Dynamical Instabilities in Extrasolar Planetary Systems

By

Kenneth K. Yu

Submitted to the Department of Physics and Astronomy in partial fulfillment
of the requirements for the degree of

Bachelor of Arts in Physics

at

NORTHWESTERN UNIVERSITY

May 17, 2003

Dynamical Instabilities in Extrasolar Planetary Systems

Kenneth K. Yu

Department of Physics and Astronomy

Northwestern University

Introduction

The existence of planets outside our solar system is one of the greatest discoveries of the 20th century, with implications extending far beyond astronomy and astrophysics. The first extrasolar planet was detected in orbit around 51 Pegasi by Mayor and Queloz in 1995 (Mayor & Queloz 1995). Today, more than one hundred planets are known to exist outside our solar system. These planets are Jupiter-like gas giants and are found in more than ninety known extrasolar planetary systems, and of these systems, twelve are known to contain multiple planets, not unlike our home system. Surprisingly, the overwhelming majority of these extrasolar planets are characterized either by highly eccentric orbits at large orbital distances, or circular orbits with extremely small orbital radii (as small as 0.02 AU, where an AU, or astronomical unit, is defined to be the mean distance between the Earth and the Sun). These observations are in marked contrast to our solar system, where multiple planets move in stable, coplanar, circular orbits about the sun and raises many interesting questions and challenges to the standard model of planetary system formation. In this thesis, I show that the puzzling range of orbital properties found in extrasolar planets, specifically, the observed orbital eccentricity distribution in extrasolar planets, can be explained naturally by dynamical instabilities inherent in multiple-planet systems. I also discuss the advantages and disadvantages of different numerical integration schemes for studying N-body dynamics. Along the way, I point out interesting results from numerical experiments that may have important implications for the stability of multi-body systems and may deserve further study.

Background

In the standard model of planetary system formation (see e.g., Lin & Ida, 1997), Jupiter-like (or "jovian") gas giants are thought to form from condensations of a gaseous disk around a newly formed host star onto rocky cores that assemble through collisions between smaller particles, or "planetesimals." In the solar system, the terrestrial planets (those planets close to the sun with rocky surfaces) are made primarily of elements of high molecular weights, which condense into planetesimals close to the star due to their high escape velocities. The jovian planets are made of elements of low molecular weights, primarily hydrogen and helium, and form from a gas accretion process onto a central core far from the host star (Jupiter is 5.2 AU from the sun). Because the planets form from a circular protostellar disk, one naturally suspects that they should have coplanar, circular orbits. This indeed is what we observe in the solar system. However, the standard model of planet formation does not adequately explain how most of the extrasolar planets discovered can be gas giants with either extremely small orbital radii (as small as 0.05 AU) or high orbital eccentricities (ranging from 0.1 to > 0.9, whereas Jupiter's eccentricity is only 0.05).

Gravitational interactions between planets and the resulting perturbation of planetary orbits have been proposed as a possible explanation for the observed orbital characteristics of extrasolar planets (Rasio & Ford 1996; Marzari & Weidenschilling 1996). In this scenario, jovian-type planets still form in the same way as described in the standard model. However, after the initial formation phase, mutual gravitational interactions between the planets perturb their orbits and bring the planetary system into a period of chaotic evolution, during which the orbits become highly irregular, and which finally results in collision or ejection of the planets. If planets are ejected from the system, conservation of energy requires that the remaining planets be brought in closer to the host star. The final outcome may be that if the remaining planet assumes an orbit that plunges extremely close to the star, which tends to be highly eccentric due to conservation of angular momentum, tidal interactions between the planet and star could result in the circularization of its orbit very close to the star. If the planets collide, they merge to form a larger body in a nearly circular orbit (Ford et al 2001).

For the case of a system of two planets, there exist analytical criteria concerning the stability of the planetary orbits (Gladman 1993). For example, the Hill stability criterion (Gladman 1993) states that two planets are stable against close encounters if initially they move in circular coplanar orbits whose semimajor axes, a_1 and a_2, differ by more than $2\sqrt{3}\,R_H$, where R_H is the Hill radius of the planets, equal to $\overline{a}[(m_1 + m_2)/3M]^{1/3}$, where $\overline{a} = (a_1 + a_2)/2$ is their mean distance from the star of mass M, and m_1, m_2 are the planetary masses. However, for a system of three planets or more, there exist no analytical criteria for stability. Thus to study the stability and evolution of these systems, numerical methods are required.

Summary of Observational Data

The detection of planets outside the Solar System continues to be one of the most exciting recent developments in astronomy and astrophysics. It is expected that the discovery of these extrasolar planets will lead to significant improvements in our understanding of many processes related to planet and star formation, as well as deeper questions such as the existence of extraterrestrial life in the Universe. Figures 1 and 2 summarize the orbital properties of 107 extrasolar planets known today. For a more detailed summary with properties of the host star, see http://www.obspm.fr/encycl/. It is expected that new discoveries will continue to arrive every few months. The majority of these planets (all but one) were detected in radial velocity surveys using spectroscopic methods (Marcy et al. 1997; Korzennik et al. 1997; Mayor & Queloz 1998; Cochran et al. 1997; Butler et al. 1998). Other techniques such as transit photometry (Borucki &

Summers 1984) and astrometry (Gatewood 1996; Pravdo & Shaklan 1996), including interferometric astrometry (Colavita & Shao 1994) have been used to make additional detections and confirm the discovery of planets in radial velocity surveys.

Unexpected Observations

The properties of the extrasolar planets are puzzling (see Fig. 1 & Fig. 2). Most are Jupiter-mass objects in very tight circular orbits or in wider eccentric orbits. The standard model for planet formation in our Solar System cannot explain their orbital properties (see, e.g., Lissauer 1993; Boss 1995; and discussion in the Background section). According to this standard model, planetary orbits should be nearly circular, and giant planets should be found at large distances (> 1 AU) from the central star, where the temperature in the protostellar nebula is low enough for icy materials to condense (Boss 1995, 1996). These simple predictions of the standard model for the formation of the Solar System are at odds with the observed parameters of most detected extrasolar planets. About half of the planets come within 1 AU of the central star. Sixteen planets (e.g. HD 187123, 51 Peg, *t* Boo, *?* And, HD 217107) are in extremely tight circular orbits with periods of only a few days. Eighteen planets (e.g., *?* Cnc, Gl 86, HD 195019, and *?* CrB) have nearly circular orbits with somewhat longer periods, on the order of tens of days. Other companions with wider orbits (e.g., HD 168443, Gl 876, HD 114762, 70 Vir, HD 210277, 16 Cyg B, & 14 Her, etc.) have very large eccentricities (between 0.1 and 0.9) as compared to planets in the solar system. A number of different theoretical models have been proposed to explain the unexpected orbital properties of these extrasolar planets. This thesis will invoke the mechanism of natural dynamical instabilities and show that it can account for the observed eccentricity distribution in extrasolar planets. First, I briefly review the proposed mechanisms.

The four major mechanisms proposed to date to account for the orbital properties of extrasolar planets are 1) secular perturbations in a hierarchical triple system, 2) interaction with a gaseous disk, 3) resonant scattering of a planetesimal disk, and 4) dynamical instabilities in systems with multiple planets.

Secular Perturbations in Hierarchical Triple Systems

If the orbit of a wide binary star system is inclined relative to a planet's orbit by more than $\sim 40^{\circ}$, the relative inclination of the binary star can couple to give a large secular increase in the eccentricity in the planet's orbit (Ford, Kozinsky, & Rasio, 2000; Holman, Touma, & Tremaine 1996; Mazeh, Krymolowski, & Rosenfeld 1996). The amplitude of the perturbation depends on the relative inclination of the orbits, but is independent of the mass of the binary star companion. Once placed on a highly eccentric orbit, the planet may plunge into the star and be destroyed, or its orbit may circularize around the star through tidal dissipation. One should note that this mechanism cannot work for all planets, because many planets are known to be around a *single* star only and this mechanism cannot work without a binary companion star.

Interaction with a Gaseous Disk

Dissipation of orbital energy through viscous drag in the protoplanetary nebula is a second possible mechanism for producing short-period planets and inducing orbital eccentricity through resonant interactions. (The orbital energy E of a planet is given by $E = -\frac{GMm}{2a}$, where G is the universal gravitational constant, M is the mass of the star, m is the mass of the planet, and a is the semimajor axis of the planet's orbit. A decrease in E implies a reduction in a.) This idea of orbital migration was proposed by William Ward (1988), and a detailed discussion of the geometry of resonances and how they give rise to an increase in orbital eccentricity during orbital migration can be found in Ward

1988. Since the orbital migration of the planet tends to accelerate with decreasing distance from the star (gravity follows the inverse square law!), the dissipation must switch off at some critical distance from the star or the planet may be tidally disrupted or swallowed by the star. Possible mechanisms for stopping the inward migration include Roche lobe overflow and tidally coupled dissipation to a rapidly rotating star (Trilling et al. 1998; Lin et al. 1996). Another possibility is that migration halts when the planet gets to the inner edge of a disk limited by a magnetosphere around the star (Lin et al. 1996).

Resonant Scattering in a Planetesimal Disk

Resonant interactions with a disk of planetesimals (planetary embryos) is another possible source of orbital migration. However, this mechanism requires a very high-mass protoplanetary disk for a Jupiter mass planet to migrate inwards all the way to $\sim$ 0.1 AU (Murray et al, 1998). The advantage of this mechanism is that inward migration is naturally halted at short distances from the star when the majority of perturbed planetesimals collide with the star rather than escape on nearly parabolic orbits. Wide, eccentric orbits can also be produced for planets more massive than $\sim$3 Jupiter mass. Because of the large mass of the disk required for this mechanism to operate, it is likely that a second or third planet would also be formed in the disk. It is not understood how a second or a third planet would affect this scenario.

Dynamical Instabilities in Systems with Multiple Planets

The fourth mechanism is based on dynamical instabilities; it operates for systems that start out containing multiple giant planets of comparable masses (Rasio & Ford, 1996). This is the mechanism that this project investigated. The orbits of the planets could become unstable naturally through mutual secular gravitational interactions. This can lead to a dynamical instability and close encounters between the two planets

(Gladman 1993; Chambers, Wetherill, & Boss, 1996). The strong interactions during close encounters can lead to the ejection or collision of the planets. If the pericenter distance of the inner planet is sufficiently small, its orbit can later circularize at an orbital separation of a few stellar radii (Rasio, Tout, Lubow & Livio 1996). This mechanism can produce eccentric orbits naturally: if one planet is ejected, the others may be left in eccentric orbits in order to conserve angular momentum (Ford et al. 2001).

Numerical Methods

As no analytical solutions to the equations of motion exist for systems with more than 2 interacting bodies (the quasi-analytic limit being the restricted three-body problem), I employed numerical integration of the equations of motion for the three-planet systems modelled, with the star-planet interactions as the dominant forces and the effects of the three planets on one another solved exactly and combined with the orbits produced by the star-planet interaction, using the Mixed Variable Symplectic (MVS) algorithm described in Chambers 1999. The calculations were performed using version 6 of the *Mercury* N-body integrator package written by John Chambers (1999). *Mercury* includes as one of its major features a *hybrid* symplectic integrator that combines an efficient variable-timestep algorithm, known as Bulirsch-Stoer (BS) integration, with MVS integration, and is capable of handling close encounters between massive bodies, i.e., Jupiter-sized planets. The most important feature of the BS algorithm for N-body simulations is that it is capable of keeping an upper bound on the local errors introduced due to taking finite timesteps by adaptively reducing the step size when interactions between the particles increase in strength. This, in addition to other properties of this method, (see e.g. Press et al. 1993 for a detailed discussion,) makes it the ideal algorithm for handling close encounters between planetary bodies. The MVS algorithm, a method based on Hamiltonian dynamics, is roughly two orders of magnitude faster than traditional integration techniques (such as Runge-Kutta or Bulirsch-Stoer) while still preserving the energy-conserving property of the system, provided that the assumption of strictly dominant star-planet interaction over planet-planet interaction is satisfied, and that the timestep of the integration remains constant (Chambers 1999). (It is worth noting that MVS is a *fixed* timestep integration scheme, a property that is critical to the energy conserving property of MVS, and hence its speed. However, as I shall show below, this also limits its usefulness.) The first assumption clearly fails when any pair of planets enters into a close encounter with each other, when planet-planet interaction can approach or even exceed star-planet interaction in magnitude. This defect can be remedied by patching together the MVS solution of the equations of motion with the solution by Bulirsch-Stoer for the close encounter portion, and then switching back into MVS upon completion of the close encounter (Chambers 1999). Because Bulirsch-Stoer is a variable timestep integration scheme, it is then necessary to bring the timestep back to the original step size used for the MVS portion of the integration. This is accomplished with smoothing functions developed by Chambers that were found to work very well in his test cases (Chambers 1999). For these reasons, the hybrid MVS-BS integration method was initially chosen for this project, for to complete a statistical exploration of the parameter space of the three-planet problem within the limits of available computing resources, high integration speed was essential. On an Intel *Pentium* 1.7 GHz Linux machine, a typical simulation of a system of three planets with initially circular orbits using hybrid MVS-BS takes about 30 minutes of CPU time for a simulated system evolution time of 10^9 years, whereas it takes traditional BS more than 30 hours to complete the same simulation. Furthermore, this number is highly variable from system to system and can grow exponentially with the system size. Thus, it was hoped that by using this hybrid MVS-BS scheme, the accuracy of the simulations could be improved during the close encounter phase, while taking advantage of the large timesteps allowed by the MVS algorithm in order to save CPU time during normal evolution of the system. However, during the actual numerical experiments, it was discovered that even with MVS coupled with Bulirsh-Stoer, the energy errors still grew unacceptably large (sometimes growing to a few times the initial energy of the system) shortly after a close encounter occurred. It was observed that in the majority of cases, this was often accompanied by ejections of at least one planet. These observations were disconcerting and puzzling. After careful examination of the events leading up to the large growth in energy errors in a large number of these systems with a variety of initial conditions, it was discovered that the energy errors arose from the fixed timestep nature of the MVS algorithm. The problem was essentially one of poor sampling of the orbits near the periastron. When interactions between planets increase the eccentricity of the orbits (which was found to be the tendency), the increased eccentricity leads to a reduced periastron distance, (since $q = a(1-e)$, where q is the periastron distance of an orbit) and the fixed timestep limitation of the MVS method naturally causes a decrease in the accuracy of the integration scheme as the portion of the orbit where a planet interacts most strongly with the star, i.e., near the periastron, becomes poorly sampled. Put another way, because the time the planet spends around the periastron is reduced, fewer integration steps are taken around the periastron as a result of the constant timestep, even though one should really take more timesteps when the orbit is closer to the star, as there is now a much stronger interaction between the star and the planet, and each step introduces a larger correction to the motion of the planet. However, MVS is not capable of taking this into account because of its fixed timestep requirement. As a result, there is a drastic decrease in the accuracy of the integration and a marked increase in energy errors. This limitation of the MVS algorithm also provides an explanation for the much larger number of ejections observed when the hybrid MVS-BS scheme was used as compared to the results obtained when the variable timestep BS scheme was used. To understand this, imagine that a simulated planet is making a high-eccentricity approach to a star, and its orbital motion is corrected periodically by MVS as it makes its approach. When the planet is far from the star, the corrections are small, and it moves rather smoothly toward the star. But as it gets closer, the corrections to its orbit begin to grow in magnitude and now it starts to follow a jagged path. Near the periastron, the corrections to its orbit become large enough that the planet may overshoot its true periastron before the next correction is introduced. By the time the next correction is applied, the planet has already gone on much farther from the star, and the resulting correction to its motion is reduced and its return trajectory from the star is now on a less eccentric orbit. The planet has now achieved a spurious orbit with a much larger periastron distance than its true orbit. The result is an accumulation of energy errors and widening orbit, leading up to ejection of the planet after multiple passes around the star.

This fixed timestep limitation of hybrid MVS-BS was a serious problem as I was interested in high-eccentricity, possibly star-grazing, orbits, which tend to have small periastron distances. In order to maintain the accuracy and integrity of the numerical experiments, the hybrid MVS-BS approach had to be abandoned, and a more traditional integration scheme, namely, Bulirsch-Stoer was adopted. Fortunately, the speed of the Bulirsch-Stoer integration scheme can be increased by a factor of two if the forces acting between the bodies can be assumed to be conservative, as is the case for Newtonian gravity. The form of Bulirsch-Stoer integration taking advantage of this assumption is called *conservative Bulirsch-Stoer*. Conservative Bulirsch-Stoer, while slightly faster than traditional BS and is equally accurate, cannot make up completely for the gain in speed in hybrid MVS-BS. The rest of this thesis will discuss results obtained using the conservative Bulirsh-Stoer integration scheme.

Results

Extensive studies have been done on systems containing two planets (see e.g., Ford et al. 2001; Rasio & Ford 1996; Marzari & Weidenschilling 1996). However, two-planet systems have certain special properties that more general N-body systems do not have. These properties include the separate conservation of energy and angular momentum in the subsystems consisting of a planet and a star (Gladman 1993; Ford et al. 2001). Thus, there is a limit to the range of results obtainable with these systems and the general applicability of those results. In order to extend our understanding of N-body dynamics to more general systems, I have chosen to investigate systems containing three planets, where special properties such as the subsystem conservation of energy and angular momentum no longer apply, and hence represent a qualitatively different type of system. The simplest case of three planets with identical masses but varying orbital parameters was investigated in detail in this project. The results obtained are discussed below.

Close Encounter Time Scale

In general, systems of multiple-interacting bodies are chaotic. Formally, the stability of an N-body system can be characterized by the Lyapunov coefficient, which is defined to be the time it takes for trajectories in phase space to diverge. This value has been calculated for the solar system, and it is estimated to be on the order of 10^8 years (see e.g., Lecar, 2001; Murray & Holman, 1999; Malhotra, 1998; and Lecar, 1996). However, as there is no evidence for significant orbit-crossing in the history of the solar system, and none are predicted to occur for billions more years in the future (apparently the solar system has been stable for billions of years, or we would not be here to study it), another criterion for characterizing the stability of a multi-planet system is needed. A good measure of the stability of a three-planet system for the dynamical properties under study is the *time to first close encounter*, or T_{CE}, and can be defined to be the time it takes from the start of the evolution of a system to the first close encounter between any pair of planets. The precise definition of the distance between planets at which point a close encounter is considered to take place is immaterial (Chambers 1999), but in the following numerical experiments, it was arbitrarily taken to be $3\times R_H$, a distance that is slightly smaller than the Hill stability criterion for systems with two planets (recall Hill's criterion states that for stability in a *two*-planet system, $\Delta \geq 2\sqrt{3}R_H$, where Δ is the separation between a pair of planets). To investigate the stability of three-planet systems as a function of the initial orbital spacing between the planets, the T_{CE} for two representative sets of systems of three identical Jupiter-mass planets, with fixed period ratios, nearly circular initial orbits (e = 0.05 for all three planets), with the inner most planet having an initially 1 or 5 AU semimajor axis, and randomized initial orbital phases and small orbital inclinations (uniformly distributed between 0 and 5^o), have been calculated using the BS2 integration scheme. The orbits were integrated up to the first close encounter, or for 1 billion years of simulated evolution time, whichever occurred first. The results are summarized in Figures 3 & 4. As can be seen clearly in the figures (note the logarithmic scale in time), the average T_{CE} grows approximately as an exponential as the period ratio is increased. Furthermore, the two plots exhibit the same reduction in T_{CE} at period ratios around 1.90, presumably due to resonant interactions between the planets, which may either increase or decrease the stability time scale, depending on the specific resonant interaction involved (see Murray & Dermott 2000, for a detailed discussion of different types of resonances). These results are consistent with those of Chambers (1999). Also apparent from these plots is the large scatter in T_{CE} as the initial orbital conditions are varied. The scatter can give rise to T_{CE}'s that differ by as much as four orders of magnitude for the same period ratio. This reflects the chaotic nature of these multiple-planet systems and suggests that in addition to the orbital spacing between planets, other factors, such as the relative orbital phases of the planets are also important in determining the stability of these systems. Nevertheless, the trend of an exponential growth in instability timescales with increasing initial orbital separation is clearly demonstrated. These relationships can be helpful in determining the plausible range of initial semimajor axes for future studies by enabling one to select model systems that have stability timescales that correspond to theoretically predicted or observationally constrained values.

There is one important, but not immediately apparent feature of the stability/close encounter time plots that may deserve further study in the future. When the T_{CE} calculations were performed, both plots were truncated at a period ratio equal to 2.0. This was compelled by a limitation in available computing time (as the period ratios approached 2.0, the integration time grew from hours to days, unwelcome but consistent with the increase in T_{CE}). At period ratio equal to 2.0, the model systems suddenly became stable on time scales $\geq 10^9$ years (no close encounters occurred during any of these runs). This was a rather unexpected result; as even though the close encounter time is expected to grow exponentially, the scatter in T_{CE} should have produced some systems with $T_{CE} \leq 1$ billion years. However, none such systems were observed. Integrations of systems with period ratios greater than 2.0 (up to 2.5) showed the same result. This suggests that some sort of protection mechanism against close encounters begins to operate when period ratios grow beyond 2.0. Two possible explanations are immediate. First, the sudden increase in stability may be due to another resonance that operates at or near 2.0. But it should be noted that increasing the period ratio from 2.00 to 2.50 showed no apparent change in system stability. If we keep in mind that the large scatter in T_{CE} should produce at least some systems with $T_{CE} \leq 1$ billion years, and that the longest T_{CE} in the systems with $a_{inner} = 1$ AU is 10 *million* years, two orders of magnitude short of the maximum integration time, one will suspect that resonance may not be the true explanation. There is another possible and more interesting explanation. It may be possible that as the period ratios are increased past 2.00, the model systems begin to approach that of a 2-planet system with the third planet acting as a small perturbation on the two-planet system, for which the Hill stability criterion may apply in an approximate sense. The conditions for stability for the two-planet system may be approximately satisfied, and thus producing a sudden increase in stability. As the Hill stability criterion is a *sufficient* condition for stability for two planet systems, this suggests that there may exist a more general stability criterion that reduces to the Hill stability criterion in the limit of small perturbation by a third body. Hill-type stability thus remains a plausible mechanism and merits careful study in the future. In addition, these results suggest that for practical purposes, there may exist an empirical stability boundary even in three-planet systems, where no analytical criterion for stability has been proposed.

Distribution of Final Semimajor Axis and Eccentricity of the Inner Most Planet

The final semimajor axes and eccentricities of the planet that was found to be the closest to the star are plotted in Figure 5 for a representative set of 500 integrations using the same initial conditions as for the 5-AU systems discussed above. Interesting structure seems to emerge in the population of systems and divides the population into four distinct groups: 1. a collection of systems that lie along the bottom of the plot, showing a strong tendency of the final semimajor axes to cluster around 2.7 AU, and is independent of the eccentricity (Type I); 2. a region between 4 and 6 AU that shows a decreasing slope in semimajor axis with increasing eccentricity (Type II); 3. a more or less uniform distribution of semimajor axes between 6 AU and 15 AU (Type III); 4. a sparse population of systems that occupy final semimajor axes $\geq$ 20 AU (Type IV). Even though the exact origins of these structures are not known and merits further study, in Type I, III, and IV systems, the semimajor axes of the planets appear to be independent of the final orbital eccentricity. This is a result that is reminiscent of the two-body problem, where it can be shown analytically that the semimajor axis is only a function of the total system energy, and is independent of the eccentricity, due to the separate conservation of energy and angular momentum. Another possible explanation for the Type I structure seen in the a vs. e distribution is the existence of a resonance protection mechanism. However, an examination of the other survivor of the systems shows little evidence for a simple integer relationship between the periods or semimajor axes for the outer survivor and inner survivor, and does not provide strong evidence for the existence of a resonance interaction. The exact nature of these structures apparent in the relationship between a and e in three-planet systems deserves further investigation.

Absence of Apsidal Resonance

Apsidal resonance is the phenomenon of phase-locking of the apsidal longitudes (the angular positions of the point of closest approach to the star of the planetary orbits as measured from some common reference direction in an inertial reference frame) of two orbits, such that the two planets have a common average rate of apsidal precession, and the angular difference of their apsidal longitudes, $\Delta v \equiv v_1 - v_2$ (where v_1 and v_2 are the longitudes of pericenter of the two planets, respectively) librates around 0^o (Malhotra, 2002). In simpler terms, the periapses of the orbits of two planets undergoing apsidal resonance tends to line up with each other. This behavior is observed in at least two systems with multiple planets in eccentric orbits (n Andromedae and HD 83443) and these systems are suspected of exhibiting secular apsidal resonance (Chiang, Tabachnik & Tremaine 2001; Wu & Goldreich 2002). A plausible mechanism that gives rise to this behavior is proposed by Renu Malhotra (Malhotra, 2002). It was shown that for two planets that are not undergoing other types of resonant interactions and whose orbits are nearly circularly and have small relative inclinations, an impulsive increase in the eccentricity of one planet, perhaps due to the ejection of a third planetary companion, can lead to the phenomenon of apsidal resonance.

Since the ejection of at least one planet was commonly observed in the three-planet systems modeled in this project, the Δv of these systems were computed for the two surviving planets in systems with zero initial eccentricity and small initial eccentricity (e = 0.05). The results are summarized as a cumulative distribution in Figure 6. The cumulative distributions of Δv is nearly linear, and hence show little preference for apsidal alignment (which would appear as a large fraction of the systems having small values of Δv). This suggests that the mechanism of an impulsive increase in orbital eccentricity of one planet may not be a good candidate for explaining the observed eccentricity distribution in naturally occurring systems.

Agreement in Eccentricity Distribution with Observations

The most important results of these studies of three planet-systems is the agreement found in the eccentricity distributions that were produced by natural dynamical interactions in these simulated systems and the eccentricity distribution observed in extrasolar planetary systems (Figure 7). As explained in the Background section, four mechanisms have been proposed to explain the eccentricity distributions in extrasolar planetary systems. By far the simplest and most general of these is the mechanism of dynamical instabilities. For systems of two planets, it had been shown that this mechanism can produce eccentricity distributions that are consistent with observed values (Ford et al. 2001). It remained to be seen that the mechanism could still be invoked to produce eccentricity distributions consistent with observations in three-planet systems. Agreement would add credence to the generality and applicability of the three-planet model, as well as provide greater insight into the dynamics of general N-body systems. To test the three-planet model, I calculated the eccentricity distributions for two representative populations of 500 three-planet systems, each with masses = 1 $M_{Jupiter}$, randomized orbital phases, zero and small eccentricity (*e*=0.05), small random inclinations (uniformly distributed between 0 and 5^{o}), fixed period ratios of 1.73 to avoid any apparent resonances while maintaining a reasonable CPU time requirement (~ 1 day of wall time per simulated system of three planets). The results are plotted with the observed eccentricity distribution in extrasolar planets as cumulative distributions and summarized in Figure 7. As is apparent upon first inspection, the agreement between the calculated values and observed values is encouraging. This result suggests that the observed eccentricity distribution in extrasolar planetary systems can be plausibly explained by dynamical instabilities, which develop very naturally in multi-body systems, and that the three-planet model is indeed a plausible model for the dynamical properties of extrasolar planetary systems.

Acknowledgments

For this work, I would like to acknowledge the guidance and support of Prof. Fred Rasio of Northwestern University, who is the supervisor of this thesis project and whose advice, suggestions, as well as care, made this project possible. I would also like to thank Eric Ford of Princeton University for providing invaluable help during the seminal stages of this project. I would also like to acknowledge the help I received from Prof. Vicky Kalogera and Prof. William Halperin. I acknowledge the support of the WCAS Undergraduate Research Council for providing partial funding for this project. Last, but not the least, I want to thank Elyssa Burg of Northwestern University, who stood by me and encouraged me as I completed the most difficult part of writing this thesis.

References

- Butler, R. P., Marcy, G. W., Vogt, S. S., Apps, K. 1998, *Publications of the Astronomical Society of the Pacific*, **110,** 1389
- Chambers, J. E. 1998, *Earth, Moon and Planets*, **81**, 3
- Chambers, J. E., 1999, *Monthly Notices of Royal Astron. Soc.*, **304**, 793
- Chambers, J. E. 2000, *Bulletin of the American Astron. Soc.*, **32**, 1050
- Chiang, E. I., Tabachink, S., Tremaine, S. 2001, *Astronomical Journal*, **122**, 1607
- Cochran, W. D., Hatzes, A. P., Butler, R. P. & Marcy, G. W. 1997, *Astrophysical Journal*, **483,** 457
- Colavita, M. M., Shao M. in *Planetary Systems: Formation, Evolution, and Detection*. Kluwer Academic, 1994
- Ford, E.B., Havlickova, M. & Rasio, F. A. 2001, *Icarus*, **150**, 303
- Ford, E.B., Kozinsky, B. & Rasio, F. A. 2000, *Astrophysical Journal*, **535**, 385
- Ford, E.B., Rasio, F.A. & Yu, K. K. 2002, *ASP Conference Series*, in press
- Gladman, B. 1993, *Icarus*, **106**, 247
- Gatewood, G.D., 1996, *BAAS188*, 40.11 Abstract
- Holman, M. 1999, *American Astronomical Society, DDA meeting #31*, #03.01
- Korzennik, S. G., Brown, T. M., Contos, A. R., Horner, S., Jha, S., Kennelly, T., Krockenberger, M., Nisenson, P, & Noyes, R. W. 1997, The 10th Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun.
- Lecar, M., Franklin, F.A., Murison, M. 1992, *Astronomical Journal*, **4**, 3, 1230
- Lecar, M. 1996, *Celestial Mechanics and Dynamical Astronomy*, **64**, 1, 163
- Lecar, M., Franklin, F.A., Holman, M.J., & Murray N.J. 2001, *Annual Review of Astronomy and Astrophysics*, **39**, 581
- Lin, D. N. C., Bodenheimer, P. & Richardson, D. C. 1996, *Nature*, **380**, 606
- Lin, D. N. C., Ida, S. 1997, *Astrophysical Journal*, **447**, 781
- Malhotra, R. 1998, *ASP Conference Series*, **149**, 37
- Mayor, M., Queloz, D., Marcy, G., Butler, R., and coauthors. 1995, *IAU Circulars*, **6521**, 1
- Mayor, M. & Queloz, D., Beuzit, J.0L., Mariotti, J.-M., Naef, D., Perrier, C., Sevan, J.-P. in *Protostars and Planets IV (V. Manings Ed.)*, 1998
- Marcy, G., Butler, R. 1997, *Bulletin of the American Astronomical Society*, **29**, 1316
- Murray, N. & Holman, M. 1999, *Science*, **283**, 1877
- Murray, N. & Dermott, S. F. in *Solar System Dynamics*, Cambridge University Press, 2000
- Pravdo, S. H., & Shaklan, S. B. 1996, *Astrophysical Journal*, **465**, 264
- Press, W. H., Tuekolsky, S. A., Vetterling, W. T., Flannery, B. P. in *Numerical Recipes in C: the Art of Scientific Computing, 2nd Ed.*, Cambridge University Press, 1993
- Rasio, F.A., Tout, C.A., Lubow, S.H. & Livio, M. 1996, *Astrophysical Journal*, **470**, 1187
- Trilling D., Benz, W., Guillot, T., Lunine, J. I., Hubbard, W. B., Burrows, A. 1998, *Astrophysical Journal*, **500**, 428
- Ward, W. R. 1988, *Icarus*, **73**, 330

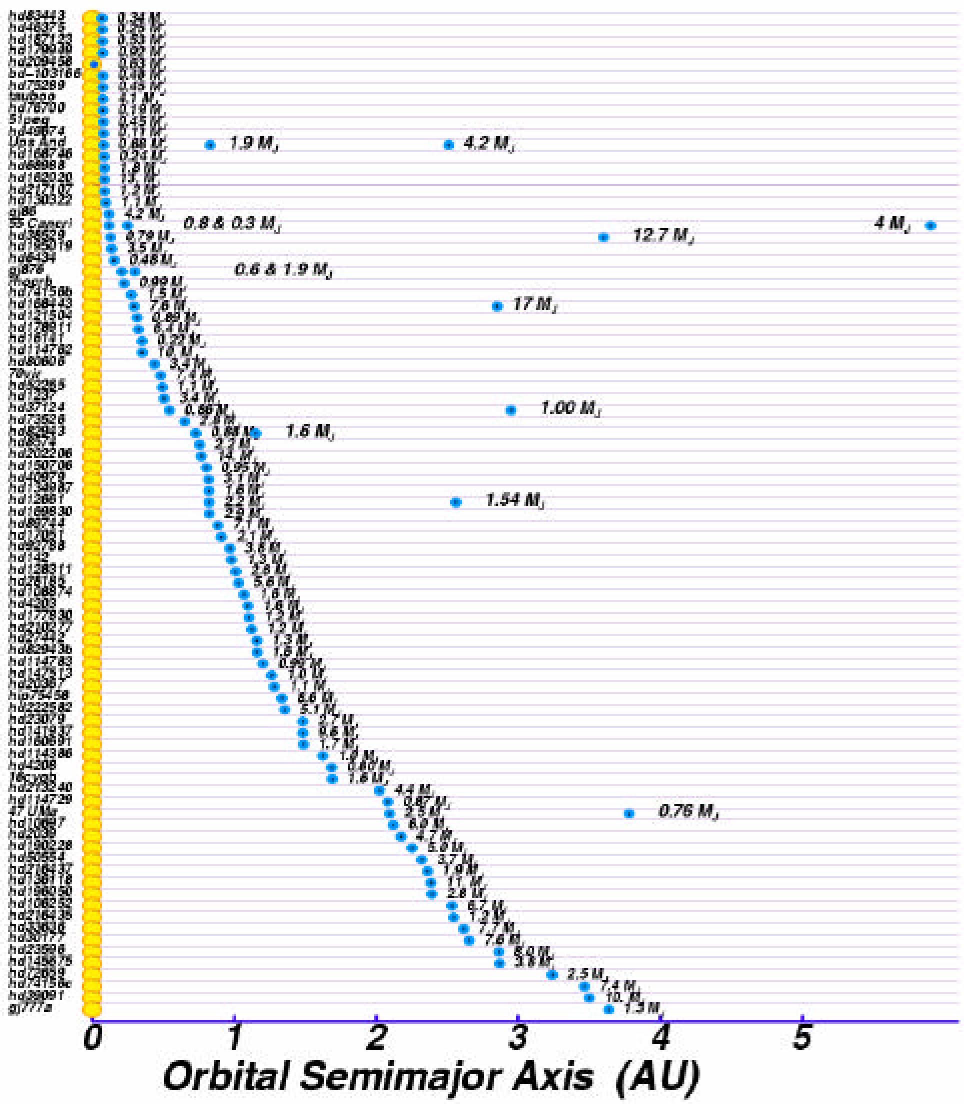

Figure 1: Size Distribution of Extrasolar Planetary Systems (from http://exoplanets.org/massradiiframe.html).

The names of the host stars are plotted along the vertical axis. The semimajor axes of the inner most planets are plotted along the horizontal axis. Blue dots indicate planets. The text next to each dot gives the mass of the planet. Extrasolar planetary systems exhibit a wide range of sizes. Ten systems shown here have multiple planetary companions.

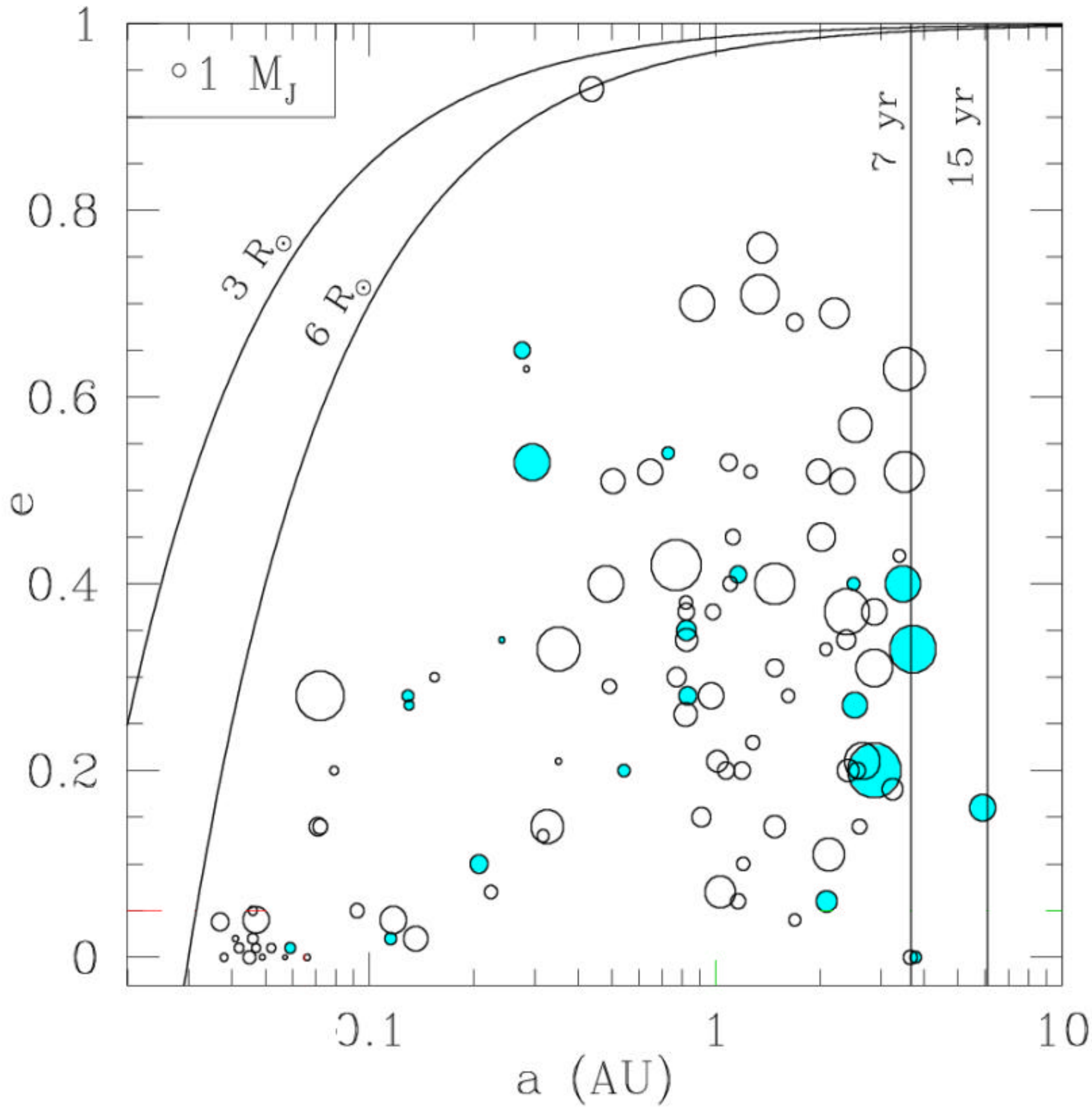

Figure 2: Dynamical Properties of Observed Extrasolar Planets.
Each circle represents a planet. Plotted along the horizontal axis are the semimajor axes of the planets in AU. Plotted along the vertical axis is the eccentricity of planetary orbits. The area of each circle is directly proportional to the mass of the planet. Solid circles represent planets in systems with more than one planet. Note that the distribution of planets into two clusters of planets: those with small semimajor axes and low eccentricity, and those with large semimajor axes with high eccentricity.

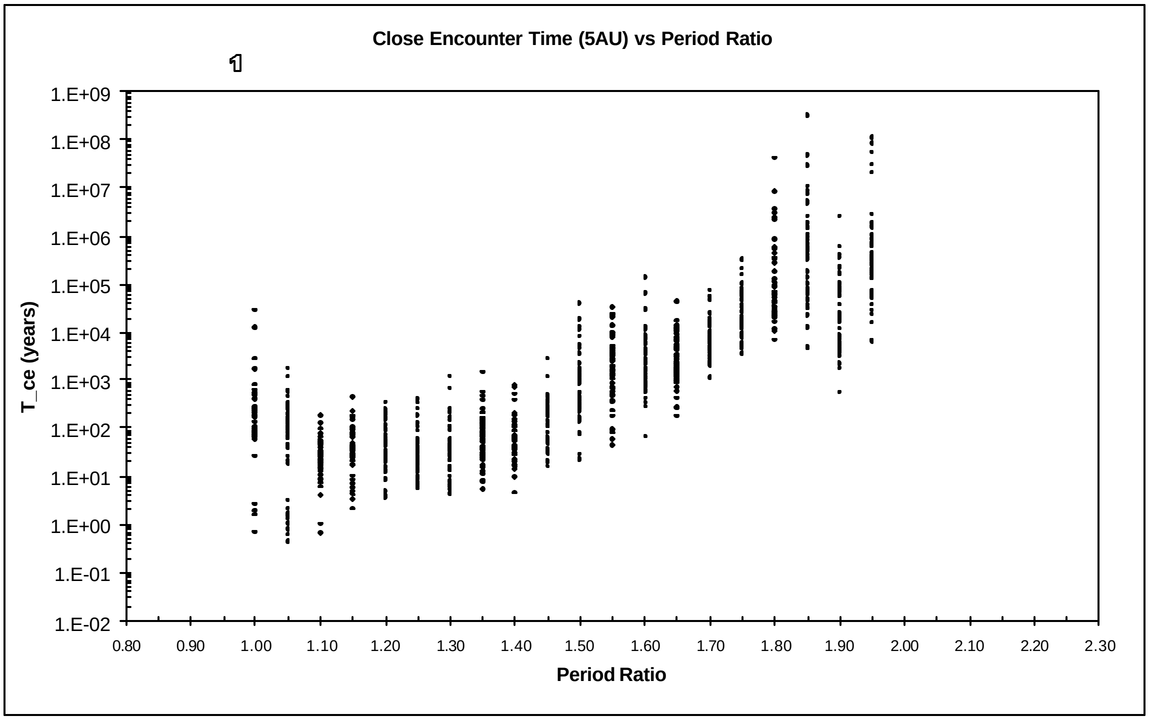

Figure 3: Stability Time Scale for $a_{inner} = 5AU$

This plot shows the close encounter time, T_{CE}, for systems whose inner planet has an initial semimajor axis of 5 AU. T_{CE} here is defined to be the time to the first close encounter between any pair of planets, where the separation between the planets becomes less than $3 \times R_H$.

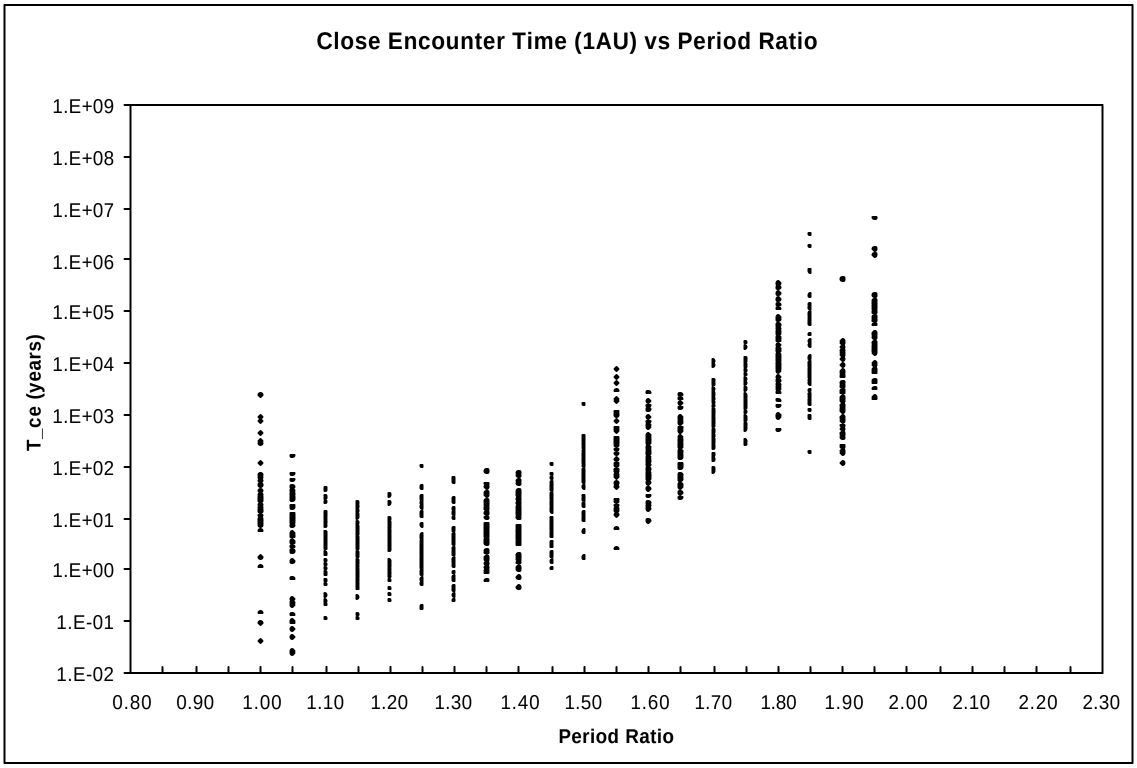

Figure 4: Stability Time Scale for $a_{inner} = 1\,AU$

This plot shows the close encounter time, T_{CE}, for systems whose inner planet has an initial semimajor axis of 1 AU. T_{CE} here is defined to be the time to the first close encounter between any pair of planets, where the separation between the planets becomes less than $3 \times R_H$. Note the striking resemblance to the T_{CE} plot for $a_{inner} = 5$ AU. The two plots exhibit the same features, but T_{CE} in the 1 AU plot is scaled down by roughly an order of magnitude.

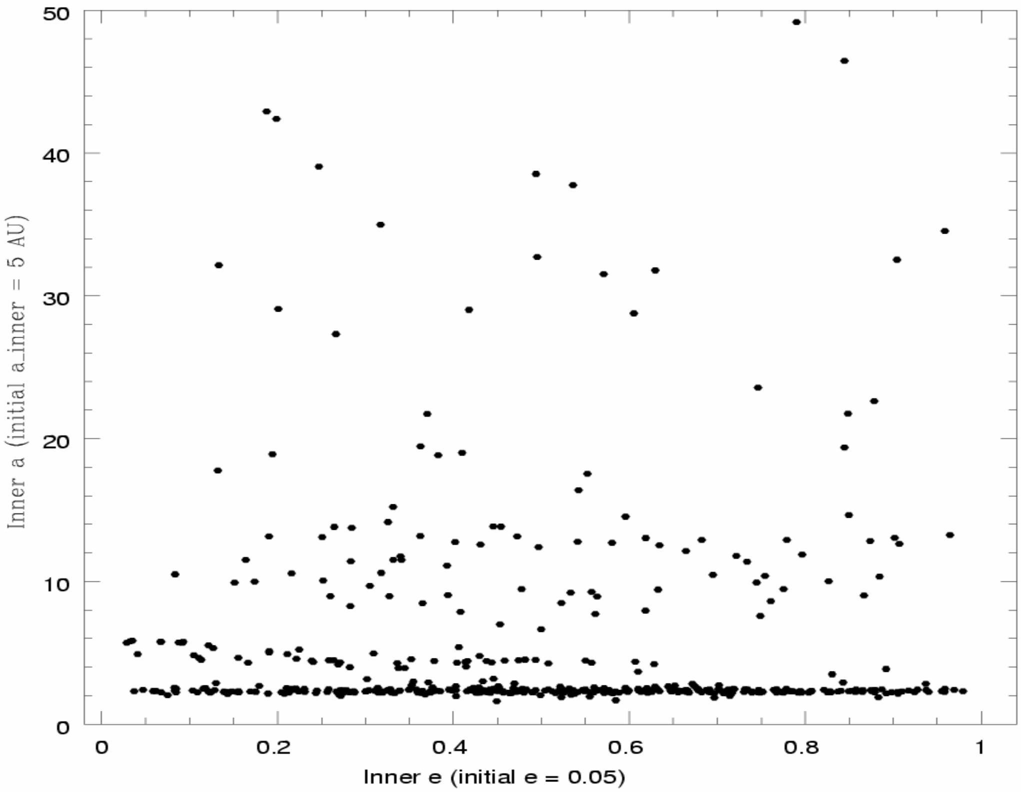

Figure 5: Scatter Plot of Final Inner-Planet Semimajor Axis vs. Eccentricity
The horizontal axis represents the eccentricity of the inner most planet at the end of a simulation. The vertical axis shows the semimajor axis of this planet in AU. Each dot represents a simulated system with identical initial orbital parameters, except for phase and inclination. Note that there is a clear structure to the scatter plot, which may be roughly grouped into four regions. 1) A collection of systems along the bottom of the plot shows a strong tendency of the final semimajor axes to cluster around 2.7 AU, and is independent of the eccentricity. 2) A region between 4 and 6 AU shows a decreasing slope in semimajor axis with increasing eccentricity. 3) A more or less uniform distribution of semimajor axes between 6 AU and 15 AU. 4) A sparse population of systems that occupy final semimajor axis $\geq$ 20 AU.

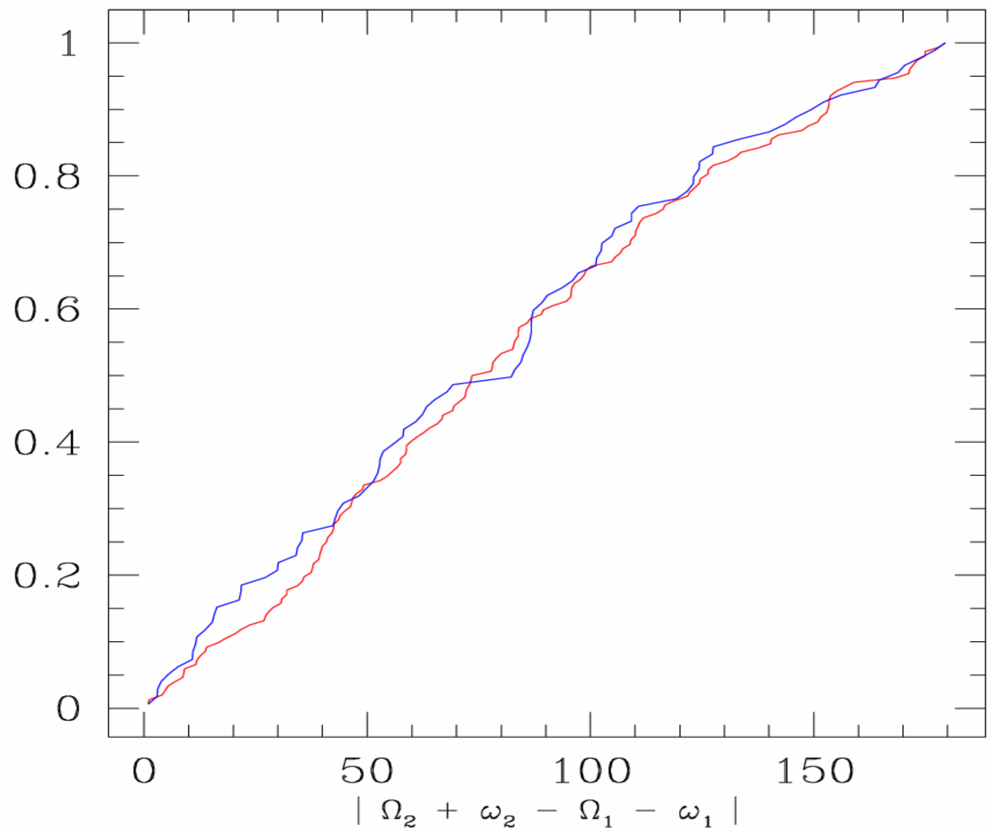

Figure 6: Distribution of Difference in Apsidal Longitudes for Systems with Two Surviving Planets.
Plotted along the horizontal axis is the absolute value of $\Delta\nu \equiv \nu_1 - \nu_2$. The vertical axis indicates the fraction of systems with apsidal-longitude difference $\leq \Delta\nu$. The red line represents values calculated for systems with zero initial eccentricity, and the blue line represents values calculated for systems with small initial eccentricity (e = 0.05). The distributions are nearly uniform and exhibit little evidence for apsidal resonance.

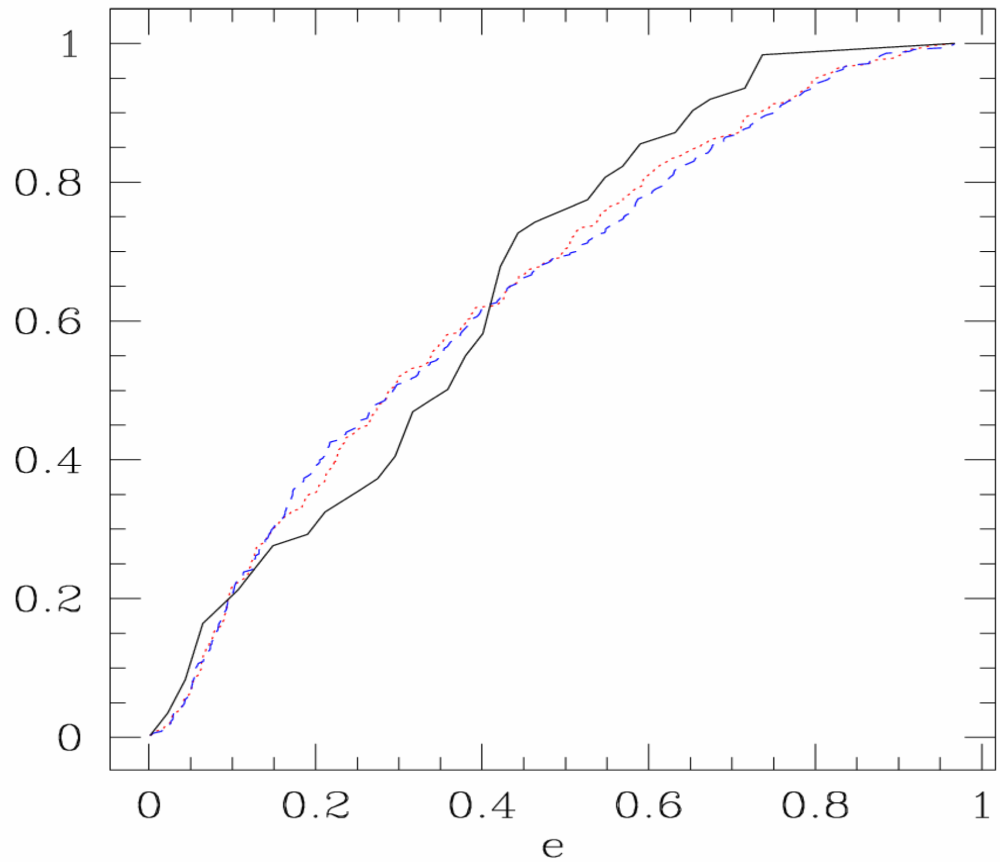

Figure 7: Cumulative Distributions of Observed and Calculated Orbital Eccentricity for the Innermost Planet.

The solid line represents the cumulative distribution of observed eccentricities in extrasolar planets. The dashed and dotted lines correspond to results obtained from the numerical integration of the equations of motion. The red line corresponds to systems with zero initial eccentricity, and the blue line corresponds to systems with small initial eccentricity (e = 0.05). The agreement between the observed values and values predicted from the model systems is striking.