Chapter 12

The Effect of Government Purchases

In this chapter we consider how governmental purchases of goods and services affect the economy. Governments tend to spend money on two things: wars and social services. Barro's Figure 12.2 shows that expenditures by the U.S. government have comprised a generally increasing fraction of GNP since 1928, but even today that fraction is nowhere near the peak it attained during WWII. This pattern is generally repeated across countries. The taste for social services seems to increase with national wealth, so the governments of richer countries tend to spend more, as a fraction of GDP, than the governments of poorer countries, especially during peacetime. Of course, there are exceptions to this pattern.

We will examine government spending in three ways:

1. We shall consider the effect of permanent changes in government spending in order to think about the secular peacetime increases in spending;
2. We shall consider temporary changes in government spending in order to think about the effect of sudden spikes like wars;
3. We shall begin an analysis of the effect of government social programs. Since government social programs (unemployment insurance, social security systems) are inextricably linked to tax systems, we will defer part of our analysis to the next chapter.

Since we have yet to fully discuss tax policy, for this chapter we will assume that the government levies a very special kind of tax: a *lump-sum tax.* That is, the government announces a spending plan and then simply removes that amount of money from the budget of the representative household. As we shall see in the next chapter, this kind of tax system does not distort the household's choices.

In the Barro textbook, the government budget constraint, in addition to lump sum taxes, also contains fiat currency. In this chapter we will assume that the government does not use the printing press to finance its purchases. In later chapters (especially Chapter 18) we will examine this effect in much greater detail.

12.1 Permanent Changes in Government Spending

Assume that the government announces a permanent level of government spending, G, to be levied each period. What is the role of these government expenditures? The government provides *productive* services, such as a court system for enforcing contracts and an interstate highway system for quickly and cheaply transporting goods. The government also provides *consumption* services such as public parks and entertainment spectacles such as trips to the moon and congressional hearings. We focus on the first role.

How should we model the productive services provided by the government? We shall analyze a model under two assumptions:

1. Government spending at some constant rate ϕ,
2. The effect of government spending G is augmented by the level of capital, K_t, so output Y increases by the amount $\phi G K_t$.

In the first case, \$100 of government spending increases output by 100ϕ regardless of the current level of capital, while in the second case, the same \$100 boosts output much more in nations with more capital.

The representative household lives forever and has preferences over consumption streams $\{C_t\}_{t=0}^{\infty}$ given by:

$$V(\{C_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t U(C_t).$$

Where $U' > 0$ and $U'' < 0$. Here $0 < \beta < 1$ reflects impatience. In addition, to keep the algebra nice, we will say that:

$$\beta = \frac{1}{1+\rho}.$$

Here β is the discount factor and ρ the discount rate.

The household has access to a productive technology mapping capital K_t into *private* output Y_t^P of:

$$Y_t^P = K_t^{\alpha}.$$

Total output (and hence income) of the household will be the sum of private output and government-augmented output, Y_t^G. Government augmented output will take on one of two values:

$$Y_t^G = \phi G, \text{ or:} \tag{12.1}$$

$$Y_t^G = \phi G K_t. \tag{12.2}$$

Equation (12.1) corresponds to the case of government spending affecting total output the same amount no matter what the level of capital. Equation (12.2) corresponds to the case of government spending affecting total output more when the level of capital is high. We shall examine the effect of G on capital accumulation, aggregate output and consumption under both of these assumptions.

The household must split total income $Y_t = Y_t^P + Y_t^G$ into consumption C_t, investment I_t and *payments to the government* of G. Recall that we assumed the government would simply levy lump-sum taxes. Now we are using that assumption. The household's resource constraint is thus:

$$C_t + I_t + G \leq Y_t. \tag{12.3}$$

Finally, there is a law of motion for the capital stock K_t. Each period, a proportion δ of the capital stock vanishes due to physical depreciation, so only the remaining $(1-\delta)$ proportion survives into the next period. In addition, capital may be augmented by investment. Thus capital evolves according to:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{12.4}$$

We assume that the representative household begins life with some initial stock of capital $K_0 > 0$.

We are interested in writing C_t as a function of next period's capital stock K_{t+1}. Combining equations (12.3) and (12.4) gives:

$$C_t = K_t^\alpha + (1-\delta)K_t - K_{t+1} - G + \phi G, \text{ or:} \tag{BC1}$$

$$C_t = K_t^\alpha + (1-\delta)K_t - K_{t+1} - G + \phi G K_t. \tag{BC2}$$

The differences between the two equations arises from which version of the government technology we use, equation (12.1) or (12.2).

Analysis with Equation (BC1)

Let us begin our analysis with the first version of the government spending technology, equation (12.1). Thus we are using as the relevant budget constraint equation (BC1). The household's problem becomes:

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\left[K_t^{\alpha} + (1-\delta)K_t - K_{t+1} - (1-\phi)G\right].$$

We take first-order conditions with respect to the choice of next period's capital K_{j+1} in some typical period j. Remember that K_{j+1} appears in two periods, j and $j+1$:

$$\beta^j U'(C_j)[-1] + \beta^{j+1} U'(C_{j+1})\left[\alpha K_{j+1}^{\alpha-1} + 1 - \delta\right] = 0.$$

For all $j = 0, 1, \ldots, \infty$. Here C_j is given by equation (BC1) above. Simplifying produces:

$$U'(C_j) = \beta U'(C_{j+1})[\alpha K_{j+1}^{\alpha-1} + 1 - \delta]. \tag{12.5}$$

For simplicity (and as in other chapters) we choose not to solve this for the transition path from the initial level of capital K_0 to the steady state level K_{ss}, and instead focus on characterizing the steady state. At a steady state, by definition the capital stock is constant:

$$K_t = K_{t+1} = K_{ss}.$$

As a result:

$$C_t = C_{t+1} = C_{ss}, \text{ and:}$$
$$I_t = I_{t+1} = I_{ss} = \delta K_{ss}.$$

Equation (12.5) at the steady-state becomes:

$$U'(C_{ss}) = \beta U'(C_{ss})[\alpha K_{ss}{}^{\alpha-1} + 1 - \delta].$$

Simplifying, and using the definition of β as $1/(1+\rho)$ produces:

$$1 + \rho = \alpha K_{ss}{}^{\alpha-1} + 1 - \delta.$$

We now solve for the steady-state capital level:

$$K_{ss} = \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}.$$

Notice immediately that, under this formulation of government spending the steady state capital level is independent of government spending. As we shall see in the next chapter, this is a direct consequence of the lump-sum tax technology. If the government had to use a distortionary tax, K_{ss} would be affected by G.

Given K_{ss}, it is easy to calculate the other variables that the household controls: steady-state private income, $Y_{ss}{}^P$, consumption C_{ss}, and investment, I_{ss}. From the technology, we know that $Y^P = K^{\alpha}$, so:

$$Y_{ss}{}^P = K_{ss}{}^{\alpha}.$$

Total output (GDP) is private output Y^P plus government output Y^G, or:

$$Y_{ss} = {K_{ss}}^{\alpha} + \phi G.$$

Consumption is, in this case, determined by the budget constraint equation (BC1). At the steady-state, then:

$$C_{ss} = {K_{ss}}^{\alpha} + (1-\delta)K_{ss} - K_{ss} - (1-\phi)G.$$

We can simplify this to produce:

$$C_{ss} = {K_{ss}}^{\alpha} - \delta K_{ss} - (1-\phi)G.$$

At the steady-state, the household must be investing just enough in new capital to offset depreciation. Substituting into the law of motion for capital provides:

$$I_{ss} = \delta K_{ss}.$$

Now we are ready to determine the effect of government spending on total output, consumption and the capital level. When we think about changing G we are comparing two different steady states. Thus there may be short-term fluctuations immediately after the government announces its new spending plan, but we are concerned here with the long-run effects.

Notice immediately that:

$$\frac{dK_{ss}}{dG} = 0, \tag{12.6}$$

$$\frac{dY_{ss}}{dG} = \frac{d}{dG}(Y^P + Y^G) = \phi, \text{ and:} \tag{12.7}$$

$$\frac{dC_{ss}}{dG} = -(1-\phi)G.. \tag{12.8}$$

That is, total output is increasing in G but consumption is decreasing in G if $\phi < 1$. Thus $\phi < 1$ is an example of *crowding out.* Think of it this way: the government spends \$1000 on a new factory, which produces 1000ϕ units of new output. The household pays the \$1000 in taxes required to construct the new factory, does not alter its capital level and enjoys the extra output of 1000ϕ as consumption. If $\phi < 1$ the household has lost consumption. Thus output has increased and consumption has decreased.

Why do we automatically assume that $\phi < 1$? This is equivalent to saying that the government is worse at building factories than the private sector. The government may be the only institution that can provide contract enforcement, police and national defense, but long history has shown that it cannot in general produce final goods as effectively as the private sector.

One final note before we turn our attention to the effect of production augmenting government spending. Government transfer payments, in which the government takes money from one agent and gives it to another, fit nicely into this category of expenditure. Transfer payments have absolutely no productive effects, and the government institutions required to administer the transfer payments systems will prevent the perfect transmission of money from one agent to another. Since we are working with a representative consumer, transfer payments appear as taxes which are partially refunded.

Analysis with Equation (BC2)

Now let us consider the effect of government spending whose benefits are proportional to capital stock. We will use precisely the same analysis as before. except that now consumption C_t as a function of capital K_t and K_{t+1} and government spending G will be given by equation (BC2) above.

The household's problem becomes:

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\left[K_t^{\alpha} + (1-\delta+\phi G)K_t - K_{t+1} - G\right].$$

We take first-order conditions with respect to the choice of next period's capital stock K_{j+1} in some typical period j. Remember the trick with these problems: K_{j+1} appears twice in the maximization problem, first negatively in period j and then positively in period $j+1$:

$$\beta^j U'(C_j)[-1] + \beta^{j+1}U'(C_{j+1})\left[\alpha K_{j+1}^{\alpha-1} + 1 - \delta + \phi G\right] = 0.$$

For all $j = 0, 1, \ldots, \infty$. C_j is given by equation (BC2). Simplifying produces:

$$U'(C_j) = \beta U'(C_{j+1})[\alpha K_{j+1}^{\alpha-1} + 1 - \delta + \phi G]. \tag{12.9}$$

Compare this with the previous simplified first-order condition, equation (12.5) above. Notice that in equation (12.5) the government spending term G does not appear. Here it does. This should alert us immediately that something new is about to happen. As before, we assume a steady state and characterize it. At the steady state:

$$U'(C_{\text{ss}}) = \beta U'(C_{\text{ss}})[\alpha K_{\text{ss}}{}^{\alpha-1} + 1 - \delta + \phi G].$$

Using our definition of β as $1/(1+\rho)$ this becomes:

$$1 + \rho = \alpha K_{\text{ss}}{}^{\alpha-1} + 1 - \delta + \phi G.$$

Hence the steady-state capital level is:

$$K_{\text{ss}} = \left(\frac{\alpha}{\rho + \delta - \phi G}\right)^{\frac{1}{1-\alpha}}.$$

Notice immediately that, under this formulation of government spending the steady-state capital level is increasing in government spending. If the government were forced to finance its spending with a distortionary tax this result might not go through.

Given the steady-state capital level, it is easy to calculate the steady-state levels of total output Y_{SS}, consumption C_{SS} and investment, I_{SS}. Since the steady-state capital level, K_{SS}, is now affected by G, both public output Y^G and private output Y^P are in turn affected by G. Given the production function, we see that:

$$Y_{SS} = {K_{SS}}^{\alpha} + \phi K_{SS} G.$$

From the budget constraint equation (BC2) above, we see that the steady state, consumption is:

$$C_{SS} = {K_{SS}}^{\alpha} - \delta K_{SS} - (1 - \phi K_{SS}) G.$$

As before, the household must be investing just enough to overcome depreciation, to keep the capital level constant:

$$I_{SS} = \delta K_{SS}.$$

Now we can reconsider the effect of government spending on total output, consumption and the capital level. Some of these derivatives are going to be fairly involved, but if we break them down into their constituent pieces they become quite manageable.

Begin by defining:

$$X \equiv \frac{\alpha}{\rho + \delta - \phi G}.$$

Note that:

$$\frac{dX}{dG} = \frac{\phi}{\rho + \delta - \phi G} X.$$

The steady-state capital stock is:

$$K_{SS} = X^{\frac{1}{1-\alpha}},$$

so the derivative of the steady-state capital stock with respect to G is:

$$\frac{dK_{SS}}{dG} = \frac{1}{1-\alpha} X^{\frac{1}{1-\alpha}-1} \frac{dX}{dG}.$$

Plugging in dX/dG yields:

$$\begin{aligned} \frac{dK_{SS}}{dG} &= \frac{1}{1-\alpha} X^{\frac{1}{1-\alpha}-1} \frac{\phi}{\rho + \delta - \phi G} X \\ &= \frac{1}{1-\alpha} \frac{\phi}{\rho + \delta - \phi G} X^{\frac{1}{1-\alpha}} \\ &= \frac{1}{1-\alpha} \frac{\phi}{\rho + \delta - \phi G} K_{SS}. \qquad (12.10) \end{aligned}$$

Armed with this result we can tackle the other items of interest. First, consider the effect of increased spending on aggregate output:

$$
\begin{aligned}
\frac{dY_{ss}}{dG} &= \frac{d}{dG}({Y_{ss}}^P + {Y_{ss}}^G) \\
&= \frac{d}{dG}\left({K_{ss}}^\alpha + \phi G K_{ss}\right) \\
&= \alpha {K_{ss}}^{\alpha-1}\frac{dK_{ss}}{dG} + \phi G \frac{dK_{ss}}{dG} \\
&= \alpha {K_{ss}}^{\alpha-1}\frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}K_{ss} + \phi G \frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}K_{ss} \\
&= \frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}\left[\alpha {K_{ss}}^\alpha + \phi G K_{ss}\right] \\
&= \frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}\left[\alpha {Y_{ss}}^P + {Y_{ss}}^G\right]. \qquad (12.11)
\end{aligned}
$$

Compare the effect of government spending on aggregate output here with the effect of government spending on aggregate output when government spending simply augments output directly, equation (12.7) above. Notice that while previously every dollar of government spending translated into ϕ dollars of extra output no matter what the output level, now government spending is more productive in richer economies.

Finally, we turn our attention to consumption. Recall that before, for $\phi < 1$, consumption decreased as government spending increased, that is, consumption was crowded out. Now we shall see that, while consumption may be crowded out, it will not necessarily be crowded out. In fact, in rich economies, increases in government spending may increase consumption. Once again, this result will hinge to a certain extent on the assumption of a perfect tax technology. Begin by writing consumption as:

$$
(12.12) \qquad C_{ss} = {K_{ss}}^\alpha - \delta K_{ss} - G + \phi G K_{ss}, \text{ so:}
$$

$$
\begin{aligned}
\frac{dC_{ss}}{dG} &= \frac{d}{dG}({K_{ss}}^\alpha + (\phi G - \delta)K_{ss} - G) \\
&= \alpha {K_{ss}}^{\alpha-1}\frac{dK_{ss}}{dG} + (\phi G - \delta)\frac{dK_{ss}}{dG} + \phi K_{ss} - 1 \\
&= \alpha {K_{ss}}^{\alpha-1}\frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}K_{ss} + (\phi G - \delta)\frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}K_{ss} + \phi K_{ss} - 1 \\
&= \frac{1}{1-\alpha}\frac{\phi}{\rho+\delta-\phi G}\left[\alpha {K_{ss}}^\alpha + (\phi - \delta)K_{ss}\right] + \phi K_{ss} - 1.
\end{aligned}
$$

The first two terms are certainly positive. The question is, are they large enough to outweigh the -1? Even if $\phi < 1$, for large values of G this may indeed be the case.

Increasing Returns to Scale and Government Spending

Thus we have seen that the effect of government spending depends crucially on assumptions about how it is transformed into output. In the next chapter we will also see that it depends on how the government raises the revenue it spends.

Our second assumption about technology, embodied in equation (BC2), generated some exciting results about government spending. It seems that, if the world is indeed like the model, there is a potential for governments to provide us with a free lunch. Take a closer look at equation (12.2). If we assumed that the representative household controlled G directly (through representative government, for example) what level would it choose? Ignore the dynamics for a moment and consider the household's consumption C^a given that it has chosen some level of G and K:

$$C^a \equiv C(K, G) = K^\alpha + \phi GK - \delta K - G.$$

Now suppose the household doubles its inputs of K and G, so it is consuming some amount C^b:

$$C^b \equiv C(2K, 2G) = 2^\alpha K^\alpha + 4\phi GK - 2\delta K - 2G.$$

For sufficiently large values of G and K it is easy to see that:

$$C^b > 2C^a.$$

In other words, by doubling G and K, the representative household could more than double net consumption. This is the standard free lunch of increasing returns to scale, in this case jointly in K and G. In the real world, are there increasing returns to scale jointly in government spending and capital? In certain areas this is almost certainly true. For example, by providing sewage and water-treatment services the government prevents epidemics and lowers the cost of clean water to consumers. This is a powerful direct benefit. This direct benefit is increasing in the population concentration (a small village probably would do fine with an outhouse, while 19th-century Chicago was periodically decimated by Cholera epidemics before the construction of the sanitary canal), and in turn encourages greater capital accumulation. No one business or household in 18th century Chicago would have found it worthwhile to build a sewage system, so it would have been difficult for private enterprise alone to have provided the improvements. Furthermore, since the Chicago sewage system depends in large measure on the Sanitary Canal, which had to be dug across previously-private land, it may have been impossible to build without the power of eminent domain.[1]

Unfortunately, there are few such clear-cut cases of increasing returns to scale combined with the requirement of government power. Why should a city government construct a stadium to lure sports teams? To build it, the government has to tax citizens who may experience no direct or indirect benefit.

[1] For more information on Chicago's sewer works, see Robin L. Einhorn, *Property Rules: Political Economy in Chicago, 1833-1872.*

Transitions in the Example Economies

We have so far ignored the problem of *transitions* in order to concentrate on steady-state behavior. But transition dynamics, describing the path that capital, consumption and the interest rate take as an economy transitions from low capital to the steady state capital level can be extremely interesting. In this subsection we will study transition dynamics by numerically simulating them on a computer.

Consider an example economy in which $G = 0.4$, $\phi = 0.1$, $\alpha = 0.25$, $\rho = 0.075$, $\delta = 0.1$ and $\beta = 1/(1+\rho)$. Using the technology from equation (BC1), the steady-state capital level is $K_{ss} = 1.6089$, using the better technology from equation (BC2), the steady-state capital level is $K_{ss} = 2.2741$. Notice that, since $G = 0.4$, government spending as a fraction of output in these example economies is 0.3436 and 0.3033, respectively.

What happens if we endow the representative consumer with an initial capital stock $K_0 = 0.03$, which is far below the eventual steady-state level? We know generally that there will be growth to the steady-state, but little more.

The evolution of the capital stock under both assumptions about the government spending technology is plotted in Figure (12.1). The solid line gives the evolution with the high-return government spending technology (that is, equation (BC2)), while the dotted line gives the evolution with the low-return technology (that is, equation (BC1)). Notice that the economy based on equation (BC2) is initially poorer and slower-growing than the other economy. This is because, at low levels of capital, government spending is not very productive and is a serious drag on the economy. As capital accumulates and the complementarities with government spending kick in, growth accelerates and the economy based on equation (BC2) surpasses the economy based on equation (BC1).

In the same way, the time path of consumption is plotted in Figure (12.2). Finally, the real interest rate in these economies is plotted in Figure (12.3). For more about how to calculate the real interest rate in these models, please see the next section.

The Real Interest Rate

Now we turn our attention to the effect of permanent changes in government spending on the equilibrium real interest rate in this model. Recall that in infinite-horizon capital accumulation models, like the one we are studying here, it usual to assume there is a closed economy, so the representative household does not have access to a bond market. In this setting, the equilibrium interest rate becomes the interest rate at which the household, if offered the opportunity to use a bond market, would not do so. In other words, there is, as usual, no net borrowing or lending in a closed economy. We will refer to this condition as a market-clearing condition in the bond market, or simply market-clearing for short.

We shall see that, during the transition period while capital is still being accumulated, the

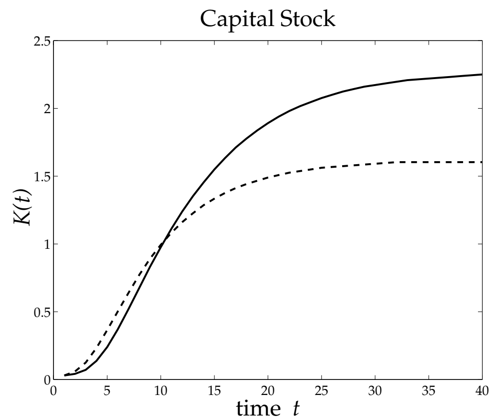

Figure 12.1: Evolution of capital stock. The solid line gives K_t assuming that government purchases affect output as in equation (BC2) and the dotted line assuming they affect output as in equation (BC1).

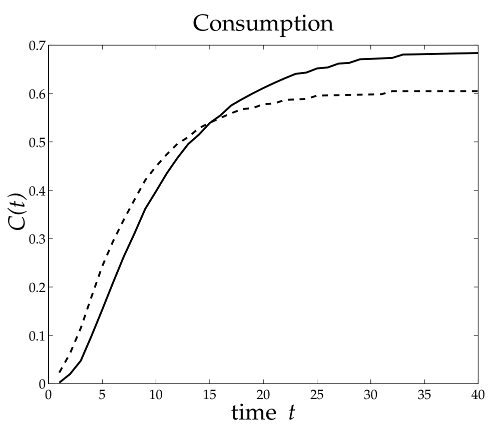

Figure 12.2: Time path of consumption. The solid line gives C_t assuming that government purchases affect output as in equation (BC2) and the dotted line assuming they affect output as in equation (BC1).

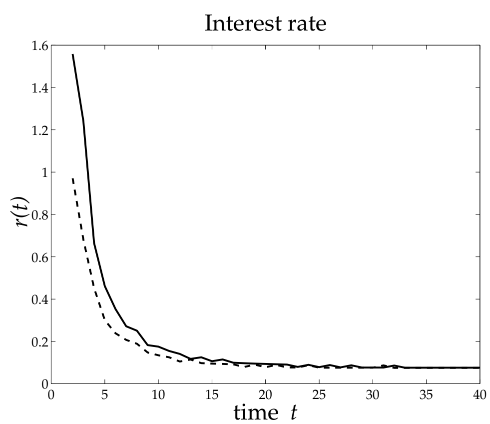

Figure 12.3: Time path of interest rates. The solid line gives r_t assuming that government purchases affect output as in equation (BC2) and the dotted line assuming they affect output as in equation (BC1).

interest rate is decreasing in the capital stock. At the steady state, however, when consumption is constant, the equilibrium interest rate will just be ρ, the discount rate. Because permanent changes in government spending lead eventually to a new steady-state, at which consumption is constant, permanent changes in the level of government spending will not affect the equilibrium interest rate at the steady state.

The easy way to see this is to notice that if the representative household has some endowment stream $\{e_t\}_{t=0}^{\infty}$, and the interest rate satisfies:

$$1 + r_t = \frac{1}{\beta}\frac{U'(C_t = e_t)}{U'(C_{t+1} = e_{t+1})},$$

then there will be no net borrowing or lending across periods. In our case the endowment stream $\{e_t\}_{t=0}^{\infty}$ is the result of a capital accumulation process which eventually reaches a steady state at which $e_t = e_{t+1} = e_{\text{ss}}$. Hence at a steady-state:

$$1 + r_{\text{ss}} = \frac{1}{\beta}\frac{U'(C_{\text{ss}})}{U'(C_{\text{ss}})} = \frac{1}{\beta} = 1 + \rho, \text{ so:}$$
$$r_{\text{ss}} = \rho.$$

No matter what the eventual steady-state level of capital, at the steady-state consumption becomes smooth, which forces the equilibrium interest rate to the discount rate. If $r_{\text{ss}} > \rho$ the household would wish to save on the bond market (consuming below endowment and thus violating market-clearing) and if $r_{\text{ss}} < \rho$ then then the household would wish to borrow on the bond market (consuming above endowment and again violating market-clearing).

12.2 Temporary Changes in Government Spending

Studying temporary changes in government spending requires studying the transition path of an economy from one steady-state to another and then back again. Imagine an economy of the type we studied in the previous section, in which the government is spending some low but constant amount G_0 each period. As time goes forward, the capital stock and consumption converge to their steady-state levels and the real interest rate converges to the discount rate. Suddenly the government must fight an expensive war. Government spending shoots up to some high level G_1 for a relatively short period of time. During the war, the capital stock will begin to transition to the steady-state implied by the new spending level G_1. Since wars tend to be short it may never get there. When the war is over, government spending drops to its accustomed pre-war level of G_0, and the capital stock slowly returns from wherever it was when the war ended to the old steady-state.

Analytically determining the trajectories of capital, consumption and the interest rate under temporary shifts in government spending is beyond the scope of this chapter. However, we can easily simulate them numerically, using precisely the same techniques we did to study the growth experience of economies.

All of the figures that follow make the following assumptions: That in periods 1-5 the economy is at its pre-war steady-state, that in periods 6-15 the economy is in a war, with increased government spending, and in periods 16-30 the economy is back at peace. During the war the economy begins its transition to a war steady-state, but the relatively short duration of the war prevents it from ever reaching that steady-state. After the war the economy transitions slowly back to its pre-war steady-state. We are also assuming that in the last period of peace before the war (period 5) the population learns of the impending war, and that in the last period of the war before peace begins again (period 15) the population learns of the coming peace.

The parameters used here are exactly those used in the section on transitions in the example economies (page 120) above. In addition, the peacetime spending level is $G_0 = 0$ and the wartime spending level is $G_1 = 0.4$.

The evolution of the capital stock under both assumptions about the government spending technology is plotted in Figure (12.4). The solid line gives the evolution with the high-return government spending technology (that is, equation (BC2)), while the dotted line gives the evolution with the low-return technology (that is, equation (BC1)).

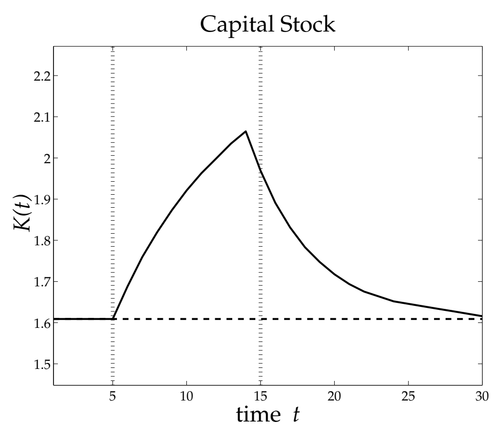

Figure 12.4: Time path of the capital stock before, during and after a war. The solid line gives K_t assuming that government purchases affect output as in equation (BC2) and the dotted line assuming they affect output as in equation (BC1).

In the same way, the time path of consumption is plotted in Figure (12.5). Finally, the *real interest rate* in these economies is plotted in Figure (12.6). It is surprising to note that sometimes the real interest rate is negative. From the section on the real interest rate (on page 120 above) we know that, given consumption decisions C_t and C_{t+1} that r_t must satisfy:

$$r_t = \frac{1}{\beta}\frac{U'(C_t)}{U'(C_{t+1})} - 1.$$

If C_{t+1} is quite small relative to C_t, then $U'(C_{t+1})$ will be large relative to $U'(C_t)$ and r_t might be negative. A negative real interest rate occurs in precisely those periods in which today's consumption must be high relative to tomorrow's, as in the last period of peacetime before the war, in order to prevent agents from carrying wealth forward into the next period. At the ends of wars, when today's consumption is low relative to tomorrow's (think March, 1945), real interest rates are quite high, to dissuade borrowing.

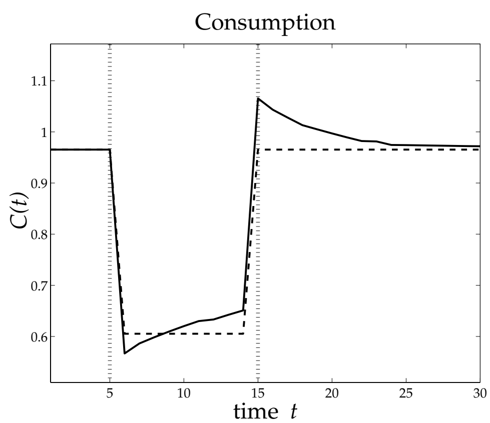

Figure 12.5: Time path of consumption before, during and after a war. The solid line gives C_t assuming that government purchases affect output as in equation (BC2) and the dotted line assuming they affect output as in equation (BC1).

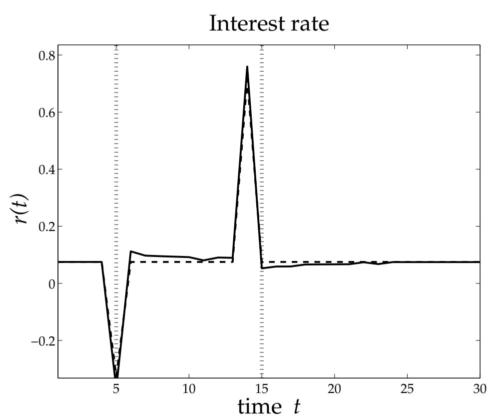

Figure 12.6: Time path of the interest rate before, during and after a war. Note the very low interest rates prevalent in the last period before the war and the generally higher interest rates during the war. The solid line gives r_t assuming that government purchases affect output as in equation (BC2) and the dotted line assuming they affect output as in equation (BC1).

In general, Barro presents evidence that, during wartime, interest rates tend to increase. That fits well with the experience of the second model presented here, the one in which government purchases affect output as in equation (BC2).

12.3 Social Security

The Social Security system is one of the largest components of U.S. government spending. There are some interesting theoretical issues associated with it that are worth examining. Social Security is an old-age pension system, in which young workers pay into a general fund with a *payroll tax* of about 7% of wages and old retirees receive payments from this same general fund. Thus although it maintains the illusion of being a national savings scheme (and many politicians and voters are convinced that it is exactly that) is in fact an *unfunded* or *pay-as-you-go* pension scheme. In an unfunded pension system, payments to retirees are paid for by taxes levied on the current young.

Other countries have adopted *funded* pension schemes, which are essentially forced savings systems. In a funded pension system, young workers are taxed, with the proceeds going to an individual account, invested in some securities (the precise type of investment mix, and whether these investments are under the control of the government or the worker vary from country to country). When workers become old and retire, they draw down their accumulated stock of savings.

Consider a world in which there are two types of agents: Young workers who earn an amount y in their working years, and old retirees who earn nothing. This is clearly a vast simplification over reality, since, in particular, the retirement date is exogenous. However, even this simple model will help us think clearly about pension schemes. A generation born in period t will have preferences over consumption while young C_0^t and old C_1^t of:

$$U(C_0^t, C_1^t) = 2\sqrt{C_0^t} + 2\beta\sqrt{C_1^t}.$$

Where $0 < \beta < 1$ reflects a preference for consumption while young.

Each period t there are N_t new young workers born, each of whom produces y with certainty in their youth. The youth population N_t evolves as:

$$N_{t+1} = (1 + n)N_t.$$

There is a bond market which pays a constant, riskless, real interest rate of $r \geq 0$, paid "overnight" on savings. Where does this bond market come from? We will not say here, leaving it simply outside of the scope of the model. If you are bothered by this, however, imagine that a certain portion of the population, instead of being workers, are entrepreneurs, who will accept funds from workers, use them as capital in a productive process of some kind, and then use the output from that production to repay the workers (now old) with interest. The interest rate gets set as the result of competition among entrepreneurs for funds.

Funded Pension Systems

Begin with an analysis of a funded system. The government levies a *tax rate* of τ on young workers' income y, taking τy. Since the young workers do not affect y, this is equivalent to a lump-sum tax. The government invests τy on behalf of the young workers, realizes the common real rate of return r on it, and returns it to the agents when they are retired. In addition, workers of generation t may save an amount $S_t \geq 0$ in the bond market on their own. Assume that τ is small relative to the savings needs of agents. This will prevent them from attempting to set $S_t < 0$, and will save us having to check corner conditions.

Given τ and S_t, we can calculate an agent's expected consumption path C_0^t, C_1^t:

$$C_0^t = (1 - \tau)y - S_t \tag{12.13}$$

$$C_1^t = (1 + r)(\tau y + S_t). \tag{12.14}$$

Because the government has taken τy from the agent while young, he is left only with $(1 - \tau)y$ to split between consumption while young and own-savings, S_t. When old, the agent gets the benefit of both public (government forced) savings τy and private (own) savings S_t. Consumption while old is merely the total volume of savings times the prevailing gross interest rate $1 + r$.

We are now ready to find S_t for this agent. The agent maximizes $U(C_0^t, C_1^t)$ where C_0^t as a function of S_t is given by equation (12.13) and C_1^t by equation (12.14). Thus the agent solves:

$$\max_{S_t} \left\{ 2\sqrt{(1-\tau)y - S_t} + 2\beta\sqrt{(1+r)(\tau y + S_t)} \right\}.$$

Assuming that the constraint $S_t \geq 0$ will not be binding, we take the derivative of this function with respect to S_t and set it to zero to find the optimal value of S_t. So:

$$\frac{-1}{\sqrt{(1-\tau)y - S_t}} + \frac{\beta\sqrt{1+r}}{\sqrt{\tau y + S_t}} = 0.$$

We cross-multiply to find:

$$\begin{aligned}
\left(\beta\sqrt{(1+r)}\right)\left(\sqrt{(1-\tau)y - S_t}\right) &= \sqrt{\tau y + S_t}, \\
[\beta^2(1+r)][(1-\tau)y - S_t] &= \tau y + S_t, \\
[\beta^2(1+r)](1-\tau)y - \tau y &= [1 + \beta^2(1+r)]S_t, \text{ and:} \\
\beta^2(1+r)y - \tau y[1 + \beta^2(1+r)] &= [1 + \beta^2(1+r)]S_t.
\end{aligned}$$

Dividing both sides by $1 + \beta^2(1 + r)$ produces:

$$S_t = \frac{\beta^2(1+r)}{1 + \beta^2(1+r)}y - \tau y. \tag{12.15}$$

Substituting back into equations (12.13) and (12.14) gives us optimal consumption choices in each period:

$$C_0^t = \frac{1}{1+\beta^2(1+r)}y,$$
$$C_1^t = \frac{\beta^2(1+r)^2}{1+\beta^2(1+r)}y.$$

Notice that the government-forced public savings policy τ does not affect the agent's choice of savings. If τ increases, the agent will merely decrease his choice of S_t.

If the government sets τ to exactly the agent's desired savings rate, that is:

$$\tau = \frac{\beta^2(1+r)}{1+\beta^2(1+r)},$$

then $S_t = 0$ and all saving is done by the government.

Unfunded Pension Systems

Now we turn our attention to unfunded pension systems (also known as *pay as you go* systems), in which the government taxes the current young workers to pay the current old retirees. The key insight will be that unfunded pension systems will dominate funded pension systems if the population is growing quickly enough.

In period t there are N_t young workers and N_{t-1} old retirees who were born in period $t-1$ and are now old. If the government taxes each young worker an amount τ it raises total revenue of:

$$G = \tau N_t y.$$

If it distributes this equally among the old, each old agent will get G/N_{t-1} or:

$$\frac{G}{N_{t-1}} \equiv g_{t-1} = \tau y \frac{N_t}{N_{t-1}}.$$

Recall that the population is growing at a rate n so that $N_t = (1+n)N_{t-1}$. Hence:

$$g_{t-1} = \tau y(1+n).$$

Notice that, since the population growth rate is constant at n, g_t does not vary with time, so we write merely g.

Consider again the agent's budget constraints as a function of τ and S_t, equations (12.13) and (12.14) above, only now using the unfunded pension system:

$$C_0^t = (1-\tau)y - S_t, \text{ and:}$$
$$C_1^t = (1+r)S_t + (1+n)\tau y.$$

We could solve this explicitly for S_t as a function of τ, y, n, r in much the same way that we did above (in fact, this is a good exercise to do on your own), but instead we are will simply provide intuition for the agent's choices.

If $n \neq r$ then the agent is no longer indifferent between public and private savings. If $n < r$, then public savings make the agent worse off. As τ increases more and more of the agent's wealth is being used in a relatively low-return activity. Agents would complain bitterly to their government about this (apparent) waste of their money.

On the other hand, if $n > r$, then the agent would prefer to save entirely by using the government pension system. Agents would demand that the system be increased until their private savings (in the relatively inefficient bond market) fell to zero.

Exercises

Exercise 12.1 (Easy)
For each of the following questions provide a brief answer.

1. (True, False or Uncertain) All things being equal, there is more total savings under a funded than under an unfunded pension system.
2. For the U.S., at the moment, is $n > r$?
3. Name three items in the Federal budget that account for more than 20% of all government expenditures (each).

Exercise 12.2 (Easy)
Assume that every dollar spent by the government augments total output by ϕ, where $0 < \phi < 1$. Assume that total private output is fixed at Y and that the government pays for its expenditures with lump-sum taxes. What is the absolute maximum amount of government spending, G? At this level, how much does the household consume and invest?

Exercise 12.3 (Moderate)
For this exercise assume that the representative household lives for only two periods and has preferences over consumption streams $\{C_0, C_1\}$ given by:

$$U(C_0) + \beta U(C_1),$$

where $\beta = 1/(1+\rho)$ and $\rho > 0$. Here assume that $U' > 0, U'' < 0$. The household has a constant endowment stream $\{Y, Y\}$ which is not affected by government spending. Any government spending must be paid for by lump-sum taxes on the representative household. There is no capital stock. This is a closed economy. Answer the following questions:

1. Assume that the government spends the same amount G each period. What is the market-clearing interest rate, r_0?

2. Assume that the government spends different amounts in each period, $\{G_0, G_1\}$ and that $G_0 > G_1$. Now what is the market-clearing interest rate r_0^*?

3. Which is greater, r_0^* or r_0? Does this fit with your intuition about the effect of temporary government spending?

Exercise 12.4 (Moderate)
Consider again the model of Section 12.3 above. Calculate S_t explicitly when the return on public savings is n and the return to private savings is r. Assume $n \neq r$ and τ is small.

Exercise 12.5 (Moderate)
Grace lives for two periods. She has preferences over consumption streams c_0, c_1 of:

$$u(c_0, c_1) = \ln(c_0) + \beta \ln(c_1),$$

where $0 < \beta \leq 1$. Grace is endowed with one unit of time each period. In the first period, she can divide her time between working in a low-wage job at a wage of $w = 1$ or attending S hours of school. Grace earns nothing while in school, but she is augmenting her *human capital.* In the second period of life, Grace spends all of her time at her high-wage job, earning AK_1 where K_1 is her human capital and $A > 1$. Human capital is augmented by schooling by the simple formula $K_1 = S$, so given a choice for S, Grace earns $1 - S$ while young and AS while old. There is no bond market.

The government is interested in helping Grace go to school. It levies a lump-sum tax of G on Grace when she is young and uses it to augment her human capital so that $K_1 = S + \phi G$ where $\phi > 0$. Answer the following questions:

1. Assume $G = 0$. Find Grace's optimal schooling choice S and human capital K_1.

2. Assume $G > 0$. Find Grace's optimal schooling choice S and human capital K_1. Remember that K_1 is affected directly by G. Show that S is decreasing in G and that K_1 is decreasing in G is $\phi < 1$.

3. Now assume that the human capital augmentation is a a straight subsidy from the government, that is, the government has taxed someone else to pay for Grace's schooling, so she is not taxed at all while young. Now how do S and K_1 vary with G?

Variable	Definition
B_t	Household savings at the end of period t (if positive), or household debt (if negative).
C, C_t	Consumption by the household (in period t).
C^a, C^b	Specific consumption levels used in an example.
$\{e_t\}_{t=0}^{\infty}$	Sequence of household endowments over time.
G	Government spending (usually assumed to be constant).
I_t	Household's investment in the capital stock at time t.
K_t, K_0	Capital stock in period t (initial capital stock).
Y_t^P	Output from private productive processes.
Y_t^G	Output from government production which is refunded to the household.
Y_t	Total output in period t, the sum of private and government output.
β	Household discount factor, usually assumed to be $1/(1+\rho)$.
ρ	Household discount rate.
δ	Depreciation rate of capital.
$V(\{C_t\}_{t=0}^{\infty})$	Household's preferences over an entire stream of consumptions.
$U(C_t), u(C_t)$	Household utility in period t from consumption in that period of C_t.
C_0^t, C_1^t	Consumption of generation born in period t while young and old.
N_t	Population in period t.
n	Growth rate of population.
r	Real interest rate.
τ	Income tax rate.
y	Household income.
S_t	Household gross private savings.
G	The government's realized revenue from taxes on young.
g_t	Government's per-capita payments to the old, G/N_t, in period $t+1$.

Table 12.1: Notation for Chapter 12