DYNAMICS OF BLACK HOLES IN DENSE STAR CLUSTERS

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ABSTRACT

In dense star clusters, the black holes (BHs) created via stellar evolution concentrate in an inner core and effectively decouple from the rest of the cluster following mass segregation. Using the most recent results on BH distributions in young stellar systems, we simulate the dynamical interactions of BHs in a variety of different clusters. In our simulations, we follow the evolution of the BH sub-cluster in a simple core/halo model, where BHs interact in the core only, assumed to have a constant density and velocity dispersion. We allow for Newtonian recoil into the halo and reintroduce the BHs according to mass segregation. Between interactions, BH-BH binaries evolve because of the emission of gravitational radiation and may eventaully merge. We account for linear momentum kicks imparted by gravitational radiation to merging binaries in some simulations and we find a steep dependence of successive mergers and BH growth on the magnitude of the recoil. We present the results of these simulations, and we derive the probability of massive BH growth and retention in the cluster. We also show that the Kozai mechanism in triples has almost no significant effect on the merger rate, BH growth, or retention in contrast to previous claims in the literature. We determine a net rate of BH-BH binary mergers detectable by current and future ground-based gravitational-wave interferometers.

1. INTRODUCTION

1.1. *Astrophysical Motivation*

Recent observations of globular clusters suggest the existence of an intermediate-mass black hole (IMBH) of mass $\sim 10^3$ - 10^4 $M_\odot$ in the center of some cluster cores. The predicted masses of these IMBHs correlate well with the mass-velocity dispersion relation of galactice nuclei (Gebhard et al. 2000). Observations and dynamical modeling of the globular clusters M15 in the Milky Way and G1 in M31 seem to be consistent with such a central massive BH (Gerssen et al. 2002, 2003; van der Marel et al. 2002; Gebhardt, Rich, & Ho 2002), although Baumgardt et al. (2003a,b) argue using *N*-body simulations that the observations could also be explained without the IMBH. Another example of a possible IMBH is found in the young star cluster MGG11 in M32, where recent observations with the *Chandra X-Ray Observatory* revealed an ultraluminous X-ray source that, if isotropic as Strohmayer & Mushotzky (2003) suggest, would be a BH with mass $\gtrsim 350\ M_\odot$ (Matsumoto et al. 2001; Strohmayer & Mushotzky 2003). Although the formation of these IMBHs is not directly constrained by observations, one proposed mechanism for their formation is a runaway merging process through stellar collisions (Colgate 1967; Ebisuzaki et al. 2001). Recent numerical studies by Portegies Zwart & McMillan (2002), Gürkan, Freitag, & Rasio (2004), and Freitag, Gürkan, & Rasio (2005) demonstrate that the collapse of the cluster core may proceed so quickly that there is rapid, runaway growth through stellar collisions before the most massive stars have evolved to supernovae ($\lesssim 3$ Myr). Other numerical evidence comes from the *N*-body simulations of Portegies Zwart et al. (2004), where they demonstrate such growth in clusters similar to MGG11 (in contrast to MGG9, in which no IMBH is seen).

Another possible formation scenario, the one we study in this paper, is an eventual runaway merging process of primordial, stellar mass black holes ($\sim 10\ M_\odot$). Unlike massive stars, their cross-section for physical collision is negligible, as they can only merge through gravitational wave emission, possibly enhanced by the dynamical interactions in the cluster (see Miller & Colbert 2004 for a thorough review and §1.2). Whether or not IMBHs exist or form through either channel, many BH-BH binaries are expected to merge within and outside the cluster, and these merging binaries constitute likely candidates for observation by ground-based laser interferometers, such as LIGO.

1.2. *BH Formation and Segregation*

In a globular cluster, it can be expected that a fraction $\sim 10^{-4}$ to 10^{-6} of the $N \sim 10^6$ initial stars will become stellar mass BH via single star stellar evolution (Sigurdsson & Hernquist 1993). Assuming that all stars with initial mass greater than 20 $M_\odot$ become BHs, the most recent Kroupa IMF (Kroupa & Weidner 2003) gives a slightly higher initial BH fraction of $N_{\rm BH} \approx 1.5 \times 10^{-3} N$, where we use $M_{\rm min} = .08\ M_\odot$ and

$M_{\rm max} = 150\ {\rm M}_\odot$ as the minimum and maximum stellar masses. When we scale this to the total mass of the cluster, $M_{\rm cl}$, we find $N_{\rm BH} \approx 3 \times 10^{-3}(M_{\rm cl}/{\rm M}_\odot)$, all of which should form before ~ 10 Myr, with the most massive BHs forming at around 3 Myr (Schaller et al. 1992).

The radial distribution of these BHs in the cluster when they form is not known, but it is reasonable to assume that they should be more centrally concentrated than the main sequence stars (MSs), due to mass segregation of the initial, higher mass progenitors (Freitag et al. 2005), and the possibility of a position dependent IMF (see, e.g., Murray & Lin 1996; Bonnell et al. 2001). A BH with mass $M_{\rm BH}$ segregates via two-body relaxation to the cluster core from the halo on the timescale

$$t_{\rm seg} \sim \frac{\langle m \rangle}{M_{\rm BH}} t_{\rm rh}, \tag{1}$$

where $t_{\rm rh}$ is the relaxation time at the half-mass radius, and $\langle m \rangle$ is the average stellar mass (Fregeau et al. 2002). Considering a typical cluster with $t_{\rm rh} \sim 1$ Gyr, we conclude that a sub-cluster of BHs should form near the center after ~ 100 Myr, when most of the BHs have segregated to the core.

In a time small compared to the overall dynamical evolution timescale of the globular cluster, the BHs should form a self-gravitating subsystem within the core, which evolves independently from the cluster. Three-body interactions, i.e., the scattering of a BH-BH binary and single BH, along with four-body interactions, i.e., the scattering of two BH-BH binaries, eventually lead to the ejection of BHs from the cluster, until there are so few that the number of interactions between BHs and field stars becomes comparable to BH-BH interactions. For simple two-component clusters, Spitzer (1969) derived through analytic methods the condition necessary to reach equipartition:

$$\left(\frac{M_2}{M_1}\right)\left(\frac{m_2}{m_1}\right)^{1.5} < 0.16, \tag{2}$$

where $M_2 < M_1$ are the total masses of the two components, and $m_2 > m_1$ the mass of each individual object. Watters, Joshi, & Rasio (2000) used an empirical approach to find the more accurate condition

$$\left(\frac{M_2}{M_1}\right)\left(\frac{m_2}{m_1}\right)^{2.4} < 0.32. \tag{3}$$

For a typical cluster with total mass $M_1 = 10^6\ {\rm M}_\odot$, BH mass $m_2 = 15\ {\rm M}_\odot$, and average star mass $m_1 = 1\ {\rm M}_\odot$, the cluster can be in equipartition, according to Equation 3, when there are about $N \lesssim 30$ BHs left. Although it is an oversimplification to assume that the BHs behave as one group of their average mass, the total number of BHs required to decouple from the cluster would likely be even less if their mass spectrum was considered (Gürkan et al. 2004).

During the mass segregation, as the BHs start entering the denser cluster core, they would likely interact with each other to form BH-BH binaries if not already in binaries. BH-MS binaries would likely end up in BH-BH binaries through three-body interactions with other BHs, since in resonant binary-single encounters the two heaviest masses stay in the binary(Sigurdsson & Phinney 1993). Therefore, as the sub-cluster begins to dynamically decouple from the rest of the cluster, there would remain a considerable number of hard BH-BH binaries. This has been demonstrated with N-body simulations by Portegies Zwart & McMillan (2000), where they found that almost all BH-binaries ejected were BH-BH.

The exact properties of this decoupled sub-cluster in relation to the total cluster are not so apparent. In our simulations we are interested in the velocity dispersion of the the BHs, $\sigma_{\rm BH}$. Numerical simulations suggest the mean kinetic energy of the dynamically decoupled massive objects is only a few times larger than for other objects in the core (Gürkan et al. 2004). Therefore, when decoupling, the BHs will have an effective velocity dispersion

$$\sigma_{\rm BH} = \sqrt{K\frac{\langle m\rangle}{\langle M_{\rm BH}\rangle}}\sigma_{\rm core}, \tag{4}$$

where $\sigma_{\rm core}$ is the velocity dispersion of the core, $\langle M_{\rm BH}\rangle$ is the average BH mass, and K is the ratio of the mean kinetic energies between the BHs and the rest of the core, and $K = 1$ corresponds to energy equipartition. Because the BHs are not in thermal equilibrium with the rest of the core, their velocity profile should evolve to be a lowered-Maxwellian distribution based on the average mass of the BHs. Of course, under their own self-gravity, they may slightly contract, increasing the sub-cluster density and velocity dispersion. Therefore we look at sub-clusters with $K = 1$, 2^2, 3^2, and 5^2.

One issue when determining if the sub-cluster of BHs entirely decouples from the rest of the stars is the presence of stars of comparable mass to the BHs in the core. We find this to be an insignificant factor in nearly all clusters. For our simulations to be valid, we must avoid runaway stellar growth. For stars of mass ≈ 100 $M_\odot$ to undergo supernovae before reaching the core the cluster would require a half-mass relaxations time $\gtrsim 300$ Myr. On the other hand, for an approximately 10 $M_\odot$ main-sequence star to segregate to the core before it goes supernova, the half-mass relaxation timescale must be $\lesssim 300$ Myr (Gürkan et al. 2004). Therefore we see that the parameter space of the half-mass relaxation time for massive stars to be present in the cluster core with a sub-cluster of BHs is effectively negligible, and do not need to treat the presence of BH mass stars in the core.

1.3. *Previous Studies*

Sigurdsson & Hernquist (1993) analytically treated the evolution and fate of 10 $M_\odot$ BHs in globular clusters. In binary-single interactions, hard binaries, those binaries with a binding energy larger than the average kinetic energy of the interloping single BH, tend to be hardened, whereas soft binaries tend to be disrupted (i.e., Heggie's Law, Heggie 1975). Qualitatively looking just at the three-body interactions in the sub-cluster, passing BHs would continuously harden the BH-BH binaries, with the release of the binding energy of the binary into kinetic energy of the components of the interaction (i.e., Newtonian recoil). Eventually, as the binary hardens, the recoil from the interactions becomes so great that the binary is ejected from the cluster. The timescale for merger from emission of gravitational radiation (GR), is usually longer than that for the next likely interaction, therefore hardened binaries usually merge only outside the cluster after they have been ejected, prohibiting subsequent mergers and growth.

In order to better understand the complex interactions of BHs in clusters, Portegies Zwart & McMillan (2000) ran N-body simulations of clusters with N = 2048 and N = 4096, where a small fraction (1% and 0.5% of N) were equal-mass BHs 10 times more massive than the other stars. They found that $\sim$ 90% of the BHs were ejected from the cluster, with $\sim$ 30% in BH-BH binaries. This is relatively consistent with our qualitative understanding of the interactions of BHs, and strongly supports the assumptions we will make in §2.

Gültekin, Miller, & Hamilton (2004), hereafter GMH04, were the first to look at the possibility of successive mergers of BHs in the core, driven by GR. In their simulations, GMH04 simulated successive interactions between a BH-BH binary with 10 $M_\odot$ and single BHs. In their simulations, they evolved the binary between interactions because of GR. They concluded that GR allowed for more mergers than previously thought possible, but the number of BHs required to form an IMBH would be much greater than believed to exist in a typical globular cluster.

To account for the limitations of binary-single interactions inducing mergers of BHs in cluster cores, Miller & Hamilton (2002b) suggested that one large seed BH may help overcome the Newtonian recoil, since the mass of the binary would be too large to have a recoil velocity greater than the escape velocity from the cluster. It has also been proposed by Miller & Hamilton (2002a) and Wen (2003) that binary-binary interactions may have a large influence on BH mergers in the cluster core. Specifically, the Kozai mechanism may allow for a slow but steady increase in a BH's mass over the evolution of the cluster, as triples formed through binary-binary interactions, undergo internal secular evolution, which can lead to the eventual merger of the inner binary. Because these hierarchal triples may merge before their next interaction they should have a higher probability of staying in the cluster.

In our simulations, we treat not only binary-single interactions, but also include four-body (binary-binary) interactions, and we also implement proper treatment of triples, including the Kozai mechanism (Wen 2003; Miller & Hamilton 2002a). We also include a more realistic IMF for the BHs (to be detailed in §2.2).

Our paper is organized as follows. In §2 we present the code and assumptions we use for this study, as well as all the initial conditions. The main results of the simulations are shown in §3. Finally we conclude our paper in §4 with a discussion of the implications of our results, and suggestions for further studies.

2. METHODS AND ASSUMPTIONS

2.1. *Numerical Methods*

We use a Monte Carlo simulation code in conjunction with the small N-body software *Fewbody* to simulate the evolution of a sub-cluster of BHs embedded in a larger star cluster. We assume that the BH sub-cluster has a constant density and velocity dispersion throughout the entire evolution. Our code determines the interaction rates of BH binaries, and calculates their interactions by direct integration by *Fewbody*. This code is a modified version of the one presented in Ivanova et al. (2005), specially adapted to treat a BH system.

Star clusters are characterized by a dense central core, surrounded by a much larger low-density halo. Consisten with this structure of a star cluster, we assume that all strong interactions occur within the core, which has a velocity dispersion $\sigma_{\rm core}$ (cf Eq. 4). If a product of an interaction has a velocity greater than the escape velocity from the core of the cluster, $v_{\rm esc}$, then it is assumed ejected from the cluster and is removed from the simulation. If it is less than $v_{\rm esc}$ but greater than the escape velocity from the core into the halo $v_{\rm halo}$, it is placed in the halo of the cluster, from where it can later reenter the core through dynamical friction with the background stars. Dynamical friction is implemented in our code by sampling from a Poisson distribution with an average timescale given in equation 1 (see §3.3 from Ivanova et al. 2005).

Between interactions, all BH-BH binaries are evolved according to the standard post-Newtonian equations (Peters 1964),

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{m_1^2 m_2^2 (m_1+m_2)}{a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right) \tag{5}$$

$$\frac{de}{dt} = -\frac{304}{15}\frac{G^3}{c^5}\frac{m_1 m_2 (m_1+m_2)e}{a^4(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right), \tag{6}$$

where m_1 and m_2 are the masses of the two BHs, a is the binary semimajor axis, and e is the orbital eccentricity.

In some simulations, we account for linear momentum kicks imparted by the GR to the remnant of the merging binary (Fitchett 1983). Because of the large theoretical uncertainty (Favata, Hughes, & Holz 2004) in the recoil velocity of "major mergers" (i.e., when the mass ratio $q = m_2/m_1 \sim 1$; here we assume $m_1 > m_2$), and the smaller but significant uncertainty from the spins of the BHs, we opted to neglect spin in determining recoil velocities in our simulations. We determine the overall recoil velocity of the merger remnant, $V_{\rm rec}$, by using the form of the equation derived by Fitchett (1983),

$$\tilde{V}_{\rm rec} = V_0 \frac{f(q)}{f_{\rm max}}\left(\frac{2GM/c^2}{r_{\rm isco}}\right)^4, \tag{7}$$

where $f(q) = q^2(1-q)/(1+q)^5$, $f_{max} \approx .0179$, V_0 is the maximum magnitude of recoil, and $r_{\rm isco}$ is the radius of innermost stable circular orbit. Fitchett (1983) found for circular orbits $V_0 \approx 1480~{\rm km~s^{-1}}$, much greater than the escape velocity from any globular cluster. Because of the uncertainty in the spin of primordial BHs, some of which may have undergone accretion, we choose to neglect spin entirely, and set $V_{\rm rec} = V_0 \cdot f(q)/f_{\rm max}$ for ease of comparing V_0 with other works. In our simulations we use V_0 near or slightly above the escape velocity of the cluster, up to 80 ${\rm km~s^{-1}}$. We do not account for GR recoil in the merger of the inner binaries that are part of hierarchical triples, because in the simulations where we include GR recoil, mergers in hierarchical triples are insignificant.

Lee (1993) shows that for the velocity dispersions ($< 100~{\rm km~s^{-1}}$) and numbers of BHs ($\lesssim 10^3$) expected in the star clusters we are investigating, the rate of two-body binary formation from gravitational bremsstrahlung is much less than that of regular (Newtonian) three-body binary formation, whereby a binary is formed with the help of a third BH, which takes away the excess energy needed to form the bound pair. Therefore, for our simulations, we only account for three-body and binary-binary (four-body) Newtonian interactions. Because the code is not capable of tracking the interactions of triples, we must break them up before the next interaction time-step. In order to determine how to destroy the triple, we check if the binary is likely to merge before its next interaction. The first method is calculated by numerically integrating Equations 5 & 6 (Peters 1964), the other is calculated from a first order Kozai mechanism approximation without post-Newtonian precession from solving Equation 8 of Wen (2003) and then appropriately scaling the timescale for merger as calculated in Miller & Hamilton (2002a). It is necessary to consider both methods because the scaling from Miller & Hamilton (2002a) overestimates the merging time in the instances when the Kozai mechanism is insignificant. In this instance, the inner binary is merely perturbed and the eccentricity doesn't fluctuate, therefore the timescale of the merger should be the same as an unperturbed binary. If the inner binary is likely to merge before the triple would interact with a field BH or BH-BH binary it is immediately merged, otherwise the triple is broken-up by keeping the inner binary, and appropriately shrinking it to conserve the energy of the system.

2.2. *Initial Conditions and Parameters*

We use the results of Belczynski, Sadowski, & Rasio (2004) (hereafter BSR04), adopting the mass and binary distributions of their standard model at 11.0 Myr for our initial conditions (see, e.g., their Fig. 2, Fig. 4, & Fig. 5). BSR04 used a population synthesis approach to follow the evolution of a large number of massive stars and binaries, as would likely form in a massive star cluster. The model we base our calculations on has an initial binary fraction $f_{\rm bin}$ = 50%, and follows the traditional Salpeter IMF for all initial stars with mass > 4 $M_{\odot}$. Although most BH binaries have MS companions, we assume that these binaries will eventually become BH-BH binaries through exchange interactions. We create BH-BH binaries in their place, and select the companion BH, such that the distribution of the mass ratio, q, is uniform throughout the range $0 < q < 1$. We then increase the separation of the BH-BH binary assuming the binary conserves energy in the exchange interaction. All wide binaries with orbital period $P > 10^4$ days are destroyed before our simulations begin. For each individual run, the mass of each BH is randomly selected with a distribution that reflects the results of BSR04, so that no two runs of any model contain the exact same BH population.

The parameters used in all our simulations can be found in Table 1. For our simulations, besides the exceptions noted in the table, we use self consistent parameters determined by following a King model, with W_0 = 7, 9, and 11. Given a total cluster mass, $M_{\rm cl}$, and core density, $n_{\rm c}$, we can calculate the 1-D velocity dispersion, $\sigma_{1,\rm core}$, the escape velocity from the center of the potential to the half-mass radius, $v_{\rm halo}$, the escape velocity of a BH in the core from entire cluster, $v_{\rm esc}$, and the escape velocity of a BH at the half-mass radius from the entire cluster, $v_{\rm haloesc}$. We analyzed relatively massive and dense clusters, precisely the types of clusters where BH growth would be expected. Specifically, we systematically varied $M_{\rm cl}$ between 5×10^5 and $2\times10^6 M_\odot$, and n_c between 10^5 and $10^7 {\rm pc}^{-3}$. Most of our simulations had 512 BHs ($N_{\rm BH}$ = 512), but for two simulations we looked at clusters with smaller and larger number of BHs. All of these parameters are listed in Table 1 for easy reference. The escape velocities are used to determine whether the product of an interaction is to remain in the cluster as prescribed in §2.1. The velocity dispersion of the BHs, $\sigma_{\rm BH}$, can be related to the 1-d velocity dispersion simply as $\sigma_{\rm BH} = \sqrt{3}\sigma_{1,\rm BH}$.

3. RESULTS

3.1. *Fate of the BHs*

The fate of the BHs in the cluster is determined mainly by the characteristic interaction rate in the core. We are most interested in knowing which cluster parameters determine if the BH sub-cluster reaches equipartition (the end of the applicability of our code) within a Hubble time, and whether some clusters will harbor an IMBH. Because of the simplicity of our model, it is easy to see the trends associated with varying just one parameter in the simulation. Figure 1 shows the number of BHs located in the cluster core and halo as a function of time for a variety of different cluster types. It is clear that most BHs remain in the cluster core, rather than in the halo of the cluster. In fact, in almost all simulations, less than about 10% of the BH-BH mergers in the cluster occur within the halo. Strong binary-binary interactions are efficient in lowering the binary fraction in the cluster core, and generally lead to a low binary fraction for most of the dynamical evolution. Of course, the low binary fraction is not entirely attributed to disrupted binaries. For the simulations in which the BH sub-cluster reaches equipartition in a Hubble time, approximately 10 − 15%

TABLE 1. SIMULATION PARAMETERS

Model Name	Structure	$M_{\rm cl}$ ($M_\odot$)	Effective N	$N_{\rm BH}$	n_c (pc^{-3})	$t_{\rm rh}$ (yr)	$\sigma_{1,\rm core}$ (km s^{-1})	$\sigma_{1,\rm BH}$ (km s^{-1})	$v_{\rm esc}$ (km s^{-1})	$v_{\rm haloesc}$ (km s^{-1})	$v_{\rm halo}$ (km s^{-1})
e5e5king7	$W_0 = 7$	5×10^5	1×10^6	512	5×10^5	1.5×10^8	14.08	14.08	55.60	38.04	40.73
v2e5k7	$W_0 = 7$	5×10^5	1×10^6	512	5×10^5	1.5×10^8	14.08	7.04	55.60	38.04	40.60
v3e5k7	$W_0 = 7$	5×10^5	1×10^6	512	5×10^5	1.5×10^8	14.08	4.69	55.56	38.02	40.70
v3e5k7ej54[a]	$W_0 = 7$	5×10^5	1×10^6	512	5×10^5	1.5×10^8	14.08	4.69	55.56	38.02	40.70
v3e5k7ej75[a]	$W_0 = 7$	5×10^5	1×10^6	512	5×10^5	1.5×10^8	14.08	4.69	55.56	38.02	40.70
v5e5k7	$W_0 = 7$	5×10^5	1×10^6	512	5×10^5	1.5×10^8	14.08	2.82	55.68	38.10	40.78
e5e5king9	$W_0 = 9$	5×10^5	1×10^6	512	1×10^5	7.1×10^8	10.13	10.13	44.04	22.63	37.90
v2e5k9	$W_0 = 9$	5×10^5	1×10^6	512	1×10^5	7.1×10^8	10.13	5.07	44.08	22.66	37.94
v3e5k9	$W_0 = 9$	5×10^5	1×10^6	512	1×10^5	7.1×10^8	10.13	3.38	44.08	22.66	37.94
e55king9	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	13.24	57.56	29.58	49.53
v2e55k9-256	$W_0 = 9$	5×10^5	1×10^6	256	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9-1024	$W_0 = 9$	5×10^5	1×10^6	1024	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9-2048	$W_0 = 9$	5×10^5	1×10^6	2048	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9-100[b]	$W_0 = 9$	5×10^5	1×10^6	513	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9-200[b]	$W_0 = 9$	5×10^5	1×10^6	513	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9e6[a]	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9e65[a]	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9e7[a]	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v2e55k9e8[a]	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	6.62	57.56	29.58	49.53
v3e55k9	$W_0 = 9$	5×10^5	1×10^6	512	5×10^5	3.2×10^8	13.24	4.41	57.52	29.56	49.50
v2e6e5k9	$W_0 = 9$	1×10^6	2×10^6	512	5×10^5	5.9×10^8	16.60	8.30	72.17	37.09	62.10
v3e6e5k9	$W_0 = 9$	1×10^6	2×10^6	512	5×10^5	5.9×10^8	16.60	5.56	72.52	37.27	62.40
v22e6e5k9	$W_0 = 9$	2×10^6	4×10^6	512	5×10^5	1.1×10^9	21.00	10.50	91.30	46.92	78.57
v32e6e5k9	$W_0 = 9$	2×10^6	4×10^6	512	5×10^5	1.1×10^9	21.00	7.00	91.30	46.92	78.57
e5king9	$W_0 = 9$	5×10^5	1×10^6	512	1×10^6	2.2×10^8	14.87	14.87	64.65	33.22	55.63
v2o5k9	$W_0 = 9$	5×10^5	1×10^6	512	1×10^6	2.2×10^8	14.87	7.44	64.69	33.25	55.67
v3o5k9	$W_0 = 9$	5×10^5	1×10^6	512	1×10^6	2.2×10^8	14.87	4.96	64.69	33.25	55.67
e5e5king11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^5	2.6×10^9	6.94	6.94	33.30	14.79	29.69
v2e5k11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^5	2.6×10^9	6.94	3.47	33.30	14.79	29.69
v3e5k11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^5	2.6×10^9	6.94	2.31	33.25	14.76	29.65
e55king11	$W_0 = 11$	5×10^5	1×10^6	512	5×10^5	1.2×10^9	9.07	9.07	43.52	19.32	38.80
v2e55k11	$W_0 = 11$	5×10^5	1×10^6	512	5×10^5	1.2×10^9	9.07	4.54	43.56	19.34	38.85
v3e55k11	$W_0 = 11$	5×10^5	1×10^6	512	5×10^5	1.2×10^9	9.07	3.02	43.47	19.30	38.76
e5king11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^6	8.3×10^8	10.19	10.19	48.89	21.71	43.59
v2o5k11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^6	8.3×10^8	10.20	5.10	48.94	21.73	43.64
v2o5k1110	$W_0 = 11$	5×10^5	1×10^6	1024	1×10^6	8.3×10^8	10.20	5.10	48.94	21.73	43.64
v2o5k1111	$W_0 = 11$	5×10^5	1×10^6	2048	1×10^6	8.3×10^8	10.20	5.10	48.94	21.73	43.64
v3o5k11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^6	8.3×10^8	10.20	3.40	48.94	21.73	43.64
v2e5e7k11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^7	2.6×10^8	14.95	7.48	71.77	31.87	64.00
v3e5e7k11	$W_0 = 11$	5×10^5	1×10^6	512	1×10^7	2.6×10^8	14.95	4.98	71.68	31.83	63.92
GMH[c]	...	...	...	512	5×10^5	...	...	5.77	50.00	...	...

NOTE. — The simulation parameters used. Starting from the top, the table is sorted according to the following parameters in their respective order: W_0, n_c, $\sigma_{1,\rm BH}$, $M_{\rm cl}$, $N_{\rm BH}$. In many cases the velocities expressed between models should be identical, but vary because of rounding errors.

[a]These models also include GR recoil, as described in §3.4

[b]These models include one primordial BH of varying mass, as detailed in §3.3

[c]The GMH models are based on the simulations in GMH04. For comparison we have done similar runs, with each model, GMHA, GMHB, and GMHC, having the same parameters, but a different mass function as explained in §3.6.

of the BHs are ejected in binaries.

For sub-clusters with low densities and high velocity dispersions, three-body binary formation becomes a relatively rare event. One would expect the cluster to self-gravitate until at least one binary forms, but this is not necessary since there always exists at least one BH-binary in the core, thereby preventing further collapse of the sub-cluster. Because of the low initial binary fraction in our simulations three-body binary

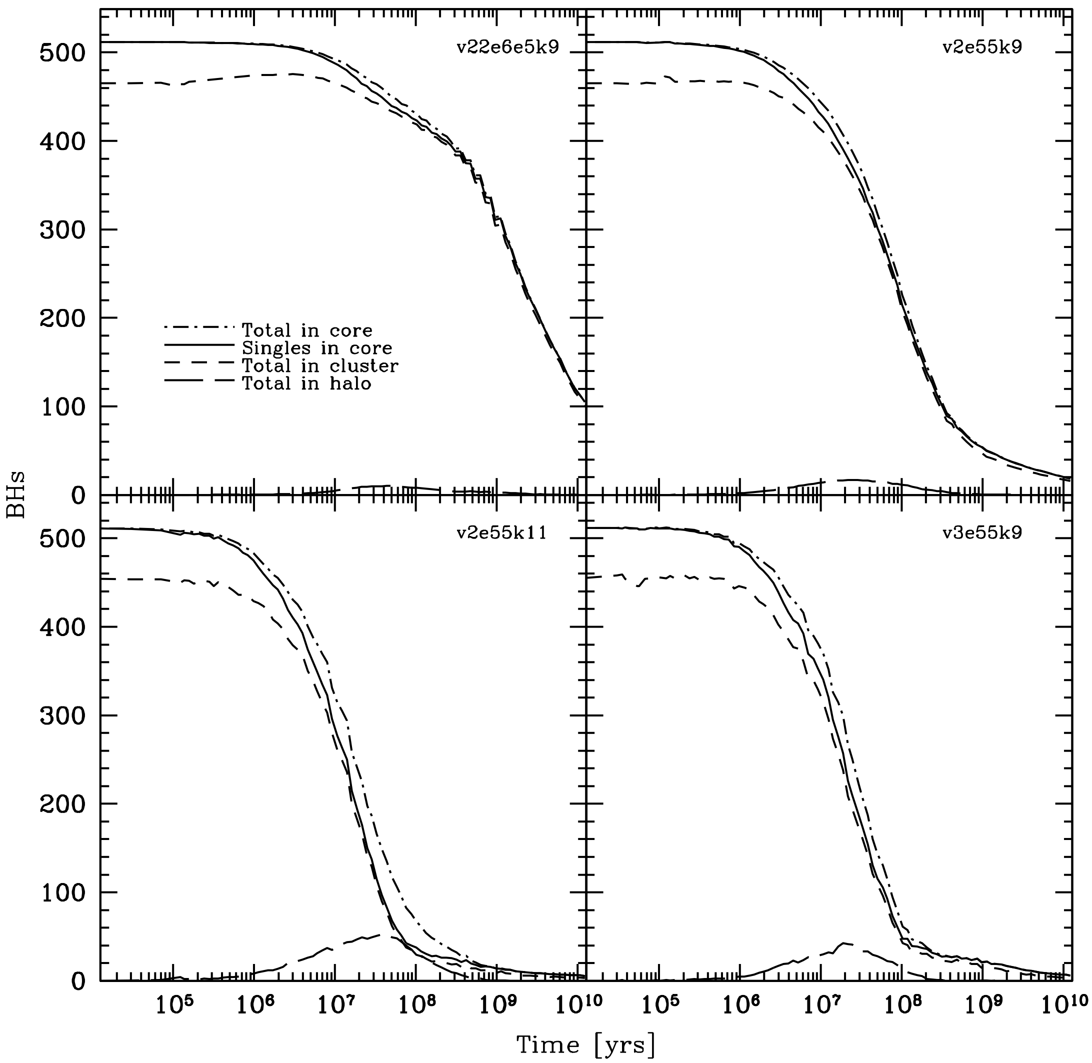

FIG. 1.— A comparison of the number of BHs in the cluster as a function of time between a few example simulations. The data for each figure is binned and then averaged over the entire model type in order to reduce the level of noise in the graph. Notice how few BHs remain in the cluster halo. Only in v2e55k11 do the numbers of BHs in the core and halo become comparable, but only when the cluster has already approximately reached equipartition. The more massive cluster v22e6e5k9 doesn't reach equipartition in a Hubble time, suggesting massive clusters with low W_0 could still contain significant numbers of single BHs. Each model has the same core density, $n_c = 5 \times 10^5$, and $K = 2^2$, except model v3e55k9, which has $K = 3^2$. Model v22e6e5k9, is a massive $W_0 = 9$ King model cluster with mass $M_{\rm cl} = 2^6 M_\odot$. Model v2e55k11 is a $W_0 = 11$ King model with mass $M_{\rm cl} = 5^5 M_\odot$. Models v2e55k9 and v3e55k9 are the same as v2e55k11, except they have $W_0 = 9$.

formation is important to the continued creation of binaries, and hence, increased interaction cross-sections of binary-single and binary-binary interactions. Since BHs are only ejected from the cluster after three- or four-body interactions, many of the BH sub-clusters which do not produce many binaries also do not dissolve within a Hubble time. If accurate, this would mean that a significant number of single BHs could still exist in clusters with similar parameters, assuming that the clusters' parameters have not changed significantly over their lifetimes.

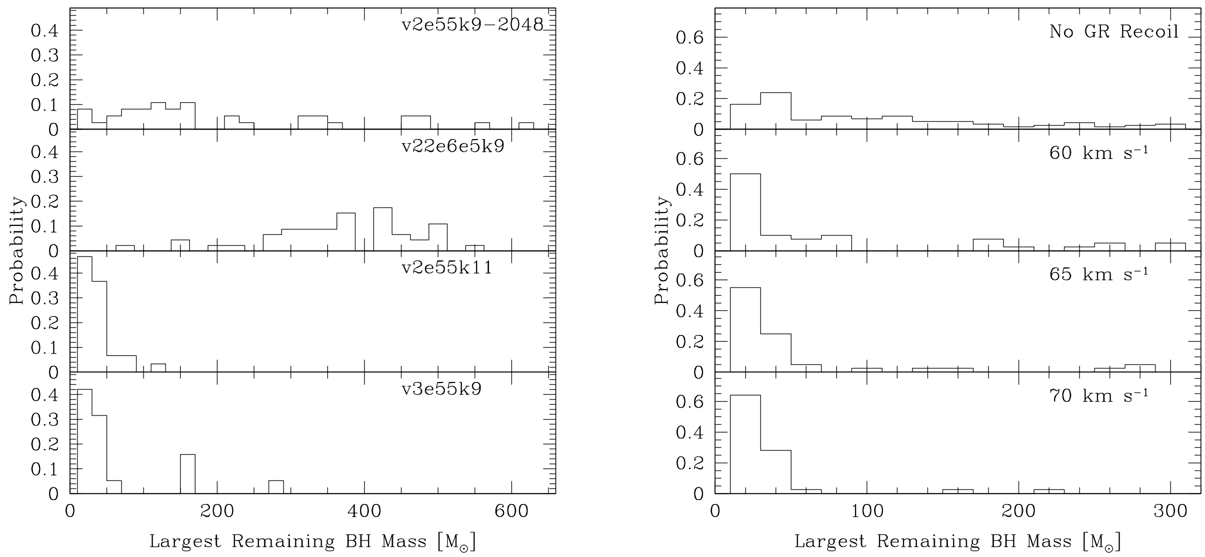

FIG. 2.— Largest remaining BH mass distribution. We use the same scale for all axes of each individual plot. The largest remaining BH is calculated at equipartition simply as the largest BH remaining in the cluster at equipartition, whether in the core or halo.

3.2. *Formation of an IMBH*

One possible mechanism for the formation of IMBHs is the successive merging of BHs to form one large BH with mass $\gtrsim 100\ M_\odot$. Table 2 shows the probability of large BHs forming for all the different models. In a given King Model, clusters with greater core densities and larger masses generally have a greater probability of forming an IMBH, which usually grows to even larger masses. Eventually though, a cluster that follows a given King model profile with a fixed mass and large enough core density will have a half-mass relaxation time, $t_{\rm rh}$, small enough such that the massive stars will likely collide in the cluster core before undergoing supernovae. This suggests that as one finds clusters with denser cores, the likelihood of an IMBH existing increases whether formed through successive BH mergers or stellar collisions. Likewise, clusters with higher W_0's also seem to have more BH growth. Figure 2 shows the mass distributions of the largest BHs formed in a few different cluster types. Note, that when clusters completely disrupt in less than a Hubble time, larger core temperatures directly correlate to more growth of a single massive BH. This is evident by directly comparing models v2e55k9 and v3e55k9, which have an average largest BH of mass 104 $M_\odot$ and 66 $M_\odot$ respectively. Although the difference in the average number of mergers in the cluster is small, v2e55k9 had about 14 mergers whereas v3e55k9 had about 13, the number and sizes of the large BHs formed are dramatically different. Model v2e55k9 had about twice the number of BHs as v3e55k9 with mass > 100 M$_\odot$ remain in the core at equipartition, and an average mass of the largest BH remaining in the core about 60% larger as well. This seems reasonable since the higher velocity dispersion, and therefore smaller interaction timescale, of the system would cause an increased number of interactions before a binary could merge, and increase the chances it is ejected before merging through GR.

3.3. *Proposed Mechanisms for BH Retention*

One proposed method for helping keep BHs in the cluster and preventing their ejection through hardening and eventual recoil is the Kozai mechanism in stable hierarchical triples (Miller & Hamilton 2002a; Wen 2003). If the relative inclination between the two orbital planes is large enough, the inner binary's eccentricity can be driven to a high value, and therefore can decrease the merging time of the inner binary below the likely interaction time of the triple with another field object. This allows the inner binary to merge without Newtonian recoil. We find that this usually has almost no significant effect on the formation of large BHs, but does account for a significant fraction of mergers in general in a few simulations with high velocity dispersions. Overall, mergers due to the Kozai mechanism always account for less than 10% of the total mergers, but less than a few percent in most models. For example, the model with the largest percentage of triple mergers, v3e5k11, exhibits almost no growth at all, with none of the 14 runs having a BH with mass greater than even 80 M$_\odot$.

Another possible mechanism to help retain large BHs is the introduction of a massive seed BH (Miller & Hamilton 2002b). The mass spectrum of BHs suggested by BSR04 generally includes a massive BH of ~ 50 M$_\odot$ in each cluster. This mass is not large enough to guarantee that the BH remains in the cluster, as the largest initial BH is in fact almost always ejected from the cluster, with little or no growth. Although Miller & Hamilton (2002b) suggest that the introduction of a seed BH with mass 50 M$_\odot$ may be sufficient to have significant growth, we find that this is still not massive enough, in agreement with GMH04.

To analyze this situation further, we looked at one model particularly efficient in forming and retaining massive BHs. In model v2e55k9-100, we introduced an initial seed BH of mass 100 M$_\odot$, and in a 200 M$_\odot$ seed BH in model v2e55k9-200. We find that even these large masses can easily be ejected from the cluster. For example, in model v2e55k9-100, the seed BH was retained 18% of the time. Even in model v2e55k9-200, with a seed BH of 200 M$_\odot$, the seed BH is retained only 35% of the time. These probabilities are still lower than those found in GMH04, where they found the BHs to be retained in a similar cluster $\sim 40\%$ and $\sim 90\%$ of the time, respectively. The main source of this discrepancy likely comes from the mass distribution of BHs in our simulations. The average mass of the BHs in our simulation is 50% higher than in GMH04, and thus should have an increased probability of ejecting the large seed BH.

3.4. *GR Recoil*

One question of great current interest concerns the linear recoil arising from the GR emitted by coalescing binary BHs. Our code allows us to prescribe a velocity for every BH-BH binary merger, and therefore, we can follow the effects of GR recoil in a cluster environment. In order to measure the effects of GR recoil, we look again at the model v2e55k9, which is particularly efficient in forming large BHs. By varying the maximum magnitude of the recoil velocity, we are able to determine the possible effects gravitational recoil has on BH-BH mergers (Favata et al. 2004).

As expected, the number of successive mergers has a strong dependence on the maximum recoil velocity, but it seems that it has only a small effect on the overall dynamics of the rest of the BH sub-cluster. Increasing the recoil has almost no noticeable effect on the total number of mergers in the cluster but can have significant consequences on the rate of BH-BH in-spirals (see §3.5). Even a maximum recoil velocity slightly larger than the escape velocity of the core, $V_0 = 1.042 \times v_{\rm esc} = 60$ km s^{-1}, will have a dramatic effect on the number of large BHs formed. The profile of the final masses of the largest BHs are vastly different as seen in Figure 2. There are about half the number of cases of BHs with masses greater than 100 $M_\odot$ with even this small escape velocity, and the average largest BH mass at equipartition with recoil is about 75% that without recoil. When we look at higher recoil velocities, the possibility of growing a large BH gets only smaller.

This is not such a surprising result, when one considers our methods in modeling GR recoil. Only mergers with a mass ratio, $q = m_1/m_2 \approx .38$, or when $f(q) \approx f_{max}$, will generally be ejected as a result of merger when V_0 is close to the core escape speed. For BHs which go through successive mergers, it is likely that, before reaching masses > 100 $M_\odot$, the BH will have already been ejected. But since this only affects BHs after they merge, it is reasonable that the number of mergers remains relatively unaffected.

Our simple model of GR recoil neglects many aspects of actual recoil, which could dramatically alter the results. Most importantly, we do not follow the evolution of the BHs' spins. Because of this, we must neglect the effects of spin on recoil, and therefore do not look at alternative paths to large BH retention. One such possibility is that a random sampling of clusters could still have a significant probability of retaining larger BHs even if the recoil velocity is much larger than the escape speed, if BHs with the approriate spins merge. On the other hand, spin breaks the symmetry of the binary, and can lead to large recoil velocities even for equal-mass binaries with $q = 1$ (Favata et al. 2004).

3.5. *Binary BHs as Sources of Gravitational Waves*

BH-BH binaries formed through dynamical interactions have the potential to be some of the best sources of gravitational waves detectable by ground-based laser interferometers. Previous studies of detection rates have led to a large range of possible values, with some of the greatest uncertainty coming from the dynamics of interacting BH binaries (Tutukov & Yungelson 1993; Portegies Zwart & McMillan 2000). In this subsection we analyze the properties important to the detection of BH-BH mergers, and determine possible maximum detection rates of BH-BH binary in-spirals from some globular cluster models.

One important factor in the detection of gravitational waves from in-spiraling BHs is the characteristics of the binary. Dynamical interactions, such as those in a cluster core, coupled with the strong dependence of merging time with eccentricity, suggest that many binaries will have highly eccentric orbits before merging. Of course, gravitational waves counteract this effect by circularizing the binaries, and therefore, it is often assumed that most binaries can be fitted with templates for mergers with no eccentricity. Figure 3 shows that there will be almost no loss in possible LIGO BH-BH binary sources with this assumption because at such a high frequency, almost all binaries are circular. However, the eccentricity distribution will likely matter for space-based detectors, which operate at lower frequencies, since the in-spiraling binaries have not been entirely circularized by the GR emission.

Another important factor in detecting such in-spirals is the chirp mass of the binary, $M_{\rm chirp} = (M_1 M_2)^{3/5}/(M_1+M_2)^{1/5}$. The chirp mass solely determines the overall magnitude of the gravitational waves of a circular binary. Because our simulations allow for successive mergers of BHs, some mergers have chirp masses well above those expected if dynamics were not included. In Figure 4 we plot the chirp masses of merging binaries for two different cluster models. Because of the realistic initial distribution of BHs, the

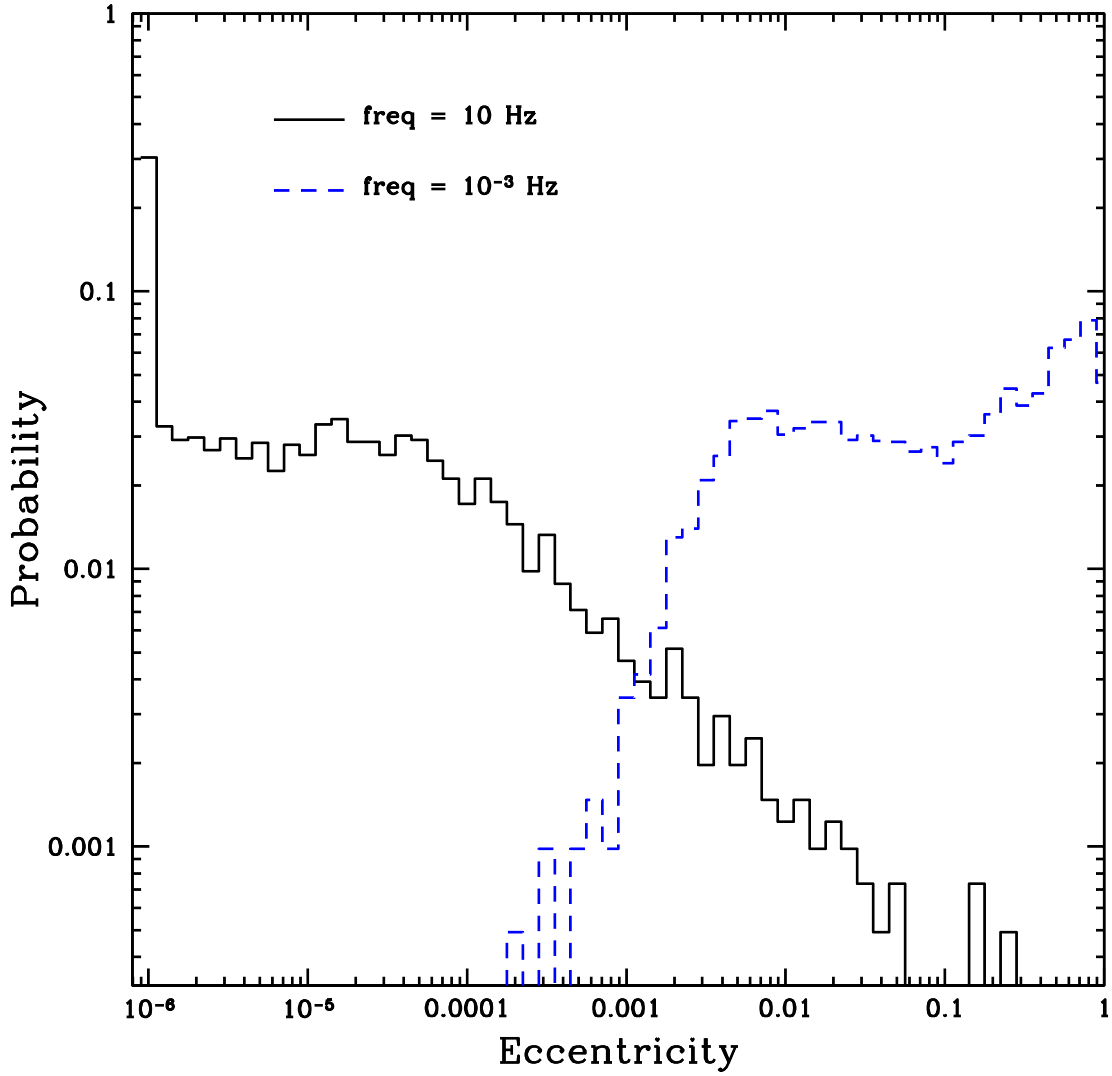

FIG. 3.— Eccentricity distribution of merging BH binaries. For this log−log plot, we show the eccentricity distribution of all binary BH mergers for model v2e55k9. We find the distribution of eccentricities is almost entirely independent of the model used. The two frequencies of the GR were chosen to show the expected eccentricity distribution of a binary when it enters the observable bands of both ground-based, ~ 10 Hz, and space based interferometers, $\sim 10^{-3}$ Hz. The low eccentricity of most binaries entering the ground-based band suggests almost no loss of detectable BH-BH binary signals if only circular templates are used for detection.

chirp masses of most mergers is above the expected value for two 10 $M_\odot$ BHs merging, 8.7 $M_\odot$. The large masses of the merging BH-BH binaries may seem to be only fortuitous, but because more massive binaries merge at lower frequencies, the larger masses do not directly correlate to higher detection rates. Quite simply, as the binary decays, it eventually reaches its last stable circular orbit, after which the BHs plunge into each other. For the most massive merging BHs, the merger may occur at frequencies before the most sensitive region of the estimated noise curves for ground-based laser interferometers. Because of the theoretical uncertainty in the waveforms of the final plunge of the BHs, one has the highest probability of

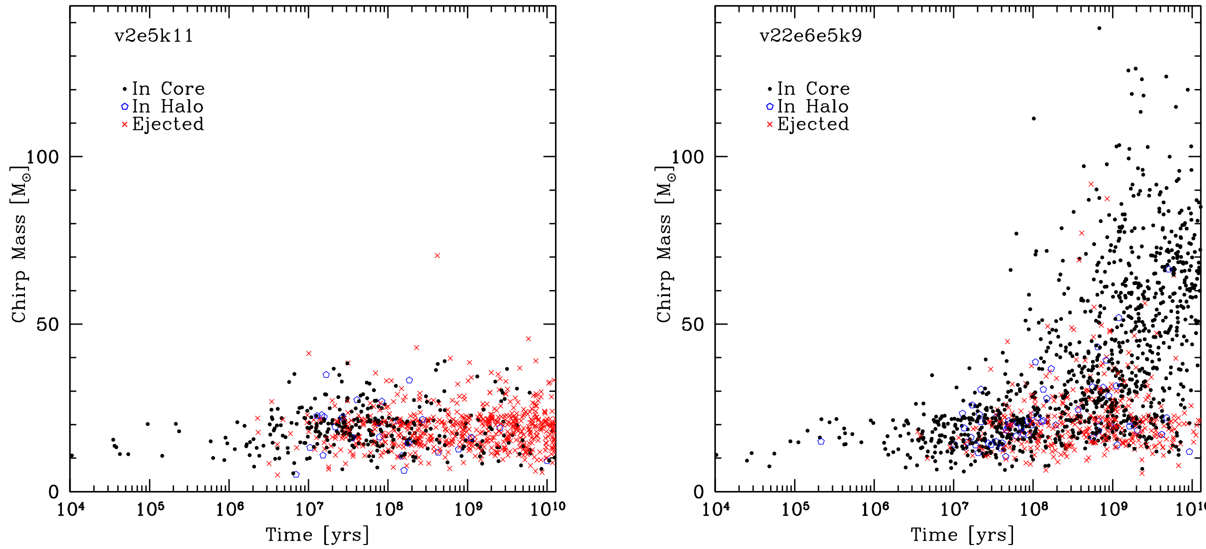

FIG. 4.— Chirp mass scatter plot versus time. This is a comparison of the two models v2e5k11 and v22e6e5k9. Plotted is a scatter plot from all mergers of 46 random runs of model v2e5k11 and all 46 runs of v22e6e5k9. Model v2e5k11 is one of the least effi cient clusters in producing large BHs and BH-BH binary mergers in general. Therefore, the distribution is most nearly that expected from the initial mass distribution of BSR04. Because of how quickly v2e5k11 evolves, $T_{\rm equi} \approx 200$ Myr, almost all mergers in later times occur outside the cluster. In comparison, v22e6e5k9 is a massive cluster, which doesn't reach equipartition before a Hubble time. There is still a signifi cant fraction of BHs in the cluster at the end of the simulation, which allows for more growth, and also more massive BH mergers. The distribution of the chirp masses for model v22e6e5k9 would be dramatically different if the magnitude of gravitational recoil were ~ 100 km s^{-1}.

detecting the merger in the in-spiral phase. For two Schwarzschild BHs the binary separation at innermost stable circular orbit is $r_{isco} = 6GM/c^2$, where $M = m_1 + m_2$. Therefore, the GR is detected only up to the cutoff frequency,

$$f_{\rm off} = \frac{c^3}{\pi 6^{3/2} GM} \frac{1}{1+z} \approx 220 \left(\frac{20{\rm M}_\odot}{M}\right)\left(\frac{1}{1+z}\right) {\rm Hz}, \tag{8}$$

where z is the redshift of the merger.

Figure 4 also shows that the rate of mergers in a given cluster is not constant in time, as assumed in Portegies Zwart & McMillan (2000). Our simulations give us a greater understanding of the dynamics of BH sub-clusters, but there are still great complications in determining the detection rate of BH binary mergers. The birth rate, density, and also the parameters of globular clusters are not well known. Despite our uncertainty in the distribution of globular clusters, our models give us an understanding of how these parameters affect the distribution of BH-BH in-spirals.

Because of the invariance of gravitational waveforms under redshift, for a given BH-BH merger at redshift $z_{\rm m}$, we can calculate the accumulated signal-to-noise ratio (SNR) from its redshifted chirp mass $\mathcal{M}_{\rm chirp} = (1+z_{\rm m})M_{\rm chirp}$, and luminosity distance $D_{\rm L} = (1+z_{\rm m})D_{\rm prop}$, where $D_{\rm prop}$ is the proper distance of the merger.

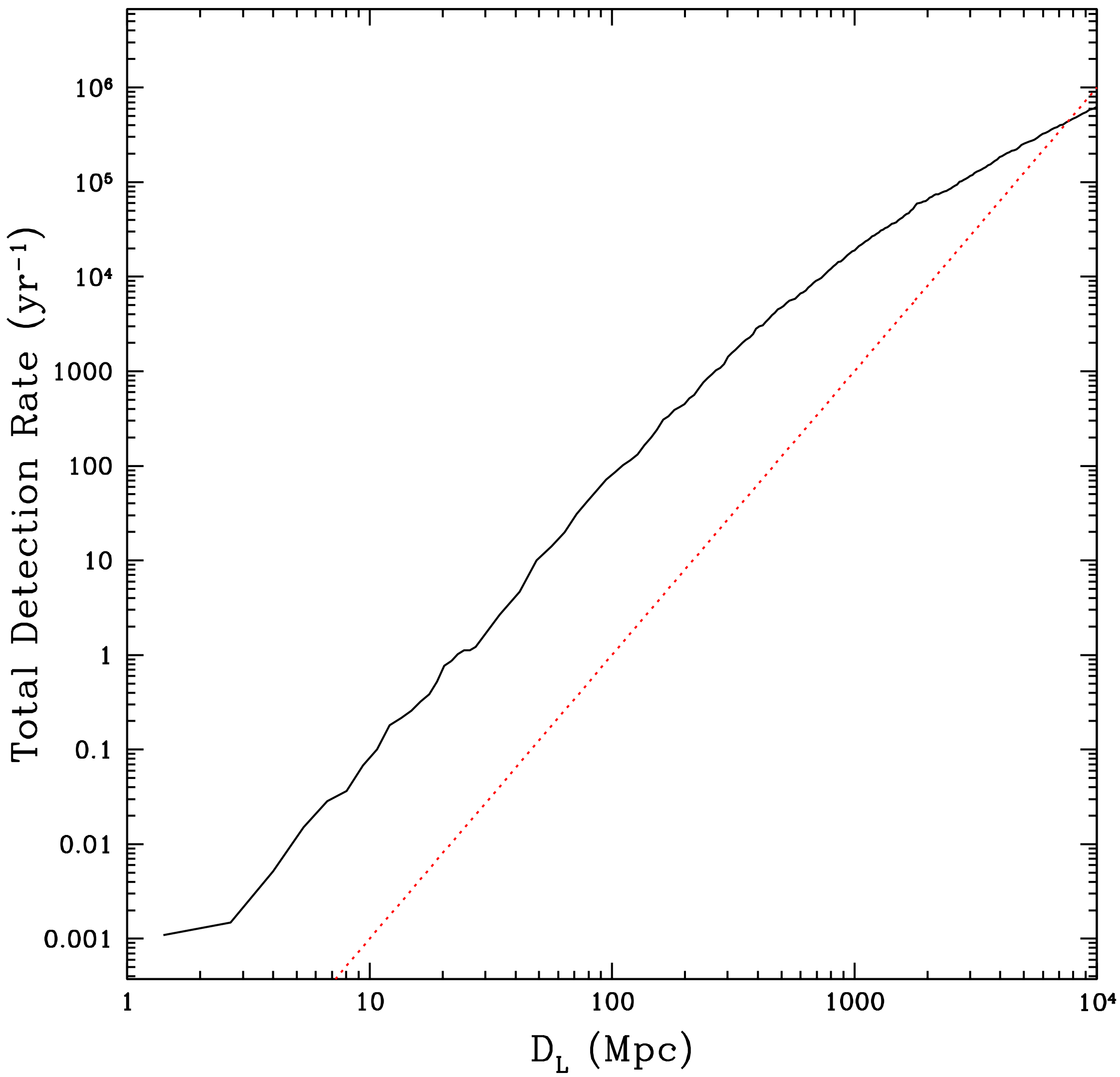

FIG. 5.— Detection rate of BH-BH in-spirals. The solid curve represents the expected detection rate of cluster v2e55k9 if it has a current density $\rho = 1$ Mpc^{-3} and formed at $z_i = 7.84$, or 13 Gyr ago. The actual rate could be $\sim$ 100 times smaller, when one considers the universal globular cluster rate found in Portegies Zwart & McMillan (2000) of 3 Mpc^{-3}, and the distribution of globular clusters within the Milky Way galaxy. The dotted curve is a power-law curve for D_L^{-3}.

The detectability of a signal can be determined by looking at the SNR from,

$$(S/N)^2(f_{\rm off}) = \frac{Q^2}{D_L^2}\mathcal{M}_{\rm chirp}{}^{5/3}\int_0^{f_{\rm off}} \frac{(f')^{-7/3}}{S_n(f')}df', \tag{9}$$

where $S_n(f')$ is the detector's noise spectrum, and Q is a function of the orientation of the merger and the detector (Cutler & Flanagan 1994). In this study, we assume Q to be, on average, the same for all signals, and use an analytic fit for the shape of Advanced LIGO's noise spectrum found in Cutler & Flanagan (1994),

$$S_{\rm n}(f') = \begin{cases} \infty & f' < 10 \text{ Hz}, \\ S_0\{(f_0/f')^4 + 2[1+(f'/f_0)^2]\} & f' \geq 10 \text{ Hz}, \end{cases} \tag{10}$$

where f_0 = 70 Hz, and $S_0 = 3 \times 10^{-48}\,\mathrm{Hz}^{-1}$ for the advanced LIGO detector.

To calculate a detection rate for a given model, we assume that the globular cluster model is formed uniformly throughout the universe at a given cosmic time corresponding to redshift $z_{\rm i}$. We then look at all BH-BH binary mergers caused by interactions before the cluster reaches equipartition, and consider it detectable if the SNR of the merger, using Equation 9, is greater than that of two in-spiraling neutron stars with $\mathcal{M}_{\rm chirp} = (1+z_{\rm n})2.4\,\mathrm{M}_\odot$, at a given luminosity distance $D_{\rm L}$. All detectable mergers are then put into bins of equal time, and the merger rate is scaled appropriately to the physical volume, higher density, and lower rates of the mergers at that cosmological time. For all cosmological calculations we use the cosmological parameters as determined in Spergel et al. (2003), with the Hubble expansion rate , $H_0 = 71\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$ and matter, radiation, and vacuum energy densities $\Omega_m = 0.27$, $\Omega_\gamma = 5 \times 10^{-5}$, and $\Omega_\Lambda = .73$, respectively. In their calculation Portegies Zwart & McMillan (2000) determined the number density of globular clusters in the universe to be $\rho = 8.4\,h^3\,\mathrm{Mpc}^{-3}$. Figure 5 shows the expected detection rates of BH binary mergers for cluster v2e55k9, if it has a current number density, $\rho = 1\ \mathrm{Mpc}^{-3}$, and formed at $z_{\rm i} = 7.84$, or 13 Gyr ago.

3.6. *Comparison with Previous Studies*

The overall characteristics of the runs coincide with previous direct N-body simulations. In the simulations of Portegies Zwart & McMillan (2000) with N = 2048 and 4096, $N_{\rm BH}$ = 20 and 40 in one simulation, approximately 60% of the BHs where ejected as single BHs, and 30% ejected as binary BHs. Our results are show agreement with these when one expects a lower binary ejection rate because of in core mergers and the lower order interactions used as compared to N-body simulations. As shown in Table 2, in most runs about 70 – 85% of the BHs were ejected as singles, and 10 – 15% were ejected in binaries.

To see what effect the mass spectrum has on our simulations, and also to compare to the results of GMH04, we use three different models. Each simulation uses the same velocity dispersion and escape velocities as GMH04 (see models GMHA, GMHB, and GMHC in Table 1), but start with different mass

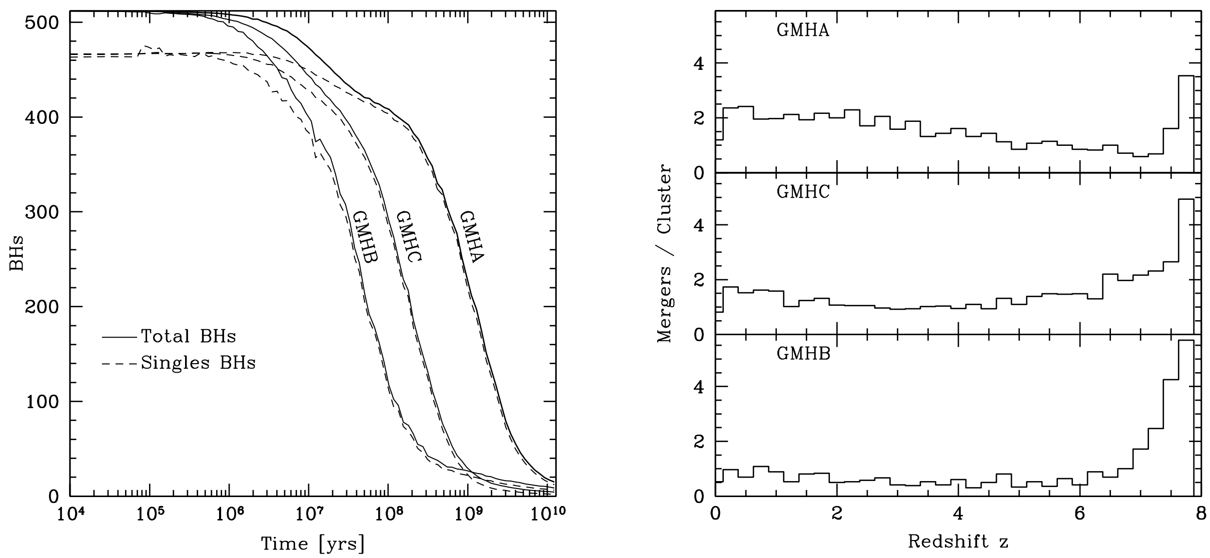

FIG. 6.— A comparison of the three different mass functions. In the figure a (on the left), the number of BHs as a function of time is plotted. Each pair of lines is labeled above by its associated model that we use to compare our results with GMH04. Model GMHA, with only 10 $M_{\odot}$ BHs, on average reaches equipartition at $\sim$ 5.2 Gyr. Model GMHC, a cluster with only 15 $M_{\odot}$ BHs, reaches equipartition at 830 Myr. Finally, GMHB, with a varied mass spectrum and average mass $\approx$ 15 $M_{\odot}$, has not only a significantly smaller number of ejected binaries, but also reaches equipartition at the earliest time, 670 Myr. In the figure b (on the right), the number of mergers at a given time, denoted by redshift, are plotted. We assume that the clusters would currently be 13 Gyr old. The model with the varied mass, GMHB, not only dissociates quicker, but has most of its mergers at an earlier time in the cluster's evolution, i.e., at a higher redshift.

functions. GMHA and GMHC both have 10 $M_{\odot}$ and 15 $M_{\odot}$ equal-mass BHs respectively. GMHB has BHs with a mass distribution as generated in our other simulations, which has a corresponding average mass of about 15 $M_{\odot}$ (see §2.1 & §2.2). As can be seen in Table 2, in our simulations, using an equal-mass model artificially increases the number of binaries ejected by almost factor of two. The models with equal-mass BHs, GMHA and GMHC, have about 20% of their BHs ejected as binaries, whereas GMHB has only about 10%. Figure 6 shows how the varied mass spectrum also causes the cluster to reach equipartition at an earlier time and changes the timescales of when mergers occur.

Another point of comparison with Portegies Zwart & McMillan (2000), is the energy distribution of ejected BH binaries. Portegies Zwart & McMillan (2000) found that the ejected BH binaries had a binding energy distribution more or less uniform in the log. We show in Figure 7 a very typical binding energy distribution of the ejected BHs. Clearly, our simulations produce a distribution much more log-normal. Every model we analyzed has a similar distribution of energy.

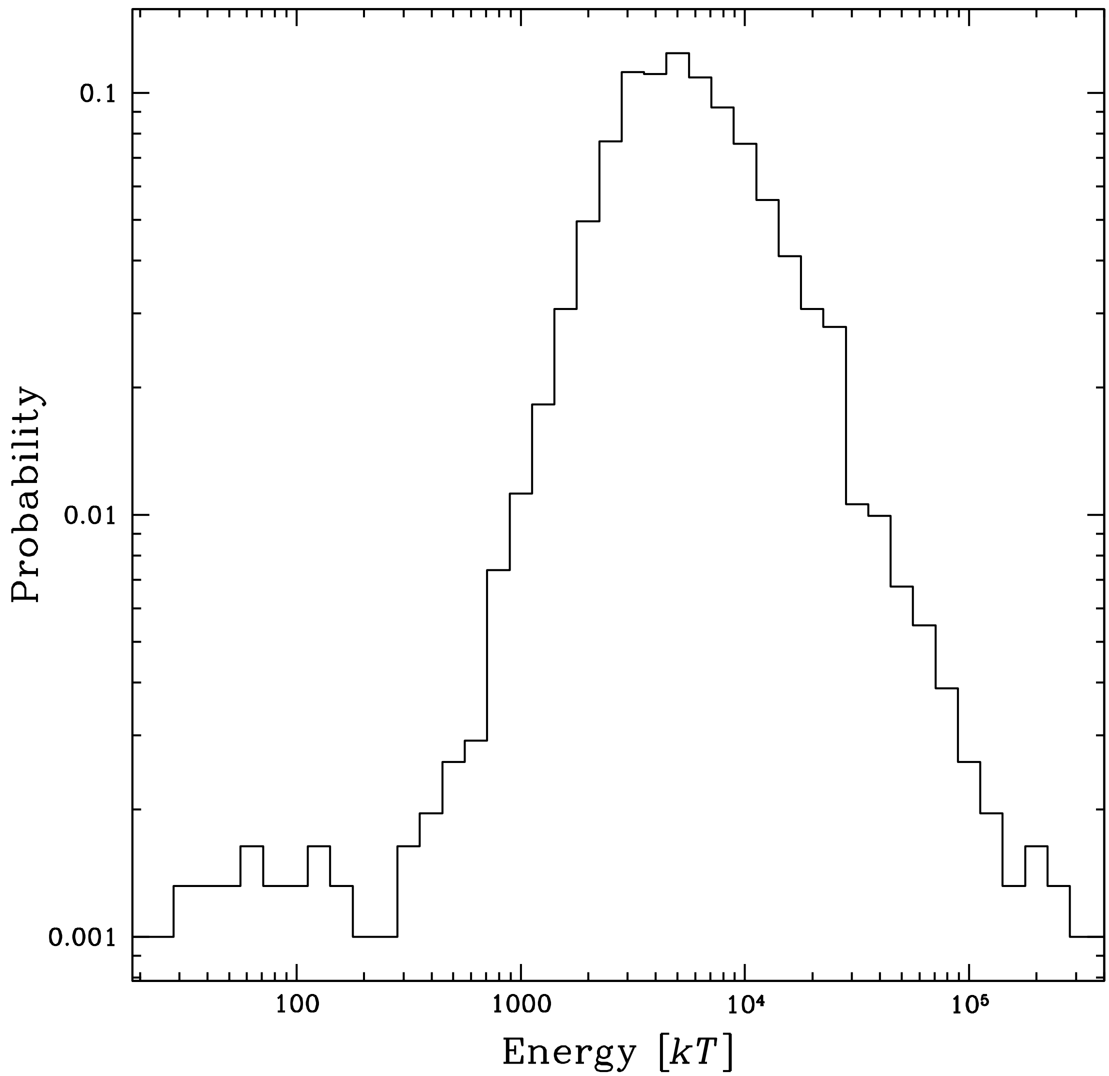

FIG. 7.— Energy distribution of ejected BHs. This is the plotted probability distribution of the energy of all ejected BH-BH binaries from 117 runs of model v2e55k9 in 13 Gyr. The energy is plotted in units of the mean kinetic energy, i.e., where $3/2kT$ is the mean stellar kinetic energy in the fi eld stars in the core of the cluster. All other models have a very similar shape to the one shown above.

4. DISCUSSION

The simulations we have done here are different from previous studies in that we not only account for binary evolution from GR between successive interactions of a fixed group of BHs, but also have used a realistic mass function based on the most recent population synthesis models. Most analytic and numerical simulations thus far assume all BHs are 10 $M_\odot$ equal-mass BHs (see, e.g., Miller & Hamilton (2002b) & Portegies Zwart & McMillan (2000)), or include just one large BH (i.e., GMH04). The mass spectrum of BSR04 gives a slightly higher average mass ($\approx$ 15 $M_\odot$), and also includes at least one significantly large

BH, which is almost always ejected early in the simulations. Even some of the largest BHs formed from mergers, with mass ≈ 120 $M_{\odot}$, are ejected from the cluster when they are formed early enough in the simulation to interact often with other large BHs. The distribution of the binary parameters has a significant effect on the interactions. The low binary fraction results in very few formed triples, and even fewer mergers while in the triple, whether enhanced by the Kozai mechanism or not.

The results of the simulations indicate a greater likelihood of moderate growth in globular clusters than N-body simulations have suggested possible, but show varied results compared with GMH04. First, our larger BH mass function not only makes the ejection of BHs with mass ≈ 100 $M_{\odot}$ very likely, but also helps in the formation of yet even larger BHs. Also, the mass spectrum shows that the chirp masses are larger than would be expected for a cluster with equal-mass BHs.

Despite the demonstrated growth, the probability for an IMBH $\sim 10^3$ $M_{\odot}$ forming directly from BH-BH mergers still seems exceedingly rare, but we should not neglect the fact that the early times at which the evolution of the sub-cluster is done allows for much further growth of the final BH through stellar collisions. Results from Baumgardt, Makino, & Ebisuzaki (2004) suggest that even a somewhat reasonable 200 $M_{\odot}$ BH could grow into a 10^3 $M_{\odot}$ after only a few billion years; well within the current ages of galactic globular clusters.

Of most importance in generating IMBH growth seems to be the initial conditions. Even with many large primordial BHs, if the interaction rate is too large the BHs will be ejected through subsequent hardening rather than merged. The Kozai mechanism, although successful in merging BHs with possibly very little recoil, does not occur often enough to prove fruitful in IMBH formation, although it still may help in globular clusters with very high binary fractions.

Like all models, our simulation has some weaknesses, especially at later times, when the growth of BHs was most significant. At this time the subsystem of BHs gets smaller in number, which means the constant conditions assumed by the simulation are likely to change. The subsystem would eventually start interacting with field stars at a rate comparable to each other and would likely start to take on the characteristics of the cluster core, eventually reaching equipartition. That is why we emphasize the values at equipartition, as opposed to the values at the end of the evolution. The constant conditions also do not allow for the evolution of the entire globular cluster. In many cases it may take as many as 1 Gyr for the evolution of the sub-cluster to complete, and in a few cases as long as a Hubble time. With possibly larger, deeper potential wells early in globular cluster evolutions, the recoil of the BH-BH binaries from hardening or gravitational wave recoil is likely to be less significant, and more growth will occur.

We also make many assumptions about the initial nature of the clusters we studies. Many of the current profiles of clusters that may harbor an IMBH, M15 for example, typically do not follow the King-model profile with flat cores, as their central density continues to increase all the way to the center (Gerssen et al. 2002, 2003). This, of course, is expected, as a large central mass such as an IMBH was found to result in such a profile (Gerssen et al. 2002, 2003; Bahcall & Wolf 1976, 1977), although Baumgardt, Makino, & Hut (2005) demonstrate through direct *N*-body simulations the power-law of the cusp is not nearly as steep as previously expected, and clusters without observed powerlaws can harbor IMBHs as well. Also, it is very likely that today's globular clusters are very much different from the clusters early in their evolution, when the dynamical processes we study were most likely to occur, and before the IMBH has already formed. Therefore, we chose a variety of different centrally concentrated King-models ($W_0 = 7, 9$, or 11) to determine the properties in our simulations.

I want to thank my thesis advisor and academic mentor Fred Rasio for all his help these past few years, and his generous support of my habitual idleness. Without him I would never have realized my potential as a researcher and would not have been able to do a project of this caliber. Also, without him, I would never have been able to finish it. I want to thank Natasha Ivanova for her guidance over the years, and for her giving me her code, and also John Fregeau for his patience editing this thesis, as well as Richard O'Shaughnessy for his support in making this paper appeasing to the LIGO community. I would also like to thank Marc Freitag for providing us with the appropriate code to determine the King Model parameters use in the simulations. Finally, I would like to thank my undergraduate cohorts at Dearborn Observatory for keeping the party going until May 9.

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TABLE 2. RESULTS

Model	Runs	Avg. Largest BH Mass($M_\odot$)	Avg. Largest BH Mergers	Largest BH Mass ($M_\odot$)	Fraction $> 100\ M_\odot$	Frac. Ejected as Singles	Frac. Ejected in Binaries	$\log_{10} T_{\rm equi}$ (yr)	Frac. of Core Mergers in Triples	Core Merger Fraction	Total Number of Mergers
e5e5king7[a] . .	99	62	0.96	164	0.05	0.15	0.03	… [a]	0.01	0.51	10.28
v2e5k7[a]	99	147	5.53	370	0.59	0.73	0.08	… [a]	0.02	0.53	30.35
v3e5k7	65	144	5.46	395	0.60	0.80	0.10	9.69	0.04	0.51	35.43
v3e5k7ej54 . .	53	123	4.45	370	0.47	0.79	0.10	9.58	0.04	0.51	36.51
v3e5k7ej75 . .	62	32	0.55	107	0.02	0.77	0.12	9.46	0.04	0.48	40.98
v5e5k7	26	81	3.15	303	0.38	0.77	0.12	8.63	0.13	0.49	44.23
e5e5king9[a] . .	99	66	1.43	246	0.14	0.33	0.05	… [a]	0.01	0.43	12.79
v2e5k9	96	71	2.26	269	0.22	0.78	0.12	9.66	0.04	0.41	28.38
v3e5k9	27	52	1.44	209	0.19	0.77	0.13	8.86	0.11	0.41	30.11
e55king9[a] . . .	100	106	3.36	330	0.40	0.49	0.05	… [a]	0.01	0.49	19.23
v2e55k9-256 .	20	61	1.70	219	0.15	0.73	0.10	9.08	0.07	0.47	18.20
v2e55k9	117	104	3.81	296	0.43	0.80	0.10	9.39	0.03	0.43	35.36
v2e55k9-1024	38	160	6.39	462	0.55	0.83	0.10	9.59	0.02	0.41	68.87
v2e55k9-2048	37	208	9.16	619	0.68	0.85	0.10	9.72	0.03	0.40	133.08
v2e55k9-100 .	57	143	4.89	405	0.49	0.81	0.09	9.38	0.02	0.47	33.04
v2e55k9-200 .	43	212	5.58	519	0.56	0.82	0.08	9.34	0.02	0.49	29.49
v2e55k9e6 . . .	40	80	2.63	308	0.23	0.79	0.11	9.38	0.03	0.44	37.15
v2e55k9e65 . .	40	56	1.60	285	0.13	0.78	0.12	9.32	0.03	0.41	38.75
v2e55k9e7 . . .	39	37	0.69	217	0.05	0.78	0.12	9.31	0.05	0.40	39.95
v2e55k9e8 . . .	40	37	0.70	186	0.05	0.77	0.12	9.26	0.04	0.40	40.45
v3e55k9	19	66	2.21	276	0.21	0.77	0.13	8.47	0.11	0.36	40.37
v2e6e5k9	46	248	10.28	445	0.89	0.81	0.06	10.04	0.02	0.59	35.33
v3e6e5k9	37	199	7.92	386	0.76	0.81	0.08	9.24	0.03	0.53	40.05
v22e6e5k9[a] . .	46	367	14.78	547	0.98	0.72	0.03	… [a]	0.02	0.76	32.81
v32e6e5k9 . . .	47	373	15.62	509	1.00	0.83	0.05	9.81	0.02	0.72	38.70
e5king9[a]	99	145	4.98	335	0.61	0.56	0.05	… [a]	0.02	0.51	21.37
v2o5k9	35	138	5.23	353	0.60	0.81	0.09	9.29	0.03	0.46	36.63
v3o5k9	21	107	4.10	308	0.43	0.78	0.12	8.31	0.07	0.41	42.10
e5e5king11 . .	99	40	0.76	142	0.05	0.76	0.12	10.09	0.03	0.41	16.23
v2e5k11	49	41	0.76	97	0.02	0.76	0.15	8.76	0.11	0.37	20.31
v3e5k11	14	42	0.71	75	0.00	0.75	0.16	8.42	0.24	0.41	25.79
e55king11 . . .	99	55	1.49	237	0.13	0.79	0.12	9.88	0.02	0.38	23.81
v2e55k11	30	40	0.60	117	0.07	0.77	0.15	8.43	0.09	0.31	29.13
v3e55k11	8	52	1.38	89	0.00	0.75	0.15	8.08	0.17	0.40	34.38
e5king11	99	62	1.71	297	0.18	0.80	0.11	9.80	0.03	0.37	27.48
v2o5k11	53	53	1.26	202	0.17	0.78	0.14	8.29	0.07	0.33	32.11
v2o5k1110 . .	20	52	1.40	210	0.20	0.80	0.14	8.51	0.06	0.30	65.60
v2o5k1111 . .	9	75	2.56	298	0.22	0.82	0.14	8.74	0.06	0.28	130.78
v3o5k11	8	61	1.25	100	0.00	0.74	0.17	7.97	0.15	0.31	39.50
v2e5e7k11 . . .	30	88	3.10	239	0.43	0.79	0.12	7.81	0.06	0.36	42.53
v3e5e7k11 . . .	8	41	0.75	56	0.00	0.74	0.16	7.46	0.14	0.34	53.25
GMHA	41	20	0.95	50	0.00	0.69	0.21	9.71	0.03	0.35	49.39
GMHB	36	68	2.25	291	0.22	0.79	0.12	8.82	0.05	0.38	31.69
GMHC	60	31	1.07	120	0.03	0.68	0.23	8.92	0.03	0.30	46.57

NOTE. — Results of all simulations. Starting from the left the columns are the model name, number of total runs made, the average mass of the largest remaining BH at equipartition, the total number of successive mergers the average largest remaining BH went through before equipartition, the largest BH ever formed which remained in the cluster, the fraction of simulations with a BH with mass $> 100\ M_\odot$ present at equipartition, the fraction of BHs ejected as singles, the fraction of BHs ejected in binaries, the log of time the cluster reached equipartition, the fraction of in core mergers in triples, fraction of total mergers which occurred in the core, and the total number of mergers on average for a single simulation. The halo merger fraction is ≈ 0.00 in all simulations.