Conformally Flat Hydrodynamic Calculations with a Spectral Methods Field Solver

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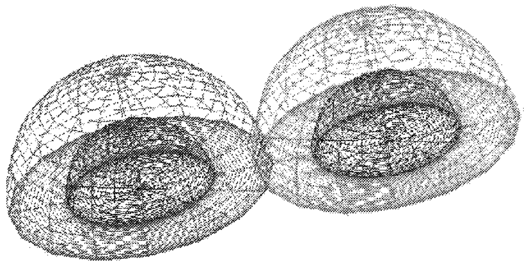
Abstract. We present the first results from our new relativistic hydrodynamics code. Using the equations of the conformally flat (CF) formalism, we solve for the fields using the LORENE libraries, developed by the Meudon group. These libraries, previously used for high-accuracy studies of equilibria, have been combined with a smoothed particle hydrodynamics (SPH) evolution treatment, to allow for a combination of high accuracy as well as computational speed. We have tested the resulting code extensively, finding that it performs well for calculations of a collapsing relativistic dust cloud as well as single-star configurations, for which the exact matter and field solution can be computed. We have also tested the code by computing the orbits of binary neutron stars, with the effects of radiation reaction not included, finding good long-term stability. We have used the code, with an approximate treatment of radiation reaction, to study the case of merging neutron star binaries, finding that we can successfully compute the fields even as the stars become tidally disrupted during the mergers.

Coalescing binary neutron star (NS) systems are a leading candidate to be the first sources detected by the current generation of gravitational wave detectors. Many calculations have been performed to study these systems, but it has long been recognized that the effects of general relativity (GR) must play an important role in the dynamics. As a result, many different gravitational formalisms have been used in hydrodynamical calculations, including a post-Newtonian treatment, developed by Blanchet et al. [1], which includes all lowest-order 1PN effects [2–5]. More recently, calculations have been performed in full general relativity [6, 7]. Unfortunately, the PN approximation breaks down during the merger when higher-order relativistic effects grow significant, and fully relativistic calculations typically introduce numerical instabilities which limit the amount of time for which a calculation will remain accurate. A middle ground is provided by the conformally flat (CF) approximation, developed originally by Wilson et al. [8], which includes much of the non-linearity inherent in GR, but results in a set of coupled, non-linear, elliptic field equations, which can be evolved stably. Such a method assumes that the spatial part of the GR metric is equal to the flat-space form, multiplied by a conformal factor which varies with space and time, the metric taking the form

$$ds^2 = -(N^2 - B_i B^i)dt^2 - 2B_i dt dx^i + A^2 \delta_{ij} dx^i dx^j. \tag{1}$$

While this approach cannot reproduce the exact GR solution for a general matter configuration, it is exact for spherically symmetric systems, and yields solutions which agree with those calculated using full GR to within a few percent for many systems of interest [9].

CP686, *The Astrophysics of Gravitational Wave Sources,* edited by J. M. Centrella



Source: Gourgoulhon et al. 2001

FIGURE 1. Radial domains used to evaluate the field equations of the CF method. Each vertex is a collocation point.

Calculations of binary NS coalescence using the CF formalism have been performed by Oechslin et al. [10], and a PN variation of the method was used by Shibata et al. [11]. The standard approach of using large computational grids to solve non-linear equations is very costly, however, both in time and computer memory. For many problems, it is much more efficient to use spectral methods to solve a system of equations, decomposing the relevant quantities into radial and angular terms, yielding extremely accurate solutions with a very small number of coefficients. We have developed a field solver based on `LORENE`, publicly available at `http://lorene.obspm.fr`, which has been used previously, among other things, to construct quasi-equilibrium binary configurations [12], and added on a smoothed particle hydrodynamics (SPH) set of evolution equations, resulting in a 3-d code which can compute the full evolution of any number of relativistic matter configurations accurately and efficiently. The field solver works by breaking up source terms into two distinct components, each centered on a star, which are further broken into radial domains, as shown in Fig. 1. In each domain, terms are evaluated at "collocation points" spaced out in the radial and angular directions to handle any convex surface. Typically, solutions accurate to one part in 10^9 can be achieved quickly, using only a $17 \times 13 \times 12$ grid. The Lagrangian nature of SPH has several advantages over Eulerian grid-based methods for these calculations, first and foremost the natural way it handles a surface; there are no particles where there is no matter.

We have performed several tests to ensure that our code works properly. Since the CF formalism is known to be exact for spherically symmetric matter configurations, we calculated models of isolated neutron stars, finding excellent agreement with the well known Oppenheimer-Volkov solutions to well within a percent for all hydrodynamic expressions and field values throughout the star.

To test the dynamical aspects of the code, we also computed the collapse of a dust cloud, i.e. pressureless matter, placed initially at rest. We compared our results to those of Petrich et al. [13], who developed a semi-analytic procedure which yields the field values at all points in spacetime, as well as the paths traced out by any given mass shell. We find that our code can reproduce the collapse extremely well, until just short of the point where the event horizon reaches the surface of the matter, showing in Fig. 2 the

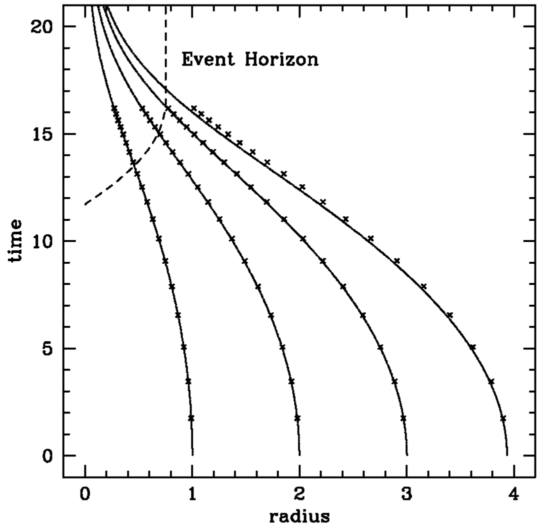


FIGURE 2. Lagrange radii in conformal coordinates versus time for particles in our calculation (points), compared to the exact semi-analytic form (solid lines), for a dust cloud of unit mass and initial radius $R_S = 5$ in Schwarzchild coordinates. The surface has an initial radius of $R_c = 3.99$ in conformal coordinates. The agreement is excellent until shortly before the event horizon (dashed line), which starts at the center and moves outward, reaches the surface of the matter.

particle position values over time compared with the exact solution. Note that the radii shown are in conformal coordinates, rather than Schwarzchild.

Since the CF formalism is time-symmetric, it does not contain terms which lead to gravitational radiation back reaction. Thus, we have tested our code by computing the evolution of quasi-equilibrium binaries. We see in Fig. 3 that the binary separation and conserved system angular momentum vary by no more than 2.5% over two orbits, and the ADM mass is nearly constant, for runs started at three different initial separations, with configurations taken from the compactness $M/R = 0.14$ calculation of Taniguchi and Gourgoulhon [14]. Similarly, a comparison of the field values and density profiles of the stars after two orbits yields very little deviation from the initial configuration. These results confirm for the first time that all equilibrium binary configurations calculated by Taniguchi and Gourgoulhon [14] are dynamically stable all the way to the appearance of a cusp.

Dissipative effects can be added to the CF formalism through a radiation reaction potential which reproduces the lowest-order energy loss rate [8]. When radiation reaction is used, we find that the binary plunges rapidly toward merger soon after the point where a cusp is reached along the quasi-equilibrium sequence. In our approach, we have found that throughout the evolution, the matter's surfaces can be modeled well enough for convergence by triaxial ellipsoids that are allowed to rotate to match the growing tidal lag angles. Field quantities are calculated by finding the SPH values for source terms at

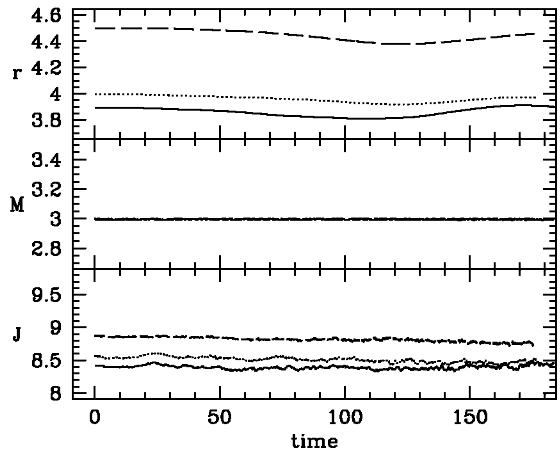


FIGURE 3. Binary separation, ADM mass, and total angular momentum for three circular orbit calculations, with initial separations $r_0 = 3.9$ (solid), 4.0 (dotted), and 4.5 (dashed). Units are the same as in Ref. [14]. The orbital period in these units is ≈ 90.

collocation points, and after the fields are calculated they are interpolated back to SPH particles, with derivatives calculated to high accuracy by the field solver, rather than particle-based techniques. For overlapping configurations, we split our source terms between the two stars, weighting the density contributions for each such that each star has a well-defined central density maximum up until the point where the central density of the system allows us to treat the object as a single rapidly spinning body.

We note that, while binary NS systems have served as the primary motivation for this code, it can be used in the study of a variety of physical systems, including collapsing matter configurations with rotation, as well as supernova progenitors.

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