

Chapter 11

Economic Growth

This chapter examines the determinants of economic growth. A startling fact about economic growth is the large variation in the growth experience of different countries in recent history. Some parts of the world, like the United States or Western Europe, experienced sustained economic growth over a period of more than 100 years, so by historical standards these countries are now enormously wealthy. This is not only true in absolute terms (i.e., GDP), but also if we measure wealth as income per capita (i.e., GDP per person). In contrast, there are countries where even today large parts of the population live close to the subsistence level, much the same as Europeans and Americans did some hundreds of years ago. Also, a group of countries that used to be relatively poor around the time of World War II managed to achieve even higher growth rates than the western industrialized countries, so their per capita incomes now approach those of western countries. Most of the members of this group are located in East Asia, like Japan, Singapore, Hong Kong, and so on.

It proves to be difficult to explain these different growth experiences within a single model. There are models that provide an explanation for the growth experience of the now industrialized countries, but most of these models fail to explain why much of the world is still poor. Models that seek to explain the difference between rich and poor countries are less successful at reproducing the growth facts for industrialized countries. We will therefore approach the topic of economic growth from a number of different angles. In Section 11.1 we present a number of facts about economic growth, facts that we will seek to explain with our growth models. Section 11.2 introduces the *Solow growth model*, a classic in the theory of economic growth. This model is quite successful at matching a number of facts about growth in industrialized countries. Section 11.3 introduces growth accounting, an empirical application of the Solow framework. This kind of accounting can be used to determine the sources of growth for a given country. In Section 11.4 we turn to the question why some countries are still poor today. A complete answer to this question is beyond the scope of this book; in fact, it is fair to say that a satisfactory answer has not been found yet.

Therefore we concentrate on only one important aspect of the growth experience of poor countries: the relationship between fertility, human capital, and growth.

11.1 Growth Facts

If we look at the group of industrialized countries only, we can identify a number of empirical regularities in the growth process. The British economist Nicholas Kaldor summarized these regularities in a number of stylized facts. Although he did that more than 50 years ago, the Kaldor facts still provide an accurate picture of growth in industrialized countries. Kaldor's first observation was that both output per worker and capital per worker grow over time. They also grow at similar rates, so the ratio of the aggregate capital stock to output or GDP does not change much over time. The return to capital, i.e., the interest that firms have to pay if they rent capital, is almost constant over time. Finally, the *labor share* and *capital share* are almost constant. The labor share is the fraction of output that goes to workers in the form of wages; it is computed as aggregate labor income divided by GDP. Similarly, the capital share is given by aggregate payments to capital divided by GDP. Notice that the Kaldor facts hold even if we consider long periods of time. For example, the capital-output ratio and the return to capital are not much different now from what they were 100 years ago, even though output is much higher now and the goods produced and the general technology have changed completely.

In addition to the Kaldor facts, another important fact about growth in the industrialized world is the convergence of per capita GDP of different countries and regions over time. For example, the relative difference in per capita GDP between the southern and northern states in the United States has diminished greatly since the Civil War. Similarly, countries like Germany and Japan that suffered greatly from World War II have grown fast since the war, so today per capita income in the United States, Japan, and Germany are similar again.

There are no empirical regularities comparable to the Kaldor facts that apply to both industrialized and developing countries. However, we can identify some factors that distinguish countries that went through industrialization and have a high income today from countries that remained relatively poor. An explanation of the role of such factors might be an important step toward understanding the large international differences in wealth. We are going to focus on the relationship between growth and fertility. Every now industrialized country has experienced a large drop in fertility rates, a process known as the *demographic transition*. All industrialized countries have low rates of population growth. Without immigration countries like Germany and Japan would actually shrink. Two centuries ago, fertility rates were much higher, as they are in most developing countries today. Today, almost all of the growth in world population takes place in developing countries. We will come back to these observations in the section on fertility and human capital, but first we present a model that accounts for the stylized facts about growth in developed countries.

11.2 The Solow Growth Model

A natural starting point for a theory of growth is the *aggregate production function*, which relates the total output of a country to the country's aggregate inputs of the factors of production. Consider the neoclassical production function:

$$(11.1) \quad Y_t = (A_t L_t)^\alpha K_{t-1}^{1-\alpha}.$$

We used a production function of this form already in the chapter on business cycles. Output depends on the aggregate labor input L_t , the aggregate capital input K_{t-1} , and a productivity parameter A_t . Of course, it is a simplification to consider only three determinants of output. We could include other factors like land or environmental quality, and our factors could be further subdivided, for example by distinguishing labor of different quality. It turns out, however, that a production function of the simple form in equation (11.1) is all we need to match the stylized facts of economic growth. The production function equation (11.1) exhibits constant returns to scale, which means that if we double both inputs, output also doubles. Our choice of a constant-returns-to-scale production function is not by accident: most results in this section hinge on this assumption.

Equation (11.1) indicates the potential sources of growth in output Y_t . Either the inputs L_t and K_{t-1} must grow, or productivity A_t must grow. If we want to explain economic growth, we need a theory that explains how the population (i.e., labor), the capital stock, and productivity change over time. The best approach would be to write down a model where the decisions of firms and households determine the changes in all these variables. The consumers would make decisions about savings and the number of children they want to have, which would explain growth in capital and population. Firms would engage in research and development, which would yield a theory of productivity growth. However, doing all those things at the same time results in a rather complicated model.

The model that we are going to present takes a simpler approach. Growth in productivity and population is assumed to be exogenous and constant. This allows us to concentrate on the accumulation of capital over time. Moreover, instead of modeling the savings decision explicitly, we assume that consumers invest a fixed fraction of output every period. Although these are quite radical simplifications, it turns out that the model is rather successful in explaining the stylized facts of economic growth in industrialized countries. It would be possible to write down a model with optimizing consumers that reaches the same conclusions. In fact, we wrote down that model already: the real business cycle model that we discussed in Chapter 9 used a neoclassical production function, and the optimal decision of the consumers was to invest a fixed fraction of their output in new capital. To keep the presentation simple, we will not go through individual optimization problems; instead, we will assume that it is optimal to save a fixed fraction of output. There are a number of names for the model. It is either referred to as the *Solow model* after its inventor Robert Solow, or as the *neoclassical growth model* after the neoclassical production function it uses, or as the *exogenous growth model* after the fact that there is no direct explanation for productivity growth.

The *law of motion* for a variable describes how the variable evolves over time. In the Solow model, the law of motion for capital is:

$$(11.2) \quad K_t = (1 - \delta)K_{t-1} + I_t,$$

where I_t is investment and δ is the depreciation rate, which is between zero and one. We assume that investment is a fixed fraction $0 < s < 1$ of output:

$$I_t = sY_t = s(A_t L_t)^\alpha K_{t-1}^{1-\alpha}.$$

Productivity and labor grow at fixed rates μ and γ :

$$\begin{aligned} A_{t+1} &= (1 + \mu)A_t, \text{ and:} \\ L_{t+1} &= (1 + \gamma)L_t. \end{aligned}$$

We now have to find out how the economy develops, starting from any initial level of capital K_0 , and then check whether the model is in line with the stylized facts of economic growth in industrialized countries.

We assume that there is a competitive firm operating the production technology. We can check one of the stylized facts, constant labor and capital share, just by solving the firm's problem. The profit maximization problem of the firm is:

$$\max_{L_t, K_{t-1}} \left\{ (A_t L_t)^\alpha K_{t-1}^{1-\alpha} - w_t L_t - r_t K_{t-1} \right\}.$$

The first-order conditions with respect to labor and capital yield formulas for wage and interest:

$$(11.3) \quad w_t = \alpha A_t^\alpha L_t^{\alpha-1} K_{t-1}^{1-\alpha}, \text{ and:}$$

$$(11.4) \quad r_t = (1 - \alpha)(A_t L_t)^\alpha K_{t-1}^{-\alpha}.$$

We can use these to compute the labor and capital shares in the economy:

$$\begin{aligned} \frac{w_t L_t}{Y_t} &= \frac{\alpha A_t^\alpha L_t^{\alpha-1} K_{t-1}^{1-\alpha} L_t}{(A_t L_t)^\alpha K_{t-1}^{1-\alpha}} = \alpha, \text{ and:} \\ \frac{r_t K_{t-1}}{Y_t} &= \frac{(1 - \alpha)(A_t L_t)^\alpha K_{t-1}^{-\alpha} K_{t-1}}{(A_t L_t)^\alpha K_{t-1}^{1-\alpha}} = 1 - \alpha, \end{aligned}$$

so the labor share is α , and the capital share is $1 - \alpha$. Thus both the labor and capital shares are indeed constant. This result is closely connected to the fact that the production function exhibits constant returns to scale. Actually, the fact that the labor and capital shares are about constant is one of the main arguments in favor of using production functions that exhibit constant returns to scale.

To continue, we have to take a closer look at the dynamics of capital accumulation in the model. It turns out that this is easiest to do if all variables are expressed in terms of units

of effective labor $A_t L_t$. The product $A_t L_t$ is referred to as effective labor because increases in A_t make labor more productive. For example, $A_t = 2$ and $L_t = 1$ amounts to the same quantity of effective labor as $A_t = 1$ and $L_t = 2$. When put in terms of units of effective labor, all variables will be constant in the long run, which will simplify our analysis.

We will use lowercase letters for variables that are in terms of effective labor. That is, $y_t = Y_t/(A_t L_t)$, $k_{t-1} = K_{t-1}/(A_t L_t)$, and $i_t = I_t/(A_t L_t)$. Substituting $Y_t = y_t A_t L_t$ and so on into the production function, equation (11.1), yields:

$$(11.5) \quad \begin{aligned} y_t A_t L_t &= (A_t L_t)^\alpha (k_{t-1} A_t L_t)^{1-\alpha}, \text{ or:} \\ y_t &= k_{t-1}^{1-\alpha}. \end{aligned}$$

From the law of motion for capital, equation (11.2), we get the law of motion in terms of effective labor:

$$(11.6) \quad \begin{aligned} k_t(1 + \mu)A_t(1 + \gamma)L_t &= (1 - \delta)k_{t-1}A_tL_t + i_tA_tL_t, \text{ or:} \\ k_t(1 + \mu)(1 + \gamma) &= (1 - \delta)k_{t-1} + i_t. \end{aligned}$$

Finally, investment is determined by:

$$(11.7) \quad i_t = sy_t = sk_{t-1}^{1-\alpha}.$$

Plugging equation (11.7) into the law of motion in equation (11.6) yields:

$$(11.8) \quad \begin{aligned} k_t(1 + \mu)(1 + \gamma) &= (1 - \delta)k_{t-1} + sk_{t-1}^{1-\alpha}, \text{ or:} \\ k_t &= \frac{(1 - \delta)k_{t-1} + sk_{t-1}^{1-\alpha}}{(1 + \mu)(1 + \gamma)}. \end{aligned}$$

This last equation determines the development of the capital stock over time. Dividing by k_{t-1} yields an expression for the growth rate of capital per unit of effective labor:

$$(11.9) \quad \frac{k_t}{k_{t-1}} = \frac{1 - \delta + sk_{t-1}^{-\alpha}}{(1 + \mu)(1 + \gamma)}.$$

The expression k_t/k_{t-1} is called the *gross* growth rate of capital per unit of effective labor. The gross growth rate equals one plus the *net* growth rate. The growth rates in Chapter 1 were net growth rates.

Since the exponent on k_{t-1} in equation (11.9) is negative, the growth rate is inversely related to the capital stock. When a country has a lower level of capital per unit of effective labor, its capital and hence its output grow faster. Thus the model explains the convergence of GDP of countries and regions over time.

Since the growth rate of capital decreases in k_{t-1} , there is some level of k_{t-1} where capital per unit of effective labor stops growing. We say that the economy reaches a *steady state*. Once the economy arrives at this steady state, it stays there forever. Figure 11.1 is a graphical representation of the growth process in this economy. For simplicity, we assume for the

moment that labor and productivity are constant, $\mu = \gamma = 0$. In that case, equation (11.8) simplifies to:

$$k_t = (1 - \delta)k_{t-1} + sk_{t-1}^{1-\alpha}, \text{ or:}$$

$$k_t - k_{t-1} = sk_{t-1}^{1-\alpha} - \delta k_{t-1}.$$

The change in capital per unit of effective labor is equal to the difference between investment and depreciation. Figure 11.1 shows the production function per unit of effective labor $y_t = k_{t-1}^{1-\alpha}$, investment $i_t = sk_{t-1}^{1-\alpha}$, and depreciation δk_{t-1} . Because the return to capital is diminishing, investment is a concave function of capital. For low values of capital, the difference between investment and depreciation is large, so the capital stock grows quickly. For larger values of capital, growth is smaller, and at the intersection of depreciation and investment the capital stock does not grow at all. The level of capital per unit of effective labor at which investment equals depreciation is the steady-state level of capital. In the long run, the economy approaches the steady-state level of capital per unit of effective labor, regardless of what the initial capital stock was. This is even true if the initial capital stock exceeds the steady-state level: capital per unit of effective labor will shrink, until the steady state is reached.

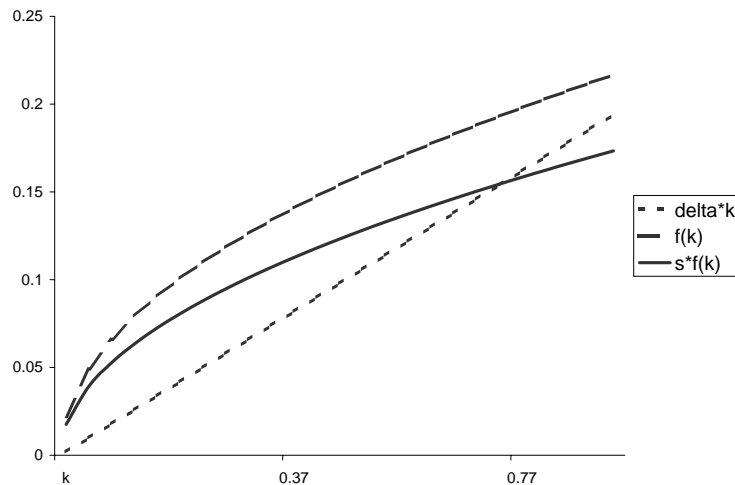


Figure 11.1: Output, Saving, and Depreciation in the Solow Model

At the steady state we have $k_t = k_{t-1}$. Using equation (11.8), we see that the steady-state level of capital per unit of effective labor \bar{k} has to satisfy:

$$\bar{k}(1 + \mu)(1 + \gamma) = (1 - \delta)\bar{k} + s\bar{k}^{1-\alpha},$$

which yields:

$$(11.10) \quad \bar{k} = \left(\frac{s}{\delta + \mu + \gamma + \mu\gamma} \right)^{1/\alpha}.$$

We can use this equation to compute output, investment, and growth in the steady state. From equation (11.5), the steady-state level of output per effective labor unit is:

$$\bar{y} = \bar{k}^{1-\alpha} = \left(\frac{s}{\delta + \mu + \gamma + \mu\gamma} \right)^{\frac{1-\alpha}{\alpha}}.$$

The level of output depends positively on the saving rate. From equation (11.7), the steady-state investment per unit of effective labor is:

$$\bar{i} = s \left(\frac{s}{\delta + \mu + \gamma + \mu\gamma} \right)^{\frac{1-\alpha}{\alpha}}.$$

The steady-state growth rate of capital is $\mu + \gamma + \mu\gamma$:

$$\frac{K_t}{K_{t-1}} = \frac{\bar{k}(1 + \mu)A_t(1 + \gamma)L_t}{\bar{k}A_tL_t} = 1 + \mu + \gamma + \mu\gamma,$$

and the growth rate of output equals $\mu + \gamma + \mu\gamma$ as well. This implies that the long-run growth rate of an economy is independent of the saving rate. With a higher saving rate, the economy approaches a higher steady state, but the long-run growth rate is determined by growth in labor and productivity only.

There are still a number of stylized facts left to be checked. First, we will verify that the return to capital is constant. From equation (11.4), the return to capital is:

$$r_t = (1 - \alpha)(A_t L_t)^\alpha K_{t-1}^{-\alpha} = (1 - \alpha) \left(\frac{K_{t-1}}{A_t L_t} \right)^{-\alpha}.$$

In the steady state, capital per unit of effective labor is a constant \bar{k} . Therefore the return to capital in steady state is:

$$\bar{r}_t = (1 - \alpha)\bar{k}^{-\alpha},$$

which is constant since \bar{k} is constant. On the other hand, the wage is growing in the steady state, since the productivity of labor increases. The steady-state wage can be computed as:

$$w_t = \alpha A_t^\alpha L_t^{\alpha-1} K_{t-1}^{1-\alpha} = \alpha A_t \left(\frac{K_{t-1}}{A_t L_t} \right)^{1-\alpha} = \alpha A_t \bar{k}^{1-\alpha}, \text{ so:}$$

$$\frac{w_{t+1}}{w_t} = \frac{A_{t+1}}{A_t} = 1 + \mu,$$

which implies that the wage grows at the rate of technological progress.

The capital-output ratio in steady state is:

$$\frac{K_{t-1}}{Y_t} = \frac{\bar{k}A_tL_t}{\bar{y}A_tL_t} = \frac{\bar{k}}{\bar{y}},$$

which is a constant. This verifies the last stylized fact of economic growth on our list.

The Solow model succeeds in explaining all stylized facts of economic growth in industrialized countries. The key element of the model is the neoclassical constant-returns production function. Since returns to capital alone are decreasing, economies grow faster at lower levels of capital, until they approach the steady state, where units of effective labor and capital grow at the same rate. The model also explains why different saving rates in different industrialized countries do not translate into long-term differences in the growth rate. The saving rate affects the level of the steady state, but it does not affect the steady-state growth rate. The capital stock cannot grow faster than effective labor for a long time because of decreasing returns to capital.

Since the Solow model does well at matching the facts of economic growth, it forms the basis of many more-advanced models in macroeconomics. For example, our real business cycle model of Chapter 9 is a Solow model enriched by optimizing consumers and productivity shocks. On the other hand, the model works well only for countries that satisfy the assumptions of constant rates of population growth and technological progress. These assumptions are justified for industrialized countries, but they are not helpful for understanding the early stages of development of a country, which are usually accompanied by the demographic transition, so exogenous and constant population growth is not a useful assumption. We will look for possible explanations of fertility decisions below, but before that, we will introduce growth accounting, a method that allows us to decompose the growth rate of a country into growth in population, capital, and productivity.

11.3 Growth Accounting

In this section we will use the general framework of the Solow model to compute a decomposition of the rate of economic growth for a given country. Consider the neoclassical production function:

$$(11.11) \quad Y_t = (A_tL_t)^\alpha K_{t-1}^{1-\alpha}.$$

We will interpret Y_t as GDP, L_t as the number of workers, K_{t-1} as the aggregate capital stock, and A_t as a measure of overall productivity. We will be concerned with measuring the relative contributions of A_t , L_t , and K_{t-1} to growth in GDP. We assume that data for GDP, the labor force, and the aggregate capital stock are available. The first step is to compute the productivity parameter A_t . Solving the production function for A_t yields:

$$A_t = \frac{Y_t^{\frac{1}{\alpha}}}{L_t K_{t-1}^{\frac{1-\alpha}{\alpha}}}.$$

If α were known, we could compute the A_t right away. Luckily, we found out earlier that α is equal to the labor share. Therefore we can use the average labor share as an estimate of α and compute the A_t .

Now that A_t is available, the growth rates in A_t , L_t and K_{t-1} can be computed.¹ We can see how the growth rates in inputs and productivity affect the growth rate of GDP by taking the natural log of the production function:

$$(11.12) \quad \ln Y_t = \alpha \ln A_t + \alpha \ln L_t + (1 - \alpha) \ln K_{t-1}.$$

We are interested in growth between the years t and $t+k$, where k is some positive integer. Subtracting equation (11.12) at time t from the same equation at time $t+k$ yields:

$$\ln Y_{t+k} - \ln Y_t = \alpha(\ln A_{t+k} - \ln A_t) + \alpha(\ln L_{t+k} - \ln L_t) + (1 - \alpha)(\ln K_{t+k-1} - \ln K_{t-1}).$$

Thus the growth rate in output (the left-hand side) is α times the sum of growth in productivity and labor, plus $1 - \alpha$ times growth in capital. Using this, we can compute the relative contribution of the different factors. The fraction of output growth attributable to growth of the labor force is:

$$\frac{\alpha[\ln L_{t+k} - \ln L_t]}{\ln Y_{t+k} - \ln Y_t}.$$

The fraction due to growth in capital equals:

$$\frac{(1 - \alpha)[\ln K_{t+k-1} - \ln K_{t-1}]}{\ln Y_{t+k} - \ln Y_t}.$$

Finally, the remaining fraction is due to growth in productivity and can be computed as:

$$\frac{\alpha[\ln A_{t+k} - \ln A_t]}{\ln Y_{t+k} - \ln Y_t}.$$

It is hard to determine the exact cause of productivity growth. The way we compute it, it is merely a residual, the fraction of economic growth that cannot be explained by growth in labor and capital. Nevertheless, measuring productivity growth this way gives us a rough idea about the magnitude of technological progress in a country.

11.4 Fertility and Human Capital

In this section we will examine how people decide on the number of children they have. Growth and industrialization are closely connected to falling fertility rates. This was true for 19th century England, where industrialization once started, and it applies in the same way to the Asian countries that only recently began to grow at high rates and catch up with

¹See Chapter 1 for a discussion of growth rates and how to compute them.

Western countries. Understanding these changes in fertility should help explain why some economies start to grow, while others remain poor.

The first economist to think in a systematic way about growth and fertility was Thomas Malthus. Back in 1798, he published his “Essay on Population”, in which his basic thesis was that fertility was checked only by the food supply. As long as there was enough to eat, people would continue to produce children. Since this would lead to population growth rates in excess of the growth in the food supply, people would be pushed down to the subsistence level. According to Malthus’s theory, sustained growth in per capita incomes was not possible; population growth would always catch up with increases in production and push per capita incomes down. Of course, today we know that Malthus was wrong, at least as far as the now industrialized countries are concerned. Still, his theory was an accurate description of population dynamics before the industrial revolution, and in many countries it seems to apply even today. Malthus lived in England just before the demographic transition took place. The very first stages of industrialization were accompanied by rapid population growth, and only with some lag did the fertility rates start to decline. We will take Malthus’s theory as a point of departure in our quest for explanations for the demographic transition.

Stated in modern terms, Malthus thought that children were a normal good. When income went up, more children would be “consumed” by parents. We assume that parents have children for their enjoyment only, that is, we abstract from issues like child labor. As a simple example, consider a utility function over consumption c_t and number of children n_t of the form:

$$u(c_t, n_t) = \ln(c_t) + \ln(n_t).$$

We assume that the consumer supplies one unit of labor for real wage w_t and that the cost in terms of goods of raising a child is p . Therefore the budget constraint is:

$$c_t + pn_t = w_t.$$

By substituting for consumption, we can write the utility maximization problem as:

$$\max_{n_t} \{\ln(w_t - pn_t) + \ln(n_t)\}.$$

The first-order condition with respect to n_t is:

$$\begin{aligned} \text{(FOC } n_t) \quad & -\frac{p}{w_t - pn_t} + \frac{1}{n_t} = 0, \text{ or:} \\ \text{(11.13)} \quad & n_t = \frac{w_t}{2p}. \end{aligned}$$

Thus the higher the real wage, the more children are going to be produced.

If we assume that people live for one period, the number of children per adult n_t determines the growth rate of population L_t :

$$\frac{L_{t+1}}{L_t} = n_t.$$

To close the model, we have to specify how the wage is determined. Malthus's assumption was that the food supply could not be increased in proportion with population growth. In modern terms, he meant that there were decreasing returns to labor. As an example, assume that the aggregate production function is:

$$Y_t = A_t L_t^\alpha,$$

with $0 < \alpha < 1$. Also assume that the real wage is equal to the marginal product of labor:

$$(11.14) \quad w_t = \alpha A_t L_t^{\alpha-1}.$$

We can combine equation (11.14) with the decision rule for the number of children in equation (11.13) to derive the law of motion for population:

$$(11.15) \quad \begin{aligned} \frac{L_{t+1}}{L_t} &= \frac{\alpha A_t L_t^{\alpha-1}}{2p}, \text{ or:} \\ L_{t+1} &= \frac{\alpha A_t L_t^\alpha}{2p}. \end{aligned}$$

Notice that this last equation looks similar to the law of motion for capital in the Solow model. The growth rate of population decreases as population increases. At some point, the population stops growing and reaches a steady state \bar{L} . Using equation (11.15), the steady-state level of population can be computed as:

$$\begin{aligned} \bar{L} &= \frac{\alpha A_t \bar{L}^\alpha}{2p}, \text{ or:} \\ \bar{L} &= \left(\frac{\alpha A_t}{2p} \right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

In the steady state, we have $L_{t+1}/L_t = n_t = 1$. We can use this in equation (11.13) to compute the wage \bar{w} in the steady state:

$$\begin{aligned} 1 &= \frac{\bar{w}}{2p}, \text{ or:} \\ \bar{w} &= 2p. \end{aligned}$$

Thus the wage in the steady state is independent of productivity A_t . An increase in A_t causes a rise in the population, but only until the wage is driven back down to its steady-state level. Even sustained growth in productivity will not raise per capita incomes. The population size will catch up with technological progress and put downward pressure on per capita incomes.

This Malthusian model successfully explains the relationship between population and output for almost all of history, and it still applies to large parts of the world today. Most developing countries have experienced large increases in overall output over the last 100 years. Unlike in Europe, however, this has resulted in large population increases rather

than in increases in per capita incomes. Outside the European world, per capita incomes stayed virtually constant from 1700 to about 1950, just as the Malthusian model predicts.

Something must have changed in Europe in the nineteenth century that made it attractive to people to have less children, causing fertility rates to fall, so per capita incomes could start to grow. While these changes are by no means fully understood, we can identify a number of important factors. We will concentrate on two of them: the time-cost of raising children, and a quality-quantity tradeoff in decisions on children.

Human capital is a key element of the model that we are going to propose. So far, we considered all labor to be of equal quality. That might be a reasonable assumption for earlier times in history, but it certainly does not apply in our time, where special qualifications and skills are important. In the model, human capital consists of two components. First, there is innate human capital that is possessed by every worker, regardless of education. We will denote this component of human capital by H_0 . This basic human capital reflects the fact that even a person with no special skill of any kind is able to carry out simple tasks that require manual labor only. In addition to this basic endowment, people can acquire extra human capital H_t through education by their parents. H_t reflects special skills that have to be taught to a worker. The total endowment with human capital of a worker is $H_0 + H_t$.

To come back to fertility decisions, we now assume that parents care both about the number n_t of their children and their "quality", or human capital $H_0 + H_{t+1}$. Preferences take the form:

$$u(c_t, n_t, H_{t+1}) = \ln(c_t) + \ln(n_t(H_0 + H_{t+1})).$$

The other new feature of this model is that parents must invest time, rather than goods, to raise children. In the Malthusian model, p units of the consumption good were needed to raise a child. We now assume that this cost in terms of goods is relatively small, so it can be omitted for simplicity. Instead, children require attention. For each child, a fraction h of the total time available has to be used to raise the child. In addition, the parents can decide to educate their children and spend fraction e_t of their time doing that. This implies that only a fraction $1 - hn_t - e_t$ is left for work. If w_t is the wage per unit of human capital when working all the time, the budget constraint is:

$$(11.16) \quad c_t = w_t(H_0 + H_t)(1 - hn_t - e_t).$$

The right-hand side says that income is the wage multiplied by human capital and the fraction of time worked. All this income is spent on consumption. We still have to specify the determination of the human capital of the children. We assume that the extra human capital of each child H_{t+1} depends on: the acquired human capital H_t of that child's parents, and the time e_t the parents spend teaching their children:

$$(11.17) \quad H_{t+1} = \gamma e_t H_t.$$

Here γ is a positive parameter. The interpretation of equation (11.17) is that parents who are skilled themselves are better at teaching their children. A person who does not have any skills is also unable to teach anything to his or her children.

We now want to determine how fertility is related to human capital in this model. If we plug the constraints in equations (11.16) and (11.17) into the utility function, the utility maximization problem becomes:

$$\max_{n_t, e_t} \{ \ln(w_t(H_0 + H_t)(1 - hn_t - e_t)) + \ln(n_t(H_0 + \gamma e_t H_t)) \}.$$

The first-order conditions with respect to n_t and e_t are:

$$\begin{aligned} \text{(FOC } n_t) \quad & -\frac{h}{1 - hn_t - e_t} + \frac{1}{n_t} = 0; \text{ and:} \\ \text{(FOC } e_t) \quad & -\frac{1}{1 - hn_t - e_t} + \frac{\gamma H_t}{H_0 + \gamma e_t H_t} = 0. \end{aligned}$$

(FOC n_t) can be rewritten as:

$$\begin{aligned} hn_t &= 1 - hn_t - e_t, \text{ or:} \\ \text{(11.18)} \quad e_t &= 1 - 2hn_t. \end{aligned}$$

Using equation (11.18) in (FOC e_t) allows us to compute the optimal fertility decision:

$$\begin{aligned} \gamma H_t(1 - hn_t - (1 - 2hn_t)) &= H_0 + (1 - 2hn_t)\gamma H_t; \text{ or:} \\ \gamma H_t hn_t &= H_0 + \gamma H_t - 2\gamma H_t hn_t; \text{ or:} \\ 3\gamma H_t hn_t &= H_0 + \gamma H_t; \text{ or:} \\ \text{(11.19)} \quad n_t &= \frac{1}{3h} \left[\frac{H_0}{\gamma H_t} + 1 \right]. \end{aligned}$$

According to equation (11.19), the key determinant of fertility is human capital H_t . If it is close to zero, the number of children is very high. If we added a cost of children in terms of goods to this model, for low values of H_t the outcomes would be identical to the Malthusian model. However, things change dramatically when H_t is high. Fertility falls, and if H_t continues to rise, the number of children reaches the steady state: $\bar{n} = 1/(3h)$. There are two reasons for this outcome. On the one hand, if human capital increases, the value of time also increases. It becomes more and more costly to spend a lot of time raising children, so parents decide to have less of them. The other reason is that people with high human capital are better at teaching children. That makes it more attractive for them to invest in the quality instead of the quantity of children.

The model sheds some light on the reasons why today fertility in industrialized countries is so much lower than that in developing countries. The theory also has applications within a given country. For example, in the United States teenagers are much more likely to become pregnant if they are school dropouts. The model suggests that this is not by accident. People with low education have a relatively low value of time, so spending time with children is less expensive for them.

The question that the model does not answer is how the transition from the one state to the other takes place. How did England manage to leave the Malthusian steady state?

In the model, only a sudden jump in H_t over some critical level could perform this task, which is not a very convincing explanation for the demographic transition. Still, the model is a significant improvement over theories that assume that population growth rates are exogenous and constant. More research on this and related questions will be needed before we can hope to find a complete explanation for the demographic transition and the wide disparity in wealth around the world.

Variable	Definition
Y_t	Aggregate output
y_t	Output per unit of effective labor
L_t	Aggregate labor input or population
K_t	Aggregate capital stock
k_t	Capital per unit of effective labor
A_t	Productivity parameter
I_t	Aggregate investment
i_t	Investment per unit of effective labor
w_t	Wage
r_t	Return on capital
δ	Depreciation rate
α	Parameter in the production function
$u(\cdot)$	Utility function
c_t	Consumption
n_t	Number of children
p	Cost of raising a child, in terms of goods
h	Cost of raising a child, in terms of time
e_t	Time spent on educating children
H_0	Innate human capital
H_t	Acquired human capital
γ	Parameter in the production function for human capital

Table 11.1: Notation for Chapter 11

Exercises

Exercise 11.1 (Easy)

Suppose the aggregate production technology is $Y = 3L^{.7}K^{.3}$ and that $L = 150$. Both the labor force and productivity are constant. Assume that the depreciation rate is 10% and that

20% of output is saved and invested each year. What is the steady-state level of output?

Exercise 11.2 (Moderate)

Assume that the Solow model accurately describes the growth experience of Kuwait. As a result of the Gulf war, much of the capital in Kuwait (oil extracting equipment, vehicles, structures etc.) was destroyed. Answer the following questions, and provide brief explanations.

- What will be the effect of this event on per capita income in Kuwait in the next five years?
- What will be the effect of this event on per capita income in Kuwait in the long run?
- What will be the effect of this event on the annual growth rate of per capita income in Kuwait in the next five years?
- What will be the effect of this event on the growth rate of per capita income in Kuwait in the long run?
- Will recovery in Kuwait occur faster if investment by foreigners is permitted, or if it is prohibited?
- Would Kuwaiti workers gain or lose by a prohibition of foreign investment? Would Kuwaiti capitalists gain or lose?

Exercise 11.3 (Moderate)

In this and the following two exercises, you will apply growth accounting to measure the determinants of growth in output per worker in a country of your choice. To start, you need to pick a country and retrieve data on real GDP per worker and capital per worker. You can get the time series you need from the Penn World Tables. See Exercise 9.1 for information about how to access this data set. You should use data for all years that are available.

In Section 11.3, we introduced growth accounting for output growth, while in this exercise we want to explain growth in output per worker. We therefore have to redo the analysis of Section 11.3 in terms of output per worker. The first step is to divide the production function in equation (11.11) by the number of workers L_t , which yields:

$$(11.20) \quad \frac{Y_t}{L_t} = \frac{(A_t L_t)^\alpha K_{t-1}^{1-\alpha}}{L_t} = \frac{A_t^\alpha L_t^\alpha K_{t-1}^{1-\alpha}}{L_t^\alpha L_t^{1-\alpha}} = A_t^\alpha \left(\frac{K_{t-1}}{L_t} \right)^{1-\alpha}.$$

Equation (11.20) relates output per worker Y_t/L_t to capital per worker K_{t-1}/L_t . If we use lower case letters to denote per-worker values ($y_t = Y_t/L_t$, $k_{t-1} = K_{t-1}/L_t$), we can write equation (11.20) as:

$$(11.21) \quad y_t = A_t^\alpha k_{t-1}^{1-\alpha}.$$

Use equation (11.21) to derive a formula for A_t and to derive a decomposition of growth in output per worker into growth in capital per worker and productivity growth. You can do that by following the same steps we took in Section 11.3.

Exercise 11.4 (Moderate)

Compute the productivity parameter A_t for each year in your sample. For your computations, assume that $1 - \alpha = .4$. This is approximately equal to the capital share in the United States, and we assume that all countries use the same production function. In fact, in most countries measures for $1 - \alpha$ are close to .4.

Exercise 11.5 (Moderate)

By using log-differences, compute the growth rate of GDP, productivity, and capital per worker for each year in your sample. Also compute the average growth rate for these three variables.

Exercise 11.6 (Moderate)

What percentage of average growth per worker is explained by growth in capital, and what percentage by productivity growth? For the period from 1965 to 1992, the average growth rate of output per worker was 2.7% in the United States, and productivity growth averaged 2.3%. How do these numbers compare to your country? Does the neoclassical growth model offer an explanation of the performance of your country relative to the United States? If not, how do you explain the differences?