Growth and Fertility in the Long Run*

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Abstract

This paper develops a theory that accounts for three stylized facts concerning growth and fertility in the long run. First, economies start in a "Malthusian Regime" with stagnant living standards and high fertility. Second, ultimately a "Growth Regime" is reached in which per capita income grows at near-constant rates, and fertility is low. Third, the speed and timing of the fertility decline during the transition from the Malthusian Regime to the Growth Regime differs across countries. I show that this development pattern can be reproduced by a theory consisting of three key elements: an agricultural production function, an industrial production function, and the quantity-quality model of fertility. The transition from Malthusian stagnation to growth and the ensuing fall in fertility is an endogenous feature of the model that occurs inevitably, regardless of initial conditions or policies. The speed of the fertility transition depends on policies that affect the opportunity cost of education, namely education subsidies and child-labor restrictions.

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1 Introduction

Stylized facts have played an important role in the theory of economic growth, both as a tool for summarizing the main features of the data, and as a benchmark against which the success of alternative models can be evaluated. A limitation of the common list of growth facts, due to Kaldor (1963), is that it describes only the recent experience of developed countries. Kaldor's list starts with the observation that output per capita tends to grow over time. While this has been the case for a number of countries since the Industrial Revolution, most of history was characterized by stagnation in living standards. In this paper I develop a theory that accounts for a set of stylized facts which describe the behavior of growth and fertility in the long run. The theory generates an endogenous transition from stagnation to growth. Kaldor did not mention fertility or population in his list of growth facts, since population growth seemed to be of minor importance for growth in industrial countries. In contrast, in my theory interactions between income and population will be of central importance for explaining long-run growth.

I will concentrate on three facts that characterize long-run behavior of growth and fertility. The first fact is that all countries initially start out with a long phase of stagnating living standards and high fertility. I will refer to this phase of development as the "Malthusian Regime," since it is consistent with Thomas Malthus' theory of income and population. In the Malthusian Regime, population and output grow at about equal rates, which results in roughly constant output per capita. The second fact is that many countries ultimately arrive in a "Growth Regime." This phase of development is summarized by the Kaldor facts. In the Growth Regime there is sustained growth in output per capita, and fertility is low. Both regimes have in common that different countries in the same regime are similar to each other. Cross-country differences in output per capita or fertility before 1750, when all countries were in the Malthusian Regime, were small by modern standards. Likewise, in our days the industrialized countries that have arrived in the Growth Regime have similar living standards and fertility rates. In contrast, during the transition from Malthusian stagnation to growth there are striking cross-country differences in growth and fertility. In this line, a third stylized fact is that the speed and timing of the fertility decline during the transition from stagnation to growth differs widely across countries. In England, for instance, fertility remained high for about 100 years after output per capita started to grow, and only then fell rapidly to the low modern level. In this century, countries like Japan or Korea have completed their fertility transition in less than 30 years after the growth takeoff. Apart from being interesting in its own right, this third fact will serve as a test of the theory: if we find the right mechanism that explains the fertility decline associated with development, we should also be able to explain why the fertility decline differed so much across countries.

I show that the stylized facts can be accounted for by a theory consisting of three key elements: An agricultural production function, an industrial production function, and a quantity-quality fertility model. Both technologies are subject to exogenous productivity growth and use skilled and unskilled labor. The defining characteristic of the agricultural production function is that it also uses land. Since both production functions exhibit constant returns to scale in all inputs, the presence of the fixed factor land implies that there are diminishing returns to labor in the agricultural technology. This property is essential for generating the Malthusian Regime. People live for two periods in the model, and are either skilled or unskilled. As adults they decide on the number and on the education level of their children, i.e., they face a quantity-quality tradeoff. If parents want their children to be skilled, they have to send them to school. Children who do not go to school can engage in child labor. The cost of education therefore has two components, the direct schooling cost and the value of the child's time, that is, their potential income from child labor. If parents decide to send their children to school, they choose a smaller number of children, since skilled children are more expensive. This property is essential for generating the fertility decline during the transition to the Growth Regime.

An important contribution of the model is that it generates an endogenous transition from Malthusian stagnation to growth. Initially, as long as productivity in the industrial sector is low, only the agricultural technology is used. Wages are constant over time, and population growth is just fast enough to offset the improvements in agricultural technology. This Malthusian Regime is stable, since fertility is positively related to income. Temporary productivity shocks that raise wages also raise population growth, until wages are driven down to the earlier level. This part of the model is consistent with the economic history of the world before the Industrial Revolution. But at some point, productivity in the industrial sector reaches a sufficiently high level for the industrial technology to be used. Since the industrial technology has constant returns, population growth no longer depresses wages, so that for the first time wages start to rise. During the transition period, the proportion of the population working in the industrial sector increases. Since the industrial technology is assumed to be more skill-intensive, the relative number of skilled workers increases. The greater demand for skill also lowers average fertility, since skilled children are more expensive and parents substitute quality for quantity. Ultimately, the economy reaches a balanced growth path. Wages increase at the rate of technological progress, and population grows at a constant rate. The timing and speed of the fertility decline during the transition to growth depends on government policies that affect the relative cost of education. If parents have to pay for schooling and child labor is unrestricted, the fertility transition starts later and progresses slowly. In contrast, with public education and restrictions on child labor, the relative cost of education is small. Parents decide to have skilled children earlier, and fertility declines rapidly.

To check whether the model outcomes are quantitatively plausible, I consider the case of two countries that started to grow at the same time, but had very different government policies: (South) Korea and Brazil. Korea has a strong public education system, and child-labor restrictions are strictly enforced. Brazil has an ineffective public education system, and there is little enforcement of child-labor restrictions. As the model predicts, the fertility decline associated with development proceeded much faster in Korea than in Brazil. A calibrated version of the model generates fertility differences that are of a similar magnitude as those observed in the data. As additional tests of the theory, I consider the implications of the model for fertility differentials within a country, and for the distribution of income. As observed in the data, the model generates large fertility differentials between groups with different education under Brazilian policy assumptions, and small differences with Korean policies. As in the data, there is little inequality with the Korean policy, but a very unequal income distribution with the Brazilian policy.

I then turn to the country where the Industrial Revolution started, England. In England the demographic transition was spread out over about 100 years. Inequality increased during early industrialization, and started falling towards the end of the 19th century, about at the same time when fertility started to fall more quickly as well. There were major changes in government policies towards education and child-labor policies in the second half of the 19th century. When these policy changes are accounted for, the calibrated model generates paths for fertility and inequality that follow a pattern similar to what is observed in the data.

While there have been models for a long time that account for either the Malthusian or the Growth Regime, economists only recently have started to develop models that are consistent with both of them. Becker, Murphy and Tamura (1990) develop a model with two steady states, one in which wages stagnate and fertility is high, and another in which wages grow and fertility is low. The model does not generate a transition from stagnation to growth, since the Malthusian steady state is locally stable. Recent papers by Galor and Weil (1998), Tamura (1998), and Jones (1999) present models that are able to generate an endogenous transition from stagnation to growth. While these models are consistent with two of the stylized facts, they do not allow for any cross-country differences in the transition. The models also cannot be used to address fertility differentials or the income distribution. The production functions used in this paper are related to Laitner (1998) and especially Hansen and Prescott (1998). Hansen and Prescott explain the transition from stagnation to growth along similar lines, but they assume a specific exogenous function for fertility, instead of endogenizing fertility decisions. Raut (1991), Kremer and Chen (1999), Dahan and Tsiddon (1998), and Veloso (1999) examine the relationship of endogenous fertility and the income distribution. However, they do not generate a transition from stagnation to growth.

Relative to the existing literature, the main contributions of this paper are threefold. First, I show that the transition from Malthusian stagnation to growth can be generated by a model consisting of fairly standard elements. From this perspective, the Industrial Revolution and the ensuing fertility decline are not as surprising as it initially appears. Second, I introduce government policies to understand cross-country differences in the transition, and I calibrate the model to check the predictions. An important novelty of my formulation of the education cost is that I consider the opportunity cost of the child, through the potential wage in child labor. Child labor has previously been neglected in the fertility literature. My results show that the effects of child-labor restrictions are actually more important than direct education subsidies. Third, my model provides a perspective on the growth-and-inequality debate. Previous authors who have studied the relation of growth to inequality generally have not considered the role of fertility decisions.

The rest of the paper is organized as follows. In the next section, I compare the growth takeoff and fertility decline in Brazil and Korea, and describe government policies in these two countries. Section 3 introduces the model and defines a recursive competitive equilibrium. Section 4 derives a number of theoretical properties of the model, and Section 5 discusses the behavior of the model in the Malthusian Regime, the Growth Regime, and the transition between the two. In Section 6 I discuss my calibration procedure, and Section 7 uses the calibrated model to explain the transition experiences of Brazil and Korea. Section 8 uses the model to understand the evolution of fertility and the income distribution in England in the last 200 years. Section 9 concludes.

2 Growth, Fertility, and Policies in Brazil and Korea

In this section I describe the facts which I will later use as a test for my theory. I start with the evidence on growth and fertility decline, and then turn to policies in the fields of education and child labor.

Growth and Fertility

During the 1960s and 1970s, Brazil and Korea shared the distinction of being "Miracle Economies." Until about 1950 there was little growth in per capita income in both Brazil and Korea. After the Japanese occupation and the Korean war South Korea was in fact one of the poorest countries on the earth. There had been some earlier progress in Brazil, but living standards were still very low compared to the world leaders. After about 1950 for Brazil and 1960 for Korea, per capita income started to take off, and growth accelerated into the 1970s. Between 1970 and 1980, both countries achieved growth rates in real GDP per capita of almost 6% per year. In the 1980s, Korea continued to grow, while income per capita leveled off in Brazil.

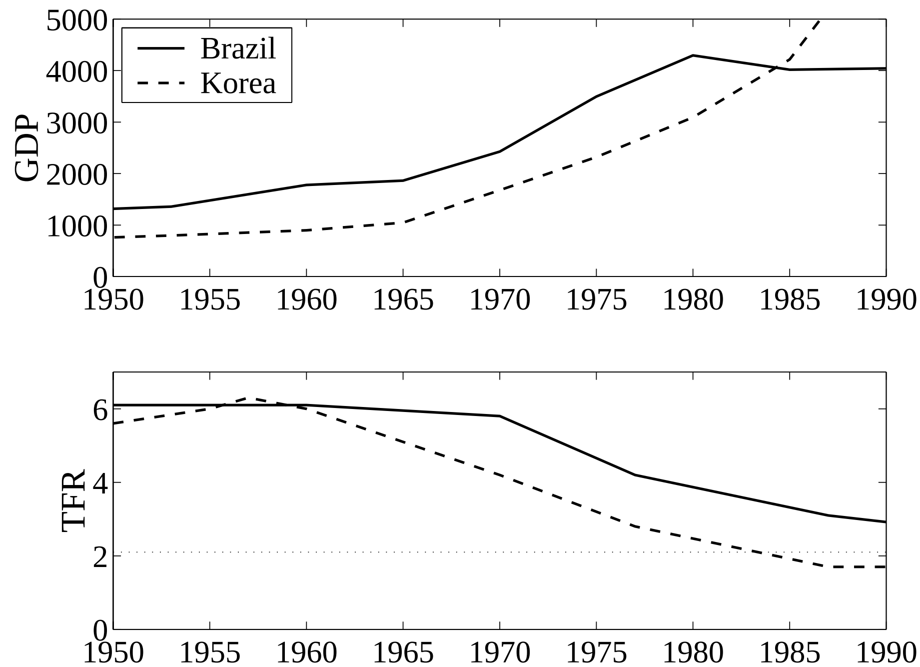

Figure 1: GDP per Capita and Total Fertility Rate in Brazil and Korea

At the time of the take-off fertility was high in both Korea and Brazil. The total fertility rate[1] was at 6.0 in both countries in 1960. Subsequently, fertility fell in both countries, but the decline was much faster in Korea. In Korea, the total fertility rate fell below the replacement level (the level at which population growth becomes zero if fertility stays constant) within about 25 years, or only one generation. In Brazil, fertility is still above the replacement level today. During most of the transition Brazilian families had on average one to two more children than Korean families. Figure 1 shows GDP per capita and the total fertility rate for the two countries. The fast fertility decline in Korea is especially remarkable given that GDP per capita was lower in Korea than in Brazil until 1984. Clearly, any theory that relies on a direct link of income per capita and fertility to explain the fertility decline is inconsistent with the evidence from Brazil and Korea.

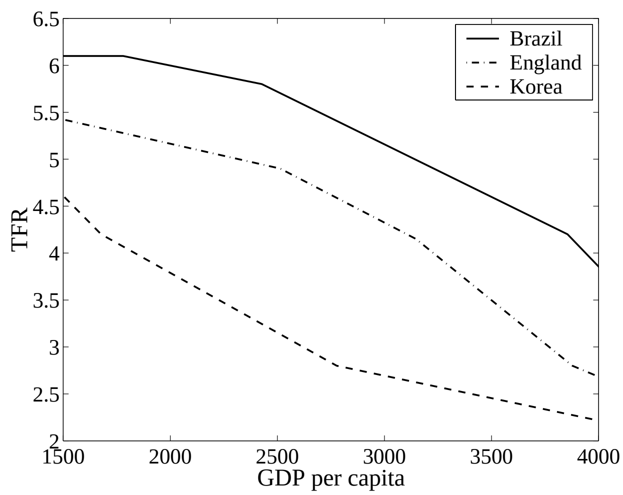

Figure 2: Fertility Decline relative to GDP per Capital in Brazil, Korea, and England

As another representation of the link of fertility to stages of development, Figure 2 plots the total fertility rate against GDP per capita. As a benchmark, I also included English data (covering the period 1820 to 1914) into the picture. As the first country to enter the Industrial Revolution, England experienced declining fertility much earlier than Brazil and Korea, and the English experience is similar to that of other European countries. Relative to income per capita, we see that the fertility decline progressed much faster in Korea than it had in England, whereas the decline is slower in Brazil. Most of the time,

[1]Total fertility rates are defined as the sum of age specific fertility rates in a given year. That is, if n_t^a is the average number of children born to women of age a in year t, the total fertility rate in year t is given by $TFR_t = \sum_{a=15}^{50} n_t^a$. The total fertility rate can be interpreted as the total number of children a woman can expect to have over her life time if age-specific fertility rates remain at the same level.

for a given level of income per capita the fertility differential between Brazil and Korea exceeds two children per family.

Policies

My model links the observed differences between Brazil and Korea to different policies in the areas of education and child labor. Korea has a strong public education system, and child labor is virtually absent. Brazil has an ineffective public education system, and child labor is widespread. The main difficulty in documenting these policies is that most of the time the differences lie not in the word of the law, but in enforcement and financing. Korea instituted a system of free, compulsory education from the ages of 6 to 12 in 1949, right after independence.[2] Implementation was delayed by the war, in which more than two-thirds of schools were destroyed. After the war the government instituted a highly successful "Compulsory Education Accomplishment Plan," and by 1959 the primary enrollment rate reached 96%. Strictly enforced compulsory education served as an effective constraint on child labor, in addition to direct child-labor restrictions.[3] According to the International Labor Organization (ILO), in 1960 only 1.1% of the children from zero to fifteen years were economically active, and by 1985 only .3% of the children between ages ten and fourteen participated in the labor market.

Year	Brazil	Korea
1960	2.6	3.2
1965	2.6	4.4
1970	2.9	5.6
1975	2.8	5.9
1980	3.0	6.8
1985	3.5	7.8

Source: Barro and Lee (1993).

Table 1: Average Years of Schooling of Population over Age 25

By the letter of the law, free and compulsory education was introduced in Brazil already in 1930.[4] In practice, however, primary schooling was simply not available in many rural areas, and to the present day the available schools are often of poor quality. In 1965 the primary enrollment rate was still below 50% in rural areas. The neglect of primary

[2]See Adams and Gottlieb (1993) or McGinn et al. (1980) for details.

[3]Margo and Fingan (1996) present evidence from U.S. data that a combination of compulsory schooling legislation with child-labor restrictions is more effective that child-labor restrictions alone.

[4]See Birdsall and Sabot (1996), Haussman and Haar (1978), or Plank (1996) for details on educational policies in Brazil.

	Primary		Secondary	
Year	Brazil	Korea	Brazil	Korea
1960	.95	.94	.11	.27
1965	1.0	1.0	.16	.35
1970	.72	1.0	.26	.42
1975	.88	1.0	.26	.56
1980	.99	1.0	.34	.76
1985	1.0	.96	.35	.95
1990	1.0	1.0	.39	1.0

Source: Barro and Lee (1993), World Tables.

Table 2: Adjusted Enrollment Ratios

	Brazil			Korea		
Year	Total	Male	Female	Total	Male	Female
1950	50.6	45.2	55.8			
1955				23.2	12.6	33.3
1960	39.3	35.6	42.6	29.4	16.6	41.8
1970	33.8	30.6	36.9	12.4	5.6	19.0
1980	25.5	23.7	27.2			
1991	20.1	19.9	20.3			
1995	16.7	10.7	20.3	2.0	.7	3.3

Source: UNESCO Statistical Yearbook, various editions.

Table 3: Adult (15+) Illiteracy Rates

education in Brazil can also be gauged from education finances. In 1960, less than 10% of public spending on education was directed to the primary sector, while the corresponding figure was about 70% for Korea. Child-labor regulation is less restrictive in Brazil in Korea. While Korea signed on to an ILO convention that rules out child labor under the age of 14, Brazil did not. The minimum age for employment in Brazil is now 12, and even this limit is not always enforced. In 1985 18.7% of the children between ages ten and fourteen participated in the labor market. Child labor is even more prevalent among male children, with 25.3% listed as economically active in 1987, and still 24.3% in 1990.

The different policies in Brazil and Korea are also reflected in educational outcomes. Table 1 shows average years of schooling for the adult population in Brazil and Korea from 1960 to 1985. In both countries average years of schooling increase over time, but the increase is much faster in Korea. In 1960, the difference in average years of schooling between Korea and Brazil was less than a year. By 1985, the difference increased to more than four years, so that Koreans on average have more than twice as much schooling as

Brazilians.

Table 2 shows adjusted enrollment ratios for primary and secondary education from 1960 to 1990. While primary enrollment rates are similar, Korea has a large and growing advantage in secondary education. The differences in enrollment rates understate the differences in education between Korea and Brazil, because they do not account for the quality of education. As a crude measure of educational success, in Table 3 I display adult illiteracy rates. In 1960, 39% of adult Brazilians were illiterate, compared to 29% of adult Koreans. In 1995, in Brazil illiteracy is still at 16.7%, while Koreans are almost completely literate with an illiteracy rate of only 2%. In Brazil illiteracy is high even among groups who went to school only recently. In 1991, illiteracy was 12.1% for the age group from 15 to 19 years. Thus even though the primary enrollment ratio is 100% in Brazil and Korea, the quality of education is clearly lower in Brazil.

The next section develops the model that I will use to link the transition experience of Brazil and Korea to the described government policies.

3 The Model

The economy is populated by overlapping generations of people who live for two periods, childhood and adulthood. Children receive education, do not enjoy any utility, and do not get to decide anything. Adults can be either skilled or unskilled, depending on their education. In each period there is a continuum of adults of each type; N_S is the measure of skilled adults, and N_U is the measure of unskilled adults. Adults decide on their consumption, labor supply, and on the number and the education of their children.

The single consumption good in this economy can be produced with two different methods. There is an agricultural technology that uses skilled labor, unskilled labor, and land as inputs, and an industrial technology that only uses the two types of labor. Production in each sector is carried out by competitive firms. I will now describe the two technologies in more detail, and then turn to the decision problem of an adult.

Technology

The agricultural technology uses the two types of labor and land. Since I want to abstract from land ownership and bequests, I assume that land is a public good. From the perspective of a small individual firm, there are constant returns to labor. However, since there is a limited amount of land, labor input by one firm imposes a negative externality on all other firms. The situation resembles a hunter-and-gatherer society, or a world of fishermen who all fish in the same sea. Output y_F (F stands for "Farm") for a firm that uses the agricultural technology and employs l_S units of skilled labor and l_U units of unskilled labor is given by:

$$y_F = \tilde{A}_F \, (l_S)^{\frac{\theta_S}{\theta_S+\theta_U}} \, (l_U)^{\frac{\theta_U}{\theta_S+\theta_U}}, \tag{1}$$

where:

$$\tilde{A}_F = A_F \, [(L_{FS})^{\theta_S}(L_{FU})^{\theta_U}]^{-\frac{1-\theta_S-\theta_U}{\theta_S+\theta_U}} \, (Z)^{1-\theta_S-\theta_U}.$$

Here A_F is a productivity parameter, L_{FS} and L_{FU} are the aggregate amounts of skilled and unskilled labor employed in the agricultural sector, and Z is the total amount of land. Thus the total amount of labor employed has a negative effect on the productivity of an individual firm. The specific form of the external effect was chosen such that the aggregate agricultural production function is given by (2) below.

Assumption 1 *The parameters θ_S and θ_U satisfy $\theta_S, \theta_U > 0$ and $\theta_S + \theta_U < 1$.*

Profit maximization implies that all firms choose the same ratio of skilled to unskilled labor. Aggregating (1) yields the following aggregate agricultural production function:

$$Y_F = A_F \, (L_{FS})^{\theta_S} \, (L_{FU})^{\theta_U} \, (Z)^{1-\theta_S-\theta_U}. \tag{2}$$

As far as the analysis in this paper is concerned, the main feature of the agricultural production function are decreasing returns to labor. The assumption of decreasing returns is essential for generating the Malthusian regime.[5]

The industrial production function, on the other hand, exhibits constant returns to scale even in the aggregate. From the perspective of an individual firm, the production function is given by:

$$y_F = A_I \, (l_S)^{1-\alpha} \, (l_U)^{\alpha}, \tag{3}$$

[5]The assumption of an external effect from labor, on the other hand, is not essential, and is used only to abstract from land ownership. Alternatives to this formulation include a socialistic society in which everyone owns an equal share of land, or an economy with a separate land-owning class. Each of these formulations would lead to the same qualitative results as the model described here.

and since there are no externalities, aggregate industrial output is:

$$Y_I = A_I \, (L_{IS})^{1-\alpha} \, (L_{IU})^{\alpha},$$

where L_{IS} and L_{IU} are aggregate amounts of skilled and unskilled labor employed in the industrial sector.

Assumption 2 *The parameter α satisfies* $0 < \alpha < 1$.

The productivities of both technologies grow at constant, though possibly different rates:

$$A'_F = \gamma_F A_F, \tag{4}$$
$$A'_I = \gamma_I A_I, \tag{5}$$

where $\gamma_F, \gamma_I > 1$. The state vector x in this economy consists of the productivity levels A_F and A_I in the agricultural and industrial sectors, and the measures N_S and N_U of skilled and unskilled people:

$$x \equiv \{A_F, A_I, N_S, N_U\}.$$

The only restriction on the state vector is that it has to consist of nonnegative numbers. Therefore the state space X for this economy is given by $\mathbf{R}^4_+$:

$$X \equiv \mathbf{R}^4_+.$$

In equilibrium wages are a function of the state. The problem of a firm in sector j, where $j \in \{F, I\}$, is to maximize profits subject to the production function, taking wages as given:

$$\max_{l_S, l_U} \left\{ y_j - w_S(x) l_S - w_U(x) l_U \right\}, \tag{6}$$

subject to (1) or (3) above. It will be shown in Proposition 1 below that firms will always be operating in the agricultural sector, while the industrial sector is only operated if the wages satisfy the following condition:

$$w_S(x)^{1-\alpha} \, w_U(x)^{\alpha} \leq A_I \, (1-\alpha)^{1-\alpha} \, \alpha^{\alpha}.$$

Profit maximization implies that wages equal marginal products in each sector. Writing labor demand as a function of the state, for the agricultural sector we get the following conditions:

$$w_S(x) = A_F \frac{\theta_S}{\theta_S + \theta_U} \frac{L_{FU}(x)^{\theta_U}}{L_{FS}(x)^{1-\theta_S}} Z^{1-\theta_S-\theta_U}, \tag{7}$$

$$w_U(x) = A_F \frac{\theta_U}{\theta_S + \theta_U} \frac{L_{FS}(x)^{\theta_S}}{L_{FU}(x)^{1-\theta_U}} Z^{1-\theta_S-\theta_U}. \tag{8}$$

If the industrial sector is operating, wages have to equal marginal products as well:

$$w_S(x) = A_I (1-\alpha) \left(\frac{L_{IU}(x)}{L_{IS}(x)}\right)^{\alpha} \qquad \text{if} \quad L_{IS}(x), L_{IU}(x) > 0, \tag{9}$$

$$w_U(x) = A_I \alpha \left(\frac{L_{IS}(x)}{L_{IU}(x)}\right)^{1-\alpha} \qquad \text{if} \quad L_{IS}(x), L_{IU}(x) > 0. \tag{10}$$

Instead of writing out the firms' problem in the definition of an equilibrium below, I will impose (7)–(10) as equilibrium conditions.

Preferences

I will now turn to the decision problem of the adults. Adults care about consumption and the number and utility of their children. In this model, there are no gender differences; every adult is able to produce children without outside help. The preference structure is an extension of Becker and Barro (1989) to the case of different types of children. Adults discount the utility of their children, and the discount factor is decreasing in the number of children. In other words, the more children an adult already has, the smaller is the additional utility from another child. I specialize the utility function to the constant-elasticity case. The utility of an adult who consumes c units of the consumption good and has n_S skilled children and n_U unskilled children is given by:

$$U(c, n_S, n_U) = c^{\sigma} + \beta(n_S + n_U)^{-\epsilon}[n_S V_S' + n_U V_U'].$$

Here V_S' is the utility skilled children will enjoy as adults, and V_U' is the utility of unskilled children, both foreseen perfectly by the parent. The parameter σ determines the elasticity of utility with respect to consumption, β is the general level of altruism, and ϵ is the elasticity of altruism with respect to the number of children. The utilities V_S' and V_U' are outside of the control of parents and are therefore taken as given. The utility of children depends on the aggregate state vector in the next period, and since there is a continuum of people, aggregates cannot be influenced by any finite number of people.

Assumption 3 *The utility parameters satisfy* $0 < \beta < 1$, $0 < \sigma < 1$, *and* $0 < \epsilon < 1$.

Adults are endowed with one unit of time, and they allocate their time between working and child-raising. Children are costly, both in terms of goods and in terms of time. Raising each child takes $\rho > 0$ units of the consumption good and fraction $\phi > 0$ of the total time available to an adult. Adults also have to decide on the education of their children. Children need a skilled teacher to become skilled. It takes fraction ϕ_S of a skilled adult's time to teach one child. Therefore, if parents want skilled children, they have to send their children to school and pay the skilled teacher. Children who do not go to school stay unskilled and work during childhood. Children can perform only the unskilled task, and one working child is equivalent to fraction ϕ_U of an unskilled adult who works full time. The parameter ϕ_U is smaller than one since children do not work from birth on, and since they are not as productive as adults. I also assume $\phi_U < \phi$, so that even after accounting for child labor there is still a net time cost associated with having unskilled children.

The budget constraint of an adult of type i, where $i \in \{U, S\}$, is given by:

$$c + (\phi w_i(x) + \rho)(n_S + n_U) + \phi_S w_S(x) n_S \leq w_i(x) + \phi_U w_U(x) n_U \tag{11}$$

The right-hand side is the full income of the adult plus the income from working unskilled children. On the left-hand side are consumption, the cost that accrues for every child (goods cost and time cost), and the cost for the education of children who go to school. For simplicity, adults are not restricted to choose integer numbers of children. Also notice that there is no uncertainty in this model. Whether a child becomes skilled does not depend on chance or unobserved abilities, but is under full control of the parent.

In equilibrium, wages and the utilities of skilled and unskilled people are functions of the state vector. The maximization problem of an adult of type i, where $i \in \{S, U\}$, is described by the following Bellman equation:

$$V_i(x) = \max_{c, n_U, n_S \geq 0} \left\{ c^\sigma + \beta (n_S + n_U)^{-\epsilon} [n_S V_S(x') + n_U V_U(x')] \right\}$$

subject to the budget constraint (11) and the equilibrium law of motion $x' = g(x)$.

The fact that only parents, not children, make educational decisions leads to a market imperfection. With perfect markets, children would be able to borrow funds to finance their own education. In equilibrium, children would have to be indifferent between going to school or not, so that net income of skilled and unskilled adults would be equalized. Since there are no differences in ability or stochastic income shocks, the market imperfection is necessary to create inequality in this model. I also rule out the possibility that parents write contracts that bind their children. Otherwise, parents could borrow funds from richer adults, and have their children pay back the loan to the children of the lender. Both assumptions are in line with reality: In the real world, children are usually not responsible for the debts of their parents, and we do not observe many children who receive loans to pay for their primary or secondary education.

Equilibrium

It will be shown in Section 4 below that the adults' problem has only corner solutions. Adults either send all their children to school, or none of them; there are never both skilled and unskilled children within the same family. It is possible, however, that adults of a specific type are just indifferent between sending all their children to school or none. In that case, some parents of a given type might decide to have skilled children, while others go for the unskilled variety. In equilibrium, the typical situation will be that all skilled parents have skilled children, while there are both unskilled parents with unskilled children and unskilled parents who send their children to school. In other words, there is upward intergenerational mobility.

In the definition of an equilibrium I have to keep track of the fractions of adults of each type who have skilled and unskilled children. The function $\lambda_{i\to j}(\cdot)$ gives the fraction of adults of type i who have children of type j, as a function of the state x. Of course, for each type of parent and for all $x \in X$ these fractions have to sum to one:

$$\lambda_{S\to S}(x) + \lambda_{S\to U}(x) = \lambda_{U\to S}(x) + \lambda_{U\to U}(x) = 1. \tag{12}$$

The policy function $n_j(i, \cdot)$ gives the number of children for i-type parents who have j-type children, as a function of the state. For example, $n_S(U, x)$ is the number of children born to an unskilled adult who decides to send the children to school. Notice that in equilibrium adults have only one type of children.

I will now introduce the remaining equilibrium conditions, starting with the determination of labor supply. Skilled adults distribute their time between working, raising and teaching their own children, and teaching children of unskilled parents. Therefore the total supply of skilled labor L_S is given by:

$$L_S(x) = [1 - (\phi + \phi_S)\ \lambda_{S\to S}(x)\ n_S(S,x) - \phi\ \lambda_{S\to U}(x)\ n_U(S,x)]\ N_S \\ - \phi_S\ \lambda_{U\to S}(x)\ n_S(U,x)\ N_U. \tag{13}$$

Notice that L_S only refers to skilled labor used for producing the consumption good; the time skilled adults spend as teachers is not counted. This is merely a matter of notational convenience, since it simplifies the market-clearing constraints for the labor market. Unskilled labor L_U is supplied by unskilled adults and by children who do not go to school:

$$L_U(x) = [1 - \phi\ \lambda_{U\to S}(x)\ n_S(U,x) - \phi\ \lambda_{U\to U}(x)\ n_U(U,x)]\ N_U \\ + \phi_U\ [\lambda_{S\to U}(x)\ n_U(S,x)\ N_S + \lambda_{U\to U}(x)\ n_U(U,x)\ N_U]. \tag{14}$$

In equilibrium, labor supply has to equal labor demand for each type of labor. I assume that skilled adults can perform both the skilled and the unskilled work, while unskilled adults can do unskilled work only. Going to school does not lead to a loss of the ability to do unskilled work. Under this assumption, the skilled wage cannot fall below the unskilled wage, because then all skilled adults would decide to do unskilled work:

$$w_S(x) \geq w_U(x). \tag{15}$$

Unless the economy starts out with a very high number of skilled adults, even without this assumption the skilled wage never falls below the unskilled wage. Still, it will be analytically convenient to impose (15). The market-clearing conditions for the labor market are:

$$L_{FS}(x) + L_{IS}(x) \leq L_S(x), \qquad = \text{ if } w_S(x) > w_U(x), \tag{16}$$

$$L_{FU}(x) + L_{IU}(x) = L_U(x) + [L_S(x) - L_{FS}(x) - L_{IS}(x)]\,. \tag{17}$$

The final equilibrium condition is the law of motion for population. Since I abstract from child mortality, the number of adults of a given type tomorrow is given by the number of children of that type today:

$$N'_S = \lambda_{S\to S}(x)\ n_S(S,x)\ N_S + \lambda_{U\to S}(x)\ n_S(U,x)\ N_U, \tag{18}$$

$$N'_U = \lambda_{S\to U}(x)\ n_U(S,x)\ N_S + \lambda_{U\to U}(x)\ n_U(U,x)\ N_U. \tag{19}$$

We now have all the ingredients at hand that are needed to define an equilibrium.

Definition 1 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium consists of value functions V_S and V_U, labor supply functions L_S and L_U, labor demand functions L_{FS}, L_{FU}, L_{IS}, and L_{IU}, wage functions w_S and w_U, mobility functions $\lambda_{S\to S}$, $\lambda_{S\to U}$, $\lambda_{U\to S}$, and $\lambda_{U\to U}$, all mapping X into $\mathbf{R}_+$, policy functions n_S and n_U mapping $\{S,U\}\times X$ into $\mathbf{R}_+$, and a law of motion g mapping X into itself, such that:*

(i) The value functions satisfy the following functional equation for $i \in \{S,U\}$:

$$V_i(x) = \max_{c,n_S,n_U\geq 0} \left\{ c^{\sigma} + \beta(n_S+n_U)^{-\epsilon}\left[n_S V_S(x') + n_U V_U(x')\right]\right\} \tag{20}$$

subject to the budget constraint (11) *and the law of motion $x' = g(x)$.*

(ii) For $i,j \in \{S,U\}$, if $\lambda_{i\to j}(x) > 0$, $n_j(i,x)$ attains the maximum in (20)*.*

(iii) The wages w_S and w_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (7)–(10) *and* (15)*.*

(iv) Labor supply L_S and L_U satisfies (13) *and* (14)*.*

(v) Labor supply L_S and L_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (16) *and* (17)*.*

(vi) The mobility functions $\lambda_{S\to S}$, $\lambda_{S\to U}$, $\lambda_{U\to S}$, and $\lambda_{U\to U}$ satisfy (12)*.*

(vii) The law of motion g for the state variable x is given by (4), (5), (18), *and* (19)*.*

Notice that the equilibrium conditions do not include a market-clearing constraint for the goods market, because it holds automatically by Walras' Law. In condition (ii) above, it is understood that parents choose only one type of children. In other words, saying that $n_S(S,x)$ attains the maximum in (20) means that $\{n_S = n_S(S,x), n_U = 0\}$ and the consumption c that results from the budget constraint maximize utility. Maximization is only required if a positive number of parents choose the type of children in question. Condition (iii) requires that wages equal marginal products and that the skilled wage does not fall below the unskilled wage, condition (iv) links labor supply to population and education time, condition (v) is the market-clearing condition for the labor market, condition (vi) requires that for each type of adult the fractions having skilled and unskilled children sum to one, and condition (vii) defines the law of motion.

Schooling Subsidies, Taxes, and Child Labor Restrictions

In the model described above, parents pay for the schooling of their children, and there are no restrictions on child labor. In this section I extend the model to allow for schooling subsidies and child-labor legislation. In the real world, most countries finance a large part of the education of their citizens, and child labor is usually subject to restrictions. In much of the analysis below I will be concerned with the effects of changes in these policies during the transition from agriculture to industry.

Incorporating child-labor restrictions is straightforward. The government can limit the amount of time that children work, which in the model amounts to lowering the parameter ϕ_U. To stay in the recursive framework, I let the government choose a function $\phi_U(\cdot)$ which determines how much time children work, depending on the state. Since restrictions can only lower the legal amount of child labor, I require $0 \leq \phi_U(x) \leq \phi_U$ for all x. In the applications below, I will consider a one-time change in child labor policy. Such a policy can be represented by using a $\phi_U(\cdot)$ function that changes once the industrial technology reaches a certain threshold level. For example, if child labor is abolished completely in the period when A_I reaches $\bar{A}_I$, the function is given by:

$$\phi_U(x) = \begin{cases} \phi_U & \text{if } A_I < \bar{A}_I, \\ 0 & \text{if } A_I \geq \bar{A}_I. \end{cases}$$

Introducing education policies is more complicated. The government cannot decree the amount of time required to teach a child. Instead, I assume that the government subsidizes a fixed amount of the schooling cost for all children at school. The expenditure is financed with a flat income tax, and budget balance is observed in every period. The government chooses a function δ that determines the fraction of the schooling cost to be paid by the government, where $0 \leq \delta(x) \leq 1$ for all x. Contingent on this function, the flat tax τ is chosen to observe budget balance. The tax rate is given by dividing the total expenditure on schooling subsidies by total wage income:

$$\tau(x) = \frac{\delta(x)\ \phi_S\ N'_S(x)\ w_S(x)}{L_S(x)\ w_S(x) + L_U(x)\ w_U(x) + \phi_S\ N'_S(x)\ w_S(x)}. \tag{21}$$

Here $N'_S(x)$ is shorthand notation for the total number of skilled children:

$$N'_S(x) = \lambda_{S \to S}(x)\ n_S(S, x)\ N_S + \lambda_{U \to S}(x)\ n_S(U, x)\ N_U.$$

Notice that for the computation of total labor income we have to add the income of the teachers to the wage of the usual workers. Teachers receive wages for their work and are taxed like all other adults in the model economy.

With taxes and the subsidy, the budget constraint of an adult of type i becomes:

$$\begin{aligned} c + (\phi\ (1-\tau(x))\ w_i(x) + \rho)\ (n_S + n_U) + (1-\delta(x))\ \phi_S w_S(x)\ n_S \\ \leq (1-\tau(x))\ (w_i(x) + \phi_U(x)\ w_U(x)\ n_U). \end{aligned} \tag{22}$$

We also have to adjust the expression for unskilled labor supply (14) for the child-labor policy:

$$\begin{aligned} L_U(x) =& [1 - \phi\ \lambda_{U\to S}(x)\ n_S(U,x) - \phi\ \lambda_{U\to U}(x)\ n_U(U,x)]\ N_U \\ &+ \phi_U(x)\ [\lambda_{S\to U}(x)\ n_U(S,x)\ N_S + \lambda_{U\to U}(x)\ n_U(U,x)\ N_U]. \end{aligned} \tag{23}$$

Apart from these changes, the definition of an equilibrium is parallel to the case without child labor and education policies.

Definition 2 (Equilibrium with Government Policy) *Given a government policy $\{\phi_U, \delta\}$, a recursive competitive equilibrium consists of a tax function τ, value functions V_S and V_U, labor supply functions L_S and L_U, labor demand functions L_{FS}, L_{FU}, L_{IS}, and L_{IU}, wage functions w_S and w_U, mobility functions $\lambda_{S\to S}$, $\lambda_{S\to U}$, $\lambda_{U\to S}$, and $\lambda_{U\to U}$, all mapping X into $\mathbf{R}_+$, policy functions n_S and n_U mapping $\{S,U\} \times X$ into $\mathbf{R}_+$, and a law of motion g mapping X into itself, such that:*

(i) The value functions satisfy the following functional equation for $i \in \{S,U\}$:

$$V_i(x) = \max_{c,n_S,n_U \geq 0} \left\{ c^\sigma + \beta(n_S + n_U)^{-\epsilon} \left[n_S V_S(x') + n_U V_U(x') \right] \right\}. \tag{24}$$

subject to the budget constraint (22) *and the law of motion $x' = g(x)$.*

(ii) For $i,j \in \{S,U\}$, if $\lambda_{i\to j}(x) > 0$, $n_j(i,x)$ attains the maximum in (24).

(iii) The tax function τ satisfies the government budget constraint (21).

(iv) The wages w_S and w_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (7)–(10) *and* (15).

(v) Labor supply L_S and L_U satisfies (13) *and* (23).

(vi) Labor supply L_S and L_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (16) *and* (17).

(vii) The mobility functions $\lambda_{S\to S}$, $\lambda_{S\to U}$, $\lambda_{U\to S}$, and $\lambda_{U\to U}$ satisfy (12).

(viii) The law of motion g for the state variable x is given by (4), (5), (18), *and* (19).

4 Analytical Results

This section derives a number of theoretical results that will be useful for describing the equilibrium behavior of the model. First, I analyze the two production sectors, and then I turn to the decision problem of an adult in the economy.

Production in the Agricultural and Industrial Sectors

I will now take a closer look at the two production sectors in the economy. The main result is that while the agricultural sector is always operating, industrial firms produce only if industrial productivity is sufficiently high relative to wages. The following proposition derives the condition that is necessary for production in industry.

Proposition 1 *Firms will be operating in the industrial sector only if the skilled and unskilled wages $w_S(x)$ and $w_U(x)$ satisfy the condition:*

$$w_S(x)^{1-\alpha}\ w_U(x)^{\alpha} \leq A_I\ (1-\alpha)^{1-\alpha}\ \alpha^{\alpha}. \tag{25}$$

Proof: The profit-maximization problem of a firm in the industrial sector is given by:

$$\max_{l_S,l_U} \left\{ A_I\ (l_S)^{1-\alpha}\ (l_U)^{\alpha} - w_S(x)\ l_S - w_U(x)\ l_U \right\}. \tag{26}$$

The first-order condition for a maximum with respect to l_U gives:

$$(l_S)^{1-\alpha} = \frac{w_U(x)(l_U)^{1-\alpha}}{A_I \alpha}.$$

Plugging this expression back into (26) yields a formulation of the profit maximization problem as a function of unskilled labor only:

$$\max_{l_U} \left\{ \frac{w_U(x)}{\alpha}\ l_U - w_S(x) \left(\frac{w_U(x)}{A_I \alpha} \right)^{\frac{1}{1-\alpha}} l_U - w_U(x)\ l_U \right\}. \tag{27}$$

Since this expression is linear in l_U, production in the industrial sector will be profitable only if we have:

$$\frac{w_U(x)}{\alpha} - w_S(x) \left(\frac{w_U(x)}{A_I \alpha} \right)^{\frac{1}{1-\alpha}} - w_U(x) \geq 0,$$

which can be rearranged to get:

$$w_S(x)^{1-\alpha}\ w_U(x)^{\alpha} \leq A_I\ (1-\alpha)^{1-\alpha}\ \alpha^{\alpha},$$

which is (25). □

The next proposition shows that in contrast to firms in the industrial sector, agricultural firms always operate.

Proposition 2 *For any skilled and unskilled wages $w_S(x)$ and $w_U(x)$ firms will be operating in the agricultural sector.*

Proof: The first-order necessary conditions for a maximum of the profit-maximization problem of a firm in agriculture are given by the wage conditions (7) and (8):

$$w_S(x) = A_F\ \frac{\theta_S}{\theta_S + \theta_U}\ \frac{L_{FU}^{\theta_U}}{L_{FS}^{1-\theta_S}}\ Z^{1-\theta_S-\theta_U}, \tag{28}$$

$$w_U(x) = A_F\ \frac{\theta_U}{\theta_S + \theta_U}\ \frac{L_{FS}^{\theta_S}}{L_{FU}^{1-\theta_U}}\ Z^{1-\theta_S-\theta_U}. \tag{29}$$

Since the problem is concave, the first-order conditions are also sufficient for a maximum. It is therefore sufficient to show that for any $w_S(x), w_U(x) > 0$ we can find values for skilled and unskilled labor supply L_{FS} and L_{FU} such that (28) and (29) are satisfied. The required values are given by:

$$L_{FS} = \left(\frac{A_F}{\theta_S + \theta_U} \right)^{\frac{1}{1-\theta_S-\theta_U}} \left(\frac{\theta_S}{w_S(x)} \right)^{\frac{1-\theta_S}{1-\theta_S-\theta_U}} \left(\frac{\theta_U}{w_U(x)} \right)^{\frac{\theta_U}{1-\theta_S-\theta_U}} Z$$

and:

$$L_{FU} = \left(\frac{A_F}{\theta_S + \theta_U} \right)^{\frac{1}{1-\theta_S-\theta_U}} \left(\frac{\theta_S}{w_S(x)} \right)^{\frac{\theta_S}{1-\theta_S-\theta_U}} \left(\frac{\theta_U}{w_U(x)} \right)^{\frac{1-\theta_U}{1-\theta_S-\theta_U}} Z,$$

which are positive for any positive wages $w_S(x)$ and $w_U(x)$. □

It is easy to check whether the industrial sector will be operated for a given supply of skilled and unskilled labor. We can use conditions (28) and (29) to compute wages in agriculture under the assumption that there is agricultural production only. If the resulting wages satisfy condition (25), the industrial technology is used. Skilled and unskilled labor is allocated so that the wage for each skill is equalized across the two sectors. If condition (25) is violated, production takes place in agriculture only.

In equilibrium, it will be the case that initially only the agricultural technology is used. Given that there is positive productivity growth in industry, at some point the industrial technology becomes sufficiently productive to be introduced alongside agriculture. An "Industrial Revolution" occurs, and ultimately the fraction of output produced in agriculture converges to zero. This behavior arises from an interaction between the properties of the two production sectors and the population dynamics in the model. Since population is determined by fertility decisions, I will now turn to the decision problem of an adult in the model economy.

The Decision Problem of an Adult

From the point of view of an adult, the utility of a potential skilled or unskilled child is given by a number that cannot be influenced. There are no individual state variables, and the utility of children is determined by fertility decisions in the aggregate, which adults take as given since there is a continuum of people. This allows us to analyze the decision problem of an adult in detail without solving for a complete equilibrium first. In this section, we will analyze the decision problem of an adult who receives wage $w > 0$ and who knows that skilled children will receive utility $V_S > 0$ in the next period, whereas unskilled children can expect $V_U > 0$. I restrict attention to positive utilities, because if children receive zero utility it is clearly optimal not to have any children. In order to keep notation simple, I will express the cost of children directly in terms of the consumption good. The cost for a skilled child is p_S, and the cost for an unskilled child is denoted as p_U. In this model, we have $p_S = \phi w + \phi_S w_S + \rho$ and $p_U = \phi w - \phi_U w_U + \rho$. Obviously, this implies that $p_S > p_U$; skilled children are always more expensive than unskilled children.

The analysis leads to two main results. The first one is that the problem of the adult has only corner solutions. That is, adults have either skilled or unskilled children, but there are no adults who have children of both kinds. The second result is that if an adult is just indifferent between skilled and unskilled children, the total expenditure on children is independent of the chosen type of children. If one type of children is more expensive, this will be made up exactly by a lower number of children.

Corner Solutions

I want to analyze the following maximization problem of an adult:

$$\max_{n_S, n_U \geq 0} \left\{ (w - p_S n_S - p_U n_U)^\sigma + \beta (n_S + n_U)^{-\epsilon} [n_S V_S + n_U V_U] \right\}. \tag{30}$$

An alternative way of formulating this problem is to imagine the adults as choosing the total education cost E they spend on raising children, and the fraction f of this cost that they spend on skilled children. The number of children is then given by $n_S = fE/p_S$ and $n_U = (1-f)E/p_U$. This formulation is more convenient to work with, and it is equivalent to the original one. In the new formulation, the maximization problem of the adult is:

$$\max_{0 \leq E \leq w, 0 \leq f \leq 1} \left\{ (w - E)^\sigma + \beta E^{1-\epsilon} (f/p_S + (1-f)/p_U)^{-\epsilon} [f V_S/p_S + (1-f) V_U/p_U] \right\}. \tag{31}$$

We are now in position to show the first main result.

Proposition 3 *Given Assumption 3, for any pair $\{E, f\}$ that attains the maximum in (31) we have either $f = 0$ or $f = 1$.*

Proof: See Appendix. □

Proposition 3 implies that adults have either skilled or unskilled children, but they never mix both types in one family. While the actual proof is a little tedious, the result is intuitive. If we had $\epsilon = 0$ (which is ruled out by Assumption 3), both the utility gained from having children and the cost of children would be linear in the numbers of the two types of children. If we have $V_S/p_S = V_U/p_U$, it has to be the case that the adult is indifferent between unskilled and skilled children, and any combination of the two. However, if we now have $\epsilon > 0$, as assumed, the term $(f/p_S + (1-f)/p_U)^{-\epsilon}$ in (31) becomes a convex function of f, and the adult will choose a corner solution.

As a graphical illustration, Figure 3 shows indifference curves describing the preferences over skilled and unskilled children for a given level of consumption. The indifference curves are concave. Since the budget line is linear, the parent always chooses a corner. In the figure, the maximal indifference curve crosses the budget line at both ends. In this situation, the parent is willing to specialize either in skilled or in unskilled children, but any convex combination of the two would be inferior.

Given the fact that there are only corner solutions, we can determine the optimal number of children by separately computing the optimal choices assuming that there are only

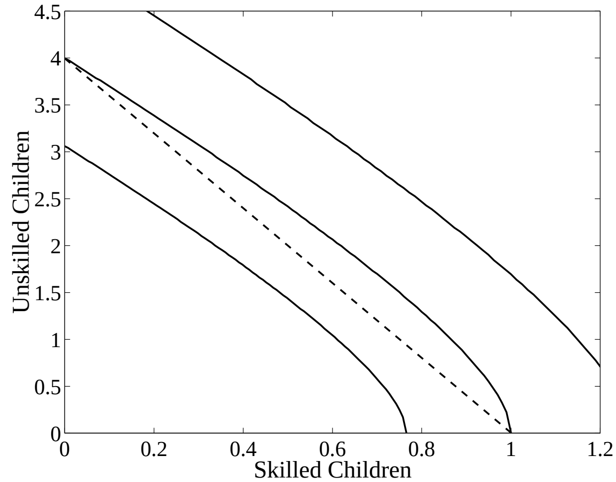

Figure 3: Indifference Curves

unskilled or only skilled children. We can then compare which type yields higher utility. Parents who decide to have children of type i solve:

$$\max_{0 \leq n_i \leq w/p_i} \left\{ [(w - p_i n_i)]^{\sigma} + \beta (n_i)^{1-\epsilon} V_i \right\}.$$

The first-order condition is:

$$-\sigma p_i (w - p_i n_i)^{\sigma - 1} + \beta(1-\epsilon)(n_i)^{-\epsilon} V_i \leq 0,$$

and this equation holds with equality if $n_i > 0$. In fact, since the marginal utility of an additional child tends to infinity if the number of children goes to zero, and the marginal utility from consumption tends to infinity if consumption goes to zero, there is an interior solution, characterized by:

$$\beta(1-\epsilon)(n_i)^{-\epsilon} V_i = \sigma p_i (w - p_i n_i)^{\sigma - 1},$$

or:

$$\beta(1-\epsilon)(w - p_i n_i)^{1-\sigma} V_i = \sigma p_i (n_i)^{\epsilon}. \tag{32}$$

We cannot solve for n_i explicitly apart from certain parameter combinations, but we can be sure that there is a unique n_i solving (32): the right-hand side equals zero if $n_i = 0$ and is strictly increasing in n_i, and the left-hand side is strictly decreasing in n_i and equals zero if $n_i = w/p_i$. The second-order condition for a maximum is given by:

$$-\sigma(1-\sigma)p_i^2(w-p_in_i)^{\sigma-2} - \beta\epsilon(1-\epsilon)(n_i)^{-\epsilon-1}V_i < 0, \tag{33}$$

which is satisfied since we assume $0 < \sigma < 1$ and $0 < \epsilon < 1$. The first-order condition is therefore necessary and sufficient for a maximum.

The next question is how the optimal number of children varies with the wage w and the utility of children V_i.

Proposition 4 *The optimal number of children n_i is increasing in V_i and in the wage w.*

Proof: Totally differentiating (32) gives:

$$n_i\ dV_i + (1-\sigma)(w-p_in_i)n_iV_i\ dw = [\epsilon V_i + (1-\sigma)p_in_iV_i]\ dn_i.$$

We therefore have:

$$\frac{dn_i}{dV_i} = \frac{n_i}{\epsilon V_i + (1-\sigma)p_in_iV_i} > 0, \tag{34}$$

and:

$$\frac{dn_i}{dw} = \frac{(1-\sigma)(w-p_in_i)n_iV_i}{\epsilon V_i + (1-\sigma)p_in_iV_i} > 0. \tag{35}$$

□

Thus children are a normal good in this model. On the other hand, if the cost of children p_i is directly proportional to the wage w, as in the case of a pure time cost for children, the optimal number of children decreases with the wage. To see this, assume that the time cost of raising one child of type i is ϕ, so that the price of children is $p_i = \phi w$. Plugging this into (32) and bringing w to the right-hand side we get:

$$\beta(1-\epsilon)(1-\phi n_i)^{1-\sigma}V_i = \sigma\phi w^{\sigma}(n_i)^{\epsilon}. \tag{36}$$

Totally differentiating yields:

$$\frac{dn_i}{dw} = -\frac{\sigma n_i}{\epsilon w} < 0. \tag{37}$$

Thus if the cost of children is a pure time cost, the substitution effect outweighs the income effect, and the optimal number of children decreases with income.

Another important property of the decision problem of an adult is that if the adult is indifferent between skilled and unskilled children, the total expenditure on children does not depend on the type of the children.

Proposition 5 *An adult is indifferent between skilled and unskilled children if and only if the costs and utilities of children satisfy:*

$$\frac{V_S}{(p_S)^{1-\epsilon}} = \frac{V_U}{(p_U)^{1-\epsilon}}. \tag{38}$$

If an adult is indifferent, the total expenditure on children does not depend on the type of children that is chosen.

Proof: It is helpful to consider the formulation of the problem in which adults choose the total education cost E, so that the number of children equals E/p_i for adults who choose to have children of type i. The maximization problem in this formulation is:

$$\max_{0\leq E\leq w/p_i} \left\{(w-E)^\sigma + \beta(E/p_i)^{1-\epsilon}V_i\right\}. \tag{39}$$

This can also be written as:

$$\max_{E\geq 0} \left\{(w-E)^\sigma + \beta(E)^{1-\epsilon}\frac{V_i}{(p_i)^{1-\epsilon}}\right\}. \tag{40}$$

Since the costs and utilities of children enter only in the last term, clearly an adult is indifferent between skilled and unskilled children if and only if:

$$\frac{V_S}{(p_S)^{1-\epsilon}} = \frac{V_U}{(p_U)^{1-\epsilon}}, \tag{41}$$

Notice that this condition does not depend on the wage of the adult. Also, if condition (41) is satisfied, adults face the same maximization problem regardless whether they decide for unskilled or skilled children. This implies that the optimal total education cost E does not depend on the type of the children. The higher cost of having skilled children will be exactly made up by a lower number of children. □

Implications for Equilibrium Behavior

Propositions 3 and 5 have important implications for intergenerational mobility in the model. Simply put, Propositions 5 states that for given utilities of skilled and unskilled children, the ratio of the prices of skilled and unskilled children determines whether parents send their children to school. As long as the wage for skilled labor is higher than the unskilled wage, skilled children are relatively cheaper for skilled parents, since $w_S > w_U$ implies:

$$\frac{\phi w_S + \phi_S w_S + \rho}{\phi w_S - \phi_U w_U + \rho} < \frac{\phi w_U + \phi_S w_S + \rho}{\phi w_U - \phi_U w_U + \rho}.$$

The term on the left-hand side is the ratio of the prices for skilled and unskilled children for skilled adults, and the right-hand side is the ratio for unskilled adults. The cost of time is higher for skilled adults, because the skilled wage is higher than the unskilled wage. Since the opportunity cost of child rearing makes up a larger fraction of the cost of unskilled children, unskilled children are relatively more expensive for skilled parents. The only case when this is not true is when the skilled and unskilled wage is the same. However, in equilibrium the skilled wage is always going to be higher, with the possible exception of the initial period. If both wages were equal in any given period, all adults in the preceding period would have decided to have unskilled children, since they are cheaper to educate. In equilibrium there always have to be some skilled children, so this situation never arises.

Since the relative price of skilled and unskilled children differs for skilled and unskilled parents, it can never be the case that both types of adults are indifferent between the two types of children at the same time. Since skilled children are relatively cheaper for skilled parents, in equilibrium there are always skilled parents who have skilled children. Otherwise, there would be no skilled children at all, which cannot happen in equilibrium. Likewise, there are always unskilled adults with unskilled children.

Taking these facts together, exactly three situations can arise in any given period. The first possibility is that skilled parents strictly prefer skilled children, while unskilled parents strictly prefer unskilled children. In that case, there is no intergenerational mobility. The second possibility is that skilled parents are indifferent between the two types of children, while all unskilled parents have unskilled children. The third option is that all skilled parents have skilled children, while the unskilled adults are indifferent between the two types. This last case is the typical one along an equilibrium path, as will be explained in more detail later. In this situation, there is upward intergenerational mobility, because some unskilled adults have skilled children, but no downward mobility.

The following corollary sums up the implications of these results for an equilibrium.

Corollary 1 *In equilibrium, for any $x \in X$ such that $w_S(x) > w_U(x)$, the following must be true:*

- *A positive fraction of skilled adults has skilled children, and a positive fraction of unskilled adults has unskilled children:*

$$\lambda_{S\to S}(x), \lambda_{U\to U}(x) > 0.$$

- *Just one type of adult can be indifferent between the two types of children:*

$$\lambda_{S\to U}(x) > 0 \text{ implies } \lambda_{U\to S}(x) = 0,$$
$$\lambda_{U\to S}(x) > 0 \text{ implies } \lambda_{S\to U}(x) = 0.$$

- *Specifically, $\lambda_{S\to U}(x) > 0$ implies:*

$$\left(\frac{\phi w_S(x) + \phi_S w_S(x) + \rho}{\phi w_S(x) - \phi_U w_U(x) + \rho}\right)^{1-\epsilon} = \frac{V_S(g(x))}{V_U(g(x))},$$

and $\lambda_{U\to S}(x) > 0$ implies:

$$\left(\frac{\phi w_U(x) + \phi_S w_S(x) + \rho}{\phi w_U(x) - \phi_U w_U(x) + \rho}\right)^{1-\epsilon} = \frac{V_S(g(x))}{V_U(g(x))}.$$

Proof: Follows directly from Proposition 5. □

5 Outline of the Behavior of the Model

Assuming that the economy starts at a time when productivity in industry is low compared to agriculture, the economy evolves through three different regimes: The Malthusian Regime, the transition, and the Growth Regime. In the Malthusian Regime the industrial technology is too inefficient to be used for some time. Therefore the model behaves like one in which there is an agricultural sector only. The economy displays Malthusian features—wages stagnate, and population growth offsets any improvements in productivity. The economy reaches a stable steady state in which wages are constant and population growth just offsets productivity growth in agriculture. If there were sudden improvements in technology, per-capita incomes would rise only temporarily, until higher population growth makes up for the higher productivity.

The transition starts when productivity in industry becomes high enough for the industrial technology to be introduced. Since the industrial technology does not exhibit decreasing returns, population growth no longer offsets productivity growth, so that wages start to rise. If productivity growth in industry is sufficiently high, the fraction of output produced in industry will increase, until the agricultural sector ultimately becomes negligible. The economy will then reach a second steady state, the Growth Regime. Here the model behaves like one in which there is the industrial technology only. Whether population growth and fertility is higher in the Growth Regime than in the Malthusian Regime is determined by the relative importance of skill in the two technologies. If the industrial technology is sufficiently skill-intensive, in the Growth Regime most children will go to school. Since schooling is costly, this will tend to lower fertility and population growth. On the other hand, as wages grow the physical cost ρ of children becomes less important. This effect makes children relatively cheaper in the growth regime, which will tend to increase fertility. Which effect dominates depends on the specific parameters chosen. During the transition, it is possible that fertility first increases in response to higher wages, but decreases later as the industrial technology starts to dominate and more children go to school.

The transition can also be influenced by public policy. Both an education subsidy and child-labor restrictions lower the relative cost of skilled children. Therefore both policies have a positive effect on the number of children going to school. The effects on fertility, however, are different. Since a subsidy lowers the cost of children, an education subsidy tends to increase fertility, even though more children are going to school. Child labor restrictions, on the other hand, increase the cost of children, and therefore lead to lower fertility. Since in this model inequality is linked to the relative cost of skilled and unskilled children, both policies decrease inequality in subsequent generations. I will now analyze the three regimes in more detail.

The Malthusian Regime

It was shown in Section 4 that if productivity in the industrial sector is low, the industrial technology is not used. If productivity is low enough so that the industrial technology will not be used any time in the near future, the economy behaves approximately like one in which the industrial technology does not exist at all. In this regime, the model exhibits Malthusian features. That is, wages are constant, and population growth just offsets productivity growth. Sudden improvements in productivity or sudden decreases in population lead to temporarily higher wages, until higher population growth drives wages back to the steady-state values.

There are two key features of the model that generate the Malthusian steady state. First, it is important that children are a normal good, as shown in Proposition 4. This property ensures that population growth increases once improvements in technology lead to higher wages. Because the agricultural technology exhibits decreasing returns, higher population growth depresses wages and pushes the economy back to the steady state. If the income effect were negative (a common assumption in fertility models that do not consider a quantity-quality tradeoff), higher wages would lead to less population growth, which increases wages even further.

The second key assumption is that there is a goods cost ρ for each child. Without this cost, it would be possible that population growth stays ahead of productivity growth, and wages converge to zero. The goods cost rules out this possibility, because at some point wages will fall to a level were the current population level can just be maintained.

Apart from the wages, the ratio of the number of skilled and unskilled adults is also constant in the Malthusian Regime.[6] If the wage of skilled adults temporarily increases for an exogenous reason, more unskilled adults will decide to have skilled children, until the ratio of skilled to unskilled adults reaches its steady-state value again.

Without making specific assumptions on parameters, it is not clear whether fertility will be higher for skilled or for unskilled adults in the Malthusian Regime. On the one hand, children are a normal good. If the only costs for children were the goods cost and the schooling cost (i.e., $\phi = \phi_U = 0$, $\phi_S, \rho > 0$), the absolute price of skilled and unskilled children would be the same for both types of parents. In that case the adults with the higher wage (the skilled adults) would have more children. On the other hand, it was shown above that if the cost of children is a pure time cost (i.e., $\phi_S = \rho = \phi_U = 0$, $\phi > 0$), higher wages cause lower fertility. This effect favors lower fertility for skilled adults.

[6]This is not true for all parameter combinations, however. If the schooling cost is very high, it is possible that the ratio of skilled to unskilled adults converges to zero, and the wage for skilled adults converges to infinity. In the numerical illustrations below I restrict attention to the case where a steady-state exists.

Finally, in equilibrium a larger fraction of skilled adults has skilled children. Since skilled children are more expensive, this effect tends to lower the fertility of skilled adults as a group. Which of these effects dominates depends on assumptions on parameters. If the parameters are chosen in a way that is consistent with a demographic transition, the last effect tends to be the most important one. I will therefore concentrate on the case in which skilled adults have less children on average in the steady state.

In the steady state, the ratio of skilled to unskilled adults is constant. Since at the same time average fertility in steady state is higher for unskilled parents, it has to be the case that a fraction of unskilled adults has skilled children. Otherwise, the fraction of skilled adults would decrease over time. Therefore unskilled adults have to be indifferent between skilled and unskilled children in the steady state. Using Corollary 1, steady state wages $\bar{w}_S$ and $\bar{w}_U$ and utilities $\bar{V}_S$ and $\bar{V}_U$ then have to satisfy the following condition:

$$\left(\frac{\phi\bar{w}_U + \phi_S\bar{w}_S + \rho}{\phi\bar{w}_U - \phi_U\bar{w}_U + \rho}\right)^{1-\epsilon} = \frac{\bar{V}_S}{\bar{V}_U}. \tag{42}$$

This equation determines the wage differential, i.e., the degree of inequality in the Malthusian Regime. Of course, without other equilibrium conditions we cannot solve for the steady-state wages directly, but (42) will be useful for comparing the degree of inequality in the Malthusian Regime and in the Growth Regime.

Within the Malthusian Regime, it is straightforward to compute an equilibrium numerically via value function iteration (some details are given in the Appendix). Steady-state values can either be inferred from the computed solution, or derived directly from steady-state conditions. No closed-form solutions are available, however. Since my main interest is in the transition from the Malthusian Regime to the Growth Regime, I defer a discussion of numerical results until the next section.

The Transition

In the Malthusian Regime, wages depend on preference parameters and the growth rate of agricultural productivity, but are independent of the level of productivity. At some point, productivity in industry will reach a level at which industrial production is profitable at the wages that prevail in the Malthusian Regime. From that time on the industrial technology will be used alongside the agricultural technology. Since population growth does not depress wages in industry, wages and income per capita start to grow.

The evolution of fertility, inequality, and intergenerational mobility depends on the specific properties of the industrial production function. I assume that production in industry is more skill-intensive than production in agriculture:

Assumption 4 *The industrial sector is more skill-intensive, i.e., the production function parameters satisfy* $\alpha < \theta_U$*, which implies* $1 - \alpha > \theta_S$*.*

Under this assumption, the introduction of the industrial sector increases the wage premium for skilled labor. This will increase the returns to education, so that more unskilled adults will choose to have skilled children. Because there are more skilled children, more skilled adults work as teachers. This in turn leaves less skilled adults to work in industry and in agriculture, which causes an additional upward effect on the skilled wage. However, increases in the skilled wage also increase the cost of education. This will make up for the higher utility of skilled children, and unskilled parents will be indifferent between skilled and unskilled children once again. The overall effect is an increase in skilled wages, and increased upward mobility in the sense that more unskilled adults send their children to school.

Government policy can have large effects during the transition. Education subsidies induce more unskilled adults to have skilled children. The wage premium will still rise initially, because many skilled adults are needed as teachers. Child-labor restrictions lower the relative cost of education and therefore also lead to increased mobility. The short-run effects on the wage premium are ambiguous. On the one hand, higher demand for skilled children requires more teachers, which increases the skill premium. On the other hand, with child-labor restrictions unskilled labor supply falls. This has a positive effect on the unskilled wage, and therefore decreases the premium for skilled labor.

The Growth Regime

As the economy continues to grow, the fraction of the population working in the agricultural sector falls, and under a wide range of conditions ultimately approaches zero[7]. In the limit, the economy behaves like one in which there is no agricultural sector at all. Also, as income grows, the goods cost ρ for children will become negligible.

[7] If productivity growth is significantly higher in the agricultural sector than in industry or if population growth turns negative, there can be situations where the fraction of output produced in the agricultural sector does not tend to zero. I concentrate on the case in which agriculture disappears. Judging from the experience of industrial countries, this seems to be the empirically interesting case.

The limit economy without agriculture can be described with just two state variables. Since the industrial production function exhibits constant returns, wages are determined by the ratio of skilled to unskilled labor supply. The only state variables are therefore the ratio of the number of skilled to unskilled adults, and the productivity level in industry. The setup can further be simplified by noting that the period utility function is of the constant-elasticity form, and that wages are linear in the productivity level. This results in equilibrium value functions that are homogeneous in the industrial productivity level:

$$V_i(A_I, N_S/N_U) = A_I^{\sigma} V_i(N_S/N_U).$$

This reduces the Growth Regime essentially to a one-dimensional system, with the ratio of skilled to unskilled adults as the state variable.

Before I turn to the properties of the Growth Regime, one further technical assumption is needed. The assumption is that the effective discount factor is not higher than one, i.e., adults cannot place higher weight on the utility of their children than on their own utility. Since the discount factor depends on the number of children, we have to consider the highest possible number of children, which is reached by an unskilled adult who spends all income on children. The resulting number of children is $1/(\phi - \phi_U)$.

Assumption 5 *The parameters β, ϵ, σ, ϕ, ϕ_U, and γ_I satisfy:*

$$\beta(\gamma_I)^{\sigma} \left(\frac{1}{\phi - \phi_U} \right)^{1-\epsilon} < 1.$$

Given this assumption, the problem of adults is always well defined.

In the Growth Regime the economy can exhibit two types of behavior. In the first case, the one that I am going to concentrate on, there is a globally stable steady state. That is, the ratio of skilled to unskilled adults reaches a fixed number, population growth and fertility are constant, and wages and consumption grow at the rate of technical progress. In the steady state, average fertility is lower for skilled than for unskilled adults. This would be true even if the schooling cost were zero and if there were no child labor, since then the only remaining cost of children would be a time cost. It was shown in Section 4 that if there is a pure time cost for having children, wages and fertility are negatively related. With positive schooling cost and child labor the relative cost of skilled children increases, and since relatively more skilled adults have skilled children, this will further increase the fertility differential between the two types of adults.

Given that fertility is higher for unskilled adults in the steady state, it has to be the case that some unskilled adults have skilled children. As in the Malthusian Regime, this is necessary because otherwise the fraction of unskilled adults would increase over time. Therefore unskilled adults are just indifferent between the two types of children. Steady state wages $\bar{w}_S$ and $\bar{w}_U$ and utilities $\bar{V}_S$ and $\bar{V}_U$ then have to satisfy the following condition:

$$\left(\frac{\phi \bar{w}_U + \phi_S \bar{w}_S}{\phi \bar{w}_U - \phi_U \bar{w}_U} \right)^{1-\epsilon} = \frac{\bar{V}_S}{\bar{V}_U}. \tag{43}$$

The difference to the parallel condition (42) for the Malthusian Regime is that the goods cost ρ for children is negligible in the Growth Regime. Comparing (42) and (43), we can infer that skilled children are relatively more expensive in the Growth Regime, thus inequality will be higher than in the Malthusian Regime.

The second possible behavior in the Growth Regime arises when the schooling cost, i.e., the parameter ϕ_S, is very high. In that case, the ratio of skilled to unskilled adults converges to zero, and the ratio of the skilled to the unskilled wage tends to infinity. The increasing wage premium makes education very expensive from the perspective of unskilled adults, and if ϕ_S is sufficiently high, this more than outweighs the high utility of potential skilled children. Since in real-world countries the number of skilled adults generally does not converge to zero, I concentrate on parameterizations for which a Growth Regime exists.

Education and child-labor policies influence the relative cost of the two types of children much in the same way as described in the discussion of the transition period. Both education subsidies and child-labor restrictions lower the relative cost of educating children. This leads to a higher fraction of skilled adults in the Growth Regime, lower inequality, and a lower fertility differential between skilled and unskilled adults.

So far, I have established that the model can generate a transition from Malthusian stagnation to growth, and that public policy has potential for explaining cross-country differences during the transition. I will now turn to the question whether the model outcomes are also quantitatively plausible when compared to real-world data. In the following sections, I calibrate the model parameters, and compare the model outcomes to evidence from Brazil and Korea.

6 Calibration

In this section I describe the procedure for calibrating the model parameters. Since I use the same parameters to compare the experience from different countries, it would be counterproductive to choose parameters to closely match data from one specific country. Rather, I chose parameters such that in the Growth Regime the model matches certain features of modern industrialized countries, while in the Malthusian Regime the model resembles a pre-modern economy. For the Growth Regime, I use on current U.S. data, while for the pre-modern economy I rely mostly on England, where the available data is of relatively high quality. Since all the data that is used for calibration stems from the two ends of the time scale, we can use the transition period for testing the model. My calibration procedure reflects the hypothesis that observed cross-country differences are mainly explained by policies, as opposed to "cultural" factors. If this is indeed the case, calibrating the model parameters with out-of-country data should work just as well as using in-country data.

I start by describing the parameter choices that are determined by features of the Growth Regime. The parameter γ_I, the rate of technological progress in the industrial sector, determines the growth rate of per-capita output in the Growth Regime. In the United States, real GDP per capita increased on average by 1.9% per year in the period from 1960 to 1992. I therefore chose the yearly growth rate of productivity in the industrial sector to be 2%. Since a model period is 25 years, this gives a value for γ_I of 1.64.

Given that there is hardly any child labor in Western industrialized countries, I calibrated the parameters for the Growth Regime under the assumption that child labor is ruled out. According to the Digest of Education Statistics (U.S. Department of Education, 1998), in the U.S. teachers on all levels of education make up 1.5% of the American population, and there are about 16 students per teacher. Since unlike in the model in the real world each teacher teaches more than one generation of students, we cannot match both values at the same time. If the ratio of teachers to population is matched, class sizes would be too big, and if class size is matched, there would be too many teachers. As a compromise, I chose ϕ_S to be 0.04, which results in a class size of 21, and 1.7% of the population are teachers. The time cost ϕ was then chosen to match the total expenditure on children in the United States, estimated for 1992 by Haveman and Wolfe (1995). According to their estimates, parental per-child expenditures were \$9,200 or about 38% of per capita GDP. I chose ϕ to be 0.155, which leads to the same ratio of per-child expenditures to GDP per capita.

Knowles (1999) calibrates the same parameter to other estimates of the cost of children, and arrives at a similar value of 0.15.

Using data from 1975, Jones (1982) finds that in Britain the difference in total fertility between women with elementary education or less and women with secondary or higher education is about 0.4, and the corresponding value for the U.S. is about 0.5. I chose the preference parameters σ, ϵ and β to be consistent with a fertility difference of 0.5 between skilled parents and unskilled parents who have unskilled children, and a total fertility rate of 2.0, which matches current fertility in the United States. The choices $\sigma = 0.5$, $\epsilon = 0.5$, and $\beta = 0.132$ are consistent with these observations.

According the Digest of Education Statistics, in 1994 total expenditures on education were 7.3% of GDP, while public expenditures were 4.8%, or roughly two thirds of the total. These numbers exclude expenditures by parents and students, like textbooks and transportation. The government therefore pays for less than two thirds of all educational expenditures. In the model, I chose the fraction δ of education cost paid for by the government to be 0.5.

The technology parameter α, the share of unskilled labor in the industrial production function, mainly determines the ratio of skilled to unskilled people in the Growth Regime. It is hard to match this ratio to data, since there are more than two skill levels in the real world. If we define skill to mean completed high school, skilled people would make up most of the population, since already today almost 90% of recent school cohorts satisfy this criterion. On the other hand, if skilled means completed college education, the number of skilled people would drop below 30%. Since college education was rare in some of the countries and time periods I am interested in, I chose a compromise with higher weight on high school. The parameter α was chosen to be .22, which results in 75% of the population in the Growth Regime to be skilled.

I now turn to the Malthusian Regime. Most parameters are identical to the Growth Regime, we only need to calibrate the agricultural technology and the child-labor parameter. The parameter γ_F, the rate of technological progress in the agricultural sector, determines fertility in the Malthusian Regime. In Britain the total fertility rate was about 4.0 in 1700, and values in other European countries were similar. I therefore chose γ_F to be consistent with a doubling of the population every period, and the resulting value is 1.32. The fixed cost for children ρ is a scale parameter, therefore the choice is arbitrary. I chose $\rho = .001$.

The only remaining parameters are the production function parameters θ_S and θ_U, and the child-labor coefficient ϕ_U. Given our limited access to economic statistics from pre-modern societies, these parameters are hard to calibrate. However, ϕ_U has to be chosen sufficiently large to allow a Malthusian Regime. If ϕ_U is small, population growth does not catch up with technological progress, and per capita output grows already before the introduction of the industrial technology. On the other hand, if ϕ_U is too large, the number of skilled agents converges to zero after the industrial technology is introduced. My choice of $\phi_U = .07$ is in the middle ground and is consistent both with Malthusian stagnation and a Growth Regime with a positive fraction of skilled agents. Adjusting for labor supply, this implies that nine children are equivalent to one unskilled worker.

In terms of the sensitivity of my results, ϕ_U is actually the most important parameter. By making ϕ_U large, we can increase the effects of eliminating child labor. It is therefore important to ascertain that ϕ_U is not unrealistically large. From the perspective of the parent, the key question is how much income children contribute to the household income. With my parameter choice, in the Malthusian Regime income from child labor makes up 25% of family income for families with working children. Horrell and Humphries (1995) find that in a sample of working-class families in England around 1800 children contribute between 27% and 32% of household income. From this perspective, my choice for ϕ_U is not too large.

For the parameters of the agricultural technology, I chose $\theta_S = .1$ and $\theta_U = .5$. With these choices, the share of output going to land rents is 40%, and skilled adults make up about 5% of the population in the Malthusian Regime. A lower bound on the land share can be derived from the English National Accounts for 1688 in Deane and Cole (1969). At that time rents made up 27% of national income. Since some of the 34% of national income going to non-rent property income and profits are also derived from land, 30% is a reasonable lower bound for the share of land. On the other hand, share-cropping contracts typically allocated 50% of output to the land owner. Since not all of national income was derived from agriculture, 50% is then an upper bound for the land share. My choice lies in the middle between these bounds.

7 Recent Transitions: Comparing Brazil and Korea

This section uses the calibrated to model to determine whether the predictions of the model are quantitatively plausible. I start by simulating the model under two different assumptions on government policies, one of which resembles Korean policies, and another that resembles Brazilian policy. In the stylized Brazilian policy, parents pay for education, and child labor is unrestricted. In the Korean simulation, the economy starts out with the same policies. Once the economy starts to leave the Malthusian Regime, however, the policy changes to one in which the government pays for 50% of the education cost for skilled children, and child labor is ruled out. Both economies start from identical conditions. The initial values for agricultural productivity and the number of skilled and unskilled adults are chosen to start the economies in the Malthusian steady state, when only the agricultural technology is used. The productivity in industry is chosen so that the transition to industry starts two periods after the start of the simulation. Apart from different education and child labor policies, the simulations for Brazil and Korea are identical.

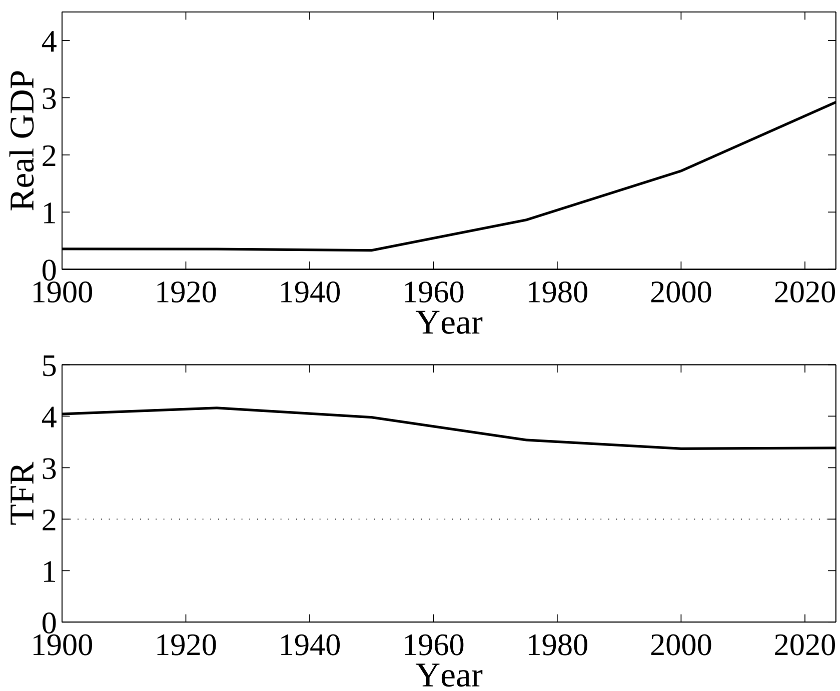

Figure 4: Outcome with “Brazilian” Policy

Figure 4 shows the simulated path for the GDP per capita and the total fertility rate under the Brazilian policy. The time axis is labeled such that each model period corresponds to 25 years, and the start of the transition to the Growth Regime is placed in the year 1950. GDP per capita starts to grow in 1950. The total fertility rate increases slightly before the start of the transition, and the declines slowly to a little above 3. Figure 5 shows the

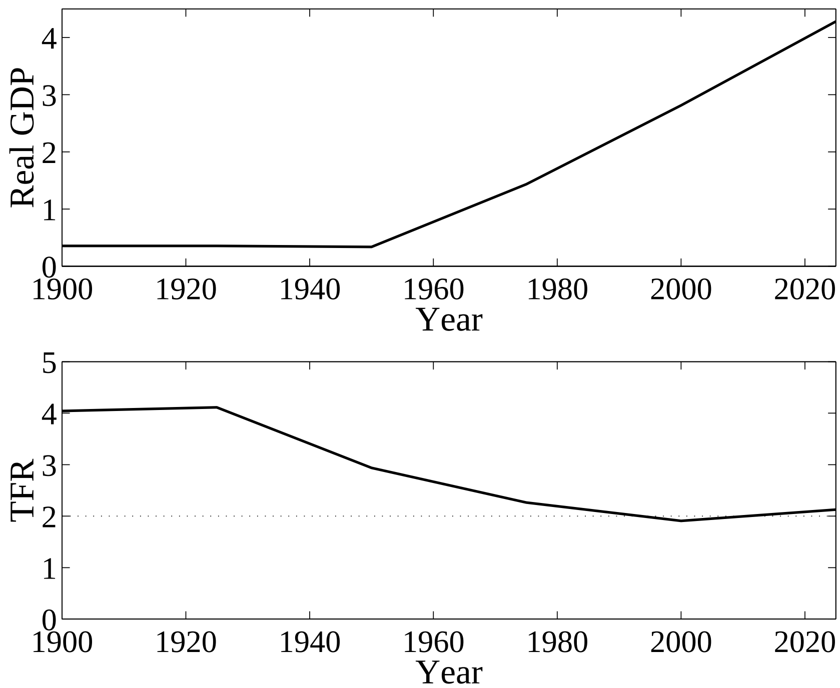

Figure 5: Outcome with "Korean" Policy

corresponding results under the Korean policy assumptions. Income per capita takes off at the same time. However, growth is faster in Korea, and in 2000 GDP per capita is about 40% higher than in Brazil. The reason for this is that the policy change increases the fraction of skilled adults in the population. Since the industrial technology is skill-intensive, this raises income per capita. As in the Brazilian simulation, the take-off in living standards is accompanied by a fertility decline. However, the decline is much faster than in Brazil. In 1950, the date of the policy change and the introduction of industrial production, Fertility is already much lower than in the Malthusian Regime. Replacement fertility is reached in less than two generations after the take-off.

The simulations match key features of the data in Figure 1. The main deviation is that in the data fertility starts out higher than in the model. The model does of good job of reproducing the much faster fertility decline in Korea, and the large fertility differential between the two countries during the transition.

The effect of government policies in the model is brought about by influencing the choice of parents between skilled and unskilled children. Figures 6 and 7 break down the population in skilled and unskilled adults, and show the fraction of each type working in the industry. In Korea, the number of skilled people rises rapidly after the policies lower the

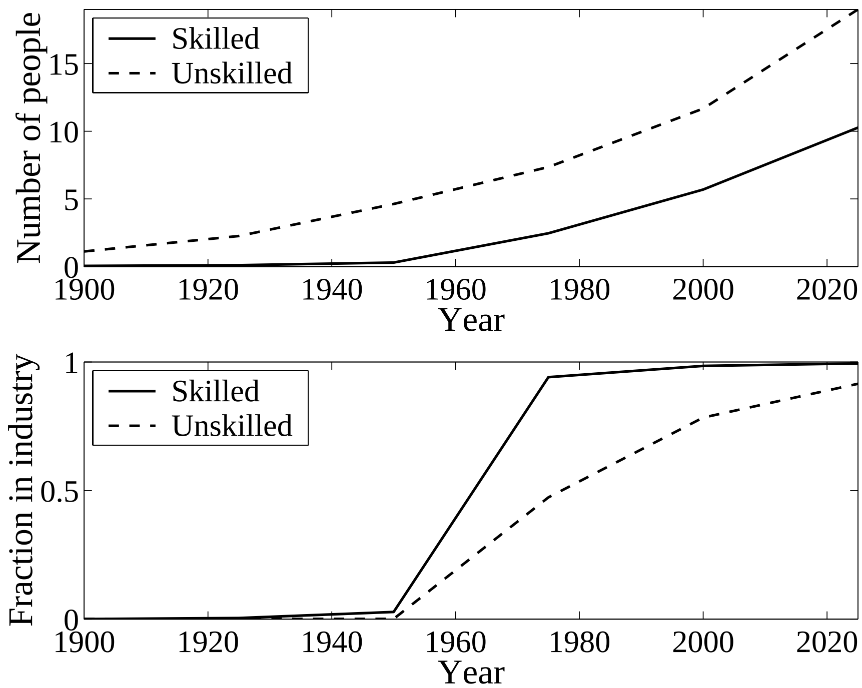

Figure 6: Outcomes with "Brazilian" Policy

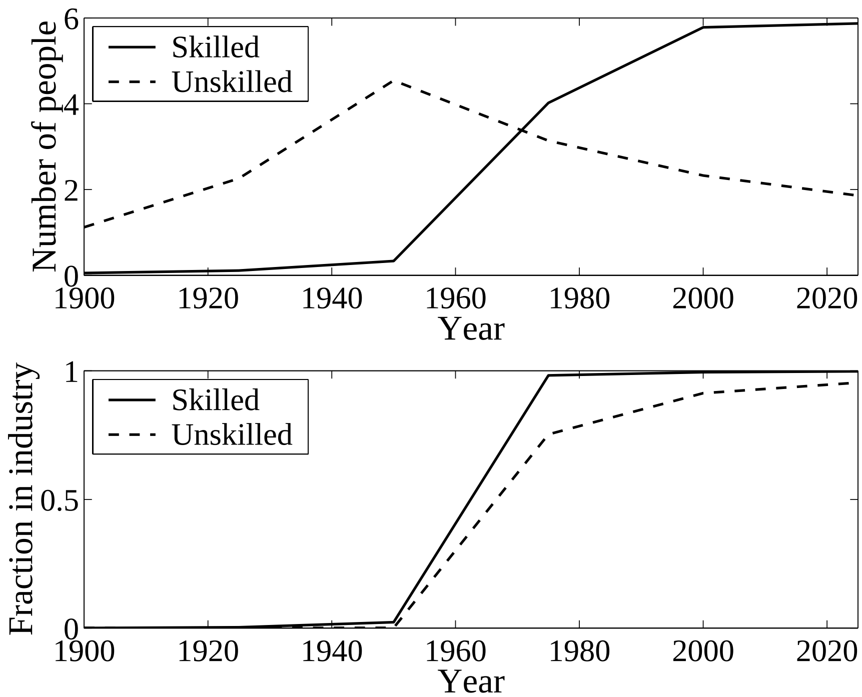

Figure 7: Outcome with "Korean" Policy

price of skilled children. Already one period after the policy change skilled adults are in the majority. In Brazil the cost of skilled children stays high, and consequently unskilled people continue to make up the majority of the population. Since unskilled people are relatively more productive in agriculture, this also implies that the transition from agriculture to industry is slower with the Brazilian policy.

Implications for Fertility Differentials

In my theory changes in overall fertility are driven by fertility differences between skilled and unskilled parents. It is therefore crucial to check whether the fertility differentials generated by the model are in line with empirical observations. Figure 8 breaks down

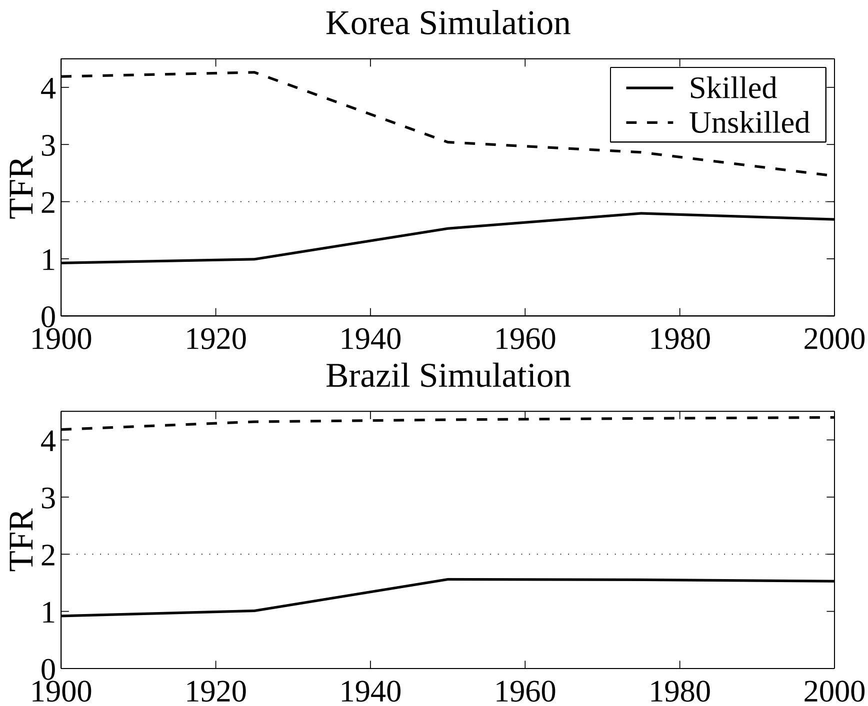

Figure 8: Fertility Differential between Skilled and Unskilled Parents

the total fertility rate for both the Brazilian and the Korean simulation by the skill of the parent. In the Korean simulation, the fertility differential between the two types of parents declines rapidly after the policy change. The abolition of child labor affects only unskilled parents, since skilled parents specialized in skilled children already before the change. Since the cost of unskilled children rises, the fertility of unskilled parents declines. For skilled parents the education subsidy implies that the cost of children declines, so that their fertility rises. The overall result is a small fertility differential between skilled and unskilled parents.

In the Brazilian simulation there is no policy change. Unskilled children are still cheap, and therefore unskilled parents continue to have many children. Average fertility actually increases slightly for both groups. The fall in the aggregate total fertility rate in Figure 4 occurs only since after the transition skilled adults make up a larger fraction of the population than they did in the Malthusian Regime. The fertility differential between skilled and unskilled parents stays large.

There is no time series data for fertility by skill for Brazil and Korea, but the available evidence indeed shows small fertility differentials in Korea and large differentials in Brazil. According to United Nations (1995), in 1986 Brazilian women without formal education had a total fertility rate of 6.7, while for women with seven or more years of education the number is 2.4. Thus the fertility differential amounts to more than four children. Alam and Casterline (1984) report that in 1974 the total fertility rate for Korean women without formal education was 5.7, while for women with seven or more years of education the rate is 3.4. This gives a fertility differential of 2.3, roughly half of the differential in Brazil.[8] Thus fertility differentials are indeed much lower in Korea than in Brazil, as predicted by the model.

Implications for Inequality

I will now turn to the implications of the model for the distribution of income. Since there are people with different skills in the model, the income distribution is generally nontrivial. The same policies that I use to understand differences in the fertility transition also affect the income distribution, which gives us another dimension along which the model can be compared to data.

Figure 9 shows actual and simulated Gini coefficients for the income distribution in Brazil and Korea. In the data, in both countries the Gini coefficient stays roughly constant throughout the transition. In Brazil the Gini is always above .5, while in Korea the value is closer to .3. The difference in inequality between Brazil and Korea is quite large. The share of income that goes to the poorest 40% of the Brazilians is only slightly higher than the share of the poorest 20% in Korea. This implies drastic differences in living standard between poor Brazilians and poor Koreans. There are also large differences at the top of

[8]Unfortunately, there is no data for Brazil and Korea for the same year. However, given the overall fall in fertility, it seems likely that the fertility differential by education in Korea was even lower in 1986 than in 1974.

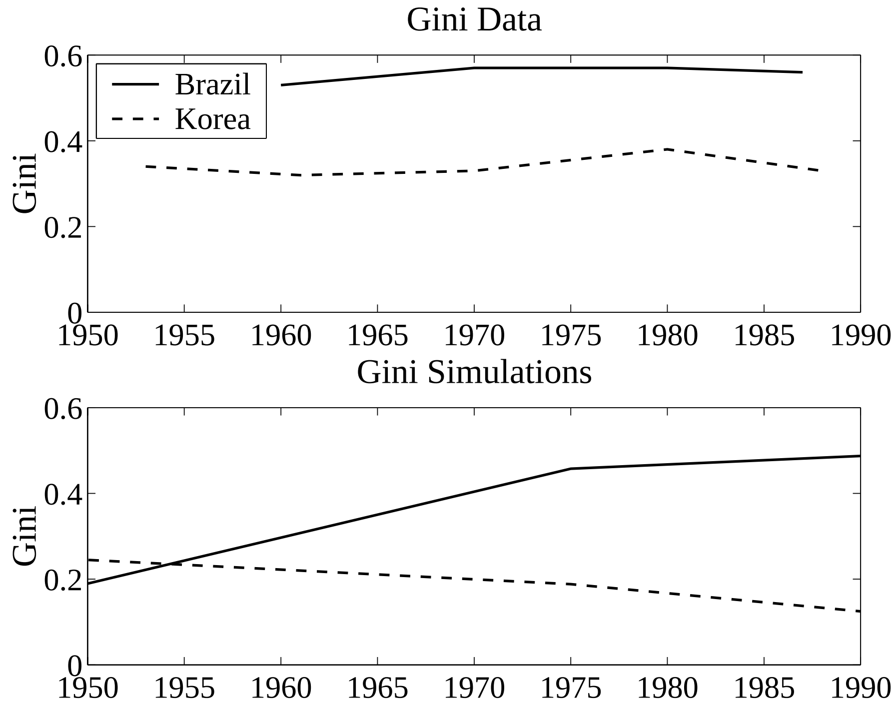

Figure 9: Actual and Simulated Gini Coefficients

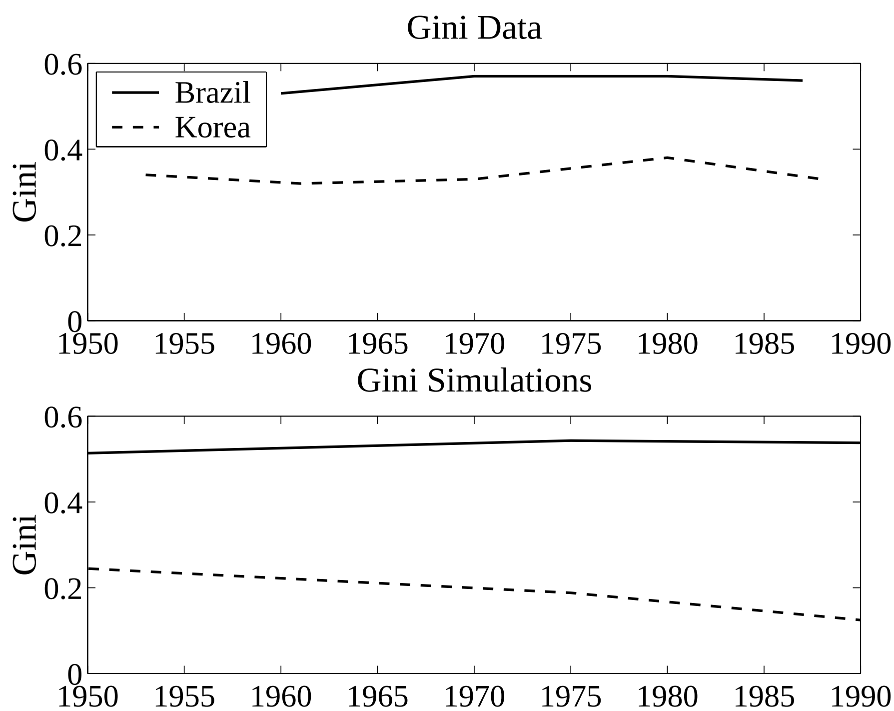

Figure 10: Ginis with Concentrated Land-Ownership in Brazil

the income distribution: In 1970, the income share of the top quintile was 61.7% in Brazil, and only 41.6% in Korea.

In the model, inequality is determined by the skill premium, which in turn depends on the relative supply of skilled and unskilled labor. The Korean policy increases the relative number of skilled adults, which offsets the increased demand for skilled labor in the industrial sector. Therefore in the Korean simulation inequality is generally low and decreases slightly during the transition. In the Brazilian simulation unskilled adults continue to make up the majority of the population. The increased demand for skill in the industry translates into an increased wage premium, so that inequality increases during the transition.

Relative to the data, the main deviation is that inequality in the Brazilian simulation starts out low and then increases, instead of being high to begin with. My interpretation is that the model captures only one aspect of inequality, namely the premium for skilled labor. Another potential source of inequality is concentrated land ownership. Historically, land ownership was very concentrated in Brazil, much more so than in Korea. The distribution of land is an important determinant of the income distribution as long as agriculture and therefore land rents make up a large fraction of GDP. As production switches to industry, the importance of land ownership declines, and the importance of skill premia increases. If land ownership is concentrated, an increase in inequality deriving from skill premia is consistent with stable overall fertility, since at the same time inequality from concentrated land ownership declines in importance. To illustrate this point, I also computed Gini coefficients for the Brazilian simulation under the assumption that land rents go to a single land owner, instead of being distributed among agricultural workers as I assumed so far. Figure 10 shows that under this assumption inequality starts out high in Brazil, and stays roughly constant over time.

These results are especially interesting since the existing literature on inequality and growth has mostly concentrated on the Kuznets curve. Kuznets' (1955) hypothesis on the connection between development and the income distribution states that inequality first increases and later decreases during industrialization. While this was the case in the countries on which his observations were based (England, Germany, and the United States), Figure 9 shows that neither Brazil and Korea exhibit a Kuznets curve. My results show that interactions of public policies with fertility decisions can generate large cross-country differences in the growth-inequality relationship. This may provide an explanation why Kuznets curves show up in some countries, but not in others.

Which Policy Matters Most?

So far I restricted my analysis of public policy to a combination of two policies, an education subsidy and child-labor restrictions. A natural question to ask is which of the two policies is more important for generating the results described above. Figure 11 shows simulated time paths for GDP per capita and fertility for an economy that only introduces education subsidies, but leaves child-labor unrestricted. The evolution of income per capita is very similar to the Korean simulation, which introduces both policies. In contrast, when there is only an education subsidy, fertility increases initially, and the subsequent decline is much smaller than in the Korean simulation with both policies. Figure 12 shows the outcome when child labor is abolished, but there are no education subsidies. The results are very similar to the Korean simulation. The main difference is that fertility falls even faster when there are no education subsidies. These results underline the importance of including the cost of the children's time in the opportunity cost of education.

A welfare analysis of the policies is made difficult by the fact that skilled and unskilled people have opposing interests. Therefore the policies cannot be ranked using the Pareto criterion. The education subsidy, however, is beneficial for adults of both skills in the period when the subsidy is introduced. Future generations of skilled people have lower utility with the policy, because of a lower skill premium. A child-labor restriction alone hurts both skills in the period when it is introduced, since the unskilled parents lose income, and the utility of skilled children falls. However, future generations of unskilled people benefit from child-labor restrictions. A combination of the two policies yields the highest growth path for GDP per capita, but future skilled generations are still better off without the policies.

8 A Nineteenth-Century Transition: England

As a final application of my theory I turn to the country where the Industrial Revolution once started, England. As the first country to start the transition from stagnation to growth, England is a natural testing ground for any theory of transition. In addition, England is one of the countries where inequality followed the inverted-U-shape pattern that was described by Simon Kuznets, whereas both in Brazil and Korea the income distribution changed little during transition. This provides an interesting test for distributional

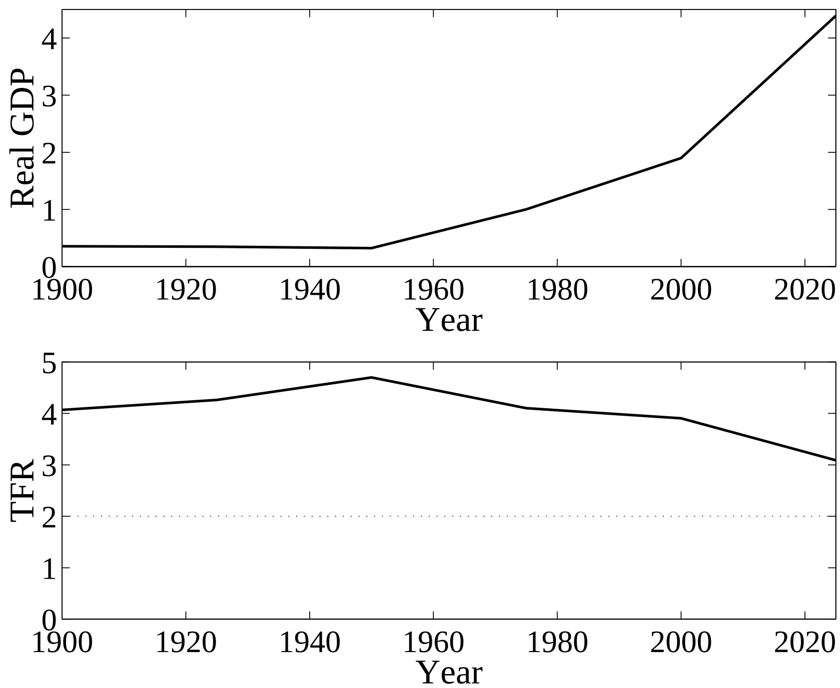

Figure 11: Only Education Subsidies

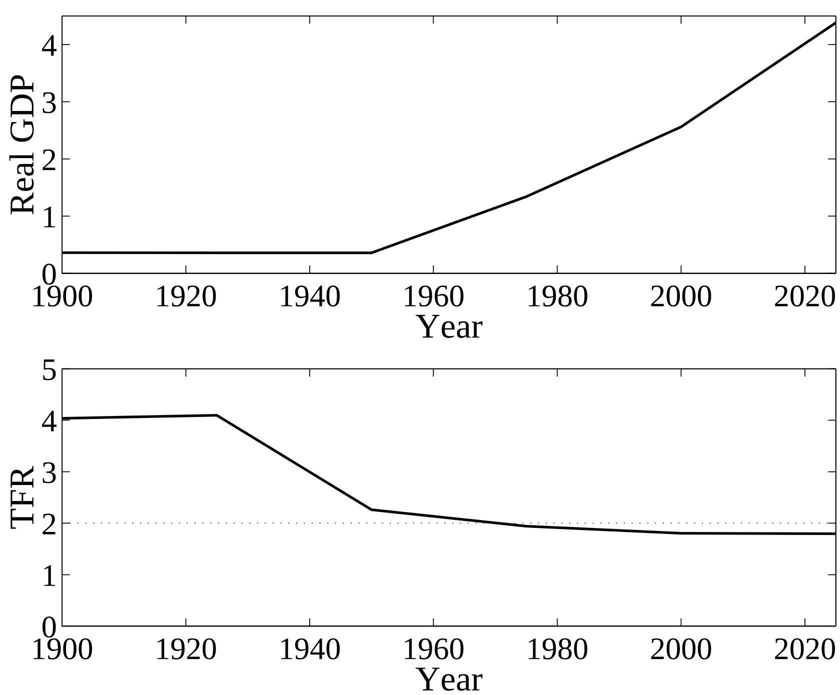

Figure 12: Only Child-Labor Restrictions

implications of the model.

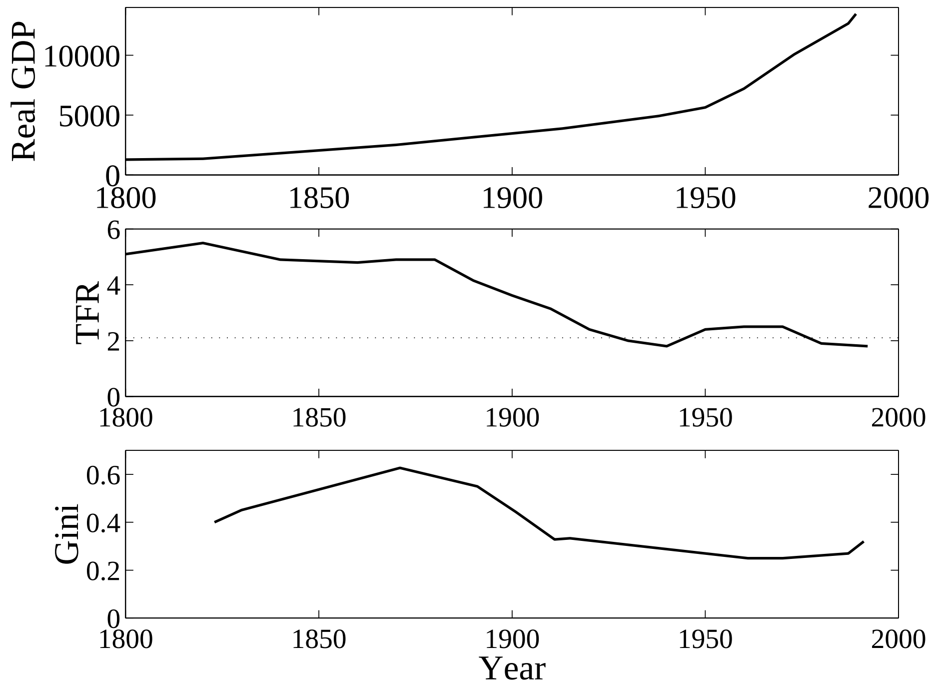

Figure 13: Income, Fertility, and Inequality in England

Facts on Fertility, Inequality, Education, and Child Labor

Figure 13 shows GDP per capita, fertility, and inequality for England from 1800 until 2000. Even though industrialization started in the late eighteenth century and per capita income was growing throughout the nineteenth century, fertility stayed high until about 1880. Then fertility started to fall rapidly around the turn of the century, and reached replacement level before 1940. The available evidence for the income distribution shows that inequality increased during the first half of the nineteenth century, and then decreased in the late nineteenth and early twentieth centuries. Williamson (1991) reports that income shares at the top of the income distribution rose from about 1760, the premium on skilled labor increased, and the income distribution widened. Inequality peaked around 1860. Afterwards, income shares at the top fell, as did the wage premium, the relative position of the unskilled workers improved, and the income distribution narrowed. The Gini coefficients in Figure 13 from Williamson (1985) are based on tax assessment data. Even though there might be substantial measurement error, the initial increase and later decrease in inequality, the "Kuznets curve," is clearly visible.

At the beginning of the Industrial Revolution, education was not widespread in England. In 1780, only about 50% of brides and grooms were able to sign their name. Educational expansion proceeded slowly in the early nineteenth century. While overall enrollment rates increased, much of the increase was initially accounted for by Sunday schools, which were less effective than day schools in providing education. Mitch (1999) reports that in 1818 most students were enrolled in private, for-profit schools, many of which were of questionable educational value. Public education also started to expand in the nineteenth century, but there were large regional differences and no universal access to affordable education. The situation changed drastically with the Victorian education reforms, starting in the 1870s. The Forster Education Act of 1870 placed primary education under public control. School boards were established, and many new schools were built. In 1880 compulsory schooling was introduced, and starting in 1891 primary education was free. While schooling quality is hard to measure, literacy data suggests that the reforms were successful. While in 1880 about 15% of grooms and 20% of brides were unable to sign their name, these numbers decreased to less than 2% until 1910.

Child-labor restrictions were expanded on a number of occasions during the nineteenth century. The first restrictions were put in place with the Factory Act of 1833. Only a small set of industries was affected, however, and Nardinelli (1980) concludes that the impact on child labor was small. The Factory Acts were amended in 1844 and 1874, when the minimum age for child laborers was raised to 10. At that time the restrictions became universal, instead of being limited to certain industries. Compulsory schooling laws also had an effect on child labor.[9] As a result, the incidence of child labor was decreasing late in the nineteenth century. Activity rates for children aged from ten to fourteen reached a peak 1861, when 29% of all children in that age group were economically active. In 1871, the number was still at 26%, but then it fell to about 20% in 1881 and 1891, 17% in 1901, and 14% in 1911.

Simulation

To analyze whether my theory is consistent with the stylized facts of the English transition, I use the same model parameters that were applied for Brazil and Korea. The policy change is the same that was used for the Korean simulation, but it now occurs three model periods, or 75 years, after the start of industrialization. In other words, in the simulation England follows the Brazilian model until late in the nineteenth century,

[9]Margo and Fingan (1996) find that child-labor restrictions are especially effective if combined with compulsory schooling laws.

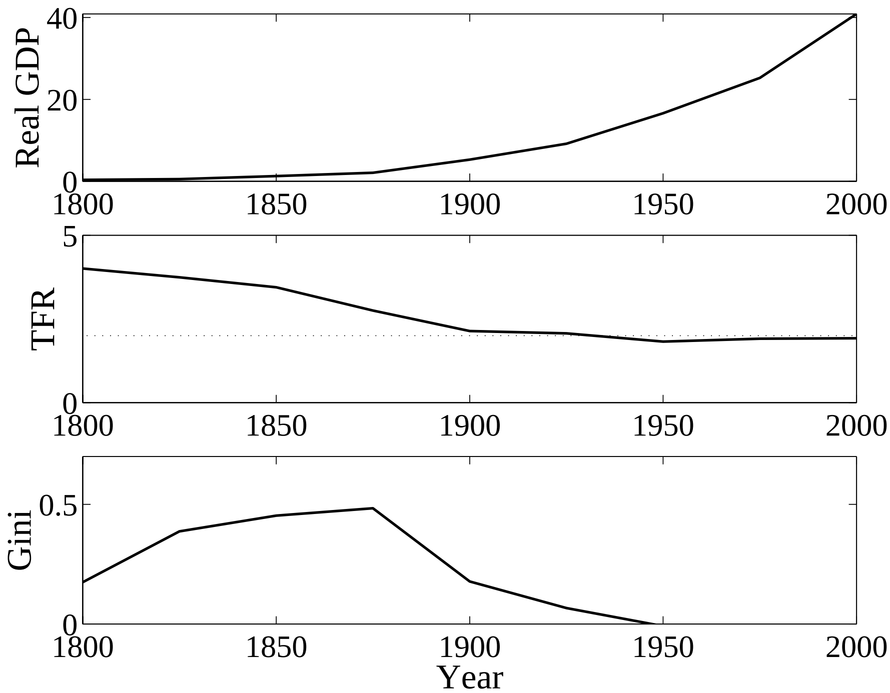

Figure 14: Simulated Income, Fertility, and Inequality

and then switches policies to a Korean regime. Figure 14 shows the results. The time axis is labeled to place the start of industrialization in 1800 and the policy change in 1875. In the simulation, fertility falls slowly until 1850, and then drops in the year of the policy change. This matches the overall pattern in the data. The main discrepancies is that the drop in fertility occurs earlier the simulation than in the data, and the model does not capture the initial increase in fertility in the 19th century.

The Gini coefficient behaves according to the Kuznets hypothesis: Inequality increases from the beginning of industrialization, and then starts decreasing late in the nineteenth century. As in the Korean simulation, fertility differentials between skilled and unskilled adults are low after the policy change, which matches the contemporary observations from Britain.

The overall results show that the model is capable of reproducing the Kuznets curve in England, together with the associated changes in fertility behavior. Even though I concentrated on the transition, it should be pointed out that the model is also consistent with the behavior of the English economy before and after the nineteenth century. Both in England and in the model wages stagnate before industrialization, and grow at roughly constant rates after the industrialization.

9 Conclusions

In this paper I develop a theory that accounts for a set of stylized facts concerning growth and fertility in the long run. The model is consistent with a phase of stagnation during which the economy exhibits Malthusian features, followed by a transition to a Growth Regime. The character of the transition depends on education and child-labor policies. Previous models that are capable of generating a demographic transition have not considered the role of policy variables. My results suggest that accounting for policy changes is important for understanding the experience of different countries during the transition from stagnation to growth. Another novelty is that the model allows an analysis of fertility differentials by income and education within a country.

I use the model to analyze the different transition experiences of Brazil and Korea. While Korea had a fast demographic transition and consistently low inequality, the demographic transition was slow in Brazil, and the income distribution was unequal. I link these differences to policies in the areas of education and child labor. Brazil has a weak education system, and child-labor restrictions are not strictly enforced. In contrast, Korea provides excellent education to its citizens, and child-labor restrictions are enforced. Simulations of a calibrated model show that these policies can explain a major part of the observed differences between Brazil and Korea. The results also indicate that education subsidies alone cannot explain the faster fertility decline in Korea. This underlines the importance of the cost of a child's time as a component of the opportunity cost of education. In a further empirical application, I show that the model can reproduce the observed patterns in inequality and fertility in nineteenth-century England, once changes in educational and child-labor policies are accounted for.

In my discussion of the transition from stagnation to growth I concentrated on cross-country differences in the fertility decline. An important question that is not addressed in this paper is why the take-off occurs at different times in different countries. The government policies that I discuss in this paper have the potential to cause a delay in the take-off of about 10 to 20 years, which is clearly too little to explain the observed differences. In order to make progress along this line, it will be necessary to move beyond the assumption of exogenous productivity growth and introduce a theory of technological progress. The results in this paper do not depend in any way on the assumption that productivity growth is exogenous. As a first step beyond exogenous growth, it is possible link the rate of technological progress to the number of skilled people in the economy. Such a model can explain why the rate of economic growth increased during the Industrial Revolution, instead of jumping to the Growth-Regime level right away. The effects of government policies on growth in GDP per capita would be amplified. In order to understand why Asian and African countries took more than a hundred years to follow the lead of Western countries, however, I suspect that a theory of technology diffusion and adoption will be needed.

A Mathematical Appendix

Nonexistence of Interior Solutions for the Adults' Problem

We consider the following maximization problem:

$$\max_{E\geq 0, 0\leq f\leq 0} \left\{ (w-E)^{\sigma} + \beta E^{1-\epsilon}(f/p_S + (1-f)/p_U)^{-\epsilon}[fV_S/p_S + (1-f)V_U/p_U] \right\}.$$

By Assumption 3, the parameters β, σ and ϵ are all strictly bigger than zero and strictly smaller than one. It is also assumed that the inequality $0 < p_U < p_S$ holds, and that we have $V_S, V_U \geq 0$. These assumptions could be relaxed, at the price of complicating the proof.

Proposition 1 *There are no interior solutions in f, i.e., if there is a solution to the maximization problem above, the optimal f is either zero or one.*

Proof: To show that there are no interior solutions, assume that we have already determined the optimal E. Given this E and the fact that the function to be maximized is twice continuously differentiable in f, if there were an interior solution, the optimal f would have to satisfy first- and second-order conditions for a maximum. I solve for the unique f which solves the first-order condition, and show that this f does not satisfy the second-order condition. This proves that there are only corner solutions.

I will name the maximand $U(\cdot)$. The first and second derivatives of U with respect to f are:

$$\begin{aligned}\frac{\partial U}{\partial f} &= \beta E^{1-\epsilon}\left[-\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)\right.\\ &\quad \left. + \left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\right],\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 U}{\partial f^2} &= \beta E^{1-\epsilon}\left[\epsilon(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)^2\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-2}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)\right] \\
&\quad -2\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1}\left(\frac{V_U}{p_U}-\frac{V_S}{p_S}\right)\Bigg] \\
&= \beta E^{1-\epsilon}\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1} \\
&\quad \left[(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)-2\left(\frac{V_U}{p_U}-\frac{V_S}{p_S}\right)\right].
\end{aligned}$$

In the first derivative, for $0 \leq f \leq 1$, the first term within the outer brackets is positive. For an interior solution to be possible, it has to be the case that $V_S/p_S < V_U/p_U$, because otherwise the second term is also positive and the first-order condition cannot be satisfied. Therefore if $V_S/p_S \geq V_U/p_U$, we are done. For the case that $V_S/p_S < V_U/p_U$, I set the first derivative equal to zero, and solve for f.

$$\begin{aligned}
0 &= \beta E^{1-\epsilon}\left[-\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)\right. \\
&\quad \left.+\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\right],
\end{aligned}$$

$$0 = -\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)+\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right) = \epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right),$$

$$\left(\frac{f}{p_S}-\frac{f}{p_U}+\frac{1}{p_U}\right)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right) = \epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{fV_S}{p_S}-\frac{fV_U}{p_U}+\frac{V_U}{p_U}\right),$$

$$(1-\epsilon)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)f = \epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\frac{V_U}{p_U}-\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$f = \frac{\epsilon\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)-\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)}{(1-\epsilon)p_U\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)}.$$

I will now plug this value for f into the second derivative to verify that the second derivative is positive, so that the critical point is not a maximum. The second derivative is positive if the following inequality holds:

$$(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)-2\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)<0,$$

or:

$$(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)<2\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right),$$

$$(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left[\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)f+\frac{V_U}{p_U}\right]<2\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left[\left(\frac{1}{p_S}-\frac{1}{p_U}\right)f+\frac{1}{p_U}\right],$$

$$(1+\epsilon)\frac{1}{p_U}\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)<(1-\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)f+2\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right).$$

Plugging in our value for f yields:

$$(1+\epsilon)\frac{1}{p_U}\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)<\frac{1}{p_U}\left[\epsilon\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)-\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\right]+2\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$\frac{1}{p_U}\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)<\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$V_U<V_S,$$

Thus if $V_S > V_U$ we are done. But if V_S were smaller than V_U, there would be only unskilled children for sure, since they are cheaper to educate. Therefore there are only corner solutions, families have either unskilled or skilled children, but they do not mix. □

B Notes on Computation

The Malthusian Regime and the Growth Regime

Within the Malthusian Regime and the Growth Regime, the model can be computed via standard value function iteration on a discretized state space. The initial guess for the value functions is computed by setting the number of children to zero, so that utility stems from consumption only. During the iterations, for a given state x, the algorithm finds a state x' in the next period such that the resulting fertility decisions of adults are consistent with state x'. After the iterations converge, the equilibrium law of motion can be used to compute steady-state values.

The Transition

In principle it is possible to compute the entire model, encompassing the Malthusian Regime, the transition, and the Growth Regime, with the same method described above. However, since the state vector is four-dimensional, computations would either take very long or would be imprecise. Therefore I use an algorithm that directly computes the equilibrium path over T periods from any starting value x_0 for the state vector.

In my computations I concentrate on transitions that start in the Malthusian Regime, where all skilled adults have skilled children. Since during the transition the returns to being skilled increase, it will be the case that during the entire transition skilled adults continue to prefer skilled children. We therefore have $\lambda_{S\to S\,t} = 1$ and $\lambda_{S\to U\,t} = 0$ for all t. Also, $\lambda_{U\to U\,t}$ is given by $\lambda_{U\to U\,t} = 1 - \lambda_{U\to S\,t}$ in all periods. Therefore it is sufficient to solve for the following equilibrium sequences: $\{x_t\}_{t=0}^T$, $\{V_{St}, V_{Ut}\}_{t=0}^T$, $\{L_{St}, L_{Ut}\}_{t=0}^T$, $\{w_{St}, w_{Ut}\}_{t=0}^T$, $\{\lambda_{U\to S\,t}\}_{t=0}^T$, and $\{n_{St}(S), n_{St}(U), n_{Ut}(U)\}_{t=0}^T$. Notice that the state vector consists of productivity and population values, $x_t = \{A_{At}, A_{It}, N_{St}, N_{Ut}\}$.

I will use superscripts to denote iterations. The algorithm starts with an initial guess $\{x_t^0\}_{t=0}^T$, $\{V_{St}^0, V_{Ut}^0\}_{t=0}^{T+1}$, $\{\lambda_{U\to S\,t}^0\}_{t=0}^T$, and $\{n_{St}^0(S), n_{St}^0(U), n_{Ut}^0(U)\}_{t=0}^T$. The productivity values in $\{x_t^0\}_{t=0}^T$ are already chosen to satisfy $A_{F\,t+1} = \gamma_F A_{F\,t}$ and $A_{I\,t+1} = \gamma_I A_{I\,t}$. We need to guess utility values up to period $T+1$, because they are needed for decisions of adults in period T. The utilities in $T+1$ are updated using the relationship $V_{i\,t+1} = \gamma_I^\sigma V_{i\,t}$ which holds in the Growth Regime. Therefore T has to be chosen large enough for the economy to be close to the Growth Regime after T periods. Whether this is the case can be checked by computing the Growth Regime as described above before computing the transition.

The algorithm proceeds in "nested iterations." At first the sequence $\{\lambda_{U\to S\,t}^0\}_{t=0}^T$ will be held constant, until the other sequences converge. The sequences are updated by computing the optimal decisions of adults, given wages and the utilities of children. Given initial guesses, the decisions of adults lead to new sequences $\{\tilde{x}_t\}_{t=0}^T$, $\{\tilde{V}_{St}, \tilde{V}_{Ut}\}_{t=0}^{T+1}$, and $\{\tilde{n}_{St}(S), \tilde{n}_{St}(U), \tilde{n}_{Ut}(U)\}_{t=0}^T$. Instead of using these sequences directly, I take a linear combination of $\{x_t^0\}_{t=0}^T$, $\{V_{St}^0, V_{Ut}^0\}_{t=0}^{T+1}$, $\{n_{St}^0(S), n_{St}^0(U), n_{Ut}^0(U)\}_{t=0}^T$ and $\{\tilde{x}_t\}_{t=0}^T$, $\{\tilde{V}_{St}, \tilde{V}_{Ut}\}_{t=0}^{T+1}$, $\{\tilde{n}_{St}(S), \tilde{n}_{St}(U), \tilde{n}_{Ut}(U)\}_{t=0}^T$ in order to prevent cycling. Then $\{\lambda_{U\to S\,t}\}_{t=0}^T$ is updated by comparing the utilities of unskilled adults with skilled and unskilled children: If utility from having skilled children is higher in period t, $\lambda_{U\to S\,t}$ is increased, and it is decreased if adults prefer unskilled children. Then the new $\{\lambda_{U\to S\,t}\}_{t=0}^T$ sequence is held constant until the other sequences converge again. Overall convergence is reached once unskilled adults are just indifferent between the two types of children along the equilibrium path. This algorithm is not guaranteed to converge, but it works well in practice.

C Parameter Values

Parameter	Value
θ_S	.1
θ_U	.5
α	.22
γ_F	1.32
γ_I	1.64
σ	.5
β	.132
ϵ	.5
ϕ	.155
ϕ_S	.04
ϕ_U	.07
ρ	.001
δ	.5

Table 4: Parameter Values for Simulations

D Data Sources

Brazil and Korea

Real GDP per capita is from Penn World Tables, Mark 5.6 (see Heston and Summers 1991). Total fertility rates (Figure 1) are from various editions of the World Bank World Tables and Chesnais (1992), Tables A2.6 and A2.7. Inequality data (Figure 9) is from Deininger and Squire (1996). For both Brazil and Korea, Gini coefficients and income shares are computed using household gross income data (before taxes) from national samples. Adjusted enrollment rates (Table 2) are from Barro and Lee (1993) and World Tables. Illiteracy rates and public expenditures on schooling are from World Tables and various editions of the UNESCO Statistical Yearbook. Average years of schooling (Table 1) are from Barro and Lee (1993). Data on child labor is from various editions of the ILO Year Book of Labor Statistics.

England

Real GDP per capita is from Maddison (1992), Table A.2 (real GDP) and Tables B.1–B.4 (population). Total fertility rates (Figure 14) are from Chesnais (1992), Table A2.1, for 1860 until present, and from Lee and Schofield (1981) for the period 1700 to 1840. The numbers by Lee and Schofield are based on family reconstitution data described in Wrigley et al. (1997). Inequality data after 1960 is from Deininger and Squire (1996). Gini coefficients and income shares are computed using personal net income data (after taxes) from national samples. Gini coefficients from 1823–1915 are estimates by Williamson (1985, Table 4.2), based on inhabited house duty tax assessment data. Literacy data is from Mitch (1992). Child labor data is from Nardinelli (1990).

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