# Newtonian and Post-Newtonian Calculations of Coalescing Compact Binaries

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**Abstract**

Coalescing binary neutron stars are important sources of gravitational waves that should become detectable with the laser interferometers now being built as part of LIGO, VIRGO and GEO. Post-Newtonian (PN) approximation methods have been used to calculate waveform templates in the low-frequency, slow-inspiral phase of the binary evolution. These theoretical templates can be used to extract parameters such as the neutron star (NS) masses and spins. In the slow-inspiral phase the two stars are still well separated and can be treated essentially as point masses. Near the end of the coalescence, however, hydrodynamic effects and the interior structure of the stars play an increasingly important role. Hydrodynamics becomes dominant when the two stars finally merge together into a single object. The shape of the corresponding burst of gravitational waves provides a direct probe into the interior structure of a NS and the nuclear equation of state (EOS). The interpretation of the gravitational waveform data will require detailed theoretical models of the complicated 3D hydrodynamic processes involved. This review summarizes recent work on the hydrodynamic aspects of NS binary coalescence. Newtonian and, more recently, relativistic calculations have been performed. The methods include both approximate quasi-analytic techniques and large-scale numerical hydrodynamics calculations on supercomputers. Also included here is a brief discussion of coalescing white dwarf (WD) binaries, which are important sources of very low-frequency gravitational waves, potentially detectable by LISA.

## 1 Introduction

The coalescence and merging of two stars into a single object is the almost inevitable endpoint of compact binary evolution. Dissipation mechanisms such as friction in common envelopes and the emission of gravitational radiation are always present and cause the binary orbit to decay. The terminal stage of this orbital decay is generally hydrodynamic in nature, with the final merging of the two stars taking place on a time scale comparable to the orbital period. In addition to the angular momentum loss to gravitational radiation, *global hydrodynamic instabilities* can drive the binary system to rapid coalescence once the tidal interaction between the two stars becomes sufficiently strong (Rasio and Shapiro 1992–1997, Lai et al. 1993a,b, 1994a,b,c, Lai and Shapiro 1995, New and Tohline 1997). The existence of these global instabilities for binary systems containing a compressible fluid was demonstrated for the first time in Rasio and Shapiro (1992) using numerical hydrodynamic calculations. In addition, the classical *analytic* work for binaries containing an *incompressible* fluid (Chandrasekhar 1969) was extended to compressible fluids in the work of Lai et al. This new analytic study confirmed the existence of dynamical and secular instabilities for sufficiently close binary systems containing polytropes.

Although the simplified analytic studies have given us much physical insight into difficult questions of global fluid instabilities, fully numerical calculations remain essential for establishing the stability limits of close binaries accurately and for following the nonlinear evolution of unstable systems all the way to complete coalescence. Given the absence of any underlying symmetry in the problem, these calculations must be done in 3D and therefore require supercomputers. A number of different groups have now performed such calculations, using a variety of numerical methods and focusing on different aspects of the problem. Nakamura and collaborators [see Nakamura (1994) and references therein] were the first to perform 3D hydrodynamic calculations of binary NS coalescence, using a traditional Eulerian finite-difference code. Rasio and Shapiro have been using the Lagrangian method SPH (Smoothed Particle Hydrodynamics) and have focused on determining the stability properties of initial binary models in strict hydrostatic equilibrium and calculating the emission of gravitational waves from the coalescence of unstable binaries. Many of the results of Rasio and Shapiro have now been independently confirmed in the work of New and Tohline (1997), who used completely different numerical methods but also focused on stability questions. Zhuge et al. (1994) have also used SPH and studied the dependence of the gravitational waveforms on the initial NS spins. Davies et al. (1994) and Ruffert et al. (1996, 1997) have incorporated a treatment of the nuclear physics in their hydrodynamic calculations (done using SPH and PPM codes, respectively) but discussed their results primarily in the context of gamma-ray burst models.

For compact binaries, relativistic effects combine nonlinearly with Newtonian tidal effects so that close binary configurations can become dynamically unstable earlier during the spiral-in phase (i.e., at larger binary separation and lower orbital frequency) than predicted by Newtonian hydrodynamics alone. The combined effects of relativity and hydrodynamics on the stability of close compact binaries have only very recently begun to be studied. Preliminary results have been obtained using both analytic approximations [basically, PN generalizations of Lai et al.; see Lai (1996), Taniguchi and Nakamura (1996), Lai and Wiseman (1997), Lombardi et al. (1997)] as well as numerical hydrodynamics calculations in 3D incorporating simplified treatments of relativistic effects (Wilson and Mathews 1995, Shibata 1996, Baumgarte et al. 1997, Mathews and Wilson 1997). A NASA Grand Challenge project is under way (E. Seidel in this volume) that will ultimately attempt a fully relativistic calculation of the final coalescence, combining the techniques of numerical relativity and numerical hydrodynamics in 3D.

This review will concentrate on the coalescence of compact binaries, containing either two NS (§2) or two WD (§3). Although relativity plays a less important role during the final merging of two WD, the very low-frequency gravitational waves emitted during the inspiral could be easily detected by space-based laser interferometers such as those planned for the LISA project (see the article by K. Danzmann in this volume). Many of the results obtained for WD binaries are also relevant to low-mass main-sequence stars in contact binaries and the important related problem of blue straggler formation in star clusters (Rasio and Shapiro 1995, Rasio 1995, Lombardi et al. 1996).

# 2 Coalescing Neutron Star Binaries

## 2.1 Astrophysical Motivation

Coalescing compact binaries are the most promising known sources of gravitational radiation that could be detected by the new generation of laser interferometers: the Caltech-MIT LIGO (Abramovici et al. 1992, Cutler et al. 1993) and the European projects VIRGO (Bradaschia et al. 1990) and GEO (Danzmann, this volume). In addition to providing a major new confirmation of Einstein's theory of general relativity, including the first direct proof of the existence of black holes (Flanagan and Hughes 1997, Lipunov et al. 1997), the detection of gravitational waves from coalescing binaries at cosmological distances could provide accurate measurements of the Hubble constant and mean density of the Universe (Schutz 1986, Chernoff and Finn 1993, Marković 1993). For recent reviews on the detection and sources of gravitational radiation, see Thorne (1995, 1996).

Expected rates of binary coalescence in the Universe, as well as expected event rates in forthcoming laser interferometers, have now been calculated by many groups. Although there is some disparity between various published results, the estimated rates are generally encouraging. Statistical arguments based on the observed local population of binary radio pulsars with probable NS companions lead to an estimate of the rate of NS binary coalescence in the Universe of order $10^{-7}\,\mathrm{yr}^{-1}\,\mathrm{Mpc}^{-3}$ (Narayan et al. 1991, Phinney 1991). Using this estimate, Finn and Chernoff (1993) predict that an advanced LIGO detector could observe as many as 70 events per year. These numbers are based on a Galactic merger rate $R \simeq 10^{-6}\,\mathrm{yr}^{-1}$ derived from radio pulsar surveys. More recently, however, van den Heuvel and Lorimer (1996) revised this number to $R \simeq 0.8 \times 10^{-5}\,\mathrm{yr}^{-1}$, using the latest galactic pulsar population model of Curran and Lorimer (1995). This value is consistent with the upper limit of $10^{-5}\,\mathrm{yr}^{-1}$ for the Galactic binary NS birth rate derived by Bailes (1996) on the basis of very general statistical considerations about pulsars. In addition, theoretical models of the binary star population in our Galaxy also suggest that the NS binary coalescence rate may be as high as $\gtrsim 10^{-6}\,\mathrm{yr}^{-1}\,\mathrm{Mpc}^{-3}$ [Tutukov and Yungelson (1993), see also the more recent studies by Portegies Zwart and Spreeuw (1996) and Lipunov et al. (1997)].

Most recent calculations of the gravitational radiation waveforms from coalescing binaries have focused on the signal emitted during the last few thousand orbits, as the frequency sweeps upward from about 10 Hz to 1000 Hz. The waveforms in this regime can be calculated fairly accurately by performing high-order PN expansions of the equations of motion for two *point masses* (Lincoln and Will 1990, Junker and Schäfer 1992, Kidder et al. 1992, Wiseman 1993, Will 1994, Blanchet et al. 1996). High accuracy is essential here because the observed signals will be matched against theoretical templates. Since the templates must cover $\gtrsim 10^3$ orbits, a phase error as small as $\sim 10^{-3}$ can prevent detection (Cutler et al. 1993, Cutler and Flanagan 1994, Finn and Chernoff 1993).

Near the end of the inspiral, when the binary separation becomes comparable to the stellar radii, hydrodynamic effects become important and the character of the waveforms will change. Special purpose narrow-band detectors that can sweep up frequency in real time will be used to try to catch the corresponding final few cycles of gravitational waves (Meers 1988, Strain and Meers 1991, Danzmann, this volume). In this terminal phase of the coalescence, the waveforms contain information not just about the effects of general relativity, but also about the internal structure of the stars and the nuclear EOS at high density. Extracting this information from observed waveforms, however, requires detailed theoretical knowledge about all relevant hydrodynamic processes.

Many models of gamma-ray bursts at cosmological distances are also based on coalescing NS-NS systems (Paczyński 1986, Eichler et al. 1989, Narayan et al. 1992, Mészáros, this volume). The isotropic angular distribution of the bursts detected by the BATSE experiment on the Compton GRO satellite (Meegan et al. 1992) strongly suggests a cosmological origin, and the rate of gamma-ray bursts detected by BATSE, of order one per day, is in rough agreement with theoretical predictions for the rate of NS binary coalescence in the Universe (cf. above). However, the complete hydrodynamic and nuclear evolution during final merging, especially in the outermost, low-density regions of the merger, must be understood in details before realistic models can be constructed for the gamma-ray emission. Numerical calculations of binary coalescence including some treatment of the nuclear physics have been performed by Davies et al. (1994) and Ruffert et al. (1996, 1997). The most recent results from these calculations indicate that, even under the most favorable conditions, the energy provided by $\nu\bar{\nu}$ annihilation is too small by at least an order of magnitude, and more probably two or three orders of magnitude, to power typical gamma-ray bursts at cosmological distances (Janka and Ruffert 1996).

### 2.2 Hydrodynamic Instabilities

Hydrostatic equilibrium configurations for binary systems with sufficiently close components can become *dynamically unstable* (Chandrasekhar 1975, Tassoul 1975). The physical nature of this instability is common to all binary interaction potentials that are sufficiently steeper than $1/r$ [see, e.g., Goldstein (1980), §3.6]. It is analogous to the familiar instability of circular orbits sufficiently close to a black hole (BH) (Shapiro and Teukolsky 1983, §12.4). Here, however, it is the *tidal interaction* that is responsible for the steepening of the effective interaction potential between the two stars and for the destabilization of the circular orbit (Lai et al. 1994a). The tidal interaction exists of course already in Newtonian gravity and the instability is therefore present even in the absence of relativistic effects. For sufficiently compact binaries, however, the combined effects of relativity and hydrodynamics lead to an even stronger tendency toward dynamical instability (see below).

The stability properties depend sensitively on the NS EOS. Close binaries containing NS with stiff EOS (adiabatic exponent $\Gamma \gtrsim 2$) are particularly susceptible to the dynamical instability. This is because tidal effects are stronger for stars containing a less compressible fluid. As the dynamical stability limit is approached, the secular orbital decay driven by gravitational wave emission can be dramatically accelerated (Lai et al. 1993b, 1994a). The two stars then plunge rapidly toward each other, and merge together into a single object in just a few rotation periods. This dynamical instability was first identified in Rasio and Shapiro (1992), where the evolution of Newtonian binary equilibrium configurations was calculated for two identical polytropes with $\Gamma = 2$. It was found that when $r \lesssim 3R$ ($r$ is the binary separation and $R$ the radius of an unperturbed NS), the orbit becomes unstable to radial perturbations and the two stars undergo rapid coalescence. For $r \gtrsim 3R$, the system could be evolved dynamically for many orbital periods without showing any sign of orbital evolution (in the absence of dissipation). Many of the results derived in Rasio and Shapiro and Lai et al. concerning the stability properties of NS binaries have been confirmed recently in completely independent work by New and Tohline (1997), using very different numerical methods (a combination of a 3-D self-consistent field code for constructing equilibrium configurations and a finite-difference code for following the dynamical evolution of the binaries).

The dynamical evolution of an unstable, initially synchronized (i.e., rigidly rotating) binary can be described typically as follows (Rasio and Shapiro 1992, 1994). During the initial, linear stage of the instability, the two stars approach each other and come into contact after about one orbital revolution. In the corotating frame of the binary, the relative velocity remains very subsonic, so that the evolution is adiabatic at this stage. This is in sharp contrast to the case of a head-on collision between two stars on a free-fall, radial orbit, where shocks are very important for the dynamics (Rasio and Shapiro 1992). Here the stars are constantly being held back by a (slowly receding) centrifugal barrier, and the merging, although dynamical, is much more gentle. After typically two orbital revolutions the innermost cores of the two stars have merged and the system resembles a single, very elongated ellipsoid. At this point a secondary instability occurs: *mass shedding* sets in rather abruptly. Material is ejected through the outer Lagrangian points of the effective potential and spirals out rapidly. In the final stage, the spiral arms widen and merge together. The relative radial velocities of neighboring arms as they merge are supersonic, leading to some shock-heating and dissipation. As a result, a hot, nearly axisymmetric rotating halo forms around the central dense core. No measurable amount of mass escapes from the system. The halo contains about 20% of the total mass and has a pseudo-barotropic structure (Tassoul 1978, §4.3), with the angular velocity decreasing as a power-law $\Omega \propto \varpi^{-\nu}$ where $\nu \lesssim 2$ and $\varpi$ is the distance to the rotation axis (Rasio and Shapiro 1992). The core is rotating uniformly near breakup speed and contains about 80% of the mass still in a cold, degenerate state.

The emission of gravitational radiation during dynamical coalescence can be calculated perturbatively using the quadrupole approximation (Rasio and Shapiro 1992). Both the frequency and amplitude of the emission peak somewhere during the final dynamical coalescence, typically just before the onset of mass shedding. Immediately after the peak, the amplitude drops abruptly as the system evolves towards a more axially symmetric state. For an initially synchronized binary containing two identical polytropes, the properties of the waves near the end of the coalescence depend very sensitively on the stiffness of the EOS. When $\Gamma < \Gamma_{\rm crit}$, with $\Gamma_{\rm crit} \approx 2.3$, the final merged configuration is perfectly axisymmetric[1] and the amplitude of the waves drops to zero in just a few periods (Rasio and Shapiro 1992). In contrast, when $\Gamma > \Gamma_{\rm crit}$, the dense central core of the final configuration remains *triaxial* (its structure is basically that of a compressible Jacobi ellipsoid; cf. Lai et al. 1993a) and therefore it continues to radiate gravitational waves. The amplitude of the waves first drops quickly to a nonzero value and then decays more slowly as gravitational waves continue to carry angular momentum away from the central core (Rasio and Shapiro 1994). Because realistic NS models give effective $\Gamma$ values precisely in the range 2–3 (Lai et al. 1994a), i.e., close to $\Gamma_{\rm crit} \approx 2.3$, a simple determination of the absence or presence of persisting gravitational radiation after the coalescence (i.e., after the peak in the emission) could place a strong constraint on the stiffness of the EOS.

### 2.3 Mass Transfer and the Dependence on the Mass Ratio

Clark and Eardley (1977) suggested that secular, *stable* mass transfer from one NS to another could last for hundreds of orbital revolutions before the lighter star is tidally disrupted. Such an episode of stable mass transfer would be accompanied by a secular *increase* of the orbital separation. Thus if stable mass transfer could indeed occur, a characteristic "reversed chirp" would be observed in the gravitational wave signal at the end of the inspiral phase (Jaranowski and Krolak 1992).

The question was reexamined recently by Kochanek (1992) and Bildsten and Cutler (1992), who both argued against the possibility of stable mass transfer on the basis that very large mass transfer rates and extreme mass ratios would be required. Moreover, in Lai et al. (1994a) it was pointed out that mass transfer has in fact little importance for most NS binaries (except perhaps those containing a very low-mass NS). This is because for $\Gamma \gtrsim 2$, *dynamical instability always arises before the Roche limit* along a sequence of binary configurations with decreasing $r$. Therefore, by the time mass transfer begins, the system is already in a state of dynamical coalescence and it can no longer remain in a nearly circular orbit. Thus stable mass transfer from one NS to another appears impossible.

In Rasio and Shapiro (1994) a complete dynamical calculation was presented for a system containing two polytropes with $\Gamma = 3$ and a mass ratio $q = 0.85$[2]. For this system it is found that the dynamical stability limit is at $r/R \approx 2.95$, whereas the Roche limit is at $r/R \approx 2.85$. The dynamical evolution turns out to be quite different from that of a system with $q = 1$. The Roche limit is quickly reached while the system is still in the linear stage of growth of the instability. Dynamical mass transfer from the less massive to the more massive star begins within the first orbital revolution. Because of the proximity of the two components, the fluid acquires very little velocity as it slides down from the inner Lagrangian point to the surface of the other star. As a result, relative velocities of fluid particles remain largely subsonic and the coalescence proceeds quasi-adiabatically, just as in the $q = 1$ case. In fact, the mass transfer appears to have essentially no effect on the dynamical evolution. After about two orbital revolutions the smaller-mass star undergoes complete tidal disruption. Most of its material is quickly spread on top of the more massive star, while a small fraction of the mass is ejected from the outermost Lagrangian point and forms a single-arm spiral outflow. The more massive star, however, remains little perturbed during the entire evolution and simply becomes the inner core of the merged configuration.

The dependence of the peak amplitude $h_{\max}$ of gravitational waves on the mass ratio $q$ appears to be very strong, and nontrivial. In Rasio and Shapiro (1994) an approximate scaling $h_{\max} \propto q^2$ was derived. This is very different from the scaling obtained for a detached binary system with a given binary separation. In particular, for two point masses in a circular orbit with separation $r$ the result would be $h \propto \Omega^2 \mu r^2$, where $\Omega^2 = G(M + M')/r^3$ and $\mu = MM'/(M + M')$. At constant $r$, this gives $h \propto q$. This linear scaling is obeyed (only approximately, because of finite-size effects) by the wave amplitudes of the various systems at the *onset* of dynamical instability. For determining the *maximum* amplitude, however, hydrodynamics plays an essential role. In a system with $q \neq 1$, the more massive star tends to play a far less active role in the hydrodynamics and, as a result, *there is a rapid suppression of the radiation efficiency as $q$ departs even slightly from unity*. For the peak luminosity of gravitational radiation Rasio and Shapiro found approximately $L_{\max} \propto q^6$. Again, this is a much steeper dependence than one would expect based on a simple point-mass estimate, which gives $L \propto q^2(1 + q)$ at constant $r$.

[2] This is the most probable value of the mass ratio in the binary pulsar PSR 2303+46 (Thorsett et al. 1993) and represents the largest observed departure from $q = 1$ in any observed binary pulsar with likely NS companion. For comparison, $q = 1.386/1.442 = 0.96$ in PSR 1913+16 (Taylor and Weisberg 1989) and $q = 1.32/1.36 = 0.97$ in PSR 1534+12 (Wolszczan 1991).

### 2.4 Measuring the Radius of a Neutron Star with LIGO/VIRGO

The most important parameter that enters into quantitative estimates of the gravitational wave emission during the final coalescence is the relativistic parameter $M/R$ for a NS (here we take $G = c = 1$). In particular, for two identical point masses we know that the wave amplitude obeys $(r_O/M)h \propto (M/R)$, where $r_O$ is the distance to the observer, and the total luminosity $L \propto (M/R)^5$. Thus one expects that any quantitative measurement of the emission near maximum should lead to a direct determination of the radius $R$, assuming that the mass $M$ has already been determined from the low-frequency inspiral waveform (Cutler and Flanagan 1994). Most current NS EOS give $M/R \sim 0.1$, with $R \sim 10\,\mathrm{km}$ nearly independent of the mass in the range $0.8M_\odot \lesssim M \lesssim 1.5M_\odot$ [see, e.g., Baym (1991), Cook et al. (1994), Lai et al. (1994a)].
However, the details of the hydrodynamics also enter into this determination. The importance of hydrodynamic effects introduces an explicit dependence of all wave properties on the internal structure of the stars (which we represent here by a single dimensionless parameter $\Gamma$), and on the mass ratio $q$. If relativistic effects were taken into account for the hydrodynamics itself, an additional, nontrivial dependence on $M/R$ would also be present. This can be written conceptually as

$$\left(\frac{r_O}{M}\right) h_{\max} \equiv \mathcal{H}(q,\Gamma,M/R) \times \left(\frac{M}{R}\right) \tag{1}$$

$$\frac{L_{\max}}{L_o} \equiv \mathcal{L}(q,\Gamma,M/R) \times \left(\frac{M}{R}\right)^5 \tag{2}$$

Combining all the results of Rasio and Shapiro, we can write, in the limit where $M/R \to 0$ and for $q$ not too far from unity,

$$\mathcal{H}(q,\Gamma,M/R) \approx 2.2\,q^2 \qquad \mathcal{L}(q,\Gamma,M/R) \approx 0.5\,q^6, \tag{3}$$

*essentially independent of* $\Gamma$ in the range $\Gamma \approx 2$–$3$ (Rasio and Shapiro 1994). This is in the case of synchronized spins. For nonsynchronized configurations, the spin frequency of the stars must be considered as additional parameters.

### 2.5 Nonsynchronized Binaries

Recent theoretical work suggests that the synchronization time in close NS binaries remains always longer than the orbital decay time due to gravitational radiation (Kochanek 1992, Bildsten and Cutler 1992). In particular, Bildsten and Cutler (1992) show with simple dimensional arguments that one would need an implausibly small value of the effective viscous time, $t_{\text{visc}} \sim R/c$, in order to reach complete synchronization just before final merging.
In the opposite limiting regime where viscosity is completely negligible, the fluid circulation in the binary system is conserved during the orbital decay and the stars behave approximately as Darwin-Riemann ellipsoids (Kochanek 1992, Lai et al. 1994a).
Of particular importance are the irrotational Darwin-Riemann configurations, obtained when two initially nonspinning (or, in practice, slowly spinning) NS evolve in the absence of significant viscosity. Compared to synchronized systems, these irrotational configurations exhibit smaller deviations from point-mass Keplerian behavior at small $r$. However,

as shown in Lai et al. (1994a) and Rasio and Shapiro (in prep.), irrotational configurations for binary NS with $\Gamma \gtrsim 2$ can nevertheless become dynamically unstable near contact. Thus the final coalescence of two NS in a nonsynchronized binary system must still be driven by hydrodynamic instabilities.

The details of the hydrodynamics are very different, however (Rasio and Shapiro, in prep.). Because the two stars appear to be counter-spinning in the corotating frame of the binary, a vortex sheet with $\Delta v = |v_+ - v_-| \approx \Omega r$ appears when the surfaces come into contact. Such a vortex sheet is Kelvin-Helmholtz unstable on all wavelengths and the hydrodynamics is therefore rather difficult to model accurately given the limited spatial resolution of 3D calculations. The breaking of the vortex sheet generates a large turbulent viscosity so that the final configuration may no longer be irrotational. In numerical simulations, however, vorticity is generated mostly through spurious shear viscosity introduced by the spatial discretization. An additional difficulty is that nonsynchronized configurations evolving rapidly by gravitational radiation emission tend to develop significant tidal lags, with the long axes of the two components becoming misaligned (Lai et al. 1994c). This is a purely dynamical effect, present even if the viscosity is zero, but its magnitude depends on the entire previous evolution of the system. Thus the construction of initial conditions for hydrodynamic calculations of nonsynchronized binary coalescence must incorporate the gravitational radiation reaction *self-consistently*. Instead, previous studies of nonsynchronized, equal-mass binary coalescence by Shibata et al. (1992), Davies et al. (1994), and Zhuge et al. (1994) used very approximate initial conditions consisting of two identical *spheres* (polytropes with $\Gamma \approx 2$) placed on an inspiral trajectory calculated for two point masses.

### 2.6 Relativistic Effects on the Stability of Compact Binaries

Most of the results discussed so far in this section are based on purely Newtonian calculations of NS binaries. Over the last year or so, various efforts have started to calculate the stability limits for NS binaries including both hydrodynamic finite-size (tidal) effects and relativistic effects. Note that, strictly speaking, equilibrium circular orbits do not exist in general relativity because of the emission of gravitational waves. However, the stability of quasi-circular orbits can still be studied in the framework of general relativity by truncating the radiation-reaction terms in a PN expansion of the equations of motion (Lincoln and Will 1990, Kidder et al. 1992, Will 1994). Alternatively, one can solve the full Einstein equations numerically in the $3+1$ formalism (see the article by Seidel in this volume) on time slices with a spatial 3-metric chosen to be conformally flat (Wilson and Mathews 1989, 1995, Wilson et al. 1996, Baumgarte et al. 1997). This effectively minimizes the gravitational wave content of space-time. The field equations then reduce to a set of coupled elliptic equations for the lapse and shift functions and the conformal factor.

Several groups are now working on PN generalizations of the semi-analytic Newtonian treatment of Lai et al. based on ellipsoids. Taniguchi and Nakamura (1996) consider NS-BH binaries and adopt a modified version of the pseudo-Newtonian potential of Paczyński and Wiita (1980) to mimic general relativistic effects near the BH. Lai and Wiseman (1997) concentrate on NS-NS binaries and the dependence of the results on the NS EOS. They add a restricted set of PN orbital terms to the dynamical equations given in Lai and Shapiro (1995) for a binary system containing two NS modeled as Riemann-S ellipsoids (cf. Lai et al. 1993a,b, 1994a,b,c), but neglect relativistic corrections to the fluid motion, self-gravity and tidal interaction. Lombardi et al. (1997) include PN corrections affecting both the orbital motion and the interior structure of the stars and explore the consequences not only for orbital stability but also for the stability of each NS against collapse. The most important result, on which these various studies all seem to agree, is that neither the relativistic effects nor the Newtonian tidal effects can be neglected if one wants to obtain a quantitatively accurate determination of the stability limits. In particular, the critical frequency corresponding to the onset of dynamical instability can be much lower than the value obtained when only one of the two effects is included. This critical frequency for the "last stable circular orbit" is a measurable quantity (with LIGO/VIRGO) and can provide direct information on the NS EOS.

A surprising result coming from the numerical $3+1$ relativistic calculations of Wilson and Mathews (1995, Mathews and Wilson 1997) is the appearance of a binary-induced collapse instability of the NS. This must be a purely relativistic effect, since the Newtonian tidal effects in fact tend to *stabilize* the NS against collapse (Lai 1996). In effect, the maximum stable mass of a NS in a relativistic close binary system could be slightly lower than that of a NS in isolation. Initially stable NS close to the maximum mass could then collapse to BH before getting to the final phase of binary coalescence. It should be noted, however, that the numerical results of Wilson and Mathews have yet to be confirmed independently by other studies. Even if it is real, the effect would be of importance only if the NS EOS is very soft, and the maximum stable mass for a NS in isolation is not much larger than $1.4M_\odot$.

## 3 Coalescing White Dwarf Binaries

### 3.1 Astrophysical Motivation

Close WD binaries are expected to be extremely abundant in our Galaxy. Iben and Tutukov (1984, 1986) predict that $\sim 20\%$ of all binary stars produce close pairs of WD at the end of their stellar evolution. The most common systems should be those containing two low-mass helium WD. Their final coalescence can produce an object massive enough to start helium burning. Bailyn (1993) suggests that extreme horizontal branch stars in globular clusters may be such helium-burning stars formed by the coalescence of two WD. Paczyński (1990) has proposed that the peculiar X-ray pulsar 1E 2259+586 may be the product of a recent WD-WD merger. Planets in orbit around a massive WD may also form following a merger (Livio et al. 1992).

Coalescing WD binaries may also be progenitors for Type Ia supernovae (Iben and Tutukov 1984, Webbink 1984, Paczyński 1985, Mochkovitch and Livio 1989, Yungelson et al. 1994). To produce a supernova, the total mass of the system must be above the Chandrasekhar mass. Given evolutionary considerations, this requires two C-O or O-Ne-Mg WD. Yungelson et al. (1994) show that the expected merger rate for close pairs of WD with total mass exceeding the Chandrasekhar mass is consistent with the rate of type Ia supernovae deduced from observations. Alternatively, a massive enough merger may collapse to form a rapidly rotating NS (Nomoto and Iben 1985, Colgate 1990). Chen and Leonard (1993) have discussed the possibility that most millisecond pulsars in globular clusters may have formed in this way. In some cases planets may form in the disk of material ejected during the coalescence and left in orbit around the central pulsar (Podsiadlowski et al. 1991). Indeed the first extrasolar planets have been discovered in orbit

around a millisecond pulsar, PSR B1257+12 (Wolszczan 1994). A merger of two highly magnetized WD might lead to the formation of a NS with extremely high magnetic field, and this scenario has been proposed as a source of gamma-ray bursts (Usov 1992).
Coalescing WD binaries are also important sources of very low-frequency gravitational waves that should be easily detectable by future space-based interferometers such as LISA (Danzmann, this volume). Evans et al. (1987) estimate a WD merger rate of order one every 5 yr in our own Galaxy. Coalescing systems closest to Earth should produce quasi-periodic gravitational waves of amplitude $h \sim 10^{-21}$ in the frequency range $\sim 10$–100 mHz. In addition, the total number ($\sim 10^4$) of close WD binaries in our Galaxy emitting at lower frequencies $\sim 0.1$–1 mHz (the emission lasting for $\sim 10^2$–$10^4$ yr before final coalescence) should provide a continuum background signal of amplitude $h_c \sim 10^{-20}$–$10^{-21}$. Individual sources should be detectable by LISA above this background when their frequency becomes $\gtrsim 10$ mHz. The detection of the final burst of gravitational waves emitted during the actual merging would provide a unique opportunity to observe in "real time" the hydrodynamic interaction between the two WD, possibly followed immediately by a supernova explosion, nuclear outburst, or some other type of electromagnetic signal.

### 3.2 Hydrodynamics of Coalescing White Dwarf Binaries

The results of Rasio and Shapiro (1995) for polytropes with $\Gamma = 5/3$ show that hydrodynamics also plays an important role in the coalescence of two WD, either because of dynamical instabilities of the equilibrium configuration, or following the onset of mass transfer. Systems with $q \approx 1$ must evolve into deep contact before they become dynamically unstable and merge. Instead, equilibrium configurations for binaries with $q$ sufficiently far from unity never become dynamically unstable. However, when these binaries reach the Roche limit, *dynamically unstable mass transfer* occurs and the less massive star is completely disrupted after a small number ($< 10$) of orbital periods [see also Benz et al. (1990)]. In both cases, the final merged configuration is an axisymmetric, rapidly rotating object with a core-halo structure similar to that obtained for coalescing NS binaries [Rasio and Shapiro (1994, 1995); see also Mochkovitch and Livio (1989)].
For two massive enough WD, the merger product may be well above the Chandrasekhar mass $M_{\rm Ch}$. The object may therefore explode as a (type Ia) supernova, or perhaps collapse to a NS. The rapid rotation and possibly high mass (up to $2M_{\rm Ch}$) of the object must be taken into account for determining its final fate. Unfortunately, this is not done in current theoretical calculations of accretion induced collapse (AIC), which always consider a nonrotating WD just below the Chandrasekhar limit accreting matter slowly and quasi-spherically (Canal et al. 1990, Nomoto and Kondo 1991, Nomoto et al. 1995). Under these assumptions it is found that collapse to a NS is possible only for a narrow range of initial conditions. In most cases, a supernova explosion follows the ignition of the nuclear fuel in the degenerate core. However, the fate of a much more massive object with substantial rotational support and large deviations from spherical symmetry (as would be formed by dynamical coalescence) may be very different.

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