HYDRODYNAMICS OF BINARY COALESCENCE

Frederic A. Rasio

Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, USA

ABSTRACT. Hydrostatic equilibrium configurations for close binary systems can become *globally unstable*. Instabilities arise from the strong tidal interaction between the two components, which tends to make the effective two-body potential steeper than $1/r$. As a result, a circular orbit can become unstable to small radial perturbations. The instability can be either secular or dynamical. In both cases it leads to the coalescence and merging of the two components on a time scale much shorter than the typical lifetime of a stable system. In a secularly unstable system, orbital decay is driven by viscous dissipation and proceeds on a time scale comparable to the synchronization time of a stable binary. In a dynamically unstable system, the two stars suddenly plunge toward each other and merge hydrodynamically in just a few orbital periods.

Introduction

The coalescence and merging of two stars into a single object is the almost inevitable endpoint of close binary evolution. Loss of angular momentum through a variety of dissipation mechanisms always leads to orbital decay. It was only recently realized that the late stages of this orbital decay can often become hydrodynamic in nature, with the final merging of the two stars occurring on a time scale much shorter than the angular-momentum loss time scale (Rasio & Shapiro 1992, 1994, hereafter RS; Lai, Rasio, & Shapiro 1993a,b, 1994,a,b, hereafter LRS). This is because *global instabilities* can drive the binary system to rapid coalescence once the tidal interaction between the two components becomes sufficiently strong. When the binary system becomes unstable, rapid orbital decay is inevitable, even in the absence of an effective angular-momentum loss mechanism. This orbital decay of unstable systems can be either *dynamical*, with the two stars coalescing in just a few orbital periods, or *secular*, i.e., driven by internal viscous dissipation in the fluid. In this latter case, the orbit decays by continuously transferring angular momentum to an internal shear flow, while the system attempts unsuccessfully to maintain uniform fluid rotation.

Binary coalescence has been associated with a number of astrophysical phenomena of great current interest. Close neutron-star binaries are most important sources of gravitational radiation in the Universe, and are the primary targets for the LIGO project (Abramovici et al. 1992). The coalescence of two neutron stars is at the basis of numerous models of γ-ray bursters (see Narayan, Paczyński, & Piran 1992 and references therein; Colpi & Rasio, these Proceedings). Double white-dwarf systems are now generally thought to be the progenitors of Type Ia supernovae (Iben & Tutukov 1984; Yungelson et al. 1994). They are also promising sources of low-frequency gravitational waves that should be easily detectable by future space-based interferometers (Evans, Iben, & Smarr 1987). In addition to producing supernovae, the coalescence of two white dwarfs may also lead in certain cases to the formation by gravitational collapse of an isolated millisecond pulsar (Chen & Leonard 1993) or the formation of blue subdwarf stars in globular clusters (Bailyn 1993). In the case of coalescing magnetized white dwarfs, a neutron star with extremely high magnetic field may form, and such an object has also been proposed as a source of γ-ray bursts (Usov 1992). Coalescing main-sequence star binaries are likely progenitors for the numerous blue stragglers found in stellar clusters (Mateo et al. 1990), and are observed directly as Algol and W UMa systems (Rucinski 1992, and these Proceedings).

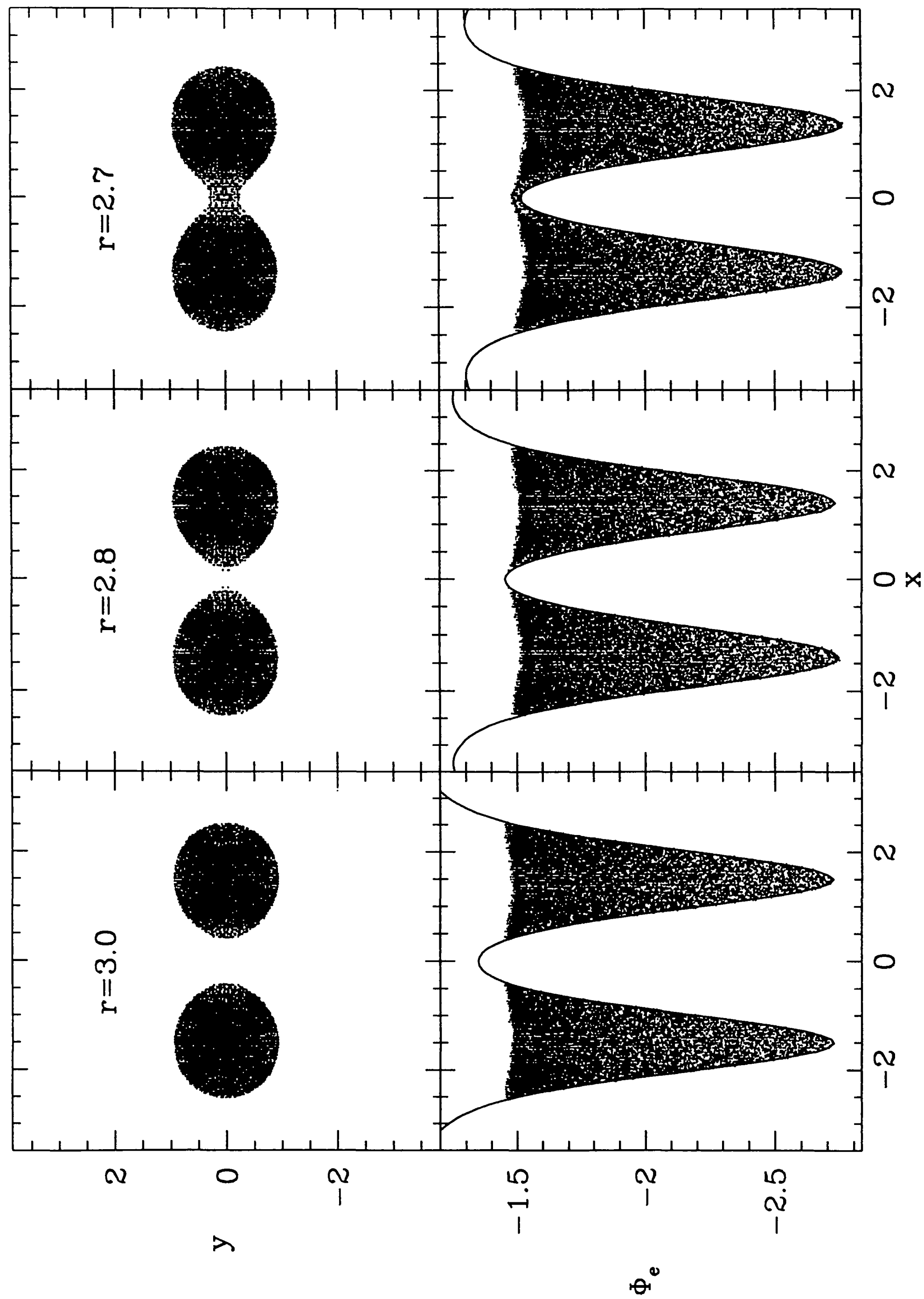
Dynamical Instabilities

When the two components of a binary system are sufficiently close, the strong tidal interaction can make the effective two-body potential governing the orbital motion deviate quite significantly from a Keplerian $1/r$ potential. Since the effective potential becomes *steeper* than $1/r$, a circular orbit can become *dynamically unstable* (see, e.g., Goldstein 1980, Chap. 2), just like general relativity can make circular orbits become unstable sufficiently close to a black hole (see, e.g., Shapiro & Teukolsky 1983, §12.4). The existence of this dynamical instability in close binary configurations containing an *incompressible fluid* has been known for a long time (Chandrasekhar 1969, 1975; Tassoul 1975). The recent work of RS and LRS shows that it extends quite far into the compressible regime, so that it can affect real stars made of compressible fluid. Self-gravity is of course essential here, and the usual Roche model (which treats close binaries as made of massless gas in the effective potential of two point masses) cannot be used. Instead, three-dimensional hydrodynamics must be used, which until very recently was beyond the capabilities of most computers.

For an incompressible fluid, the onset of instability corresponds to a binary configuration which is still slightly detached (Tassoul 1975). The same is true for binary neutron stars with stiff equations of state (RS). However, for most normal stars with centrally concentrated mass profiles, significant deviations from Keplerian behavior occur only when the outer layers of the two stars are overlapping, i.e., for contact configurations. As an example, consider a sequence of equilibrium binary configurations for two identical polytropes with $\Gamma = 5/3$ (Fig. 1). These could be simple models for two low-mass white dwarfs or main-sequence stars (although in reality these systems are unlikely to have a mass ratio close to unity; Similar polytropic configurations with higher values of Γ have been considered by RS for modeling neutron star binaries, where the observed mass ratios are very close to 1). The dynamical stability of these equilibrium models can be tested directly by integrating numerically on a supercomputer the hydrodynamical equations in three dimensions. The results of such integrations, using the smoothed particle hydrodynamics (SPH) method (see Monaghan 1992 for a recent review), are shown in Figures 2–4.

As predicted by the linear perturbation calculations of LRS, binary systems with sufficiently close components are found to be dynamically unstable (Fig. 2). Numerically, one can continue the integrations well into the nonlinear regime and determine the final (stable) equilibrium configuration of the merger product (Figs. 3 and 4). It is tempting to envision (cf. Rasio 1993) that the secular evolution of a contact main-sequence star binary through loss of angular momentum (by gravitational radiation or via magnetized winds), possibly accelerated by viscous dissipation (see below), could bring the two components sufficiently close together to make the system unstable. The object depicted in Figure 4 could then be a young blue straggler. For two white dwarfs, the merger product may well be above the Chandrasekhar mass, and should therefore explode as a (Type Ia) supernova, or perhaps collapse to a neutron star. The rapid rotation (cf. Fig. 4) and possibly high mass (up to $2M_{Ch}$) of the object must be taken into account for determining its final fate. This has not been done in recent theoretical calculations (Nomoto 1987; Isern, these Proceedings), where a quasi-spherical (nonrotating) white dwarf just below the Chandrasekhar limit is assumed to accrete slowly from a disk.

Figure 1. (following two pages) Sequence of corotating binary configurations with decreasing separation r, for two identical polytropes with $\Gamma = 5/3$. Exact solutions of the hydrostatic equilibrium equation were calculated numerically in three dimensions using the smoothed particle hydrodynamics (SPH) method. On top, projections of all SPH particles into the orbital (x, y) plane are shown. At the bottom, the effective potential (exact gravitational potential of the fluid plus centrifugal potential) $\Phi_e(x, y = 0, z = 0)$ along the binary axis is shown (solid lines), together with the positions of all SPH particles in the (x, Φ_e) space. Units are such that $G = M = R = 1$, where M and R are the mass and radius of one unperturbed (spherical) star. Contact configurations are obtained for $r/R \lesssim 2.8$. Configurations with $r/R \lesssim 2.7$ are secularly unstable; they become dynamically unstable for $r/R \lesssim 2.5$. For $r/R \lesssim 2.3$, the fluid overflows through the outer Lagrangian points and hydrostatic equilibrium solutions no longer exist.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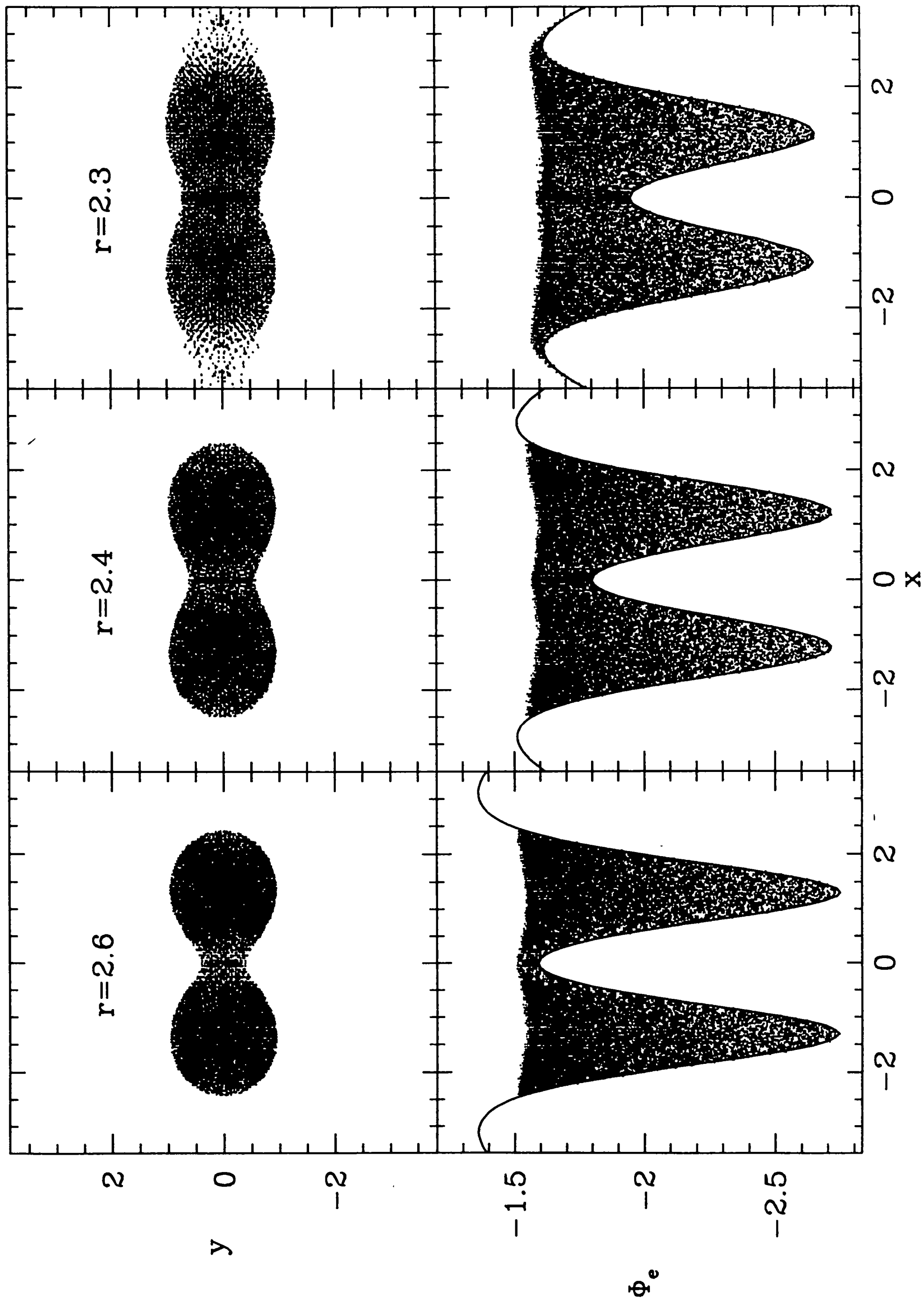
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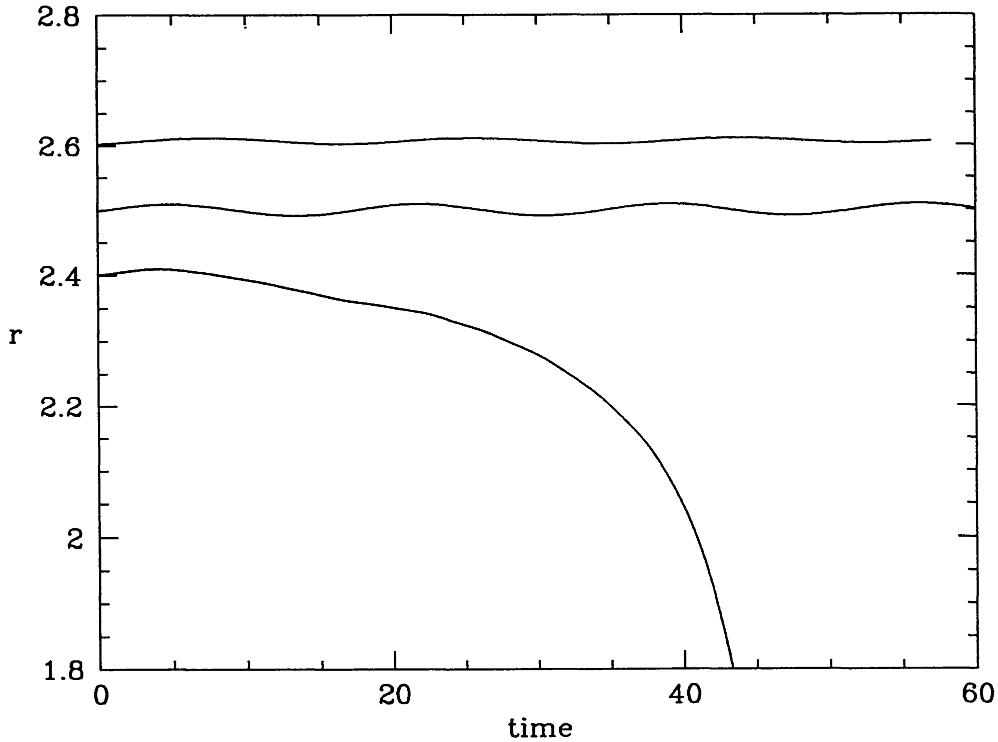
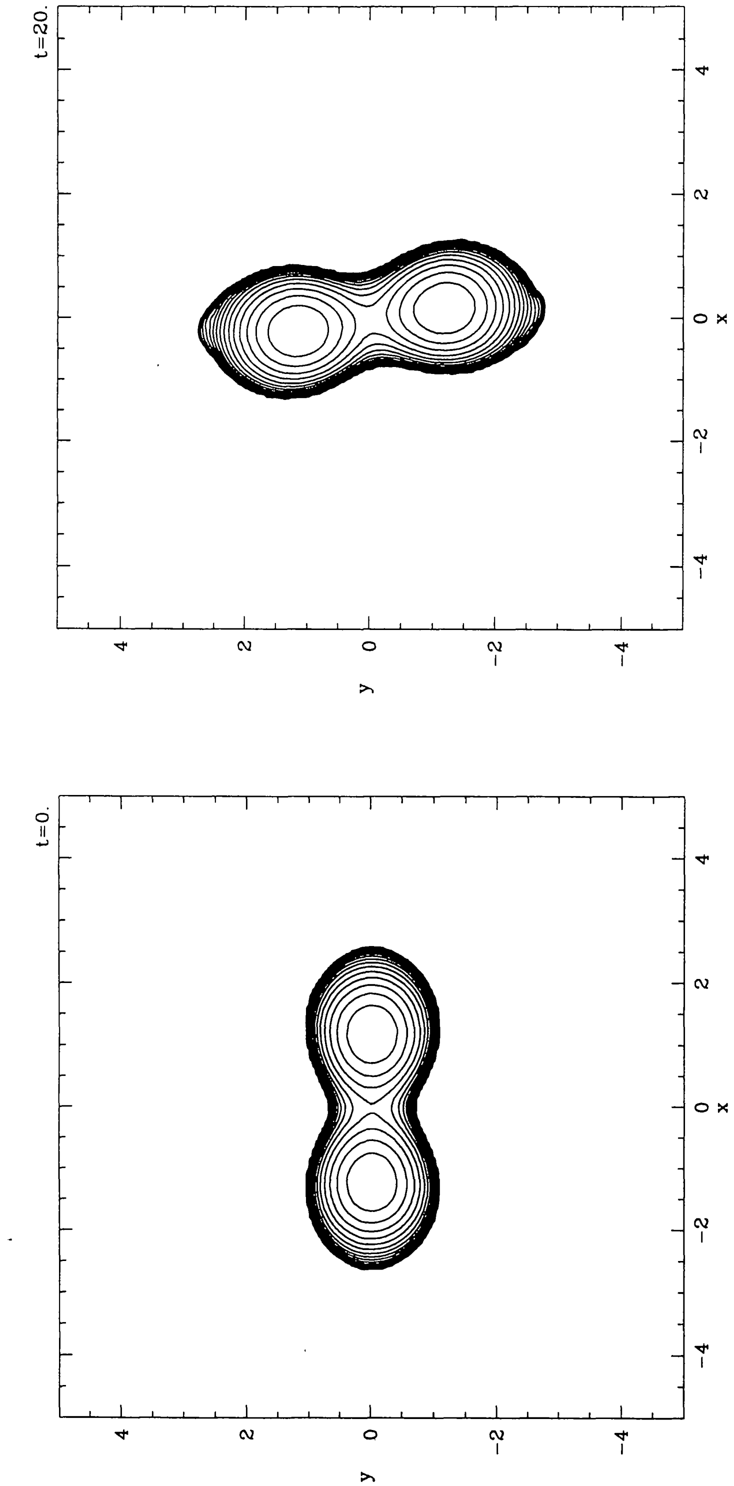


Figure 2. (above) Time evolution of the binary separation during hydrodynamical integrations starting from several of the equilibrium models shown in Fig. 1. Time is in units of $(R^3/GM)^{1/2}$, which gives an orbital period $P_{orb} \approx 20$ for all models. The binaries with initial $r/R = 2.5$ and 2.6 are clearly stable: the separation exhibits only very small-amplitude epicyclic oscillations (excited by small numerical deviations from strict equilibrium in the initial conditions). The equilibrium solution with initial $r/R = 2.4$, however, is dynamically unstable: the two components spiral in rapidly, on a time scale comparable to P_{orb}.

Figure 3. (following three pages) Dynamical evolution of the unstable contact binary with $r/R = 2.4$ shown in Figs. 1 and 2. Contours of equal density in the orbital plane are shown at various times. The scale is logarithmic, with 4 contours per decade covering 4 decades down from the maximum. The orbital rotation is counterclockwise. Between $t = 0$ and $t \approx 20$, the two stars approach each other quasi-statically, sliding down the end of the sequence shown in Fig. 1 in about one orbital period. At $t \approx 20$, the mass-shedding limit for binary configurations is reached: mass and angular momentum must flow out of the central region through spiral arms. As they grow in length, the spiral arms also widen and, eventually, they merge together to form a smooth outer disk ($t \approx 60$). The redistribution of mass and angular momentum in the system leads to an axisymmetric structure in stable hydrostatic equilibrium at the end of the calculation.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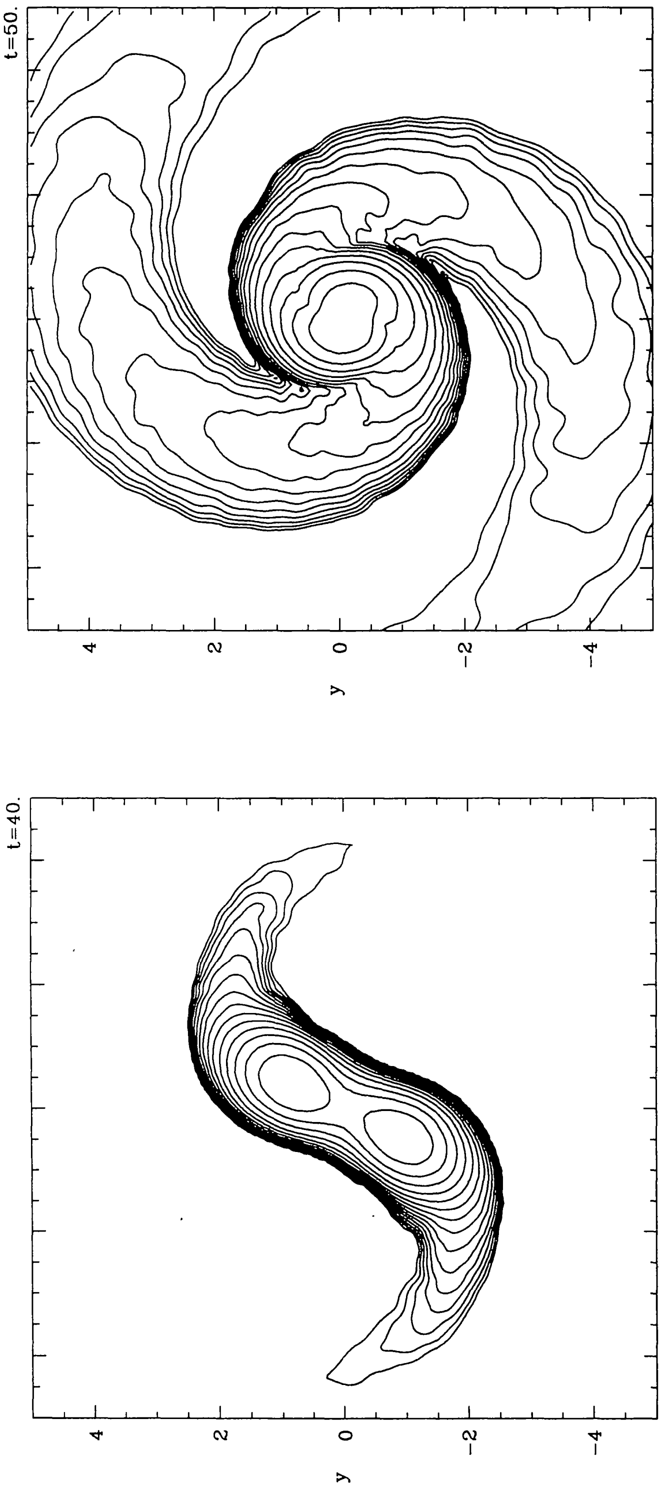
−4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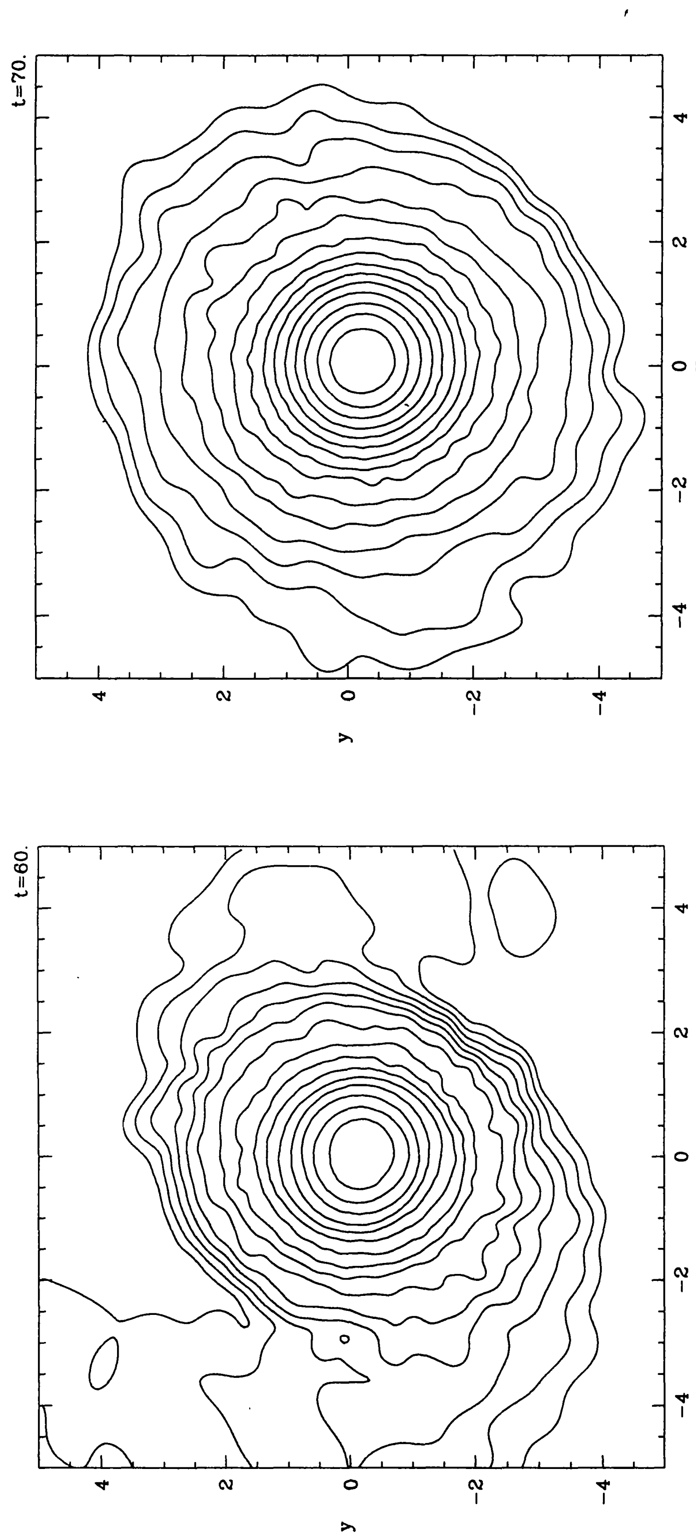
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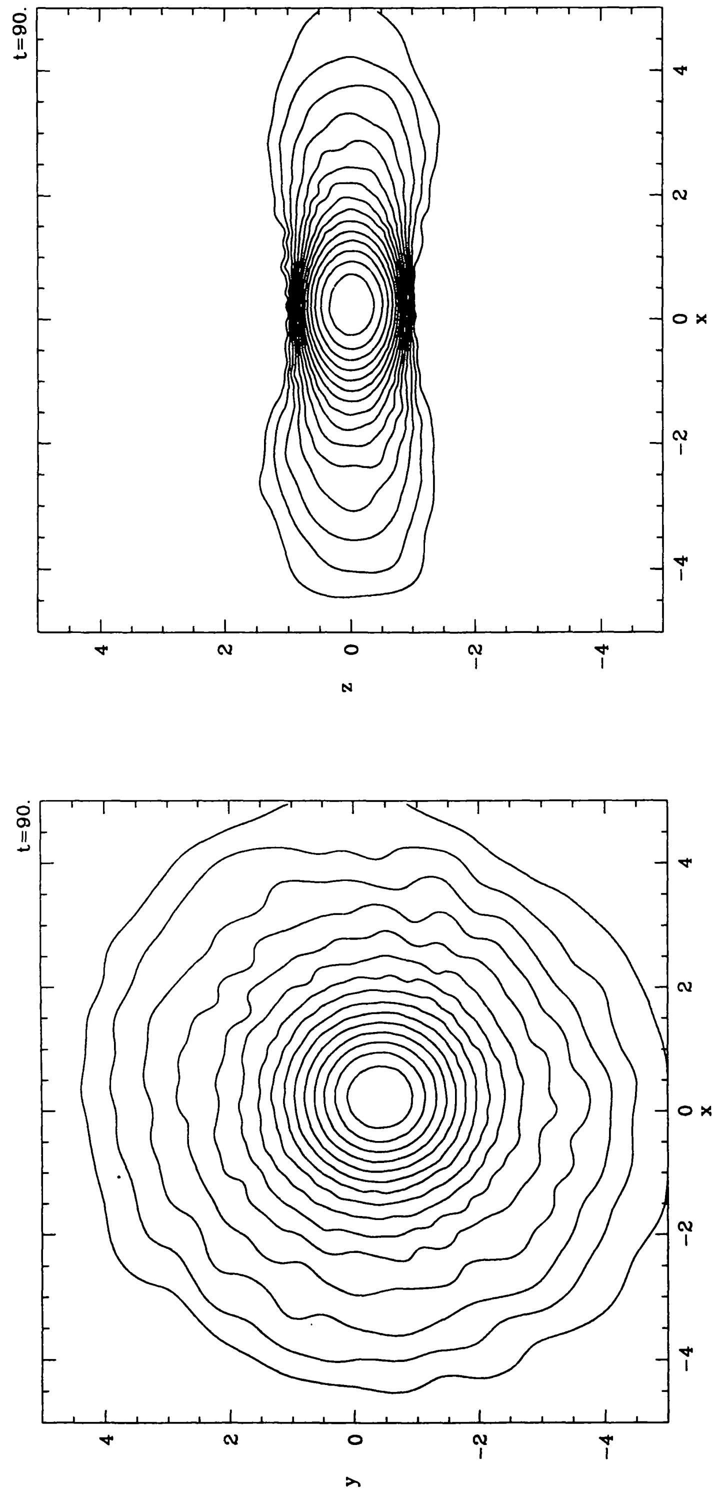

t=90.
x
z
t=90.
x
y

Figure 4. (previous page) Final merged configuration resulting from the dynamical evolution shown in Fig. 3. Density contours in the orbital (x, y) plane and the meridional (x, z) plane are shown. Conventions are as in Fig. 3. The object is rapidly rotating and highly flattened. The rotation is nearly uniform in the central core (containing about 80% of the mass), but very differential in the outer disk. The ratio of kinetic energy of rotation to gravitational binding energy is $T/|W| \approx 0.12$.

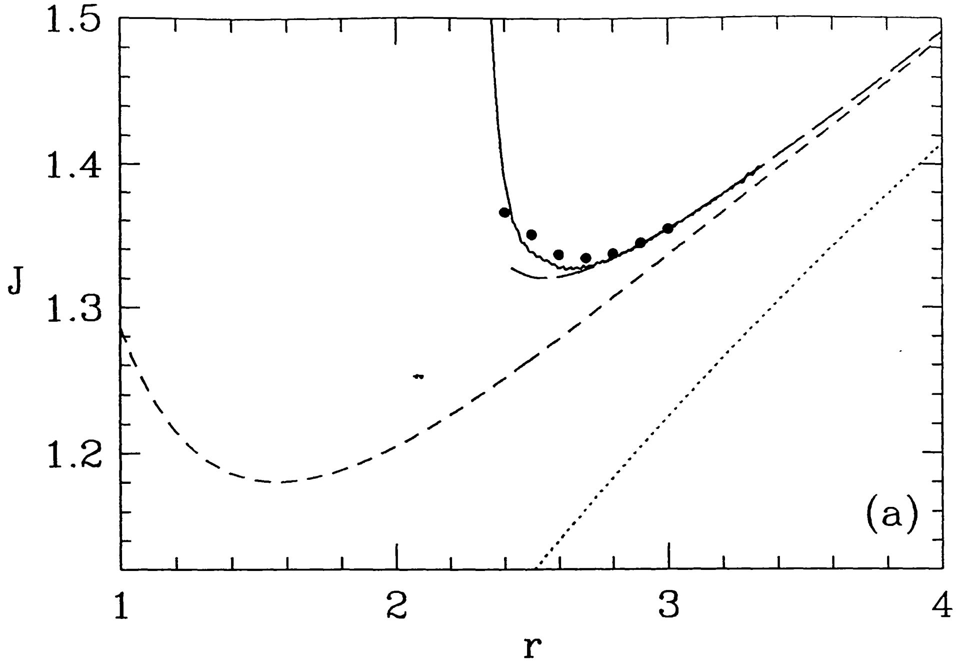
Secular Instabilities

Well before a close binary system becomes dynamically unstable (and even if it never does), another type of global instability can affect its evolution. It has been referred to by various names, such as secular instability (LRS), tidal instability (Counselman 1973; Hut 1980), and Darwin instability (Levine et al. 1993). Its physical origin is very easy to understand (Fig. 5). There exists a *minimum* value of the total angular momentum for a *synchronized* close binary. This is simply because the spin angular momentum, which *increases* as r decreases, can become comparable to the orbital angular momentum for sufficiently small r. A system that reaches the minimum of J cannot evolve further by angular momentum loss and remain synchronized. Instead, the combined action of tidal forces and viscous dissipation will drive the system *out* of synchronization and cause rapid orbital decay as angular momentum is continually transferred from the orbit to the spins.

The orbital decay of a secularly unstable binary proceeds on the (de)synchronization time scale, which is typically much shorter than the time scale associated with any angular-momentum loss mechanism (LRS). For example, for two low-mass main-sequence stars with large convective envelopes, this time scale could be as short as 10^4 yr (Zahn 1977). This may explain why the components of W UMa binaries are always observed to be in very shallow contact (Rucinski 1992): If the two components ever got closer together because of mass exchange or loss of angular momentum, the system would become secularly unstable and the two stars would quickly coalesce. The exact location of the secular stability limit (and the corresponding maximum value of the degree of contact for a stable, long-lived system) should depend sensitively on the internal structure of the binary, and, in particular, the distribution of specific entropy in the system. Thus a large sample of well-determined contact-binary parameters could be used to place interesting constraints on their internal structure.

Secular instabilities can also be important for the orbital evolution of close binaries containing a compact object in orbit around a more massive, extended star, as in high-mass X-ray binaries (Levine et al. 1993). In this case the Roche limit configuration can be secularly unstable (LRS), and it is possible that both mass transfer and orbital decay could be driven purely by internal viscous dissipation and tidal effects, rather than by stellar evolution or angular momentum loss.

Figure 5. (following page) Variation of the total angular momentum J as a function of binary separation r along the equilibrium sequence considered previously (Fig. 1). Here J is in units of $(GM^3R)^{1/2}$ and r is in units of the stellar radius R as before. The solid dots are the fully numerical results corresponding to the models of Fig. 1; the solid curves are from a simplified numerical calculation (RS). The long-dashed curves are from the quasi-analytic models of LRS, representing the stars as compressible ellipsoids; the short-dashed lines are for two rigid spheres, and the dotted line is for two point masses in a Keplerian orbit. The minimum of J at $r/R \approx 2.7$ gives the secular stability limit for synchronized configurations. In (b), a blow-up of the region around this minimum is shown. Binary systems sliding down the equilibrium $J(r)$ curve as they lose angular momentum (e.g., through gravitational radiation) must eventually attain this minimum. At that point, internal viscous dissipation will take over and drive the subsequent orbital decay on a much shorter time scale. The system can then minimize its energy (at constant angular momentum) by evolving *out* of synchronization as the orbit decays. The two stars ultimately merge, either dynamically (Fig. 3) if a dynamical instability is encountered at smaller r before the onset of mass shedding (which would be the case for the two polytropes considered here), or quasi-statically otherwise. Note that the simple two-sphere model (considered in all previous studies of the secular instability; see, e.g., Counselman 1973, Hut 1980) can give largely inaccurate results for close binaries: it predicts the minimum of J at $r/R \approx 1.6$ in this case (short-dashed line), far below the exact value determined numerically.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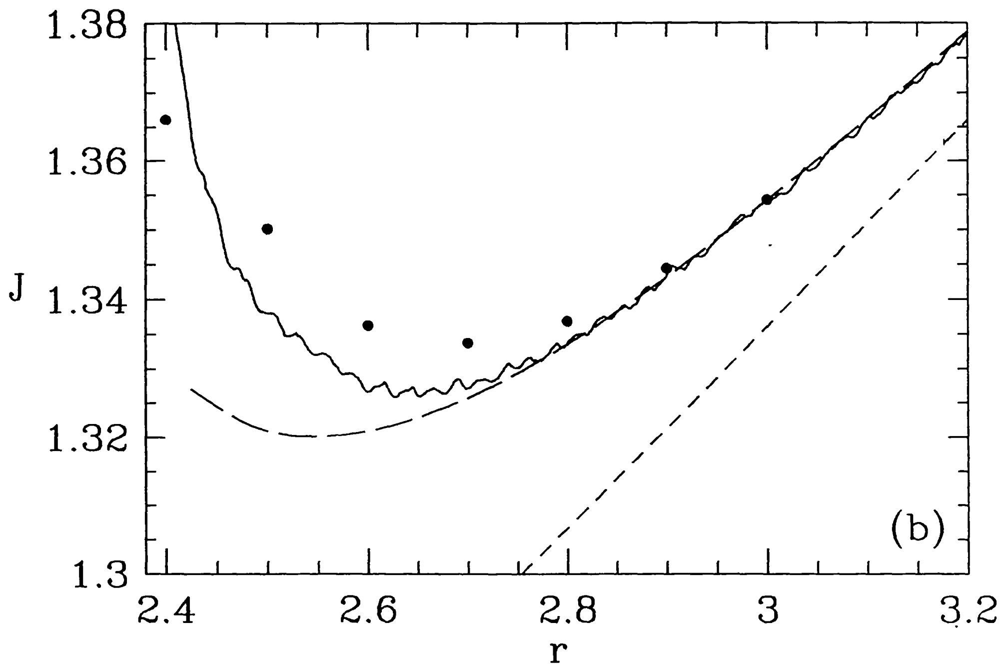
1.4
1.3
1.2
J
(a)
1
2
3
4
r


1.38
1.36
1.34
1.32
1.3
J
(b)
2.4
2.6
2.8
3
3.2
r

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