Chapter 15

Comparative Advantage and Trade

Most people would rather have a job making computer chips rather than potato chips. This may be rational, but the speed with which people state their preference belies a common misconception. In fact, the one occupation is not necessarily more profitable than the other. Haitians, for example, can make more money per hour growing and harvesting peanuts than they could make building computers. Economists use the terms absolute advantage and comparative advantage in discussing such issues.

A worker (or a country of workers) has an *absolute advantage* in production of a particular good if that worker (or country) can produce the good using fewer inputs than the competition. For example, in producing a good that requires labor only, the worker who can make a unit of the good in the least amount of time has an absolute advantage in the production of that good. The United States has an absolute advantage in producing a number of goods, since its work force is extremely productive and its economy is very well organized. Guatemala has an absolute advantage in the production of bananas, because of the country's climate. Kuwait has an absolute advantage in the production of crude oil, since its plentiful reserves make it easier to extract oil.

The term *comparative advantage* dates back to Ricardo.[1] Suppose a worker (or a country of workers) can make some good x and sell it for p_x dollars per unit on an open market. Obviously, the worker would like to sell the good for as high a price p_x as possible. If p_x is low enough, the worker will switch to production of some other good. We say that the worker has a comparative advantage in the production of x if the worker (or country) will find it profitable to make x at lower p_x than that at which the competition will find it profitable. This will be made quite a bit clearer when we formalize the concept in Section 15.2.

For now, let's think about the production of a particular good: the amount you learn in

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[1] Ricardo, David. *The Principles of Political Economy and Taxation*. 1817.

your macroeconomics course. Suppose you have a truly gifted instructor. This instructor could explain the textbook page by page, and teach that material better than the black and white textbook, i.e., the instructor has an absolute advantage over the textbook when it comes to expositing material linearly. Nonetheless, this would not be the best use of the instructor's time. He or she could be more productive by conducting in-class discussions and answering your questions. The textbook is terrible at answering your questions; your only hope is to look things up in the index and search through the text looking for an answer. The key point is that the instructor's time will be put to best use by doing the activity that he or she is relatively better at. This is the activity in which the instructor has a comparative advantage.

15.1 Two Workers under Autarky

We now move to a concrete model so as to be precise about the meaning of comparative advantage. There are two workers, Pat (P) and Chris (C), and two goods, wine (w) and beer (b). In this section we introduce the baseline case in which Pat and Chris live in autarky, i.e., they are not permitted to trade with each other. In the next section, we allow them to trade. It is the possibility of trade that raises the issue of comparative advantage.

Pat and Chris have H hours to devote to production each day. Use n_w^P to denote the number of hours that Pat needs to make a jug of wine. Similarly, use n_b^P for the number of hours that Pat needs to make a jug of beer. Replacing P with C gives us the time requirements of Chris. (Throughout this chapter, superscripts will denote whether the variable pertains to Pat or Chris, and subscripts will distinguish between variables for wine and beer.)

Pat's utility is: $U(c_w^P, c_b^P) = (c_w^P)^\gamma (c_b^P)^{1-\gamma}$, where c_w^P and c_b^P are Pat's consumption of wine and beer, respectively, and γ is some number between zero and one. Let h_w^P and h_b^P be the number of hours per day that Pat spends on production of wine and beer, respectively. That means that Pat will produce h_w^P / n_w^P jugs of wine each day. (For example, if $n_w^P = 4$, then it takes Pat 4 hours to make a jug of wine. If $h_w^P = 8$, then Pat spends 8 hours on wine production, so Pat makes 2 jugs of wine.)

Putting all this together, we get Pat's maximization problem:

$$\max_{c_w^P, c_b^P, h_w^P, h_b^P} \left\{ (c_w^P)^\gamma (c_b^P)^{1-\gamma} \right\}, \text{ such that:}$$

$$h_w^P + h_b^P = H,$$

$$c_w^P = \frac{h_w^P}{n_w^P}, \text{ and:}$$

$$c_b^P = \frac{h_b^P}{n_b^P}.$$

Substituting all the constraints into the objective yields:

$$\max_{h_b^P}\left\{\left(\frac{H-h_b^P}{n_w^P}\right)^{\gamma}\left(\frac{h_b^P}{n_b^P}\right)^{1-\gamma}\right\}.$$

Taking the first-order condition with respect to h_b^P and solving yields: $h_b^{P\star} = (1-\gamma)H$. Plugging this back into the time constraint gives us: $h_w^{P\star} = \gamma H$. (You should check to make sure you know how to derive these.) These optimal time allocations do not hinge on n_w^P and n_b^P because Pat's preferences are homothetic; it's not a general result.

We assume that Chris has the same preferences as Pat. All the math is the same; we just replace each instance of P with C. Chris's optimal choices are: $h_b^{C\star} = (1-\gamma)H$ and $h_w^{C\star} = \gamma H$, just like for Pat.

Suppose $H = 12$; each day Pat and Chris have 12 hours in which to work. Further, assume $\gamma = 1/3$. This implies that $h_b^{P\star} = h_b^{C\star} = 8$ and $h_w^{P\star} = h_w^{C\star} = 4$. Now suppose Pat can make a jug of wine in 4 hours and a jug of beer in 6 hours. Chris can make a jug of wine in 3 hours and a jug of beer in 1 hour. Translating these values to our variables gives the values in Table 15.1. Since Chris can make a jug of in fewer hours than Pat, Chris has an absolute advantage in wine production. Chris also has an absolute advantage in beer production.

	Hours per Jug	
	Wine	Beer
Pat	$n_w^P = 4$	$n_b^P = 6$
Chris	$n_w^C = 3$	$n_b^C = 1$

Table 15.1: Time Requirements

Plugging these values into our formulae above, we get the consumptions of Pat and Chris. Namely, $c_w^{P\star} = h_w^{P\star}/n_w^P = 4/4 = 1$, etc. Table 15.2 contains the rest of the consumption values.

	Consumption	
	Wine	Beer
Pat	$c_w^{P\star} = 1$	$c_b^{P\star} = 4/3$
Chris	$c_w^{C\star} = 4/3$	$c_b^{C\star} = 8$

Table 15.2: Consumption under Autarky

15.2 Two Workers Who Can Trade

We now turn to a world in which Pat and Chris can trade with each other. The open-market prices of beer and wine are p_b and p_w, respectively. We assume that Pat and Chris take these prices as given. There are two pieces to the maximization problem that they face. First, given these prices, they want to choose a way to allocate their time to production of the two goods so as to maximize their income. Second, for any given income, they want to choose how to divide up their consumption between the two goods. Thankfully, we can consider the problems separately. We address production first.

Production under Trade

Recall, n_w^P is the number of hours that Pat needs to make a jug of wine. This means that Pat can make $1/n_w^P$ jugs of wine per hour. Similarly, n_b^P is the number of hours for Pat to make a jug of beer, so Pat can make $1/n_b^P$ jugs of beer per hour.

If Pat chooses to make wine, Pat's hourly wage will be the number of jugs times the price: $(1/n_w^P)(p_w)$. Similarly, if Pat makes beer, Pat's hourly wage will be: $(1/n_b^P)(p_b)$. Pat will choose which to produce based solely on which wage is highest. Accordingly, Pat makes wine if:

$$\left(\frac{1}{n_w^P}\right) p_w > \left(\frac{1}{n_b^P}\right) p_b \quad \text{or:}$$

$$\frac{p_w}{p_b} > \frac{n_w^P}{n_b^P}. \tag{15.1}$$

This makes sense. Pat is more inclined to make wine if the price of wine p_w is higher or if Pat is able to make more wine per hour (i.e., n_w^P is smaller). Pat is less inclined to make wine if the price of beer p_b is higher or if Pat is able to make more beer per hour (i.e., n_b^P is smaller).

We get a similar relation for Chris, who makes wine if:

$$\frac{p_w}{p_b} > \frac{n_w^C}{n_b^C}. \tag{15.2}$$

To figure out the aggregate supply decisions of Pat and Chris, we conduct a thought experiment. First we suppose that the relative price of wine p_w/p_b is very low. Then we ask what happens as the relative price rises. When p_w/p_b is very low, both will make beer because the return to making wine is low relative to the return to making beer. As the price rises, eventually one of the two workers will find it profitable to switch to wine production. Eventually the price will rise enough so that both will make wine.

The numbers on the right-hand sides of equations (15.1) and (15.2) are the relative efficiencies of Pat and Chris at wine production, respectively. Whoever has the smaller number on the right-hand side is said to have a comparative advantage in wine production. This is because that worker will find it profitable to make wine even at low relative prices for wine.

Using the numbers from the previous section, we see that Pat makes wine if:

$$\frac{p_w}{p_b} > \frac{4}{6},$$

and Chris makes wine if:

$$\frac{p_w}{p_b} > \frac{3}{1}.$$

Since $4/6 < 3$, Pat has a comparative advantage in the production of wine. Recall, Chris has an absolute advantage in the production of both wine and beer. The idea of comparative advantage is that Chris has a more significant absolute advantage in beer production. There will be relative prices levels at which Chris will not make wine even though Chris has an absolute advantage in that market, since it will be even more profitable for Chris to specialize in beer.

We could just as well have looked at production in terms of beer. This is just the flip side of the wine market; the roles of w and b are just reversed. Pat makes beer if:

$$\frac{p_b}{p_w} > \frac{n_b^P}{n_w^P},$$

and Chris makes beer if:

$$\frac{p_b}{p_w} > \frac{n_b^C}{n_w^C}.$$

Be sure you understand how these equations relate to equations (15.1) and (15.2). It turns out that in a market with two producers and two goods, if one producer has a comparative advantage in one market, then the other will have a comparative advantage in the other market. Using the example numbers above, Pat has a comparative advantage in wine production, so Chris has a comparative advantage in beer production.

Using our example numbers, we can construct the aggregate supply curve for wine produced by Pat and Chris. On the vertical axis we put the relative price of wine: p_w/p_b. On the horizontal axis we put the quantity of wine supplied. See Figure 15.1.

When the relative price of wine is really low, neither Pat nor Chris produce wine, so the quantity supplied is zero. As the relative price of wine rises above 4/6, it suddenly becomes profitable for Pat to make wine instead of beer. Since Pat works H = 12 hours and can make $1/n_w^P$ = 1/4 jugs per hour, Pat's supply of wine is 3 jugs. (At a relative price of 4/6, Pat is indifferent between producing wine and beer, so Pat's wine production could be anything between 0 jugs and 3 jugs.) At prices between 4/6 and 3, Pat makes three jugs, and Chris still finds it profitable to make beer only.

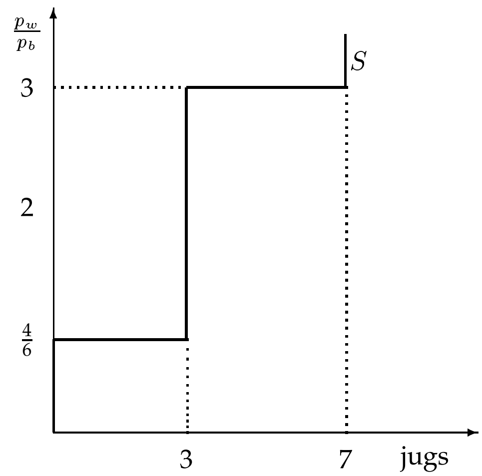

Figure 15.1: The Supply of Wine by Pat and Chris Together

When the relative price rises to 3, Chris finds it profitable to switch to beer production. Since Chris can make $(H)(1/n_w^C) = 4$ jugs per day, aggregate production jumps up to 7 jugs. (At a relative price of 3, it is now Chris who is indifferent between producing wine and beer, so Chris could produce anything from 0 to 4 jugs of wine, and aggregate production could be anything from 3 to 7 jugs.) As the relative price continues to rise above 3, both Pat and Chris reap higher profits, but the quantity supplied does not change, since both have already switched to produce wine exclusively.

Consumption under Trade

We can use the production numbers and prices from the previous section to calculate the dollar incomes of both Pat and Chris. Let m^P and m^C be the incomes of Pat and Chris, respectively. For example, if $p_w = 2$ and $p_b = 1$, then $p_w/p_b = 2$, and Pat will make wine only. Since Pat can make 3 jugs of wine per day, Pat's income will be: $m^P = (2)(3) = 6$. Similarly, given our sample parameters, Chris makes beer only and earns an income of: $m^C = 12$.

In the general case, our task now is to determine the optimal choices of consumption for Pat and Chris when their incomes are m^P and m^C, respectively. This is just a standard consumer-choice problem. Pat's maximization problem is:

$$\max_{c_w^P, c_b^P} \left\{ (c_w^P)^\gamma (c_b^P)^{1-\gamma} \right\}, \text{ such that:}$$

$$c_w^P p_w + c_b^P p_b = m^P.$$

Pat's optimal choices are:

$$c_w^{P\star} = \frac{\gamma m^P}{p_w}, \text{ and:} \tag{15.3}$$

$$c_b^{P\star} = \frac{(1-\gamma)m^P}{p_b}. \tag{15.4}$$

(See Exercise 15.1 for the derivations.) The choices of Chris are analogous, with C replacing P.

Equilibrium under Trade

We now have all the pieces to determine the equilibrium prices $p_w^\star$ and $p_b^\star$. Given two candidate values for these prices, we use equations (15.1) and (15.2) to determine which goods Pat and Chris produce. Multiplying each worker's production by the prices gives each worker's income. We then use equations (15.3) and (15.4), and the equivalent versions for Chris, to determine what Pat and Chris will consume at those prices. If the sum of the their production equals the sum of their consumption for each good, then these candidate prices are an equilibrium.

Actually finding the correct candidate equilibrium prices is somewhat complicated. We consider possible prices in regions dictated by the supply curve. Consider the supply curve derived from the sample parameter values we have been using in this chapter. We might start by supposing p_w/p_b is between 0 and 4/6. It turns out that this would make supply of wine smaller than demand, so that can't be an equilibrium. Then we might suppose that $p_w/p_b = 4/6$. These prices too lead to excess demand.

It turns out that the equilibrium occurs at $p_w/p_b = 2$. For example, $p_w^\star = 2$ and $p_b^\star = 1$ is an equilibrium. At these prices: Pat makes 3 jugs of wine and no beer; and Chris makes 12 jugs of beer and no wine. Their optimal consumptions are in the Table 15.3. At these prices aggregate consumption of each good equals aggregate production of each good, so this is an equilibrium.

	Consumption	
	Wine	Beer
Pat	$c_w^{P\star} = 1$	$c_b^{P\star} = 4$
Chris	$c_w^{C\star} = 2$	$c_b^{C\star} = 8$

Table 15.3: Consumption under Trade

When we compare Table 15.3 with Table 15.2 we see that there are gains from trade. Pat's consumption of wine and Chris's consumption of beer are the same in each case, but under trade, Pat gets to consume 2/3 extra units of wine, and Chris gets to consume 2/3 extra units of wine. Accordingly, both Pat and Chris are made better off by trade. This is literally the single most important result from the theory of international trade. Free trade allows each worker (or country) to specialize in production of the good in which the worker (or country) has a comparative advantage. As a result, free trade is generally Pareto improving.

There is open debate among economists about just how often trade is, or can be, Pareto improving. The most difficult aspect of analysis along these lines is that citizens of a given country are not affected equally. Consider peanut exports from Haiti to the United States. Free trade in peanuts almost certainly makes just about every Haitian better off. Similarly, most consumers in the United States are made better off by free trade, because Haitian peanuts cost less than those produced in the United States, but U.S. peanut producers are almost certainly made worse off by unfettered imports of Haitian peanuts. Accordingly, free trade in peanuts would not be Pareto improving. That said, almost all economists agree that it could be made Pareto improving, simply by having U.S. consumers reimburse U.S. peanut growers for their losses due to imports. Such a move can be made Pareto improving because the benefits to consumers because of cheaper peanuts far outweigh the high profits U.S. peanut growers receive from blocking Haitian imports.

There are other arguments about whether free trade is Pareto improving. For example, many economists think that free trade can damage what are called "infant industries". If South Korean makers of automobiles had faced unfettered imports when they first started production, those auto makers might never have had enough time to learn how to make competitive products. By shielding their producers from competition when they were just getting started, South Korea may have allowed a productive and efficient industry to develop. Now that South Korea's auto industry is no longer "infant", free trade can almost certainly be made Pareto improving, but that industry might never have existed without some protection in the early years.

Variable	Definition
P	Designates that a variable pertains to Pat
C	Designates that a variable pertains to Chris
w	Designates that a variable relates to wine
b	Designates that a variable relates to beer
n_i^j	Hours worker j requires to make a jug of good i
H	Total hours available for work in a day
h_i^j	Hours worker j spends making good i
p_i	Price of a jug of good i
m^j	Dollar income of worker j
c_i^j	Consumption of good i by worker j

Table 15.4: Notation for Chapter 15

Exercises

Exercise 15.1 (Moderate)
Derive the equations for $c_w^{P\star}$, $c_b^{P\star}$, $c_w^{C\star}$, and $c_b^{C\star}$ from Section 15.2. See equations 15.3 and 15.4.

Exercise 15.2 (Moderate)
Pat and Chris work 8 hours each day. They each try to make as much money as possible in this time. Pat can make a jug of wine in 2 hours and a jug of beer in 1 hour. Chris can make a jug of wine in 6 hours and a jug of beer in 2 hours. Pat and Chris are the only producers of wine in this economy. The price of wine is p_w, and the price of beer is p_b. The daily demand for wine is:

$$Q_w^D = 11 - 2\left(\frac{p_w}{p_b}\right).$$

1. Graph the aggregate supply curve for wine.
2. Graph the demand curve for wine (on the same graph).
3. Determine the equilibrium relative price of wine (i.e., the value of p_w/p_b that causes supply to equal demand).
4. Calculate the equilibrium values of: (i) the amount of wine made by Pat; (ii) the amount of beer made by Pat; (iii) the amount of wine made by Chris; and (iv) the amount of beer made by Chris.
5. Does either Pat or Chris have an absolute advantage in wine production? If so, which does?
6. Does either Pat or Chris have a comparative advantage in wine production? If so, which does?