Chapter 13

The Effect of Taxation

Taxes affect household behavior via income and substitution effects. The income effect is straightforward: as taxes go up, households are poorer and behave that way. For example, if leisure is a normal good, then higher taxes will induce consumers to consume less leisure. The substitution effect is trickier, but it can be much more interesting. Governments levy taxes on observable and verifiable actions undertaken by households. For example, governments often tax consumption of gasoline and profits from sales of capital assets, like houses. These taxes increase the costs to the households of undertaking the taxed actions, and the households respond by adjusting the actions they undertake. This can lead to outcomes that differ substantially from those intended by the government.

Since optimal tax policy is also a subject of study in microeconomics and public finance courses, we shall concentrate here on the effect of taxation on labor supply and capital accumulation. When modeling labor supply decisions we are going to have a representative agent deciding how to split her time between labor supply and leisure. Students might object on two grounds: First, that the labor supply is quite inelastic (since everyone, more or less, works, or tries to) and second, that everyone puts in the same number of hours per week, and the variation in leisure comes not so much in time as in expenditure (so that richer people take more elaborate vacations).

The representative household stands for the decisions of millions of underlying, very small, households. There is, to name only one example, mounting evidence that households change the timing of their retirement on the basis of tax policy. As taxes increase, more and more households choose to retire. At the level of the representative household, this appears as decreasing labor supply. As for the observation that everyone puts in either 40 hours a week or zero, this misses some crucial points. The fact is that jobs differ significantly in their characteristics. Consider the jobs available to Ph.D. economists: they range from Wall Street financial wizard, big-time university research professor, to small-time college instructor. The fact is that a Wall Street financial wizard earns, on her first day on the job, two or three times as much as a small-time college instructor. Of course, college teachers have a much more relaxed lifestyle than financiers (their salary, for example, is computed assuming that they only work nine months out of the year). The tax system can easily distort a freshly-minted Ph.D.'s choices: Since she consumes only the after-tax portion of her income, the Wall Street job may only be worth 50% more, after taxes, than the college instructor's job. The point is not that every new economics Ph.D. would plump for the college instructor's job, but that, as the tax on high-earners increased, an increasing fraction would. Again, we can model this with a representative household choosing how much leisure to consume.

We begin with a general overview of tax theory, discuss taxation of labor, then taxation of capital and finally consider attempts to use the tax system to remedy income (or wealth) inequality.

13.1 General Analysis of Taxation

In this section we will cast the problem of taxation in a very general framework. We will use this general framework to make some definitions and get some initial results.

Notation

Assume that the household take some observed action a in A (this discussion generalizes to the case when a is a vector of choices). For example, a could be hours worked, number of windows in one's house, or the number of luxury yachts the household owns (or, if a is a vector, all three). The set A is the set of allowed values for a, for example 0 to 80 hours per week, $\{0, 1, 2, \ldots, 500\}$ windows per house or 0 to ten luxury yachts (where we are assuming that no house may have more than 500 windows and no household can use more than 10 luxury yachts).

The government announces a *tax policy* $\mathcal{H}(a; \psi)$, where $\mathcal{H}(\psi) : A \to \mathbb{R}$. That is, a tax policy is a function mapping observed household choices into a *tax bill* which the household has to pay (if positive), or takes as a subsidy to consumption (if negative). The term ψ (which may be a vector) is a set of parameters to the tax policy (for example, deductions). The household is assumed to know the function $\mathcal{H}(a; \psi)$ and ψ before it takes action a.

An example of a tax policy $\mathcal{H}$ is the *flat income tax.* In a flat income tax, households pay a fixed fraction of their income a in taxes, so $\psi = \tau$, where τ is the flat tax rate. A more complex version of the flat income tax allows for *exemptions* or *deductions,* which are simply a portion of income exempt from taxation. If the exempt income is E, then the parameters to the tax system are $\psi = \{E, \tau\}$ and $\mathcal{H}(a;\psi)$ is:

$$\mathcal{H}(a;\psi) = \begin{cases} 0, & a \leq E \\ \tau(a-E), & a \geq E. \end{cases}$$

Definitions

We can use our notation to make some useful definitions. The *marginal tax rate* is the tax paid on the next increment of a. So if one's house had 10 windows already and one were considering installing an 11th window, the marginal tax rate would be the increase in one's tax bill arising from that 11th window. More formally, the marginal tax rate at a is:

$$\frac{\partial \mathcal{H}(a;\psi)}{\partial a}.$$

Here we are assuming that a is a scalar and smooth enough so that $\mathcal{H}(a;\psi)$ is at least once continuously differentiable. Expanding the definition to cases in which $\mathcal{H}(a;\psi)$ is not smooth in a (in certain regions) is straightforward, but for simplicity, we ignore that possibility for now.

The *average tax rate* at a is defined as:

$$\frac{\mathcal{H}(a;\psi)}{a}.$$

Note that a flat tax with $E = 0$ has a constant marginal tax rate of τ, which is just equal to the average tax rate.

If we take a to be income, then we say that a tax system is *progressive* if it exhibits an increasing marginal tax rate, that is if $\mathcal{H}'(a;\psi) > 0$. In the same way, a tax system is said to be *regressive* if $\mathcal{H}'(a;\psi) < 0$.

Household Behavior

Let us now turn our attention to the household. The household has some technology for producing income $\mathcal{Y}$[1] that may be a function of the action a, so $\mathcal{Y}(a)$. If a is hours worked, then $\mathcal{Y}$ is increasing in a, if a is hours of leisure, then $\mathcal{Y}$ is decreasing in a and if a is house-windows then $\mathcal{Y}$ is not affected by a. The household will have preferences directly over action a and *income net of taxation* $\mathcal{Y}(a) - \mathcal{H}(a;\psi)$. Thus preferences are:

$$U\left[a, \mathcal{Y}(a) - \mathcal{H}(a;\psi)\right].$$

There is an obvious maximization problem here, and one that will drive all of the analysis in this chapter. As the household considers various choices of a (windows, hours, yachts),

[1] We use the notation $\mathcal{Y}$ here to mean income to emphasize that income is now a function of choices a.

it takes into consideration both the direct effect of a on utility and the indirect effect of a, through the tax bill term $\mathcal{Y}(a) - \mathcal{H}(a;\psi)$. Define:

$$V(\psi) \equiv \max_{a \in A} U\left[a, \mathcal{Y}(a) - \mathcal{H}(a;\psi)\right].$$

For each value of ψ, let $a_{\max}(\psi)$ be the choice of a which solves this maximization problem. That is:

$$V(\psi) = U\left[a_{\max}(\psi), \mathcal{Y}(a_{\max}(\psi)\right] - \mathcal{H}\left[a_{\max}(\psi);\psi)\right].$$

Assume for the moment that U, $\mathcal{Y}$ and $\mathcal{H}$ satisfy regularity conditions so that for every possible $|psi$ there is only one possible value for $a_{\max}$.

The government must take the household's response $a_{\max}(\psi)$ as given. Given some tax system $\mathcal{H}$, how much revenue does the government raise? Clearly, just $\mathcal{H}\left[a_{\max}(\psi);\psi\right]$. Assume that the government is aware of the household's best response, $a_{\max}(\psi)$, to the government's choice of tax parameter ψ. Let $\mathcal{T}(\psi)$ be the revenue the government raises from a choice of tax policy parameters ψ:

$$\mathcal{T}(\psi) = \mathcal{H}\left[a_{\max}(\psi);\psi\right]. \tag{13.1}$$

Notice that the government's revenue is just the household's tax bill.

The functions $\mathcal{H}(a;\psi)$ and $\mathcal{T}(\psi)$ are closely related, but you should not be confused by them. $\mathcal{H}(a;\psi)$ is the tax system or tax policy: it is the legal structure which determines what a household's tax bill is, given that household's behavior. Households choose a value for a, but the tax policy must give the tax bill for all possible choices of a, including those that a household might never choose. Think of $\mathcal{H}$ as legislation passed by Congress. The related function $\mathcal{T}(\psi)$ gives the government's actual revenues under the tax policy $\mathcal{H}(a;\psi)$ when households react optimally to the tax policy. Households choose the action a which makes them happiest. The mapping from tax policy parameters ψ to household choices is called $a_{\max}(\psi)$. Thus the government's actual revenue given a choice of parameter ψ, $\mathcal{T}(\psi)$, and the legislation passed by Congress, $\mathcal{H}(a;\psi)$, are related by equation (13.1) above.

The Laffer Curve

How does the function $\mathcal{T}(\psi)$ behave? We shall spend quite a bit of time this chapter considering various possible forms for $\mathcal{T}(\psi)$. One concept to which we shall return several times is the *Laffer curve.* Assume that, if a is fixed, that $\mathcal{H}(a;\psi)$ is increasing in ψ (for example, ψ could be the tax rate on house windows). Further, assume that if ψ is fixed, that $\mathcal{H}(a;\psi)$ is increasing in a. Our analysis would go through unchanged if we assumed just the opposite, since these assumptions are simply naming conventions.

Given these assumptions, is $\mathcal{T}$ necessarily increasing in ψ? Consider the total derivative of $\mathcal{T}$ with respect to ψ. That is, compute the change in revenue of an increase in ψ, taking in to account the change in the household's optimal behavior:

$$\frac{d\mathcal{T}(\psi)}{d\psi} = \frac{\partial \mathcal{H}\left[a_{\max}(\psi);\psi\right]}{\partial a}\frac{\partial a_{\max}(\psi)}{\partial \psi} + \frac{\partial \mathcal{H}\left[a_{\max}(\psi);\psi\right]}{\partial \psi}. \tag{13.2}$$

The second term is positive by assumption. The first term is positive if $a_{\max}$ is increasing in ψ. If $a_{\max}$ is decreasing in ψ, and if the effect is large enough, then the government revenue function may actually be decreasing in ψ despite the assumptions on the tax system $\mathcal{H}$. If this happens, we say that there is a Laffer curve in the tax system.

A note on terms: the phrase "Laffer curve" has become associated with a bitter political debate. We are using it here as a convenient shorthand for the cumbersome phrase, "A tax system which exhibits decreasing revenue in a parameter which increases government revenue holding household behavior constant because the household adjusts its behavior in response". Do tax systems exhibit Laffer curves? Absolutely. For example, a Victorian-era policy which levied taxes on the number of windows (over some minimum number designed to exempt the middle class) in a house, over a span of years, resulted in grand houses with very few windows. As a result, the *hoi polloi* began building more modest homes also without windows and windowlessness became something of a fashion. Increases in the window tax led, in the long term, to decreases in the revenue collected on the window tax. The presence of a Laffer curve in the U.S. tax system is an empirical question outside the scope of this chapter.

Finally, the presence of a Laffer curve in a tax system does not automatically mean that a tax cut produces revenue growth. The parameter set ψ must be in the downward-sloping region of the government revenue curve for that to be the case. Thus the U.S. tax system could indeed exhibit a Laffer curve, but only at very high average tax rates, in which case tax cuts (given the current low level of taxation) would lead to decreases in revenue.

Lump-sum Taxes

Now consider the results if the government introduced a tax system with the special characteristic that the tax bill did not depend on the household's decisions. That is,

$$\frac{\partial \mathcal{H}(a;\psi)}{\partial a} = 0,$$

for all choices of ψ. Notice that the household's optimal decisions may still change with ψ, but that the government's revenue will not vary as $a_{\max}$ varies. Let us determine what happens to the derivative of the government revenue function $\mathcal{T}$ from equation (13.2) above:

$$\begin{aligned}\frac{d\mathcal{T}(\psi)}{d\psi} &= \frac{\partial \mathcal{H}\left[a_{\max}(\psi);\psi\right]}{\partial a}\frac{\partial a_{\max}(\psi)}{\partial \psi} + \frac{\partial \mathcal{H}\left[a_{\max}(\psi);\psi\right]}{\partial \psi} \\ &= (0)\left[\frac{\partial a_{\max}(\psi)}{\partial \psi}\right] + \frac{\partial \mathcal{H}\left[a_{\max}(\psi);\psi\right]}{\partial \psi} \\ &= \frac{\partial \mathcal{H}\left[a_{\max}(\psi);\psi\right]}{\partial \psi}.\end{aligned}$$

This is always greater than zero by assumption. Hence there is never a Laffer curve when the tax system has the property that $\partial\mathcal{H}/\partial a = 0$, that is, with lump-sum taxes.

Taxes which do not vary with household characteristics are known as *poll taxes* or *lump-sum taxes.* Poll taxes are taxes that are levied uniformly on each person or "head" (hence the name). Note that there is no requirement that lump sum taxes be uniform, merely that household actions cannot affect the tax bill. A tax lottery would do just as well. In modern history there have been relatively few examples of poll taxes. The most recent use of poll taxes was in England, where they were used from 1990-1993 to finance local governments. Each council (roughly equivalent to a county) divided its expenditure by the number of adult residents and delivered tax bills for that amount. Your correspondent was, at the time, an impoverished graduate student living in the Rotherhithe section of London, and was presented with a bill for £350 (roughly \$650 at the time). This policy was deeply unpopular and led to the "Battle of Trafalgar Square"—the worst English riot of the 20th century. It is worth noting that this tax did not completely meet the requirements of a lump sum tax since it did vary by local council, and, in theory, households could affect the amount of tax they owed by moving to less profligate councils, voting Conservative or rioting. These choices, though, were more or less impossible to implement in the short-term, and most households paid.

Lump-sum taxes, although something of a historical curiosity, are very important in economic analysis. As we shall see in the next section, labor supply responds very differently to lump-sum taxes than to income taxes.

The Deadweight Loss of Taxation

Lump sum taxes limit the amount of *deadweight loss* associated with taxation. Consider the effect of an increase in taxes which causes an increase in government revenue: revenue increases slightly and household income net of taxes decreases by slightly more than the revenue increase. This difference is one form of deadweight loss, since it is revenue lost to both the household and the government.

It is difficult to characterize the deadweight loss of taxation with the general notation we have established here (we will be much more precise in the next section). However, we will be able to establish that the deadweight loss is increasing in the change of household behavior. That is, the more sensitive $a_{\max}(\psi)$ is to ψ, the larger the deadweight loss.

Consider a tax policy $\mathcal{H}(a;\psi)$ and two different parameter sets for the tax policy, ψ_0 and ψ_1. Assume that, for fixed a, $\mathcal{H}(a;\psi_0) < \mathcal{H}(a;\psi_1)$. The household's utility at each of the tax parameters is:

$$
\begin{aligned}
V(\psi_0) &= U\left(a_{\max}(\psi_0), \mathcal{Y}\left[a_{\max}(\psi_0)\right] - \mathcal{H}\left[a_{\max}(\psi_0), \psi_0\right]\right), \text{ and:}\\
V(\psi_1) &= U\left(a_{\max}(\psi_1), \mathcal{Y}\left[a_{\max}(\psi_1)\right] - \mathcal{H}\left[a_{\max}(\psi_1), \psi_1\right]\right).
\end{aligned}
$$

The claim is that the change in household net income exceeds the change in government revenue, or:

$$
\begin{aligned}
(13.3)\quad \left(\mathcal{Y}\left[a_{\max}(\psi_0)\right] - \mathcal{H}\left[a_{\max}(\psi_0), \psi_0\right]\right) - \left(\mathcal{Y}\left[a_{\max}(\psi_1)\right] - \mathcal{H}\left[a_{\max}(\psi_1), \psi_1\right]\right)&\\
> \mathcal{T}(\psi_1) - \mathcal{T}(\psi_0).&
\end{aligned}
$$

Recall that $\mathcal{T}(\psi) = \mathcal{H}\left[a_{\max}(\psi); \psi\right]$. Equation (13.3) is true only if:

$$
\mathcal{Y}[a_{\max}(\psi_0)] > \mathcal{Y}[a_{\max}(\psi_1)].
$$

That is, the more household gross (that is, pre-tax) income falls in response to the tax, the greater the deadweight loss. But since household gross income is completely under the household's control through choice of a, this is tantamount to saying that the more a changes, the greater the deadweight loss. This is a very general result in the analysis of taxation: the more the household can escape taxation by altering its behavior, the greater the deadweight loss of taxation.

If we further assume that there are no pure income effects in the choice of a, then lump-sum taxes will not affect the household's choice of a and there will be no deadweight loss to taxation (a formal proof of this point is beyond the scope of this chapter). The assumption of no income effects is relatively strong, but, as we shall see later, even without it lump-sum taxes affect household behavior very differently than income taxes.

13.2 Taxation of Labor

In this section we shall assume that households choose only their effort level or labor supply L. We will assume that they have access to a technology for transforming labor into the consumption good of wL. Think of w as a wage rate. Although we will not clear a labor market in this chapter, so w is not an endogenous price, we can imagine that all households have a backyard productive technology of this form.

Households will enjoy consumption and dislike effort, but will be unable to consume without expending effort. They will balance these desires to arrive at a labor supply decision. Government taxation will distort this choice and affect labor supply.

A Simple Example

As a first step, consider a household with a utility function over consumption C and effort L of the form:

$$U(C, L) = 2\sqrt{C} - L.$$

The household's income takes the form:

$$\mathcal{Y}(L) = wL.$$

Assume that there is a simple flat tax, so the tax policy is:

$$\mathcal{H}(L; \tau) = \tau \mathcal{Y}(L).$$

Hence the household's budget constraint becomes:

$$C = \mathcal{Y}(L) - \mathcal{H}(L; \tau) = wL - \tau wL = (1 - \tau)wL.$$

Substituting this budget constraint into the household's utility function produces:

$$V(\tau) = \max_L \left\{ 2\sqrt{(1 - \tau)wL} - L \right\}.$$

This function is just the household's utility given a tax rate τ. We can solve the maximization problem to find $V(\tau)$ directly. Take the derivative with respect to the single choice variable, labor supply L, and set it to zero to find:

$$\sqrt{\frac{(1 - \tau)w}{L}} - 1 = 0.$$

Solving for L produces:

$$L(\tau) = (1 - \tau)w.$$

We can substitute the labor supply decision $L(\tau)$ back into the government's tax policy to find the government's revenue function:

$$\mathcal{T}(\tau) = \mathcal{H}\left[L(\tau); \tau\right] = \tau w L(\tau) = w^2 \tau (1 - \tau).$$

Does this system exhibit a Laffer curve? Indeed it does. Clearly, $\mathcal{T}(\tau)$ in this case is simply a parabola with a maximum at $\tau = 0.5$. (See Figure (13.1)).

The effect of the income tax was to drive a wedge between the productivity of the household (constant at w) and the payment the household received from its productive activity. The household realized an *effective wage rate* of $(1 - \tau)w$. As the flat tax rate τ moved to unity, the effective wage rate of the household falls to zero and so does its labor supply. Compare this tax structure with one in which the household realizes the full benefit of its effort, after paying its fixed obligation. Thus we turn our attention next to a lump-sum tax.

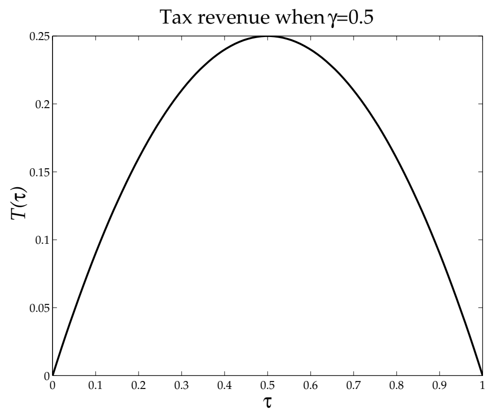

Figure 13.1: A government revenue function that exhibits a Laffer curve.

A Lump-sum Tax

Now let us introduce a lump-sum tax of amount τ_L.[2] No matter what income the household accumulates, it will be forced to pay the amount τ_L. On the other hand, after paying τ_L, the household consumes all of its income. Previously, with the income tax, the household faced an effective wage rate of $(1-\tau)w$, which decreased as τ increased. Now the household's effective wage will be w (after the critical income of τ_L is reached). Does this mean that effort will be unaffected by τ_L? Recall from the previous section that this will only happen if there are no wealth effects. Examining the utility function reveals that it is not homogeneous of degree 1 in wealth, hence we can expect labor supply to vary to with τ_L. In particular, since leisure is a normal good, we will expect that labor supply will be increasing in τ_L. The household's budget constraint, with this tax policy, becomes:

$$C = wL - \tau_L,$$

so the household's maximization problem is:

$$V(\tau_L) = \max_L \left\{ 2\sqrt{wL - \tau_L} - L \right\}.$$

The first-order condition for optimality is:

$$\frac{w}{\sqrt{wL - \tau_L}} - 1 = 0.$$

Solving for L produces:

$$L(\tau_L) = w + \frac{\tau_L}{w}.$$

[2]The notation τ_L is meant to imply lump-sum tax: there is a surfeit of notation involving τ and L in this chapter. Please refer to the table at the end if you become confused.

We see that labor supply is in fact increasing in the lump-sum tax amount τ_L. The household increases its labor supply by just enough to pay its poll tax obligation. What is the government revenue function? It is, in this case, simply:

$$\mathcal{T}(\tau_L) = \tau_L.$$

So there is no Laffer curve with a lump-sum tax (of course).

General Labor Supply and Taxation

With the assumption of a square-root utility function, we were able to derive very interesting closed-form solutions for labor supply and the government revenue function. Our results, though, were hampered by being tied to one particular functional form. Now we introduce a more general form of preferences (although maintaining the assumption of linear disutility of effort). We shall see that a Laffer curve is not at all a predestined outcome of income taxes. In fact, when agents are very risk-averse, and when zero consumption is catastrophic, we shall see that the Laffer curve vanishes from the income tax system.

Consider agents with preferences over consumption C and labor supply L of the form:

$$U(C, L) = \begin{cases} \frac{C^\gamma}{\gamma} - L, & \gamma \neq 0, \quad \gamma \leq 1 \\ \ln(C) - L, & \gamma = 0. \end{cases} \tag{13.4}$$

Notice the immediate difference when $0 < \gamma \leq 1$ and when $\gamma \leq 0$. In the former case, a consumption of zero produces merely zero utility, bad, but bearable; while in the latter case, zero consumption produces a utility of negative infinity, which is unbearable. Agents will do anything in their power to avoid any possibility of zero consumption when $\gamma \leq 0$. Recall that in our previous example (when $\gamma = 0.5$) labor supply dropped to zero as the income tax rate increased to unity. Something very different is going to happen here.

Given a distortionary income tax rate of τ, the household's budget constraint becomes:

$$C = (1 - \tau)wL,$$

as usual. The household's choice problem then becomes:

$$V(\tau) = \max_L \left\{ \frac{[(1 - \tau)wL]^\gamma}{\gamma} - L \right\}.$$

The first-order necessary condition for maximization is:

$$[(1 - \tau)w]^\gamma L^{\gamma - 1} - 1 = 0.$$

This in turn implies that:

$$L^{1-\gamma} = [(1 - \tau)w]^\gamma, \text{ so:}$$
$$L = [(1 - \tau)w]^{\frac{\gamma}{1-\gamma}}.$$

Notice that if $\gamma < 0$, then L is decreasing in w.

The government revenue function $\mathcal{T}(\tau)$ is:

$$\mathcal{T}(\tau) = \tau w L(\tau) = \tau(1-\tau)^{\frac{\gamma}{1-\gamma}} w^{\frac{1}{1-\gamma}}.$$

The question becomes, when does this tax system exhibit a Laffer curve? This is tantamount to asking when, if ever, the government revenue function is decreasing in τ. We begin by taking the derivative of $\mathcal{T}$ with respect to τ:

$$\begin{aligned}
\mathcal{T}'(\tau) &= w^{\frac{1}{1-\gamma}} \left[(1-\tau)^{\frac{\gamma}{1-\gamma}} - \frac{\gamma}{1-\gamma}\tau(1-\tau)^{\frac{\gamma}{1-\gamma}-1}\right] \\
&= w^{\frac{1}{1-\gamma}}(1-\tau)^{\frac{\gamma}{1-\gamma}-1}\left[1-\tau-\frac{\gamma}{1-\gamma}\tau\right] \\
&= w^{\frac{1}{1-\gamma}}(1-\tau)^{\frac{\gamma}{1-\gamma}-1}\left[1-\frac{1}{1-\gamma}\tau\right].
\end{aligned}$$

Notice immediately that the derivative $\mathcal{T}'(\tau)$ has the same sign as the term:

$$\left[1-\frac{1}{1-\gamma}\tau\right],$$

since the term outside of the brackets is positive by the assumptions that $w > 0$ and $\gamma < 1$. Thus $\mathcal{T}'(\tau)$ will be negative only if:

$$\begin{aligned}
1-\frac{1}{1-\gamma}\tau &\leq 0, \text{ or:} \\
\tau &\geq 1-\gamma.
\end{aligned}$$

The tax rate τ must satisfy $0 \leq \tau \leq 1$. Thus we notice two things immediately: (1) If $\gamma \leq 0$ there is no Laffer curve, and (2) If $0 < \gamma < 1$, then there is a Laffer curve, and the peak occurs at $\tau = 1-\gamma$.

What is the real-world significance of this sharp break in behavior at $\gamma = 0$? Agents with $\gamma \leq 0$ are very risk-averse and are absolutely unwilling to countenance zero consumption (the real world equivalent would be something like bankruptcy). In addition, their labor supply is decreasing in the wage rate w. In contrast, agents with $\gamma > 0$ are less risk-averse (although by no means risk neutral), are perfectly willing to countenance bankruptcy and have labor supply curves which are increasing in the wage rate w. In a world with many households, each of whom has a different value of γ, and a government which imposes a common tax rate τ, we would expect greater distortions among those households that are less risk-averse and harder-working.

Finally, the reader may find it an instructive exercise to repeat this analysis with a lump-sum tax. Households will all respond to a lump-sum tax by increasing their labor effort by precisely the same amount, τ_L/w, no matter what their value of γ.

13.3 Taxation of Capital

Now we turn our attention to the problem of a household which owns some capital stock and a technology for transforming capital into output. We shall see that, if households are allowed to deduct investment from their taxes (that is, if investment is tax-exempt), then there will not be a Laffer curve in income taxes. If on the other hand investment is not tax-deductible, or only a portion is, then there will be a Laffer curve in income taxation.

The household lives forever and has preferences over consumption streams $\{C_t\}_{t=0}^{\infty}$ of:

$$\sum_{t=0}^{\infty} \beta^t u(C_t),$$

where $\beta = 1/(1+\rho)$. Here $\rho > 0$ is the discount rate.

The household begins life with an initial stock of capital $K_0 > 0$. In addition, the income of the household each period, Y_t, is:

$$Y_t = K_t^{\alpha},$$

where K_t is the household's capital stock in period t and α is a production parameter satisfying $0 < \alpha < 1$. The capital stock evolves according to the law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t,$$

where I_t is investment in physical capital (a choice of the household) and δ is the depreciation rate of capital. The economy is closed, so there is no bond market.

Assume that the government's tax policy is a flat tax on income from capital. We will consider two forms: one in which investment is exempt and one in which it is not. Thus without the exemption, in period t, the legislative tax code requires households to pay:

$$\mathcal{H}_t(K_t;\tau) = \tau K_t^{\alpha}.$$

If investment is exempt, then the legislative tax code requires, in period t, that households pay:

$$\mathcal{H}_t(K_t;\tau) = \tau(K_t^{\alpha} - I_t).$$

The household chooses K_t (so it plays the role of a) in response to changes in the tax code. We will study the steady-state capital level as a function of taxes $K_{\text{ss}}(\tau)$. Thus the steady-state revenue raised each period by the government is:

$$\mathcal{T}_{ss}(\tau) = \mathcal{H}_{ss}\left[K_{\text{ss}}(\tau);\tau\right].$$

This will vary depending on whether investment is deductible or not.

The household's budget constraint is:

$$C_t + I_t + (\text{tax bill})_t = Y_t.$$

Begin by assuming that investment is non-deductible. The tax bill then becomes the tax rate τ times income Y_t, or:

$$(\text{tax bill})_t = \tau Y_t.$$

Hence the household's budget constraint becomes:

$$C_t = (1-\tau)Y_t - I_t. \tag{13.5}$$

Now assume that investment is tax deductible. The government levies a tax at rate τ on every dollar earned above investment. This also sometimes called *paying for investment with pre-tax dollars.* That is:

$$(\text{tax bill})_t = \tau(Y_t - I_t).$$

The household's budget constraint now becomes:

$$C_t = (1-\tau)(Y_t - I_t). \tag{13.6}$$

We shall see that, because the tax system in equation (13.5) raises the implicit price of investment, the steady-state level of capital will be distorted away from its first-best level. Thus as the tax rate increases, investment and the steady-state capital level fall, so there is a Laffer curve lurking in the tax system. In contrast, the tax system in equation (13.6) leaves the implicit price of investment in terms of output unaffected by the tax rate, hence we shall see that the steady-state capital level will be unaffected by the tax rate. As a result, the Laffer curve is banished from the system, and government revenues become linear in the tax rate τ.

Analysis When Investment is Not Exempt

We want to collapse the budget constraint in equation (13.5) and the law of motion for capital into one equation, giving consumption C_t as a function of K_t and K_{t+1}, where in period t the household takes K_t as given and chooses K_{t+1}. Thus write consumption C_t as:

$$C_t = (1-\tau)K_t^\alpha + (1-\delta)K_t - K_{t+1}. \tag{13.7}$$

Here we have substituted in K_t^α for income Y_t from the agent's technology.

The household's problem thus becomes:

$$\max_{\{K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t u(C_t),$$

where C_t is given in equation (13.7). Take the derivative with respect to the capital choice K_{j+1} for some arbitrary period j (where we avoid taking derivatives with respect to capital in period T because t is the time index in the summation). Remember the trick to these problems: K_{j+1} will appear in period j and period $j+1$. Hence optimality requires:

$$\beta^j u'(C_j)[-1] + \beta^{j+1} u'(C_{j+1})[\alpha(1-\tau)K_{j+1}^{\alpha-1} + 1 - \delta] = 0.$$

Divide by the common factor β^j. Now assume that a steady-state has been reached. At a steady-state $K_t = K_{t+1} = K_{\text{ss}}$ and $C_t = C_{t+1} = C_{\text{ss}}$. Hence:

$$u'(C_{\text{ss}}) = \beta u'(C_{\text{ss}})[\alpha(1-\tau){K_{\text{ss}}}^{\alpha-1} + 1 - \delta].$$

Recall that $\beta^{-1} = 1 + \rho$. Divide both sides by $\beta u'(C_{\text{ss}})$ to find:

$$1 + \rho = \alpha(1-\tau){K_{\text{ss}}}^{\alpha-1} + 1 - \delta.$$

Hence:

$$K_{\text{ss}} = (1-\tau)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}.$$

Notice immediately that the steady-state capital level is decreasing in the tax rate τ. Gross income each period at the steady-state is:

$$Y_{\text{ss}} = (1-\tau)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Let $\mathcal{T}_t(\tau)$ be the tax revenue of the government each period when the tax rate is τ. At the steady-state:

$$\mathcal{T}_t(\tau) = \tau Y_{\text{ss}} = \tau(1-\tau)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{13.8}$$

Since $0 < \alpha < 1$, $\mathcal{T}_t$ is a parabola with a peak at $\tau = 1 - \alpha$. Thus this tax system exhibits a Laffer curve.

Analysis When Investment is Exempt

As before, we begin by collapsing the budget constraint (now equation (13.6)) and the law of motion for capital into one equation. Thus write:

$$C_t = (1-\tau)(K_t^\alpha + (1-\delta)K_t - K_{t+1}).$$

Notice the difference that exempting investment makes. The entire right-hand-side is now multiplied by $1-\tau$, so the household cannot escape taxation by altering its mix of investment and consumption.

Once again, we choose sequences of capital to maximize:

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t),$$

where C_t is given above. The derivative with respect to K_{j+1} is now:

$$\beta^j u'(C_j)[-(1-\tau)] + \beta^{j+1} u'(C_{j+1})[(1-\tau)(\alpha K_{j+1}^{\alpha-1} + 1 - \delta)] = 0.$$

Divide the equation by the common factor $(1-\tau)\beta^j$. Notice that the tax rate has vanished. Now assume a steady-state. Hence:

$$u'(C_{\mathrm{ss}}) = \beta u'(C_{\mathrm{ss}})[\alpha {K_{\mathrm{ss}}}^{\alpha-1} + 1 - \delta].$$

Solving for K_{ss} produces:

$$K_{\mathrm{ss}} = \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}.$$

Notice that the steady-steady state capital level is unaffected by the tax rate τ. Gross income at the steady state is Y_{ss} and is given by:

$$Y_{\mathrm{ss}} = {K_{\mathrm{ss}}}^{\alpha}.$$

Again, this is not a function of τ.

The government's period-by-period revenue function $\mathcal{T}_t(\tau)$ is now simply:

$$\mathcal{T}_t(\tau) = \tau(Y_{\mathrm{ss}} - I_{\mathrm{ss}}),$$

where I_{ss} is the steady-state investment level (which is tax-exempt). We can find I_{ss} by solving the law of motion for capital:

$$K_{t+1} = (1-\delta)K_t + I_t$$

for the steady-state level of capital:

$$\begin{aligned} K_{\mathrm{ss}} &= (1-\delta)K_{\mathrm{ss}} + I_{\mathrm{ss}}, \text{ so:} \\ I_{\mathrm{ss}} &= \delta K_{\mathrm{ss}}. \end{aligned}$$

Hence $\mathcal{T}_t$ becomes:

$$\mathcal{T}_t(\tau) = \tau(Y_{\mathrm{ss}} - I_{\mathrm{ss}}) = \tau\left({K_{\mathrm{ss}}}^{\alpha} - \delta K_{\mathrm{ss}}\right) = \tau\left[\left(\frac{\alpha}{\rho+\delta}\right)^{\frac{\alpha}{1+\alpha}} - \delta\left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1+\alpha}}\right].$$

Thus government revenue each period is just a linear function of the tax rate τ, and there is no Laffer curve in this tax system.

13.4 Redistribution and Taxation

Now we turn our attention, as promised, to fiscal policies aimed at redistribution. We shall write down a model with two agents. One agent will be low-productivity and the other agent will be high-productivity. Without government intervention, there will be income inequality in this model. Why is the government interested in redistributing income? For now, let us simply take it as given that the fiscal authority will attempt to remedy income equality by taxes and transfers. We might expect the government to address the underlying causes of the agents' productivity gap, but since they are likely the result of, in a best case, different schooling histories, they are not likely to be remedied over the short-term.

Agents of type i, $i = a, b$ have a common utility function over consumption C^i and labor effort L^i of the (familiar) form:

$$\frac{C^{i\gamma}}{\gamma} - L^i,$$

where $\gamma < 1$. There is a technology transforming labor effort into the consumption good of the form:

$$Y^i = w^i L^i,$$

where $i = a, b$. Assume that $w^a > w^b$ so agents of type a are more productive than type b agents.

The government will tax type a agents at a rate τ in order to make a lump-sum transfer to type b agents of v. Hence agents face budget constraints of the form:

$$\begin{aligned} C^a &= (1-\tau)w^a L^a, \text{ and:} \\ C^b &= w^b L^b + v. \end{aligned}$$

Agents of type a face precisely the same problem that we solved in Section 13.2. Agents of type b face a "negative lump sum tax" of v. There is thus nothing unfamiliar about this problem.

The government has a budget constraint which requires it to balance transfer payments v with tax revenue $\mathcal{T}^a$ from agents of type a. Assume that there are an equal number of type a and type b agents, so the government budget constraint becomes:

$$v = \mathcal{T}^a = \tau w^a L^a(\tau).$$

From our analysis in Section 13.2 above, we know that $L^a(\tau)$ is:

$$L^a(\tau) = [(1-\tau)w^a]^{\frac{\gamma}{1-\gamma}}.$$

Thus:

$$\mathcal{T}^a(\tau) = \tau(1-\tau)^{\frac{\gamma}{1-\gamma}} w^{a\,\frac{1}{1-\gamma}}.$$

Agents of type b get a lump-sum subsidy of $v = \mathcal{T}^a$. Solving for their optimal labor supply gives:

$$L^b = w^{b\,\frac{1}{1-\gamma}} - \frac{v}{w^b}.$$

Thus agents of type b certainly decrease their effort as v increases.

We know that if $\gamma > 0$ that agents of type a will also decrease their effort as τ increases. Hence if $\gamma > 0$, redistribution will certainly lower both agent's labor supply and total national output.

If $\gamma < 0$, an increase in the tax on type a agents will increase their labor supply. This effect will, we shall see, never be large enough to overcome the decrease in type b labor effort. As a result, increases in redistribution will again lower national income. To see this, begin by calculating the effect on L^a and L^b of an increase in τ:

$$\begin{aligned}
\frac{\partial L^a}{\partial \tau} &= w^{a\,\frac{\gamma}{1-\gamma}} \left[\frac{\gamma}{1-\gamma}(1-\tau)^{\frac{\gamma}{1-\gamma}-1}(-1)\right] \\
&= -(1-\tau)^{\frac{\gamma}{1-\gamma}-1} w^{a\,\frac{\gamma}{1-\gamma}} \left[\frac{\gamma}{1-\gamma}\right] \\
&= -L^a \frac{\gamma}{1-\gamma}\frac{1}{1-\tau}. \qquad (13.9)\\
\frac{\partial L^a}{\partial \tau} &= -\frac{w^a}{w^b}\left[L^a - \tau\frac{\gamma}{1+\gamma}\frac{L^a}{1-\tau}\right] \\
&= -L^a\frac{w^a}{w^b}\left[1 - \frac{\gamma}{1-\gamma}\frac{\tau}{1-\tau}\right]. \qquad (13.10)
\end{aligned}$$

Armed with these derivatives, we can consider the effect on total national output (GDP) of an increase in τ:

$$\frac{dY}{d\tau} = \frac{dY^a}{d\tau} + \frac{dY^b}{d\tau} = \frac{d}{d\tau}w^aL^a + \frac{d}{d\tau}w^bL^b = -\frac{\gamma}{1-\gamma}\frac{1}{1-\tau}w^aL^a - w^aL^a\left[1 - \frac{\gamma}{1-\gamma}\frac{\tau}{1-\tau}\right].$$

We divide by w^aL^a, to find that $dY/d\tau < 0$ if and only if:

$$\begin{aligned}
-\frac{\gamma}{1-\gamma}\frac{1}{1-\tau} - \left[1 - \frac{\gamma}{1-\gamma}\frac{\tau}{1-\tau}\right] &< 0, \text{ or:} \\
-\frac{\gamma}{1-\gamma}\frac{1}{1-\tau} + \frac{\gamma}{1-\gamma}\frac{\tau}{1-\tau} - 1 &< 0, \text{ or:} \\
-\frac{\gamma}{1-\gamma}\frac{1}{1-\tau}(1-\tau) &< 1, \text{ or:} \\
-\frac{\gamma}{1-\gamma} &< 1.
\end{aligned}$$

Since we are assuming here that $\gamma < 0$, this is always true. In this model of labor supply, redistribution financed by distortionary taxes leads to a decrease in total national income.

Variable	Definition
$\mathcal{H}(a;\psi)$	Tax policy or system: legal mapping from actions of household a and parameters ψ to tax liability of household.
$\mathcal{T}(\psi)$	Realized revenue of the government under tax policy $\mathcal{H}$ with parameters ψ. The household is assumed to be using its best response, $a_{\max}(\psi)$.
a, A	Action of household a must lie in the set of possible actions A.
ψ	Vector of parameters in tax system or policy $\mathcal{H}$.
$a_{\max}(\psi)$	Household's optimal choice of action a under tax policy $\mathcal{H}$ with parameters ψ.
$\mathcal{Y}(a)$	Household's gross (pre-tax) income as a function of household action choice a.
$U\left[a, \mathcal{Y}(a) - \mathcal{H}(a;\psi)\right]$	Household's utility over action a and net (post-tax) income $\mathcal{Y} - \mathcal{H}$.
$V(\psi)$	Household's indirect utility with parameters ψ: $U\left(a_{\max}(\psi), \mathcal{Y}(a_{\max}(\psi) - \mathcal{H}\left[a_{\max}(\psi);\psi\right)\right]\right)$.
τ, E	Parameters of a flat tax system: the flat tax rate and the exemption.

Table 13.1: General Tax Notation for Chapter 13.

Exercises

Exercise 13.1 (Hard)
Consider an economy with infinitely many agents, each of whom is very very small. An agent i has preferences over consumption c^i and labor effort ℓ^i of:

$$u^i(c^i, \ell^i) = c^i - \gamma^i \ell^i.$$

The preference parameter γ^i is distributed uniformly on the interval $[0, 1]$. So the fraction of agents with preference parameters γ less than some number x is just x, for $0 \le x \le 1$. For example, exactly half of the population have values of γ less than or equal to 0.5.

Agents may only choose whether or not to work, not how many hours to work. If an agent chooses to work, she supplies exactly one unit of labor effort to the common backyard technology transforming labor effort into output as $y^i = \ell^i$. If an agent chooses not work, her labor effort is zero, she produces nothing and consumes nothing. All agents have the same backyard technology.

The government levies a flat income tax at a rate $0 \le \tau \le 1$. The tax rate τ is common to all

Variable	Definition
$u(C_t)$	One-period utility function when consumption is C_t
C_t, c_t	Aggregate consumption, household consumption.
I_t	Household investment in capital at time t.
K_t	Capital stock at time t.
L	Representative household's labor supply.
w	Wage rate on labor.
τ_L	Lump sum tax amount.
α, δ	Production parameters: the marginal product of capital and the depreciation rate.
γ	Preference parameter.
w^a, w^b	Wage rates of agents of type a and type b.
C^a, C^b	Consumption of agents of type a and type b.
L^a, L^b	Labor supply of agents of type a and type b.
v	Lump-sum transfer from type a agents to type b agents.

Table 13.2: Other Notation for Chapter 13

agents (that is, all agents face the same tax rate). Answer the following questions:

1. Given the tax rate τ, how much does an agent consume if she works? If she does not work?
2. For agent i, with preference parameter γ^i, calculate the utility of working (so that labor supply is $\ell^i = 1$) and of not working (so that labor supply is $\ell^i = 0$). What determines whether or not an agent works?
3. Given τ, find the largest value of γ such that agents prefer to work, or are at least indifferent between working and not working. Call this critical value $\gamma^*(\tau)$.
4. Given τ, multiply revenue per worker by the fraction of agents willing to work. Call this $\mathcal{T}(\tau)$, the government revenue function. Draw $\mathcal{T}(\tau)$ as a function of τ.
5. Is there a Laffer curve in the tax system?

Exercise 13.2 (Easy)
Suppose I ran the following regression:

$$G_t = b_0 + b_1\tau_t + u_t,$$

for $t = 1948, 1949, \dots, 1997$, and where G_t are real government receipts and τ_t are some measure of the marginal tax rates faced by a typical American for the indicated years. Suppose that my estimated coefficient $\hat{b_1}$ is negative and statistically significant. Would you conclude from this that there is a Laffer curve in the U.S. economy, and that we are on its downward-sloping portion? Why or why not?

Exercise 13.3 (Moderate)
The representative household lives for one period and has preferences over consumption C and labor supply L as follows:

$$U(C, L) = 2\sqrt{C} - L.$$

The government levies a flat tax at rate τ and a lump-sum tax of S. Money spent on the lump-sum tax is exempt from the flat tax (that is, the lump-sum tax is paid with *pre-tax* dollars). The household can transform labor effort into the consumption good at a rate of one-to-one (the wage rate is unity). Answer the following questions:

1. What are the parameters of this tax system? What is the action chosen by the household?
2. Write down the tax system function $\mathcal{H}(a; \psi)$ in this case (replace a with the household's action and ψ with the parameters of the tax system).
3. If the household works some amount L, write down its tax bill.
4. Write down the household's consumption as a function of L, τ and S.
5. Solve the household's problem. What is $L(\tau, S)$?
6. Find the government's revenue function $\mathcal{T}(\psi)$.

Exercise 13.4 (Easy)
Assume that a household lives for one period and has preferences over consumption c and labor supply ℓ as follows:

$$4\sqrt{c} - \ell.$$

The household earns a constant wage of $w = 1$ for each unit worked. There is a flat tax of rate τ. Answer the following questions:

1. Given that the household works an amount ℓ, find the household's tax bill, $\mathcal{H}(\ell; \tau)$ and consumption, $c(\ell, \tau)$.
2. Find the household's optimal choice of labor effort, $\ell(\tau)$.
3. Find the government revenue function $\mathcal{T}(\tau)$.

4. The government wishes to raise an amount 3/4 in tax revenue from this household. Which tax rate(s) can the government use. Assume that the government is benevolent (that is, nice). Which tax rate does the government use?

Exercise 13.5 (Hard)
Tammy lives for two periods. In the first period of life she works ℓ hours at a wage of w per hour. In the second period of life she is retired and supplies no labor. There is a perfect bond market on which Tammy saves an amount b in the first period of life, which earns a net interest rate of $r = 0$ (for a gross interest rate $1 + r = 1$), so Tammy's savings pay off $(1 + r)b = b$ in the second period of life. Tammy has preferences over consumption sequences $\{c_1, c_2\}$ and effort ℓ as follows:

$$u(\{c_1, c_2\}, \ell) = \sqrt{c_1} + \sqrt{c_2} - \ell.$$

The government taxes Tammy at a rate τ_1 on income earned in the first period of life, $w\ell$. Tammy owes no tax in the second period of life.

1. Write down Tammy's budget constraint in each period of her life, substitute out the savings term b and show that Tammy's present-value budget constraint is:

$$c_1 + \frac{1}{1+r}c_2 = c_1 + c_2 \leq (1 - \tau_1)w\ell.$$

2. Calculate Tammy's optimal choices of consumption, work, and savings: c_1, c_2, ℓ, and b, as a function of the tax rate τ_1.

3. How much revenue does the government raise as a function of the tax rate τ_1, $\mathcal{T}(\tau_1)$. Find τ_1^*, the tax rate at which government revenue is maximized. What is the maximum amount of revenue the government can raise (that is, what is $\mathcal{T}_1(\tau_1^*)$)?

4. Is there a Laffer curve?

Exercise 13.6 (Hard)
Use the same preferences and technology as in Exercise (13.5) above, except that here we call the government tax rate τ_2 instead of τ_1. The government will allow Tammy to *exempt* her savings b from taxes (as in a 401(k) plan), so she owes tax at a rate τ_2 in the first period only on the portion of her income that she does not save, $w\ell - b$. Tammy still owes no tax in the second period of life. If Tammy saves an amount b she consumes $(1 - \tau_2)(w\ell - b)$ in period 1 and b in period 2.

1. Write down Tammy's present-value budget constraint. How is it different from the one you calculated in part (1) of Exercise (13.5) above?

2. Solve Tammy's optimization problem. What are her optimal choices of consumption, work, and savings c_1, c_2, ℓ, and b as a function of the tax rate τ_2?

3. Now how much revenue does the government raise as a function of the tax rate τ_2 (call the revenue function $\mathcal{T}_2(\tau_2)$? What is the maximum amount of revenue the government can raise?

4. Is there a Laffer curve?

5. How do your answers differ from those in Exercise (13.5)? Why?