Estimation, Solution and Analysis of Equilibrium Monetary Models

Assignment 5: The Japanese Economic Growth Slowdown in the 1990s and the Hours Worked Hypothesis.

The Japanese economic growth slowdown of the 1990s has attracted a great deal of attention. No doubt, the Japanese slowdown reflects many causes, including a weak banking sector[1] and perhaps bad monetary policy. This tutorial explores a factor that was suggested by Hayashi and Prescott[2], and which can be evaluated in a slightly modified version of the neoclassical growth model.[3] The 'hours worked hypothesis' is based on three observations: (i) in the late 1980s the Japanese passed legislation which had the effect of encouraging a reduction in working hours, (ii) per capita working hours fell during the decade of the 1990s by about 10 percent, and (iii) the output growth shortfall was about 1 percent a year, putting Japan about 10 percent below trend by 1990s. These are intriguing observations because if steady state hours worked fall by x percent in a neoclassical growth model, then we expect steady state output to fall by about x percent too. Moreover, along the transition, we would expect investment and employment to be below their new steady state levels. Qualitatively at least, this may sound like the Japanese economy in the 1990s. The purpose of this tutorial is to use the neoclassical growth model to see how far, quantitatively, this idea gets us towards constructing a quantitative account of Japan in the 1990s.

The hours worked hypothesis seems unlikely to explain the boom in investment and output that occured in Japan in the late 1980s. In this tutorial, we investigate whether this can be interpreted as an overinvestment boom in the sense of the analysis in assignment 4. However, the analysis there suggests that key features of overinvestment booms are missing unless we investigate them using a model with monetary policy. We find other problems with this hypothesis below. Still, the nonmonetary analysis of this tutorial sets up a useful benchmark.

The conclusion of this analysis is as follows. The hours worked hypothesis ap-

[1]See, for example, Caballero, Hoshi and Kashyap," Zombie Lending and Depressed Restructuring in Japan," 2004, University of Chicago Graduate School of Business, and Levon Barseghyan, "Non Performing Loans, Prospective Bailouts, and Japan's Slowdown" , 2003, Cornell University Department of Economics.

[2]See Hayashi and Prescott, 'The 1990s in Japan: A Lost Decade', http://minneapolisfed.org/research/wp/wp607.html

[3]The tutorial is based on ongoing research by Christiano and Fujiwara. Very substantial programming assistance was provided by Etienne Gagnon.

pears to provide a nice account of the fact that hours worked, output, investment and consumption fell to a lower growth path in the 1990s. However, the hypothesis does not help explain the boom of the late 1980s and early 1990s. We explored the potential for the overinvestment hypothesis to account for this, but ran into some trouble. In particular, that hypothesis implies that employment should have been high in the boom. However, employment appears not to have changed much in this period. One possibility is that a zero-income-effect on leisure utility function might work better. With this type of utility function, labor supply is only a function of the current real wage (that is, if utility is separable).[4]

Hopefully, this exercise has exhibited the potential for quantitative general equilibrium analysis to be helpful in evaluating alternative hypotheses about the data.

1. The Model

The first subsection describes the agents and technology. The next subsection derives the equations that characterize equilibrium. The following three sections then carry out the three steps to implement the linearization strategy for solving the model: (i) compute the steady state, (ii) linearize the equilibrium equations, (iii) solve the linearized system. Although the technology shock is explicitly introduced in the description of the model setup, it is not incorporated into the subsequent discussion, because this is already discussed in previous assignments. The last section describes our way of capturing the Japanese laws which encouraged a reduction in work effort. It is possible to do the assignment by just looking at the first subsection ('Basic Setup') and the last ('Experiment....').

1.1. Basic Setup

The preferences of the representative agent are as follows:

$$\sum \beta^t \frac{\left[\left(C_t - bC_{t-1}\right)\left(T_t - h_t\right)^{\psi}\right]^{1+\sigma}}{1+\sigma},$$

where T_t is the time endowment. Notice that this has a time subscript. This is because we will model the Japanese legislation designed to reduce work hours in

[4] This was suggested by Isabel Correia.

a reduced form way as a reduction in T_t. We posit that $\hat{T}_t$, the percent deviation of T_t from steady state, evolves as follows:

$$\hat{T}_t = \rho_T \hat{T}_{t-1}.$$

Here, C_t and h_t denote consumption and hours worked, respectively. The resource constraint is

$$a\left(u_t\right)\bar{K}_t + I_t + C_t \leq K_t^{\alpha}\left(z_t h_t\right)^{1-\alpha}, \tag{1.1}$$

where K_t denotes capital services. Also, the exogenous shock to technology evolves as follows:

$$z_t = z_{t-1}\mu_z \exp(\varepsilon_t), \tag{1.2}$$

where ε_t evolves as the technology shock in assignment 4:

$$\varepsilon_t = \rho_z \varepsilon_{t-1} + u_{t-p} + \xi_t,$$

where u_t and ξ_t are uncorrelated white noise processes (sorry for the switch in notation). With this setup, a shock to u_t shifts up $E_t\varepsilon_{t+p}$. As with any expectation, the higher value of ε_{t+p} need not actually be realized. That depends on the realization of ξ_{t+p}.

The technology for capital accumulation is:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + (1 - S\left(\frac{I_t}{I_{t-1}}\right))I_t,$$

where $\bar{K}_t$ is the physical stock of capital. Also, S and S' are zero on a steady state growth path and $\chi > 0$, where χ is S'', evaluated in steady state.

The stock of physical capital, $\bar{K}_t$, and the services of that stock, K_t, are related as follows:

$$K_t = u_t \bar{K}_t,$$

where u_t is unity in steady state. The cost of capital utilization, in units of the consumption goods, is given by:

$$a\left(u_t\right)\bar{K}_t,$$

where a is zero in steady state and $a', a'' > 0$. We define the curvature of a in steady state to be:

$$\sigma_a = \frac{a''}{a'}.$$

1.2. Equilibrium Conditions

The equilibrium allocations are the solution to a particular planning problem. In Lagrangian form, this problem is:

$$\begin{aligned}
&\sum \beta^t \{ u(C_t, C_{t-1}, h_t; T_t) \\
&+\lambda_t \left[\left(u_t \bar{K}_t \right)^{\alpha} (z_t h_t)^{1-\alpha} - C_t - \left(a\left(u_t\right) \bar{K}_t + I_t \right) \right] \\
&+\mu_t \left[(1-\delta)\bar{K}_t + (1 - S\left(\frac{I_t}{I_{t-1}}\right)) I_t - \bar{K}_{t+1} \right] \},
\end{aligned}$$

where

$$u(C_t, C_{t-1}, h_t; T_t) = \frac{\left[\left(C_t - bC_{t-1}\right)\left(T_t - h_t\right)^{\psi} \right]^{1+\sigma}}{1+\sigma}.$$

The first order condition with respect to C_t is:

$$u_{1,t} - \lambda_t + \beta u_{2,t+1} = 0,$$

or,

$$\begin{aligned}
&\left(C_t - bC_{t-1}\right)^{\sigma} \left(T_t - h_t\right)^{\psi(1+\sigma)} \\
-&\beta b (C_{t+1} - bC_t)^{\sigma} \left(T_{t+1} - h_{t+1}\right)^{\psi(1+\sigma)} = \lambda_t.
\end{aligned} \tag{1.3}$$

The first order condition with respect to h_t is:

$$u_{3,t} + (1-\alpha)\lambda_t \left(u_t \bar{K}_t \right)^{\alpha} z_t^{1-\alpha} h_t^{-\alpha} = 0, \tag{1.4}$$

or,

$$\psi (C_t - bC_{t-1})^{1+\sigma} \left(T_t - h_t\right)^{[\psi(1+\sigma)-1]} = (1-\alpha)\lambda_t \left(u_t \bar{K}_t \right)^{\alpha} z_t^{1-\alpha} h_t^{-\alpha}.$$

The first order condition with respect to I_t is:

$$\begin{aligned}
&-\lambda_t + \mu_t (1 - S\left(\frac{I_t}{I_{t-1}}\right)) \\
&-\mu_t S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \\
&+\beta \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 0.
\end{aligned} \tag{1.5}$$

To help interpret this equation, it is useful divide through by λ_t and the result in assignment 4:

$$P_{K',t} = \frac{\frac{dU_t}{d\bar{K}_{t+1}}}{\frac{dU_t}{dC_t}} = \frac{\mu_t}{\lambda_t},$$

to substitute in $P_{K',t}$. Then,

$$P_{K',t} = \frac{1}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}} - \frac{P_{K',t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2}{\frac{\lambda_t}{\beta\lambda_{t+1}}}.$$

See assignment 4 for an extended discussion of this expression.

The first order condition for $\bar{K}_{t+1}$ is:

$$-\mu_t + \beta\lambda_{t+1}\left[\alpha u_{t+1}^a \bar{K}_{t+1}^{\alpha-1}\left(z_{t+1}h_{t+1}\right)^{1-\alpha} - a\left(u_{t+1}\right)\right] + \beta(1-\delta)\mu_{t+1} = 0, \quad (1.6)$$

which can be written:

$$1 = \beta\frac{\lambda_{t+1}}{\lambda_t}\left\{\frac{\left[\alpha u_{t+1}^a \bar{K}_{t+1}^{\alpha-1}\left(z_{t+1}h_{t+1}\right)^{1-\alpha} - a\left(u_{t+1}\right)\right] + (1-\delta)P_{K',t+1}}{P_{K',t}}\right\}.$$

The object in braces is the rate of return on capital.

Finally, the first order condition for u_t is, after dividing through by $\bar{K}_t$

$$\alpha u_t^{\alpha-1}\bar{K}_t^{\alpha-1}\left(z_t h_t\right)^{1-\alpha} = a'\left(u_t\right). \quad (1.7)$$

In a decentralized competitive market, the rental rate on capital services, r_t^k, is:

$$r_t^k = \alpha K_t^{\alpha-1}\left(z_t h_t\right)^{1-\alpha}.$$

So, the first order condition for u_t is to equate the rental rate on capital services to the marginal cost of providing those services.

We follow the discussion in assignment 3 and scale the variables that grow in steady state. Let:

$$\begin{aligned} c_t &= \frac{C_t}{z_t} \\ i_t &= \frac{I_t}{z_t} \\ \bar{k}_{t+1} &= \frac{\bar{K}_{t+1}}{z_t} \\ \tilde{\lambda}_t &= \frac{\lambda_t}{z_t^\sigma}. \end{aligned}$$

We now display the scaled Euler equations. The resource constraint is:

$$(1)\ \left(\frac{u_t\bar{k}_t}{\mu_z}\right)^{\alpha} h_t^{1-\alpha} = c_t + a\left(u_t\right)\frac{1}{\mu_z}\bar{k}_t + i_t,$$

The capital law of motion is:

$$(2)\ \frac{(1-\delta)}{\mu_z}\bar{k}_t + (1 - S\left(\frac{\mu_z i_t}{i_{t-1}}\right))i_t = \bar{k}_{t+1}.$$

The consumption Euler equation is:

$$\begin{aligned}(3)\ (c_t - \frac{b}{\mu_z}c_{t-1})^{\sigma}\left(T_t - h_t\right)^{\psi(1+\sigma)}&\\ -\beta b\mu_z^{\sigma}(c_{t+1} - \frac{b}{\mu_z}c_t)^{\sigma}\left(T_{t+1} - h_{t+1}\right)^{\psi(1+\sigma)} &= \tilde{\lambda}_t,\end{aligned}$$

The labor Euler equation is

$$(4)\ \psi(c_t - \frac{b}{\mu_z}c_{t-1})^{1+\sigma}\left(T_t - h_t\right)^{[\psi(1+\sigma)-1]} = (1-\alpha)\,\tilde{\lambda}_t\left(\frac{u_t\bar{k}_t}{\mu_z}\right)^{\alpha} h_t^{-\alpha}$$

The investment Euler equation is:

$$\begin{aligned}(5)\ -\tilde{\lambda}_t + \tilde{\lambda}_t\tilde{P}_{K',t}(1 - S\left(\frac{\mu_z i_t}{i_{t-1}}\right))&\\ -\tilde{\lambda}_t\tilde{P}_{K',t}S'\left(\frac{\mu_z i_t}{i_{t-1}}\right)\frac{\mu_z i_t}{i_{t-1}}&\\ +\beta\mu_z^{\sigma}\tilde{\lambda}_{t+1}\tilde{P}_{K',t+1}S'\left(\frac{\mu_z i_{t+1}}{i_t}\right)\left(\frac{\mu_z i_{t+1}}{i_t}\right)^2 &= 0.\end{aligned}$$

The Euler equation for capital is:

$$\begin{aligned}(6)\ -\tilde{\lambda}_t\tilde{P}_{K',t} + \beta\mu_z^{\sigma}\tilde{\lambda}_{t+1}[\alpha u_{t+1}^{a}\bar{k}_{t+1}^{\alpha-1}\left(\mu_z h_{t+1}\right)^{1-\alpha} - a\left(u_{t+1}\right)]&\\ +\beta(1-\delta)\mu_z^{\sigma}\tilde{\lambda}_{t+1}\tilde{P}_{K',t+1} &= 0.\end{aligned}$$

The Euler equation for capital utilization is:

$$(7)\ \alpha u_t^{\alpha-1}\left(\frac{1}{\mu_z}\bar{k}_t\right)^{\alpha-1}\mu_{\Upsilon}\left(h_t\right)^{1-\alpha} - a'\left(u_t\right) = 0.$$

1.3. Steady State

The steady state is easy to compute. The capital accumulation equation is:

$$i = \bar{k}\left[1 - \frac{(1-\delta)}{\mu_z}\right]. \tag{1.8}$$

The consumption euler equation is:

$$c^{\sigma}(1 - \frac{b}{\mu_z})^{\sigma}\,(T-h)^{\psi(1+\sigma)}\,[1 - \beta b \mu_z^{\sigma}] = \tilde{\lambda}. \tag{1.9}$$

The capital labor ratio, after some manipulation, is:

$$\frac{h}{\bar{k}} = \left[\frac{1 - \beta(1-\delta)\mu_z^{\sigma}}{\mu_z^{1-\alpha}\beta\mu_z^{\sigma}\alpha}\right]^{\frac{1}{1-\alpha}}$$

Then, divide the consumption euler equation by the labor euler equation:

$$\frac{(T-h)\,[1 - \beta b\,(\mu_z)^{\sigma}]}{\psi c(1 - \frac{b}{\mu_z})} = \frac{\left(\frac{h}{\bar{k}}\right)^{\alpha}}{(1-\alpha)\left(\frac{1}{\mu_z}\right)^{\alpha}}. \tag{1.10}$$

Combining the resource constraint and the investment equation

$$\left(\frac{1}{\mu_z}\right)^{\alpha}\left(\frac{h}{\bar{k}}\right)^{1-\alpha}\bar{k} = c + \bar{k}\left[1 - \frac{(1-\delta)}{\mu_z}\right],$$

we obtain the following expression for consumption:

$$c = \left[\left(\frac{1}{\mu_z}\right)^{\alpha}\left(\frac{h}{\bar{k}}\right)^{1-\alpha} - \left(1 - \frac{(1-\delta)}{\mu_z}\right)\right]\bar{k}. \tag{1.11}$$

Use this to substitute out for consumption in (1.10) :

$$\frac{\left(\frac{T}{\bar{k}} - \frac{h}{\bar{k}}\right)[1 - \beta b \mu_z^{\sigma}]}{\left[\left(\frac{1}{\mu_z}\right)^{\alpha}\left(\frac{h}{\bar{k}}\right)^{1-\alpha} - \left(1 - \frac{(1-\delta)}{\mu_z}\right)\right]\psi(1 - \frac{b}{\mu_z})} = \frac{\left(\frac{h}{\bar{k}}\right)^{\alpha}}{(1-\alpha)\left(\frac{1}{\mu_z}\right)^{\alpha}}.$$

Solve this for $\bar{k}$:

$$\frac{T}{\bar{k}} = \frac{\psi(1 - \frac{b}{\mu_z})\left(\frac{h}{\bar{k}}\right)^{\alpha}\left[\left(\frac{1}{\mu_z}\right)^{\alpha}\left(\frac{h}{\bar{k}}\right)^{1-\alpha} - \left(1 - \frac{(1-\delta)}{\mu_z}\right)\right]}{(1-\alpha)\left(\frac{1}{\mu_z}\right)^{\alpha}[1 - \beta b \mu_z^{\sigma}]} + \frac{h}{\bar{k}}.$$

With $\bar{k}$ in hand, we can compute h from $\left(h/\bar{k}\right)\bar{k}$. Then, c is obtained from (1.11) and i is obtained from (1.8). The multiplier, $\tilde{\lambda}$, can be obtained from (1.9). Then, $\tilde{\mu}$ is just $\tilde{\lambda}$.

The derivative of the utilization function, a', is:

$$\alpha\left(\frac{1}{\mu_z}\right)^{\alpha-1}\left(\frac{h}{\bar{k}}\right)^{1-\alpha}=a'.$$

1.4. Linear Approximation

Linearly expanding the resource constraint:

$$(1)\ c\hat{c}_t+a'\frac{1}{\mu_z}\bar{k}\hat{u}_t+i\hat{\imath}_t-\left(\frac{\bar{k}}{\mu_z}\right)^{\alpha}h^{1-\alpha}\left[\alpha\left(\hat{u}_t+\hat{\bar{k}}_t\right)+(1-\alpha)\hat{h}_t\right]=0.$$

where

$$\hat{x}_t\equiv\frac{dx_t}{x}.$$

Note:

$$dx_t=x\hat{x}_t.$$

Linearly expanding what capital accumulation equation:

$$(2)\ \hat{\bar{k}}_{t+1}-\frac{(1-\delta)}{\mu_z}\hat{\bar{k}}_t-\frac{i}{\bar{k}}\hat{\imath}_t=0$$

Linearly expanding the consumption Euler equation:

$$\begin{aligned}(3)\ \sigma c^{\sigma}(1-\frac{b}{\mu_z})^{\sigma-1}(T-h)^{\psi(1+\sigma)}\left(\hat{c}_t-\frac{b}{\mu_z}\hat{c}_{t-1}\right)&\\ +\psi(1+\sigma)c^{\sigma}(1-\frac{b}{\mu_z})^{\sigma}(T-h)^{\psi(1+\sigma)-1}\left(T\hat{T}_t-h\hat{h}_t\right)&\\ -\sigma\beta b(\mu_z)^{\sigma}c^{\sigma}(1-\frac{b}{\mu_z})^{\sigma-1}(T-h)^{\psi(1+\sigma)}(\hat{c}_{t+1}-\frac{b}{\mu_z}\hat{c}_t)&\\ -\psi(1+\sigma)\beta b(\mu_z)^{\sigma}c^{\sigma}(1-\frac{b}{\mu_z})^{\sigma}(T-h)^{\psi(1+\sigma)-1}\left(T\hat{T}_{t+1}-h\hat{h}_{t+1}\right)-\tilde{\lambda}\hat{\tilde{\lambda}}_t&=0\end{aligned}$$

The labor equation is:

$$(4)\ (1+\sigma)\psi(1-\frac{b}{\mu_z})^{\sigma}(T-h)^{[\psi(1+\sigma)-1]}c^{1+\sigma}\left(\hat{c}_t-\frac{b}{\mu_z}\hat{c}_{t-1}\right)$$

$$+\psi\left[\psi\left(1+\sigma\right)-1\right](1-\frac{b}{\mu_z})^{1+\sigma}\left(T-h\right)^{[\psi(1+\sigma)-2]}c^{1+\sigma}\left(T\hat{T}_t-h\hat{h}_t\right)$$
$$-\left(1-\alpha\right)\tilde{\lambda}\left(\frac{\bar{k}}{\mu_z}\right)^{\alpha}h^{-\alpha}\left[\hat{\tilde{\lambda}}_t+\alpha\hat{\bar{k}}_t+\alpha\hat{u}_t-\alpha\hat{h}_t\right] \;=\; 0$$

The investment equation is:

$$
\begin{aligned}
(5) -\hat{\tilde{\lambda}}_t+\tilde{P}_{K'}\left[\hat{\tilde{\lambda}}_t+\hat{\tilde{P}}_{K',t}\right] & \\
-\tilde{P}_{K'}S''\left(\mu_z\right)^2\left[\hat{\imath}_t-\hat{\imath}_{t-1}\right] & \\
+\beta\mu_z^{\sigma}\tilde{P}_{K'}S''\left(\mu_z\right)^3\left[\hat{\imath}_{t+1}-\hat{\imath}_t\right] & = 0.
\end{aligned}
$$

The capital euler equation is:

$$
\begin{aligned}
&(6)\;\; -\tilde{\lambda}\tilde{P}_{K'}\left[\hat{\tilde{\lambda}}_t+\hat{\tilde{P}}_{K',t}\right]\\
&+\beta\mu_z^{\sigma}\tilde{\lambda}a'\hat{\tilde{\lambda}}_{t+1}\\
&+\beta\mu_z^{\sigma}\tilde{\lambda}[a'\left(\alpha\hat{u}_{t+1}+\left(\alpha-1\right)\hat{\bar{k}}_{t+1}+(1-\alpha)\hat{h}_{t+1}\right)-a'\hat{u}_{t+1}]\\
&+\beta(1-\delta)\mu_z^{\sigma}\tilde{\lambda}\tilde{P}_{K'}\left[\hat{\tilde{\lambda}}_{t+1}+\hat{\tilde{P}}_{K',t+1}\right]
\end{aligned}
$$

The utilization euler equation is:

$$(7)\;\; \left(\alpha-1\right)\left[\hat{u}_t+\hat{\bar{k}}_t-\hat{h}_t\right]-a'\sigma_a\hat{u}_t=0.$$

1.5. Solving the Linearized System

1.5.1. Canonical Form

The matrix representation of the above 7 equations is:

$$\alpha_0 z_{t+1}+\alpha_1 z_t+\alpha_2 z_{t-1}+\beta_0 s_{t+1}+\beta_1 s_t=0, \tag{1.12}$$

where

$$z_t=\begin{pmatrix}\hat{\bar{k}}_{t+1}\\ \hat{c}_t\\ \hat{\imath}_t\\ \hat{\tilde{\lambda}}_t\\ \hat{\tilde{P}}_{K',t}\\ \hat{u}_t\\ \hat{h}_t\end{pmatrix},\;\; s_t=\hat{T}_t.$$

The variable, s_t, evolves as follows:

$$s_t = P s_{t-1}, \tag{1.13}$$

where $P = \rho_T$.

1.5.2. Solution to Canonical Form

We seek a solution of the following form:

$$z_t = A z_{t-1} + B s_t, \tag{1.14}$$

where A and B are to be determined. Substituting (1.13) and (1.14) into (1.12) we find:

$$\begin{aligned} & \alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \\ = & \left[\alpha_0 A^2 + \alpha_1 A + \alpha_0\right] z_{t-1} + F s_t = 0, \end{aligned}$$

for all z_{t-1} and s_t. For this equation to be zero for all possible z_{t-1} and s_t, we must have that the matrix coefficients on z_{t-1} and s_t are both exactly zero:

$$\begin{aligned} \alpha_0 A^2 + \alpha_1 A + \alpha_0 &= 0, \\ F &= 0, \end{aligned}$$

where

$$F = (\beta_0 + \alpha_0 B) P + (\beta_1 + \alpha_1 B + \alpha_0 A B). \tag{1.15}$$

In addition, for analysis to be interesting, it must be that A has eigenvalues less than unity in absolute value, for otherwise, the system would be predicted to evolve away from steady state if z_{t-1} or $s_t \neq 0$. But, the system has been linearized around steady state, so that the equations are not necessarily meaningful far from steady state (for example of this, see the two-sector model in Chapter 6 of Stokey and Lucas.) Another reason to focus on A matrices with eigenvalues less than unity in absolute value is that the theorems in Stokey and Lucas (chapter 6) tell us that that corresponds to the linearization about the true solution, in the case of the neoclassical growth model. Those theorems also say that there is only one matrix A that satisfies this condition. Often, in economic models, A satisfying the eigenvalue condition is unique.

The matrix A can be found using standard software (see findandcheckA.m, on the web site). To find B, the vectorization operator is useful. Recall that the vectorization operator, $vec(\cdot)$, takes the columns of a matrix and stacks them into a colum vector:

$$vec(X) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ where } X = [x_1, x_2, ..., x_n] \, .$$

In MATLAB, this operation is achieved by $reshape(X, n \times m, 1)$, where m is the number of rows of X. Two properties of the vectorization operator include additivity, $vec(a + b) = vec(a) + vec(b)$, and

$$vec(A_1 A_2 A_3) = (A_3' \otimes A_1) \, vec(A_2).$$

Write

$$F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_7 \end{bmatrix},$$

so that

$$\begin{aligned} vec(F') &= \begin{bmatrix} F_1' \\ F_2' \\ \vdots \\ F_7' \end{bmatrix} = vec \left[P'\beta_0' + P'B'\alpha_0' + \beta_1' + B'\alpha_1' + B'A'\alpha_0' \right] \\ &= vec\left(P'\beta_0' + \beta_1'\right) + vec\left(P'B'\alpha_0' + IB'\alpha_1' + IB'A'\alpha_0'\right) \\ &= vec\left(P'\beta_0' + \beta_1'\right) + vec\left(P'B'\alpha_0'\right) + vec\left(IB'\alpha_1'\right) + vec\left(IB'A'\alpha_0'\right) \\ &= vec\left(P'\beta_0' + \beta_1'\right) + \{(\alpha_0 \otimes P') + (\alpha_1 \otimes I) + (\alpha_0 A \otimes I)\} \, vec(B') \\ &= d + q\delta, \end{aligned}$$

say, where $\otimes$ denotes the Kronecker product. Also, I is the identity matrix with dimension equal to that of s_t. In addition,

$$\begin{aligned} d &= vec\left(P'\beta_0' + \beta_1'\right) \\ q &= (\alpha_0 \otimes P') + (\alpha_1 \otimes I) + (\alpha_0 A \otimes I) \\ \delta &= vec(B'). \end{aligned}$$

Simply compute $\delta = -q^{-1}d$ and construct B from δ. To see how to construct B from δ, write:

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_7 \end{bmatrix},$$

so that

$$\delta = vec(B') = \begin{bmatrix} B_1' \\ B_2' \\ \vdots \\ B_7' \end{bmatrix}.$$

To obtain B from δ using MATLAB, execute $B = reshape(\delta, m, n_1)'$, where n_1 is the number of equations (i.e., the left dimension of B) and m is the number of shocks (i.e., the right dimension of B). After doing these equations, it is of course wise to do a check and verify that with the computed B, the matrix F in (1.15) is indeed zero. If it is, then by construction all the calculations have been done correctly. (The MATLAB routine, findandcheckB.m, computes B.)

1.6. Experiment: Reduction in Incentive to Work

The idea is that the Japanese laws encouraging a reduction in work effort corresponded to a sudden drop in the steady state value of T_t. We model the laws as coming in slowly. Thus, in the first period (the late 1980s) the economy was on the old steady state growth path. Suddenly, the steady state time endowment drops, with the actual time endowment converging slowly to that new steady state according to the rate implied by the magnitude of P. The actual time endowment in the old steady state is 1 and in the new steady state it is 0.93, corresponding to a drop of a little over 7 percent. We model the time endowment as:

$$\hat{T}_t = \rho \hat{T}_{t-1},$$

with

$$\hat{T}_t = \frac{T_t - T}{T}.$$

In periods $t < t^*$, this is zero, and in t^*, $\hat{T}_{t^*} = (1 - 0.93)/0.93$, reflecting that the actual time endowment in t^* is still the old steady state, while the new time endowment is lower. Thus, in the period when the laws are passed, $\hat{T}_t$ is suddenly very high: the actual time endowment is now above steady state.

To simulate this system, we first compute a sequence, $\hat{T}_t$, for $t = t^*, t^*+1, ...$. Then, we solve

$$z_t = Az_{t-1} + B\hat{T}_t, \tag{1.16}$$

for $t = t^*$, $t^*+1, ...$. Note that to solve this for t^*, we require z_{t^*-1}. Writing this out explicitly, we obtain:

$$z_{t^*-1} = \begin{pmatrix} \hat{\bar{k}}_{t^*} \\ \hat{c}_{t^*-1} \\ \hat{\imath}_{t^*-1} \\ \hat{\bar{\lambda}}_{t^*-1} \\ \hat{\bar{P}}_{K',t^*-1} \\ \hat{u}_{t^*-1} \\ \hat{h}_{t^*-1} \end{pmatrix}.$$

Some of these variables may not enter the system. If so, the corresponding column in A will be composed of zeroes. To construct z_{t^*-1}, consider $\hat{\bar{k}}_{t^*}$,

$$\hat{\bar{k}}_{t^*} = \frac{\bar{k}_{t^*} - \bar{k}}{\bar{k}}.$$

We set $\bar{k}_{t^*}$ to the *previous* steady state, and we set $\bar{k}$ to the *new* steady state. In particular, $\hat{\bar{k}}_t$ is zero for $t < t^*$ and then suddenly at t^*, $\hat{\bar{k}}_t$ jumps, since the new steady state is a lower capital stock. All the other variables in z_{t^*-1} should be treated in exactly the same way. For example,

$$\hat{c}_{t^*-1} = \frac{c_{t^*-1} - c}{c},$$

where c_{t^*-1} is the old steady state and c is the new steady state.

With this setting for z_{t^*-1} and with $\hat{T}_t$, $t = t^*, t^*+1, ...$ in hand, it is possible to simulate (1.16) for $t = t^*$, $t^*+1, ...$. What we are actually interested in, of course, is the original unscaled variables. So, after this simulation of the scaled variables, they should be converted to unscaled and then graphed.

One possible way to graph things is as follows. A given page might contain many pictures: consumption, investment, output, the price of capital, the real interest rate, $u'(c_t)/[\beta u'(c_{t+1})]$, hours worked, etc. Each picture should have two lines. One depicts the variables on the old steady state growth path. The second shows the convergence to the new steady state growth path starting at t^*.

Hopefully, the second picture should resemble the Japanese economy in several respects: a weak stock market, low real rate, low output, etc.

2. Questions

Following are exercises to be implemented using the code, japan.m, which is contained in the set of MATLAB programs associated with this assignment, and which is available online. The benchmark parameter values are:

$$\begin{aligned} \alpha &= 0.36,\ b = 0.6,\ \beta = 1.03^{-.25},\ \chi = 8,\ \delta = 0.02,\ \gamma = 1,\ \psi = 2.3,\ \mu_z = 1.005 \\ \rho_z &= 0.95,\ \rho_T = 0.85,\ \sigma_a = 0.01. \end{aligned}$$

According to these parameters values, the steady state growth rate of output is 0.5 percent per quarter, or 2 percent per year (see μ_z, which is called *xbar* in japan.m.) The period in which the time endowment shock occurs is 50 (time is in units of quarters). The news that technology will jump in the future occurs in period 40. The number of periods in the future that technology is expected to jump is 8 (this is the value of p). The amount by which technology is expected to jump is $\hat{z}$. Initially, we will set $\hat{z} = 0$. (Recall, $100\times\hat{z}$ represents the percent by which z is expected to be above steady state.) We will begin by focussing on the drop in steady state hours.

The periods in the model are matched up with calender time as follows. Period 0 corresponds to 1980 and period 40 corresponds to 1990. Crudely, the idea is that the arrival of information that future technology will be high triggers the investment and output boom. A couple years later the laws are imposed encouraging a reduction in work effort. The graphs generated by japan.m also include actual Japanese data as a useful benchmark.

1. Produce graphs for the benchmark parameterization. Note that the long run effects on output and employment match up reasonably well with what happened (this is why this experiment was done in the first place!). However, there are some notable misses. Consumption in the data is rather slow to come off its early 1990s peak relative to the model. The price of capital is very high, which is quite the reverse of what actually happened in Japan (only the data for the model on $P_{K'}$ are shown, the Japanese data are not).

2. Note how the rate of capital utilization in the model is quite low in the 1990s, and how this matches reasonably well with the Japanese data. Still,

our analysis in assignment 3 suggests that variable capital utilization may be behind the model's counterfactual jump in investment. So, consider changing σ_a to 10,000, thus shutting down variable capital utilization. Note that now investment does fall, though the price of capital continues to rise sharply.

3. A useful benchmark is to eliminate the 'special' frictions in the model, setting $\chi = 0.01$, $b = 0$, $\sigma_a = 10,000$, and setting $\rho_T = 0.05$. Now the 'standard' dynamics of the neoclassical growth model emerge: investment and employment immediately overshoot the new steady state growth path, and converge from below and consumption and the capital stock converge to the new path from below. The employment and output aspects of this seem particularly counterfactual.

4. Attempt an improvement over the experiment in question 3 by setting $b = 0.6$ (to ensure that consumption falls less quickly) and $\rho_T = 0.85$ (to ensure that employment falls more slowly). Redo the calculations. Note than now employment and investment actualy jump! Why is this? (Hint: with T_t expected to decline over time, now is relatively cheap - in utility terms - time to work. Might as well work a little extra, and store the results by increasing investment.)

5. The analysis of question 4 suggests that in addition to habit persistence and a slow convergence of the time endowment, adjustment costs in investment may be important. Set $\chi = 8$ again, and redo the calculations. Note that now we probably have about the best 'fit' of the model.

6. So far, nothing we have done explains the burst of investment and output - and to some extent consumption - that occured in the late 1980s. Possibly this phenomenon can be understood as an overinvestment boom like the one studied in assignment 4. To explore this further, let $\hat{z} = 0.20$. This means that in period 40 people suddenly expect that 8 periods in the future the state of technology will jump 20 percent. Redo the calculations with this change, and with the remaining parameters set as they are in question 5. Note how this produces a jump in investment and output. However, employment rises counterfactually, and the price of capital falls. The analysis of assignment 4 suggests that the fall in the price of capital would be reversed if we incorporated money and monetary policy. In addition, the rise in investment would be increased (possibly by as much as a factor of four,

according to preliminary analysis with Motto and Rostagno of the ECB), as would the rise in output. These would all put the model into closer conformity with the data. However, a significant problem is that these changes would amplify the response of employment, which is already too strong.