_{f,t}, \hat{p}_t, K_{w,t}, F_{w,t}, \tilde{\pi}_t^w, R_t^k, S_t, a(u_t), \hat{w}_t, \pi_{w,t}, gdp_t.$$

D. Resource Constraint in the Medium-sized Model

We begin by deriving a relationship between total output of the homogeneous good, Y_t , and aggregate factors of production. We first consider the production of the homogenous output good:

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 [(z_t H_{i,t})^{1-\alpha} K_{i,t}^\alpha - z_t^+ \varphi] di \\ &= \int_0^1 \left[z_t^{1-\alpha} \left(\frac{K_{i,t}}{H_{i,t}} \right)^\alpha H_{i,t} - z_t^+ \varphi \right] di \\ &= z_t^{1-\alpha} \left(\frac{K_t}{H_t} \right)^\alpha \int_0^1 H_{i,t} di - z_t^+ \varphi, \end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} K_t^\alpha H_t^{1-\alpha} - z_t^+ \varphi.$$

The demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_f}{\lambda_f-1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\hat{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_f}{\lambda_f-1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f-1}} \left(\hat{P}_t \right)^{\frac{\lambda_f}{1-\lambda_f}},$$

say, where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_f}{1-\lambda_f}} di \right]^{\frac{1-\lambda_f}{\lambda_f}}. \quad (\text{D.1})$$

Dividing by P_t ,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} di \right]^{\frac{1-\lambda_f}{\lambda_f}},$$

or,

$$\mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right)^{\lambda_f} + \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}}. \quad (\text{D.2})$$

The preceding discussion implies:

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f-1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f-1}} [z_t^{1-\alpha} K_t^\alpha H_t^{1-\alpha} - z_t^+ \varphi],$$

or, after scaling by z_t^+ ,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f-1}} \left[\left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha H_t^{1-\alpha} - \varphi \right],$$

where

$$k_t = \bar{k}_t u_t.$$

Finally, we adjust hours worked in the resource constraint so that it corresponds to the total number of people working, as in (F.6):

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f-1}} \left[\left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha \left[\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right]^{1-\alpha} - \varphi \right].$$

It is convenient to also have an expression that exhibits the uses of the homogeneous output,

$$z_t^+ y_t = G_t + C_t + \tilde{I}_t,$$

or, after scaling by z_t^+ :

$$y_t = g_t + c_t + i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}}.$$

E. Optimal Price Setting in the Medium-sized Model

The profit function of the i^{th} intermediate good firm with the substituted demand function is given by,

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{1-\frac{\lambda_f}{\lambda_f-1}} - s_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{\frac{-\lambda_f}{\lambda_f-1}} \right\},$$

or,

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ (X_{t,j} \tilde{p}_t)^{1-\frac{\lambda_f}{\lambda_f-1}} - s_{t+j} (X_{t,j} \tilde{p}_t)^{\frac{-\lambda_f}{\lambda_f-1}} \right\},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j} \tilde{p}_t, \quad X_{t,j} \equiv \begin{cases} \frac{\tilde{\pi}_{t+j} \cdots \tilde{\pi}_{t+1}}{\pi_{t+j} \cdots \pi_{t+1}}, & j > 0 \\ 1, & j = 0 \end{cases}.$$

The i^{th} firm maximizes profits by choice of $\tilde{p}_t$. The fact that this variable does not have an index, i , reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_f}{\lambda_f-1}+1}$, rearranging, and scaling we obtain:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} [\tilde{p}_t X_{t,j} - \lambda_f s_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+,t+j} y_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda_f s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} X_{t,j}} = \frac{K_t^f}{F_t^f},$$

say, where

$$\begin{aligned} K_t^f &\equiv E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda_f s_{t+j} \\ F_t^f &= E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} X_{t,j}. \end{aligned}$$

These objects have the following convenient recursive representations:

$$\begin{aligned} E_t \left[\psi_{z^+,t} y_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{t+1}^f - F_t^f \right] &= 0 \\ E_t \left[\lambda_f \psi_{z^+,t} y_t s_t + \beta \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{t+1}^f - K_t^f \right] &= 0. \end{aligned}$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)} \\ &= \left[(1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p (\tilde{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{(1-\lambda_f)}. \end{aligned} \quad (\text{E.1})$$

After dividing by P_t and rearranging:

$$\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{t,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} = (\tilde{p}_t)^{\frac{1}{1-\lambda_f}}. \quad (\text{E.2})$$

This completes the derivations of optimal decisions with respect to firms price setting.

F. Optimal Wage Setting in the Medium-sized Model

The objective function with the substituted labor demand function looks as follows:

$$\begin{aligned} E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i & \left[-A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{1+\phi} \right. \\ & \left. + v_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right]. \end{aligned}$$

Recalling the scaling of variables, (B.2), we have

$$\begin{aligned} \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_{t+i}^+ P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} \\ &= \frac{W_t \left(\tilde{W}_t / W_t \right)}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t \right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}, \end{aligned} \quad (\text{F.1})$$

where

$$\begin{aligned} X_{t,i} &= \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}, \quad i > 0 \\ &= 1, \quad i = 0. \end{aligned} \quad (\text{F.2})$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \rightarrow \infty$. Thus,

$$X_{t,i} = \frac{(\pi_t \cdots \pi_{t+i-1})^{\kappa_w} (\pi^i)^{(1-\kappa_w)} \mu_{z^+}^i}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}.$$

In steady state,

$$X_{t,i} = \frac{(\pi^i)^{\kappa_w} (\pi^i)^{(1-\kappa_w)} \mu_{z^+}^i}{\pi^i \mu_{z^+}^i} = 1.$$

Simplifying using the scaling notation,

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{1+\phi} \right. \\ \left. + v_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{1+\phi} \right. \\ \left. + \psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{1+\phi} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} \right. \\ \left. + \psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right].$$

Differentiating with respect to w_t ,

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{1+\phi} \lambda_w (1+\phi) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)-1} \right. \\ \left. + \psi_{z^+,t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right] = 0.$$

Dividing and rearranging,

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi} \right. \\ \left. + \frac{\psi_{z^+,t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\phi)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right] = 0. \quad (\text{F.3})$$

Solving for the wage rate:

$$\begin{aligned}
w_t^{\frac{1-\lambda_w(1+\phi)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}} \\
&= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}
\end{aligned}$$

where

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi} \\
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.
\end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\phi)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi} \\
&= H_t^{1+\phi} + \beta \xi_w \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\phi} \\
&\quad + (\beta \xi_w)^2 \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{(\pi_t \pi_{t+1})^{\kappa_w} (\pi^2)^{(1-\kappa_w)} \mu_{z^+}^2}{\pi_{t+2} \pi_{t+1} \mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\phi} \\
&\quad + \dots
\end{aligned}$$

or,

$$\begin{aligned}
K_{w,t} &= H_t^{1+\phi} + E_t \beta \xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} \{ H_{t+1}^{1+\phi} \\
&\quad + \beta \xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\pi_{t+1}^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+2} \mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\phi} + \dots \} \\
&= H_t^{1+\phi} + \beta \xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} K_{w,t+1} \\
&= H_t^{1+\phi} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} K_{w,t+1},
\end{aligned} \tag{F.4}$$

using,

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1} \mu_{z^+,t+1} \pi_{t+1}}{\bar{w}_t}.$$

Also,

$$\begin{aligned} F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \\ &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \\ &\quad + \beta \xi_w \frac{\psi_{z^+,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\pi_t^{\kappa_w} \pi^{1-\kappa_w} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \\ &\quad + (\beta \xi_w)^2 \frac{\psi_{z^+,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \\ &\quad \times \left(\frac{(\pi_t \pi_{t+1})^{\kappa_w} (\pi^2)^{(1-\kappa_w)} \mu_{z^+}^2}{\pi_{t+2} \pi_{t+1} \mu_{z^+,t+2} \mu_{z^+,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \\ &\quad + \dots \end{aligned}$$

or,

$$\begin{aligned} F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \\ &\quad + \beta \xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\pi_t^{\kappa_w} \pi^{1-\kappa_w} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} \left\{ \frac{\psi_{z^+,t+1}}{\lambda_w} H_{t+1} \right. \\ &\quad \left. + \beta \xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\pi_{t+1}^{\kappa_w} \pi^{1-\kappa_w} \mu_{z^+}}{\pi_{t+2} \mu_{z^+,t+2}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z^+,t+2}}{\lambda_w} H_{t+2} \right. \\ &\quad \left. + \dots \right\} \\ &= \frac{\psi_{z^+,t}}{\lambda_w} H_t + \beta \xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}, \end{aligned}$$

so that

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}. \quad (\text{F.5})$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1 - \xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w}.$$

Dividing both sides by W_t and rearranging,

$$w_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w}.$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(1+\phi)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t , and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \hat{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{F.6}$$

where

$$\hat{w}_t \equiv \frac{\hat{W}_t}{W_t}, \quad \hat{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\hat{W}_t = \left[(1 - \xi_w) \left(\hat{W}_t \right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} \hat{W}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

This completes the derivations of the optimal wage setting.

Table 1: Non-Estimated Parameters in Medium-sized DSGE Model

Parameter	Value	Description
α	0.25	Capital share
δ	0.025	Depreciation rate
β	0.999	Discount factor
π	1.0083	Gross inflation rate
η_g	0.2	Government consumption to GDP ratio
$p_{k'}$	1	Relative price of capital
κ_w	1	Wage indexation to π_{t-1}
λ_w	1.01	Wage markup
ξ_w	0.75	Wage stickiness
μ_z	1.0041	Gross neutral tech. growth
μ_ψ	1.0018	Gross invest. tech. growth

Table 3: Medium-sized DSGE Model Steady State at Posterior Mean for Parameters

Variable	Standard Model	Description
k/y	7.73	Capital to GDP ratio (quarterly)
c/y	0.56	Consumption to GDP ratio
i/y	0.24	Investment to GDP ratio
H	0.63	Steady state labor input
R	1.014	Gross nominal interest rate (quarterly)
R^{real}	1.006	Gross real interest rate (quarterly)
r^k	0.033	Capital rental rate (quarterly)
A_L	2.25	Slope, labor disutility

Table 2: Priors and Posteriors of Parameters for Medium-sized DSGE Model

Parameter		Prior		Posterior ^a
		Distribution [bounds]	Mean, Std.Dev. [5% and 95%]	Mean, Std.Dev. [5% and 95%]
<i>Price setting parameters</i>				
Price Stickiness	ξ_p	Beta	0.50, 0.15	0.64, 0.03
		[0, 0.8]	[0.23, 0.72]	[0.59, 0.70]
Price Markup	λ_f	Gamma	1.20, 0.15	1.17, 0.08
		[1.01, ∞]	[1.04, 1.50]	[1.04, 1.28]
<i>Monetary authority parameters</i>				
Taylor Rule: Interest Smoothing	ρ_R	Beta	0.80, 0.10	0.87, 0.02
		[0, 1]	[0.62, 0.94]	[0.84, 0.89]
Taylor Rule: Inflation Coefficient	r_π	Gamma	1.60, 0.15	1.43, 0.10
		[1.01, 4]	[1.38, 1.87]	[1.26, 1.60]
Taylor Rule: GDP Coefficient	r_y	Gamma	0.20, 0.15	0.04, 0.02
		[0, 2]	[0.03, 0.49]	[0.00, 0.07]
<i>Household parameters</i>				
Consumption Habit	b	Beta	0.75, 0.15	0.76, 0.02
		[0, 1]	[0.47, 0.95]	[0.73, 0.79]
Inverse Labor Supply Elasticity	ϕ	Gamma	0.30, 0.20	0.14, 0.03
		[0, ∞]	[0.06, 0.69]	[0.09, 0.19]
Capacity Adjustment Costs Curv.	σ_a	Gamma	1.00, 0.75	0.32, 0.09
		[0, ∞]	[0.15, 2.46]	[0.17, 0.46]
Investment Adjustment Costs Curv.	S''	Gamma	12.00, 8.00	16.49, 3.44
		[0, ∞]	[2.45, 27.43]	[11.1, 21.9]
<i>Shocks</i>				
Autocorr. Investment Technology	ρ_ψ	Uniform	0.50, 0.29	0.60, 0.08
		[0, 1]	[0.05, 0.95]	[0.47, 0.72]
Std.Dev. Neutral Tech. Shock (%)	σ_z	Inv. Gamma	0.20, 0.10	0.22, 0.02
		[0, ∞]	[0.10, 0.37]	[0.19, 0.25]
Std.Dev. Invest. Tech. Shock (%)	σ_ψ	Inv. Gamma	0.20, 0.10	0.16, 0.02
		[0, ∞]	[0.10, 0.37]	[0.12, 0.20]
Std.Dev. Monetary Shock (APR)	σ_R	Inv. Gamma	0.40, 0.20	0.45, 0.03
		[0, ∞]	[0.21, 0.74]	[0.39, 0.51]

^a Based on standard random walk metropolis algorithm. 600 000 draws, 100 000 for burn-in, acceptance rate 27%.

$$\text{Taylor Rule: } R_t = r_\pi \pi_{t+1} + r_c \hat{c}_t$$

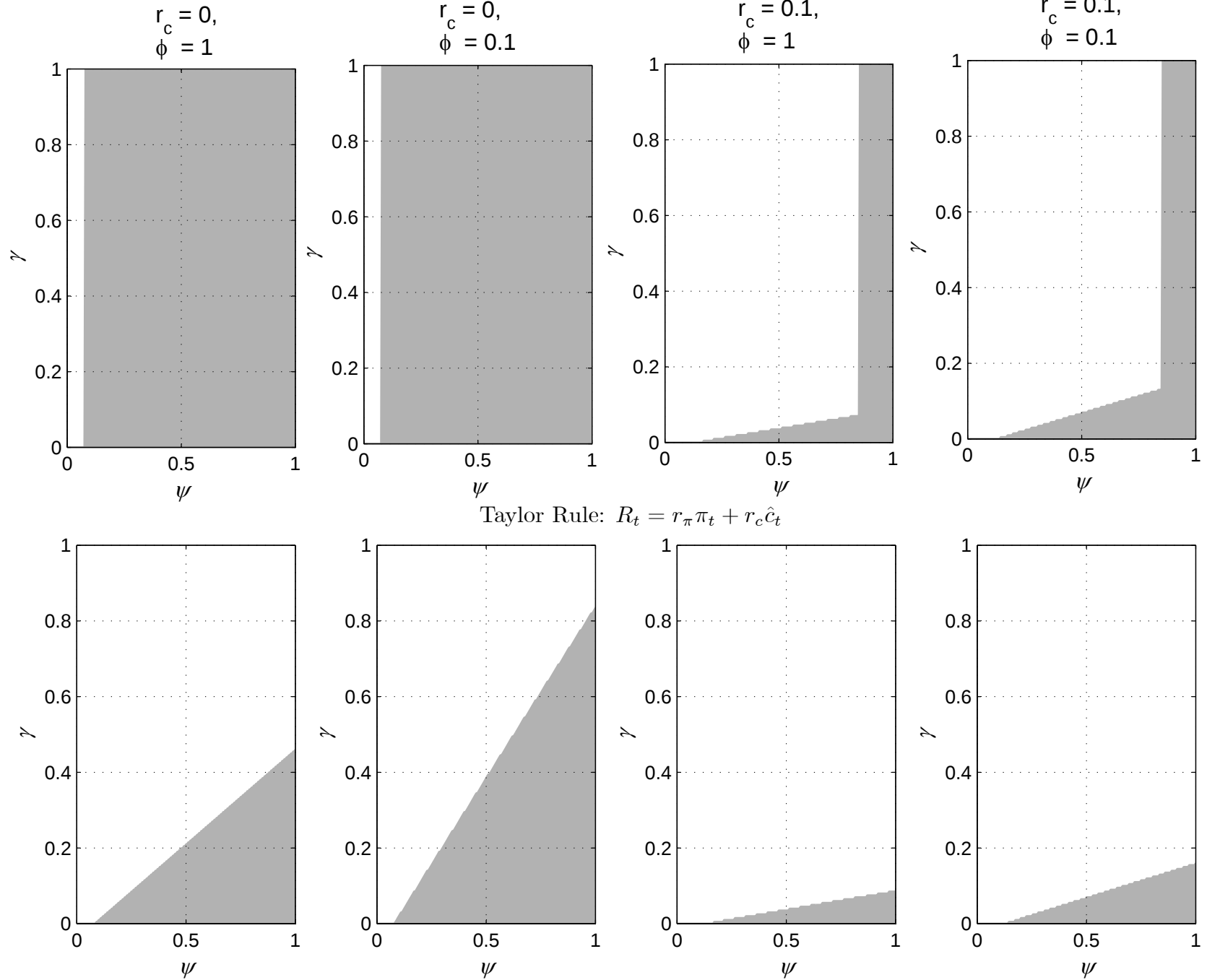
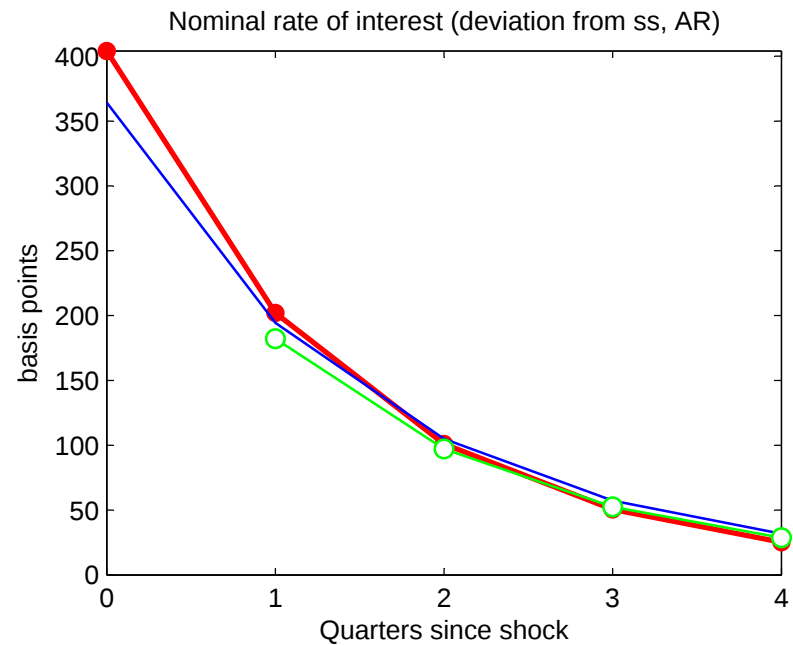
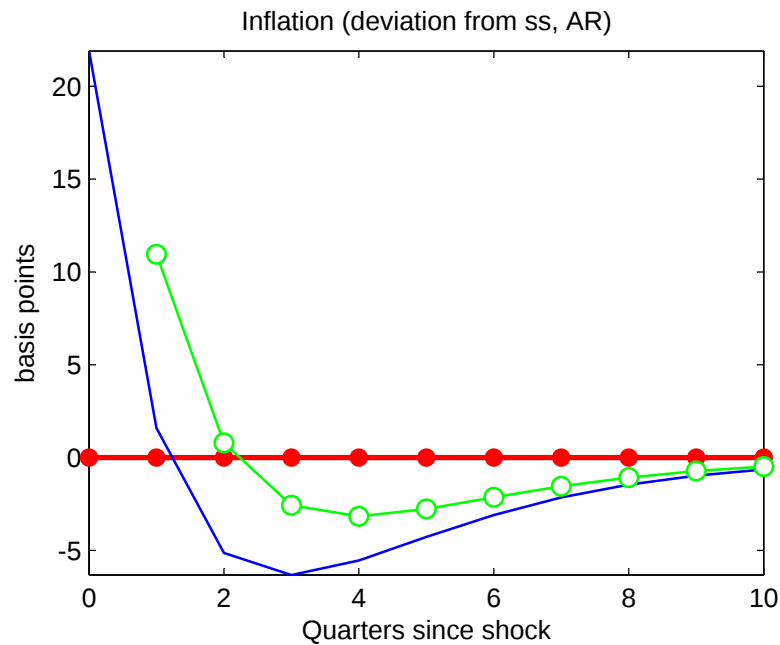
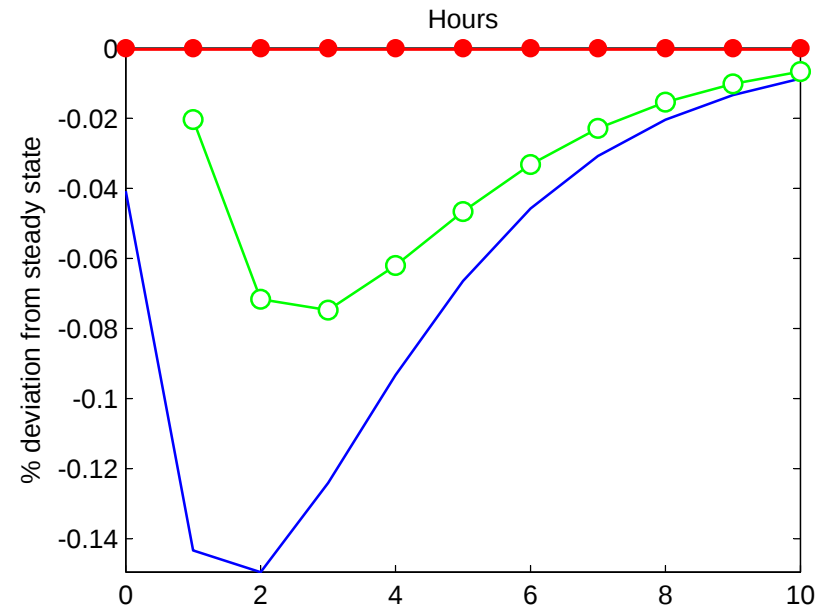
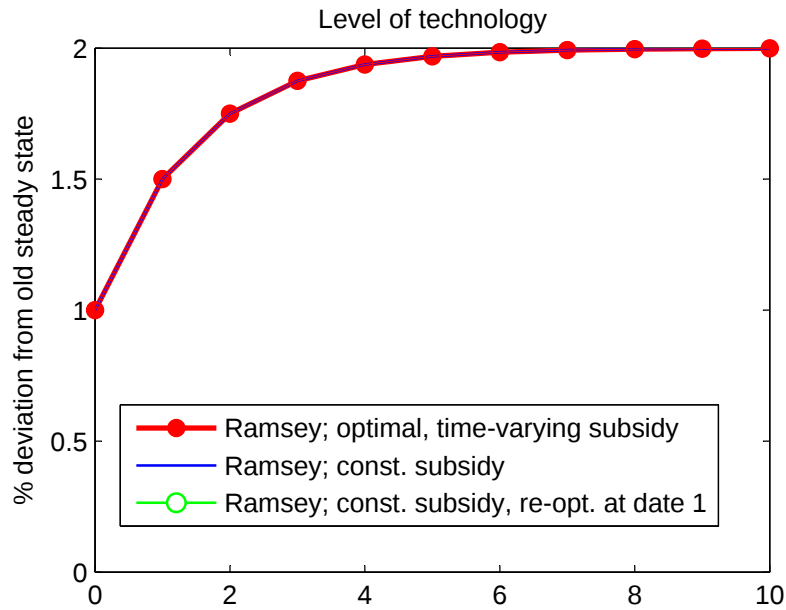


Figure 1: Indeterminacy Regions for Model with Working Capital Channel and Materials Inputs

Figure 2: Dynamic Properties of Ramsey Equilibrium Under Two Treatments of Subsidy Rate



Note: 'Ramsey, constant subsidy' refers to the Ramsey equilibrium when the subsidy rate is kept constant at a rate that extinguishes distortions due to monopoly power and the working capital channel in steady state.

Figure 3: Dynamic Responses of Variables to a Monetary Policy Shock

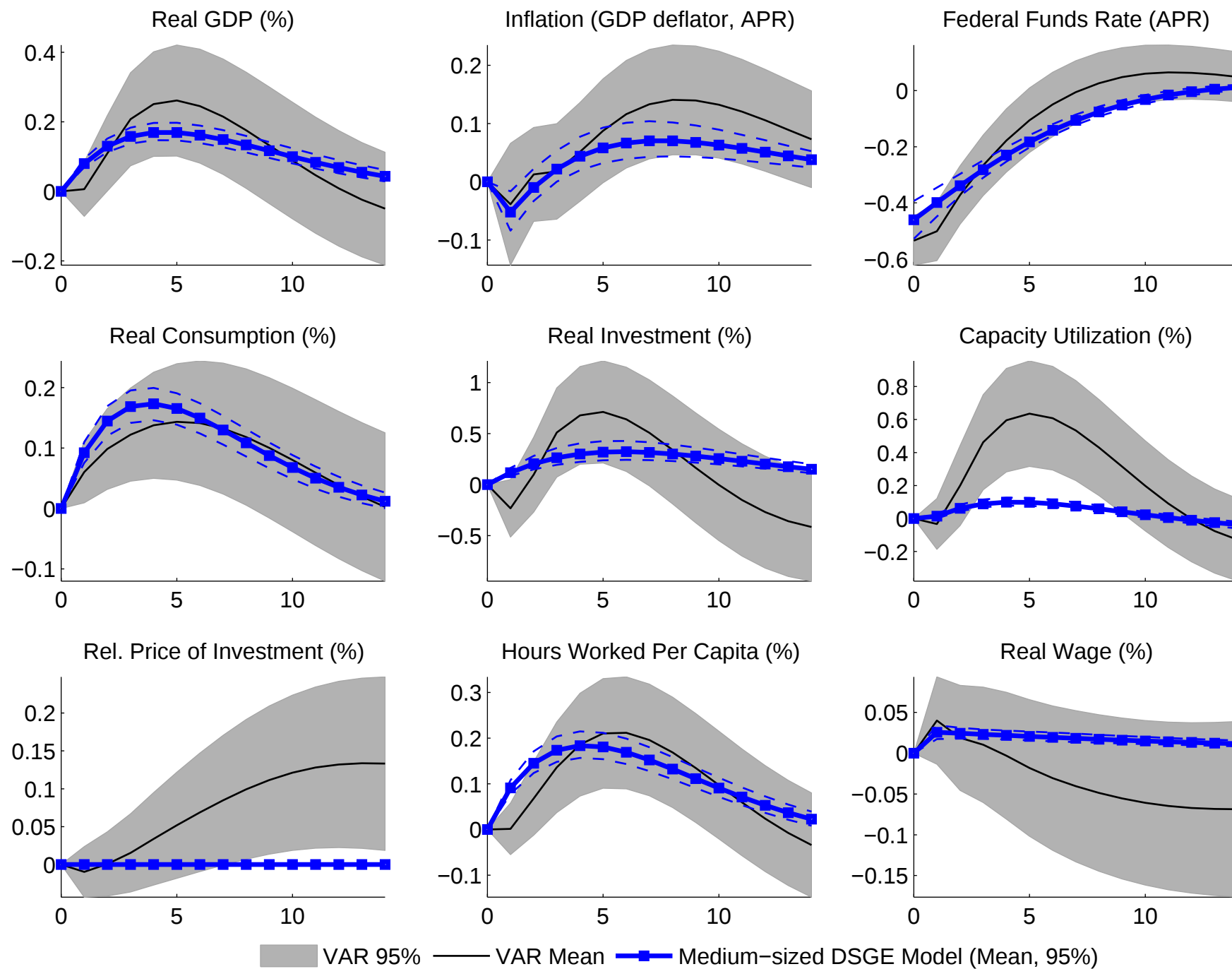


Figure 4: Dynamic Responses of Variables to a Neutral Technology Shock

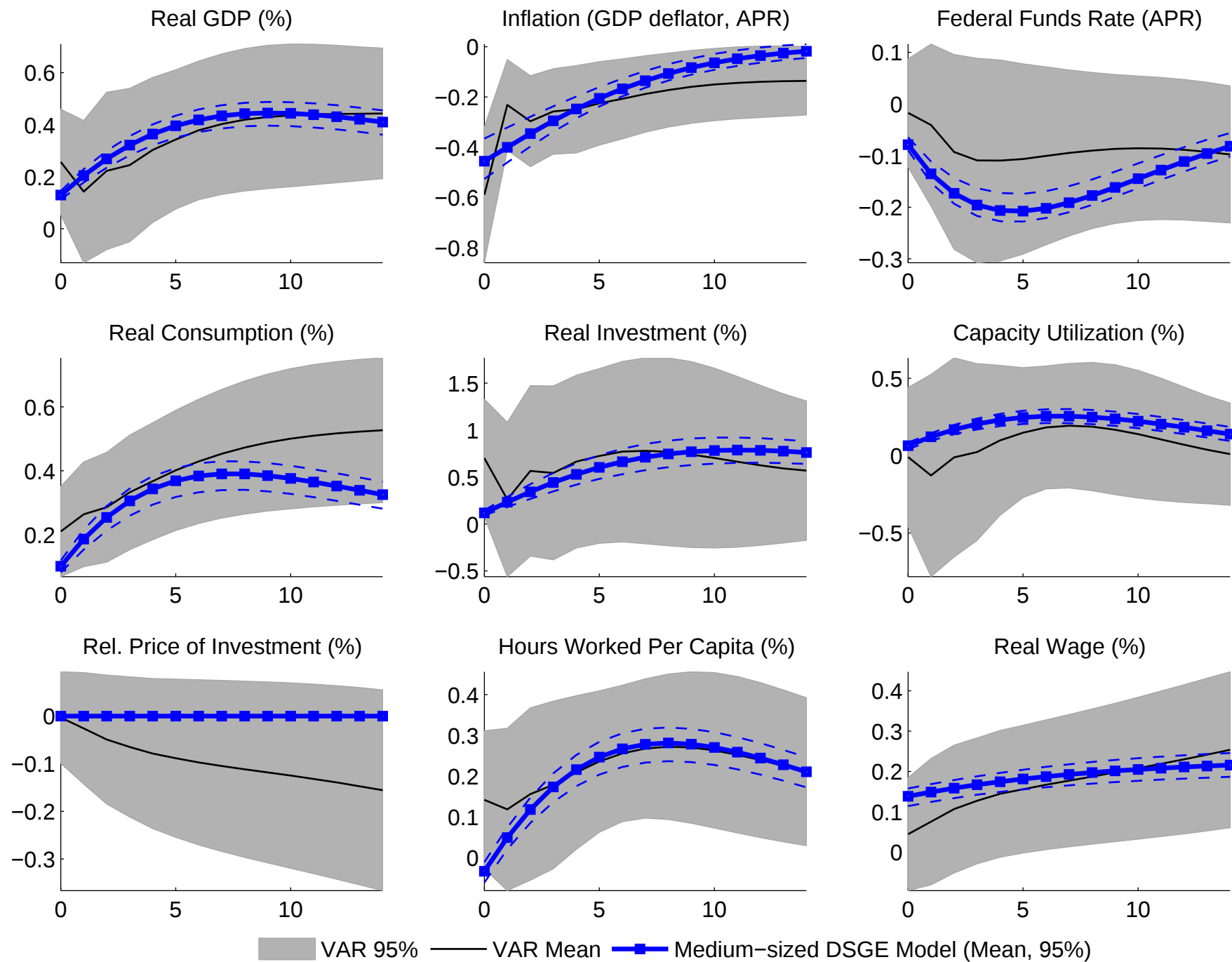


Figure 5: Dynamic Responses of Variables to an Investment Specific Technology Shock

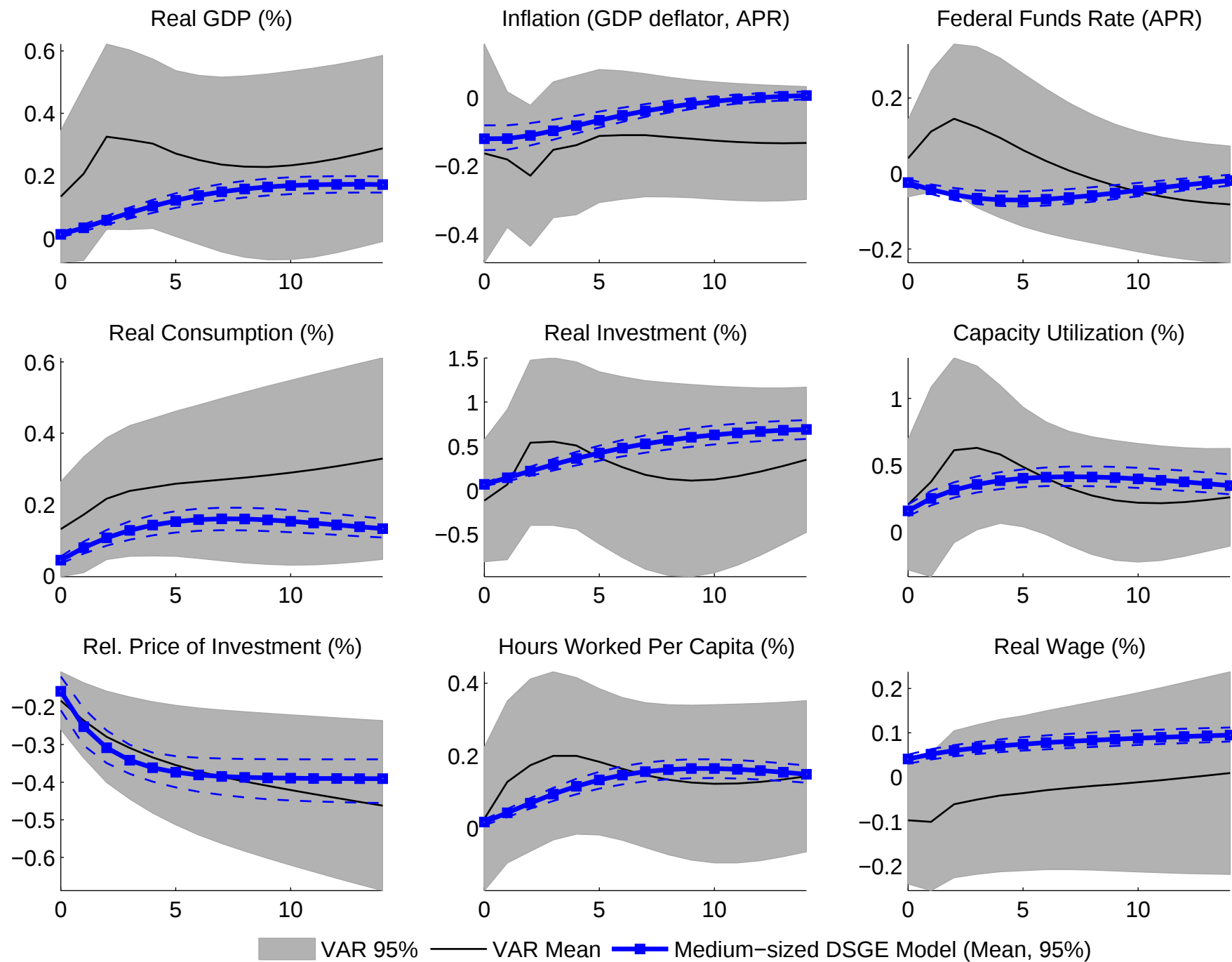


Figure 6: VAR Specification Sensitivity: Response to a Monetary Policy Shock

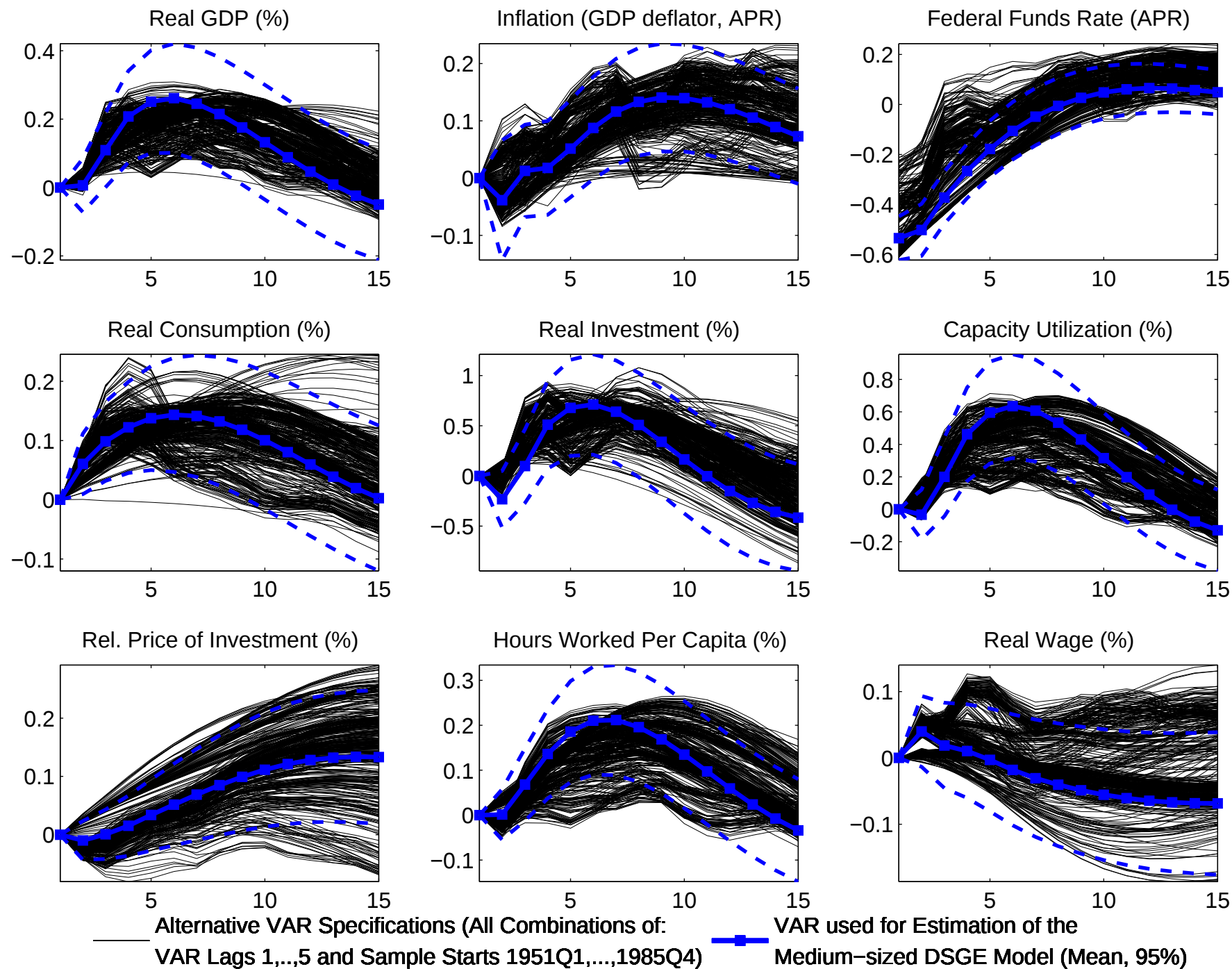


Figure 7: VAR Specification Sensitivity: Neutral Technology Shock

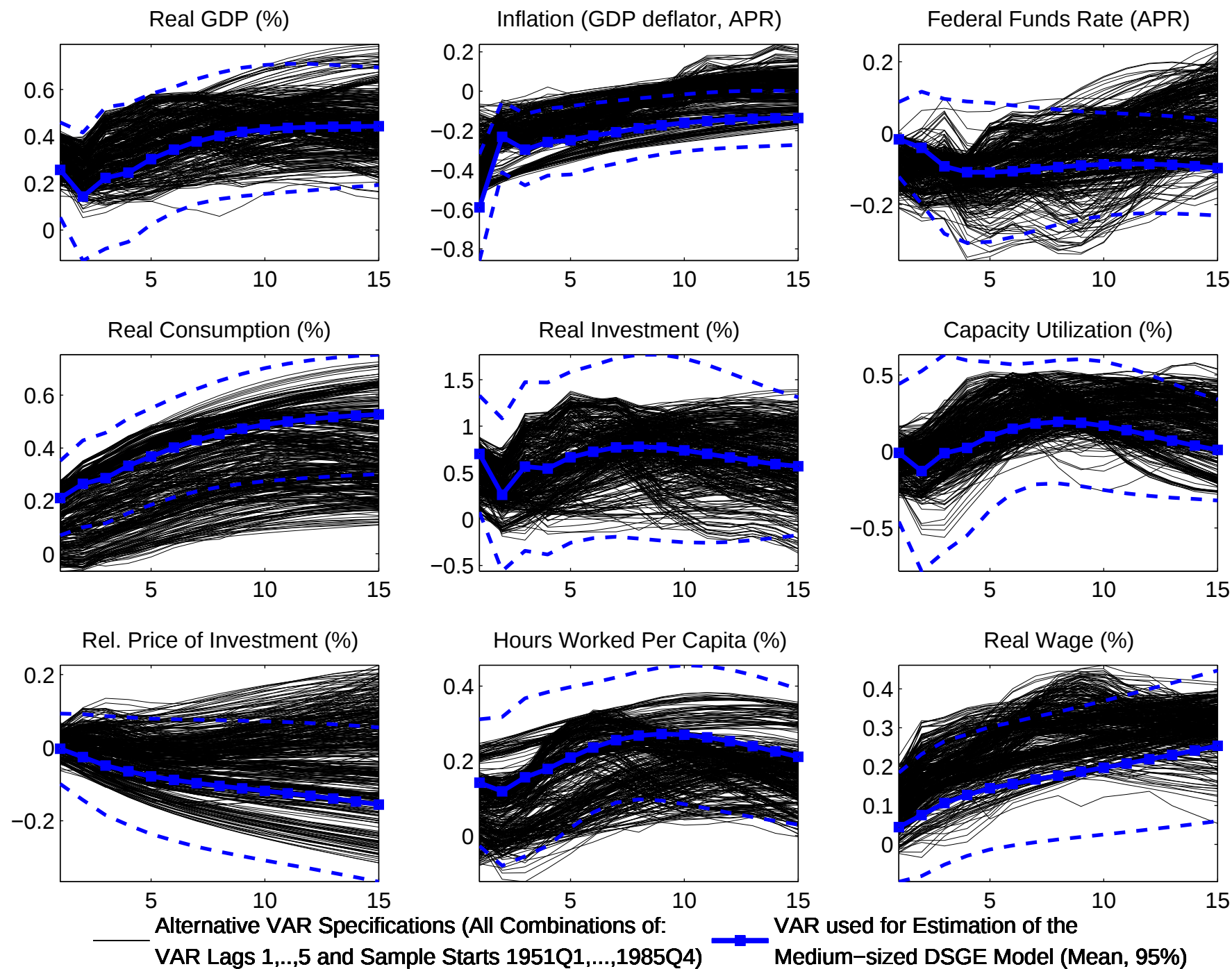


Figure 8: VAR Specification Sensitivity: Investment Specific Technology Shock

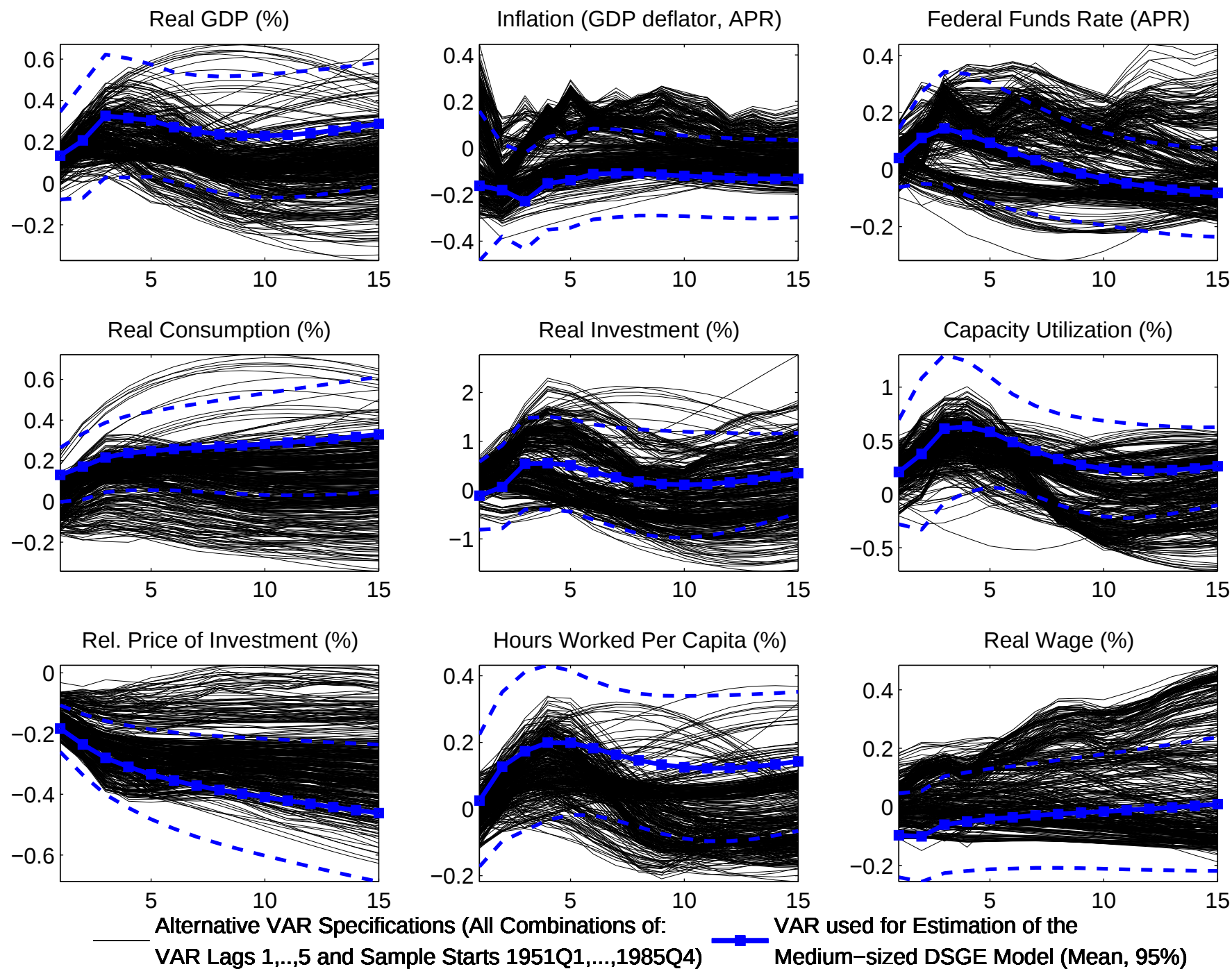


Figure 9: Priors and Posteriors of Estimated Parameters of the Medium-Sized DSGE Model

