Long-Term Stability of the Ups And Planetary System

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Abstract. Since the announcement of the triple planet system orbiting Upsilon Andromedae by Butler et al. in 1999 [1] the best-fit orbital parameters of the system have varied significantly with additional observations over the years. We have performed long-term numerical integrations and a new stability analysis using the most recent best-fit orbital parameters. All of our integrations run for a minimum of 10^8 years, unless a close encounter between two planets or the collision of one planet with the central star occurs first. We vary systematically the value of the unknown inclination angle with respect to the plane of the sky as well as the relative inclinations. Based on these numerical results we are able to provide improved constraints on angles of inclination and the masses of the planets. We also find evidence against the claim that the middle and outer planets exhibit a secular resonance.

1. MOTIVATION

The planetary system around Ups And was the first of only two known triple planet systems to be discovered (the other is 55 Cnc). It is one of only thirteen known multiple planet systems, to date. Few thorough studies with long-term numerical integrations for two or three giant planets have been done ([2], [3], [4]). Previous studies concerning the stability of the Ups And system have addressed only the short-term stability, and they were carried out several years ago with values of the orbital parameters that differ noticeably from the most up-to-date best-fit values (e.g., Stepinski et al. 2000 [5], Chiang et al. 2001 [8]). By studying the long-term stability of the system we can in principle better constrain the masses and orbital parameters of the planets as well as assess the role played by possible secular resonances ([8]).

2. METHOD

We used the most recent orbital parameters available (Table 1.) from directly measurable quantities [1].

Keeping the measurable quantities constant, we varied the overall inclination, i, and the relative inclinations (with respect to the overall inclination), e.g., $\cos i_r =$

[1] Taken from “A Triple Planet System Orbiting Upsilon Andromedae”, California and Carnegie Planet Search Team, `http://www.exoplanets.org/esp/upsandb/upsandb.shtml`, 03 July 2003

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TABLE 1. Orbital elements for the Ups And planetary system.

	Period (days)	e	$m \sin i$ **(M_J)**	a **(AU)**	ϖ **(degrees)**	$T_{\rm peri}$ **(JD)**	K **(m/s)**
b	4.6171	0.012	0.69	0.059	73	2450002.093	70.2
c	241.5	0.28	1.89	0.829	250	2450160.5	53.9
d	1284	0.27	3.75	2.53	260	2450064.0	61.1

$\cos i_c \cos i_d + \sin i_d \sin i_c \cos(\Omega_d - \Omega_c)$ for planets c and d. For the integrations reported here we kept $\Delta\Omega = 0$ so the equation for relative inclination reduces to the difference between the angles of inclination of two orbits. Unless otherwise noted, the inner planet is taken to be at $i + i_r$ relative to the middle planet, and the outer planet is taken to be $i - i_r$ relative to the middle planet. The total mass is $m_{\rm tot}(i_{\rm tot}) = m_{\rm obs}/\sin i$, where $i_{\rm tot} \equiv i \pm i_r$.

The code used to do our integrations is the multivariable symplectic (MVS) integrator from Mercury [6]. The symplectic integrator is particularly helpful for our integrations since it is fast for relatively large time steps while maintaining good energy conservation (to $\sim 10^{-8}$) for essentially regular orbits. Our integrations were stopped at the time these orbits first became irregular (i.e., at the onset of instability), before the fixed time step of the MVS integrator would cause energy errors to grow unacceptably large. For all integrations presented here, time steps were kept constant at $0.5\,$d, or about 10% of the period of the inner planet.

We integrated all three planets in the Ups And system either until a close encounter had occurred, defined as the approach of two bodies in the system to within 3 times the Hill radius, or until a maximum time of $10^8\,$yr had been reached (for most cases, but see below). Even though the integration time is roughly an order of magnitude smaller than the estimated age of the central star ($\simeq 1-3\times10^9\,$yr [7]), this was the longest reasonable integration time considering the available computational resources.

Two integrations that seemed to border on stability limits for the given system were extended to beyond the standard integration time of $10^8\,$yr used for all other integrations. By this we attempted to approach the age of the central star, since we know the planetary system must be near the same age, and we tried to elucidate the drastic jump between times to first close encounter produced by two inclinations about one degree apart (see below).

3. RESULTS

Integrations for small relative inclination angles are shown in Fig. 1. When $i_r = \pm1^\circ$ the system becomes unstable at an overall inclination of $i = 31^\circ$ after about $10^4\,$yr, while the integration with $i = 32^\circ$ was extended to more than $9.7\times10^8\,$yr without producing a close encounter. The same was true for the system with $i_r = \pm3.5^\circ$ from the middle planet's orbital plane, between 34° and 35°, except that, to date, this integration has only reached $\sim 5\times10^8\,$yr. Integrations for systems with the inner and outer planets' relative inclinations $i_r = \pm5^\circ, \pm10^\circ$, and $\pm15^\circ$ were stable for at least $10^8\,$yr when the middle

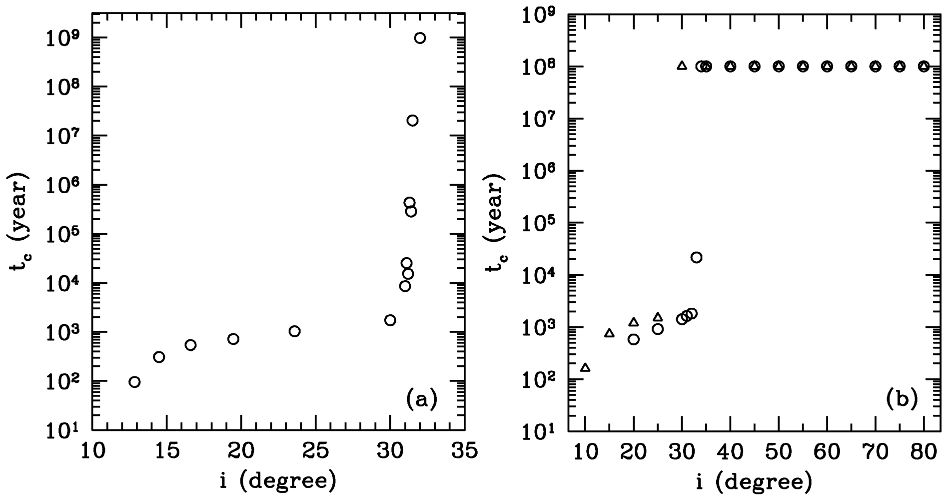

FIGURE 1. For small angles of relative inclination, the time to first close encounter, t_c, is shown as a function of overall inclination, i. (a) Relative inclinations $i_r = \pm 1°$. The circles indicate the time at which the first close encounter occurred in the system, except the last circle at an overall inclination of 32°, which represents an integration that is still presently stable and running. (b) Relative inclinations $i_r = \pm 3.5°$. These relative inclinations are more representative of the range found in our Solar System. The circles indicate integrations that had relative inclinations $i_r = +3.5°$ between the inner and middle planets, and $i_r = -3.5°$ between the middle and outer planets. The triangles represent integrations with relative inclinations in the opposite configuration, with $i_r = -3.5°$ betweent the inner and middle planets and $i_r = +3.5°$ between the middle and outer planets.

planet had an overall inclination $i \geq 35°, 40°$, and 45°, respectively.

For systems with relative inclinations $i_r = \pm 3.5°$ between each pair of planets, Fig. 1b shows that configurations where the inner planet has a negative relative inclination and the outer planet has the positive relative inclination retained stability when the middle planet had an overall inclination $i \geq 30°$, whereas with the inclinations of the inner and outer planets reversed, stability was retained only when the overall inclination of the middle planet is $i \geq 34°$.

Integrations where either the inner or middle planet was given a relative inclination $i_r = +30°$ and the remaining two planets had equal overall inclinations $i \geq 35°$ produced stable systems (Fig. 2a). However, the systems where the outer planet had a relative inclination $i_r = +30°$ with respect to the inner two planets were stable when the overall inclination was only $i \geq 15°$.

In the past, others to study this system (e.g. Lissauer and Rivera [2], Stepinski et al. [5], and Chiang et al. [8]) have found secular resonances between the middle and outer planets, where $|\Delta\omega| = |\omega_d - \omega_c| \ll 1$ for all time. Secular resonance in these cases has been used to explain the unusual proximity of ω_c to ω_d, and also to distinguish likely orbital configurations. In particular, approximating the orbit of the inner planet as circular, Chiang et al. [8] found that relative inclinations $i_r \leq 20°$, where $i_r = i_d - i_c = i_b - i_c$ (so

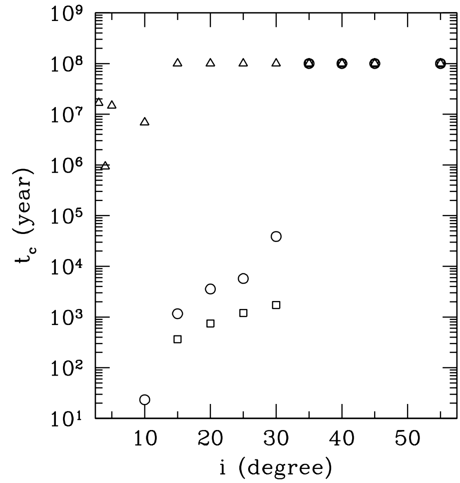

FIGURE 2. Same as Fig. 1, but with only one planet inclined with $i_r = +30°$ with respect to the overall inclination. The squares correspond to systems where the inner planet was inclined relative to the outer two planets, the circles correspond to systems where the middle planet was inclined, and the triangles correspond to systems where the outer planet was inclined.

that the inner and outer planets are in the same plane), limit variations in eccentricity, thus conserving the stability of the system. Using the same initial conditions cited by Chiang et al. we were able to reproduce their results and find secular resonance in the same orbital configurations. However, in our integrations with up-to-date orbital parameters, no secular resonances were detected, suggesting that the proximity of the observed longitudes of pericenter is coincidental and that there is therefore no apparent correlation between secular resonance and long-term stability for any inclinations considered here.

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