(In)efficiency in Information Acquisition and Aggregation through Prices*

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Abstract

The paper studies the interaction between traders' acquisition of private information and the aggregation of information in financial markets. We consider a canonical market microstructure in which partially-informed traders compete in schedules and prices partially aggregate the traders' private information. Before submitting their demand schedules, traders acquire information about the long-term profitability of the traded asset. We show that, when the errors in the traders' signals are correlated, policies that induce the traders to submit the efficient schedules when the traders' private information is exogenous do not necessarily induce them to collect the efficient amount of private information. In particular, we identify conditions under which such policies induce over-investment (alternatively, under-investment) in information acquisition, relative to what is efficient. We find that, as information technology reduces the cost of acquiring information, the economy eventually moves to a regime with excessive information acquisition. Finally, we show that, generically, there exist no policies based on the price of the financial asset and the volume of individual trades that implement efficiency in both information acquisition and trading. Such an impossibility result, however, turns into a possibility result if taxes/subsidies can condition directly on the information acquired by the traders.

Keywords: information acquisition, aggregation through prices, information externalities, team problem.

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1 Introduction

Improvements in information technology are reducing the cost of acquiring and processing information. This cost reduction naturally raises the question of whether such improvements are likely to contribute to higher welfare or benefit a few at the expenses of society at large. Such concerns are at the heart of various policy proposals to "put sand in the wheels" of financial markets as a way of limiting speculative trading facilitated by asymmetric information (the Tobin tax being a prominent example).

In this paper, we present a tractable framework to study both the positive and normative issues of interest. In particular, we characterize the sources of inefficiency in the collection of private information prior to trading and relate them to possible inefficiencies in the limit orders that traders submit in financial markets, given available information. We first show that, when traders' private information is exogenous, the equilibrium usage of information is inefficient. That is, the limit orders that traders submit in equilibrium fail to maximize welfare, given the dispersion of private information. However, the inefficiency can be corrected with appropriate (non-linear) taxes/subsides on the trades. We then show that, when the traders' private information is endogenous, policies that induce efficiency in the usage of information (equivalently, that induce the traders to submit the efficient limit orders) need not induce the traders to collect information efficiently. We identify conditions under which traders over-invest in information acquisition as well as conditions under which they under-invest. Finally, we show that, generically, there exists no tax/subsidy scheme, measurable in the price of the financial asset and in the volume of individual trades, that induces efficiency in both information acquisition and trading.

Our model is a canonical linear-quadratic-Gaussian financial microstructure à la Grossman and Stiglitz 1980 in which a unit-mass continuum of traders compete by submitting a collection of generalized limit orders (equivalently, a demand schedule). The traders face uncertainty about the asset's fundamental value, as well as the value that other investors in the market (noisy traders, high-frequency traders, hedge-fund managers and the like) assign to the asset. Before submitting their generalized limit orders, each trader receives a private signal about the asset's fundamental value whose noise is endogenous and correlated across traders. Such a correlation may originate, for example, in the traders paying attention to common sources of information, which have source-specific noise. This generalization, which is more in line with what seems relevant in practice, has important implications for the (in)efficiency of the equilibrium acquisition and usage of information, as we discuss further on.[1]

Our first main result is that, except in very special cases, and absent policy interventions,

[1]Typically, the literature assumes that the noise in the agents' signals is iid across agents.

the market does not use the information it collects efficiently. As in Vives 2017, the inefficiency originates in the interaction between two externalities. First, traders do not account for how their orders affect the co-movement between the equilibrium market-clearing price (and hence the equilibrium asset allocations) and the various fundamental shocks that are responsible for different agents' payoffs (a pecuniary externality). Second, traders do not account for the fact that a collective change in limit orders may induce a change in the information contained in the equilibrium price, which in turn affects other agents' ability to align their trades with the asset's fundamental value (a learning externality).

The pecuniary externality makes the traders overreact to their private information, whereas the learning externality makes them under-react to it. The knife-edge case in which the two externalities cancel each other out obtains when the equilibrium demand schedules are perfectly inelastic (such as in a Cournot game in which traders are restricted to submitting market orders). When the equilibrium schedules are downward sloping, the pecuniary externality dominates and the equilibrium trades feature excessive sensitivity to the traders' private information. When, instead, the equilibrium schedules are upward sloping, the learning externality dominates and the sensitivity of the equilibrium limit orders to the traders' private information is inefficiently low. Interestingly, as the precision of the traders' private information grows (for example, due to a reduction in the cost of information facilitated by technological progress), the pecuniary externality gains weight in relation to the learning externality.[2] We show that, no matter whether traders over- or under-respond to their private information, the aforementioned inefficiencies in the equilibrium usage of information can always be corrected using a (non-linear) tax-subsidy scheme contingent on both the equilibrium price of the asset and the volume of individual trades.

Our second main result is that inducing the traders to trade efficiently does not guarantee that they acquire private information efficiently prior to trading. In particular, suppose that the planner could enforce the efficient usage of information by constraining the traders to submit the efficient demand schedules. The traders would then over-invest in information acquisition when the efficient schedules are downward sloping and under-invest in information acquisition when they are upward sloping. In other words, when the pecuniary externality prevails in the usage of information, so that the traders over-respond to their private information, inducing the traders to trade efficiently induces them to over-invest in the acquisition of information. When, instead, the learning externality prevails, in the absence of any policy

[2] Provided that the noise in the traders' information is not too large, when the precision of the traders' information is relatively low, the learning externality dominates and the demand schedules are upward sloping, whereas the opposite is true (i.e., the pecuniary externality dominates and the demand schedules are downward sloping) for high levels of precision.

intervention, traders under-respond to private information. In this case, forcing them to trade efficiently would induce them to under-invest in the acquisition of private information. The inefficiencies in the collection of information thus parallel those in the usage of information. Importantly, these results hinge on the noise in the agents' information being correlated among traders. If such noise were uncorrelated, holding fixed the efficient demand schedules, the only effect of an increase in the precision of the traders' private information on welfare would be through the reduction in the dispersion of individual trades around the average trade. However, when the traders submit the efficient limit orders, the private and the social value of reducing such a dispersion coincide, in which case efficiency in the usage of information implies efficiency in the acquisition of information.

We also show that, if traders could be trusted to submit the efficient demand schedules, then an appropriate tax-subsidy scheme, linear in individual expenditures on asset purchases, would induce the efficient collection of private information.

Next, we show that, absent any policy intervention, as information technology makes the collection of information cheaper, the economy eventually enters into a regime of over-investment in information acquisition and excessive sensitivity of the equilibrium trades to private information. In other words, the secular trend of improvement in information technology[3] may have the undesirable effect of enticing over-investment in information acquisition and over-reaction to it in the trading of financial assets. Hence, putting "sand in the wheels" of financial markets can be particularly valuable precisely when the cost of information acquisition is low.

Finally, we show that, generically, there do not exist (differentiable) tax-subsidy schemes contingent on the price of the traded asset and on the volume of individual trades that induce efficiency in both the collection and the usage of information. This impossibility result, however, turns into a possibility result if the tax-subsidy scheme can condition directly on the quality of information acquired by the traders. In other words, when individual investments in information acquisition are verifiable (as when the traders purchase information from known sources), efficiency in both information acquisition and trading can be induced with policies with familiar contingencies. When, instead, such investments are not verifiable, the policy maker may either have to compromise between inducing efficiency in information acquisition and in trading, or resort to non-standard policy interventions.[4]

[3]See, for example, Nordhaus 2015 on the sharp decline in the cost of computation (and therefore of information processing). See also Gao and Huang 2020 and Goldstein, Yang, and Zuo 2020 for the effects of the dissemination of corporate disclosures over the internet on the production of information by corporate outsiders.

[4]The latter may include making the tax on individual trades depend on other traders' limit orders and/or conditioning on the fundamental value of the asset, beyond what can be learned through the asset's price,

Related Literature

The paper is related to several strands of the literature. The first strand is the literature investigating the sources of inefficiency in the equilibrium usage of private information when information is exogenous. See, among others, Vives 1988, Angeletos and Pavan 2007, Amador and Weill 2012, Myatt and Wallace 2012, and Vives 2017. Among these works, the closest to ours is Vives 2017 who also studies inefficiency in information aggregation through prices when traders submit demand schedules, but in a setting in which the traders' private information is exogenous.

The second strand is the literature on information acquisition in financial markets. See Diamond and Verrecchia 1981 and Verrecchia 1982 for earlier contributions. More recently, Peress 2010 examines the trade off between risk sharing and information production, whereas Manzano and Vives 2011 studies information acquisition in markets with correlated noise, while Kacperczyk, Van Nieuwerburgh, and Veldkamp 2016 studies information acquisition in markets with multiple risky assets. Dávila and Parlatore 2019 study the effect of trading costs on information aggregation and acquisition. None of these papers, however, studies inefficiencies in information acquisition and how the latter relate to inefficiencies in trading.[5]

In particular, Vives 1988 shows that, in a Cournot economy in which a continuum of privately-informed traders with conditionally independent signals submit market orders, both the decentralized acquisition of information and the equilibrium trades are efficient. In the present paper, we show that the same result extends to economies in which the information collected in equilibrium is subject to correlated noise, provided that the traders are restricted to submitting market orders instead of richer supply/demand functions. When traders submit market orders neither the pecuniary externality nor the learning externality of conditioning on prices is present and efficiency obtains.

Colombo, Femminis, and Pavan 2014 show that efficiency in actions does not imply efficiency in information acquisition when payoffs depend on the dispersion of individual actions around the average action. In the present paper, we consider a richer setting in which agents compete in schedules and where information is partially aggregated in the equilibrium price. We show that, even in the absence of externalities from the dispersion of individual actions

which often appears unfeasible as it requires to wait till such value is publicly revealed.

[5]See also the literature on the Grossman-Stiglitz paradox, namely on the (lack of) incentives to acquire information when prices are fully revealing (see Grossman and Stiglitz 1980, and Vives 2014 for a potential resolution of the paradox). Related is also the literature on strategic complementarity/substitutability in information acquisition (see, among others, Ganguli and Yang 2009, Hellwig and Veldkamp 2009, Manzano and Vives 2011, Myatt and Wallace 2012, and Pavan and Tirole 2021).

around the mean action, efficiency in information usage does not imply efficiency in information acquisition when the noise in the agents' signals is correlated. As anticipated above, that noise is correlated is important. As shown in Vives 2017, when the noise in the agents' signals is independent across agents, policies that correct inefficiencies in the usage of information induce efficiency in the acquisition of information, despite the (imperfect) aggregation of information made possible by the limit orders. Efficiency in the usage of information also implies efficiency in information acquisition in the macro business-cycle economies considered in Angeletos, Iovino, and La'O (2020). In these economies, prices imperfectly aggregate information, as in our paper, but agents have access to complete markets that fully insure them against any idiosyncratic consumption risk. In contrast, in our economy, traders consume the returns to their own investments and policies that correct inefficiencies in the usage of information need not induce efficiency in the collection of information.[6]

The third strand is the recent literature analyzing the impact of technological progress on the collection of information and its usage in financial markets. Farboodi, Matray, and Veldkamp 2018 show that the growth of big data, combined with the size distribution of firms, can lead to a decline in price informativeness for smaller firms. Peress 2005 shows that a declining cost of information collection is outweighed by a parallel decline in the cost of entry to financial markets and the interaction between the two can explain several empirical anomalies. Malikov 2019 shows that falling information costs can actually contribute to a rise in passive investment by reducing the cost of, and therefore the returns to, stock picking. Several papers (see, among others, Azarmsa 2019, Mihet 2018, and Kacperczyk, Nosal, and Stevens 2019) show that technological progress that facilitates the collection of information can lead to increasing levels of inequality. Unlike most of the work in this literature, we focus on the normative implications of technological improvements in the collection of information.

Finally, in this paper, we assume that higher investments in information acquisition can reduce the agents' exposure to correlated noise in information. Recent work by Woodford 2012a, Woodford 2012b, and Nimark and Sundaresan 2019 shows that rational inattention can also explain the agents' exposure to correlated noise and that the equilibrium of a rationally-inattentive economy shares several features with those of an economy in which the agents' use of information is "biased" in the sense of prospect theory. Particularly related in this respect is Frydman and Jin 2020. That paper demonstrates how rational inattention can

[6] A similar conclusion holds in Colombo, Femminis, and Pavan 2021. That paper considers an economy in which agents can perfectly insure against any idiosyncratic consumption risk, but where production is affected by investment spillovers. They show, among other things, that familiar taxes-subsidies linear in revenues that correct for market power induce efficiency in production but not in the acquisition of information. Instead, more sophisticated Pigouvian taxes where the marginal rates depend on aggregate output can induce efficiency in both the usage and the acquisition of information.

lead to endogenous bias in valuation, and that the noise in perception is closely linked to the bias in perception. Our paper shares with this literature the property that investments in information acquisition also affect the agents' exposure to correlated noise, something that, from the perspective of an outside observer, may look like a bias in decision making.

Organization. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 compares the equilibrium to the efficient usage of information, identifies the sources of the inefficiency, and shows how certain tax-subsidy schemes may restore efficiency in trade. Section 4 identifies inefficiencies in information acquisition and discusses possible policy corrections. Section 5 concludes. All proofs are in the Appendix at the end of the document.

2 Model

In this section, we lay out the model and describe the traders' choice of demand schedule and information-acquisition problems.

2.1 Trading environment

The market is populated by a continuum of traders, indexed by $i \in [0,1]$, and a representative investor, who can be interpreted as representing a sector of competitive liquidity suppliers, trading a homogenous and perfectly divisible asset. Let x_i denote the demand for the asset by trader i and $\tilde{x} = \int_0^1 x_i di$ the aggregate demand for the asset by all the traders. The representative investor's payoff from supplying $\tilde{x}$ units of the asset at price p is given by

$$\Pi = (p - \alpha + u)\,\tilde{x} - \beta\frac{\tilde{x}^2}{2},$$

where α and β are positive scalars, and where $u \sim N(0, \sigma_u^2)$. The term $\alpha - u$ proxies for the investor's opportunity cost from unloading the asset. The term $\beta\tilde{x}^2/2$, instead, is a quadratic trading cost that proxies for the investor's risk aversion, or, more generally, for all sort of possible limits to arbitrage opportunities.

Each trader i's payoff from purchasing x_i units of the asset at price p is given by

$$\pi_i = (\theta - p)\,x_i - \lambda\frac{x_i^2}{2},$$

where λ is a positive scalar, and where $\theta \sim N(0, \sigma_\theta^2)$. The term θ proxies for the traders' gross common value from purchasing the asset, whereas the term $\lambda\frac{x_i^2}{2}$ is a quadratic trading cost whose role parallels the corresponding term in the representative investor's payoff.

To simplify the derivation of the equilibrium formulas, we assume that the variables θ and u are independently distributed. The results, however, extend to the case where they are imperfectly correlated. For notational purposes, given any Gaussian random variable h with variance σ_h^2, hereafter we denote by $\tau_h \equiv 1/\sigma_h^2$ the variable's precision.

2.2 Information

For simplicity, we assume that the representative investor knows u (alternatively, we interpret $\alpha - u$ as the investor's expected opportunity cost from selling the asset). The traders, instead, do not know θ. They privately collect information about θ prior to submitting their generalized demand schedules, but also condition the latter on the information that the market-clearing price contains about θ (that is, account for the fact that the equilibrium price imperfectly aggregates the traders' dispersed information about θ).

Formally, we assume that each trader observes a (possibly large) collection of Gaussian signals about θ differing in their noises and in the extent to which such noises are correlated among traders. Such signals are summarized in a uni-dimensional statistic

$$s_i = \theta + \epsilon_i$$

where

$$\epsilon_i = f(y_i)(\eta + e_i)$$

is a combination of idiosyncratic and correlated noise. Precisely, the noise variable $\eta \sim N(0, \sigma_\eta^2)$ is perfectly correlated among the traders and can be thought of as originating in the imperfect information contained in the common sources the traders attain to (as, e.g., in Myatt and Wallace 2012). The variables $e_i \sim N(0, \sigma_e^2)$, instead, are i.i.d. among the traders and can be interpreted as idiosyncratic disturbances. The variables $(\theta, u, \eta, (e_i)_{i\in[0,1]})$ are jointly independent. The total noise in trader i's information ϵ_i depends on (η, e_i) but also on the agent's information acquisition activity $y_i \in \mathbb{R}_+$. Depending on the context, the latter can be interpreted as the amount of information acquired by the individual on a market or by the attention allocated to exogenous sources of information. The cost of y_i is given by a differentiable function $\mathcal{C}(y_i)$, with $\mathcal{C}'(y_i), \mathcal{C}''(y_i) > 0$ for all $y_i > 0$.

The idea behind the above information structure is that traders learn from a variety of (Gaussian) signals of different precision whose noise is imperfectly correlated among the traders. It is well known that, in Gaussian environments, each trader's belief about θ is summarized by a sufficient statistic whose noise is a combination of a perfectly correlated term and a perfectly idiosyncratic one. That is, any symmetric cross-sectional distribution of Gaussian posteriors can be generated by giving each agent a perfectly private and a perfectly public signal. Because, in this environment, the traders condition on the price, they do not need to form expectations about the aggregate action in the market. As a result, any such pair of signals can in turn be summarized in a one-dimensional statistic of the form $s_i = \theta + k_\eta \eta + k_e e_i$, for some scalars k_η, k_e. For our purposes, we assume that these coefficients are functions of the traders' information acquisition policy. More acquisition reduces the trader's exposure to both types of noise. For tractability, we assume that the marginal effect of more acquisition on the reduction of the influence of both noises is the same, with the function f taking the form $f(y) = y^{-1/2}$. Such an assumption allows us to express the precision of the combined noise term ϵ as

$$\tau_\epsilon(y) = \frac{y\tau_e\tau_\eta}{\tau_e + \tau_\eta}.$$

2.3 Timing

At $t = 0$, traders simultaneously make their information acquisition decisions y_i. A $t = 1$, traders privately observe their signals s_i. At $t = 2$, both the traders and the representative investor simultaneously submit their schedules. At $t = 3$, the market clears, the equilibrium price is formed, the equilibrium trades are implemented, and payoffs are realized.

2.4 Equilibrium schedules

The representative investor's (inverse) supply of the asset is given by $p = \alpha - u + \beta\tilde{x}$. Equivalently, his supply schedule is given by

$$\tilde{x} = \frac{1}{\beta}(p + u - \alpha).$$

Given her private information $I_i = (y_i, s_i)$, trader i's demand schedule maximizes, for each price p, the trader's expected payoff

$$\mathbb{E}\left[(\theta - p)\, x_i - \lambda\frac{x_i^2}{2}|I_i, p\right]$$

taking into account how the price $p = P(\theta, u, \eta)$ co-moves with the traders' fundamental value θ, the representative investor's supply shock u, and the common noise η in the traders' information. The solution to this problem is the demand schedule given by

$$X(p; I_i) = \frac{1}{\lambda}(\mathbb{E}[\theta|I_i, p] - p) \tag{1}$$

where $\mathbb{E}[\theta|I_i, p]$ denotes the trader's expectation of θ given the quality of the trader's information, as proxied by y_i, the realization s_i of the trader's signal, and the price p.

2.5 Information acquisition

At $t = 0$, each trader selects y_i to maximize his expected profit

$$\mathbb{E}\left[\left(\theta - p - \frac{\lambda}{2}X(p; I_i)\right) X(p; I_i)|y_i\right] - \mathcal{C}(y_i)$$

where the expectation is over (s_i, θ, p), given y_i. Following the pertinent literature, we focus on equilibria and on team-efficient allocations (defined below) in which the market-clearing price $p = P(\theta, u, \eta)$ is an affine function of all aggregate variables (θ, u, η), and where all agents acquire information of the same quality (equivalently, pay the same attention to all relevant sources), and follow the same rule to map their information into the demand schedules.

3 Inefficiency in the Usage of Information

Fixing the precision of the traders' private information τ_ϵ (equivalently, their information acquisition activity $y_i = y$ all i), we start by solving for the equilibrium schedules (equivalently, for the decentralized equilibrium usage of information). We then compare the equilibrium schedules to their efficient counterparts (equivalently, to the decentralized efficient use of information), and discuss the nature of the inefficiency in the usage of information, and possible policies alleviating the inefficiency.

3.1 Equilibrium usage of information

In any symmetric equilibrium in which the price is an affine function of (θ, u, η), each trader's demand schedule is an affine function of her private signal s_i and the price p. That is,[7]

$$x_i = X(p; I_i) = a^* s_i + \hat{b}^* - \hat{c}^* p$$

for some scalars $(a^*, \hat{b}^*, \hat{c}^*)$ that depend on the exogenous parameters of the model, as well as on the quality of the agents' information $y_i = y$. Aggregating across traders, we then have that the cumulative aggregate demand is equal to

$$\tilde{x} = \int x_i di = a^*(\theta + f(y)\eta) + \hat{b}^* - \hat{c}^* p.$$

Notice that, given the general informational structure assumed above, although idiosyncratic errors in signals wash out in the aggregate demand,[8] the quality of the agents' information (parametrized by y) impacts the aggregate demand through its effect on the traders' exposure to common correlated noise η. Combining the above expression with the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$ from the representative investor, we then have that the equilibrium price must satisfy

$$p = \frac{1}{1+\beta\hat{c}^*}\Big(\alpha - u + \beta\hat{b}^* + \beta a^*(\theta + f(y)\eta)\Big) = \frac{\alpha + \beta\hat{b}^*}{1+\beta\hat{c}^*} + \frac{\beta a^*}{1+\beta\hat{c}^*} z, \tag{2}$$

where

$$z \equiv \theta + f(y)\eta - \frac{u}{\beta a^*}. \tag{3}$$

The information about θ contained in the price is thus the same as the one contained in the endogenous public signal

$$z = \theta + \omega,$$

where

$$\omega \equiv f(y)\eta - \frac{u}{\beta a^*}.$$

[7] The reason why we are denoting the sensitivity $\hat{c}^*$ of the demand schedules to the price and the constant $\hat{b}^*$ in the demand schedules with the ^ is that, in the Appendix, we use the notation (a, b, c) to denote the sensitivity of the induced trades (the volume of the asset purchased/sold) to the traders' private information and the endogenous signal generated by the price. We do not use ^ for the sensitivity a^* of the demand schedules to the traders' private information s_i because that sensitivity is the same no matter whether one looks at the submitted demand schedules or the at the induced trades.

[8] This is so since we make the convention that the analog of the SLLN holds for a continuum of independent random variables with uniformly bounded variances. The last property holds as long as the y_i's have a common lower bound strictly larger than 0.

Note that, fixing y, the precision

$$\tau_\omega(a^*) \equiv \frac{\beta^2 a^{*2} y \tau_u \tau_\eta}{(\beta^2 a^{*2} \tau_u + y\tau_\eta)}$$

of the noise ω in the endogenous signal z depends on the traders' demand schedules only through the sensitivity a^* of the latter to their private information s_i.[9]

When the price takes the form in (2), the conditional expectation of θ given s_i and p is given by

$$\mathbb{E}[\theta|s_i, p] = \mathbb{E}[\theta|s_i, z] = \gamma_1(\tau_\omega(a^*))s_i + \gamma_2(\tau_\omega(a^*))z \tag{4}$$

where, for ant τ_ω, the function γ_1 and γ_2 are given by[10]

$$\gamma_1(\tau_\omega) \equiv \frac{\tau_\epsilon y \tau_\eta \left(y\tau_\eta - \tau_\omega\right)}{y^2\tau_\eta^2(\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega \tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right)} \tag{5}$$

and

$$\gamma_2(\tau_\omega) \equiv \frac{\tau_\omega y \tau_\eta \left(y\tau_\eta - \tau_\epsilon\right)}{y^2\tau_\eta^2(\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega \tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right)}. \tag{6}$$

That is, each trader's expectation of θ is a weighted average of her private signal, s_i, and the endogenous public signal contained in the price, z. Note that, in the expressions above and throughout the rest of the section, we dropped the dependence of $\tau_\epsilon(y)$ on y to ease the exposition. Using (4) and (1), we then have that the coefficients $(a^*, \hat{b}^*, \hat{c})$ in the affine strategy describing the equilibrium demand schedules satisfy

$$a^* = \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta \left(y\tau_\eta - \tau_\omega(a^*)\right)}{y^2\tau_\eta^2(\tau_\omega(a^*) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^*) \tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right)}, \tag{7}$$

$\hat{c}^* = \hat{C}(a^*)$, and $\hat{b}^* = \hat{B}(a^*)$ where, for any a,

$$\hat{C}(a) \equiv -\frac{\left(1 - \lambda a \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a}{\beta(\beta+\lambda)a + \beta\left[\left(1 - \lambda a \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right]}, \tag{8}$$

and

$$\hat{B}(a) \equiv \frac{\alpha}{\beta + \lambda}\left(\lambda \hat{C}(a) - 1\right). \tag{9}$$

The equilibrium sensitivity to private information, a^*, is thus given by the unique positive

[9] Because y is held fixed, to alleviate the notation, we are dropping the dependence of $\tau_\omega(a^*; y)$ on y.

[10] Again, we are dropping the dependence of $\gamma_1(\tau_\omega(a^*; y); y)$ and $\gamma_2(\tau_\omega(a^*; y); y)$ on y to ease the notation.

root to the cubic equation

$$0 = \lambda\beta^2\tau_u a^3 \left[y^3\tau_\eta^3 + y^2\tau_\eta^2 (\tau_\epsilon + \tau_\theta) - y\tau_\eta\tau_\epsilon (\tau_\theta + 2y\tau_\eta)\right] + \lambda a y^3\tau_\eta^3 (\tau_\epsilon + \tau_\theta) - \tau_\epsilon y^3\tau_\eta^3 \tag{10}$$

whereas $\hat{c}^*$ and $\hat{b}^*$ are given by the functions (8) and (9). In the Appendix, we verify that the unique positive root to the above cubic expression is such that $a^* < \frac{1}{\lambda}$. We then have the following result:

Proposition 1. *Suppose $y_i = y$ for all i, with y fixed exogenously. There exits a unique symmetric equilibrium. The sensitivity of the traders' equilibrium demand schedules to their private information, a^*, is given by the unique positive root to equation 10 and is such that $0 < a^* < \frac{1}{\lambda}$.*

3.2 Efficient Use of Information

We now characterize the inefficiencies in the equilibrium usage of information that arise when the precision of the traders' private information is exogeneous.

3.2.1 Welfare losses

Ex-post total welfare is given by

$$W \equiv \int_0^1 \left(\theta x_i - \frac{\lambda}{2}x_i^2\right) di + \left(u - \alpha - \beta\frac{\tilde{x}}{2}\right)\tilde{x}.$$

The integral term is the total benefit that the traders derive from purchasing the asset, net of the transaction costs. The remaining term is the gross payoff that the representative investor derives from selling the asset, once again, net of the transaction cost. It is easy to see that, under complete information, the allocation (equivalently, the trades) that maximizes total surplus are given by $x_i = x^o$ for all i, with[11]

$$x^o \equiv \frac{\theta + u - \alpha}{\beta + \lambda}. \tag{11}$$

Under the first-best allocation, (ex-post) total welfare is then given by

$$W^o \equiv \left(\theta - \frac{\lambda}{2}x^o\right)x^o + \left(u - \alpha - \beta\frac{x^o}{2}\right)x^o = \left(\theta + u - \alpha - \frac{\beta + \lambda}{2}x^o\right)x^o.$$

[11]See the Appendix for the derivation.

Next, let

$$WL \equiv \mathbb{E}[W^o] - \mathbb{E}[W]$$

denote the (ex-ante) expected welfare losses that arise when the traders purchase the asset in quantities different from the first-best level, due to imperfect information. Under any strategy profile for the agents in which $X(p; I_i)$ is affine in s_i and p, the welfare losses can be expressed as follows (the derivations are in the Appendix):

$$WL = \frac{(\beta + \lambda)\mathbb{E}[(\tilde{x} - x^o)^2] + \lambda\mathbb{E}[(x_i - \tilde{x})^2]}{2}. \tag{12}$$

The term $\mathbb{E}[(\tilde{x} - x^o)^2]$ captures the losses due to the aggregate trades being different from the first-best level. The term $\mathbb{E}[(x_i - \tilde{x})^2]$, instead, captures the losses due to the dispersion of the individual trades around the average level.

3.2.2 Team Problem

Let the efficient use of information be the traders' strategy (that is, the collection of demand schedules) that minimizes the ex-ante welfare losses subject to the constraint that the traders' demand schedules (equivalently, the trades) be affine in their private signal and the price. We do not include the representative investor in the team's definition for two reasons: (a) we are interested in isolating the inefficiencies that pertain to the traders' usage of information, and (b) we have in mind markets where policy interventions can manipulate the behavior of certain investors (the traders in our model) but not others (e.g., noisy traders, liquidity suppliers, or foreign investors). Accordingly, $(a^T, \hat{b}^T, \hat{c}^T)$ identifies the efficient use of information if, whenever all traders submit demand schedules $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$, the welfare loses are as small as under any other affine strategy $x_i = a' s_i + \hat{b}' - \hat{c}' p$.[12] Paralleling the analysis of the equilibrium use of information presented above, we have that, when all traders submit the above demand schedules, the information contained in the market-clearing price is the same as the one in the endogenous signal

$$z = \theta + \omega(a)$$

where

$$\omega = f(y)\eta - \frac{u}{\beta a}$$

has the same structure as in the equilibrium usage of information.

Lemma 1. *For any sensitivity a of the demand schedules to the traders' private information,*

[12] Again, we use ˆ to distinguish the efficient demand schedules from the efficient trades.

the values of $\hat{c}$ and $\hat{b}$ in the demand schedules that minimize the welfare losses are given by the same functions (8) and (9) that define the equilibrium usage of information.

Using Lemma 1, the welfare losses can then be expressed as a function of a and $\tau_\omega(a)$ as follows (see the Appendix for the formal proof):[13]

$$WL(a,\tau_\omega(a)) = \frac{\left[\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2\left(\beta+\lambda\right)\tau_\omega(a)} + \frac{\lambda^2 a^2}{2\left(\beta+\lambda\right)y\tau_\eta} + \frac{\lambda a\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{\left(\beta+\lambda\right)y\tau_\eta}$$

$$+\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2\left(\beta+\lambda\right)\tau_\theta} + \frac{\lambda a^2}{2y\tau_e}. \tag{13}$$

The last term $\lambda a^2/y\tau_e$ in $WL(a,\tau_\omega(a))$ represents the welfare losses due to the dispersion of individual trades around the average trade. The other terms represent the losses due to the volatility of the aggregate volume of trade around its first-best level. Both losses are computed under the optimal choice of $\hat{c}$ and $\hat{b}$, using the result in Lemma 1.

The efficient level of a, which we denote by a^T, is thus the value of a that minimizes $WL(a,\tau_\omega(a))$. Now let

$$\begin{aligned}
\Delta(a) &\equiv -\frac{\tau_\epsilon\beta^2 y^4\tau_\eta^4\tau_u\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2\left(\beta^2a^2\tau_u+y\tau_\eta\right)^2\left(\tau_\omega(a)+\tau_\theta\right)},\\
\Xi(a) &\equiv \frac{y\tau_\epsilon\tau_\eta^2\beta\left(\tau_\omega(a)+\tau_\theta\right)}{\lambda\tau_e},\\
\tau_\omega(a) &\equiv \frac{\beta^2a^2y\tau_\eta\tau_u}{\beta^2a^2\tau_u+y\tau_\eta}.
\end{aligned}$$

We then have the following result:

Proposition 2. *Suppose that $y_i = y$ for all i, with y fixed exogenously. The team problem has a unique solution. The efficient sensitivity a^T of the traders' demand schedules to their private information is given by the unique solution to*

$$a=\frac{1}{\lambda}\frac{\tau_\epsilon y\tau_\eta(y\tau_\eta-\tau_\omega(a))}{y^2\tau_\eta^2\left(\tau_\omega(a)+\tau_\epsilon+\tau_\theta\right)-\tau_\omega(a)\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)+\Xi(a)+\Delta(a)} \tag{14}$$

and is such that $0 < a^T < \frac{1}{\lambda}$. Given a^T, the other two parameters that define the efficient

[13] Note that, given $(a,\tau_\theta,\tau_\eta,y,\tau_e)$, τ_u affects WL only through its effect on $\tau_\omega(a)$. Hence, holding $(a,\tau_\theta,\tau_\eta,y,\tau_e)$ fixed, changes in $\tau_\omega(a)$ can be thought of as originating in changes in τ_u.

demand schedules, $\hat{c}^T$ and $\hat{b}^T$, are then given by the same functions in (8) and (9) that describe the corresponding coefficients of the demand schedules under the decentralized equilibrium usage of information.

When, for any a, b and c are set optimally, the welfare losses $WL(a, \tau_\omega(a))$ are a convex function of a reaching a minimum at $a = a^T$, with $0 < a^T < \frac{1}{\lambda}$. Note that the equation that determines the value of a^T differs from the one that yields the equilibrium value of a^* only by the two terms $\Xi(a)$ and $\Delta(a)$ in the denominator of the right-hand side of (14). The first term, $\Xi(a)$, is a *pecuniary externality* that arises because the traders do not internalize that their demand schedules impact the co-movement between the market-clearing price and the aggregate shocks (u, θ, η), which, in turn, impacts the way the equilibrium trades co-move with these variables, the volatility of the aggregate volume of trade, and ultimately the representative investor's payoff. The term $\Xi(a)$ is always positive, thus contributing to an over-reaction of the equilibrium trades to private information. The second term, $\Delta(a)$, is essentially a scaling of

$$\frac{\partial WL(a, \tau_\omega(a)))}{\partial \tau_\omega(a)} \frac{\partial \tau_\omega(a)}{\partial a}.$$

Therefore, this term can be thought of as a proxy for the information externality that arises because traders do not account for the fact that their demand schedules impact the informativeness of the equilibrium price and therefore the possibility for other traders and for the representative investor to respond to the various shocks. This term is always negative thus contributing to under-reaction of the equilibrium demand schedules to private information.

To shed more light on the role of these externalities, it is useful to consider a *fictitious environment* in which traders are naive in that they do not recognize the information contained in the market-clearing price. Such a benchmark is similar in spirit to the (fully) cursed equilibrium of Eyster and Rabin (2005). To facilitate the comparison to the true economy, further assume that, in this fictitious environment, each trader, in addition to receiving the private signal $s_i = \theta + \underbrace{f(y)\eta + f(y)e_i}_{\equiv \epsilon_i}$, as in the baseline model, also observes an exogenous public signal $z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$ whose structure is the same as the one of the endogenous public signal generated by the market-clearing price, but with the endogenous noise $-u/\beta a$ replaced by the exogenous noise χ, with the latter drawn from a Normal distribution with mean zero and variance τ_χ^{-1} independently of all other variables (this shock is the same for all traders).

As we show in the Appendix, in this fictitious environment, the (cursed) equilibrium sensitivity of the traders' demand schedules to their private information s_i is given by

$$a_{exo}^{*} \;=\; \frac{1}{\lambda}\frac{\tau_{\epsilon} y\tau_{\eta}\left(y\tau_{\eta}-\tau_{\zeta}\right)}{y^{2}\tau_{\eta}^{2}(\tau_{\zeta}+\tau_{\epsilon}+\tau_{\theta})-\tau_{\zeta}\tau_{\epsilon}(\tau_{\theta}+2y\tau_{\eta})}. \tag{15}$$

Note that the formula in (15) is similar to the one in the baseline economy, except for the fact that the precision $\tau_{\omega}(a)$ of the endogenous public signal contained in the market-clearing price is replaced by the precision τ_{ζ} of the exogenous public signal about θ.

Now suppose that the planner can select a but, given the latter, is constrained to choose (b,c,d) so as to maintain the same relationship between a and (b,c,d) as in the cursed equilibrium.[14] The level of a that maximizes ex-ante welfare is then equal to

$$a_{exo}^{T} \;=\; \frac{1}{\lambda}\frac{\tau_{\epsilon} y\tau_{\eta}(y\tau_{\eta}-\tau_{\zeta})}{y^{2}\tau_{\eta}^{2}\left(\tau_{\zeta}+\tau_{\epsilon}+\tau_{\theta}\right)-\tau_{\zeta}\tau_{\epsilon}\left(\tau_{\theta}+2y\tau_{\eta}\right)+\frac{y\tau_{\epsilon}\tau_{\eta}^{2}\beta\left(\tau_{\zeta}+\tau_{\theta}\right)}{\lambda\tau_{e}}}. \tag{16}$$

Again, the formula for a_{exo}^{T} is similar to the one for a^{T} in the baseline model, except for the fact that $\tau_{\omega}(a)$ is replaced by τ_{ζ} and the term $\Delta(a)$ in the denominator of the expression giving the socially-optimal level of a in the baseline model is equal to zero, reflecting the fact that the agents do lot learn from the price. Note that $y\tau_{\epsilon}\tau_{\eta}^{2}\beta\left(\tau_{\zeta}+\tau_{\theta}\right)/\lambda\tau_{e}$ has exactly the same form as the pecuniary externality $\Xi(a)$ in the baseline model. Hence, in this fictitious economy, the (cursed) equilibrium demand schedules unambiguously feature an excessively high sensitivity to private information relative to the solution to the planner's problem: $a_{exo}^{T} > a_{exo}^{*}$. Furthermore, when the precision of the exogenous public signal in the cursed economy is the same as the precision of the endogenous public signal under the solution to the planner's problem in our baseline model (that is, when $\tau_{\zeta}=\tau_{\omega}(a^{T})$), the values of a^{T} and a_{exo}^{T} are easily comparable and a_{exo}^{T} coincides with the solution to the equation $\partial WL(a,\tau_{\omega}(a))/\partial a = 0$. Relative to the solution to the planner's problem in the (cursed) equilibrium economy, the planner in our model does recognize the value of increasing the precision of information contained in the price and thus demands that traders increase the sensitivity of their limit orders to their private information ($a^{T} > a_{exo}^{T}$).

In our baseline model, both the traders and the planner account for the information contained in the price. Whether the sensitivity of the equilibrium demand schedules to the traders'

[14] Whereas, in the baseline economy, choosing b and c to satisfy the same relationship between a and (b,c) as in equilibrium is without loss of optimality for the planner, this need not be true in the fictitious economy. However, imposing the restriction permits us to isolate the relevant effects. It is also possible to show that, given a, a planner who expects p to be orthogonal to (θ,η), as do the traders, optimally chooses (b,c,d) to satisfy the same relationship between these coefficients and a as in the cursed equilibrium (the proof is available upon request).

private information is excessively high or excessively low (compared to the efficient level a^T) then depends on which of the above two externalities prevails. Comparing (7) and (14), we see that the sign of $(a^* - a^T)$ equals the sign of $\Xi(a^T) + \Delta(a^T)$. When $\Xi(a^T) + \Delta(a^T) = 0$, the two externalities described above cancel each other out, the submitted schedules are inelastic (i.e., $\hat{c}^T = 0$) and $a^* = a^T$ (see Lemma 2 in the Appendix). When $\Xi(a^T) + \Delta(a^T) > 0$, the pecuniary externality dominates, $\hat{c}^T > 0$ (the efficient demands are downward sloping) and the equilibrium features an excessive response to private information. When, instead, $\Xi(a^T) + \Delta(a^T) < 0$, the learning externality dominates, $\hat{c}^T < 0$ (the efficient demands slope upwards) and the equilibrium response to private information is insufficiently low. It is worth noting that if the traders were restricted to submitting market orders (like in a Cournot model), then the usage of information would be efficient since the two externalities would not be present (See Subsection 6.2 for a formal proof of this result).

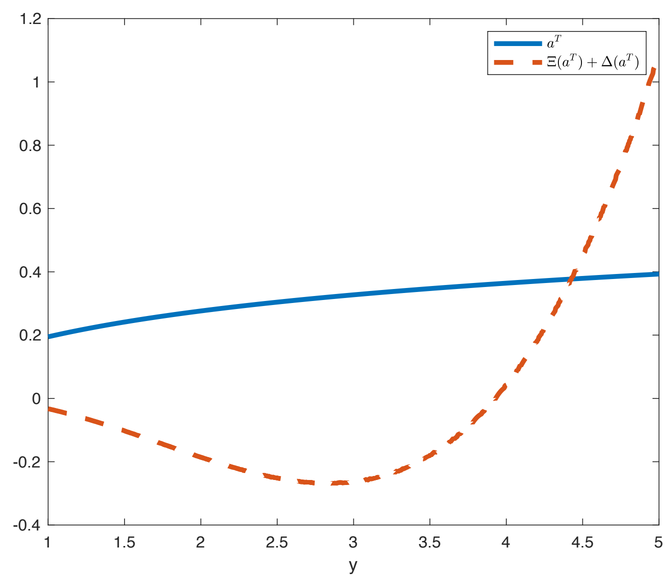

Figure 1: The blue solid line corresponds to a^T while the orange dashed line represents the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$. The parameter values used for this simulation are: $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

Using simulations (see Figure 1), it is possible to show that $\Xi(a^T) + \Delta(a^T)$ is U-shaped with respect to y. As y increases, information is more precise and the efficient response a^T to the private signal increases. Such an increase in a^T increases $\Xi(a^T) + \Delta(a^T)$ (both Ξ and Δ increase with a). However, the increase in y has also a direct negative effect on $\Xi(a) + \Delta(a)$ for given a because better information makes the learning externality less salient and increases the pecuniary externality (that is, it makes Δ more negative). This second, direct, effect dominates when y is small, whereas the first, indirect, effect dominates when y is large.

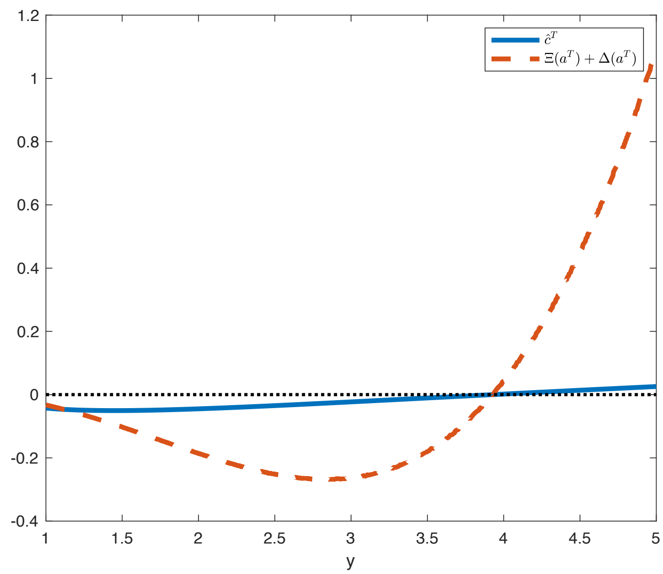

Figure 2: The blue solid line corresponds to $\hat{c}^T$ while the orange dashed line represents the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$. The two curves switch signs for the same value of y. The parameter values used for this simulation are: $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

What is more, for sufficiently large τ_u, $\hat{c}^T$ goes from negative to positive exactly when $\Xi(a^T) + \Delta(a^T)$ does, as it can be seen from Figure 2. For small values of y, i.e., when private information is imprecise, the learning externality dominates, i.e., $\Xi(a^T) + \Delta(a^T) < 0$, and the efficient demand schedules are upward sloping, i.e., $\hat{c}^T < 0$. For large values of y, instead, the pecuniary externality dominates, i.e., $\Xi(a^T) + \Delta(a^T) > 0$, and the efficient demand schedules are downward sloping, i.e., $\hat{c}^T > 0$. The two externalities cancel each other out, i.e., $\Xi(a^T) + \Delta(a^T) = 0$, when, and only when, the efficient demand schedules, which coincide with the equilibrium ones in this case, are perfectly inelastic, i.e., $\hat{c}^T = 0$.

Next, we discuss policies that correct the inefficiency in the usage of information.

Proposition 3. *Suppose $y_i = y$ for all i, with y fixed exogenously. The efficient use of information can be implemented with a policy that charges the traders a total tax bill equal to $T(x_i, p) = \frac{\delta}{2} x_i^2 + (pt_p - t_0)\, x_i$, where*

$$\delta = \frac{\lambda \left(\Xi(a^T) + \Delta(a^T) \right)}{y^2 \tau_\eta^2 (\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^T) \tau_\epsilon \left(\tau_\theta + 2y\tau_\eta \right)},$$

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda+\delta+\beta}{\beta+\lambda}\left[\left(1-\lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta} - \beta a^T\right] - \beta a^T}{\frac{\beta}{\beta+\lambda}\left[\left(1-\lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta} - \beta a^T\right] + \beta a^T},$$

and

$$t_0 = (1+t_p)\alpha - \frac{\alpha\left(\lambda+\delta+(1+t_p)\beta\right)}{\beta+\lambda}.$$

The efficient use of information can thus be induced through a combination of a linear-quadratic tax on the volume of trade $\frac{\delta}{2}x_i^2 - t_0 x_i$ along with a proportional subsidy/tax on the price paid. The role of δ is to manipulate the traders' adjustment cost (from λ to $\lambda+\delta$). This manipulation suffices to induce the traders to submit demand schedules whose sensitivity to their exogenous private information is equal to the efficient level a^T. The role of the proportional subsidy/tax on the price is to guarantee that, once the sensitivity a coincides with the efficient level a^T, the sensitivity of the equilibrium demand schedules to the price coincides with the efficient level $\hat{c}^T$. Finally, the role of the linear tax $t_0 x_i$ on the individual volume of trade is to guarantee that the fixed part of the affine demand schedule also coincides with the efficient level $\hat{b}^T$.

4 Inefficiency in Information Acquisition

We now investigate whether efficiency in information usage implies efficiency in the collection of private information. We start by considering the case where efficiency in usage is induced by the planner controlling directly the agents' usage of information, that is, by imposing that the traders submit demand schedules of the form $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$. We then consider the case where efficiency in usage is induced through the policy in Proposition 3. In both cases, we find that agents do not acquire information efficiently.

Let y^T denote the socially optimal precision of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the precision of private information is y^T. Next, let $\mathbb{E}[W^T; \bar{y}]$ denote ex-ante gross welfare when all traders acquire information of quality $\bar{y}$ but then submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$).[15] Finally, let $\mathbb{E}[\pi_i^T; y_i, \bar{y}]$ be the ex-ante gross profit of a trader acquiring information of quality y_i when all other traders acquire information of quality $\bar{y}$, and all traders (including i) submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). We then have the following result:

[15] The welfare function is gross of the cost of information acquisition.

Proposition 4. *Let y^T denote the socially optimal quality of private information and suppose that all traders are constrained to submit the efficient demand schedules for information of quality y^T. When $\hat{c}^T > 0$ (i.e., when the pecuniary externality dominates and the efficient demand schedules are downward sloping),*

$$\left.\frac{\partial \mathbb{E}[\pi_i^T; y_i, \bar{y}]}{\partial y_i}\right|_{y_i=\bar{y}=y^T} > \left.\frac{\partial \mathbb{E}[W^T; \bar{y}]}{\partial \bar{y}}\right|_{\bar{y}=y^T}$$

whereas the opposite inequality holds when $\hat{c}^T < 0$ (i.e., when the information externality dominates and the efficient demand schedules are upward sloping).

By the envelope theorem, y^T solves the optimality condition

$$\left.\frac{\partial \mathbb{E}[W^T; \bar{y}]}{\partial \bar{y}}\right|_{\bar{y}=y^T} = \mathcal{C}'(y^T). \tag{17}$$

Because $\mathbb{E}[\pi_i^T; y_i, \bar{y}]$ is strictly concave in y_i, the result in the proposition implies that, when $\hat{c}^T > 0$ (i.e., when the pecuniary externality dominates and the efficient demand schedules are downward sloping), a trader who expects all other traders to acquire information of quality y^T has incentives to acquire information of quality higher than y^T. When, instead, $\hat{c}^T < 0$ (i.e., when the information externality dominates and the efficient demand schedules are upward sloping), the individual has incentives to acquire information of quality lower than y^T. In the special case in which $\hat{c}^T = 0$ (that is, the efficient demand schedules are invariant to price), the optimal choice for the individual is to acquire information of efficient quality, that is, $y_i = y^T$.

The next result shows that the properties of individual best responses identified in Proposition 4 are inherited in equilibrium (i.e., at the fixed point).

Proposition 5. *Let y^T denote the socially optimal quality of private information and suppose that all traders are constrained to submit the efficient demand schedules for information of quality y^T. When $\hat{c}^T > 0$ (downward-sloping efficient demand schedules), the quality of private information acquired in equilibrium is higher than y^T, whereas the opposite is true when $\hat{c}^T < 0$ (upward-sloping efficient demand schedules).*

Note that the last two results hinge on $\tau_\eta \in (0, +\infty)$. When $\tau_\eta = 0$, the noise in the agents' private signals is infinite, making the signals worthless both for the individual traders and for the planner. When, instead, $\tau_\eta \to +\infty$, the correlated noise in the agents' private signals disappears, in which case the aggregate volume of trade becomes invariant to the quality of the traders' private information. In this case, holding the demand schedules fixed, the only effect of an increase in the quality of the traders' private information on welfare is through the reduction in the dispersion of individual trades around the average trade. Because this effect is weighted equally by the planner and by the individual traders, the private and the social value of information coincide in this case, which guarantees that traders acquire information of efficient quality.

The above results thus establish that, when the planner forces the traders to submit the efficient limit orders, the traders respond by over-investing (alternatively, under-investing) in information acquisition when the efficient demand schedules are downward sloping (alternatively, upward sloping). This happens because of two reasons. First, agents do not internalize that the quality of information they acquire affects the dispersion of individual trades around the average trade, which contributes to a welfare loss. Second, when the noise in the agents' information is correlated, the agents fail to internalize that a variation in the quality of their private information changes the covariance of the aggregate volume of trade with all the aggregate shocks and hence affects the impact that non-fundamental volatility has on welfare.

Another way to see things is that, when $\hat{c}^T > 0$ (downward-sloping demand schedules), agents would like to over-respond to private information (relative to what is efficient) but cannot because the planner is forcing them to submit the efficient limit orders. They then react by over-investing in information acquisition. When, instead, $\hat{c}^T < 0$ (upward-sloping demand schedules) traders would like to under-respond to private information and hence under-invest in information acquisition when forced to submit the efficient schedules.

Recall that, for small y, the learning externality dominates and demand schedules are upward sloping whereas, for large y, the pecuniary externality dominates and demand schedules are downward sloping. The above results thus also imply that, as technological progress makes information cheaper (that is, as the cost of information acquisition decreases), y increases and eventually the economy enters into a regime where too much information is acquired in equilibrium.

We then conjecture that the following result is also true but did not establish it formally:

Conjecture: *When $\hat{c}^T > 0$ (downward-sloping efficient demand schedules), the equilibrium in the laissez-faire economy features excessive acquisition and excessive usage of private information, i.e., $y^* > y^T$ and $a^* > a^T$, whereas the opposite is true when $\hat{c}^T < 0$ (upward-sloping efficient demand schedules). When $\hat{c}^T = 0$, both the acquisition and usage of private information in the laissez-faire equilibrium are socially efficient, as it is the case when traders are restricted to submitting market orders.*

We now address the question of whether efficiency in information acquisition can be induced through an appropriate design of fiscal policy, in a world where the traders can be forced to submit the efficient demand schedules (we address the more relevant case in which efficiency in trade must also be induced through an appropriate fiscal policy at the end of the section).

Proposition 6. *Let y^T denote the socially optimal quality of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the quality of information is y^T. Suppose all traders are constrained to submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$ but can choose the quality of private information. The traders can be induced to acquire information of quality y^T by charging them a tax bill $T(p, x_i) = \hat{t}_p p x_i$ with*

$$\hat{t}_p = \frac{\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T}{\beta a^T},$$

where γ_2 is the function defined in (6).

That is, efficiency in information acquisition can always be induced through a simple tax/subsidy proportional to the traders' total expenditure px_i (equivalently, with a familiar proportional tax/subsidy on the price of the asset, similar to those often discussed in the policy debate). The result, however, hinges on the possibility for the planner to force the traders to submit the efficient demand schedules (for quality of information y^T).

We now address the more relevant question of whether efficiency in both information acquisition and information usage can be induced through an appropriate design of the fiscal policy. In the previous section, we showed that, when the quality of information is exogenous, efficiency in information usage can be induced through a linear-quadratic tax on trades paired with a tax/subsidy on the price (both rebated in a lump-sum manner, if desired). Based on other results in the literature, one may conjecture that the same policy also induces efficiency in information acquisition. The next result shows that the conjecture is wrong.

Proposition 7. *Generically (i.e., with the exception of a set of parameters of zero Lebesgue measure), there exists no (differentiable) policy $T(x_i, p)$ that implements efficiency in both information acquisition and information usage.*

The result is established in the Appendix by showing that any smooth policy that induces the traders to submit the efficient demand schedules once they collect the efficient amount of private information y^T must coincide with the one in Proposition 3 (applied to $\bar{y} = y^T$), except for terms that play no role for incentives. However, any such policy necessarily induces the traders to misperceive the marginal value of their private information (around the efficient level y^T) and hence to collect an inefficient amount of private information. To induce efficiency in both information acquisition and trading, the planner may thus need to resort to unorthodox policies where the tax bill is non-smooth in (x_i, p) and/or is contingent on information other than the individual volume of trade x_i and the price of the financial asset p (e.g., T may depend on the distribution of trades in the market and/or on the ex-post profitability θ of the asset).

The following result is also an immediate implication of the above observations:

Corollary 1. *Let y^T denote the socially optimal quality of private information and T the fiscal policy in Proposition 3 (applied to $\bar{y} = y^T$) that induces the traders to submit the efficient demand schedules when y_i is exogenously fixed at $y_i = y^T$ for all i. When the quality of private information is endogenous, under the same policy, traders acquire information of quality other than y^T and then submit demand schedules other than the efficient ones.*

Our final result shows that, when the acquisition of information is verifiable (e.g., when traders acquire information of verifiable quality from known sources), then the above impossibility result turns into a possibility one: efficiency in both information acquisition and information usage can be obtained by pairing the policy in Proposition 3 with a tax on the expenditure on information acquisition.

Proposition 8. *Let y^T denote the socially optimal quality of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the quality of information is y^T. Let T^{tot} denote the fiscal policy defined by*

$$T^{tot}(x_i, p, y_i) = \frac{\delta}{2} x_i^2 + (pt_p - t_0)\, x_i - Ay_i$$

where (δ, t_p, t_0) are as in Proposition 3 and

$$A \quad = \quad \frac{\left(a^T + c^T\right)}{2\tau_\eta \left(y^T\right)^2} \left[(\beta + \lambda)c^T - (\beta t_p + \delta)a^T\right] - \frac{\delta \left(a^T\right)^2}{2\tau_e \left(y^T\right)^2}.$$

The above policy induces efficiency in both information acquisition and information usage.

Simulations establish that $A > 0$ if $\hat{c}^T > 0$ and $A < 0$ if $\hat{c}^T < 0$. That is, information should be taxed when the efficient demand schedules are downward sloping and subsidized when they are upward sloping, reflecting the fact that, in the absence of policy interventions, agents over-invest in information acquisition in the former case and underinvest in the latter.

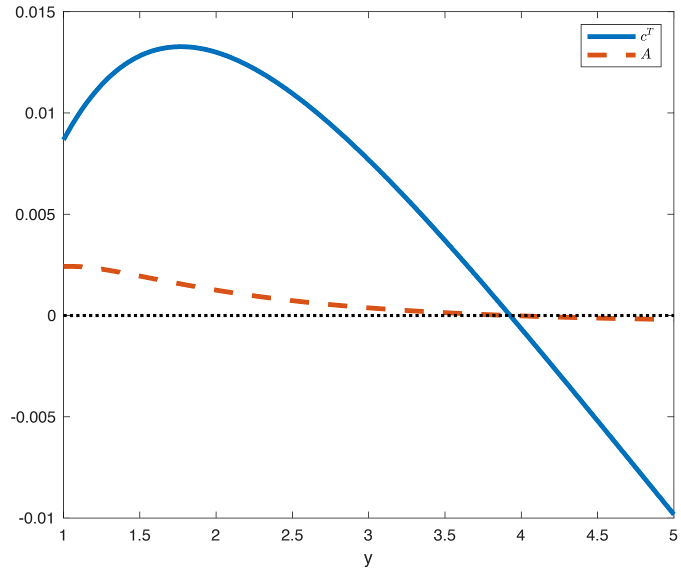

Figure 3: The parameter values used for this simulation are: $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$. The blue solid line corresponds to $\hat{c}^T$ whereas the orange dashed line represents the tax A on information acquisition. The two curves switch signs for the same value of y.

5 Conclusions

We investigate the sources of inefficiency in the usage and collection of information in financial markets. We show that, when the private information the traders possess prior to submitting their limit orders is exogenous, inefficiency in trading can be corrected with a an appropriate design of fiscal policies. When information is endogenous, however, no (smooth) policy exists in which the total tax bill is a function of the price of the financial asset and on the volume of individual trades that induces efficiency in both information usage and information acquisition.

A key driver for the identified inefficiencies is the correlation in the noise in the agents' private information. In future work it would be interesting to extend the analysis to a broader class of economies in which financial decisions interact with real decisions and in which agents trade multiple assets over multiple periods.

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6 Appendix

6.1 Omitted Proofs and Extended Derivations

Proof of Proposition 1.

As explained in the main text, when the traders submit affine demand schedules with parameters $(a, \hat{b}, \hat{c})$, the equilibrium price is equal to

$$p = \frac{\alpha + \beta\hat{b}}{1+\beta\hat{c}} + \frac{\beta a}{1+\beta\hat{c}}\left(\theta + f(y)\eta - \frac{u}{\beta a}\right).$$

The information about θ contained in the equilibrium price is thus the same as the one contained in a public signal

$$z = \theta + \omega,$$

with noise

$$\omega \equiv f(y)\eta - \frac{u}{\beta a}$$

of precision[16]

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 y \tau_u \tau_\eta}{(\beta^2 a^2 \tau_u + y\tau_\eta)}.$$

In turn, this implies that the equilibrium trades $x_i = as_i + \hat{b} - \hat{c}p$ are affine functions of the traders' exogenous private information s_i and the endogenous public information z contained in the price. That is, when the endogenous public information contained in the price is equivalent to z, a trader with private signal s_i purchases an amount of the asset equal to

$$x_i = as_i + b + cz$$

where

$$b = \hat{b} - \hat{c}\frac{\alpha + \beta\hat{b}}{1+\beta\hat{c}} \tag{18}$$

and

$$c = -\frac{\beta a\hat{c}}{1+\beta\hat{c}}. \tag{19}$$

For each vector $(a, \hat{b}, \hat{c})$ describing the traders' demand schedules, there thus exists a unique vector (a, b, c) describing the traders' equilibrium trades as a function of their (exogenous) private information, s_i, and (endogenous) public information, z, and vice versa. Hereafter, we find it more convenient to characterize the equilibrium use of information in terms of the vector (a, b, c) describing the equilibrium trades. When the individual trades are given by $x_i = as_i + b + cz$, the aggregate trade is equal to

$$\tilde{x} = \int x_i di = a(\theta + f(y)\eta) + b + cz.$$

[16] To derive $\tau_\omega(a)$ we use the fact that $f(y) = 1/\sqrt{y}$.

Using the fact that $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$, we thus have that

$$\tilde{x} = a(z + \frac{u}{\beta a}) + b + cz = (a+c)z + \frac{u}{\beta} + b.$$

Using the expression for the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$ from the representative investor, we then have that the equilibrium price can be expressed as a function of a and the endogenous public signal z as follows:

$$p = \alpha + \beta b + \beta(a+c)z. \tag{20}$$

Next, observe that

$$\begin{aligned}
\mathbb{E}[\theta|I_i,p] = \mathbb{E}[\theta|s_i,z] &= \begin{bmatrix} Cov(\theta,s_i) & Cov(\theta,z) \end{bmatrix} \begin{bmatrix} Var(s_i) & Cov(s_i,z) \\ Cov(s_i,z) & Var(z) \end{bmatrix}^{-1} \begin{bmatrix} s_i - \mathbb{E}[s_i] \\ z - \mathbb{E}[z] \end{bmatrix} \\
&= \begin{bmatrix} \sigma_\theta^2 & \sigma_\theta^2 \end{bmatrix} \begin{bmatrix} \sigma_\theta^2 + \sigma_\epsilon^2 & \sigma_\theta^2 + f(y)^2\sigma_\eta^2 \\ \sigma_\theta^2 + f(y)^2\sigma_\eta^2 & \sigma_\theta^2 + \sigma_\omega^2(a) \end{bmatrix}^{-1} \begin{bmatrix} s_i - \mathbb{E}[s_i] \\ z - \mathbb{E}[z] \end{bmatrix},
\end{aligned}$$

where $\sigma_\theta^2 \equiv \tau_\theta^{-1}$, $\sigma_\omega^2(a) \equiv \tau_\omega(a)^{-1}$, $\sigma_\eta^2 \equiv \tau_\eta^{-1}$, and $\sigma_\epsilon^2 \equiv \tau_\epsilon^{-1}$. Substituting for the inverse of the variance-covariance matrix, we have that

$$\begin{aligned}
\mathbb{E}[\theta|s_i,z] &= \frac{1}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\theta^2 + \sigma_\omega^2(a)) - (\sigma_\theta^2 + f(y)^2\sigma_\eta^2)^2} \times \\
&\quad \begin{bmatrix} \sigma_\theta^2 & \sigma_\theta^2 \end{bmatrix} \begin{bmatrix} \sigma_\theta^2 + \sigma_\omega^2(a) & -(\sigma_\theta^2 + f(y)^2\sigma_\eta^2) \\ -(\sigma_\theta^2 + f(y)^2\sigma_\eta^2) & \sigma_\theta^2 + \sigma_\epsilon^2 \end{bmatrix} \begin{bmatrix} s_i - \mathbb{E}[s_i] \\ z - \mathbb{E}[z] \end{bmatrix}.
\end{aligned}$$

Expanding the quadratic form, we have that

$$\begin{aligned}
\mathbb{E}[\theta|s_i,z] &= \frac{\sigma_\theta^2\left(\sigma_\omega^2(a) - f(y)^2\sigma_\eta^2\right)}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\theta^2 + \sigma_\omega^2(a)) - (\sigma_\theta^2 + f(y)^2\sigma_\eta^2)^2}(s_i - \mathbb{E}[s_i]) \\
&\quad + \frac{\sigma_\theta^2\left(\sigma_\epsilon^2 - f(y)^2\sigma_\eta^2\right)}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\theta^2 + \sigma_\omega^2(a)) - (\sigma_\theta^2 + f(y)^2\sigma_\eta^2)^2}(z - \mathbb{E}[z]).
\end{aligned}$$

Simplifying, and using the fact that $\sigma_\theta^2 \equiv \tau_\theta^{-1}$, $\sigma_\omega^2(a) \equiv \tau_\omega(a)^{-1}$, $\sigma_\eta^2 \equiv \tau_\eta^{-1}$, $\sigma_\epsilon^2 \equiv \tau_\epsilon^{-1}$, and $f(y) = 1/\sqrt{y}$, we have that

$$\begin{aligned}
\mathbb{E}[\theta|s_i,z] &= \frac{\frac{1}{\tau_\theta}\left(\frac{1}{\tau_\omega(a)} - \frac{1}{y\tau_\eta}\right)}{(\frac{1}{\tau_\theta} + \frac{1}{\tau_\epsilon})(\frac{1}{\tau_\theta} + \frac{1}{\tau_\omega(a)}) - (\frac{1}{\tau_\theta} + \frac{1}{y\tau_\eta})^2}(s_i - \mathbb{E}[s_i]) \\
&\quad + \frac{\frac{1}{\tau_\theta}\left(\frac{1}{\tau_\epsilon} - \frac{1}{y\tau_\eta}\right)}{(\frac{1}{\tau_\theta} + \frac{1}{\tau_\epsilon})(\frac{1}{\tau_\theta} + \frac{1}{\tau_\omega(a)}) - (\frac{1}{\tau_\theta} + \frac{1}{y\tau_\eta})^2}(z - \mathbb{E}[z]),
\end{aligned}$$

from which we obtain that

$$\begin{aligned}\mathbb{E}[\theta|s_i,z] &= \frac{y^2\tau_\eta^2\tau_\epsilon\left(1-\frac{\tau_\omega(a)}{y\tau_\eta}\right)}{y^2\tau_\eta^2(\tau_\epsilon+\tau_\omega(a)+\tau_\theta)-\tau_\epsilon\tau_\omega(a)(\tau_\theta+2y\tau_\eta)}(s_i-\mathbb{E}[s_i]) \\ &\quad+\frac{y^2\tau_\eta^2\tau_\omega(a)\left(1-\frac{\tau_\epsilon}{y\tau_\eta}\right)}{y^2\tau_\eta^2(\tau_\epsilon+\tau_\omega(a)+\tau_\theta)-\tau_\epsilon\tau_\omega(a)(\tau_\theta+2y\tau_\eta)}(z-\mathbb{E}[z]).\end{aligned}$$

Finally, using the fact that $\mathbb{E}[s_i]=\mathbb{E}[z]=0$, we have that

$$\mathbb{E}[\theta|s_i,z] = \gamma_1(\tau_\omega(a))s_i+\gamma_2(\tau_\omega(a))z$$

where[17]

$$\gamma_1(\tau_\omega(a)) \equiv \frac{\tau_\epsilon y\tau_\eta\left(y\tau_\eta-\tau_\omega(a)\right)}{y^2\tau_\eta^2\left(\tau_\omega(a)+\tau_\epsilon+\tau_\theta\right)-\tau_\omega(a)\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)}$$

and

$$\gamma_2(\tau_\omega(a)) \equiv \frac{\tau_\omega(a)\left(y^2\tau_\eta^2-\tau_\epsilon y\tau_\eta\right)}{y^2\tau_\eta^2\left(\tau_\omega(a)+\tau_\epsilon+\tau_\theta\right)-\tau_\omega(a)\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)} = \left(1-\gamma_1(\tau_\omega(a))\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}.$$

Now recall that the equilibrium trades must satisfy

$$x_i = \frac{1}{\lambda}\left(\mathbb{E}[\theta|s_i,z]-p\right).$$

Using the fact that $p=\alpha+\beta b+\beta(a+c)z$, and the characterization of $\mathbb{E}[\theta|s_i,z]$ from above, we thus have that

$$x_i = \frac{1}{\lambda}\left[\gamma_1(\tau_\omega(a))s_i-(\alpha+\beta b)+\left(\gamma_2(\tau_\omega(a))-\beta(a+c)\right)z\right].$$

We conclude that (1) the sensitivity of the equilibrium trades to private information must satisfy

$$a=\frac{\gamma_1(\tau_\omega(a))}{\lambda}, \tag{21}$$

(2) the sensitivity of the equilibrium trades to the endogenous public information must satisfy

$$c=\frac{1}{\lambda}\left(\gamma_2(\tau_\omega(a))-\beta(a+c)\right),$$

and (3) the constant b in the equilibrium trades must satisfy

$$b=-\frac{\alpha+\beta b}{\lambda}. \tag{22}$$

Replacing

$$\tau_\omega(a) \equiv \frac{\beta^2a^2y\tau_\eta\tau_u}{\beta^2a^2\tau_u+y\tau_\eta}$$

into the expression for $\gamma_1(\tau_\omega(a))$ and using (21), we thus have that the equilibrium sensitivity

[17] Consistently with the rest of the analysis above, because y is held constant, we drop it from the arguments of γ_1 and γ_2 to ease the notation.

to private information must coincide with the solution to the following equation

$$a=\frac{1}{\lambda}\frac{\tau_\epsilon y^2\tau_\eta^2(\beta^2a^2\tau_u+y\tau_\eta)-\beta^2a^2y\tau_u\tau_\eta\tau_\epsilon y\tau_\eta}{y^2\tau_\eta^2\left(\beta^2a^2y\tau_u\tau_\eta+(\tau_\epsilon+\tau_\theta)\left(\beta^2a^2\tau_u+y\tau_\eta\right)\right)-\beta^2a^2y\tau_u\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)} \tag{23}$$

Using the fact that

$$\gamma_2(\tau_\omega(a))=\left(1-\gamma_1(\tau_\omega(a))\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}$$

along with the fact that $\gamma_1(\tau_\omega(a))=\lambda a$, in turn we have that

$$c=\frac{1}{\lambda}\left[\left(1-\lambda a\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\beta(a+c)\right]$$

from which we obtain that the sensitivity of the equilibrium trades to the endogenous public signal must satisfy

$$c=\frac{1}{\beta+\lambda}\left[\left(1-\lambda a\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\beta a\right]. \tag{24}$$

Finally, rearranging (22), we have that the constant b in the equilibrium trades is given by

$$b=-\frac{\alpha}{\beta+\lambda}. \tag{25}$$

Now using (23), along with the fact that the sensitivity a of the equilibrium trades to private information coincides with the sensitivity of the equilibrium demand schedules to private information, we thus have that a^* is given by the unique positive root to the cubic equation

$$0 \;=\; \lambda\beta^2\tau_u a^3\left[y^3\tau_\eta^3+y^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta\right)-y\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)\right]+\lambda a y^3\tau_\eta^3\left(\tau_\epsilon+\tau_\theta\right)-\tau_\epsilon y^3\tau_\eta^3,$$

as claimed in the proposition. Finally, inverting the relationship between c and $\hat{c}$ and b and $\hat{b}$ in (19) and (18), we obtain that the other two parameters defining the equilibrium demand schedules are given by

$$\hat{c}^*=-\frac{c}{\beta\left(a^*+c\right)}=-\frac{\frac{1}{\beta+\lambda}\left(\gamma_2(\tau_\omega(a^*))-\beta a^*\right)}{\beta a^*+\beta\frac{1}{\beta+\lambda}\left(\gamma_2(\tau_\omega(a^*))-\beta a^*\right)}$$

and

$$\hat{b}^*=\left(1+\beta\hat{c}^*\right)b+\alpha\hat{c}^*=-\left(1+\beta\hat{c}^*\right)\frac{\alpha}{\beta+\lambda}+\alpha\hat{c}^*=\frac{\alpha}{\beta+\lambda}\left(\lambda\hat{c}^*-1\right).$$

Replacing

$$\gamma_2(\tau_\omega(a^*))=\left(1-\gamma_1(\tau_\omega(a^*))\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\omega(a^*)+\tau_\theta}=\left(1-\lambda a^*\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\omega(a^*)+\tau_\theta}$$

into the above expressions, we obtain the result in the proposition. Q.E.D.

Analysis of Equation 10.

In a cubic equation of the form $Ax^3+Bx^2+Cx+D=0$, if

$$\Delta\equiv 18ABCD-4B^3D+B^2C^2-4AC-27A^2D^2<0$$

then the equation has a unique real root. In our case, $B = 0$ and $C > 0$ and, as a result, $\Delta = -4AC - 27A^2D^2$. Furthermore (using the fact that $\tau_\epsilon \equiv \frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}$), we have that

$$\begin{aligned} A &= \lambda\beta^2\tau_u\left(y^3\tau_\eta^3 + y^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta\right) - y\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)\right) \propto y\tau_\eta\left(y^2\tau_\eta^2 + y\tau_\eta\tau_\theta - \tau_\epsilon\tau_\theta - \tau_\epsilon y\tau_\eta\right) \\ &\propto (\tau_\theta+y\tau_\eta)(y\tau_\eta-\tau_\epsilon) \propto y\tau_\eta - \frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta} \propto \frac{\tau_\eta}{\tau_e+\tau_\eta} > 0. \end{aligned}$$

Therefore $\Delta < 0$, and hence the cubic equation has a unique real root. Furthermore, because D is negative, the unique real root is positive. Replacing $a = \frac{1}{\lambda}$ into the cubic equation, we have that

$$\begin{aligned} &\beta^2\tau_u\frac{1}{\lambda^2}\left(y^3\tau_\eta^3 + y^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta\right) - y\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)\right) + y^3\tau_\eta^3\left(\tau_\epsilon+\tau_\theta\right) - \tau_\epsilon y^3\tau_\eta^3 \\ =&\beta^2\tau_u\frac{y\tau_\eta}{\lambda^2}\left(y^2\tau_\eta^2 + y\tau_\eta\tau_\theta - \tau_\epsilon\tau_\theta - \tau_\epsilon y\tau_\eta\right) + y^3\tau_\eta^3\tau_\theta > 0. \end{aligned}$$

This implies that $0 < a^* < \frac{1}{\lambda}$. Q.E.D.

Derivation of Condition 11.

Recall that $W = \int_0^1\left(\theta x_i - \frac{\lambda}{2}x_i^2\right)di - \left(\alpha - u + \beta\frac{\tilde{x}}{2}\right)\tilde{x}$. Because $\int_0^1\left(x_i^2\right)di > \left(\int_0^1 x_i di\right)^2$, we have that W is maximal when $x_i = x^o$ for all i, with

$$x^o \equiv \frac{\theta-\alpha+u}{\beta+\lambda}.$$

Q.E.D.

Derivation of Condition 12.

Recall that $WL = \mathbb{E}[W^o] - \mathbb{E}[W]$. Using the fact that

$$W^o = \theta x^o - \frac{\lambda}{2}(x^o)^2 - \left(\alpha - u + \beta\frac{x^o}{2}\right)x^o$$

along with the fact that $x^o = \frac{\theta-\alpha+u}{\beta+\lambda}$, we then have that

$$W^o = \theta x^o - \frac{\lambda}{2}(x^o)^2 - \left(\alpha - u + \beta\frac{x^o}{2}\right)x^o = \frac{\beta+\lambda}{2}(x^o)^2.$$

It follows that

$$\begin{aligned} WL &= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right] - \mathbb{E}\left[\int_0^1\left(\theta x_i - \frac{\lambda}{2}x_i^2\right)di - \left(\alpha - u + \beta\frac{\tilde{x}}{2}\right)\tilde{x}\right] \\ &= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right] - \mathbb{E}\left[\left(\theta-\alpha+u\right)\tilde{x} - \beta\frac{\tilde{x}^2}{2} - \frac{\lambda}{2}\int_0^1 x_i^2 di\right]. \end{aligned}$$

Using again the characterization of the FB allocation, $x^o = \frac{\theta-\alpha+u}{\beta+\lambda}$, and the fact that

$$\mathbb{E}\left[\int_0^1 x_i^2 di\right] = \mathbb{E}\left[\mathbb{E}[x_i^2|\tilde{x}]\right]$$

we have that

$$
\begin{aligned}
WL &= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right] - \frac{1}{2}\mathbb{E}\left[2\left(\beta+\lambda\right)\tilde{x}x^o - \beta\tilde{x}^2 - \lambda\int_0^1 x_i^2 di\right] \\
&= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right] + \frac{1}{2}\mathbb{E}\left[(\beta+\lambda)\tilde{x}^2 - 2x^o\tilde{x}(\beta+\lambda) - \lambda\tilde{x}^2 + \lambda\mathbb{E}[x_i^2|\tilde{x}]\right] \\
&= \frac{(\beta+\lambda)\mathbb{E}[(\tilde{x}-x^o)^2] + \lambda\mathbb{E}[(x_i-\tilde{x})^2]}{2}.
\end{aligned}
$$

Q.E.D.

Proof of Lemma 1.

The same arguments as in the proof of Proposition 1 imply that, when the traders submit demand schedules of the form $x_i = as_i + \hat{b} - \hat{c}p$, for some $(a, \hat{b}, \hat{c})$, the trades induced by market clearing can be expressed as a function of the endogenous public information z generated by the market-clearing price by letting $x_i = as_i + b + cz$ where

$$z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$$

is the endogenous information about θ contained in the equilibrium price, and where the noise in the endogenous signal has precision

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 y \tau_u \tau_\eta}{\beta^2 a^2 \tau_u + y\tau_\eta}.$$

Furthermore, the values of b and c are given by (18) and (19). Using the above representation, we have that the aggregate volume of trade when the demand schedules are given by $(a, \hat{b}, \hat{c})$ is given by $\tilde{x} = a(\theta + f(y)\eta) + b + cz$ and hence ex-ante welfare is given by

$$\mathbb{E}[W] = \mathbb{E}\left[\left(\theta - \alpha + u\right)\left(a(\theta + f(y)\eta) + b + cz\right) - \beta\tfrac{(a(\theta+f(y)\eta)+b+cz)^2}{2} - \int_0^1 \tfrac{\lambda}{2}\left(as_i + b + cz\right)^2 di\right].$$

Note that

$$\frac{\partial\mathbb{E}[W]}{\partial b} = \mathbb{E}\left[\left(\theta - \alpha + u\right) - \beta\left(a(\theta + f(y)\eta) + b + cz\right) - \lambda\left(as + b + cz\right)\right] = -\alpha - (\beta+\lambda)b,$$

$$\frac{\partial^2\mathbb{E}[W]}{\partial b^2} = -(\beta+\lambda) < 0,$$

$$\frac{\partial\mathbb{E}[W]}{\partial c} = \mathbb{E}\left[z\left(\theta - \alpha + u\right) - \beta\left(a(\theta + f(y)\eta) + b + cz\right)z - \lambda z\left(as + b + cz\right)\right],$$

$$\frac{\partial^2\mathbb{E}[W]}{\partial c^2} = \mathbb{E}\left[-\beta z^2 - \lambda z^2\right] < 0,$$

and $\partial^2\mathbb{E}[W]/\partial c\partial b = 0$. Hence $\mathbb{E}[W]$ is concave in b and c . For any a, the optimal values of b and c are thus given by the FOCs $\partial\mathbb{E}[W]/\partial b = 0$ and $\partial\mathbb{E}[W]/\partial c = 0$ from which we obtain that

$$b = -\frac{\alpha}{\beta+\lambda}$$

and

$$\mathbb{E}\left[z\left(\theta + u\right) - \beta\left(a(\theta + f(y)\eta)\right)z - \beta cz^2 - \lambda azs - \lambda cz^2\right] = 0.$$

The last condition can be rewritten as

$$Cov\left[\left(\theta + u - \beta a(\theta + f(y)\eta)\right), z\right] - (\beta + \lambda)\, cVar(z) - \lambda a Cov(z, s) = 0$$

from which we obtain that

$$c = \frac{Cov\left[\left(\theta + u - \beta a(\theta + f(y)\eta)\right), z\right]}{(\beta + \lambda)\, Var(z)} - \frac{\lambda a Cov(z, s)}{(\beta + \lambda)\, Var(z)}.$$

Using the fact that $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$ and $s = \theta + \frac{1}{\sqrt{y}}(\eta + e)$, we have that

$$Var(z) = \frac{1}{\tau_\theta} + \frac{1}{\tau_\omega(a)} = \sigma_\theta^2 + \sigma_\omega^2(a),$$

where $\sigma_\theta^2 = 1/\tau_\theta$ and $\sigma_\omega^2(a) = 1/\tau_\omega(a)$. Furthermore,

$$\begin{aligned} Cov\left[\left(\theta + u - \beta a(\theta + f(y)\eta)\right), z\right] &= Cov\left[\left(\theta + u - \beta a(\theta + f(y)\eta)\right), \theta + f(y)\eta - \frac{u}{\beta a}\right] \\ &= Cov\left[\theta(1 - \beta a), \theta\right] + Cov\left[u, -\frac{u}{\beta a}\right] - Cov\left[\beta a f(y)\eta, f(y)\eta\right] \\ &= (1 - \beta a)\sigma_\theta^2 - \frac{\sigma_u^2}{\beta a} - \beta a f(y)^2 \sigma_\eta^2, \end{aligned}$$

and

$$Cov\left[z, s\right] = \sigma_\theta^2 + f(y)^2 \sigma_\eta^2.$$

Hence,

$$\begin{aligned} c &= \frac{(1 - \beta a)\sigma_\theta^2 - \frac{\sigma_u^2}{\beta a} - \beta a f(y)^2 \sigma_\eta^2}{(\beta + \lambda)\left(\sigma_\theta^2 + \sigma_\omega^2(a)\right)} - \frac{\lambda a(\sigma_\theta^2 + f(y)^2 \sigma_\eta^2)}{(\beta + \lambda)\left(\sigma_\theta^2 + \sigma_\omega^2(a)\right)} \\ &= \frac{1}{\beta + \lambda}\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right]. \end{aligned}$$

We conclude that, given a, the optimal values for c and b are given by the same functions in (24) and (25) that characterize the parameters c and b as a function of a under the equilibrium usage of information. To go from the optimal trades to the demand schedules that implement them, it then suffices to use the functions defined by (18) and (19). We thus conclude that, for any choice of a^T, the optimal values of $\hat{c}^T$ and $\hat{b}^T$ are given by the functions (8) and (9), as claimed. Q.E.D.

Derivation of Condition 13.

As shown above, the welfare losses can be expressed as

$$WL = \frac{\beta + \lambda}{2}\mathbb{E}[(\tilde{x} - x^o)^2] + \frac{\lambda}{2}\mathbb{E}[(x_i - \tilde{x})^2],$$

where x^0 is given by (11). We have also shown above that, for any vector $(a, \hat{b}, \hat{c})$ describing the demand schedules, there exists a unique vector (a, b, c) describing the induced trades $x_i = as_i + b + cz$ at the market-clearing price, and vice versa, where $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$ is the endogenous signal contained in the market-clearing price. This also means, when the traders submit the demand schedules corresponding to the vector $(a, \hat{b}, \hat{c})$, the aggregate volume of trade at the market-clearing price can be expressed as a function of (θ, η, z) as follows: $\tilde{x} = a(\theta + f(y)\eta) + b + cz$. Therefore, the dispersion of individual trades around the aggregate trade can be expressed as

$$\mathbb{E}[(x_i - \tilde{x})^2] = \mathbb{E}[a^2 f(y)^2 e_i^2] = \frac{a^2}{y\tau_e}.$$

Next, use the fact that, for any a, the optimal values of c and b are given by (24) and (25), along with the fact that $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$, and the fact that $f(y) = 1/\sqrt{y}$, to obtain that

$$\begin{aligned} \tilde{x} &= a(\theta + f(y)\eta) + b + cz \\ &= \frac{\lambda a(\theta + f(y)\eta) + u - \alpha + \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} z}{\beta + \lambda}. \end{aligned}$$

Combining the expression for $\tilde{x}$ derived above with the expression for x^0 in (11), we have that

$$\mathbb{E}[(\tilde{x} - x^o)^2] = \mathbb{E}\left[\left(\frac{\lambda a(\theta + f(y)\eta) + u - \alpha + \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} z}{\beta + \lambda} - \frac{\theta - \alpha + u}{\beta + \lambda}\right)^2\right].$$

Simplifying, we have that

$$\mathbb{E}[(\tilde{x} - x^o)^2] = \mathbb{E}\left[\left(\frac{\lambda a f(y)\eta}{\beta+\lambda} + \frac{\left(1-\lambda a-\lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}(z-\theta)}{\beta+\lambda} - \frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]\theta}{\beta+\lambda}\right)^2\right].$$

Using the fact that $f(y) = 1/\sqrt{y}$, and that $\mathbb{E}[\omega\theta] = \mathbb{E}[\eta\theta] = 0$, we then have that

$$\begin{aligned} \mathbb{E}[(\tilde{x} - x^o)^2] &= \frac{\left(\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}\right)^2}{(\beta + \lambda)^2 \tau_\omega(a)} + \frac{\lambda^2 a^2 + 2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}}{(\beta + \lambda)^2 y\tau_\eta} \\ &+ \frac{\left(1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}\right)^2}{(\beta + \lambda)^2 \tau_\theta}. \end{aligned}$$

Replacing the expressions for $\mathbb{E}[(x_i - \tilde{x})^2]$ and $\mathbb{E}[(\tilde{x} - x^o)^2]$ derived above into the formula for the welfare losses, we then have that, for any a, when $\hat{b}$ and $\hat{c}$ are set optimally, the welfare losses can be expressed as

$$\begin{aligned} WL(a, \tau_\omega(a)) &= \frac{\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}\right]^2}{2(\beta + \lambda) \tau_\omega(a)} + \frac{\lambda^2 a^2 + 2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}}{2(\beta + \lambda) y\tau_\eta} \\ &+ \frac{\left[1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}\right]^2}{2(\beta + \lambda) \tau_\theta} + \frac{\lambda a^2}{2y\tau_e}. \end{aligned}$$

as claimed in the main text. Q.E.D.

Proof of Proposition 2.

As shown above, once b and c are set optimally as a function of a to minimize the welfare losses, the latter can be expressed as a function of a and $\tau_\omega(a)$, with the formula for $WL(a, \tau_\omega(a))$ given by (13), with $\tau_\omega(a)=\frac{\beta^2 a^2 \tau_u \tau_\eta y}{\beta^2 a^2 \tau_u + y\tau_\eta}$. The socially optimal level of a is thus the one that minimizes $WL(a, \tau_\omega(a))$ and is given by the FOC

$$\frac{dWL(a,\tau_\omega(a))}{da} = \frac{\partial WL(a,\tau_\omega(a))}{\partial a} + \frac{\partial WL(a,\tau_\omega(a)))}{\partial \tau_\omega(a)}\frac{\partial \tau_\omega(a)}{\partial a} = 0.$$

Note that

$$\begin{aligned}\frac{\partial WL(a,\tau_\omega(a))}{\partial a} = & -\frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\left(\lambda\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\omega(a)} \\ & +\frac{\lambda^2 a+\lambda\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\lambda^2 a\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{(\beta+\lambda)\,y\tau_\eta} \\ & +\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]\left(-\lambda+\lambda\left(\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\theta}+\frac{\lambda a}{y\tau_e}.\end{aligned}$$

Next note that

$$\begin{aligned}\frac{\partial WL(a,\tau_\omega(a))}{\partial \tau_\omega(a)} = & \frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{2\,(\beta+\lambda)}\frac{\tau_\theta-\tau_\omega(a)}{(\tau_\omega(a)+\tau_\theta)^3}+\frac{\lambda a\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)}{(\beta+\lambda)\,y\tau_\eta}\frac{\tau_\theta}{(\tau_\omega(a)+\tau_\theta)^2} \\ & -\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]}{(\beta+\lambda)\,\tau_\theta}\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\theta}{(\tau_\omega(a)+\tau_\theta)^2}.\end{aligned}$$

Finally note that

$$\frac{\partial \tau_\omega(a)}{\partial a} = \frac{2\beta^2 a y^2\tau_\eta^2\tau_u}{(\beta^2 a^2\tau_u+y\tau_\eta)^2}.$$

Using the above expressions we obtain that

$$\begin{aligned}\frac{dWL(a,\tau_\omega(a))}{da} = & -\frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\left(\lambda\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\omega(a)}+\frac{\lambda a}{y\tau_e}+L(a) \\ & +\frac{\lambda^2 a+\lambda\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\lambda^2 a\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{(\beta+\lambda)\,y\tau_\eta} \\ & +\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]\left(-\lambda+\lambda\left(\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\theta}\end{aligned}$$

where

$$L(a) \equiv \frac{\beta^2 a y^2 \tau_\eta^2 \tau_u}{(\beta^2 a^2 \tau_u + y\tau_\eta)^2} \left\{ \frac{\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{(\beta+\lambda)} \frac{\tau_\theta - \tau_\omega(a)}{\left(\tau_\omega(a)+\tau_\theta\right)^3} + \frac{2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)}{(\beta+\lambda)\, y\tau_\eta} \frac{\tau_\theta}{\left(\tau_\omega(a)+\tau_\theta\right)^2} \right.$$
$$\left. - \frac{2\left[1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]}{(\beta+\lambda)\,\tau_\theta} \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\theta}{\left(\tau_\omega(a)+\tau_\theta\right)^2} \right\}.$$

Hence, the first-order-condition $dWL(a, \tau_\omega(a))/da = 0$ is equivalent to

$$\begin{aligned} 0 &= \lambda a \tau_\epsilon \left((y\tau_\eta + \tau_\theta)^2 \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} \right) + \lambda a y \tau_\eta \tau_\epsilon \left(\tau_\omega(a)+\tau_\theta\right) - 2\lambda a \tau_\epsilon \left(y\tau_\eta + \tau_\theta\right) \tau_\omega(a) \\ &\quad + \lambda a \tau_\epsilon \frac{\left(\tau_\omega(a)+\tau_\theta\right)}{\tau_\theta} \left(y\tau_\eta - \left(y\tau_\eta + \tau_\theta\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} \right)^2 + \lambda a y \tau_\eta \tau_\epsilon \frac{y\tau_\eta \left(\tau_\omega(a)+\tau_\theta\right)(\beta+\lambda)}{\lambda y \tau_e} \\ &\quad + y\tau_\eta \tau_\epsilon \frac{(\beta+\lambda)\left(\tau_\omega(a)+\tau_\theta\right) y\tau_\eta L(a)}{\lambda} - y\tau_\eta\tau_\epsilon \left(y\tau_\eta - \tau_\omega(a)\right) \end{aligned}$$

from which we obtain that

$$\begin{aligned} y\tau_\eta\tau_\epsilon \left(y\tau_\eta - \tau_\omega(a)\right) &= \lambda a \left\{ y^2\tau_\eta^2 \tau_\epsilon - \tau_\omega(a)\tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right) + \left(\tau_\omega(a)+\tau_\theta\right) y^2 \tau_\eta^2 \right. \\ &\quad \left. + y\tau_\eta\tau_\epsilon \frac{y\tau_\eta \left(\tau_\omega(a)+\tau_\theta\right)\beta}{\lambda y \tau_e} + y\tau_\eta\tau_\epsilon \frac{(\beta+\lambda)\left(\tau_\omega(a)+\tau_\theta\right) y\tau_\eta L(a)}{\lambda^2 a} \right\}. \end{aligned}$$

Letting

$$\Delta(a) \equiv -\tau_\epsilon \frac{y^2\tau_\eta^2}{\lambda^2 a} \frac{\beta^2 a y^2 \tau_\eta^2 \tau_u}{(\beta^2 a^2 \tau_u + y\tau_\eta)^2} \frac{\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{\left(\tau_\omega(a)+\tau_\theta\right)} < 0$$
$$\Xi(a) \equiv \frac{\tau_\epsilon y \tau_\eta^2 \left(\tau_\omega(a)+\tau_\theta\right)\beta}{\lambda \tau_e} > 0,$$

we conclude that a^T must solve

$$a = \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta (y\tau_\eta - \tau_\omega(a))}{y^2\tau_\eta^2 \left(\tau_\epsilon + \tau_\theta + \tau_\omega(a)\right) - \tau_\omega(a)\tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right) + \Xi(a) + \Delta(a)}.$$

It is straightforward to verify that

$$\begin{aligned} \left.\frac{dWL(a, \tau_\omega(a))}{da}\right|_{a=\frac{1}{\lambda}} &= \frac{\lambda\tau_\theta}{(\beta+\lambda)\, y\tau_\eta (\tau_\omega(a)+\tau_\theta)} \frac{y\tau_\eta}{\beta^2 a^2 \tau_u + y\tau_\eta} \\ &\quad \left(1 - \frac{\beta^2 a^2 \tau_u}{(\beta^2 a^2 \tau_u + y\tau_\eta)} \times \frac{\tau_\theta}{(\tau_\omega(a)+\tau_\theta)} \right) + \frac{\lambda a}{y\tau_e} > 0 \end{aligned}$$

and that

$$\left.\frac{dWL(a,\tau_\omega(a))}{da}\right|_{a=0} = \frac{\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\left(-\lambda\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\omega(a)} + \frac{\lambda\left(\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,y\tau_\eta}$$
$$+\frac{\left(1-\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)\left(-\lambda+\lambda\left(\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\theta}$$
$$\propto \frac{\tau_\omega(a)}{y\tau_\eta}-1 = -\frac{y\tau_\eta}{\beta^2a^2\tau_u+y\tau_\eta}<0,$$

which implies that $0<a^T<\frac{1}{\lambda}$, as claimed in the proposition. Q.E.D.

Derivation of (15) and (16).

In the cursed economy, each trader receives a private signal $s_i = \theta + \underbrace{f(y)\eta + f(y)e_i}_{\equiv\epsilon_i}$ and a public signal $z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv\zeta}$, and believes p to be orthogonal to (θ, η).

Following steps similar to those leading to Proposition 1, we have that

$$\mathbb{E}[\theta|s_i,z] \;=\; \gamma_1 s_i + \gamma_2 z,$$

where

$$\gamma_1 \equiv \frac{\tau_\epsilon y\tau_\eta\,(y\tau_\eta-\tau_\zeta)}{y^2\tau_\eta^2(\tau_\zeta+\tau_\epsilon+\tau_\theta)-\tau_\zeta\tau_\epsilon(\tau_\theta+2y\tau_\eta)}$$

and

$$\gamma_2 \equiv \frac{y\tau_\eta\tau_\zeta\,(y\tau_\eta-\tau_\epsilon)}{y^2\tau_\eta^2(\tau_\zeta+\tau_\epsilon+\tau_\theta)-\tau_\epsilon\tau_\zeta(\tau_\theta+2y\tau_\eta)}$$
$$=\left(1-\gamma_1\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}.$$

Now observe that the cursed equilibrium demand schedules must satisfy

$$x_i \;=\; \frac{1}{\lambda}\left(\mathbb{E}[\theta|s_i,z]-p\right). \tag{26}$$

We thus have that

$$a^*_{exo} \;=\; \frac{\gamma_1}{\lambda},$$

which, using the formula for γ_1, is equivalent to the expression in (15).

Now suppose that the planner can select a but, given the latter, is constrained to choose (b,c,d) to maintain the same relationship between a and (b,c,d) as in the cursed equilibrium. Use (26) to observe that, in the cursed equilibrium,

$$b^*_{exo} = 0,$$
$$c^*_{exo} = \frac{\gamma_2}{\lambda},$$

and

$$d^*_{exo} = \frac{1}{\lambda}.$$

Furthermore, because

$$\gamma_2 = \left(1 - \gamma_1 \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta},$$

and because $\gamma_1 = a^*_{exo}\lambda$, we have that, in the cursed equilibrium, the relationship between a and (b, c, d) is given by

$$b^*_{exo} = 0,$$

$$c^*_{exo} = \frac{1}{\lambda}\left(1 - \lambda a^*_{exo} \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta},$$

and

$$d^*_{exo} = \frac{1}{\lambda}.$$

The above properties imply that, in the cursed economy, for any choice of a, the planner is constrained to select demand schedules of the form

$$x_i = \frac{1}{\lambda}\left(\lambda a s_i + \left(1 - \frac{\lambda a\left(y\tau_\eta + \tau_\theta\right)}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta} z - p\right). \tag{27}$$

The planner then chooses a to minimize the welfare losses

$$WL \quad = \quad \frac{(\beta + \lambda)\mathbb{E}[(\tilde{x} - x^o)^2] + \lambda\mathbb{E}[(x_i - \tilde{x})^2]}{2}$$

taking into account the market-clearing condition.

Following steps similar to those in the baseline economy and using the market-clearing condition, we have that, when the traders' demand schedules are given by (27),

$$\begin{aligned}\frac{(\beta + \lambda)}{2}\mathbb{E}[(\tilde{x} - x^o)^2] \quad &= \quad \frac{\left(\left(1 - \frac{\lambda a(y\tau_\eta + \tau_\theta)}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}\right)^2}{(\beta + \lambda)^2 \tau_\zeta} + \frac{\lambda^2 a^2 + 2\lambda a\left(1 - \frac{\lambda a(y\tau_\eta + \tau_\theta)}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}}{(\beta + \lambda)^2 y\tau_\eta} \\ &+ \frac{\left(1 - \lambda a - \left(1 - \frac{\lambda a(y\tau_\eta + \tau_\theta)}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}\right)^2}{(\beta + \lambda)^2 \tau_\theta}\end{aligned}$$

and

$$\frac{\lambda\mathbb{E}[(x_i - \tilde{x})^2]}{2} \quad = \quad \frac{\lambda a^2}{2y\tau_e}.$$

This means that, for any a, the welfare losses are equal to

$$
\begin{aligned}
WL &= \frac{\left[\left(1-\frac{\lambda a(y\tau_\eta+\tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\zeta} + \frac{\lambda^2 a^2 + 2\lambda a\left(1-\frac{\lambda a(y\tau_\eta+\tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}}{2(\beta+\lambda)y\tau_\eta} \\
&+ \frac{\left[1-\lambda a-\left(1-\frac{\lambda a(y\tau_\eta+\tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\theta} + \frac{\lambda a^2}{2y\tau_e}.
\end{aligned}
$$

Following steps similar to those in the proof of Proposition 2, we then have that the value of a that minimizes the above welfare losses is equal to

$$
a^T_{exo} = \frac{1}{\lambda}\frac{\tau_\epsilon y\tau_\eta(y\tau_\eta-\tau_\zeta)}{y^2\tau_\eta^2(\tau_\epsilon+\tau_\theta+\tau_\zeta)-\tau_\zeta\tau_\epsilon(\tau_\theta+2y\tau_\eta)+\frac{\tau_\epsilon y\tau_\eta^2(\tau_\zeta+\tau_\theta)\beta}{\lambda\tau_e}}
$$

as claimed in (16). Q.E.D.

Lemma 2. *$c^*=0$ if and only if $\Xi(a^*)+\Delta(a^*)=0$.*

Proof of Lemma 2. Recall that c^* is given by

$$
\begin{aligned}
c^* &= \frac{1}{\beta+\lambda}\left(\left(1-\lambda a^*-\lambda a^*\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\theta+\tau_\omega(a^*)}-\beta a^*\right) \\
&= \frac{1}{\beta+\lambda}\left(\gamma_2(a^*)-\beta a^*\right),
\end{aligned}
$$

whereas the externalities are given by

$$
\begin{aligned}
\Delta(a) &\equiv -\frac{\tau_\epsilon\beta^2 a y^4\tau_\eta^4\tau_u\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 a(\beta^2a^2\tau_u+y\tau_\eta)^2(\tau_\omega(a)+\tau_\theta)}, \\
\Xi(a) &\equiv \frac{\tau_\epsilon y\tau_\eta^2(\tau_\omega(a)+\tau_\theta)\beta}{\lambda\tau_e}.
\end{aligned}
$$

We prove the lemma in two steps. First we show that, if $c^*=0$, then $\Xi(a^*)+\Delta(a^*)=0$. To see this, use the formula for c^* from above to verify that, when $c^*=0$, then $\beta a^*=\gamma_2(a^*)$. Using the fact that

$$
\begin{aligned}
a^* &= \frac{1}{\lambda}\frac{\tau_\epsilon y\tau_\eta(y\tau_\eta-\tau_\omega(a^*))}{y^2\tau_\eta^2(\tau_\omega(a^*)+\tau_\epsilon+\tau_\theta)-\tau_\omega(a^*)\tau_\epsilon(\tau_\theta+2y\tau_\eta)}, \\
\gamma_2(a^*) &= \frac{\tau_\omega(a^*)\left(y^2\tau_\eta^2-\tau_\epsilon y\tau_\eta\right)}{y^2\tau_\eta^2(\tau_\omega(a^*)+\tau_\epsilon+\tau_\theta)-\tau_\omega(a^*)\tau_\epsilon(\tau_\theta+2y\tau_\eta)}, \\
\tau_\epsilon &= \frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}, \\
\tau_\omega(a^*) &= \frac{\beta^2a^{*2}y\tau_\eta\tau_u}{\beta^2a^{*2}\tau_u+y\tau_\eta},
\end{aligned}
$$

we then have that, when $c^* = 0$, then

$$\beta = \frac{\gamma_2(a^*)}{a^*} = \lambda \frac{\tau_\omega(a^*)\left(y^2\tau_\eta^2 - \tau_\epsilon y \tau_\eta\right)}{\tau_\epsilon y \tau_\eta \left(y\tau_\eta - \tau_\omega(a^*)\right)} = \lambda \frac{\tau_\omega(a^*)\left(y\tau_\eta - \frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\right)}{\frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\left(y\tau_\eta - \tau_\omega(a^*)\right)}.$$

Using the formula for $\tau_\omega(a^*)$ we then have that

$$\beta = \lambda \frac{\frac{\beta^2 a^{*2} y\tau_\eta\tau_u}{\beta^2 a^{*2}\tau_u + y\tau_\eta}\left(\frac{y\tau_\eta^2}{\tau_e+\tau_\eta}\right)}{\frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\left(y\tau_\eta - \frac{\beta^2 a^{*2} y\tau_\eta\tau_u}{\beta^2 a^{*2}\tau_u + y\tau_\eta}\right)} = \lambda \frac{\beta^2 a^{*2} y\tau_\eta\tau_u\tau_\eta}{\tau_e\left(y^2\tau_\eta^2\right)} = \lambda \frac{\beta^2 a^{*2}\tau_u}{\tau_e y}$$

from which we obtain that

$$\beta = \frac{y\tau_e}{\lambda a^{*2}\tau_u}.$$

Furthermore, using the expression for c^* above, we have that, when $c^* = 0$,

$$\left(1 - \lambda a^* - \lambda a^* \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\theta + \tau_\omega(a^*)} = \beta a^*.$$

Replacing the above expression into the formula for the two externalities, we thus have that

$$\Delta(a^*) + \Xi(a^*) = \frac{\tau_\epsilon y \tau_\eta^2\left(\tau_\omega(a^*) + \tau_\theta\right)\beta}{\lambda\tau_e} - \tau_\epsilon \frac{y^2\tau_\eta^2}{\lambda^2 a^*}\frac{\left(\tau_\theta + \tau_\omega(a^*)\right)}{a^*\tau_u}.$$

Using the expression for β above, we then have that

$$\begin{aligned}\Delta(a^*) + \Xi(a^*) &= \frac{\tau_\epsilon y \tau_\eta^2\left(\tau_\omega(a^*) + \tau_\theta\right)\frac{y\tau_e}{\lambda a^2\tau_u}}{\lambda\tau_e} - \frac{\tau_\epsilon y^2\tau_\eta^2}{\lambda^2 a^*}\frac{\left(\tau_\theta + \tau_\omega(a^*)\right)}{a^*\tau_u} \\ &= \frac{\tau_\epsilon y^2\tau_\eta^2}{\lambda^2 a^*}\left(\frac{\left(\tau_\omega(a^*) + \tau_\theta\right)}{a^*\tau_u} - \frac{\left(\tau_\theta + \tau_\omega(a^*)\right)}{a^*\tau_u}\right) = 0.\end{aligned}$$

Next, we prove the converse. We show that, if $\Delta(a^*) + \Xi(a^*) = 0$, then $c^* = 0$. To see this note that, when the sum of the two externalities is zero, then

$$\Delta(a^*) + \Xi(a^*) = \frac{\tau_\epsilon y \tau_\eta^2\left(\tau_\omega(a^*) + \tau_\theta\right)\beta}{\lambda\tau_e} - \frac{\tau_\epsilon y^4\tau_\eta^4\beta^2 a^*\tau_u\left(1 - \lambda a^* - \lambda a^*\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 a^*(\beta^2 a^{*2}\tau_u + y\tau_\eta)^2\left(\tau_\omega(a^*) + \tau_\theta\right)} = 0.$$

Using the various expressions above we then have that

$$0 = \frac{\left(\tau_\omega(a^*) + \tau_\theta\right)\beta}{y\tau_e} - \frac{1}{\lambda a^*}\frac{\tau_\omega(a^*)^2}{\beta^2 a^{*3}\tau_u}\frac{\left(1 - \lambda a^* - \lambda a^*\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\tau_\omega(a^*) + \tau_\theta}$$

$$
\begin{aligned}
0 &= \frac{\beta a^*}{y\tau_e} - \frac{1}{\gamma_1(a^*)}\frac{\gamma_2(a^*)^2}{\beta^2 a^{*2}\tau_u} \\
\beta a^* &= \frac{\tau_\omega(a^*)\left(y\tau_\eta - \tau_\epsilon\right)}{\tau_\epsilon\left(y\tau_\eta - \tau_\omega(a^*)\right)}\frac{y\tau_e}{\beta^2 a^{*2}\tau_u}\gamma_2(a^*) \\
&= \frac{\beta^2 a^{*2}\tau_u}{\tau_e y}\frac{y\tau_e}{\beta^2 a^{*2}\tau_u}\gamma_2(a^*) \\
&= \gamma_2(a^*).
\end{aligned}
$$

Hence $\beta a^* = \gamma_2(a^*)$. But this means that $c^* = 0$. Q.E.D.

Proof of Proposition 3.

Under the proposed policy, each trader's demand schedule must satisfy the optimality condition

$$X_i(p; I_i) = \frac{1}{\lambda + \delta}\left(\mathbb{E}[\theta | I_i, p] - (1 + t_p)p + t_0\right).$$

For any vector $(a, \hat{b}, \hat{c})$, when all traders submit affine demand schedules $x_i = as_i + \hat{b} - \hat{c}p$, the equilibrium price then continues to satisfy the same representation as in (2) but with $(a^*, \hat{b}^*, \hat{c}^*)$ replaced by $(a, \hat{b}, \hat{c})$. This also means that the equilibrium trades can be expressed as a function of the endogenous public signal z, as in the laissez-faire equilibrium with no policy. Letting $x_i = as_i + b + cz$ denote the trades generated by the demand schedules $x_i = as_i + \hat{b} - \hat{c}p$ (with z representing the endogenous public signal contained in the market-clearing price), we then have that the functions that map the coefficients $\hat{c}$ and $\hat{b}$ in the demand schedules into the coefficients c and b in the induced trades continue to be given by (8) and (9). Using the fact that $\mathbb{E}[\theta | s_i, z] = \gamma_1(\tau_\omega(a))s_i + \gamma_2(\tau_\omega(a))z$, with the functions γ_1 and γ_2 as defined in (5) and (6), along with the fact that the market-clearing price satisfies $p = \alpha + \beta b + \beta(a + c)z$ as shown in (20), we then have that the equilibrium trades must satisfy

$$
\begin{aligned}
x_i &= \frac{1}{\lambda + \delta}\left[\gamma_1(\tau_\omega(a))s_i + \gamma_2(\tau_\omega(a))z - (1 + t_p)\alpha - (1 + t_p)\beta b - (1 + t_p)\beta(a + c)z + t_0\right] \\
&= \frac{1}{\lambda + \delta}\left\{\gamma_1(\tau_\omega(a))s_i - (1 + t_p)\left(\alpha + \beta b\right) + \left[\gamma_2(\tau_\omega(a)) - (1 + t_p)\beta(a + c)\right]z + t_0\right\}.
\end{aligned}
$$

The sensitivity of the equilibrium trades to private information s_i under the proposed policy thus satisfies $a = \gamma_1(\tau_\omega(a))/(\lambda + \gamma)$. Using the formula for γ_1 in (5), we then have that the equilibrium a under the proposed policy is the unique solution to the following equation:

$$a = \frac{1}{\lambda + \delta}\frac{\tau_\epsilon y^2\tau_\eta^2 - \tau_\omega(a)\tau_\epsilon y\tau_\eta}{y^2\tau_\eta^2(\tau_\omega(a) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)}.$$

The equilibrium c, instead, is given by the unique solution to

$$c = \frac{1}{\lambda + \delta}\left[\gamma_2(\tau_\omega(a)) - (1 + t_p)\beta(a + c)\right]$$

which is equal to

$$c = \frac{\gamma_2(\tau_\omega(a)) - (1+t_p)\beta a}{\lambda + \delta + (1+t_p)\beta}.$$

Finally, the equilibrium b is given by the unique solution to

$$b = \frac{-(1+t_p)(\alpha + \beta b) + t_0}{\lambda + \delta}$$

which is equal to

$$b = \frac{t_0 - (1+t_p)\alpha}{\lambda + \delta + (1+t_p)\beta}.$$

Now recall that the sensitivity a^T of the efficient trades to private information is given by the unique solution to

$$a = \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta (y\tau_\eta - \tau_\omega(a))}{y^2 \tau_\eta^2 (\tau_\epsilon + \tau_\theta + \tau_\omega(a)) - \tau_\omega(a)\tau_\epsilon (\tau_\theta + 2y\tau_\eta) + \Xi(a) + \Delta(a)}.$$

Therefore, the equilibrium value a under the proposed policy coincides with the efficient level a^T if and only if δ satisfies

$$\begin{aligned} &(\lambda + \delta)\left[y^2\tau_\eta^2 \left(\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta\right) - \tau_\omega(a^T)\tau_\epsilon (\tau_\theta + 2y\tau_\eta)\right] \\ &= \lambda \left[y^2\tau_\eta^2 \left(\tau_\epsilon + \tau_\theta + \tau_\omega(a^T)\right) - \tau_\omega(a^T)\tau_\epsilon (\tau_\theta + 2y\tau_\eta) + \Xi(a^T) + \Delta(a^T)\right], \end{aligned}$$

from which we obtain that

$$\delta = \frac{\lambda\left(\Xi(a^T) + \Delta(a^T)\right)}{y^2\tau_\eta^2(\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^T)\tau_\epsilon (\tau_\theta + 2y\tau_\eta)}.$$

Now recall that, given a^T, the other two coefficients c^T and b^T describing the efficient trades are given by the functions in (24) and (25), implying that

$$c^T = \frac{1}{\beta + \lambda}\left(\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T) + \tau_\theta} - \beta a^T\right)$$

and

$$b^T = -\frac{\alpha}{\beta + \lambda}.$$

Hence, for the equilibrium levels of c and b under the proposed policy to coincide with the efficient levels it must be that

$$\frac{\gamma_2(\tau_\omega(a^T)) - (1+t_p)\beta a^T}{\lambda + \delta + (1+t_p)\beta} = \frac{1}{\beta + \lambda}\left(\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T) + \tau_\theta} - \beta a^T\right)$$

and

$$\frac{t_0 - (1+t_p)\alpha}{\lambda + \delta + (1+t_p)\beta} = -\frac{\alpha}{\beta + \lambda}$$

It is easy to see that the above two equations are satisfied when

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda+\delta+\beta}{\beta+\lambda}\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right] - \beta a^T}{\beta\left\{\frac{1}{\beta+\lambda}\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a\right] + a^T\right\}}$$

and

$$t_0 = (1+t_p)\alpha - \frac{\alpha\left[\lambda + \delta + (1+t_p)\beta\right]}{\beta + \lambda}.$$

Q.E.D.

Proof of Proposition 4.

When all traders other than i acquire information of quality $\bar{y}$ and then submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, irrespectively of the information acquired by trader i and of the demand schedule submitted by the latter, the equilibrium price is given by

$$p(\theta, u, \eta; \bar{y}) = \alpha + \beta b^T + \beta(a^T + c^T)z(\theta, u, \eta; \bar{y})$$

where b^T and c^T are the coefficients obtained from $(a^T, \hat{b}^T, \hat{c}^T)$ using the functions (18) and (19), and where[18]

$$z(\theta, u, \eta; \bar{y}) \equiv \theta + f(\bar{y})\eta - u/\beta a^T.$$

Furthermore, the aggregate level of trade is equal

$$\tilde{X}(\theta, u, \eta; \bar{y}) = a^T[\theta + f(\bar{y})\eta] + b^T + c^T z(\theta, u, \eta; \bar{y})$$

whereas the equilibrium trade for agent i when he acquires information of quality y_i and then submits the demand schedule corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ is equal to

$$X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T z(\theta, u, \eta; \bar{y}).$$

It follows that, when all traders other than i acquire information of quality $\bar{y}$, trader i acquires information of quality y_i and all traders, including trader i, submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, trader i's ex-ante gross payoff is equal to

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}\left[(\theta - p(\theta, u, \eta; \bar{y}))\, X_i(\theta, u, \eta, e_i; \bar{y}, y_i) - \frac{\lambda}{2} X_i^2(\theta, u, \eta, e_i; \bar{y}, y_i)\right].$$

Using the fact that the market clearing price must also satisfy

$$p = \alpha - u + \beta \tilde{X}(\theta, u, \eta; \bar{y}),$$

we have that

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}_{\theta,u,\eta}\left[\left(\theta - \alpha + u - \beta\tilde{X}(\theta, u, \eta; \bar{y})\right)\mathbb{E}[x_i|\theta, u, \eta; \bar{y}, y_i] - \frac{\lambda}{2}\mathbb{E}\left[x_i^2|\theta, u, \eta; \bar{y}, y_i\right]\right]$$

or, equivalently,

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}_{\theta,u,\eta}\left[\left(\theta - \alpha + u - \beta\tilde{X}(\theta, u, \eta; \bar{y})\right)\mathbb{E}[x_i|\theta, u, \eta; \bar{y}, y_i] - \tfrac{\lambda}{2}Var[x_i|\theta, \eta, u; \bar{y}, y_i]\right.$$

$$\left. - \tfrac{\lambda}{2}\left(\mathbb{E}[x_i|\theta, \eta, u; \bar{y}, y_i]\right)^2\right],$$

[18] Observe that the functions (18) and (19) do not depend on y and hence c^T and b^T do not depend on y.

where

$$\mathbb{E}[x_i|\theta,u,\eta;\bar{y},y_i] \equiv \mathbb{E}[X_i(\theta,u,\eta,e_i;\bar{y},y_i)|\theta,u,\eta;\bar{y},y_i]$$

and

$$\mathbb{E}[x_i^2|\theta,u,\eta;\bar{y},y_i] \equiv \mathbb{E}\left[\left(X_i(\theta,u,\eta,e_i;\bar{y},y_i)\right)^2|\theta,u,\eta;\bar{y},y_i\right]$$

and

$$Var[x_i|\theta,\eta,u;\bar{y},y_i] \equiv \mathbb{E}\left[\left(X_i(\theta,u,\eta,e_i;\bar{y},y_i)-\mathbb{E}[x_i|\theta,u,\eta;\bar{y},y_i]\right)^2|\theta,u,\eta;\bar{y},y_i\right].$$

Using the fact that

$$\mathbb{E}[x_i|\theta,u,\eta;\bar{y},y_i] = a^T[\theta+f(y_i)\eta]+b^T+c^Tz(\theta,u,\eta;\bar{y})$$

and

$$Var[x_i|\theta,\eta,u;\bar{y},y_i] = \frac{\left(a^Tf(y_i)\right)^2}{\tau_e},$$

we have that

$$\begin{aligned}
\frac{\partial\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i} &= \mathbb{E}_{\theta,\eta,u}\left[\left(\theta-\alpha+u-\beta\tilde{X}(\theta,u,\eta;\bar{y})\right)a^Tf'(y_i)\eta\right]-\lambda\frac{\left(a^T\right)^2}{\tau_e}f(y_i)f'(y_i)\\
&\quad -\lambda\mathbb{E}_{\theta,\eta,u}\left[\left(a^T[\theta+f(y_i)\eta]+b^T+c^Tz(\theta,u,\eta;\bar{y})\right)a^Tf'(y_i)\eta\right]\\
&= -a^T\beta\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta,u,\eta;\bar{y})\eta\right]f'(y_i)-\lambda\frac{\left(a^T\right)^2}{\tau_e}f(y_i)f'(y_i)\\
&\quad -\lambda\left(a^T\right)^2f(y_i)f'(y_i)\frac{1}{\tau_\eta}-\lambda a^Tc^T\mathbb{E}_{\theta,\eta,u}\left[z(\theta,u,\eta;\bar{y})\eta\right]f'(y_i).
\end{aligned}$$

Using the fact that

$$\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta,u,\eta;\bar{y})\eta\right] = a^Tf(\bar{y})\frac{1}{\tau_n}+c^T\mathbb{E}_{\theta,\eta,u}\left[z(\theta,u,\eta;\bar{y})\eta\right]$$

and

$$\mathbb{E}_{\theta,\eta,u}\left[z(\theta,u,\eta;\bar{y})\eta\right] = f(\bar{y})\frac{1}{\tau_n},$$

we then have that

$$\begin{aligned}
\frac{\partial\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i} &= -a^T\beta\left[a^Tf(\bar{y})\frac{1}{\tau_n}+c^Tf(\bar{y})\frac{1}{\tau_n}\right]f'(y_i)-\lambda\frac{\left(a^T\right)^2}{\tau_e}f(y_i)f'(y_i)\\
&\quad -\lambda\left(a^T\right)^2f(y_i)f'(y_i)\frac{1}{\tau_\eta}-\lambda a^Tc^Tf(\bar{y})\frac{1}{\tau_n}f'(y_i), \qquad (28)
\end{aligned}$$

from which we obtain that

$$\begin{aligned}
\left.\frac{\partial\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i}\right|_{y_i=\bar{y}} &= -a^T\beta\left[a^Tf(\bar{y})\frac{1}{\tau_n}+c^Tf(\bar{y})\frac{1}{\tau_n}\right]f'(\bar{y})-\lambda\frac{\left(a^T\right)^2}{\tau_e}f(\bar{y})f'(\bar{y})\\
&\quad -\lambda\left(a^T\right)^2f(\bar{y})f'(\bar{y})\frac{1}{\tau_\eta}-\lambda a^Tc^Tf(\bar{y})\frac{1}{\tau_n}f'(\bar{y})\\
&= -f(\bar{y})f'(\bar{y})a^T\left\{\lambda\frac{a^T}{\tau_e}+(\beta+\lambda)(a^T+c^T)\frac{1}{\tau_\eta}\right\}. \qquad (29)
\end{aligned}$$

Next, observe that, when trader i also acquires information of quality $\bar{y}$ and all traders submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, then

$$\mathbb{E}[\pi_i^T; \bar{y}, \bar{y}] = \mathbb{E}_{\theta,u,\eta}\left[\left(\theta - \alpha + u - \beta\tilde{X}(\theta,u,\eta;\bar{y})\right)\tilde{X}(\theta,u,\eta;\bar{y}) - \frac{\lambda}{2}\frac{\left(a^T f(\bar{y})\right)^2}{\tau_e} - \frac{\lambda}{2}\left(\tilde{X}(\theta,u,\eta;\bar{y})\right)^2\right].$$

The ex-ante payoff of the representative investor when all traders acquire information of quality $\bar{y}$ and submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$ is equal to

$$\begin{aligned}\mathbb{E}[\Pi; \bar{y}] &= \mathbb{E}_{\theta,u,\eta}\left[\left(p(\theta,u,\eta;\bar{y}) - \alpha + u\right)\tilde{X}(\theta,u,\eta;\bar{y}) - \frac{\beta}{2}\left(\tilde{X}(\theta,u,\eta;\bar{y})\right)^2\right] \\ &= \frac{\beta}{2}\mathbb{E}_{\theta,u,\eta}\left[\left(\tilde{X}(\theta,u,\eta;\bar{y})\right)^2\right],\end{aligned}$$

where we used the fact that $p(\theta,u,\eta;\bar{y}) = \alpha - u + \beta\tilde{X}(\theta,u,\eta;\bar{y})$. We thus have that, when all traders acquire information of quality $\bar{y}$ and submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, ex-ante welfare is given by

$$\begin{aligned}\mathbb{E}[W^T; \bar{y}] &= \mathbb{E}[\pi_i^T; \bar{y}, \bar{y}] + \mathbb{E}[\Pi; \bar{y}] \\ &= \mathbb{E}_{\theta,u,\eta}\left[\left(\theta - \alpha + u\right)\tilde{X}(\theta,u,\eta;\bar{y}) - \frac{\lambda}{2}\frac{\left(a^T f(\bar{y})\right)^2}{\tau_e} - \frac{\lambda+\beta}{2}\left(\tilde{X}(\theta,u,\eta;\bar{y})\right)^2\right].\end{aligned}$$

Hence,

$$\frac{d\mathbb{E}[W^T;\bar{y}]}{d\bar{y}} = \mathbb{E}_{\theta,\eta,u}\left[(\theta-\alpha+u)\frac{\partial\tilde{X}(\theta,u,\eta;\bar{y})}{\partial\bar{y}} - \frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda+\beta)\tilde{X}(\theta,u,\eta;\bar{y})\frac{\partial\tilde{X}(\theta,u,\eta;\bar{y})}{\partial\bar{y}}\right],$$

where

$$\frac{\partial\tilde{X}(\theta,u,\eta;\bar{y})}{\partial\bar{y}} = (a^T + c^T)f'(\bar{y})\eta.$$

It follows that

$$\frac{d\mathbb{E}[W^T;\bar{y}]}{d\bar{y}} = -\frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda+\beta)(a^T+c^T)f'(\bar{y})\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta,u,\eta;\bar{y})\eta\right].$$

Using the fact that

$$\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta,u,\eta;\bar{y})\eta\right] = (a^T+c^T)f(\bar{y})\frac{1}{\tau_n},$$

we thus have that

$$\frac{d\mathbb{E}[W^T;\bar{y}]}{d\bar{y}} = -\frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda+\beta)\left(a^T+c^T\right)^2 f'(\bar{y})f(\bar{y})\frac{1}{\tau_n}. \qquad (30)$$

Comparing (29) with (30), we thus have that, when $c^T < 0$,

$$\left.\frac{\partial\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i}\right|_{y_i=\bar{y}} > \frac{d\mathbb{E}[W^T;\bar{y}]}{d\bar{y}},$$

whereas the opposite inequality holds when $c^T > 0$. Finally, use Condition (19) to observe that $\hat{c}^T = -\frac{c^T}{\beta(a^T+c^T)}$ and Condition (24), along with the formula for $\tau_\omega(a)$, to observe that $a^T + c^T > 0$. Jointly, the last two conditions imply that $sgn(\hat{c}^T) = -sgn(c^T)$ thus completing the proof. Q.E.D.

Proof of Proposition 5.

We start by establishing the (global) concavity of $\mathbb{E}[\pi_i^T;\bar{y},y_i]$ and $\mathbb{E}[W^T;\bar{y}]$ in y_i and $\bar{y}$, respectively (Recall that the coefficients defining the traders' demand are kept constant in both of these functions at (a^T,b^T,c^T), where (a^T,b^T,c^T) is the vector defining the efficient trades when the quality of private information is y^T). Using (28), we have that

$$\begin{aligned}\frac{\partial^2\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i^2} &= -a^T\beta f(\bar{y})\frac{1}{\tau_\eta}\left(a^T+c^T\right)f''(y_i)-\lambda\left(a^T\right)^2\left[\frac{1}{\tau_e}+\frac{1}{\tau_\eta}\right]\frac{\partial}{\partial y_i}\left(f(y_i)f'(y_i)\right)\\ &\quad -\lambda a^Tc^Tf(\bar{y})\frac{1}{\tau_\eta}f''(y_i)\\ &= -a^Tf(\bar{y})\frac{1}{\tau_\eta}\left[\beta\left(a^T+c^T\right)+\lambda c^T\right]f''(y_i)-\lambda\left(a^T\right)^2\left[\frac{1}{\tau_e}+\frac{1}{\tau_\eta}\right]\frac{\partial}{\partial y_i}\left(f(y_i)f'(y_i)\right).\end{aligned}$$

Now observe that $f''(y_i)=3\sqrt{y_i}/4y_i^3>0$ and $\frac{\partial}{\partial y_i}\left(f(y_i)f'(y_i)\right)=1/y_i^3>0$. Hence,

$$\frac{\partial^2\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i^2} = -\frac{a^T}{y_i^3\tau_\eta}\left[\frac{3\sqrt{y_i}}{4\sqrt{\bar{y}}}\left(\beta a^T+(\beta+\lambda)\,c^T\right)+\lambda a^T\frac{\tau_\eta+\tau_e}{\tau_e}\right].$$

Now recall that, irrespective of the sign of c^T, $a^T>0$ and $a^T+c^T>0$, where the last inequality is established in the proof of Proposition 4. Hence, when $c^T\geq 0$, for any $(\bar{y},y_i)$, $\partial^2\mathbb{E}[\pi_i^T;\bar{y},y_i]/\partial y_i^2<0$. To see that the same inequality holds when $c^T<0$, recall that

$$c^T=\frac{1}{\beta+\lambda}\left[\left(1-\lambda a^T-\lambda a^T\frac{\tau_\theta}{y^T\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta}-\beta a^T\right].$$

Hence,

$$\beta a^T+(\beta+\lambda)\,c^T=\left(1-\lambda a^T-\lambda a^T\frac{\tau_\theta}{y^T\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta},$$

which, using

$$\tau_\omega(a^T)=\frac{\beta^2\left(a^T\right)^2y^T\tau_\eta\tau_u}{\beta^2\left(a^T\right)^2\tau_u+y^T\tau_\eta},$$

we can rewrite as

$$\beta a^T+(\beta+\lambda)\,c^T=\left[\left(1-\lambda a^T\right)y^T\tau_\eta-\lambda a^T\tau_\theta\right]\frac{\beta^2\left(a^T\right)^2\tau_u}{\beta^2\left(a^T\right)^2\tau_u\left(y^T\tau_\eta+\tau_\theta\right)+y^T\tau_\eta\tau_\theta}.$$

Hence,

$$sgn\left(\beta a^T+(\beta+\lambda)\,c^T\right)=sgn\left(\left(1-\lambda a^T\right)y^T\tau_\eta-\lambda a^T\tau_\theta\right).$$

Now recall that

$$a^T=\frac{1}{\lambda}\frac{\tau_\epsilon y^T\tau_\eta(y^T\tau_\eta-\tau_\omega(a^T))}{\left(y^T\right)^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta+\tau_\omega(a^T)\right)-\tau_\omega(a^T)\tau_\epsilon\left(\tau_\theta+2y^T\tau_\eta\right)+\Xi(a^T)+\Delta(a^T)} \tag{31}$$

with $\tau_\epsilon=\left(y^T\tau_e\tau_\eta\right)/\left(\tau_e+\tau_\eta\right)$ and observe that the numerator in (31) is positive. Because $a^T>0$, as shown above, this means that the denominator in (31) is also positive. Using the fact that

$$\left(1-\lambda a^{T}\right) y^{T}\tau_{\eta}-\lambda a^{T}\tau_{\theta}=\frac{y^{T}\tau_{\eta}Q}{\left(y^{T}\right)^{2}\tau_{\eta}^{2}\left(\tau_{\epsilon}+\tau_{\theta}+\tau_{\omega}(a^{T})\right)-\tau_{\omega}(a^{T})\tau_{\epsilon}\left(\tau_{\theta}+2y^{T}\tau_{\eta}\right)+\Xi(a^{T})+\Delta(a^{T})}$$

where

$$Q=y^{T}\tau_{\eta}\left(y^{T}\tau_{\eta}-\tau_{\epsilon}\right)\left(\tau_{\theta}+\tau_{\omega}(a^{T})\right)+\Xi(a^{T})+\Delta(a^{T}),$$

we thus have that

$$sgn\left(\left(1-\lambda a^{T}\right)y^{T}\tau_{\eta}-\lambda a^{T}\tau_{\theta}\right)=sgn\left(Q\right).$$

Now, using the fact that $\tau_{\epsilon}=\left(y\tau_{e}\tau_{\eta}\right)/\left(\tau_{e}+\tau_{\eta}\right)$, we have that Q can be rewritten as

$$Q=\left(y^{T}\tau_{\eta}\right)^{2}\frac{\tau_{\eta}}{\tau_{e}+\tau_{\eta}}\left(\tau_{\theta}+\tau_{\omega}(a^{T})\right)+\Xi(a^{T})+\Delta(a^{T})$$

and hence $sgn\left(Q\right)>0$ if $\Xi(a^{T})+\Delta(a^{T})>0$. The latter property holds because, as explained in the main text, when $c^{T}<0$, then $\hat{c}^{T}>0$ in which case $\Xi(a^{T})+\Delta(a^{T})>0$. We conclude that, no matter the sign of c^{T}, for any $\bar{y}$, $\mathbb{E}[\pi_{i}^{T};\bar{y},y_{i}]$ is strictly concave in y_{i}.

Next, consider the concavity of $\mathbb{E}[W^{T};\bar{y}]$ in $\bar{y}$. Using (30), we have that

$$\begin{aligned}\frac{d^{2}\mathbb{E}[W^{T};\bar{y}]}{d\bar{y}^{2}} &= -\left[\frac{\lambda\left(a^{T}\right)^{2}}{\tau_{e}}+\left(\lambda+\beta\right)\left(a^{T}+c^{T}\right)^{2}\frac{1}{\tau_{n}}\right]\frac{\partial}{\partial\bar{y}}\left(f(\bar{y})f'(\bar{y})\right)\\ &< 0,\end{aligned}$$

where again the inequality follows from the fact that $\frac{\partial}{\partial\bar{y}}\left(f(\bar{y})f'(\bar{y})\right)>0$. Hence $\mathbb{E}[W^{T};\bar{y}]$ is strictly concave in $\bar{y}$.

Because $\mathbb{E}[\pi_{i}^{T};\bar{y},y_{i}]$ is strictly concave in y_{i}, in equilibrium, all traders acquire information of quality y^{*} such that

$$\left.\frac{\partial\mathbb{E}[\pi_{i}^{T};\bar{y},y_{i}]}{\partial y_{i}}\right|_{y_{i}=\bar{y}=y^{*}}=\mathcal{C}'(y^{*}).$$

Now recall that the socially optimal quality of information satisfies

$$\left.\frac{d\mathbb{E}[W^{T};\bar{y}]}{d\bar{y}}\right|_{\bar{y}=y^{T}}=\mathcal{C}'(y^{T}).$$

Because $\mathbb{E}[W^{T};\bar{y}]$ is strictly concave in $\bar{y}$, the result in Proposition 4, then imply that, when $\hat{c}^{T}<0$, $y^{T}>y^{*}$, whereas, when $\hat{c}^{T}>0$, $y^{T}<y^{*}$. Q.E.D.

Proof of Proposition 6.

Under the proposed policy, each trader i's ex-ante gross expected payoff when all traders other than i collect information of quality $\bar{y}$, trader i collects information of quality y_{i}, and all traders (including i) submit the efficient demand schedules $(a^T, \hat{b}^T, \hat{c}^T)$ is equal to

$$\begin{aligned}\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] &= \mathbb{E}\left[\theta x_i - (1+\hat{t}_p) p x_i - \frac{\lambda}{2} x_i^2\right] \\ &= \mathbb{E}\left[\theta x_i - (1+\hat{t}_p)\left(\alpha - u + \beta \tilde{x}\right) x_i - \frac{\lambda}{2} x_i^2\right]\end{aligned}$$

with

$$x_i = X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{\left[\theta + f(y_i)e_i + f(y_i)\eta\right]}_{s_i} + b^T + c^T\left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T}\right),$$

$$p = P(\theta, u, \eta; \bar{y}) = \alpha - u + \beta X(\theta, u, \eta; \bar{y}),$$

and

$$\tilde{x} = X(\theta, u, \eta; \bar{y}) = a^T(\theta + f(\bar{y})\eta) + b^T + c^T\left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T}\right),$$

and where b^T and c^T are the coefficients describing the equilibrium trades obtained from $\hat{b}^T$ and $\hat{c}^T$ using (18) and (19). Hence,

$$\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] = N - \beta(a^T + c^T)a^T \frac{1+\hat{t}_p}{\sqrt{\bar{y}}\sqrt{y_i}\tau_\eta} - \frac{\lambda c^T a^T}{\sqrt{\bar{y}}\sqrt{y_i}\tau_\eta} - \frac{\lambda}{2}\frac{\left(a^T\right)^2}{y_i \tau_\eta} - \frac{\lambda}{2}\frac{\left(a^T\right)^2}{y_i \tau_e}$$

where N is a function of all variables that do not interact with y_i. It follows that

$$\frac{\partial \mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p]}{\partial y_i} = \frac{\beta(1+\hat{t}_p)(a^T + c^T)a^T}{2\tau_\eta y_i \sqrt{\bar{y} y_i}} + \frac{\lambda a^T}{2\tau_\eta y_i \sqrt{y_i}}\left(\frac{a^T}{\sqrt{y_i}} + \frac{c^T}{\sqrt{\bar{y}}}\right) + \frac{\lambda\left(a^T\right)^2}{2y_i^2 \tau_e}.$$

Because $\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] - \mathcal{C}(y_i)$ is concave in y_i, for $y_i = \bar{y} = y^T$ to be sustained in equilibrium it is both necessary and sufficient that

$$\frac{\partial \mathbb{E}[\pi_i^T(y^T, y^T); \hat{t}_p]}{\partial y_i} = \mathcal{C}'(y^T)$$

which is equivalent to

$$\frac{\left[\beta(1+\hat{t}_p) + \lambda\right](a^T + c^T)a^T}{2\tau_\eta} + \frac{\lambda\left(a^T\right)^2}{2\tau_e} = \mathcal{C}'(y^T)\left(y^T\right)^2.$$

Using the fact that y^T satisfies

$$\frac{(\beta+\lambda)(a^T + c^T)^2}{2\tau_\eta} + \frac{\lambda\left(a^T\right)^2}{2\tau_e} = \mathcal{C}'(y^T)\left(y^T\right)^2,$$

we have that the proposed policy implements the efficient acquisition of private information when

$$\hat{t}_p = \frac{(\beta+\lambda)c^T}{\beta a^T}.$$

Using the fact that

$$c^T = \frac{1}{\beta+\lambda}\left(\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T\right)$$

we then have that the optimal $\hat{t}_p$ can be rewritten as

$$\hat{t}_p = \frac{\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T}{\beta a^T}$$

as claimed in the proposition. Q.E.D.

Proof of Proposition 7.

Assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Given any policy $T(x_i, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$V(y^T, y_i) \equiv \sup_{g(\cdot)} \left\{\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] - \mathcal{C}(y_i)\right\}$$

where $g : \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and z, and where

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha - u + \beta\tilde{x})g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2\right] - \mathbb{E}\left[T\left(g(s_i, z), \alpha - u + \beta\tilde{x}\right)\right].$$

Note that the definition of $\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)]$ uses the fact that the market-clearing price is given by $p = \alpha - u + \beta\tilde{x}$ with

$$\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$$

where b^T and c^T are the coefficients describing the equilibrium trades obtained from $\hat{b}^T$ and $\hat{c}^T$ using (18) and (19), and where

$$z \equiv \theta + f(y^T)\eta - \frac{u}{\beta a^T}.$$

It also uses the fact that, when all other traders submit the efficient demand schedules, any demand schedule for trader i (that is, any mapping from (s_i, p) into x_i) can be expressed as a function $g(s_i, z)$ of (s_i, z).[19]

For the policy $T(x_i, p)$ to implement the efficient acquisition and usage of information, it must be that, when $y_i = y^T$, the function $g(s_i, z)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that the equilibrium price can be expressed as $p = \alpha + \beta b^T + \beta(a^T + c^T)z$, and the fact that

$$\mathbb{E}\left[\theta | s_i, z\right] = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z,$$

we thus have that, for a differentiable policy T to implement the efficient trades, it must satisfy

$$\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right)$$

$$-\frac{\partial T\left(a^T s_i + b^T + c^T z, \alpha + \beta b^T + \beta(a^T + c^T)z\right)}{\partial x} = 0$$

[19]It suffices to use (20) to observe that $p = \alpha + \beta b^T + \beta(a^T + c^T)z$.

for all (s_i, z). Next, observe that

$$\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z$$

$$= \left[\gamma_1(\tau_\omega(a^T))\tfrac{x-b^T-c^Tz}{a^T} + \gamma_2(\tau_\omega(a^T))\left[\tfrac{p-\alpha-\beta b^T}{\beta(a^T+c^T)}\right]\right]\Big|_{x=a^Ts_i+b^T+c^Tz, p=\alpha+\beta b^T+\beta(a^T+c^T)z}$$

$$= \left[\gamma_1(\tau_\omega(a^T))\tfrac{x-b^T}{a^T} - \tfrac{\gamma_1(\tau_\omega(a^T))c^T}{a^T}\left[\tfrac{p-\alpha-\beta b^T}{\beta(a^T+c^T)}\right] + \gamma_2(\tau_\omega(a^T))\left[\tfrac{p-\alpha-\beta b^T}{\beta(a^T+c^T)}\right]\right]\Big|_{x=a^Ts_i+b^T+c^Tz, p=\alpha+\beta b^T+\beta(a^T+c^T)z}$$

But this means that $T(x,p)$ must be a polynomial of second order of the form

$$T(x_i, p) = \frac{\delta}{2}x_i^2 + (pt_p - t_0)\, x_i + K, \tag{32}$$

for some (δ, t_p, t_0, K), where K is a constant which plays no role for incentives. In the proof of Proposition 3, we showed that there exists a unique vector (δ, t_p, t_0) that induces the traders to submit the efficient demand schedules when the precision of their private information is y^T (the vector in Proposition 3 applied to $y = y^T$). Thus if a policy T induces efficiency in both information acquisition and information usage, it must be of the form in (32), with (δ, t_p, t_0) as in Proposition 3 applied to $y = y^T$. When the policy takes this form, for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = as_i + b + cz$, for some (a, b, c), implying that

$$\begin{aligned}\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \;=\; \mathbb{E}\Bigg[&(\theta + t_0)\,(as_i + b + cz) - (1+t_p)\left(\alpha - u + \beta\left[a^T(\theta + f(y^T)\eta) + b^T + c^Tz\right]\right) \times \\ &\times (as_i + b + cz) - \frac{\lambda+\delta}{2}\,(as_i + b + cz)^2\Bigg].\end{aligned}$$

Letting M be a function of all variables that do not interact with y_i, we then have that, when $g(s_i, z) = as_i + b + cz$, for some (a, b, c),

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \;=\; M - \beta(1+t_p)(a^T + c^T)a\frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda+\delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_\eta} - \frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_e}.$$

Using the envelope theorem, we then have that

$$\frac{\partial V(y^T, y^T)}{\partial y_i} = \frac{\left[\beta(1+t_p) + \lambda + \delta\right](a^T + c^T)a^T}{2\tau_\eta\,(y^T)^2} + \frac{(\lambda+\delta)\,(a^T)^2}{2\tau_e\,(y^T)^2} \;-\; \mathcal{C}'(y^T).$$

Note that in writing the above derivative, we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to the efficient trades $a^Ts_i + b^T + c^Tz$. Recall that the efficient y^T is given by the solution to the following equation

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta\left(y^T\right)^2}+\frac{\lambda\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2} \quad = \quad \mathcal{C}'(y^T).$$

Hence, for the policy of Proposition 3 (applied to $\bar{y}=y^T$) to implement the efficient acquisition of private information, it must be that

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{\tau_\eta}+\frac{\lambda\left(a^T\right)^2}{\tau_e} \quad = \quad \frac{\left[\beta(1+t_p)+\lambda+\delta\right](a^T+c^T)a^T}{\tau_\eta}+\frac{(\lambda+\delta)\left(a^T\right)^2}{\tau_e}$$

or, equivalently,

$$(a^T+c^T)\tau_e\left[(\beta+\lambda)c^T-(\beta t_p+\delta)a^T\right] \quad = \quad \delta\left(a^T\right)^2\tau_\eta.$$

One can verify that the values of δ and t_p from Proposition 3 do not solve the above equation except for a non-generic set of parameters. Q.E.D.

Proof of Proposition 8.

Paralleling the derivations in the proof of Proposition 7, when the policy takes the proposed form and all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, the affine orders corresponding to the coefficients $(a^T,\hat{b}^T,\hat{c}^T)$ for quality of information y^T), the expected net payoff for trader i when he chooses information of quality y_i is maximized by submitting an affine demand schedule $x_i=as_i+\hat{b}-\hat{c}p$ which induces trades $x_i=as_i+b+cz$ that are affine in (s_i,z), where $z=\theta+f(y^T)\eta-u/\beta a^T$ is the endogenous signal contained in the market-clearing price.

Using this result, let

$$\hat{V}(y^T,y_i)\equiv\sup_{a,b,c}\left\{\mathbb{E}[\tilde{\pi}_i(y^T,y_i);a,b,c]-\mathcal{C}(y_i)+Ay_i\right\}$$

denote the maximal payoff that trader i can obtain by acquiring information of precision y_i when all other traders acquire information of precision y^T and then submit the efficient demand schedules for information of quality y^T. As shown in the proof of Proposition 7, the expected gross payoff that trader i obtains by inducing the affine trades $x_i=as_i+b+cz$ when he chooses information of quality y_i is equal to

$$\mathbb{E}[\tilde{\pi}_i(y^T,y_i);a,b,c]=M-\beta(1+t_p)(a+c)a\frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta}-\frac{(\lambda+\delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta}-\frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_\eta}-\frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_e},$$

where M is a term collecting all variables that do not interact with y_i. Using the envelope theorem, we have that

$$\frac{\partial\hat{V}(y^T,y^T)}{\partial y_i}=\frac{\left[\beta(1+t_p)+\lambda+\delta\right](a^T+c^T)a^T}{2\tau_\eta\left(y^T\right)^2}+\frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2} \quad - \quad \mathcal{C}'(y^T)+A.$$

Again, in writing the above derivative we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to $a^T s_i + b^T + c^T z$. Using the fact that y^T satisfies

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta \left(y^T\right)^2} + \frac{\lambda \left(a^T\right)^2}{2\tau_e \left(y^T\right)^2} \;=\; \mathcal{C}'(y^T),$$

we thus have that the proposed policy induces the efficient acquisition of private information only if the following condition holds

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta} + \frac{\lambda \left(a^T\right)^2}{2\tau_e} \;=\; \frac{\left(\beta(1+t_p)+\lambda+\delta\right)(a^T+c^T)a^T}{2\tau_\eta} + \frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e} + A\left(y^T\right)^2$$

from which we obtain that

$$A \;=\; \frac{a^T+c^T}{2\tau_\eta \left(y^T\right)^2}\left[(\beta+\lambda)c^T - (\beta t_p+\delta)a^T\right] - \frac{\delta\left(a^T\right)^2}{2\tau_e \left(y^T\right)^2}.$$

Finally, one can verify numerically that the function $\hat{V}(y^T, y_i)$ is globally quasi-concave in y_i. We thus conclude that the proposed policy implements the efficient acquisition and usage of information. Q.E.D.

6.2 Extension: Cournot Case (traders submitting market orders)

In this subsection we show that, in a Cournot equilibrium, there is no inefficiency in either the collection or usage of information. The environment is the same as in the baseline model except for the fact that traders are restricted to submitting market orders instead of a collection of limit orders (equivalently, a demand schedule).

6.2.1 Efficiency in Usage

Suppose that $y_i = y$ for all i. In any symmetric equilibrium in which the price is affine in (θ, u, η), each trader's market order is an affine function of her private signal. That is,

$$x_i = as_i + b$$

for some scalars (a, b) that depend on the exogenous parameters of the model. Aggregate demand is then equal to

$$\tilde{x} = \int x_i di = a(\theta + f(y)\eta) + b.$$

Combining the above expression with the inverse aggregate supply function

$$p = \alpha - u + \beta\tilde{x}$$

from the representative investor, we then have that the equilibrium price must satisfy

$$p = \alpha - u + \beta b + \beta a(\theta + f(y)\eta). \tag{33}$$

For each s_i, the equilibrium market order $x_i = as_i + b$ must maximize trader i's expected profits

$$\Pi_i \;=\; \mathbb{E}\left[(\theta - p)\, x_i - \lambda\frac{x_i^2}{2}|s_i\right] - \mathcal{C}(y_i),$$

where $x_i = a_i s_i + b$.

Following steps similar to those in the baseline model, we have that, for any s_i, the derivative of Π_i with respect to x_i, evaluated at $x_i = a_i s_i + b$, must be equal to zero, which yields[20]

$$\mathbb{E}\left[\theta|s_i\right] - \alpha - \beta b - \beta a\mathbb{E}\left[\theta + f(y)\eta|s_i\right] \;=\; \lambda(as_i + b).$$

We conclude that the equilibrium value of b, which we denote by b^*, is equal to $b^* = -\frac{\alpha}{\beta+\lambda}$. To obtain the equilibrium value of a, which we denote by a^*, we replace $\mathbb{E}\left[\theta|s_i\right] = \frac{\tau_\epsilon}{\tau_\epsilon+\tau_\theta}s_i$ and $\mathbb{E}\left[\eta|s_i\right] \frac{f(y)\frac{1}{\tau_\eta}\tau_\theta\tau_\epsilon}{\tau_\epsilon+\tau_\theta}s_i$ into the above FOC from which we obtain that

$$a^* \;=\; \frac{\tau_\epsilon}{\lambda\left(\tau_\epsilon + \tau_\theta\right) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}}.$$

Next, we can derive the expression for the welfare losses when agents do not condition on the price. When the market orders are affine with coefficients a and b,

$$x_i - \tilde{x} \;=\; a(s_i - \theta - f(y)\eta)$$

from which we obtain that

$$\mathbb{E}[(x_i - \tilde{x})^2] = \mathbb{E}[a^2 f(y)^2 e_i^2] = \frac{a^2}{y\tau_e}.$$

as in the baseline model. Recall that the first-best action is $x^o = \frac{\theta-\alpha+u}{\beta+\lambda}$. One can then show that, for any (a, b), the welfare losses are equal to

$$WL = \frac{(\beta+\lambda)\mathbb{E}[(\tilde{x}-x^o)^2]+\lambda\mathbb{E}[(x_i-\tilde{x})^2]}{2} =$$

$$\frac{1}{2(\beta+\lambda)^2}\left(\frac{(\beta a+\lambda a-1)^2}{\tau_\theta} + \frac{(\beta+\lambda)^2a^2}{y\tau_\eta} + \frac{1}{\tau_u} + b^2(\beta+\lambda)^2 + \alpha^2 + 2\alpha b(\beta+\lambda)\right).$$

For any a, the value of b that minimizes the welfare losses is thus given by the FOC

$$\frac{\partial WL}{\partial b} = b + \frac{\alpha}{\beta+\lambda} = 0.$$

We conclude that the optimal value of b is the equilibrium one: $b^T = b^* = -\frac{\alpha}{\beta+\lambda}$. Replacing

[20]Note that $\mathbb{E}\left[u|s_i\right] = 0$.

the above into the expression for the welfare losses, we have that the latter can be expressed as a function of a as follows

$$WL(a;y) \quad = \quad \frac{1}{2}\left(\frac{(\beta a+\lambda a-1)^2}{(\beta+\lambda)\tau_\theta}+\frac{(\beta+\lambda)a^2}{y\tau_\eta}+\frac{1}{(\beta+\lambda)\tau_u}+\frac{\lambda a^2}{y\tau_e}\right).$$

Differentiating $WL(a;y)$ with respect to a and setting the derivative equal to zero gives us the socially-optimal value of a, which we denote by a^T:

$$\frac{\partial WL(a^T;y)}{\partial a} \quad = \quad \frac{(\beta a^T+\lambda a^A-1)}{\tau_\theta}+\frac{(\beta+\lambda)a^T}{y\tau_\eta}+\frac{\lambda a^T}{y\tau_e}=0$$

from which we obtain that

$$a^T=\frac{\tau_\epsilon}{\lambda\tau_\epsilon+\beta\tau_\epsilon+\lambda\tau_\theta+\frac{\beta\tau_\epsilon\tau_\theta}{y\tau_\eta}}=a^*.$$

We thus conclude that there is no inefficiency in the usage of information in the Cournot game.

6.2.2 Efficiency in Acquisition

We first characterize the equilibrium acquisition of private information. When each trader $j\neq i$ chooses $y_j=y$ and then submits the equilibrium affine market order $x_j=as_j+b$ for quality of information y, and trader i instead acquires information of quality y_i and then, after observing s_i, submits the market order x_i, his expected payoff is equal to

$$\Pi_i \quad = \quad \mathbb{E}\left[(\theta-p)\,x_i-\lambda\frac{x_i^2}{2}|s_i,y_i\right]-\mathcal{C}(y_i)$$

where $p=\alpha-u+\beta\tilde{x}$, with $\tilde{x}=a(y)(\theta+f(y)\eta)+b$, with

$$a=a(y)=\frac{\tau_\epsilon}{\lambda\left(\tau_\epsilon+\tau_\theta\right)+\beta\tau_\epsilon+\beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}}$$

and $b=-\frac{\alpha}{\beta+\lambda}$, as shown above. For any (s_i,y_i), the optimal market order for trader i is given by the FOC with respect to x_i which yields $x_i=a_is_i+b$ with

$$a_i=a_i(y,y_i)=\frac{y_i\tau_e\tau_\eta(1-\beta a(y))-\beta a(y)\frac{\sqrt{y_i}}{\sqrt{y}}\tau_\theta\tau_e}{\lambda\left(y_i\tau_e\tau_\eta+\tau_\theta(\tau_e+\tau_\eta)\right)}$$

and $b=-\frac{\alpha}{\beta+\lambda}$. That is, for any (y,y_i), trader i's expected profits when all other traders acquire information of quality y and then submit the equilibrium market orders for quality of information y, and trader i instead acquires information of quality y_i and then submits the market order that maximizes his payoff (the one described above) is given by

$$
\begin{aligned}
\Pi_i(y, y_i) &= \mathbb{E}\left[(\theta - \alpha + u - \beta\,(a\theta + af(y)\eta + b))\,(a_i s_i + b) - \lambda \frac{(a_i s_i + b)^2}{2}; y, y_i \right] - \mathcal{C}(y_i) \\
&= \frac{a_i - \beta a a_i}{\tau_\theta} - \frac{\beta a a_i}{\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda a_i^2}{2}\left(\frac{1}{\tau_\theta} + \frac{1}{y_i\tau_\eta} + \frac{1}{y_i\tau_e}\right) - \mathcal{C}(y_i) - \alpha b + (1-\beta)b^2
\end{aligned}
$$

where we used the shortcuts $a = a(y)$ and $a_i = a_i(y, y_i)$ and the fact that $s_i = \theta + f(y_i)(\eta + e_i)$.

Replacing a_i with $a_i(y, y_i)$ and a with $a(y)$, and using the Envelope Theorem, we then have that

$$
\frac{\partial \Pi_i(y, y_i)}{\partial y_i} = \frac{1}{2}\frac{\beta a(y) a_i(y, y_i)}{y_i\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda\,(a_i(y, y_i))^2}{2}\left(-\frac{1}{y_i^2\tau_\eta} - \frac{1}{y_i^2\tau_e}\right) - \mathcal{C}'(y_i).
$$

When y is equal to the equilibrium level, which we denote by y^*, it must be that

$$
\frac{\partial \Pi_i(y^*, y^*)}{\partial y_i} = 0
$$

which, using the fact $a_i(y^*, y^*) = a(y^*)$ yields

$$
\mathcal{C}'(y^*) = \frac{1}{2}\left(\frac{(\beta + \lambda)\,(a(y^*))^2}{(y^*)^2\tau_\eta} + \frac{\lambda\,(a(y^*))^2}{(y^*)^2\tau_e}\right).
$$

Next, we characterize the socially-optimal value of y. Because for any y, the socially-optimal usage of information coincides with the equilibrium, as shown above, using the Envelope Theorem, we have that the optimal value of y, which we denote by y^T is given by the condition

$$
\frac{\partial WL(a(y^T); y^T)}{\partial y} = \frac{1}{2}\left(-\frac{(\beta + \lambda)\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_\eta} - \frac{\lambda\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_e}\right) + \mathcal{C}'(y^T) = 0.
$$

We conclude that the the optimal value of y, which we denote by y^T, is given by the solution to the following condition

$$
\mathcal{C}'(y^T) = \frac{1}{2}\left(\frac{(\beta + \lambda)\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_\eta} + \frac{\lambda\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_e}\right).
$$

It is immediate to see that $y^T = y^*$, implying that the equilibrium acquisition of information is also efficient. Q.E.D.