

THE DYNAMICAL EFFECTS OF WHITE DWARF BIRTH KICKS IN GLOBULAR STAR CLUSTERS

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ABSTRACT

Recent observations of the white dwarf (WD) populations in the Galactic globular cluster NGC 6397 suggest that WDs receive a kick of a few km s^{-1} shortly before they are born. Using our Monte Carlo cluster evolution code, which includes accurate treatments of all relevant physical processes operating in globular clusters, we study the effects of the kicks on their host cluster and on the WD population itself. We find that in clusters whose velocity dispersion is comparable to the kick speed, WD kicks are a significant energy source for the cluster, prolonging the initial cluster core contraction phase significantly so that at late times the cluster core-to-half-mass radius ratio is a factor of up to ~ 10 larger than in the no-kick case. WD kicks thus represent a possible resolution of the large discrepancy between observed and theoretically predicted values of this key structural parameter. Our modeling also reproduces the observed trend for younger WDs to be more extended in their radial distribution in the cluster than older WDs.

Key words: globular clusters: general – methods: numerical – stellar dynamics – white dwarfs

1. WHITE DWARF BIRTH KICKS

Recent *Hubble Space Telescope* observations of the Galactic globular cluster NGC 6397 reveal that its youngest white dwarfs (WDs) are significantly more extended in their radial distribution within the cluster than their older visible counterparts (Davis et al. 2008b). Since the progenitors of the WDs are some of the most massive stars in the cluster at $\sim 0.8 M_{\odot}$, and since the older WDs (of mass $\sim 0.5 M_{\odot}$) have had time to diffuse to larger radii than where they formed due to mass segregation, one would naively expect the opposite—namely that the youngest WDs should be more centrally concentrated than their older counterparts. One possible interpretation is that WDs receive a kick of a few km s^{-1} shortly before they are born (Davis et al. 2008b; Heyl 2007, 2008a, 2008b). Indeed, in open star clusters, where the velocity dispersion is $\sim 1 \text{ km s}^{-1}$ and the putative WD kick would be enough to eject it from the cluster, there is an observed relative dearth of WDs (Fellhauer et al. 2003; Kalirai et al. 2001; Weidemann 1977).

The mechanism generating WD “natal” kicks is likely asymmetric mass loss late in the asymptotic giant branch (AGB) phase. Although AGB winds have not been directly observed, the effects of the resulting mass loss have been, and there are good theoretical reasons to expect such winds exist (Vassiliadis & Wood 1993). Alternatively, the kick could be produced as a result of small asymmetry during the helium core flash (Ivanova, private communication). Independent of precisely when in the evolution of a star the kick occurs, the observed rotation rates of WDs are consistent with nonaxisymmetric mass loss at some point in their evolution (Spruit 1998).

Analogous to the hydrogen-burning main sequence in stars, star clusters are thought to eventually enter a long lived binary-burning phase in which inelastic dynamical scattering interactions of binaries with other stars generate enough energy to prevent the cluster core from collapsing (e.g., Binney & Tremaine 2008). The physics of the binary-burning phase has been studied intensively. It is only recently, however, that different numerical

methods have begun to agree on the structural properties of a cluster in the binary burning phase (at least in the equal-mass case; Heggie et al. 2006; Fregeau & Rasio 2007). Unfortunately, the predicted value of the core-to-half-mass radius ratio r_c/r_h in the binary-burning phase is at least ~ 10 times smaller than what is observed for the majority of Galactic globular clusters (e.g., Fregeau & Rasio 2007). Although differences in the observational and theoretical definitions of the core radius may account for part of the difference (Hurley 2007), there is still a significant discrepancy. If the bulk of Galactic clusters are indeed currently in the binary-burning phase, a core radius larger than expected from binary burning suggests the existence of an alternate core energy source. In this vein, it has been suggested that perhaps tens of Galactic clusters contain a central intermediate-mass black hole (IMBH; Trenti 2006), powering their cores to the observed sizes. Or the required energy could come from expedited stellar evolution due to stellar collisions (Chatterjee et al. 2008), or prolonged mass segregation of compact objects (Merritt et al. 2004; Mackey et al. 2007). Alternatively, it could be that most clusters are simply not currently in the binary burning phase, in which case their initial properties may play an important role (Fregeau 2008).

Since for a standard Kroupa et al. (1993) initial mass function, roughly 15% of stars in a cluster will become WDs within 12 Gyr, a WD birth kick of a few km s^{-1} represents a potentially significant core energy source (roughly 15% percent of the total cluster energy for a WD kick speed comparable to the velocity dispersion)—one that could resolve the discrepancy between theory and observations on the structural parameters of clusters. In this Letter we consider the dynamical effects of WD birth kicks by simulating the evolution of star clusters in which WDs receive kicks of a few km s^{-1} at birth. Our simulation method treats all relevant physics in clusters self-consistently, as described below. We find that WD birth kicks can significantly prolong the initial cluster evolutionary phase of core contraction, resulting in core sizes at late times that are larger than for clusters without kicks. We also find that birth kicks produce an observational signature that is consistent with the observed radial distributions of WDs in NGC 6397.

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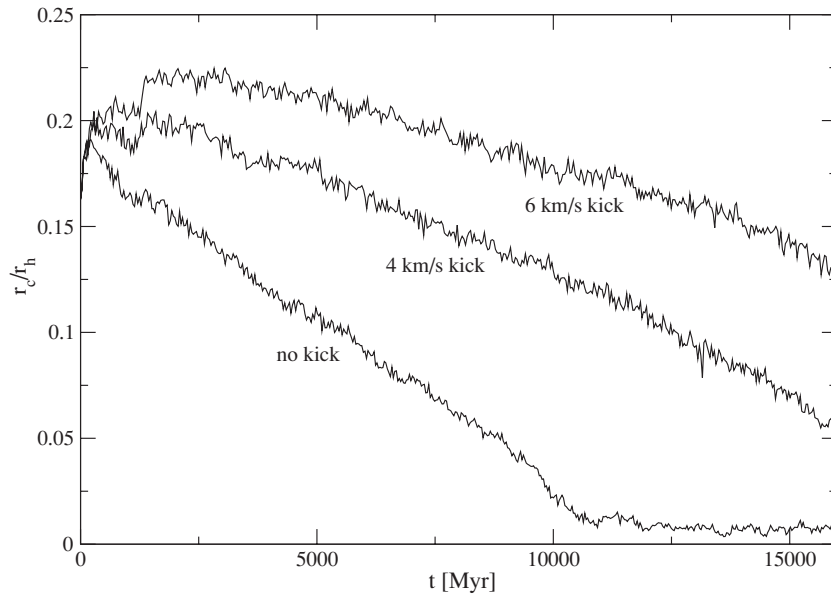


Figure 1. Evolution of core-to-half-mass radius ratio, r_c/r_h , for models with no WD birth kicks, 4 km s^{-1} birth kicks, and 6 km s^{-1} birth kicks.

2. SIMULATIONS

To simulate the evolution of clusters with WDs that experience birth kicks, we use the well tested Hénon Monte Carlo method, as described in detail elsewhere (Joshi et al. 2000, 2001; Fregeau et al. 2003; Fregeau & Rasio 2007; J. M. Fregeau & F. A. Rasio 2009, in preparation). Briefly, it treats the effects of two-body relaxation in the orbit-averaged approximation and, since it provides for a physical realization of the cluster at each time step, has been extended to include a treatment of all other relevant physics, including strong dynamical interactions of binaries, physical stellar collisions, stellar evolution of single stars and binaries, and the effects of a Galactic tide. It has shown excellent agreement with direct N -body methods for the evolution of clusters with dynamically significant populations of binaries (Fregeau & Rasio 2007), and those in which the effects of stellar evolution are dominant (J. M. Fregeau & F. A. Rasio 2009, in preparation). To model the effects of WD birth kicks we simply give a randomly oriented velocity kick to a star as soon as it becomes a WD. For simplicity, the kick is set to a fixed value between 2 and 9 km s^{-1} for each simulation.

Modeling individual clusters with any degree of detail is a difficult endeavor (e.g., Hurley et al. 2008; Giersz & Heggie 2009). While we do not attempt to model NGC 6397 in this Letter, we find that our models do have roughly the same core velocity dispersion at late times, and the model with 4 km s^{-1} WD kicks has roughly the same ratio of core-to-half-mass radius at very late times (to within a factor of 2 of the value in Noyola & Gebhardt 2006). Our standard initial model is a $W_0 = 7.5$ King model with a virial radius of 5 pc, consisting of $N = 3 \times 10^5$ objects, 1% of which are binaries (similar to NGC 6397’s inferred initial binary fraction; Davis et al. 2008a). No initial mass segregation is assumed. Single stars and binary primaries are chosen from a Kroupa et al. (1993) distribution from 0.15 to $18.5 M_\odot$. Binary secondaries are chosen from a flat mass ratio distribution with a minimum at $0.15 M_\odot$, semimajor axes are chosen uniformly in the log from $5(R_1 + R_2)$ (where R_i are the stellar radii) to the hard–soft boundary, and eccentricities are chosen from a thermal distribution truncated at the upper end to prevent pericenter approaches smaller than $5(R_1 + R_2)$.

Cluster metallicity is set to $Z = 0.001$, as in Hurley et al. (2008). Although the metallicity of NGC 6397 is a factor of ~ 5 smaller, as described in Hurley et al. (2008) stellar evolutionary processes do not depend strongly on metallicity below $Z \approx 0.001$. The Galactic tide represents that of a point-mass host galaxy of mass $10^{11} M_\odot$, with the cluster in a circular orbit at 6.9 kpc.

Figure 1 shows the evolution of r_c/r_h for our standard model with no WD birth kicks, 4 km s^{-1} birth kicks, and 6 km s^{-1} birth kicks. Around 11 Gyr, the model without kicks enters the binary burning phase, during which time $r_c/r_h \approx 0.01$. (Note that we use the standard three-dimensional, N -body definition of the core radius; J. M. Fregeau & F. A. Rasio 2009, in preparation.) The models with kicks show no evidence of reaching a binary burning phase in their 16 Gyr of evolution. Instead, they show a prolonged, gradual contraction of the core with time, similar to the behavior found in N -body simulations by Hurley (2007). As expected, the model with 6 km s^{-1} birth kicks displays an appreciably larger core radius over time compared to the 4 km s^{-1} kick model. At late times, the models with kicks display large values of r_c/r_h , with values up to ~ 10 times larger than the model without kicks. We note that for the model with 4 km s^{-1} kicks we find a value of $r_c/r_h \approx 0.11$ at an age of 12 Gyr, which is significantly larger than the observed value of $r_c/r_h \approx 0.03$ for NGC 6397 (Noyola & Gebhardt 2006). The 4 km s^{-1} kick model reaches values of $r_c/r_h \approx 0.05$ at unphysically late times, but further simulations are required to determine if a better match to NGC 6397 can be made with a suitable choice of initial conditions.

Figure 2 shows the cumulative three-dimensional radial distributions of WDs at ~ 12 Gyr in the models with no kick and with 6 km s^{-1} kicks, broken down into “young” and “old” populations. The young population consists of those WDs with $L \geq 1.2 \times 10^{-4} L_\odot$ (corresponding to ages ≤ 4 Gyr for a $0.5 M_\odot$ WD; Bergeron et al. 1995), while the old population consists of WDs with $1.9 \times 10^{-5} \leq L \leq 7.9 \times 10^{-5} L_\odot$ (corresponding to ages between 6 and 10 Gyr for a $0.5 M_\odot$ WD). In Davis et al. (2008b) analogous populations are chosen with the restriction that the young WDs have ages less than ~ 3 times the local relaxation time of 0.29 Gyr, so that mass segregation has had

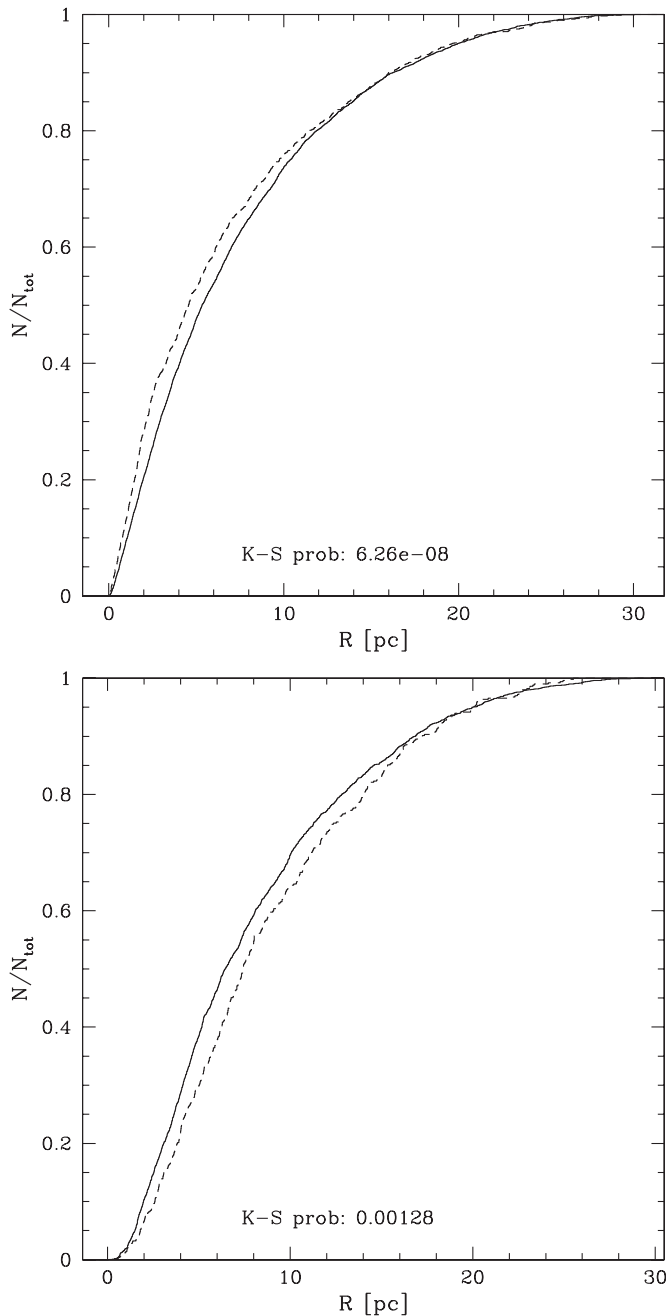


Figure 2. Cumulative three-dimensional radial distributions of old (solid curve) and young (dashed curve) WD populations at ~ 12 Gyr in models without WD birth kicks (top panel), and with kicks of 6 km s^{-1} (bottom panel). The young population consists of those WDs with $L \geq 1.2 \times 10^{-4} L_{\odot}$ (corresponding to ages ≤ 4 Gyr for a $0.5 M_{\odot}$ WD), while the old population consists of WDs with $1.9 \times 10^{-5} \leq L \leq 7.9 \times 10^{-5} L_{\odot}$ (corresponding to ages between 6 and 10 Gyr for a $0.5 M_{\odot}$ WD). The probability for obtaining the two-sided K-S statistic for each pair of distributions is also shown. In the no kick case the young population is statistically significantly more centrally concentrated than the old population, while in the 6 km s^{-1} kick case it has a more extended radial distribution.

minimal effect on their radial distribution. The effects of mass segregation decay exponentially in time (Fregeau et al. 2002), so they should be apparent for any division between young and old WDs. To maximize the evidence of the effect, we take the cut at roughly a half-mass relaxation time, which is ≈ 5 Gyr in our models at an age of 12 Gyr. The top panel shows the cumulative radial distribution of young and old WDs in the model with no WD birth kicks. The young population is statistically significantly more centrally concentrated than the

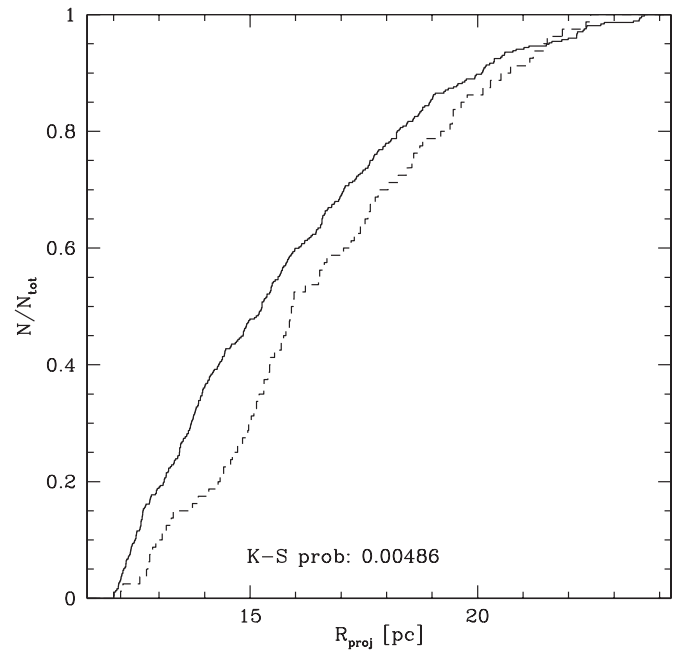


Figure 3. Cumulative projected radial distributions at ~ 12 Gyr between 12 and 24 pc (1.7 and $3.5 r_h$ in our model) of old (solid curve) and young (dashed curve) WD populations in the model with 4 km s^{-1} WD birth kicks. WD age cuts are the same as in the previous figure.

old population, as might be expected when one considers that the progenitors of the young WDs are $\sim 0.8 M_{\odot}$ stars and hence more massive than the $\sim 0.5 M_{\odot}$ “old” WDs which have had time to mass segregate out to their larger cluster radii. The bottom panel shows the cumulative distributions for the model with 6 km s^{-1} kicks. The young population is clearly and significantly more radially extended than the old population. This is consistent with the results of Heyl (2007, 2008), and of course with the observational results of Davis et al. (2008b).

Davis et al. (2008b) observed the positions of the WDs in their sample in *projected* radius between ~ 3 and ~ 6 pc (1.7 and $3.5 r_h$). Figure 3 shows the cumulative projected radial distributions at ~ 12 Gyr between 12 and 24 pc (1.7 and $3.5 r_h$ in our model) of old and young WDs in the model with 4 km s^{-1} WD birth kicks. The young WDs are clearly more radially extended than the old population, in agreement with what Davis et al. (2008b) found in NGC 6397. Of course, while observers can only take snapshots of stellar positions at the current time, with numerical modeling we have the luxury of making many statistical realizations of a model. Figure 3 shows one of many possible realizations—some yield young WD populations that are even more radially extended, some less. In no case where we looked did we find one where the old population is statistically significantly more radially extended than the young population. We note that the average mass of the young WD population plotted in this figure is $0.55 M_{\odot}$ and that of the old population is $0.56 M_{\odot}$, confirming that mass segregation cannot explain the observed difference in radial distributions. For Figure 2 the average masses in the young and old WD populations are 0.558 and $0.563 M_{\odot}$, respectively, for the top panel and 0.55 and $0.57 M_{\odot}$ for the bottom panel.

In attempting to make quantitative conclusions about the WD kick speed, we are faced with the lack of detailed agreement between our model and NGC 6397. In the cursory mapping of parameter space we have done for this Letter (consisting of ~ 50 simulations), we have found that even very small kicks

of 2 km s^{-1} are sufficient to “puff up” the core noticeably. On the other hand, the projected radial distribution of WDs in the same field as the observations and with the same luminosity cuts is somewhat more sensitive to the kick speed—too low and the effect is not apparent, too high and too few WDs remain for meaningful statistical comparisons. Without getting mired in detailed modeling we simply note that, as seen in Figure 2, an extended radial distribution for the young WDs compared to the older WDs is a robust consequence of WD birth kicks. Furthermore, a kick speed of ~ 3 to 5 km s^{-1} , as inferred from observations by Davis et al. (2008b), is sufficient to prolong the initial evolutionary process of core contraction and result in a core radius at the current time that is significantly larger than that expected from simple binary burning.

3. DISCUSSION

Through realistic numerical simulations of globular cluster evolution, we have shown that WD birth kicks of a few km s^{-1} act as an energy source that can “puff up” a cluster’s core potentially significantly. In a cluster whose velocity dispersion is comparable to the WD kick speed, r_c/r_h can be increased by a factor of up to ~ 10 at late times relative to the no-kick case. Coincidentally, theoretical predictions of r_c/r_h for a cluster whose core is supported against collapse purely by inelastic scattering interactions of binaries (binary burning) have recently been shown to be up to a factor of ~ 10 smaller than observed for the bulk of Galactic globulars (e.g., Fregeau & Rasio 2007). WD kicks offer a natural resolution of this large discrepancy for clusters whose velocity dispersions are a few km s^{-1} , and obviate the need for alternate energy sources whose existence remains speculative, such as IMBHs.

Although the large core radii of some Galactic globulars appear to be naturally explained by central IMBHs (Trenti 2006), there is still only circumstantial evidence for their existence in clusters. The evidence that exists is indirect (from cluster velocity dispersion profiles), and for M15 and G1 can be explained equally well via standard cluster evolution models which contain a population of neutron stars or WDs in the core (Baumgardt et al. 2003a, 2003b). Furthermore, we should point out that the numerical models used to infer the existence of central IMBHs in Trenti (2006) contain just 20,000 particles, and have IMBHs that are a fraction $M_{\text{BH}}/M_{\text{clus}} = 0.02$ of the cluster mass. Several independent simulations have shown that the mass of the stellar progenitor to the IMBH that results from a runaway collision scenario (by far the most likely scenario for forming an IMBH in a cluster) is generically a fraction of $M_{\text{BH}}/M_{\text{clus}} = 0.002$ of the total cluster mass, independent of total cluster mass, initial central density, properties of the mass function, and the degree of initial mass segregation (Portegies Zwart & McMillan 2002; Gürkan et al. 2004; Freitag et al. 2006b, 2006a). The mass of the IMBH formed is likely to be less than the mass of its stellar progenitor, due to winds or other mechanisms (Yungelson et al. 2008). Adopting $M_{\text{BH}}/M_{\text{clus}} = 0.002$ as an upper limit, and applying the theoretical scaling $r_c/r_h \propto (M_{\text{BH}}/M_{\text{clus}})^{3/4}$ from Heggie et al. (2007) to the value $r_c/r_h \approx 0.3$ found in the simulations of Trenti (2006), the maximum core-to-half-mass radius ratio for a cluster whose core is powered by a central IMBH should thus be $r_c/r_h \approx 0.05$. This is appreciably smaller than the value $r_c/r_h \approx 0.3$ used by Trenti (2006) to infer the presence of IMBHs, and is consistent only with the $\sim 20\%$ of clusters with small r_c/r_h that are classified observationally as core collapsed.

We have also shown that the cluster radial distribution of WDs with birth kicks is consistent with the observations of Davis et al. (2008b). While this certainly supports the interpretation of kicks as the source of the radial distribution, it is far from a direct detection. More detailed observations are needed to confirm the existence of the kick. In fact, we will soon have relatively precise proper motion measurements for the bulk of the WDs in the NGC 6397 field, which, with the help of detailed simulations, will help pin down the properties of the WD kick. Based on our current understanding we can already make a few simple predictions, though. There is no reason to believe the magnitude of the WD kick depends on the properties of the host clusters in any significant way. We thus expect two measurable consequences. In clusters whose velocity dispersion is significantly larger than the putative kick speed of a few km s^{-1} (e.g., 47 Tuc), the radial distribution of the young WDs should follow or be more centrally concentrated than that of the old WDs. Observations of WD radial profiles in the high-velocity dispersion cluster Omega Cen ($\sim 20 \text{ km s}^{-1}$ in the core) are consistent with this picture (Calamida et al. 2008). In clusters whose velocity dispersion is less than a few km s^{-1} (e.g., NGC 288), there should be a measurable paucity of WDs. In fact, in the $\sim 1 \text{ km s}^{-1}$ velocity dispersion environments of open clusters, there is thought to be a dearth of WDs (Williams 2002).

Finally, we note that if the kick does occur during the extended asymptotic giant branch (eAGB) phase, there are at least two potential complications to our modeling. Stellar winds during the eAGB phase may have velocities of just $\sim 10 \text{ km s}^{-1}$ (Marshall et al. 2004). Since the velocity dispersion is comparable to this value (or just slightly smaller), there is the very real possibility that the stellar wind does not escape the cluster. For clusters whose velocity dispersion is comparable to the WD kick speed, the “heating” of the cluster via the kick is direct, and the retained wind mass should not alter our results appreciably. However, for clusters whose velocity dispersion is less than the kick speed, the “heating” is indirect (via cluster mass loss, since the WDs are escaping the cluster), and the retention of the wind mass by the cluster may appreciably reduce the “heating” effect. Another potential complication is that if the mass loss during the eAGB phase takes place on a timescale comparable to or longer than the typical stellar orbital timescale of $\sim 10^5 \text{ yr}$, the dynamical result is not that of a kick (as we have assumed here), but a weaker, adiabatic modification of the orbit. Mass loss rates of up to $10^{-4} M_{\odot} \text{ yr}^{-1}$ have been detected in AGB stars (implying a mass loss timescale $\lesssim 10^4 \text{ yr}$), but the general characteristics of mass loss in this phase are far from settled.

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