HYDRODYNAMICAL INSTABILITY AND ORBITAL EVOLUTION OF CLOSE BINARY SYSTEMS

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1. INTRODUCTION

Essentially all recent theoretical work on close binary systems has been done in the Roche approximation, where the components are modeled as non self-gravitating gas in hydrostatic equilibrium in the effective potential of a point-mass system (e.g., Paczyński 1971). This model applies well to very compressible objects with centrally concentrated mass profiles, such as giants and early-type main-sequence stars. Some theoretical work has also been done in the completely opposite limit of binaries containing a self-gravitating, incompressible fluid (Chandrasekhar 1969). However, many binary systems of astrophysical interest contain stars that are neither very centrally concentrated nor homogeneous. In particular, low-mass white dwarf and main-sequence stars have effective polytropic indices $n \simeq 1.5$, and neutron stars typically have $n \sim 0.5 - 1$.

In our recent papers (Lai, Rasio & Shapiro 1993a,d), we have presented a comprehensive analytic study of the equilibrium and stability properties of close binary systems containing polytropic components. In addition to providing compressible generalizations for all the classical incompressible configurations as discussed in Chandrasekhar (1969), our method can also be applied to more general binary models where the stellar masses, radii, spins, entropies, and polytropic indices are all allowed to vary over a wide range and independently for each component. As a result, a variety of dynamical behaviors for various types of binary systems can be identified. Most importantly, we find that for sufficiently incompressible systems, both secular and dynamical instabilities can develop before a Roche limit or contact is reached along a sequence of models with decreasing binary separation. These instabilities result from Newtonian tidal interactions between equilibrium stars.

The development of a dynamical instability can have a profound effect on the terminal evolution of coalescing binaries (Lai, Rasio & Shapiro 1993b,c). In particular, it causes binary neutron stars whose orbits decay via gravitational wave emission to undergo rapid merging just prior to contact. The final coalescence can take place on a timescale much shorter than the energy dissipation time scale. For the coalescence of binary neutron stars, the radial infall velocity at contact is comparable to the free-fall velocity. As a result, the imploding stars will experience appreciable shock heating as they come into contact. Some high-mass X-ray binaries are expected to eventually evolve to compact binaries containing neutron stars or white dwarfs (van den Heuvel 1991). Our results are therefore important to determining the final evolution of such systems.

2. COMPRESSIBLE DARWIN-RIEMANN BINARY MODELS

A binary system in steady state is characterized by conserved global quantities such as masses M, M' for the two components, the fluid circulations C, C', and total angular momentum J. The total energy the system E can always be written as a functional of the fluid density and velocity fields $\rho(\mathbf{x})$ and $\mathbf{v}(\mathbf{x})$. In principle, an equilibrium configuration can be determined by extremizing this energy functional with respect to all variations of $\rho(\mathbf{x})$ and $\mathbf{v}(\mathbf{x})$ that leave the conserved quantities unchanged. The essence of our method is to replace the infinite number of degrees of freedom contained in $\rho(\mathbf{x})$ and $\mathbf{v}(\mathbf{x})$ by a limited number of parameters $\alpha_1, \alpha_2, \ldots$, in such a way that the total energy becomes a function of these parameters,

$$E = E(\alpha_1, \alpha_2, \ldots; M, C, J, \cdots). \tag{1}$$

An equilibrium configuration is then determined by extremizing the energy according to

$$\frac{\partial E}{\partial \alpha_i} = 0, \quad i = 1, 2, \ldots \tag{2}$$

where the partial derivatives are taken holding M, $J, \cdots$ constant.

Under the combined effects of centrifugal and tidal forces, the stars in a binary assume nonspherical shapes, which we model as ellipsoids. Moreover, we assume that the surfaces of constant density within each star are self-similar ellipsoids, and the density profile $\rho(m)$, where m is the mass interior to an isodensity surface, is identical to that of a spherical polytrope of the same mass. The independent variables $\{\alpha_i\}$ which specify the structure of our binary models are the binary separation r and the three axes of the two ellipsoids a_i, a_i' ($i = 1, 2, 3$). The velocity field of the fluid is modeled as either uniform rotation or uniform vorticity, with the spin axis perpendicular to the orbital plane. Thus both synchronized and nonsynchronized systems are considered.

3. STABILITY LIMITS AND ROCHE LIMIT

When viscosity is negligible, the fluid circulations of the stars are individually conserved. Figure 1 illustrates three different dynamical behaviors for equilibrium binary sequences with constant circulations. Such sequences are especially relevant for binary systems whose orbits decay due to gravitational radiation. This is because the gravitational radiation reaction forces conserve the fluid circulation (Miller 1974).

The three types of behaviors are (cf. Fig. 1):

(a) For sufficiently compressible systems (large n), the stars behave like two point masses. The energy E decreases monotonically as r decreases, and stable equilibrium solution exists all the way to contact.

(b) For more incompressible systems (smaller n), tidal interaction plays an important role in determining the binary equilibrium. As r decreases, E reaches a minimum before contact. Such a turning point in the equilibrium energy curve exactly coincides with the point of onset of dynamical instability. Beyond this stability limit, all equilibrium solutions become unstable. The physical nature of this instability is common to all binary interaction potentials that are sufficiently steeper than $1/r$. It is analogous to the familiar instability at $r = 6M$ of circular orbits for test particles around a Schwarzschild black hole. Here, however, it is the purely Newtonian tidal effects that are responsible for the steepening of the effective binary interaction potential and for the destabilization of the circular orbit.

(c) When the masses of the two components are different, the binary can encounter a Roche limit before contact. The Roche limit is the point where the binary separation has a minimum value below which no equilibrium solution exists. Typically, both the stability limit and Roche limit occur around orbital separation $r_m \sim 3(M'/M)^{1/3} R$, where R is the stellar radius. But note that the dynamical stability limit is always encountered prior to the Roche limit.

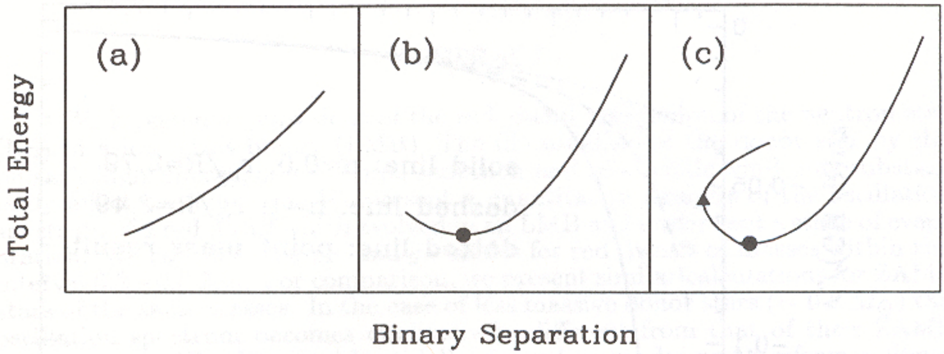

Fig. 1. General classification of equilibrium binary sequences according to terminal configurations and stability limit. The existence and ordering of the stability limit (round dots) and Roche limit (triangle) is shown schematically along equilibrium energy curves $E(r)$. All curves terminate at the point when the two stars contact.

In the opposite limit, when the effective viscosity is sufficiently large to maintain synchronization of spins and orbital motion, corotating sequences are relevant. Depending on the polytropic indices and the mass ratio, different types of equilibrium behaviors similar to Figure 1 can also be identified. However, for a corotating sequence, the minimum in the energy curve corresponds to the secular stability limit, while the true dynamical instability occurs at somewhat smaller orbital separation along the sequence.

4. ORBITAL EVOLUTION AND BINARY COALESCENCE

The importance of the dynamical instability can be easily seen. When the energy loss timescale of the system is much longer than the dynamical timescale, the orbital evolution is quasi-static. The rate of change of the orbital separation is given by $\dot{r} = \dot{E}/(dE/dr)$. As the binary approaches the dynamical stability limit r_m, where $dE/dr \to 0$, we have $\dot{r} \to \infty$. Clearly, the quasi-static description is not valid near r_m.

Figure 2 shows the results of our dynamical calculation for the terminal evolution of binary neutron stars due to gravitational radiation. The development of a dynamical instability causes a rapid acceleration of the coalescence, and the radial infall velocity at contact can be a significant fraction ($\sim 10\%$) of the tangential orbital velocity.

The effects of viscosity can also be considered. Since viscous forces conserve the total angular momentum, a binary system evolving through viscosity only will follow a sequence of configurations with constant J. Such viscous evolution may be responsible for the orbital period changes detected in some high-mass X-ray binaries such as Cen X-3, SMC X-1, LMC X-4 (Levine et al 1993). The orbital evolution of the binary system depends on its initial angular momentum J_i. The binary either evolves toward a stable synchronized state, or is driven to coalescence by viscous dissipation. Again, we find that for sufficiently incompressible systems, the binary can encounter a dynamical instability before the final merger (Lai, Rasio & Shapiro 1993d).

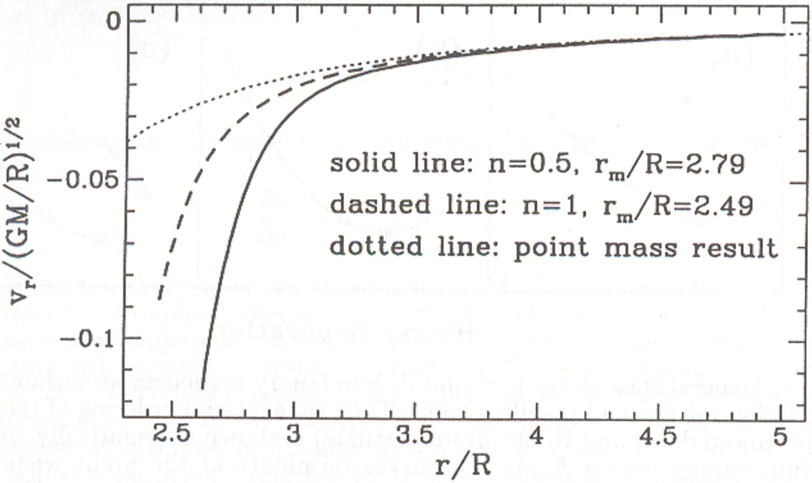

Fig. 2. The infall radial velocity of a coalescing neutron star binary. Results for binary models with different polytropic indices n are shown. The two neutron stars are assumed to be identical, with mass $M = 1.4 M_\odot$, radius $R = 10$ km, and both have zero spin. Here r_m is the dynamical stability limit.

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