Motivation

- Coordination: central to many socio-economic environments
- Damages to society of mkt coordination on undesirable actions can be severe
  - Monte dei Paschi di Siena (MPS)
  - creditors + speculators with heterogenous beliefs about size of *nonperforming loans*
  - default by MPS: major crisis in Eurozone (and beyond)
- Government intervention
  - limited by legislation passed in 2015
- Persuasion (Stress Tests): *instrument of last resort*
Questions

- Structure of optimal stress tests?
  - What information should be passed on to mkt?
- “Right” notion of transparency?
- Optimality of
  - pass/fail policies
  - monotone rules
- Properties of persuasion in global games?
Related literature


- **Persuasion in Games**: Alonso and Camara (2013), Barhi and Guo (2016), Taneva (2016), Mathevet, Perego, Taneva (2019)...


Plan

- Basic Model
- Perfect Coordination Property
- Pass/Fail Policies
- Monotone Policies
- Enrichments
- Micro-foundations
Policy maker (PM)

Agents $i \in [0, 1]$

Actions

$$a_i = \begin{cases} 1 & \text{(pledge)} \\ 0 & \text{(not pledge)} \end{cases}$$

$A \in [0, 1]$: aggregate pledge

Default outcome: $r \in \{0, 1\}$, with $r = 0$ in case of default

Default rule

$$r = \begin{cases} 0 & \text{if } A \leq 1 - \theta \\ 1 & \text{if } A > 1 - \theta \end{cases}$$

“fundamentals” $\theta$ parametrize liquidity, performing loans, etc.

Supermodular game w. dominance regions: $(-\infty, 0]$ and $(1, +\infty)$

$\theta$ drawn from abs. continuous cdf $F$, smooth density $f$
PM’s payoff

$$U^P(\theta, A) = \begin{cases} W(\theta) > 0 & \text{if } r = 1 \\ L(\theta) < 0 & \text{if } r = 0 \end{cases}$$

Agents’ payoff from not pledging (safe action) normalized to zero

Agents’ payoff from pledging

$$u = \begin{cases} g(\theta) > 0 & \text{if } r = 1 \\ b(\theta) < 0 & \text{if } r = 0 \end{cases}$$
Beliefs

- \( x \equiv (x_i)_{i \in [0,1]} \in \mathbf{X} \): signal profile with each
  \[ x_i \sim p(\cdot | \theta) \]
  i.i.d., given \( \theta \).

- \( p(x|\theta) \) strictly positive over an open interval \( \varrho_{\theta} \equiv (\varrho_{\theta}, \bar{\varrho}_{\theta}) \) containing \( \theta \).

- \( \mathbf{X}(\theta) \subset \mathbb{R}^{[0,1]} \): collection of signal profiles consistent with \( \theta \)

- Example 1: \( x_i = \theta + \sigma \zeta_i \) with \( \zeta_i \sim N(0, 1) \)
- Example 2: \( x_i = \theta + \sigma \zeta_i \) with \( \zeta_i \sim U(-1, 1) \)
Disclosure Policies (Stress Tests)

- **Stress Test** \( \Gamma = (S, \pi) \)
  - \( S \): set of scores/grades/disclosures
  - \( \pi : \Theta \rightarrow \Delta(S) \)
Timing

1. PM announces $\Gamma = (S, \pi)$ and commits to it

2. $(\theta, x)$ realized

3. $s \in S$ drawn from $\pi(\theta)$ and publicly announced

4. Agents simultaneously choose whether or not to pledge

5. Default outcome and payoffs
Solution Concept: MARP

- Robust/adversarial approach

- PM does not trust her ability to coordinate mkt on her favorite course of action

- **Most Aggressive Rationalizable Profile (MARP):**
  minimizes PM’s payoff across all profiles surviving *iterated deletion of interim strictly dominated strategies* (IDISDS)

- \(a^\Gamma \equiv (a^\Gamma_i)_{i \in [0,1]}\): MARP consistent with \(\Gamma\) (\(a^\Gamma_i\): complete plan of action)
Definition 1

$\Gamma = \{S, \pi\}$ satisfies **PCP** if, for any $\theta \in \Theta$, any exogenous information $x \in \mathbf{X}(\theta)$, any $s \in \text{supp}(\pi(\theta))$, and any pair of individuals $i, j \in [0, 1]$, $a_i^\Gamma(x_i, s) = a_j^\Gamma(x_j, s)$, where $a^\Gamma \equiv (a_i^\Gamma)_{i\in[0,1]}$ is MARP consistent with $\Gamma$. 
Theorem 1

Given any (regular) $\Gamma$, there exists (regular) $\Gamma^*$ satisfying PCP and s.t., at any $\theta$, default probability under $\Gamma^*$ same as under $\Gamma$.

- Regularity: MARP well defined (formal proof)
Perfect Coordination Property [PCP]

- Policy $\Gamma^* = (S^*, \pi^*)$ removes any **strategic uncertainty**

- It preserves **structural uncertainty**

- Under $\Gamma^*$, agents know actions all other agents but not **beliefs rationalizing such actions**

- Inability to predict beliefs that rationalize other agents’ actions essential to minimization of default risk

- “Right” form of transparency
  - conformism in beliefs about mkt response
  - ...not in beliefs about “fundamentals”
Optimal policy combines:

- public **Pass/Fail** announcement
  - eliminate strategic uncertainty
- additional disclosures necessary to guarantee that, when \( r = 1 \) is announced (i.e., when bank passed test), all agents pledge under MARP
When is optimal policy binary?

**Theorem 2**

Assume $p(x|\theta)$ satisfies MLRP. Given any policy $\Gamma$ satisfying PCP, there exists a **binary policy** $\Gamma^* = (\{0, 1\}, \pi^*)$ also satisfying PCP and s.t., for any $\theta$, prob of default under $\Gamma^*$ same as under $\Gamma$.

- MARP in threshold strategies: signals other than regime outcome can be dropped (averaging over $s$) without affecting incentives.

- Result hinges on Log-SM of $p(x|\theta)$, i.e., on MLRP:
  - co-movement between state $\theta$ and beliefs.

(Example)
Optimality of Monotone Tests

Figure: Optimal Monotone Policy.
Optimality of Monotone Tests

\[ \pi(1|\theta) \]

\[ \{ \theta : x^{**} \in q_\theta \} \]

Figure: Optimal Monotone Policy.
Foundation for Monotone Tests

**Condition M:** Following properties hold:

1. \[ \inf \{ \theta \in \Theta : x^{**} \in q_{\theta} \} \leq 0; \]
2. \[ p(x|\theta) \text{ and } |u(\theta, 1 - P(x|\theta))| \text{ (weakly) log-supermodular over} \]
   \[ \{ (\theta, x) \in [0, 1] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0 \}; \]
3. \[ \forall \theta_0, \theta_1 \in [0, 1], \text{ with } \theta_0 < \theta_1, \forall x \leq \bar{x}_G \text{ s.t. (a) } u(\theta_1, 1 - P(x|\theta_1)) \leq 0 \text{ and} \]
   \[ (b) x \in q_{\theta_0}, \]
   \[
   \frac{U^P(\theta_1, 1) - U^P(\theta_1, 0)}{U^P(\theta_0, 1) - U^P(\theta_0, 0)} \geq \frac{p(x|\theta_1) u(\theta_1, 1 - P(x|\theta_1))}{p(x|\theta_0) u(\theta_0, 1 - P(x|\theta_0))} \]

**Theorem 3**

Suppose \( p(x|\theta) \) log-supermodular, Condition M holds. Given any \( \Gamma \), there exists deterministic binary monotone \( \Gamma^* = (\{0, 1\}, \pi^*) \) satisfying PCP and yielding payoff weakly higher than \( \Gamma \).
Sub-optimality of Monotone Tests

Example 1

Suppose that, for any $\theta$,
(a) $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$, and $L(\theta) = L$;
(b) $\theta \sim U[-K, 1 + K]$, $K \in \mathbb{R}_{++}$;
(c) $x_i = \theta + \sigma \epsilon_i$, with $\sigma \in \mathbb{R}_+$ and $\epsilon_i \sim U[-1, 1]$, with $\sigma < K/2$.

There exists $\sigma^\# \in (0, K/2)$ such that, for all $\sigma \in (0, \sigma^\#)$, there exists a (deterministic) non-monotone policy satisfying PCP that yields payoff strictly higher than optimal monotone policy.
Optimality of Monotone Tests

\[ \pi(1|\theta) \]

Figure: Optimal Monotone Policy.
Sub-optimality of Monotone Tests

Let $\theta^{MS} \in (0, 1)$ be implicitly defined by

$$\int_0^1 u(\theta^{MS}, l)dl = 0$$  \hspace{1cm} (1)

Let $D^\Gamma \equiv \{d_i = (\theta_i, \bar{\theta}_i) : i = 1, ..., N\}$ be partition of $[0, \theta^{MS}]$ induced by $\Gamma$ with

$$\Delta(\Gamma) \equiv \max_{i=1,...,N} |\bar{\theta}_i - \theta_i|$$

denoting its mesh, where $\pi(s|\theta) = \pi(s|\theta')$ for all $\theta, \theta' \in d_i$ and $\pi(s|\theta) \pi(s|\theta'') = 0$ for all $i, \theta \in d_i, \theta'' \in d_{i+1}$

Example 2

Suppose $\theta \sim U[\mathbb{R}]$ and $x_i = \theta + \sigma \varepsilon_i$, with $\varepsilon_i \sim N(0, 1)$. Assume that, for any $\theta$, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$ and $L(\theta) = L$. There exists $\tilde{\sigma} > 0$ and $\mathcal{E} : (0, \tilde{\sigma}] \rightarrow \mathbb{R}_+$, with $\lim_{\sigma \rightarrow 0^+} \mathcal{E}(\sigma) = 0$, s.t, for any $\sigma \in (0, \tilde{\sigma}]$, the following is true: given any deterministic binary policy $\Gamma$ satisfying PCP and s.t. $\Delta(\Gamma) > \mathcal{E}(\sigma)$, there exists another deterministic binary policy $\Gamma^*$ with $\Delta(\Gamma^*) < \mathcal{E}(\sigma)$ that also satisfies PCP and yields policy maker payoff strictly higher than $\Gamma$. 
Extensions

- Default iff $R(\theta, A, z) \leq 0$
  - $z$ drawn from $Q_\theta$: residual uncertainty

- PM’s payoff

\[
\hat{U}^P(\theta, A, z) = \begin{cases} 
\hat{W}(\theta, z) & \text{if } r = 1 \\
\hat{L}(\theta, z) & \text{if } r = 0
\end{cases}
\]

- Agents’ payoffs

\[
\hat{u}(\theta, A, z) = \begin{cases} 
\hat{g}(\theta, A, z) & \text{if } r = 1 \\
\hat{b}(\theta, A, z) & \text{if } r = 0
\end{cases}
\]

- Expected payoff differential: $u(\theta, A)$
Generalizations

Theorem 4

(a) Given any $\Gamma$, there exists $\Gamma^*$ satisfying PCP and s.t., for any $\theta$, agents’ expected payoff under $a^\Gamma^*$ is at least as high as under $a^\Gamma$.

(b) Suppose $p(x|\theta)$ satisfies MLRP; then $\Gamma^*$ binary. PM’s payoff under $\Gamma^*$ at least as high as under $\Gamma$.

(c) Suppose condition M holds. Then $\Gamma^*$ monotone.

- PCP: announcement of sign of agents’ expected payoff under MARP
Micro-foundations and comparative statics

- Former liabilities: $D$
- Bank’s legacy asset delivers
  - $l(\theta) \in \mathbb{R}$ end of period 1
  - $C(\theta)$ end of period 2
- Bank can issue (i) new shares OR (ii) short-term debt
- Potential investors submit market orders
- Noise traders $z \sim Q_\theta$
Micro-foundations and MCS

- $Y(p, \theta, z)$: exogenous demand for shares (alternatively, debt)

- Market clearing price $p^*(\theta, A, z)$ solves

\[ q + 1 - A = A + Y(p^*, \theta, z). \]

- Default:

\[ R(\theta, A, z) = l(\theta) + \rho_S q p^*(\theta, A, z) - D \leq 0 \]
Micro-foundations and MCS

Analysis can be used to study

- effect of different recapitalization policies
  - \((q_E, q_D)\)
- role of uncertainty for toughness of optimal stress tests
  - uncertainty about bank’s profitability: \(\sigma\)
  - uncertainty about macro variables: \(z\)

**Proposition 1**

*There exists \(\bar{\sigma} > 0\) such that, for any \(\sigma, \sigma' \in (0, \bar{\sigma}]\), with \(\sigma' > \sigma\):\n\[ \theta_E^*(\sigma') < \theta_E^*(\sigma) \text{ and } \theta_D^*(\sigma') > \theta_D^*(\sigma). \]"
Robust/adversarial information design in coordination games with heterogeneously informed agents

Application: Stress tests
- Perfect coordination property ("right" notion of transparency)
- Pass/Fail tests
- Monotone rules

Extension 1: PM uncertain about mkt prior beliefs
- robust-undominated design (see also Dworczak & Pavan (2021))

Extension 2: Elicitation and Persuasion (see Inostroza (2021))
THANKS!
PCP Proof

Let \( r(\omega; a^\Gamma) \in \{0, 1\} \) be default outcome at \( \omega \equiv (\theta, x, s) \) when agents play according to \( a^\Gamma \)

Let \( \Gamma^* = \{S^*, \pi^*\} \) be s.t. \( S^* = S \times \{0, 1\} \) and
\[
\pi^* ((s, r(\omega; a^\Gamma)) | \theta) = \pi(s | \theta), \text{ all } (\theta, s) \text{ s.t. } \pi(s | \theta) > 0
\]

After receiving \( s^* \equiv (s, 1) \), agents use Bayes’ rule to update beliefs about \( \omega \equiv (\theta, x, s) \):
\[
\partial \Lambda_i^{\Gamma^+} (\omega | x_i, (s, 1)) = \frac{1 \{r(\omega; a^\Gamma) = 1\}}{\Lambda_i^{\Gamma}(1 | x_i, s)} \partial \Lambda_i^{\Gamma} (\omega | x_i, s)
\]

where
\[
\Lambda_i^{\Gamma}(1 | x_i, s) \equiv \int_{\{\omega: r(\omega; a^\Gamma) = 1\}} d\Lambda_i^{\Gamma}(\omega | x_i, s)
\]
Let $a^n_\Gamma, a^n_{\Gamma^*}$ be most aggressive profile surviving $n$ round of IDISDS under $\Gamma$ and $\Gamma^*$, respectively.

**Definition 2**
Strategy profile $a^n_{\Gamma^*}$ less aggressive than $a^n_\Gamma$ iff, for any $i \in [0, 1],$

$$a^n_\Gamma, i(x_i, s) = 1 \Rightarrow a^n_{\Gamma^*}, i(x_i, (s, 1)) = 1$$

**Lemma 1**
For any $n$, $a^n_{\Gamma^*}$ less aggressive than $a^n_\Gamma$
Induction

Let $a_0^\Gamma = a_0^{\Gamma*}$ be strategy profile where all agents refrain from pledging, regardless of their (endogenous and exogenous) information.

Suppose that $a_{(n-1)}^{\Gamma*}$ is less aggressive than $a_{(n-1)}^\Gamma$.

Note that $r(\omega | a^\Gamma) = 0 \Rightarrow r(\omega | a_{(n-1)}^\Gamma) = 0$.

Hence, $r(\omega; a^\Gamma) = 1$ “removes” from support of agents’ beliefs states $(\theta, x, s)$ for which default occurs under $a_{(n-1)}^\Gamma$. 
Because

- payoffs from pledging in case of default are negative
- payoff from pledging under $\Gamma^*$ when agents follow $a_{(n-1)}^{\Gamma}$

$$U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^{\Gamma}) = \frac{\int_\omega u(\theta, A(\omega; a_{(n-1)}^{\Gamma})) 1\{r(\omega; a^{\Gamma}) = 1\} d\Lambda_i^{\Gamma}(\omega | x_i, s)}{\Lambda_i^{\Gamma}(1 | x_i, s)}$$

$$> \frac{\int_\omega u(\theta, A(\omega; a_{(n-1)}^{\Gamma})) d\Lambda_i^{\Gamma}(\omega | x_i, s)}{\Lambda_i^{\Gamma}(1 | x_i, s)}$$

$$= \frac{U_i^{\Gamma}(x_i, s; a_{(n-1)}^{\Gamma})}{\Lambda_i^{\Gamma}(1 | x_i, s)}$$

Hence, $U_i^{\Gamma}(x_i, s; a_{(n-1)}^{\Gamma}) > 0 \Rightarrow U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^{\Gamma}) > 0$
PCP Proof

- That \( a^\Gamma_{(n-1)} \) less aggressive than \( a^\Gamma_{(n-1)} \) along with supermodularity of game implies that

\[
U_i^{\Gamma^*}(x_i, (s, 1); a^\Gamma_{(n-1)}) > 0 \Rightarrow U_i^{\Gamma^*}(x_i, (s, 1); a^\Gamma_{(n-1)}) > 0
\]

- As a consequence,

\[
a^\Gamma_{(n),i}(x_i, s) = 1 \Rightarrow a^{\Gamma^*}_{(n),i}(x_i, (s, 1)) = 1
\]

- This means that \( a^{\Gamma^*}_{(n)} \) less aggressive than \( a^\Gamma_{(n)} \).
Above lemma implies MARP under $\Gamma^*$, $a^{\Gamma^*} \equiv a_{(\infty)}^{\Gamma^*}$, less aggressive than MARP under $\Gamma$, $a^{\Gamma} \equiv a_{(\infty)}^{\Gamma}$.

In turn, this implies that $r(\omega; a^{\Gamma}) = 1$ makes it common certainty that $r(\omega; a^{\Gamma^*}) = 1$.

Hence, all agents pledge after hearing that $r(\omega; a^{\Gamma}) = 1$.

Similarly, $r(\omega; a^{\Gamma}) = 0$ makes it common certainty that $\theta \leq 1$. Under MARP, all agents refrain from pledging when hearing that $r(\omega; a^{\Gamma}) = 0$. 

PCP Proof
Example

- Assume $b = -g$
- Pledging rationalizable iff $Pr(r = 1) \geq 1/2$
Example

No disclosure: under MARP, $a_i^\Gamma(x_i) = 0$, all $x_i$
Example

Suppose PM informs agents of whether $\theta$ is extreme or intermediate

$\Pr(x^L|\theta)$: dashed \hspace{1cm} $\Pr(x^H|\theta)$: solid

- $\Pr(x^L|\theta)$: $\frac{1}{3}$ in $[\frac{1}{3}, \frac{2}{3}]$
- $\Pr(x^H|\theta)$: $\frac{2}{3}$ in $[\frac{1}{3}, \frac{2}{3}]$

$a_i^{\Gamma}(x_i, s) = 1$, all $(x_i, s)$
If, instead, PM only recommends to pledge (equivalently, $\Gamma$ is pass/fail):

\[ a_i^\Gamma(x_i, 1) = 0 \text{ for all } x_i \]

- **Suboptimality of P/F policies (+ failure of RP)**