Persuasion in Global Games with Application to Stress Testing

Nicolas Inostroza — U of Toronto
Alessandro Pavan — Northwestern

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Motivation

- Coordination: central to many socio-economic environments
- Damages to society of mkt coordination on undesirable actions can be severe
  - Monte dei Paschi di Siena (MPS)
  - creditors + speculators with heterogenous beliefs about size of *nonperforming loans*
  - default by MPS: major crisis in Eurozone (and beyond)
- Government intervention
  - limited by legislation passed in 2015
- Persuasion (Stress Tests): **instrument of last resort**

Questions

- Structure of optimal stress tests?
    - What information should be passed on to mkt?
- "Right" notion of transparency?
- Optimality of
    - pass/fail policies
    - monotone rules
- Properties of persuasion in global games?

Related literature

- **Persuasion and Information design:** Myerson (1986), Aumann and Maschler (1995), Calzolari and Pavan (2006,a,b), Glazer and Rubinstein (2004, 2012), Rayo and Segal (2010), Kamenica and Gentzkow (2011), Ely (2016), Bergemann and Morris (2017), Lipnowski and Mathevet (2017), Mathevet, Pearce, Stacchetti (2017), Dworczak and Martini (2019), Dworczak and Pavan (2021)...
- **Persuasion in Games:** Alonso and Camara (2013), Barhi and Guo (2016), Taneva (2016), Mathevet, Perego, Taneva (2019)...
- **Persuasion with ex-ante heterogenous receivers:** Bergemann and Morris (2016), Kolotilin et al (2016), Laclau and Renou (2017), Chan et al (2016), Basak and Zhou (2017), Che and Horner (2017), Doval and Ely (2017), Guo and Shmaya (2017)...
- **Financial Regulation and Stress Testing:** Goldstein and Leitner (2015), Goldstein and Sapra (2014), Alvarez and Barlevy (2017), Bouvard et al. (2015), Goldstein and Huang (2016), Williams (2017), Quigley & Walther (2020), Leitner & Williams (2020), Inostroza (2021)...
- **Global Games w. Endogenous Info:** Angeletos, Hellwig and Pavan (2006, 2007), Angeletos and Pavan (2013), Edmond (2013), Iachan and Nenov (2015), Denti (2020), Yang (2016), Cong, Grenadier & Hu (2020), Morris and Yang (2020), Li, Song & Zhao (2020), Morris, Oyama & Takahashi (2020), Alonso & Zachariades (2021)...

Plan

- Basic Model
- Perfect Coordination Property
- Pass/Fail Policies
- Monotone Policies
- Enrichments
- Micro-foundations

Stylized Global Game of Regime Change

- Policy maker (PM)
- Agents $i \in [0, 1]$
- Actions

$$a_i = \begin{cases} 1 & (\textit{pledge}) \\ 0 & (\textit{not pledge}) \end{cases}$$

- $A \in [0, 1]$: aggregate pledge
- Default outcome: $r \in \{0, 1\}$, with $r = 0$ in case of default
- Default rule

$$r = \begin{cases} 0 & \textit{if} \quad A \leq 1 - \theta \\ 1 & \textit{if} \quad A > 1 - \theta \end{cases}$$

- "fundamentals" θ parametrize liquidity, performing loans, etc.
- Supermodular game w. dominance regions: $(-\infty, 0]$ and $(1, +\infty)$
- θ drawn from abs. continuous cdf F, smooth density f

Stylized Global Game of Regime Change

- PM's payoff

$$U^P(\theta, A) = \begin{cases} W(\theta) > 0 & \text{if } r = 1 \\ L(\theta) < 0 & \text{if } r = 0 \end{cases}$$

- Agents' payoff from not pledging (safe action) normalized to zero
- Agents' payoff from *pledging*

$$u = \begin{cases} g(\theta) > 0 & \text{if } r = 1 \\ b(\theta) < 0 & \text{if } r = 0 \end{cases}$$

Beliefs

- $\mathbf{x} \equiv (x_i)_{i\in[0,1]} \in \mathbf{X}$: signal profile with each
$$x_i \sim p(\cdot|\theta)$$
i.i.d., given θ.

- $p(x|\theta)$ strictly positive over an open interval $\varrho_\theta \equiv (\underline{\varrho}_\theta, \bar{\varrho}_\theta)$ containing θ.

- $\mathbf{X}(\theta) \subset \mathbb{R}^{[0,1]}$: collection of signal profiles consistent with θ

- Example 1: $x_i = \theta + \sigma\xi_i$ with $\xi_i \sim N(0,1)$
- Example 2: $x_i = \theta + \sigma\xi_i$ with $\xi_i \sim U(-1,1)$

Disclosure Policies (Stress Tests)

- *Stress Test* $\Gamma = (S, \pi)$
  - S: set of scores/grades/disclosures
  - $\pi : \Theta \rightarrow \Delta(S)$

Timing

1. PM announces $\Gamma = (S, \pi)$ and commits to it
2. $(\theta, \mathbf{x})$ realized
3. $s \in S$ drawn from $\pi(\theta)$ and publicly announced
4. Agents simultaneously choose whether or not to pledge
5. Default outcome and payoffs

Solution Concept: MARP

- Robust/adversarial approach
- PM does not trust her ability to coordinate mkt on her favorite course of action
- **Most Aggressive Rationalizable Profile** (MARP):

  minimizes PM's payoff across all profiles surviving *iterated deletion of interim strictly dominated strategies* (IDISDS)
- $a^{\Gamma} \equiv (a_i^{\Gamma})_{i\in[0,1]}$: MARP consistent with Γ (a_i^{Γ}: complete plan of action)

Perfect Coordination Property [PCP]

Definition 1

$\Gamma = \{S, \pi\}$ satisfies **PCP** if, for any $\theta \in \Theta$, any exogenous information $\mathbf{x} \in \mathbf{X}(\theta)$, any $s \in supp(\pi(\theta))$, and any pair of individuals $i, j \in [0, 1]$, $a_i^\Gamma(x_i, s) = a_j^\Gamma(x_j, s)$, where $a^\Gamma \equiv (a_i^\Gamma)_{i \in [0,1]}$ is MARP consistent with Γ

Perfect Coordination Property [PCP]

Theorem 1

*Given any (regular) Γ, there exists (regular) Γ^* satisfying **PCP** and s.t., at any θ, default probability under Γ^* same as under Γ.*

- Regularity: MARP well defined
- formal proof

Perfect Coordination Property [PCP]

- Policy $\Gamma^* = (S^*, \pi^*)$ removes any **strategic uncertainty**
- It preserves **structural uncertainty**
- Under Γ^*, agents know actions all other agents but not *beliefs rationalizing such actions*
- Inability to predict beliefs that rationalize other agents' actions essential to minimization of default risk
- "Right" form of transparency
  - conformism in beliefs about mkt response
  - ...not in beliefs about "fundamentals"

PCP: Lesson

Optimal policy combines:

- public **Pass/Fail** announcement
  - eliminate strategic uncertainty
- additional disclosures necessary to guarantee that, when $r = 1$ announced (i.e., when bank passed test), all agents pledge under MARP

Pass/fail Policies

- When is optimal policy binary?

Theorem 2

Assume $p(x|\theta)$ satisfies MLRP. Given any policy Γ satisfying PCP, there exists ***binary policy*** $\Gamma^* = (\{0, 1\}, \pi^*)$ *also satisfying PCP and s.t., for any θ, prob of default under Γ^* same as under Γ.*

- MARP in threshold strategies: signals other than regime outcome can be dropped (averaging over s) without affecting incentives
- Result hinges on Log-SM of $p(x|\theta)$, i.e., on MLRP
  - co-movement between state θ and belies

(Example)

Optimality of Monotone Tests

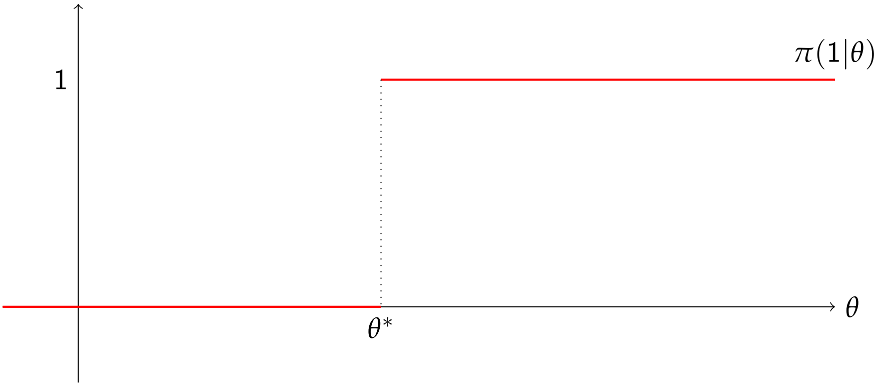


Figure: Optimal Monotone Policy.

Optimality of Monotone Tests

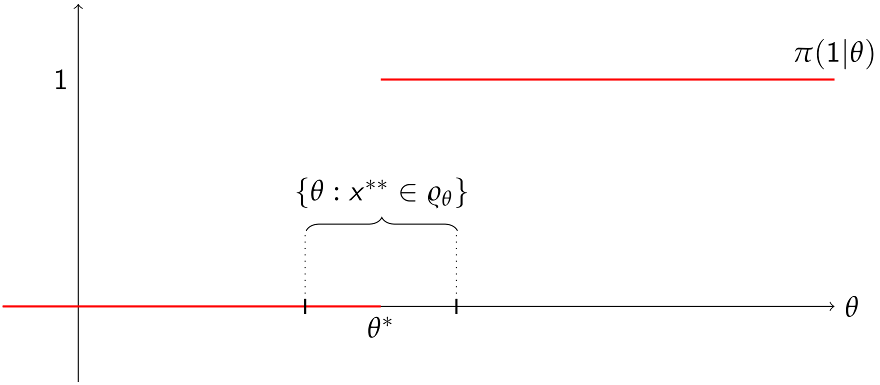


Figure: Optimal Monotone Policy.

Foundation for Monotone Tests

$$\bar{x}_G \equiv \sup \left\{ x : \int_{\Theta} u\left(\theta, 1 - P\left(x|\theta\right)\right) \mathbb{I}(\theta \geq 0) p\left(x|\theta\right) dF\left(\theta\right) \leq 0 \right\}$$

Condition M: Following properties hold:

1. $\inf \left\{\theta \in \Theta : x^{**} \in \varrho_{\theta}\right\} \leq 0;$
2. $p\left(x|\theta\right)$ and $|u(\theta, 1 - P(x|\theta))|$ (weakly) log-supermodular over $\left\{(\theta, x) \in [0,1] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\right\};$
3. $\forall \theta_0, \theta_1 \in [0,1]$, with $\theta_0 < \theta_1$, $\forall x \leq \bar{x}_G$ s.t. (a) $u(\theta_1, 1 - P(x|\theta_1)) \leq 0$ and (b) $x \in \varrho_{\theta_0}$,

$$\frac{U^P(\theta_1, 1) - U^P(\theta_1, 0)}{U^P(\theta_0, 1) - U^P(\theta_0, 0)} > \frac{p\left(x|\theta_1\right) u\left(\theta_1, 1 - P\left(x|\theta_1\right)\right)}{p\left(x|\theta_0\right) u\left(\theta_0, 1 - P\left(x|\theta_0\right)\right)}$$

Theorem 3

Suppose $p(x|\theta)$ log-supermodular, Condition M holds. Given any Γ, there exists deterministic binary monotone $\Gamma^* = (\{0,1\}, \pi^*)$ satisfying PCP and yielding payoff weakly higher than Γ.

Sub-optimality of Monotone Tests

Example 1

Suppose that, for any θ,

(a) $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$, and $L(\theta) = L$;

(b) $\theta \sim U[-K, 1+K]$, $K \in \mathbb{R}_{++}$;

(c) $x_i = \theta + \sigma\epsilon_i$, with $\sigma \in \mathbb{R}_+$ and $\epsilon_i \sim U[-1, 1]$, with $\sigma < K/2$.

There exists $\sigma^\# \in (0, K/2)$ such that, for all $\sigma \in (0, \sigma^\#)$, there exists (deterministic) *non-monotone* policy satisfying PCP that yields payoff strictly higher than optimal monotone policy.

Optimality of Monotone Tests

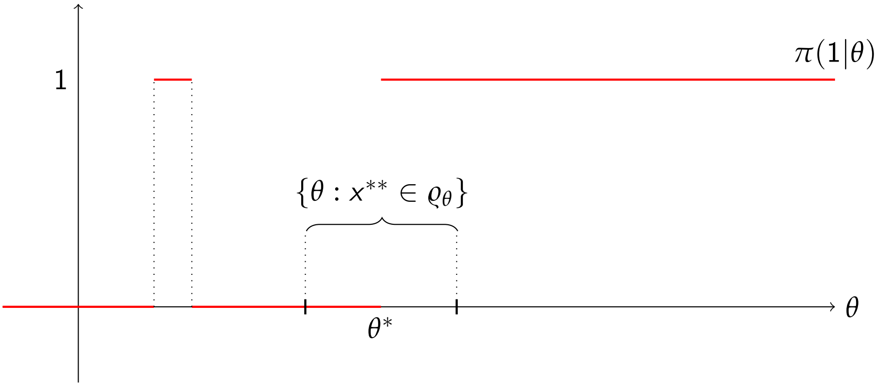


Figure: Optimal Monotone Policy.

Sub-optimality of Monotone Tests

- Let $\theta^{MS} \in (0,1)$ be implicitly defined by $\int_0^1 u(\theta^{MS}, l)dl = 0$
- $D^\Gamma \equiv \left\{d_i = (\underline{\theta}_i, \bar{\theta}_i] : i = 1, ..., N\right\}$: partition of $[0, \theta^{MS}]$ induced by deterministic Γ
- $\Delta(\Gamma) \equiv \max_{i=1,\ldots,N} |\bar{\theta}_i - \underline{\theta}_i|$: mesh of D^Γ

Example 2

Suppose $\theta \sim U[\mathbb{R}]$ and $x_i = \theta + \sigma\varepsilon_i$, with $\varepsilon_i \sim N(0,1)$. Assume that, for any θ, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$ and $L(\theta) = L$. There exists $\bar{\sigma} > 0$ and $\mathcal{E} : (0, \bar{\sigma}] \to \mathbb{R}_+$, with $\lim_{\sigma \to 0^+} \mathcal{E}(\sigma) = 0$, s.t, for any $\sigma \in (0, \bar{\sigma}]$, the following is true: given any deterministic binary Γ satisfying PCP and s.t. $\Delta(\Gamma) > \mathcal{E}(\sigma)$, there exists another deterministic binary Γ^* with $\Delta(\Gamma^*) < \mathcal{E}(\sigma)$ that also satisfies PCP and yields payoff strictly higher than Γ.

Extensions

- Default iff $R(\theta, A, z) \leq 0$
  - z drawn from Q_θ: residual uncertainty
- PM's payoff

$$\hat{U}^P(\theta, A, z) = \begin{cases} \hat{W}(\theta, A, z) & \text{if } r = 1 \\ \hat{L}(\theta, A, z) & \text{if } r = 0 \end{cases}$$

- Agents' payoffs

$$\hat{u}(\theta, A, z) = \begin{cases} \hat{g}(\theta, A, z) & \text{if } r = 1 \\ \hat{b}(\theta, A, z) & \text{if } r = 0 \end{cases}$$

- Expected payoff differential: $u(\theta, A)$

Generalizations

Condition FB. For any x, $u(\theta, 1 - P(x|\theta)) \geq 0$ (alternatively, $u(\theta, 1 - P(x|\theta)) \leq 0$) implies $u(\theta'', 1 - P(x|\theta'')) > 0$ for all $\theta'' > \theta$ (alternatively, $u(\theta', 1 - P(x|\theta')) < 0$ for all $\theta' < \theta$).

Condition PCP. For any $\Lambda \in \Delta(\Delta(\Theta))$ consistent with F

$$\int \left(\int_{-\infty}^{\theta^G} U^P(\theta, 0) G(\mathrm{d}\theta) + \int_{\theta^G}^{+\infty} U^P(\theta, 1) G(\mathrm{d}\theta) \right) \Lambda(\mathrm{d}G)$$

$$\int \left(\int U^P(\theta, 1 - P(\xi^G|\theta)) G(\mathrm{d}\theta) \right) \Lambda(\mathrm{d}G)$$

ξ^G: MARP given G

$\theta^G \equiv \inf \left\{ \theta : u(\theta, 1 - P(\xi^G|\theta)) \geq 0 \right\}$

Generalizations

Theorem 4

(a) Given any Γ*, there exists* Γ^* *satisfying PCP and s.t., for any* θ*, agents' expected payoff under* a^{Γ^*} *is at least as high as under* a^{Γ}*.*

(b) Suppose $p(x|\theta)$ *satisfies MLRP; then* Γ^* *binary. PM's payoff under* Γ^* *at least as high as under* Γ*.*

(c) Suppose condition M holds. Then Γ^* *monotone.*

- PCP: announcement of sign of agents' expected payoff under MARP

Micro-foundations and comparative statics

- Former liabilities: D
- Bank's legacy asset delivers
  - $I(\theta) \in \mathbb{R}$ end of period 1
  - $C(\theta)$ end of period 2
- Bank can issue (i) **new shares** OR (ii) **short-term debt**
- Potential investors submit market orders
- Noise traders $z \sim Q_\theta$

Micro-foundations and MCS

- $Y(p, \theta, z)$: exogenous demand for shares (alternatively, debt)
- Market clearing price $p^\star(\theta, A, z)$ solves

$$q + 1 - A = A + Y(p^\star, \theta, z).$$

- Default:

$$R(\theta, A, z) = I(\theta) + \rho_S q p^\star(\theta, A, z) - D \leq 0$$

Micro-foundations and MCS

Analysis can be used to study

- effect of different recapitalization policies
  - (q_E, q_D)
- role of uncertainty for toughness of optimal stress tests
  - uncertainty about bank's profitability: σ
  - uncertainty about macro variables: z

Proposition 1

There exists $\bar{\sigma} > 0$ such that, for any $\sigma, \sigma' \in (0, \bar{\sigma}]$, with $\sigma' > \sigma$:
$\theta_E^*(\sigma') < \theta_E^*(\sigma)$ and $\theta_D^*(\sigma') > \theta_D^*(\sigma)$.

Conclusions

- Robust/adversarial information design in coordination games with heterogeneously informed agents
- Application: Stress tests
  - Perfect coordination property ("right" notion of transparency)
  - Pass/Fail tests
  - Monotone rules
- Extension 1: PM uncertain about mkt prior beliefs
  - robust-undominated design (see also Dworczak & Pavan (2021))
- Extension 2: Elicitation and Persuasion (see Inostroza (2021))

THANKS!

PCP Proof

- Let $r(\omega; a^\Gamma) \in \{0,1\}$ be default outcome at $\omega \equiv (\theta, \mathbf{x}, s)$ when agents play according to a^Γ
- Let $\Gamma^* = \{S^*, \pi^*\}$ be s.t. $S^* = S \times \{0,1\}$ and

  $\pi^*((s, r(\omega; a^\Gamma))|\theta) = \pi(s|\theta)$, all (θ, s) s.t. $\pi(s|\theta) > 0$
- After receiving $s^* \equiv (s, 1)$, agents use Bayes' rule to update beliefs about $\omega \equiv (\theta, \mathbf{x}, s)$:

$$\partial\Lambda_i^{\Gamma^*}(\omega|x_i, (s,1)) = \frac{1\{r(\omega; a^\Gamma) = 1\}}{\Lambda_i^\Gamma(1|x_i, s)} \partial\Lambda_i^\Gamma(\omega|x_i, s)$$

where

$$\Lambda_i^\Gamma(1|x_i, s) \equiv \int_{\{\omega : r(\omega; a^\Gamma) = 1\}} d\Lambda_i^\Gamma(\omega|x_i, s)$$

PCP Proof

- Let $a^{\Gamma}_{(n)}$, $a^{\Gamma^*}_{(n)}$ be most aggressive profile surviving n round of IDISDS under Γ and Γ^*, respectively

Definition 2

Strategy profile $a^{\Gamma^*}_{(n)}$ less aggressive than $a^{\Gamma}_{(n)}$ iff, for any $i \in [0,1]$,

$$a^{\Gamma}_{(n),i}(x_i, s) = 1 \Rightarrow a^{\Gamma^*}_{(n),i}(x_i, (s,1)) = 1$$

Lemma 1

For any n, $a^{\Gamma^*}_{(n)}$ less aggressive than $a^{\Gamma}_{(n)}$

PCP Proof

- Induction
- Let $a_0^{\Gamma} = a_0^{\Gamma^*}$ be strategy profile where all agents refrain from pledging, regardless of their (endogenous and exogenous) information
- Suppose that $a_{(n-1)}^{\Gamma^*}$ less aggressive than $a_{(n-1)}^{\Gamma}$
- Note that $r(\omega|a^{\Gamma}) = 0 \Rightarrow r(\omega|a_{(n-1)}^{\Gamma}) = 0$ ($a_{(n-1)}^{\Gamma}$ more aggressive than $a^{\Gamma} = a_{\infty}^{\Gamma}$)
- Hence, $r(\omega; a^{\Gamma}) = 1$ "removes" from support of agents' beliefs states $\omega = (\theta, \mathbf{x}, s)$ for which default occurs under $a_{(n-1)}^{\Gamma}$

PCP Proof

- Payoffs from pledging in case of default are negative
- Payoff from **pledging** under Γ^* when agents follow $a^{\Gamma}_{(n-1)}$

$$
\begin{aligned}
U_i^{\Gamma^*}(x_i,(s,1);a^{\Gamma}_{(n-1)}) &= \frac{\int_\omega u(\theta,A(\omega;a^{\Gamma}_{(n-1)}))1\{r(\omega;a^{\Gamma})=1\}d\Lambda_i^{\Gamma}(\omega|x_i,s)}{\Lambda_i^{\Gamma}(1|x_i,s)} \\
&> \frac{\int_\omega u(\theta,A(\omega;a^{\Gamma}_{(n-1)}))d\Lambda_i^{\Gamma}(\omega|x_i,s)}{\Lambda_i^{\Gamma}(1|x_i,s)} \\
&= \frac{U_i^{\Gamma}(x_i,s;a^{\Gamma}_{(n-1)})}{\Lambda_i^{\Gamma}(1|x_i,s)}
\end{aligned}
$$

- Hence, $U_i^{\Gamma}(x_i,s;a^{\Gamma}_{(n-1)}) > 0 \Rightarrow U_i^{\Gamma^*}(x_i,(s,1);a^{\Gamma}_{(n-1)}) > 0$

PCP Proof

- That $a^{\Gamma^*}_{(n-1)}$ less aggressive than $a^{\Gamma}_{(n-1)}$ along with supermodularity of game implies that

$$U_i^{\Gamma^*}(x_i, (s, 1); a^{\Gamma}_{(n-1)}) > 0 \Rightarrow U_i^{\Gamma^*}(x_i, (s, 1); a^{\Gamma^*}_{(n-1)}) > 0$$

- As a consequence,

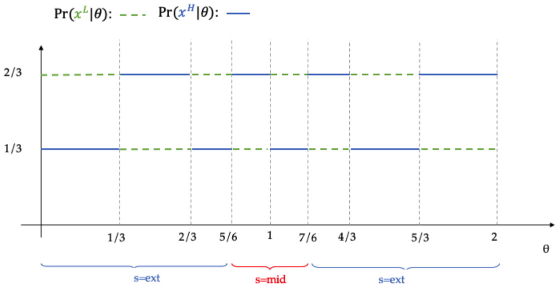
$$a^{\Gamma}_{(n),i}(x_i, s) = 1 \; \Rightarrow \; a^{\Gamma^*}_{(n),i}(x_i, (s, 1)) = 1$$

- This means that $a^{\Gamma^*}_{(n)}$ less aggressive than $a^{\Gamma}_{(n)}$.

PCP Proof

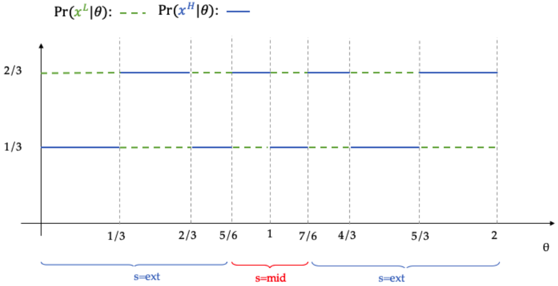
- Above lemma implies MARP under Γ^*, $a^{\Gamma^*} \equiv a^{\Gamma^*}_{(\infty)}$, less aggressive than MARP under Γ, $a^{\Gamma} \equiv a^{\Gamma}_{(\infty)}$
- In turn, this implies that $r(\omega; a^{\Gamma}) = 1$ makes it common certainty that $r(\omega; a^{\Gamma^*}) = 1$
- Hence, all agents pledge after hearing that $r(\omega; a^{\Gamma}) = 1$
- Similarly, $r(\omega; a^{\Gamma}) = 0$ makes it common certainty that $\theta \leq 1$. Under MARP, all agents refrain from pledging when hearing that $r(\omega; a^{\Gamma}) = 0$

Example



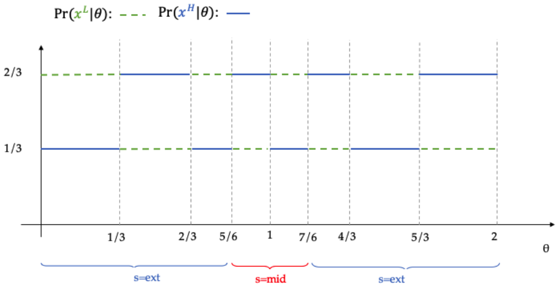
- Assume $g(\theta) = k$, $b(\theta) = -k$
- Pledging rationalizable iff $Pr(r = 1) \geq 1/2$

Example



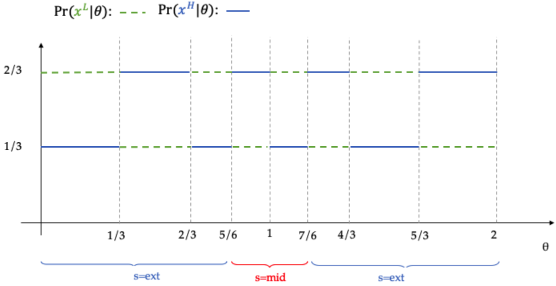
- No disclosure: under MARP, $a_i^\Gamma(x_i) = 0$, all x_i

Example



- Suppose PM informs agents of whether θ is extreme or intermediate
- $a_i^\Gamma(x_i, s) = 1$, all (x_i, s)

Example



- If, instead, PM only recommends to pledge (equivalently, Γ is pass/fail): $a_i^{\Gamma}(x_i, 1) = 0$ for all x_i
- **Suboptimality of P/F policies (+ failure of RP)**