Keeping the Agents in the Dark: Private Disclosures in Competing Mechanisms

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Competing Mechanisms

- **Competing mechanisms**
    - oligopoly
    - insurance
    - regulation
    - taxation
    - political economy
    - auctions
    - finance
    - search
    - ...

Take-aways of theory literature

- **Non validity of (standard) Revelation Principle** (McAfee, 1993, Peck, 1997,...)
    - Endogenous private information
    - Out-of-equilibrium allocations
    - Agent(s) as correlation device
- **Universal mechanisms** (Epstein and Peters, 1999,...)
    - "Type" should include market information
    - Universal language
    - Universal revelation principle
- **Folk theorems** (Yamashita, 2010, Peters & Troncoso Valverde, 2013,...)
    - Any IC allocation yielding each principal payoff above "min-max-min" can be supported in eq.

What is a mechanism?

$$\phi : M \to \Delta(\mathcal{A})$$

What is missing? Private Disclosures

- Different agents may be informed differently about implications of their actions
- Examples:
  - seller informs bidders asymmetrically about elements of an auction (e.g., reservation price)
  - planner informs citizens asymmetrically about features of public good

Mechanisms with private disclosures

- Set of private disclosures to agent i: S_i
- $S = S^1 \times \cdots \times S^I$
- Joint distribution: $\sigma \in \Delta(S)$

$$\phi : S \times M \to \Delta(\mathcal{A})$$

- Each $s = (s^1, ..., s^I)$ indexes standard mechanism

$$\phi(s) : M \to \Delta(\mathcal{A})$$

- s^i: (hierarchy of) beliefs over mapping from actions to outcomes

This paper

- **Non validity of folk theorems**
  - non-robustness of eq. allocations sustained with "standard" mechanisms
  - impossibility to sustain payoffs above min-max-min
    - no matter richness of M
- **Non-universality of standard mechanisms**
  - some eq. allocations with private disclosures cannot be sustained without
    - no matter richness of M

Literature – Incomplete

- **Universal revelation principle**
  - Epstein & Peters (1999)
- **Folk theorems**
  - Yamashita (2010), Peters & Troncoso Valverde (2013)...
- **Common agency (single agent)**
  - Martimort and Stole (2002), Peters (2001), Calzolari and Pavan (2009,2010)...
- **Moral-hazard games**
  - Attar, Campioni, Piaser (2019)...
- **Information transmission between principals, MD w. aftermarkets**
  - Calzolari and Pavan (2006a,b) Dworczak (2020)...
- **Information design**
  - ...Rayo and Segal (2010), Kamenica and Gentzkow (2012)...

Plan

1. Introduction
2. Anti folk-theorem
3. Non-universality of standard mechanisms

Anti-folk theorems

Recommendation mechanisms

- **Theorem (Yamashita, 2010)**: Assume 3 or more agents. Every deterministic IC allocation yielding each principal payoff above appropriate min-max-min supportable in eq.

Recommendation mechanisms

- **Yamashita's idea**
  - principals can punish deviations by having agents "vote" on standard DRM

$$d_j : \Omega \to \mathcal{A}_j$$

  - $\Omega = \Omega^1 \times ... \times \Omega^I$: exogenous payoff space (ω^i observed privately by agent i)
  - $\mathcal{A}_j$: allocations for principal j
  - D_j: set of all standard DRMs
  - recommendation mechanism: $M_j^i = D_j \times \Omega^i$

$$\phi_j^r(m_j^1, \ldots, m_j^I) \equiv \begin{cases} \hat{d}_j(\omega^1, \ldots, \omega^I) & \text{if } \left|\{i : m_j^i = (\hat{d}_j, \omega^i)\}\right| \geq I - 1 \\ \bar{a}_j & \text{otherwise} \end{cases}$$

  - no agent pivotal in choice of direct mechanism

Primitive game

- Agents: A1, A2, A3
- Principals: P1 and P2
- P1's allocations $X = \{x_1, x_2\}$
- P2's allocations $Y = \{y_1, y_2\}$
- A1's exogenous type $\omega^1 \in \Omega^1 = \{\omega_L, \omega_H\}$
- A2's exogenous type $\omega^2 \in \Omega^2 = \{\omega_L, \omega_H\}$
- A3: no exogenous private info
- A1's and A2's type perfectly correlated

Payoffs

- Payoffs (u_{P2}, u^{A1}, u^{A2})

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$5, 8, 8$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$5, 8, 8$

G^M: Game in standard mechanisms

- $t = 0$: A1 and A2 learns ω^1 and ω^2
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: agents send messages
- $t = 3$: principals' decisions determined by $(\phi_j(m_j))_j$

Folk theorem

Claim 1

Suppose $M_j^i \supset D_j \times \Omega^i$, $i = 1, 2, 3$, $j = 1, 2$, with M finite. Any payoff for P2 in [5, 6]

can be supported in eq.

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$5, 8, 8$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$5, 8, 8$

Equilibrium supporting min-max-min payoff

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$\mathbf{5}, \mathbf{8}, \mathbf{8}$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$\mathbf{5}, \mathbf{8}, \mathbf{8}$

- Equilibrium outcome

$$z(\omega_L, \omega_L) = (x_1, y_1), \quad z(\omega_H, \omega_H) = (x_2, y_2)$$

- On path, both P1 and P2 post *recommendation mechanisms* (ϕ_1^r, ϕ_2^r)
- In subgame (ϕ_1^r, ϕ_2^r), all agents recommend DRMs

$$d_1^*(\omega) \equiv \begin{cases} x_1 & \text{if } \omega = (\omega_L, \omega_L) \\ x_2 & \text{otherwise} \end{cases} \qquad d_2^*(\omega) \equiv \begin{cases} y_1 & \text{if } \omega = (\omega_L, \omega_L) \\ y_2 & \text{otherwise} \end{cases}$$

and A1 and A2 report truthfully to both principals

Equilibrium supporting min-max-min payoff

- Suppose P2 deviates to $\phi_2 : M_2 \to \Delta(Y)$
- Let $p(m_2) = \Pr(y_1 | m_2)$

$$\overline{p} = p(\overline{m}_2) \geq p(m_2) \qquad \forall m_2$$

$$\underline{p} = p(\underline{m}_2^1, \underline{m}_2^2, \overline{m}_2^3) \leq p(m_2^1, m_2^2, \overline{m}_2^3) \qquad \forall (m_2^1, m_2^2)$$

Equilibrium supporting min-max-min payoff

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	**5, 8, 8**	5, 1, 1
x_2	6, 4.5, 4.5	6, 4.5, 4.5

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	6, 4.5, 4.5	6, 4.5, 4.5
x_2	5, 1, 1	**5, 8, 8**

- **Case 1**: $\overline{p} \geq 1/2$
  - all agents recommend $d_1^*(\omega) \equiv \begin{cases} x_1 & \text{if } \omega = (\omega_L, \omega_L) \\ x_2 & \text{otherwise} \end{cases}$
  - A3 sends $\overline{m}_2^3$
    - (ω_L, ω_L): $8\overline{p} + (1 - \overline{p}) \geq 4.5 \Rightarrow$ truthful reporting + $\overline{m}_2^i$ optimal
    - (ω_H, ω_H): no agent can unilaterally change P1's decision
  - P2's payoff: 5

Equilibrium supporting min-max-min payoff

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	**5, 8, 8**	5, 1, 1
x_2	6, 4.5, 4.5	6, 4.5, 4.5

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	6, 4.5, 4.5	6, 4.5, 4.5
x_2	5, 1, 1	**5, 8, 8**

- **Case 2**: $\overline{p} < 1/2$
  - all agents recommend $d_1(\omega) \equiv \begin{cases} x_2 & \text{if } \omega = (\omega_H, \omega_H) \\ x_1 & \text{otherwise} \end{cases}$
  - A3 sends $\overline{m}_2^3$
    - (ω_L, ω_L): no agent can unilaterally change P1's decision
    - (ω_H, ω_H): $\underline{p} + 8(1 - \underline{p}) \geq 4.5 \Rightarrow$ truthful reporting + $\underline{m}_2^i$ optimal
  - P2's payoff: 5

G^{SM}: game with private disclosures

- $t = 0$: A1 and A2 learns ω^1 and ω^2
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: **agents receive signals**
- $t = 3$: agents send messages
- $t = 4$: principals' decisions determined by $(\phi_j(s_j, m_j))_j$

Private Disclosures

Claim 2

Suppose that $M_j^i \supset D_j \times \Omega^i$, $i = 1, 2, 3$, $j = 1, 2$ and $\left|S_2^1\right| \geq 2$, with M and S finite. PBE set of G^{SM} non empty. In any PBE of G^{SM}, P2's payoff strictly above 5 (min-max-min).

Claim 2: Proof

- Wlog, assume $\{1,2\} \subset S_2^1$
- Let $\overline{\gamma}_2$ be mechanism that
  - w.p. $\alpha \in (\frac{1}{2}, 1)$ sends $s_2^1 = 1$ to A1 and selects y_1
  - w.p. $1-\alpha$ sends $s_2^1 = 2$ to A1 and selects y_2
  - no signal sent to A2 and A3
  - no use of messages
- Proof establishes that

$$\inf_{\gamma_1 \in \Gamma_1} \inf_{\beta \in B^*(\gamma_1, \overline{\gamma}_2)} \sum_{\omega \in \Omega} \Pr(\omega) \sum_{(x,y) \in X \times Y} z_{(\gamma_1, \overline{\gamma}_2, \beta)}(x, y \mid \omega) u_{P2}(x, y, \omega) > 5$$

- ...+ eq. existence for G^{SM}

Claim 2: Proof

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$5, 8, 8$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$5, 8, 8$

- Decisions implemented in $\overline{\gamma}_2$ invariant to $m_2 \Rightarrow$ no signals for P1
- P2's payoff $= 5 \Rightarrow x_1$ in (ω_L, ω_L) and x_2 in (ω_H, ω_H)
- (ω_L, ω_L):
  - after receiving $s_2^1 = 2$, A1 wants to min $\Pr(x_1)$
  - hence, $\phi_1(m_1^1, m_1^2, m_1^3) = x_1$ all m_1^1, all (m_1^2, m_1^3) sent by A2 and A3
- (ω_H, ω_H):
  - after receiving $s_2^1 = 1$, A1 wants to max $\Pr(x_1)$
  - hence, $\phi_1(m_1^1, m_1^2, m_1^3) = x_2$ all m_1^1, all (m_1^2, m_1^3) sent by A2 and A3
- So A1 must not affect P1's decision
- Because A3 does not know state and P2 gets 5 only when P1's decision perfectly correlated with state, A2 must have full control over P1's decision
  - because $\Pr(y_1) > 1/2$, in state (ω_H,ω_H) A2 has profitable deviation

Role of Private Disclosures in Example 1

- Information P2 privately discloses to A1 makes A1 an "ally" of P2
- Importance of asymmetric disclosures:
  - If same information disclosed also to A2 and A3, agents can discipline each other, thus implementing incentive-compatible punishments for P2

Robustness and Anti-folk theorem

Proposition 1

Equilibria of games in which principals restricted to standard mechanisms need not be robust. Furthermore, eq. payoffs of such games need not be sustainable when principals can engage in private disclosures.

Robustness and Anti-folk theorem

- Result extends to
  - contracts-on-contracts
  - reciprocal mechanisms
  - rich randomizing devices
  - direct communication between principals

Plan

Non-universality of standard mechanisms

Primitive Game

- **Primitive game**
  - Agents: $A1$ and $A2$
  - Principals: P1 and P2
  - P1's allocations $X = \{x_1, x_2, x_3, x_4\}$
  - P2's allocations $Y = \{y_1, y_2\}$
  - A2's exogenous type $\omega^2 \in \Omega^2 = \{\omega_L, \omega_H\}$, $\Pr(\omega_H) = 3/4$

Payoffs

- Payoffs (u_{P2}, u^{A1}, u^{A2})
- P1's payoff: constant
- $\zeta < 0$

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$10, 3, 3$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$10, 7.5, 7.5$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$10, 7.5, 5$
x_2	$10, 3, 9$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

G^{SM}: game with private disclosures

- No signals for P1
- Signals for P2: $S_2^1 = S_2^2 = \{1, 2\}$
- No messages for P2
- Messages for P1:
  - $M_1^1 = S_2^1$ (for A1)
  - $M_1^2 = \Omega^2 \times S_2^2$ (for A2)
- Hence,
  - P2 sends signals to both agents and asks for no messages
  - P1 sends no signals but asks for P2's signals (and ω)

G^{SM}: game with private disclosures

- $t = 0$: A2 learns ω^2
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: agents receive signals (from P2)
- $t = 3$: agents send messages (to P1)
- $t = 4$: principals' decisions determined by $(\phi_j(s_j, m_j))_j$

Equilibrium outcome of G^{SM}

Claim 3

There exists PBE of G^{SM} supporting

$$z(\omega_L) \equiv \frac{2}{3}(x_3, y_1) + \frac{1}{3}(x_4, y_2)$$
$$z(\omega_H) \equiv \frac{2}{3}(x_2, y_1) + \frac{1}{3}(x_1, y_2)$$

and giving P2 a payoff of 10.

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Claim 3

- P2 posts mechanism $\gamma_2^* = (\sigma_2^*, \phi_2^*)$ s.t.

$$\sigma_2^*(1,1) = \sigma_2^*(2,2) = \frac{1}{3}$$

$$\sigma_2^*(1,2) = \sigma_2^*(2,1) = \frac{1}{6}$$

$$\phi_2^*(s) = \begin{cases} y_1 & \text{if } s \in \{(1,1),(2,2)\} \\ y_2 & \text{if } s \in \{(1,2),(2,1)\} \end{cases}$$

- Each agent believes
  - P2 will implement y_1 with prob $\frac{2}{3}$
  - other agent received same signal as his with prob $\frac{2}{3}$

Proof of Claim 3

- P1's mechanism

$$\phi_1^*(m) = \begin{cases} x_3 & \text{if } m \in \{(1,1,\omega_L),(2,2,\omega_L)\} \\ x_4 & \text{if } m \in \{(1,2,\omega_L),(2,1,\omega_L)\} \\ x_2 & \text{if } m \in \{(1,1,\omega_H),(2,2,\omega_H)\} \\ x_1 & \text{if } m \in \{(1,2,\omega_H),(2,1,\omega_H)\} \end{cases}$$

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	**10, 3, 3**	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	**10, 7.5, 7.5**

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	**10, 7.5, 5**
x_2	**10, 3, 9**	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Claim 3

- If A1 is truthful, regardless of his signal,

$$\frac{1}{4}\left(\frac{2}{3}u^{A1}(x_3, y_1, \omega_L) + \frac{1}{3}u^{A1}(x_4, y_2, \omega_L)\right) + \frac{3}{4}\left(\frac{2}{3}u^{A1}(x_2, y_1, \omega_H) + \frac{1}{3}u^{A1}(x_1, y_2, \omega_H)\right) = 3\frac{2}{3} + 7.5\frac{1}{3} = \mathbf{4.5}$$

- If A1 misreports, regardless of his signal,

$$\frac{1}{4}\left(\frac{2}{3}u^{A1}(x_4, y_1, \omega_L) + \frac{1}{3}u^{A1}(x_3, y_2, \omega_L)\right) + \frac{3}{4}\left(\frac{2}{3}u^{A1}(x_1, y_1, \omega_H) + \frac{1}{3}u^{A1}(x_2, y_2, \omega_H)\right) = \frac{2}{3} + 5.5\frac{1}{3} = 2.5$$

Proof of Claim 3

- If type ω_L of A2 reports truthfully (both ω^2 and s_2^2), regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_3, y_1, \omega_L) + \frac{1}{3}u^{A2}(x_4, y_2, \omega_L) = 3\frac{2}{3} + 7.5\frac{1}{3} = 4.5$$

- If type ω_L of A2 truthfully reports ω^2 but misreports s_2^2, regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_4, y_1, \omega_L) + \frac{1}{3}u^{A2}(x_3, y_2, \omega_L) = 3.5$$

- If type ω_L of A2 misreports ω^2 but truthfully reports s_2^2, regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_2, y_1, \omega_L) + \frac{1}{3}u^{A2}(x_1, y_2, \omega_L) = 5\frac{2}{3} + 3.5\frac{1}{3} = \frac{8.5}{3}$$

- If type ω_L of A2 misreports both ω^2 and s_2^2, regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_1, y_1, \omega_L) + \frac{1}{3}u^{A2}(x_2, y_2, \omega_L) = \frac{2}{3} + 8\frac{1}{3} = \frac{10}{3}$$

Proof of Claim 3

- If type ω_H of A2 reports truthfully (both ω^2 and s_2^2,), regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_2, y_1, \omega_H) + \frac{1}{3}u^{A2}(x_1, y_2, \omega_H) = 9\frac{2}{3} + 5\frac{1}{3} = \frac{23}{3}$$

- If type ω_H of A2 truthfully reports ω^2 but misreports s_2^2, regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_1, y_1, \omega_H) + \frac{1}{3}u^{A2}(x_2, y_2, \omega_H) = 6$$

- If type ω_H of A2 misreports ω^2 but truthfully reports s_2^2, regardless of s^2

$$\frac{2}{3}u^{A2}(x_3, y_1, \omega_H) + \frac{1}{3}u^{A2}(x_4, y_2, \omega_H) = 7\frac{2}{3} + 9\frac{1}{3} = \frac{23}{3}$$

- If type ω_H of A2 misreports both ω^2 and s_2^2, regardless of s_2^2,

$$\frac{2}{3}u^{A2}(x_4, y_1, \omega_H) + \frac{1}{3}u^{A2}(x_3, y_2, \omega_H) = 6\frac{2}{3} + 7\frac{1}{3} = \frac{19}{3}$$

Indispensability of signals

- G^M: arbitrary game with standard mechanisms $\phi_j : M_j \to \Delta(\mathcal{A}_j)$

Claim 4

No matter richness of M, there exists **no** PBE of G^M supporting

$$z(\omega_L) \equiv \frac{2}{3}(x_3, y_1) + \frac{1}{3}(x_4, y_2)$$
$$z(\omega_H) \equiv \frac{2}{3}(x_2, y_1) + \frac{1}{3}(x_1, y_2)$$

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Claim 4

- Let $\mu \in \Delta(\Phi_1 \times \Phi_2)$ and $\lambda = (\lambda^1, \lambda^2)$ continuation eq. for G^M
- **Step 1**: For μ-almost all $\phi \in supp[\mu]$, $\lambda(\phi)$-almost all (m_1, m_2),

$$(\phi_1(m_1), \phi_2(m_2)) \in \overline{\mathrm{Int}\Delta(X)} \times \overline{\mathrm{Int}\Delta(Y)}$$

  - deterministic response to messages

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Claim 4

- **Step 2**: For μ-almost all $\phi = (\phi_1, \phi_2)$, IC for A2 requires that

$$\Pr(x_3, y_1|\omega_L; \phi, \lambda) = 1 - \Pr(x_4, y_2|\omega_L; \phi, \lambda) = 2/3$$

$$\Pr(x_2, y_1|\omega_H; \phi, \lambda) = 1 - \Pr(x_1, y_2|\omega_H; \phi, \lambda) = 2/3$$

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Claim 4

- **Step 3**: For μ-almost all ϕ, there exists no pair of behavioral strategies inducing

$$\Pr(x_3, y_1|\omega_L; \phi, \lambda) = 1 - \Pr(x_4, y_2|\omega_L; \phi, \lambda) = 2/3$$
$$\Pr(x_2, y_1|\omega_H; \phi, \lambda) = 1 - \Pr(x_1, y_2|\omega_H; \phi, \lambda) = 2/3$$

  - messages A2 sends in state ω_H must have no bite
    - else ω_L can pick distribution induced by ω_H, invert correlation while preserving marginals and do strictly better
  - ...but then A1 has profitable deviation

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Indispensability of signals

Proposition 2

There exist eq. allocations in games with private disclosures that cannot be supported as eq. allocations in **any** game with standard mechanisms.

- No matter
  - richness of M
  - availability of **public** randomizing devices

Indispensability of signals

- Result extends to
  - correlation in choice of mechanisms
  - reciprocal mechanisms
  - correlation in agents' messages
- It does **not** extend to
  - direct (but private) communication between principals

Role of private disclosures in Example 2

- Correlate principals' decisions with agents' private information, **respecting IC**
- Supplement absence of direct communication among principals
- Different from action recommendations
- Disclosures may but need not change agents' beliefs
  - encrypted "keys"

Conclusions

- Private disclosures
  - irrelevant in
    - single principal
    - competing principals with single agent (common agency)
  - important role when multiple principals contract w. multiple agents
- Non-robustness of equilibria with standard mechanisms
- Anti-folk theorems: payoffs above min-max-min not sustainable in eq.
- Non-universality of standard mechanisms

Ongoing Work

- Canonical mechanisms?
    - canonical signal spaces?
    - canonical message spaces?
    - canonical extensive form?
    - $\phi : M \to \Delta(A)$
    - $\phi : S \times M \to \Delta(A)$
    - $\phi : S \times M \times S \to \Delta(A)$
    - $\phi : S \times M \times S \cdots M \to \Delta(A)$

Most Important Slide

THANKS!

$$\phi : S \times M \to \Delta(\mathcal{A})$$