Chapter 8

Inflation

This chapter examines the causes and consequences of inflation. Sections 8.1 and 8.2 relate inflation to money supply and demand. Although the presentation differs somewhat from that in Barro's textbook, the results are similar. In Section 8.3 we extend Barro's analysis with a closer look at the real effects of inflation.

8.1 Money Supply and Demand

In most countries, the general level of prices tends to increase over time. This phenomenon is known as *inflation*. In this section we will link inflation to changes in the quantity of money in an economy.

The quantity of money is determined by money supply and demand. Before we can find out how supply and demand are determined, we have to make precise what exactly is meant by money. Money is defined as the *medium of exchange* in an economy. *Currency* (bank notes and coins) is a medium of exchange, but there are other commodities that fulfill this function as well. For example, deposits on checking accounts can be used as a medium of exchange, since a consumer can write a check in exchange for goods. There are other assets where it is not so clear whether they should be considered money or not. For example, savings deposits can be used as a medium of exchange by making transfers or withdrawals, but the main purpose of savings accounts is to serve as a store of value. In order to deal with these ambiguities, economists work with a number of different definitions of money. These definitions are often referred to as *monetary aggregates*. One of the most important monetary aggregates is called M1; this measure consists of the currency in circulation plus checking deposits at banks. Broader aggregates like M2 and M3 also contain savings and time deposits.[1] As a convention, in this chapter we will identify money with M1, although most of the analysis would also work if we had broader aggregates in mind.

Having defined money, let us turn to money supply. Since we use M1 as our definition of money, we have to find the determinants of the supply of currency and checking deposits. In most countries, the supply of currency is under control of the central bank. For example, in the United States the Federal Reserve is responsible for supplying currency. If the central bank decides to increase the supply of currency, all it needs to do is to print more bank notes and hand them out, most of the time to private banks. Conversely, the central bank can decrease the supply of currency by buying back its own money. The determination of the supply of checking deposits is a more difficult question. Even though the central bank does not directly control checking deposits at private banks, a number of monetary-policy instruments give the central bank indirect control over bank deposits. To explain exactly how this works is beyond the scope of the chapter. We will come back to this question in Chapter 17, which takes a closer look at central-bank policy and its relation to the banking industry. For the purposes of this chapter, we will simply assume that both currency and checking deposits are under direct control of the central bank. This approximation works well enough for a first analysis of inflation. From now on, we will use M_t to denote the overall quantity of money supplied by the central bank in year t. For convenience, we will measure M_t in dollars.

Let us now take a look at money demand. Money is demanded by households and firms. Households need money in order to purchase consumption goods. Firms need money to purchase inputs to production and to make change at cash registers. For a given year t, we will use Y_t to denote the total amount of purchases, measured in terms of consumption goods. For example, on Crusoe's island Y_t would be the number of coconuts consumed in year t. If we are thinking about a whole country, we can interpret Y_t as real GDP. Since Y_t is in terms of goods, we have to multiply it by the price level P_t to get the total amount of purchases in terms of dollars, P_tY_t. Actual money demand is lower than P_tY_t, because money can be used more than once in a year. The *velocity of money* is defined as the average number of times a piece of money turns over in a year. The more often money turns over, the less money is needed to carry out the planned purchases. Using V_t to denote velocity, actual money demand is given by P_tY_t/V_t. For example, if $V_t = 1$, then each unit of money will be used only once. This corresponds to a situation in which all purchases are carried out at the same time, so P_tY_t dollars will be needed. On the other hand, if each month only 1/12 of all purchases are made, only $P_tY_t/12$ dollars will be required, and V_t will be 12.

In equilibrium, money supply M_t and money demand P_tY_t/V_t have to be equal. If we set them equal and multiply by velocity V_t, we arrive at the *quantity equation*:

$$M_tV_t = P_tY_t.$$

The quantity equation relates the quantity of money M_t to the price level P_t. Still, as of now it does not provide an explanation for inflation, because we have not yet explained

[1] See Chapter 4 of Barro for precise definitions of these aggregates.

how velocity V_t and the the amount of purchases Y_t are determined.

8.2 The Quantity Theory

Our task is to add theoretical underpinnings to the quantity equation in order to better understand inflation. The best way to proceed would be to write down a model that explains how the decisions of optimizing agents determine velocity V_t and output Y_t. We will do that in the following section, but as a first step we will start with a simpler approach. We assume that velocity and output in each year are given constants that are determined independently of the money supply M_t and the price level P_t. Further, we assume that velocity does not change over time. Therefore we can drop the time subscript and use V to denote velocity. The central bank controls money supply M_t, so the price level P_t is the only free variable. Given these assumptions, the quantity equation implies that the central bank has perfect control over the price level. If the central bank changes money supply, the price level will change proportionally. We can see that by solving the quantity equation for P_t:

$$P_t = M_t V / Y_t. \tag{8.1}$$

Let us now see what this implies for inflation. The inflation rate π_t in a given year t is defined as the relative change in the price level from t to $t+1$, or:

$$\pi_t = \frac{P_{t+1} - P_t}{P_t}.$$

This can also be written as:

$$1 + \pi_t = \frac{P_{t+1}}{P_t}. \tag{8.2}$$

Taking the ratio of equation (8.1) for two consecutive years, we get:

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+1} V Y_t}{M_t V Y_{t+1}}.$$

We know from equation (8.2) that P_{t+1}/P_t equals $1+\pi_t$, and the V terms cancel, so we have:

$$1 + \pi_t = \frac{M_{t+1}}{M_t} \frac{Y_t}{Y_{t+1}}. \tag{8.3}$$

We now take logs of both sides and use an approximation: $\ln(1+x) \approx x$ when x is not very large. Accordingly, equation (8.3) becomes:

$$\pi_t \approx [\ln M_{t+1} - \ln M_t] - [\ln Y_{t+1} - \ln Y_t].$$

This says that the inflation rate approximately equals the difference between the growth rate of money supply and the growth rate of output.[2] If output grows while the money

[2] See Chapter 1 for a general discussion of growth rates.

supply is constant, prices will have to fall so that money demand $P_t Y_t / V$ also stays constant. If money supply grows while output does not, prices will have to increase so that money demand increases in line with supply. Since the theory emphasizes the role of the quantity of money for the determination of inflation, it is known as the *quantity theory of money*.

Across countries and over time in a given country, we usually observe much higher variation in the growth rate of the money supply than in the rate of output growth. This indicates that variations in inflation are primarily attributable to variations in the rate of money growth. Empirical data gives strong support to this hypothesis. For example, Figure 7.1 in Barro shows that the money growth rate is almost perfectly proportional to the inflation rate in a sample of 80 countries.

While the quantity theory successfully explains the cause of inflation, it is not very helpful if we want to determine the consequences of inflation. In deriving the quantity theory, we assumed that money and prices were independent of all other variables in the economy. In the real world, high inflation is generally considered to be undesirable. If we want to understand why inflation might be bad, we have to determine the effects of inflation on real variables like output and consumption. This cannot be done within the quantity theory, since it assumed from the outset that such real effects did not exist. Instead, we need to go beyond the simplifying assumptions of the quantity theory.

To some degree we already did that in the discussion of money demand in Chapter 4, where we derived the optimal time T between a consumer's trips to the bank to get money. That time T between trips to get money was closely related to velocity V. In fact, $V = 2/T$.[3] In Chapter 4 we saw that the decision on T depended on the planned consumption expenditure and the nominal interest rate. Therefore the assumption of a constant velocity V that we made for the quantity theory was not correct. On the other hand, from an empirical point of view, the assumption of constant velocity seems to work relatively well as long as inflation rates are moderate.

The other assumption that we made for the quantity theory was that output Y_t was determined independently of monetary policy and inflation. We need to relax this assumption if we want to determine the real effects of inflation. In the next section, we will build a complete general equilibrium model that allows us to derive the impact of inflation on output and consumption.

[3]Velocity is given by $V = PY/M$. In Chapter 4, we derived that the average money holdings of a consumer were given by $\bar{m} = PcT/2$, where c was consumption. If we aggregate this over many consumers, the left-hand side becomes the aggregate money stock M, and individual consumption c sums to total output Y, so $M = PYT/2$. Plugging this into the formula for velocity yields $V = 2/T$.

8.3 A Cash-in-Advance Economy

In this section we derive the real effects of inflation. Unlike in the previous section, we will use a complete equilibrium model with optimizing consumers, because we want to understand how economic agents decide on consumption and output in the presence of inflation. The model builds on the general equilibrium framework developed in earlier chapters, but this model also contains a monetary sector.

This model is based on many identical consumers who live forever. In such a case, we say that consumers are *infinitely lived*. Since everyone is the same, it suffices to examine the choices of a single, representative consumer. The representative consumer has to decide on consumption c_t, labor supply l_t, savings s_{t+1}, and money holdings m_{t+1}. The utility function is:

$$\sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \ln(1 - l_t)], \tag{8.4}$$

where β is a discount factor between zero and one. There is only one good in the economy, and the consumer can produce the good with the technology $y_t = l_t$, i.e., output equals labor input.

Monetary policy is conducted in a particularly simple way in this economy. There is no banking sector that intermediates between the central bank and consumers. Instead, the central bank hands out money directly to consumers. Monetary policy consists of printing money and giving it as a transfer τ_t to each consumer. When the central bank wants to contract the money supply, it taxes each consumer by making τ_t negative.

We will use R_t to denote the nominal interest rate on savings and P_t to denote the price of the consumption good in period t. The time-t budget constraint of the consumer is:

$$m_{t+1} + s_{t+1} = m_t + (1 + R_t)s_t + P_t l_t + \tau_t - P_t c_t. \tag{8.5}$$

On the left-hand side are the amounts of money and savings that the consumer carries into the next period. Therefore they are indexed by $t + 1$. On the right-hand side are all the receipts and payments during the period. The consumer enters the period with money m_t and savings plus interest $(1 + R_t)s_t$, both of which he carries over from the day before. During the day, the consumer also receives income from selling produced goods $P_t l_t$ and the transfer τ_t from the central bank. The only expenditures are purchases of the consumption good, $P_t c_t$. All funds that are left after the household purchases the consumption good are either invested in savings s_{t+1} or are carried forward as money m_{t+1}.

So far, there is no explanation for why the consumer would want to hold money. After all, savings earn interest, and money does not. In order to make money attractive, we assume that cash is required for buying the consumption good. The consumer cannot consume his own production and has to buy someone else's production in the market with cash. This introduces a new constraint faced by the consumer: expenditure on consumption goods has to be less than or equal to money holdings:

$$P_t c_t \leq m_t. \tag{8.6}$$

Since money that is to be used for buying consumption goods has to be put aside one period before it is spent, equation (8.6) is also called the *cash-in-advance constraint*, which explains the name of the model. From here on we will assume that equation (8.6) holds with equality. This will be the case as long as the nominal interest rate is positive, because in that situation it is more profitable to invest additional funds in savings instead of holding them as cash.

In this economy consumption equals output, so the cash-in-advance constraint aggregates up to be the quantity equation. This formulation implicitly assumes that velocity is one. A more sophisticated model would incorporate some version of the money-demand model of Chapter 4, allowing velocity to vary with inflation. However, such a model would be more complicated without adding much to our explanation of the real effects of monetary policy.

One way of understanding the cash-in-advance constraint is to think of the consumer as a family consisting of two members, a worker and a shopper. Each morning, the worker goes to his little factory, works, and sells the production to other consumers. Only late at night does the worker come home, so the income cannot be used for buying consumption goods that same day. The shopper also leaves each morning, taking the cash that was put aside the night before to do that day's shopping. Since the shopper does not see the worker during the day, only money that was put aside in advance can be used to make purchases.

The problem of the representative consumer is to maximize utility subject to the cash-in-advance constraint and the budget constraint:

$$\max_{\{c_t, l_t, s_{t+1}, m_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \ln(1 - l_t)], \text{ subject to:}$$
$$P_t c_t = m_t, \text{ and:}$$
$$m_{t+1} + s_{t+1} = m_t + (1 + R_t)s_t + P_t l_t + \tau_t - P_t c_t.$$

In this model, the consumer's problem is much easier to analyze once we have the market-clearing conditions in place. Therefore we will complete the description of the economy first and derive the optimal decisions of the consumer later.

The next element of the model that needs to be specified is the monetary policy of the central bank. Instead of looking at aggregate money supply M_t, we will formulate monetary policy in terms of money per consumer m_t. This is merely a matter of convenience. We could recover the aggregate quantity of money by multiplying m_t by the number of consumers. However, since we are using a representative consumer, it is easier to formulate monetary policy on the level of individual consumers in the first place. We will assume a particularly simple policy: the central bank increases the money supply at a constant rate g. If the central bank wants to increase the money supply, it gives new cash to consumers.

Money supply in the next period is the sum of money supply in the current period and the transfer to the consumer. The money supply will grow at rate g if the transfer τ_t is given by $\tau_t = gm_t$, so we have:

$$m_{t+1} = m_t + \tau_t = (1 + g)m_t.$$

To close the model, we have to specify the three market-clearing conditions that must hold at each date t. The constraint for clearing the goods market states that consumption has to equal production:

$$c_t = l_t.$$

Clearing the credit market requires that total borrowing be equal to total savings. Since everyone is identical, there cannot be both borrowers and savers in the economy. In equilibrium savings have to be zero. Therefore the market-clearing constraint is:

$$s_t = 0.$$

In fact, we could omit savings from the model without changing the results. The only reason that we include savings is that this allows us to determine the nominal interest rate, which will play an important role in determining the real effects of monetary policy.

Finally, clearing the money market requires that the amount of cash demanded be the household equals the money supplied by the central bank. Since we use the same symbol m_t to denote money demand and supply, this market-clearing constraint is already incorporated in the formulation of the model.

An equilibrium for this economy is an allocation $\{c_t, l_t, s_t, m_t, \tau_t\}_{t=0}^{\infty}$ and a set of prices $\{P_t, R_t\}_{t=0}^{\infty}$ such that:

- Given the prices and transfers, $\{c_t, l_t, s_t, m_t\}_{t=0}^{\infty}$ is a solution to the household's problem; and
- All markets clear.

While this setup with infinitely-lived consumers might look complicated, having people live forever is actually a simplification that makes it easy to solve the model. The special feature of this framework is that the world looks the same in every period. The consumer always has infinitely many periods left, and the only thing that changes is the amount of money the consumer brings into the period (savings do not change since they are zero in equilibrium). The price level turns out to be proportional to the money stock, so the consumer always buys the same amount of the consumption good. In equilibrium, consumption c_t, labor l_t, and the nominal interest rate R_t are constant. Therefore we will drop time subscripts and denote interest by R and the optimal choices for consumption and labor by $c^\star$ and $l^\star$. Of course, we still need to show formally that $c^\star$, $l^\star$, and R are constant. This result will follow from the first-order conditions of the household's problem. We will plug in constants for consumption, labor, and interest, and we will be able to find prices such that the first-order conditions are indeed satisfied. For now, we just assume that $c^\star$ is constant.

As a first step in the analysis of the model, we examine the connection between monetary policy and inflation. This can be done in the same fashion as in the section on the quantity theory, without solving the consumer's problem explicitly.

The cash-in-advance constraint with constant consumption $c^\star$ is:

$$P_t c^\star = m_t. \tag{8.7}$$

The inflation rate π is defined by $1 + \pi = P_{t+1}/P_t$. Thus we can derive an equation for inflation by taking the ratio of the equation (8.7) for two consecutive periods:

$$1 + \pi = \frac{P_{t+1}}{P_t} = \frac{m_{t+1}}{m_t}.$$

Now we can use the fact that the money stock grows at a constant rate:

$$1 + \pi = \frac{m_{t+1}}{m_t} = \frac{m_t + \tau_t}{m_t} = \frac{(1+g)m_t}{m_t} = 1 + g.$$

Thus the inflation rate is equal to the growth rate of money supply. It is not surprising that we get this result. As in the quantity theory, we assume that velocity is constant. Since the cash-in-advance constraint is the quantity equation in this model, we had to come to the same conclusions as the quantity theory.

The main question that is left is how the level of consumption $c^\star$ (and hence equilibrium output) depends on inflation and monetary policy. To answer this question, we need to solve the household's problem.

We will use the Lagrangian method. The formulation of the Lagrangian differs from the one we used in the infinite-period model in Section 3.3, because here we multiply the Lagrange multipliers by the discount factor. This alternative formulation does not change results, and is mathematically more convenient. We use $\beta^t \mu_t$ for the multiplier on the time-t cash-in-advance constraint and $\beta^t \lambda_t$ as the multiplier on the time-t budget constraint. The Lagrangian for the household's problem is:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [&\ln(c_t) + \ln(1 - l_t) + \mu_t(m_t - P_t c_t) \\ &+\lambda_t(m_t + (1 + R_t)s_t + P_t l_t + \tau_t - P_t c_t - m_{t+1} - s_{t+1})].\end{aligned}$$

The first-order conditions with respect to c_t, l_t, s_{t+1} and m_{t+1} are:

$$\text{(FOC } c_t\text{)} \qquad \beta^t \frac{1}{c_t^\star} - \beta^t(\mu_t + \lambda_t)P_t = 0;$$

$$\text{(FOC } l_t\text{)} \qquad -\beta^t \frac{1}{1 - l_t^\star} + \beta^t \lambda_t P_t = 0;$$

$$\text{(FOC } s_{t+1}\text{)} \qquad -\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}(1 + R_{t+1}) = 0; \text{ and:}$$

$$\text{(FOC } m_{t+1}\text{)} \qquad -\beta^t \lambda_t + \beta^{t+1}(\mu_{t+1} + \lambda_{t+1}) = 0.$$

We now guess that in equilibrium, consumption, labor, and interest are constants $c^\star$, $l^\star$, and R. If this guess were wrong, we would run into a contradiction later. (Take our word for it: this is not going to happen.) With consumption, labor, and interest being constants, the first-order conditions simplify to the following expressions:

$$\frac{1}{c^\star} = (\mu_t + \lambda_t)P_t, \tag{8.8}$$

$$\frac{1}{1 - l^\star} = \lambda_t P_t, \tag{8.9}$$

$$\lambda_t = \beta\lambda_{t+1}(1 + R), \text{and:} \tag{8.10}$$

$$\lambda_t = \beta(\mu_{t+1} + \lambda_{t+1}). \tag{8.11}$$

If we now solve equation (8.9) for λ_t and plug the result into equation (8.10), we get:

$$\frac{1}{(1 - l^\star)P_t} = \frac{1}{(1 - l^\star)P_{t+1}}\beta(1 + R), \text{ or:}$$

$$\frac{P_{t+1}}{P_t} = \beta(1 + R).$$

The left-hand side equals one plus the inflation rate. We determined already that the inflation rate is equal to the growth rate of money supply in this economy. Therefore we can express the nominal interest rate as:

$$1 + R = \frac{1 + \pi}{\beta} = \frac{1 + g}{\beta}. \tag{8.12}$$

This says that the nominal interest rate R moves in proportion to the growth rate g of money. Dividing the nominal interest rate by inflation yields the real interest rate r:[4]

$$1 + r = \frac{1 + R}{1 + \pi} = \frac{1}{\beta}.$$

This expression should look familiar. It is a version of the Euler equation (3.16) that we derived for in the infinite-period model of Chapter 3. In the model we are considering here, consumption is constant, so the marginal utilities drop out. To interpret this equation, keep

[4] See Barro, Chapter 4 for a discussion of real versus nominal interest rates.

in mind that there cannot be any borrowing in equilibrium because there is no one from whom to borrow. If β is low, then consumers are impatient. Therefore the interest rate has to be high to keep consumers from borrowing.

We still have to trace out the effect of inflation on consumption. By using equations (8.8) and (8.9), we can eliminate the multipliers from equation (8.11):

$$\frac{1}{(1-l^\star)P_t} = \beta \frac{1}{c^\star P_{t+1}}.$$

From the goods market-clearing constraint, we know that $c^\star = l^\star$. Therefore we get:

$$\frac{P_{t+1}}{P_t} = \beta \frac{1-c^\star}{c^\star}.$$

The left-hand side is equal to the inflation rate (which itself equals the money growth rate). We can use that fact to solve for $c^\star$:

$$\begin{aligned} 1+g &= \beta \frac{1-c^\star}{c^\star}, \\ c^\star + gc^\star &= \beta - \beta c^\star, \text{so:} \\ c^\star &= \frac{\beta}{1+g+\beta}. \end{aligned} \tag{8.13}$$

This equation implies that consumption depends negatively on money growth, so consumption and inflation move in opposite directions. The intuition for this result is that inflation distorts the incentives to work. Income from labor cannot be used immediately for purchases of consumption, since consumption goods are bought with cash that has been put aside in advance. The labor income of today can be spent only tomorrow. When inflation is high, cash loses value over night. The higher inflation, the higher are prices tomorrow, and the fewer consumption goods can be bought for the same amount of labor. This implies that high rates of inflation decrease the incentive to work. Since consumption is equal to labor in equilibrium, consumption is low as well.

Given this relationship between consumption and inflation, which money growth rate should the central bank choose? In equilibrium, labor and consumption are equal. We can use this fact to find the optimal consumption, and then go backwards to compute the optimal money growth rate. The utility of consuming and working some constant $c = l$ forever is:[5]

$$\sum_{t=0}^{\infty} \beta^t [\ln(c) + \ln(1-c)] = \frac{1}{1-\beta}[\ln(c) + \ln(1-c)].$$

We will use $\hat{c}^\star$ to denote the optimal consumption. The first-order condition with respect to c is:

$$0 = \frac{1}{\hat{c}^\star} - \frac{1}{1-\hat{c}^\star}.$$

[5] Here we are using the formula for the sum of an infinite geometric series: $\sum_{n=0}^{\infty} a^n = 1/(1-a)$.

Solving for $\hat{c}^\star$ yields:

$$\hat{c}^\star = \frac{1}{2}. \tag{8.14}$$

Equation (8.13) gives us an expression for $\hat{c}^\star$ as a function of g. Combining this with equation (8.14) yields an equation involving the optimal rate of growth of the money stock $g^\star$:

$$\frac{1}{2} = \frac{\beta}{1 + g^\star + \beta}.$$

Solving this for $g^\star$ gives us:

$$g^\star = \beta - 1. \tag{8.15}$$

Since β is smaller than one, this equation tells us that $g^\star$ is negative: the optimal monetary policy exhibits shrinking money supply. Using equation (8.12) and our expression for $g^\star$, we can compute the optimal nominal interest rate:

$$1 + R = \frac{1 + g^\star}{\beta} = \frac{1 + (\beta - 1)}{\beta} = \frac{\beta}{\beta} = 1.$$

This implies $R = 0$, i.e., the nominal interest rate is zero. The intuition behind this result is as follows. The inefficiency in the model originates with the cash-in-advance constraint. The consumers are forced to hold an inferior asset, cash, for making purchases. If money were not needed for buying consumption goods and nominal interest rates were positive, everyone would save instead of holding cash. But if nominal interest rates were zero, cash and savings would earn the same return. Because prices fall in the equilibrium we calculated above, a consumer who holds money can buy more goods with this money in the future than he can buy now. This implies that the real interest rate on money is positive. Therefore incentives are not distorted if the nominal interest rate is zero. The recommendation of setting nominal interest rates to zero is known as the *Friedman rule*, after the Chicago economist Milton Friedman, who first came up with it. In Section 19.4, we will derive the Friedman rule once again within a different framework.

To summarize, the main outcomes of the cash-in-advance model are that: (1) the rate of money growth equals the inflation rate; (2) nominal interest rates move in proportion to inflation; and (3) output is negatively related to inflation. Empirical findings in the real world are consistent with these findings. The correlation of money growth and inflation was already addressed in the section on the quantity theory. Also, most of the variation in interest rates across countries can be explained by differences in inflation, which supports the second result. As to the third result, we observe that countries with very high inflation tend to do worse economically than countries with moderate inflation. However, within a set of countries with moderate inflation, the evidence is not conclusive.

There are a number of advanced issues concerning monetary policy and inflation that we will pick up later in this book. Chapter 18 is concerned with the coordination of monetary and fiscal policy, and in Chapter 19 we will return to the question of optimal monetary policy. While the prime emphasis of the cash-in-advance model is the inefficiency of holding cash instead of interest-bearing assets, Chapter 19 turns to the issue of expected versus unexpected inflation. You can think of the cash-in-advance model as describing the long-run consequences of expected inflation, while Chapter 19 considers the short-run consequences of a monetary policy that is not known in advance.

Variable	Definition
M_t	Aggregate quantity of money or cash
Y_t	Output
P_t	Price level
V	Velocity of money
π_t	Inflation rate
β	Discount factor of consumer
c_t	Consumption of consumer
l_t	Labor of consumer
$1 - l_t$	Leisure of consumer
m_t	Money or cash per consumer
s_t	Savings of consumer
τ_t	Central bank transfer of money to consumer
R_t	Nominal interest rate
r_t	Real interest rate
g	Growth rate of money supply

Table 8.1: Notation for Chapter 8

Exercises

Exercise 8.1 (Easy)
Consider an economy where velocity V equals 5, output grows at three percent a year, and money supply grows at five percent a year. What is the annual inflation rate?

Exercise 8.2 (Hard)
In the quantity theory, we assumed that velocity was constant. In reality, the velocity of money varies across countries. Would you expect countries with high inflation to have higher or lower velocity than low-inflation countries? Justify your answer. (Hint: You should draw both on Chapter 4 and Chapter 8 to answer this question.)