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HYDRODYNAMIC INSTABILITY AND COALESCENCE OF CLOSE BINARY SYSTEMS

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ABSTRACT

Close binary systems in hydrostatic equilibrium can become unstable. The stability limit for circular orbits occurs at the orbital separation that simultaneously minimizes the total equilibrium energy and angular momentum in the system. The occurrence of such a minimum is a purely Newtonian hydrodynamic effect resulting from tidal interactions. Its existence is independent of the degree of synchronization, assuming alignment of spin and orbital angular momentum. The development of an instability can drastically affect the terminal evolution of coalescing binary systems. In particular, it can cause a rapid acceleration of the coalescence, such that the final merging takes place on a time scale much shorter than the energy dissipation time scale. For orbital decay by gravitational wave emission of a system containing two identical stars of mass m and radius R, the radial infall velocity at contact is given by $v_r/(Gm/R)^{1/2} \sim 10^{-2}$–$10^{-1}$ for $0 < Gm/(Rc^2) < 0.1$. Expressed as a fraction of the Keplerian orbital velocity at the stellar surface, the radial velocity approaches a finite limiting value as $Gm/(Rc^2) \to 0$.

Subject headings: binaries: close — hydrodynamics — radiation mechanisms: gravitational — stars: neutron

1. INTRODUCTION

Close binary systems are traditionally studied in the Roche approximation, where they are modeled as massless gas in hydrostatic equilibrium in the effective potential of a point-mass system (Kopal 1959). This model applies well to very centrally condensed objects, with effective polytropic indices $n \geq 3$, such as red giants and main-sequence stars with radiative envelopes. In the opposite limit of incompressible configurations ($n = 0$), the tensor virial method has been used to calculate binary equilibrium solutions and their stability properties (Chandrasekhar 1969). Little is known, however, about the intermediate category of stars with $0 < n < 3$. Many binary systems contain stars that belong precisely to this category. In particular, all low-mass white dwarfs and main-sequence stars have $n \approx 1.5$, and neutron stars probably have $n \approx 0.5$–1.

In a recent paper (Lai, Rasio, & Shapiro 1993a, hereafter LRS), we used an energy variational method to construct hydrostatic equilibrium solutions for Newtonian polytropes in binary systems and study their stability. Both synchronized and nonsynchronized systems were considered. We found that close binary systems containing mildly compressible components ($n \lesssim 2$) become *unstable* when the binary separation decreases below a certain critical value. The instability sets in when the orbital separation is such that the total equilibrium energy and angular momentun of the system are both *minimum*. For binaries in which one component is much more compact than the other, this minimum always occurs before the Roche limit is reached. For binaries containing two identical stars, the minimum occurs before the stars come into contact.

The importance of this instability for coalescing binaries is easy to realize. The total equilibrium energy $E_{eq}(r)$ must attain a minimum simultaneously with $J_{eq}(r)$ since $dE_{eq}/dr = \Omega\, dJ_{eq}/dr$, where Ω is the orbital angular velocity (Ostriker & Gunn 1969; LRS). If quasi-circular equilibrium were maintained during the coalescence, one would naïvely estimate the secular radial infall as $\dot{r} = \dot{E}/(dE_{eq}/dr)$, which diverges as $dE_{eq}/dr \to 0$, no matter how small the energy dissipation rate $\dot{E}$ is.

The purpose of this *Letter* is to clarify the meaning of this instability and to estimate the resulting large (but finite) value of the radial infall velocity near the end of the coalescence. We focus on the case where the orbital decay is driven by gravitational radiation, but our results apply qualitatively to any mechanism that can extract energy and angular momentum from the system. We present a complete calculation of the orbital decay for a simple, generic model of a close binary. More detailed calculations based on the full equilibrium solutions of LRS will be presented elsewhere (Lai, Rasio, & Shapiro 1993b).

Our results could affect a number of problems of great current interest involving the coalescence of close binary stars. Blue stragglers are likely to be formed by the merging of contact main-sequence star binaries (Mateo et al. 1990). Double white dwarf systems coalescing by gravitational wave emission could be the progenitors of blue subdwarfs in globular clusters (Bailyn 1993). Perhaps most importantly, coalescing neutron-star binaries are now thought to be the most promising sources of gravitational radiation that may be detected by gravitational-wave interferometers such as LIGO (Thorne 1987; Abramovici et al. 1992). They have also been proposed as a possible source of extragalactic gamma-ray bursts (e.g., Paczyński 1991).

2. HYDRODYNAMIC INSTABILITIES IN BINARY SYSTEMS

Consider two stars of mass m and m' in a circular orbit of radius r. For large r, the stars behave like point masses, and the total equilibrium energy of the system is simply $-mm'/(2r)$ + const (we adopt units such that $G = c = 1$). As r decreases,

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tidal effects become increasingly important. For simplicity, let m' be a point mass. The height h of the tidal bulge raised on m by m' is given by $h/R \sim (m'/m)(R/r)^3$, where R is the stellar radius. The tidal deformation increases the self-energy of m by an amount $\sim (m^2/R)(h/R)^2 \sim m'^2R^5/r^6$ (making m less bound). It also makes the gravitational interaction between m and m' more attractive, and the corresponding energy is $\sim -m'Q/r^3 \sim -\kappa m'^2R^5/r^6$, where $Q \sim \kappa mRh$ is the quadrupole moment of m and κ is a constant of order unity (smaller for a more compressible star). However, the tidal interaction also leads to a larger orbital angular velocity, and this increases the orbital kinetic energy by an amount larger than the decrease in the interaction energy. Therefore, the net effect of the tidal interaction is to *increase* the total equilibrium energy of the system by an amount $\Delta E_{\rm tide} \sim \kappa m'^2R^5/r^6$. If the spin and orbital motion are synchronized to some degree, with $\Omega_s = f_s\Omega$ and $f_s \leq 1$, then the total energy is increased further, by an amount $\Delta E_{\rm spin} \sim \kappa mR^2f_s^2\Omega^2 \sim \kappa m(m+m')f_s^2R^2/r^3$. When r becomes sufficiently small, $\Delta E_{\rm tide}$ and $\Delta E_{\rm spin}$ become important and the total equilibrium energy $E_{\rm eq}(r)$ can *increase* as r decreases. As a consequence, *there exists a critical binary separation* $r = r_m$ *where* $E_{\rm eq}(r)$ *attains a minimum.*

In our detailed study of binary equilibrium configurations (LRS), we have calculated this critical binary separation r_m for a variety of different systems including Roche binaries (a polytrope in circular orbit about a point-mass companion) and Darwin binaries (two identical polytropes in circular orbit about each other). We have also considered their nonsynchronized generalizations, the so-called irrotational Roche-Riemann and Darwin-Riemann binaries. Such irrotational configurations can result from the coalescence of an initially wide binary containing nonspinning stars with small viscosity (Kochanek 1992, see also Bildsten & Cutler 1992). Typically, for all types of systems, we find that $r_m \sim 3q^{1/3}\ R$, where $q = m'/m$. As an illustration, Figure 1 shows the equilibrium energy curves $E_{\rm eq}(r)$ for various configurations with mass ratio $q = 1$ and polytropic index $n = 1$. For both Roche and Roche-Riemann binaries, we find that, as the separation r decreases,

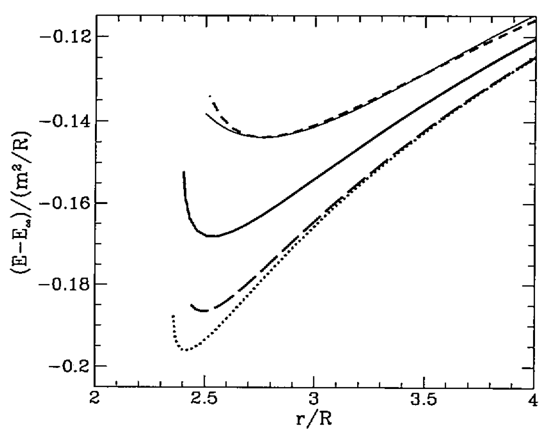

FIG. 1.—Equilibrium energy curves obtained by LRS for various binary configurations with mass ratio $q = 1$ and polytropic index $n = 1$. The quantity E_∞ is the total energy of the system when the binary separation $r \to \infty$. The Roche sequence (*solid line*), irrotational Roche-Riemann sequence (*dotted line*), Darwin sequence (*short-dashed line*) and irrotational Darwin-Riemann sequence (*long-dashed line*) are shown. The thin line near the top shows the simple analytic model (eq. [1]) for $\alpha = 6$. The curves for Roche and Roche-Riemann configurations terminate at the Roche limit, while those for Darwin and Darwin-Riemann binaries terminate when the stars are in contact.

the minimum of $E_{\rm eq}$ *is always reached before the Roche limit, independent of both n and q.* The Roche limit corresponds to the smallest binary separation for which an equilibrium configuration exists around a point mass. For Darwin and Darwin-Riemann binaries, equilibrium solutions exist up to the point at which the stars come into contact. For these systems we find that when the stars are sufficiently incompressible ($n \lesssim 2$), the minimum of $E_{\rm eq}$ is always reached before they come into contact. The reason is that when n is small, the coefficient κ is large enough that $\Delta E_{\rm tide}$ and $\Delta E_{\rm spin}$ already become appreciable at separations exceeding contact.

What is the meaning of this minimum? It is well known that any turning point (maximum or minimum of some global equilibrium quantity) along a one-parameter sequence of hydrostatic equilibrium configurations signals the onset of instability (see, e.g., Shapiro & Teukolsky 1983). Here this instability is associated with perturbations of the orbital separation r. For $r \leq r_m$, circular orbits are unstable because of deviations of the effective interaction potential between the two stars from a simple $1/r$ law. This is in complete analogy with the familiar instability of circular orbits for test masses around a Schwarzschild black hole. All circular orbits with $r < 6M$, where M is the black-hole mass, are unstable in this case. The equilibrium energy per unit mass for such orbits is given by $\tilde{E}_{\rm eq} = (r-2M)[r(r-3M)]^{-1/2}$, and $\tilde{E}_{\rm eq}(r)$ has a minimum at $r = 6M$.

3. ORBITAL DECAY DRIVEN BY GRAVITATIONAL RADIATION

Motivated by the results of LRS and § 2, we model the equilibrium energy of the system by

$$E_{\rm eq} = -\frac{mm'}{2r} + \frac{1}{2\alpha}\frac{mm'r_m^{\alpha-1}}{r^\alpha}, \tag{1}$$

where r_m is the stability limit ($E_{\rm eq}$ is minimum at $r = r_m$) and α is a parameter $\gtrsim 3$. Both r_m and α are mainly determined by the internal structure of the stars and the degree of synchronization. While the exact form of the equilibrium energy curve for a realistic system is certainly more complicated, we find that expression (1) has all the essential features and can reproduce quite accurately the results of LRS (see Fig. 1), as well as those of detailed numerical solutions (Lai et al. 1993b).

As long as the orbital decay remains quasi-static, the radial infall velocity v_r is given by

$$v_r \equiv \dot{r} = \dot{E}_{\rm GW}\left(\frac{dE_{\rm eq}}{dr}\right)^{-1}, \tag{2}$$

where $\dot{E}_{\rm GW}$ is the energy-loss rate, which we calculate for simplicity in the quadrupole approximation for point masses,

$$\dot{E}_{\rm GW} = -\frac{32}{5}\frac{(mm')^2(m+m')}{r^5}. \tag{3}$$

Substituting equations (1) and (3) into equation (2), we find

$$v_r = v_{\rm pt}\left(1 - \frac{r_m^{\alpha-1}}{r^{\alpha-1}}\right)^{-1}, \tag{4}$$

where $v_{\rm pt}$ is the radial infall velocity for two point masses,

$$v_{\rm pt} = -\frac{64}{5}\frac{mm'(m+m')}{r^3} \tag{5}$$

Clearly, when the binary separation approaches r_m, the radial infall velocity can become much larger than it would be in the

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absence of an instability. Although gravitational radiation dissipates energy on a time scale $t_E \equiv |(mm'/2r)/\dot{E}_{\rm GW}|$, the orbital decay time scale $t_r \equiv |r/v_r| = t_E(1 - r_m^{\alpha-1}/r^{\alpha-1})$ can be much shorter when approaching $r = r_m$.

Equations (2) and (4) become invalid when the rate of increase of infall kinetic energy, $\mu v_r(dv_r/dt)$, where μ is the reduced mass, becomes comparable to $(dE_{\rm eq}/dr)\, v_r$. Using equations (1)–(5), we find that the critical separation $r_c > r_m$ where this first occurs is given by

$$\delta_c \equiv \frac{r_c - r_m}{r_m} \approx 4(\alpha - 1)^{-3/4} q^{1/2} (1 + q)^{1/4} \left(\frac{r_m}{m}\right)^{-5/4} . \quad (6)$$

Since $r_m \sim 3Rq^{1/3}$ and $R/m \gg 1$, we see that r_c is in fact extremely close to r_m.

To properly calculate the orbital evolution for $r < r_c$, when the kinetic energy of radial infall becomes important, we now write the total energy of the system, *not necessarily in equilibrium*, as

$$E = \frac{\mu}{2} v_r^2 + \frac{J^2}{2\mu r^2} - \frac{mm'}{r} - \frac{mm' r_m^{\alpha-1}}{\alpha(\alpha - 2) r^\alpha} , \quad (7)$$

where J is the total angular momentum. The last term in equation (7) represents the effective tidal interaction. We are assuming that the internal dynamical time of the stars is much shorter than the orbital decay time. For circular orbits ($v_r = 0$), the equilibrium condition $(\partial E/\partial r)_{J,m,m'} = 0$ yields for the equilibrium angular momentum $J_{\rm eq}$ of the system,

$$J_{\rm eq}^2 = \mu \left[mm'r + \frac{mm' r_m^{\alpha-1}}{(\alpha - 2) r^{\alpha-2}} \right] . \quad (8)$$

If we substitute this expression for J in equation (7), we recover expression (1) for the total equilibrium energy.

Taking the time derivative of equation (7), we obtain the evolution equation

$$\mu \frac{dv_r}{dt} - \frac{J^2}{\mu r^3} + \frac{mm'}{r^2} + \frac{mm' r_m^{\alpha-1}}{(\alpha - 2) r^{\alpha+1}} = 0 , \quad (9)$$

where we have used the relation

$$\frac{dJ}{dt} = \frac{1}{\Omega} \dot{E}_{\rm GW} = \frac{\mu r^2}{J} \dot{E}_{\rm GW} . \quad (10)$$

Equations (9) and (10), together with $dr/dt = v_r$, are the equations describing the evolution of the system. They can be integrated numerically given initial conditions at any separation r_i such that $(r_i - r_m)/r_m \gg \delta_c$ (see eq. [6]). We calculate v_r and dv_r/dt at $r = r_i$ from equation (4) and then use equation (9) to obtain the initial value of J. In Figure 2, we show the results for a typical system with $q = 1$, $r_m/R = 2.8$, $\alpha = 6$, and $R/m = 10$, 20, 10^2, and 10^4. The integration is terminated when the stars are in contact, which we take to be at $r_f = 2.5R$ (a typical value; see LRS). For comparison, the evolution of two point masses is also shown for $R/m = 10$. We see that for $r < r_m$, the orbital evolution departs quite severely from that of point masses. The difference is largest when R/m is large.

From equation (6), we can derive an approximate relation for the radial infall velocity at $r = r_m$. By continuity, since $\delta_c \ll 1$, $v_r(r_m)$ should be approximately equal to $v_r(r_c)$, given by equation (4). We find

$$v_r(r_m) \sim v_r(r_c) \approx -3q^{1/2}(1 + q)^{3/4}(\alpha - 1)^{-1/4} \left(\frac{r_m}{m}\right)^{-7/4} . \quad (11)$$

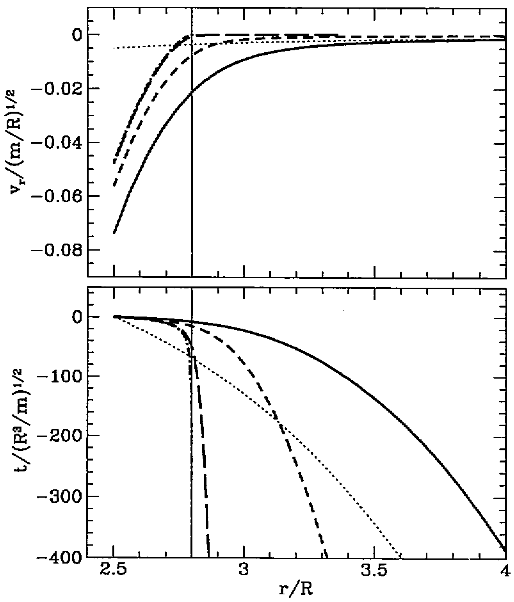

FIG. 2.—Terminal evolution of an unstable binary system with mass ratio $q = 1$, coalescing by gravitational wave emission. The stability limit is at $r = r_m = 2.8R$ (*vertical lines*) and $\alpha = 6$. All curves end at $r = r_f = 2.5R$ at time $t = 0$. Results are given for $R/m = 10$ (*solid lines*), 20 (*short-dashed lines*), 10^2 (*long-dashed lines*), and 10^4 (*dotted-dashed lines*). For comparison, the thin dotted lines show the results for two point masses with $R/m = 10$ (eq. [5]).

An exact expression can be obtained for $v_r(r_f)$ in the limit where $R/m \to \infty$. In this limit energy and angular momentum are conserved during the evolution for $r < r_m$. We therefore simply set $J^2 = J_{\rm eq}^2(r_m)$ in equation (9). We then obtain

$$v_r(r_f) = -\left[\frac{2y^\alpha}{\alpha(\alpha - 2)} - \frac{(\alpha - 1)}{(\alpha - 2)} y^2 + 2y - \frac{(\alpha - 1)}{\alpha}\right]^{1/2} \times (1 + q)^{1/2} \left(\frac{r_m}{m}\right)^{-1/2}$$
$$\simeq -\left(\frac{\alpha - 1}{3}\right)^{1/2} \delta^{3/2} (1 + q)^{1/2} \left(\frac{r_m}{m}\right)^{-1/2} , \quad (R/m \to \infty) , \quad (12)$$

where we have defined $y \equiv r_m/r_f \equiv 1 + \delta$ and for the second equality we have assumed $\delta \ll 1$. Note that the ratio $v_r(r_f)/(m/R)^{1/2}$ is independent of R/m in this limit, as shown in Figure 2. This is because the motion for $r < r_m$ is driven by a purely dynamical instability.

4. DISCUSSION

What is the final fate of such an unstable system? For Roche-type binaries, we have shown that the Roche limit cannot be approached quasi-statically. Instead, an appreciable radial infall velocity develops, leading to rapid mass transfer. Depending on how angular momentum is redistributed as a result of mass transfer, the dynamical response of the system can be stable or unstable (Hut & Paczyński 1984). If it is unstable, the star is tidally disrupted in just a few orbital periods. If it is stable, steady mass transfer could proceed, but on a time scale determined by the growth rate of the instability rather than the energy dissipation time scale. For Darwin-type

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binaries, there is no reason for the radial infall to be interrupted when contact is first established, since hydrostatic equilibrium solutions continue to exist beyond this point (Hachisu 1986). The likely outcome is then the formation of a merged object in just a few orbital periods. This has been demonstrated with three-dimensional numerical simulations by Rasio & Shapiro (1992) for neutron star binaries (see also Oohara & Nakamura 1993) and is discussed by Rasio (1993) for main-sequence star binaries in the context of blue straggler formation.

For binary neutron stars and neutron-star–black-hole binaries, relativistic effects are also likely to be important in determining the final evolution of the system (Lincoln & Will 1990; Kidder, Will, & Wiseman 1992). The last stable circular orbit due to general relativity for a neutron star of mass m orbiting a black hole of mass m' is at $r_{GR} \sim 6(m + m')$, while the hydrodynamic stability limit is at $r_m \sim 2(1 + q)^{1/3}R$ (see LRS). Therefore we expect hydrodynamics to remain important as long as $1 + q \lesssim 6[(R/m)/10]^{3/2}$. For two identical neutron stars, Kidder et al. (1992) find $r_{GR} \approx 14m$, while LRS obtain $r_m \approx 2.8R$. In this case hydrodynamic effects remain important as long as $R/m \gtrsim 5$. Note that, since general relativity, just like tidal interactions, causes a strengthening of the gravitational interaction between the two stars at small separation, the minimum of $E_{eq}(r)$ in a relativistic fluid model should occur at a somewhat larger binary separation. Therefore, the inequalities given above tend to *underestimate* the importance of hydrodynamics. From these inequalities, we conclude that the instability discussed here could play an important role in many systems containing neutron stars and stellar-mass black holes.

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REFERENCES

Abramovici, A. et al. 1992, Science, 256, 325
Bailyn, C. D. 1993, in Proc. Dynamics of Globular Clusters: a Workshop in Honor of Ivan R. King, ed. S. Djorgovski & G. Meylan, ASP Conf. Ser., in press
Bildsten, L., & Cutler, C. 1992, ApJ, 400, 175
Chandrasekhar, S. 1969, Ellipsoidal Figures of Equilibrium (New Haven: Yale Univ. Press)
Hachisu, I. 1986, ApJS, 62, 461
Hut, P., & Paczyński, B. 1984, ApJ, 284, 675
Kidder, L. E., Will, C. M., & Wiseman, A. G. 1992, Class. Quantum Gravity, 9, L125
Kochanek, C. S. 1992, ApJ, 398, 234
Kopal, Z. 1959, Close Binary Systems (London: Chapman & Hall)
Lai, D., Rasio, F. A., & Shapiro, S. L. 1993a, ApJS, (LRS)
Lai, D., Rasio, F. A., & Shapiro, S. L. 1993b, in preparation
Lincoln, C. W. & Will, C. M. 1990, Phys. Rev. D, 42, 1123
Mateo, M., Harris, H. C., Nemec, J., & Olszewski, E. W. 1990, AJ, 100, 469
Oohara, K., & Nakamura, T. 1993, Prog. Theor. Phys., in press
Ostriker, J. P., & Gunn, J. E. 1969, ApJ, 157, 1395
Paczyński, B. 1991, Acta Astron., 41, 257
Rasio, F. A. 1993, in Proceedings of STScI Workshop on Blue Stragglers, ed. M. Livio, & R. Saffer, ASP Conf. Ser., in press
Rasio, F. A., & Shapiro, S. L. 1992, ApJ, 401, 226
Shapiro, S. L., & Teukolsky, S. A. 1983, Black Holes, White Dwarfs, and Neutron Stars (New York: Wiley)
Thorne, K. S. 1987, in 300 Years of Gravitation, ed. S. W. Hawking & W. Israel (Cambridge Univ. Press), 330

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