# Binary Evolution and Neutron Stars in Globular Clusters 

Natalia Ivanova, John M. Fregeau, and Frederic A. Rasio<br>Department of Physics and Astronomy, Northwestern University, 2131 Tech Drive, Evanston, IL 60208


#### Abstract

We investigate the dynamical formation and evolution of binaries containing neutron stars in dense globular clusters. Our numerical simulations combine a simple Monte Carlo prescription for stellar dynamics, a sophisticated binary population synthesis code, and a small- $N$-body integrator for computing 3 -body and 4 -body interactions. Our results suggest that there is no "retention problem," i.e., that, under standard assumptions, globular clusters can retain enough neutron stars to produce the observed numbers of millisecond pulsars. We also identify the dominant evolutionary and dynamical channels through which globular clusters produce their two main types of binary millisecond pulsars.


## 1. Introduction

Globular clusters (GCs) have proven to be a gold mine for studies of radio millisecond pulsars (MSPs), both isolated and in binaries: observations show that the number of MSPs formed per unit mass in GCs is much higher than in the galactic field. The companion-mass and orbital-period distributions of the observed binary MSPs in GCs and in the field are rather different (see Fig. 1). Clearly, the mechanisms responsible for MSP formation in GCs and in the field must also be different. Indeed, from the earliest X-ray observations, it was suggested that compact binaries in GCs must be formed through dynamical interactions in the high-density environment (Clark 1975).

Close stellar encounters can lead mainly to: (i) the destruction of wide binaries; (ii) hardening of close binaries (following "Heggie's law"; Heggie 1975); (iii) exchange interactions, through which low-mass companions tend to be replaced by a more massive participant in the encounter; (iv) physical collisions and mergers in binary-single or binary-binary encounters. In addition, both single stars and binaries can be ejected from the cluster through recoil following an encounter or supernova (SN) explosion. All these processes naturally introduce a set of key theoretical questions: (i) how many neutron stars (NSs) will remain in the cluster, (ii) how many of them can be recycled, (iii) how many can obtain or retain a binary companion and (iv) what are the expected characteristics of NS binaries? Answers to these questions rely strongly on our understanding of the binary stellar evolution coupled with cluster dynamics.

In our current work we use a new Monte Carlo approach (Sec. 2) based on one of the most advanced binary population synthesis codes currently available (StarTrack; see Belczynski, Kalogera \& Bulik 2002). The cluster is modeled as


Figure 1. Orbital period vs companion mass for all observed binary MSPs in the galactic field (open circles), in 47 Tuc (solid squares) and in other globular clusters (solid triangles).
a fixed background: all relevant parameters such as central density and velocity dispersion, and half-mass relaxation time (but not total mass), are kept constant throughout each dynamical simulation. This assumption is well justified physically for GCs with significant fractions of primordial binaries, which can be fitted by standard King models (see, e.g., Fregeau et al. 2003) and has been used in many previous theoretical studies of binary interactions and cluster dynamics (e.g., Hut, McMillan \& Romani 1992). The most important dynamical processes treated by the code include mass segregation and evaporation, physical collisions, tidal captures, and binary-single and binary-binary encounters. Each dynamical encounter involving a binary is calculated using Fewbody, a new numerical toolkit for direct $N$-body integrations of small- $N$ gravitational dynamics that is particularly suited to performing 3 -body and 4 -body integrations with high accuracy (Sec. 6).

Our results on the retention of NSs and on the statistics of NS populations in GCs are described in Sec. 3. In Sec. 4 we discuss the main types of NS binaries that are formed dynamically, and their survival probability and further evolution in the cluster. We then compare these results to the observed MSP cluster population.

## 2. Method

We start our typical simulations with $N \sim 10^{5}$ stars and with the initial binary fraction in the range $50 \%$ - 100\%. This high primordial binary fraction (much higher than assumed in all previous studies) is needed in order to match the observed binary fractions in GC cores today (Ivanova et al. 2004). Here we consider two representative core densities: $\rho_{\mathrm{c}}=10^{4.5} M_{\odot} \mathrm{pc}^{-3}$ ("typical" cluster model) and $\rho_{\mathrm{c}}=10^{5.1} M_{\odot} \mathrm{pc}^{-3}$ (" 47 Tuc" cluster model). The one-dimensional velocity
dispersion in the core is assumed to be $\sigma_{1}=10 \mathrm{~km} / \mathrm{s}$ and the corresponding escape speed from the cluster $v_{\text {esc }}=60 \mathrm{~km} / \mathrm{s}$.

Following Rasio, Phahl \& Rappaport (2000), we assume that the probability for an object of mass $m$ to enter the cluster core after a time $t_{\mathrm{s}}$ follows a Poisson distribution, $p\left(t_{\mathrm{s}}\right)=\left(1 / t_{\mathrm{sc}}\right) \exp \left(-t_{\mathrm{s}} / t_{\mathrm{sc}}\right)$, where the characteristic masssegregation timescale is given by $t_{\mathrm{sc}}=10(\langle m\rangle / m) t_{\mathrm{rh}}$ (Fregeau et al. 2002). The half-mass relaxation time is taken to be constant for a given cluster and for all simulations here we set $t_{\mathrm{rh}}=1 \mathrm{Gyr}$. We adopt the broken power-law IMF of Kroupa (2002) for single stars and primaries, a flat binary-mass-ratio distribution for secondaries, and the distribution of initial binary periods constant in logarithm between contact and $10^{7} \mathrm{~d}$.

To evolve single stars, we use the analytic fits of Hurley, Pols, \& Tout (2000). The binary evolution is calculated employing the binary population synthesis code StarTrack, which has been used previously in many studies of double compact objects (e.g., Belczynski et al. 2002) and X-ray binaries (e.g., Belczynski et al. 2004). This code allows us to follow the evolution of binaries with a large range of stellar masses and metallicities. It incorporates detailed treatments of stable or unstable conservative or non-conservative, mass transfer (MT) episodes, mass and angular momentum loss through stellar winds (dependent on metallicity) and gravitational radiation, asymmetric core collapse events with a realistic spectrum of compact object masses, and the effects of magnetic braking and tidal circularization.

The evolution of single and binary stars can be altered drastically by dynamical encounters with other objects. In our simulations, we consider such outcomes of dynamical encounters as physical collisions, tidal captures, destruction of binaries, companion exchanges, triple formation, etc. A particularly important type of outcome that we take into account is the dynamical common envelope (CE) phase that follows a physical collision between a compact object and a red giant (RG). In particular, if this compact object is a NS, a binary containing the NS and a white dwarf (WD) companion can be formed (Rasio \& Shapiro 1990, 1991; Davies et al. 1991). In this case, during the CE phase, the NS may accrete enough material from the envelope to become recycled as a MSP (Bethe \& Brown 1998).

## 3. Numbers and Characteristics of Retained Neutron Stars

We adopt the double-peaked distribution of natal NS kick velocities from Arzoumanian, Chernoff, \& Cordes (2002). With this choice we find that $6 \%$ of all NSs are retained in our typical cluster simulation, i.e., 240 NSs in a cluster with a current total mass of $200,000 M_{\odot}$. For our 47 Tuc model (with $\sim 1.5 \times 10^{6} M_{\odot}$ ), the number of retained NSs is 1750 . Moreover, if NSs can be formed through accretion induced collapse (AIC) of WDs, the fraction of NSs remaining in the GC can be as high as $\sim 10 \%$.

Throughout most of the cluster evolution, retained NSs (single or in binaries) are more massive than other objects. Their interaction cross section is large and they actively participate in dynamical encounters. For example, the probability for any single NS to have a physical collision with a $\sim 1 M_{\odot}$ MS star in our typical cluster model during 14 Gyr is about $10 \%$. Being in a binary
increases the probability of interaction. As a result, $85 \%$ of all retained NSs experienced some kind of strong interaction (binary destruction, acquiring a new companion, significant binary orbit change, or physical collision) after the formation of the NS. Most of these interactions involve binaries ( $60 \%$ binary-single and $35 \%$ binary-binary) and $5 \%$ were single-single encounters (mainly physical collisions, and a negligible number of tidal captures). In about $20 \%$ of binary encounters with a NS as a participant, a physical collision happened; $45 \%$ of the NSs that interacted acquired a new binary companion; and $20 \%$ of them acquired a new companion twice. As a result, at the end of the simulation (cluster age of 14 Gyr ), only $\sim 5 \%$ of retained NS in binaries remain in their original binary system. In our typical model, $25 \%$ of all NSs remaining in the cluster at 14 Gyr are in binaries (essentially all formed by dynamical encounters) while $75 \%$ of NSs are single; about $5 \%$ of all NSs are MSPs and $40 \%$ of MSPs are in binaries; $75 \%$ of all MSPs were first recycled in the original primordial binary. In our 47 Tuc model the fraction of recycled NSs is higher, about $13 \%$, and the total number of MSPs is 230, with 90 binary MSPs. These numbers are comparable to the estimated numbers of MSPs in 47 Tuc derived from observations (Heinke et al. 2003). Thus we conclude that there is no "retention problem" in our model.

## 4. Properties of Dynamically Formed Binaries

As mentioned above, our results show that most NS-binaries present today in a globular cluster were formed dynamically. The survival and fate of these binaries depends on the characteristic time to the next encounter (the collision time $t_{\text {coll }}$ ) and on the probability for the NS to remain in the binary after the next encounter. The latter depends on the hardness of the binary and on the masses of the participating stars. In particular, soft binaries (having binding energy smaller than the average kinetic energy "at infinity" of an encounter) will be easily destroyed ("ionized"). In a hard binary, encounters can lead to further hardening and/or exchanges. The NS is often the most massive participant, in which case it will likely remain in the binary). As the NS-binary hardens, the probability of a physical collision increases (Fregeau et al. 2004), and destruction can then occur through mergers.

In Figure 2 we show all binaries that were formed during the entire evolution of our model with 47 Tuc-like parameters. Most of the binaries were formed with a WD or a MS star as a companion to the NS, following an exchange interaction. These binaries are typically formed with high eccentricities and can shrink their orbit very fast through tidal dissipation and/or gravitational radiation. Not all of these systems will survive after the onset of MT: if a WD-companion is more massive than about $0.6 M_{\odot}$, the MT is unstable and the binary will merge. A significant fraction of the hardest binaries (hardness ratio $\gtrsim 100 \mathrm{kT}$ ) with very long collision times ( $\gtrsim 1 \mathrm{Gyr}$ ) were formed through physical collisions of a NS and a RG. In these collisions a dynamical CE occurs and a close binary with a MSP and a WD or He-star companion is formed.

In our typical cluster model, $35 \%$ of all NSs dynamically acquired a binary companion and about half of these acquired a new companion at least twice (on average, every NS that formed a binary had 3.5 exchange interactions).


Figure 2. Orbital periods vs companion masses for all dynamically formed NS binaries during the entire evolution of our 47 Tuc model, normalized to a total mass of $3 \times 10^{5} M_{\odot}$. Values plotted are at the time of formation (squares: MS companions; stars: RGs; triangles: He stars; circles: WDs). Open symbols are for binaries formed by exchange interactions, filled symbols for physical collisions. Dashed lines have constant $t_{\text {coll }}$ and dotted lines have constant binary hardness (where " 1 kT " is a marginally hard binary).


Figure 3. Orbital periods vs companion masses for all NS binaries in our 47 Tuc model at 14 Gyr. Squares represent NSs in binaries with MS companions, circles with WD companions. Filled symbols represent binary MSPs.

Most of the resulting binaries are hard, but $60 \%$ of them are destroyed by a subsequent encounter, and $10 \%$ are destroyed by evolutionary mergers (this fraction increases for denser clusters, where, on average, harder binaries are formed).

As shown in Figure 2, most of the dynamically formed binaries have relatively short collision times $\left(t_{\text {coll }} \lesssim 1 \mathrm{Gyr}\right)$, while many of those with longer collision times will merge once MT starts. As a result, very few of these NSbinaries remain at 14 Gyr (see Fig. 3). Most of the remaining NS in binaries have a WD as a companion. Binaries containing a recycled NS are divided into two main groups. The first has very short periods and very small companion masses (lower-left corner of Fig. 3): these systems evolved through stable MT, which followed an exchange interaction of the NS into a WD-WD or WD-MS binary. The resulting NS-WD binary does not necessarily have a very short period after the exchange. However, high eccentricities help shrink the binary orbits. These NS-WD binaries also often experience subsequent encounters which lead to further hardening and eccentricity pumping, and possible further exchanges. The other group, with WD companions of $\sim 0.2 M_{\odot}$ and longer periods in the range $0.1-0.3 \mathrm{~d}$, comes mainly from physical collisions between a NS and a RG. The resulting binary MSP distribution looks remarkably similar to the observed one (see Fig. 1).

## 5. Conclusions

The formation and evolution of NS binaries in a dense stellar system almost always involves dynamical interactions. In a typical cluster, we find that only a few percent of NS binaries are of primordial origin. Our simulations also suggest that, with a realistic NS kick velocity distribution, globular clusters can retain a
sufficient number of NSs to explain the observed numbers of MSPs. For a cluster like 47 Tuc, we predict $\sim 200$ MSPs, with roughly half of them in binaries. We can produce the two main types of observed binary MSPs: with $\sim 0.2 M_{\odot}$ WD companions, and with very low-mass companions ( $\sim 0.02 M_{\odot}$ ) and ultra-short periods. The first type comes mainly from NS-RG collisions followed by CE evolution, the second from exchange interactions.

## 6. Appendix: Fewbody

Small- $N$ gravitational dynamics, such as binary-binary and binary-single interactions, are important for the formation and evolution of binaries in GCs, as well as in driving the global cluster evolution (see, e.g., Heggie \& Hut 2003). Fewbody is a new, freely available ${ }^{1}$ numerical toolkit for simulating small $-N$ gravitational dynamics (Fregeau et al. 2004). It is a general $N$-body dynamics code, although it was written for the purpose of performing scattering experiments, and therefore has several features that make it well-suited for that purpose. Fewbody may be used standalone to compute individual small- $N$ interactions, but is generally used as a library of routines from within larger numerical codes that, e.g., calculate cross sections, or evolve GCs via Monte Carlo methods.

Fewbody uses an adaptive integrator, optionally with global pairwise Kusta-anheimo-Stiefel regularization (Heggie 1974; Mikkola 1985), to advance the particle positions with time. It uses a binary-tree algorithm to classify the $N$-body system into a set of hierarchies, and uses the approximate analytical criterion of Mardling \& Aarseth (2001) for the dynamical stability of triples, applied at each level in the binary tree, to approximately assess the stability of each hierarchy. It also uses a binary-tree algorithm to speed up integration of $N$-body systems with multiple timescales, by isolating weakly perturbed hierarchies (typically hard binaries) from the integrator. Fewbody uses a set of simple rules to automatically terminate calculations when the separately bound hierarchies comprising the system will no longer interact with each other or evolve internally. Finally, Fewbody performs collisions between stars during the integration in the "stickystar" approximation, with the radius of a collision product parameterized by a single expansion factor.

As a simple example of the use of Fewbody, consider the survivability of a pulsar-black hole (BH) binary in a GC core. Although there is now ample evidence for the possible existence of intermediate-mass black holes (IMBHs) in the cores of some GCs (e.g., van der Marel et al. 2002; Gerssen et al. 2002, 2003), a stellar-mass BH has never been observed, directly or indirectly, in a Galactic GC. It has been suggested that BH-PSR binaries should exist in GCs, and that they may provide the most likely means of detection of a stellar-mass BH in a cluster (Sigurdsson 2003). An obvious question suggests itself: What is the survivability of such binaries in clusters? Due to their large interaction cross section, binaries are likely to interact with other stars (or binaries if the binary fraction is large enough). A binary-single interaction can destroy the BH-PSR binary in several ways. For example, a main sequence (MS) field star

[^0]Table 1. Normalized destruction cross sections and characteristic lifetimes of BH-PSR binaries in globular clusters. $\widetilde{\sigma}_{\text {destr }}$ is the normalized total cross section for destruction (clean exchange and the merger of the MS and NS), $\widetilde{\sigma}_{\text {exch }}$ is the cross section for clean exchange (an exchange in which there were no collisions), $\widetilde{\sigma}_{N S}-\mathrm{MS}$ is the cross section for the collision of the NS and MS, and hence the formation of a Thorne-Żytkow-like object, and $\tau$ is the characteristic lifetime as a function of the number density $n_{5}=n / 10^{5} p c^{-3}$.

| $m_{\mathrm{BH}}\left(M_{\odot}\right)$ | $a(\mathrm{AU})$ | $\widetilde{\sigma}_{\text {destr }} /\left(\pi a^{2}\right)$ | $\widetilde{\sigma}_{\text {exch }} /\left(\pi a^{2}\right)$ | $\widetilde{\sigma}_{\mathrm{NS}-\mathrm{MS}} /\left(\pi a^{2}\right)$ | $\tau(\mathrm{yr})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1 | $0.30 \pm 0.02$ | $0.25 \pm 0.02$ | $0.024 \pm 0.006$ | $2.3 \times 10^{8} n_{5}^{-1}$ |
| 3 | 0.1 | $1.16 \pm 0.08$ | $0.28 \pm 0.04$ | $0.72 \pm 0.06$ | $1.7 \times 10^{9} n_{5}^{-1}$ |

can exchange into the binary, ejecting the NS and leaving a $\mathrm{BH}-\mathrm{MS}$ binary; or the MS field star and NS can collide, perhaps yielding a short-lived Thorne-Żytkow-like object (Thorne \& Żytkow 1977).

For hard binaries in GCs, the binary interaction cross section is dominated by gravitational focusing, and so we define the normalized cross section as $\widetilde{\sigma}=$ $\sigma\left(v_{\infty} / v_{c}\right)^{2}$, where $\sigma$ is the (non-normalized) cross section, $v_{\infty}$ is the relative velocity between the binary and intruder at infinity, and $v_{c}$ is the critical velocity. The critical velocity is defined so that the total energy of the binary-single system is zero at $v_{\infty}=v_{c}$. Using Fewbody, we have performed many binarysingle scattering encounters between a BH-PSR binary and a MS intruder star to calculate binary destruction cross sections. We vary the mass of the BH and semimajor axis of the binary, but keep all other parameters fixed. We set $m_{\mathrm{NS}}=1.4 M_{\odot}, R_{\mathrm{NS}}=15 \mathrm{~km}, m_{\mathrm{MS}}=1 M_{\odot}, R_{\mathrm{MS}}=1 R_{\odot}, e=0.9$, and $v_{\infty}=7 \mathrm{~km} / \mathrm{s}$. Table 1 lists normalized cross sections for two different BHPSR binaries: one containing a typical primordial stellar-mass BH , with $m_{\mathrm{BH}}=$ $10 M_{\odot}$ and $a=1 \mathrm{AU}$; and the other containing a lower-mass BH with $m_{\mathrm{BH}}=$ $3 M_{\odot}$ and $a=0.1 \mathrm{AU}$. It is clear that for the wider binary, with $a=1 \mathrm{AU}$, the destruction cross section is dominated by clean exchange. However, for the tighter binary, with $a=0.1 \mathrm{AU}$, the destruction cross section is dominated by NS-MS mergers.

Using the cross section, one may calculate the destruction rate per binary, and hence the lifetime, as $\tau=1 / R_{\text {destr }}=1 /\left(n \sigma_{\text {destr }} v_{\infty}\right)$, where $n$ is the number density of single stars, $\sigma_{\text {destr }}$ is the destruction cross section, and $v_{\infty}$ is the relative velocity between the binary and single star at infinity. In a dense cluster core the lifetime can be relatively short, as shown in Table 1. BH-PSR binaries with $m_{\mathrm{BH}}=10 M_{\odot}$ are expected to form at the rate of $\sim 1$ per GC. Adopting $n=10^{4} \mathrm{pc}^{-3}$ as a typical core number density implies that there are likely to be $\sim 10$ observable BH-PSR binaries with primordial BHs in our Galactic GC system. With $m_{\mathrm{BH}}=3 M_{\odot}, \mathrm{BH}-\mathrm{PSR}$ binaries are likely to form only in the densest cluster cores. Assuming again that $\sim 1$ is formed per dense cluster, we adopt the lifetime listed in Table 1, and now find $\sim 1$ observable BH-PSR binaries.

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[^0]:    ${ }^{1}$ See http://www.mit.edu/~fregeau, or search the web for "Fewbody".

