Keeping the Agents in the Dark: Private Disclosures in Competing Mechanisms

Andrea Attar, Eloisa Campioni, Thomas Mariotti, Alessandro Pavan
Competing mechanisms

- oligopoly
- insurance
- regulation
- taxation
- political economy
- auctions
- finance
- search
- ...

Competing Mechanisms
Take-aways of theory literature

- **Non validity of (standard) Revelation Principle** (McAfee, 1993, Peck, 1997,...)
  - Endogenous private information
  - Out-of-equilibrium allocations
  - Agent(s) as correlation device

- **Universal mechanisms** (Epstein and Peters, 1999,...)
  - “Type” should include market information
  - Universal language
  - Universal revelation principle

- **Folk theorems** (Yamashita, 2010, Peters & Troncoso Valverde, 2013,...)
  - Any IC allocation yielding each principal payoff above “min-max-min” can be supported in eq.
What is a mechanism?

\[ \phi : M \rightarrow \Delta(\mathcal{A}) \]
Different agents may be informed differently about implications of their actions

Examples:

- seller informs bidders asymmetrically about elements of an auction (e.g., reservation price)
- planner informs citizens asymmetrically about features of public good
Mechanisms with private disclosures

- Set of private disclosures to agent $i$: $S_i$

- $S = S^1 \times \cdots \times S^I$

- Joint distribution: $\sigma \in \Delta(S)$

\[ \phi : S \times M \rightarrow \Delta(\mathcal{A}) \]

- Each $s = (s^1, \ldots, s^I)$ indexes standard mechanism

\[ \phi(s) : M \rightarrow \Delta(\mathcal{A}) \]

- $s^i$: (hierarchy of) beliefs over mapping from actions to outcomes
This paper

- **Non validity of folk theorems**
  - non-robustness of eq. allocations sustained with “standard” mechanisms
  - impossibility to sustain payoffs above min-max-min
    - no matter richness of $M$

- **Non-universality of standard mechanisms**
  - some eq. allocations with private disclosures cannot be sustained without
    - no matter richness of $M$
Literature – Incomplete

- **Universal revelation principle**
  - Epstein & Peters (1999)

- **Folk theorems**
  - Yamashita (2010), Peters & Troncoso Valverde (2013)...

- **Common agency (single agent)**

- **Moral-hazard games**
  - Attar, Campioni, Piaser (2019)...

- **Information transmission between principals, MD w. aftermarkets**
  - Calzolari and Pavan (2006a,b) Dworczak (2020)...

- **Information design**
  - ...Rayo and Segal (2010), Kamenica and Gentzkow (2012)...

1. Introduction

2. Anti folk-theorem

3. Non-universality of standard mechanisms
Anti-folk theorems
**Theorem (Yamashita, 2010):** Assume 3 or more agents. Every deterministic IC allocation yielding each principal payoff above appropriate min-max-min supportable in eq.
Recommendation mechanisms

- **Yamashita’s idea**

  - principals can punish deviations by having agents “vote” on standard DRM

    \[ d_j : \Omega \rightarrow A_j \]

  - \( \Omega = \Omega^1 \times \ldots \times \Omega^I \): exogenous payoff space (\( \omega^i \) observed privately by agent \( i \))

  - \( A_j \): allocations for principal \( j \)

  - \( D_j \): set of all standard DRMs

  - recommendation mechanism: \( M^j_i = D_j \times \Omega^i \)

\[ \phi^r_j(m^j_1, \ldots, m^j_I) \equiv \begin{cases} \hat{d}_j(\omega^1, \ldots, \omega^I) & \text{if } \left| \{ i : m^j_i = (\hat{d}_j, \omega^i) \} \right| \geq I - 1 \\ \bar{a}_j & \text{otherwise} \end{cases} \]

- no agent pivotal in choice of direct mechanism
Primitive game

- Agents: A1, A2, A3
- Principals: P1 and P2

P1’s allocations $X = \{x_1, x_2\}$

P2’s allocations $Y = \{y_1, y_2\}$

A1’s exogenous type $\omega^1 \in \Omega^1 = \{\omega_L, \omega_H\}$

A2’s exogenous type $\omega^2 \in \Omega^2 = \{\omega_L, \omega_H\}$

A3: no exogenous private info

A1’s and A2’s type perfectly correlated
Payoffs \( (u_{P2}, u^{A1}, u^{A2}) \)

\[ \omega = (\omega_L, \omega_L) \]

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\[ \omega = (\omega_H, \omega_H) \]

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$G^M$: Game in standard mechanisms

- $t = 0$: A1 and A2 learns $\omega^1$ and $\omega^2$
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: agents send messages
- $t = 3$: principals’ decisions determined by $(\phi_j(m_j))_j$
Folk theorem

Claim 1

Suppose $M^i_j \supset D_j \times \Omega^i$, $i = 1, 2, 3$, $j = 1, 2$, with $M$ finite. Any payoff for P2 in [5, 6] can be supported in eq.

\[
\omega = (\omega_L, \omega_L)
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Equilibrium supporting min-max-min payoff

\[ \omega = (\omega_L, \omega_L) \]

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- **Equilibrium outcome**

\[ z(\omega_L, \omega_L) = (x_1, y_1), \quad z(\omega_H, \omega_H) = (x_2, y_2) \]

- **On path, both P1 and P2 post recommendation mechanisms** \((\phi^r_1, \phi^r_2)\)

- **In subgame** \((\phi^r_1, \phi^r_2)\), all agents recommend DRMs

\[ d^*_1(\omega) \equiv \begin{cases} x_1 & \text{if } \omega = (\omega_L, \omega_L) \\ x_2 & \text{otherwise} \end{cases} \quad d^*_2(\omega) \equiv \begin{cases} y_1 & \text{if } \omega = (\omega_L, \omega_L) \\ y_2 & \text{otherwise} \end{cases} \]

and A1 and A2 report truthfully to both principals
Suppose P2 deviates to $\phi_2 : M_2 \rightarrow \Delta(Y)$

Let $p(m_2) = \Pr(y_1|m_2)$

$$\bar{p} = p(m_2) \geq p(m) \quad \forall m$$

$$p = p(m_{1_2}, m_{2_2}, m_{3_2}) \leq p(m_{1_2}, m_{2_2}, m_{3_2}) \quad \forall (m_{1_2}, m_{2_2})$$
Equilibrium supporting min-max-min payoff

\[ \omega = (\omega_L, \omega_L) \]

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**Case 1: \( \bar{p} \geq 1/2 \)**

- all agents recommend \( d_1^*(\omega) \equiv \begin{cases} x_1 \text{ if } \omega = (\omega_L, \omega_L) \\ x_2 \text{ otherwise} \end{cases} \)

- A3 sends \( \bar{m}_2^3 \)

- \((\omega_L, \omega_L): 8\bar{p} + (1 - \bar{p}) \geq 4.5 \Rightarrow \text{truthful reporting} + \bar{m}_2^i \text{ optimal} \)

- \((\omega_H, \omega_H): \text{no agent can unilaterally change P1’s decision} \)

- P2’s payoff: 5
Equilibrium supporting min-max-min payoff

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- **Case 2:** \( \bar{p} < \frac{1}{2} \)
  - all agents recommend \( d_1(\omega) \equiv \begin{cases} x_2 & \text{if } \omega = (\omega_H, \omega_H) \\ x_1 & \text{otherwise} \end{cases} \)
  - A3 sends \( \overline{m}_2^3 \)
    - \((\omega_L, \omega_L)\): no agent can unilaterally change P1’s decision
    - \((\omega_H, \omega_H)\): \( p + 8(1 - p) \geq 4.5 \) \( \Rightarrow \) truthful reporting + \( m_i^j \) optimal
  - P2’s payoff: 5
$G^{SM}$: game with private disclosures

- $t = 0$: A1 and A2 learn $\omega^1$ and $\omega^2$
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: agents receive signals
- $t = 3$: agents send messages
- $t = 4$: principals’ decisions determined by $(\phi_j(s_j, m_j))_j$
Claim 2

Suppose that $M_i^j \supset D_j \times \Omega_i$, $i = 1, 2, 3$, $j = 1, 2$ and $|S_2^1| \geq 2$, with $M$ and $S$ finite. PBE set of $G^{SM}$ non empty. In any PBE of $G^{SM}$, P2’s payoff strictly above 5 (min-max-min).
Claim 2: Proof

- Wlog, assume \( \{1, 2\} \subset S^1_2 \)

- Let \( \overline{\gamma}_2 \) be mechanism that
  - w.p. \( \alpha \in (\frac{1}{2}, 1) \) sends \( s^1_2 = 1 \) to A1 and selects \( y_1 \)
  - w.p. \( 1 - \alpha \) sends \( s^1_2 = 2 \) to A1 and selects \( y_2 \)
  - no signal sent to A2 and A3
  - no use of messages

- Proof establishes that
  \[
  \inf_{\gamma_1 \in \Gamma_1} \inf_{\beta \in B^*(\gamma_1, \overline{\gamma}_2)} \sum_{\omega \in \Omega} \Pr(\omega) \sum_{(x, y) \in X \times Y} z(\gamma_1, \overline{\gamma}_2, \beta)(x, y \mid \omega) u_{P2}(x, y, \omega) > 5
  \]

- ...+ eq. existence for \( G^{SM} \)
Claim 2: Proof

\[ \omega = (\omega_L, \omega_L) \]

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\[ \omega = (\omega_H, \omega_H) \]

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- Decisions implemented in \(\overline{\gamma}_2\) invariant to \(m_2 \Rightarrow \) no signals for P1
- P2's payoff = 5 \(\Rightarrow\) \(x_1\) in \((\omega_L, \omega_L)\) and \(x_2\) in \((\omega_H, \omega_H)\)
- \((\omega_L, \omega_L)\):
  - after receiving \(s_2^1 = 2\), A1 wants to \(\min\) \(Pr(x_1)\)
  - hence, \(\phi_1(m_1^1, m_2^2, m_3^3) = x_1\) all \(m_1^1\), all \((m_2^2, m_3^3)\) sent by A2 and A3

- \((\omega_H, \omega_H)\):
  - after receiving \(s_2^1 = 1\), A1 wants to \(\max\) \(Pr(x_1)\)
  - hence, \(\phi_1(m_1^1, m_2^2, m_3^3) = x_2\) all \(m_1^1\), all \((m_2^2, m_3^3)\) sent by A2 and A3

- So A1 must not affect P1's decision
- Because A3 does not know state and P2 gets 5 only when P1's decision perfectly correlated with state, A2 must have full control over P1's decision
  - because \(Pr(y_1) > 1/2\), in state \((\omega_H, \omega_H)\) A2 has profitable deviation
Role of Private Disclosures in Example 1

- Information P2 privately discloses to A1 makes A1 an “ally” of P2

- Importance of asymmetric disclosures:
  - If same information disclosed also to A2 and A3, agents can discipline each other, thus implementing incentive-compatible punishments for P2
Proposition 1

Equilibria of games in which principals restricted to standard mechanisms need not be robust. Furthermore, eq. payoffs of such games need not be sustainable when principals can engage in private disclosures.
Robustness and Anti-folk theorem

Result extends to

- contracts-on-contracts
- reciprocal mechanisms
- rich randomizing devices
- direct communication between principals
1. Introduction
2. Anti folk-theorem
3. Non-universality of standard mechanisms
Non-universality of standard mechanisms
Primitive Game

**Primitive game**

- **Agents:** $A_1$ and $A_2$

- **Principals:** $P_1$ and $P_2$

- **$P_1$’s allocations** $X = \{x_1, x_2, x_3, x_4\}$

- **$P_2$’s allocations** $Y = \{y_1, y_2\}$

- **$A_2$’s exogenous type** $\omega^2 \in \Omega^2 = \{\omega_L, \omega_H\}$, $\Pr(\omega_H) = 3/4$
Payoffs \((u_{P2}, u^{A1}, u^{A2})\)

P1's payoff: constant

\[ \zeta < 0 \]

### Payoffs

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$G^{SM}$: game with private disclosures

- No signals for P1
- Signals for P2: $S_2^1 = S_2^2 = \{1, 2\}
- No messages for P2
- Messages for P1:
  - $M_1^1 = S_2^1$ (for A1)
  - $M_1^2 = \Omega^2 \times S_2^2$ (for A2)

Hence,
- P2 sends signals to both agents and asks for no messages
- P1 sends no signals but asks for P2’s signals (and $\omega$)
$G^{SM}$: game with private disclosures

- $t = 0$: A2 learns $\omega^2$
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: agents receive signals (from P2)
- $t = 3$: agents send messages (to P1)
- $t = 4$: principals’ decisions determined by $(\phi_j(s_j, m_j))_j$
Equilibrium outcome of $G^{SM}$

**Claim 3**

There exists PBE of $G^{SM}$ supporting

$$z(\omega_L) \equiv \frac{2}{3}(x_3, y_1) + \frac{1}{3}(x_4, y_2)$$

$$z(\omega_H) \equiv \frac{2}{3}(x_2, y_1) + \frac{1}{3}(x_1, y_2)$$

and giving P2 a payoff of 10.

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Proof of Claim 3

- P2 posts mechanism $\gamma_2^* = (\sigma_2^*, \phi_2^*)$ s.t.

$$\sigma_2^*(1, 1) = \sigma_2^*(2, 2) = \frac{1}{3}$$
$$\sigma_2^*(1, 2) = \sigma_2^*(2, 1) = \frac{1}{6}$$

$$\phi_2^*(s) = \begin{cases} y_1 & \text{if } s \in \{(1, 1), (2, 2)\} \\ y_2 & \text{if } s \in \{(1, 2), (2, 1)\} \end{cases}$$

- Each agent believes
  - P2 will implement $y_1$ with prob $\frac{2}{3}$
  - other agent received same signal as his with prob $\frac{2}{3}$
Proof of Claim 3

- P1's mechanism

\[ \phi_1^*(m) = \begin{cases} 
  x_3 & \text{if } m \in \{(1, 1, \omega_L), (2, 2, \omega_L)\} \\
  x_4 & \text{if } m \in \{(1, 2, \omega_L), (2, 1, \omega_L)\} \\
  x_2 & \text{if } m \in \{(1, 1, \omega_H), (2, 2, \omega_H)\} \\
  x_1 & \text{if } m \in \{(1, 2, \omega_H), (2, 1, \omega_H)\} 
\end{cases} \]

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Proof of Claim 3

- If $A_1$ is truthful, regardless of his signal,
  \[ \frac{1}{4} \left( \frac{2}{3} u^{A_1}(x_3, y_1, \omega_L) + \frac{1}{3} u^{A_1}(x_4, y_2, \omega_L) \right) + \frac{3}{4} \left( \frac{2}{3} u^{A_1}(x_2, y_1, \omega_H) + \frac{1}{3} u^{A_1}(x_1, y_2, \omega_H) \right) = \frac{2}{3} + 7.5 \frac{1}{3} = 4.5 \]

- If $A_1$ misreports, regardless of his signal,
  \[ \frac{1}{4} \left( \frac{2}{3} u^{A_1}(x_4, y_1, \omega_L) + \frac{1}{3} u^{A_1}(x_3, y_2, \omega_L) \right) + \frac{3}{4} \left( \frac{2}{3} u^{A_1}(x_1, y_1, \omega_H) + \frac{1}{3} u^{A_1}(x_2, y_2, \omega_H) \right) = \frac{2}{3} + 5.5 \frac{1}{3} = 2.5 \]
Proof of Claim 3

- If type $\omega_L$ of A2 reports truthfully (both $\omega^2$ and $s_2^2$), regardless of $s_2^2$,
  \[ \frac{2}{3} u^{A2}(x_3, y_1, \omega_L) + \frac{1}{3} u^{A2}(x_4, y_2, \omega_L) = \frac{3}{3} + 7.5 \frac{1}{3} = 4.5 \]

- If type $\omega_L$ of A2 truthfully reports $\omega^2$ but misreports $s_2^2$, regardless of $s_2^2$,
  \[ \frac{2}{3} u^{A2}(x_4, y_1, \omega_L) + \frac{1}{3} u^{A2}(x_3, y_2, \omega_L) = 3.5 \]

- If type $\omega_L$ of A2 misreports $\omega^2$ but truthfully reports $s_2^2$, regardless of $s_2^2$,
  \[ \frac{2}{3} u^{A2}(x_2, y_1, \omega_L) + \frac{1}{3} u^{A2}(x_1, y_2, \omega_L) = 5.2 + 3.5 \frac{1}{3} = \frac{8.5}{3} \]

- If type $\omega_L$ of A2 misreports both $\omega^2$ and $s_2^2$, regardless of $s_2^2$,
  \[ \frac{2}{3} u^{A2}(x_1, y_1, \omega_L) + \frac{1}{3} u^{A2}(x_2, y_2, \omega_L) = \frac{2}{3} + 8 \frac{1}{3} = \frac{10}{3} \]
Proof of Claim 3

- If type $\omega_H$ of A2 reports truthfully (both $\omega^2$ and $s^2_2$), regardless of $s^2_2$,
  \[
  \frac{2}{3} u^{A2}(x_2, y_1, \omega_H) + \frac{1}{3} u^{A2}(x_1, y_2, \omega_H) = 9 \frac{2}{3} + 5 \frac{1}{3} = \frac{23}{3}
  \]

- If type $\omega_H$ of A2 truthfully reports $\omega^2$ but misreports $s^2_2$, regardless of $s^2_2$,
  \[
  \frac{2}{3} u^{A2}(x_1, y_1, \omega_H) + \frac{1}{3} u^{A2}(x_2, y_2, \omega_H) = 6
  \]

- If type $\omega_H$ of A2 misreports $\omega^2$ but truthfully reports $s^2_2$, regardless of $s^2_2$,
  \[
  \frac{2}{3} u^{A2}(x_3, y_1, \omega_H) + \frac{1}{3} u^{A2}(x_4, y_2, \omega_H) = 7 \frac{2}{3} + 9 \frac{1}{3} = \frac{23}{3}
  \]

- If type $\omega_H$ of A2 misreports both $\omega^2$ and $s^2_2$, regardless of $s^2_2$,
  \[
  \frac{2}{3} u^{A2}(x_4, y_1, \omega_H) + \frac{1}{3} u^{A2}(x_3, y_2, \omega_H) = 6 \frac{2}{3} + 7 \frac{1}{3} = \frac{19}{3}
  \]
Indispensability of signals

- $G^M$: arbitrary game with standard mechanisms $\phi_j : M_j \rightarrow \Delta(A_j)$

Claim 4

No matter richness of $M$, there exists no PBE of $G^M$ supporting

\[
\begin{align*}
    z(\omega_L) &\equiv \frac{2}{3}(x_3, y_1) + \frac{1}{3}(x_4, y_2) \\
    z(\omega_H) &\equiv \frac{2}{3}(x_2, y_1) + \frac{1}{3}(x_1, y_2)
\end{align*}
\]

\[
\begin{array}{|c|c|c|}
\hline
& y_1 & y_2 \\
\hline
x_1 & \zeta, 4, 1 & \zeta, 8, 3.5 \\
\hline
x_2 & \zeta, 2, 5 & \zeta, 9, 8 \\
\hline
x_3 & 10, 3, 3 & \zeta, 5.5, 3.5 \\
\hline
x_4 & \zeta, 1, 3.5 & 10, 7.5, 7.5 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
& y_1 & y_2 \\
\hline
x_1 & \zeta, 1, 6 & 10, 7.5, 5 \\
\hline
x_2 & 10, 3, 9 & \zeta, 5.5, 6 \\
\hline
x_3 & \zeta, 8, 7 & \zeta, 4.5, 7 \\
\hline
x_4 & \zeta, 9, 6 & \zeta, 3, 9 \\
\hline
\end{array}
\]
Proof of Claim 4

- Let $\mu \in \Delta(\Phi_1 \times \Phi_2)$ and $\lambda = (\lambda^1, \lambda^2)$ continuation eq. for $G^M$

- Step 1: For $\mu$-almost all $\phi \in supp[\mu]$, $\lambda(\phi)$-almost all $(m_1, m_2)$,
  
  $$(\phi_1(m_1), \phi_2(m_2)) \in \overline{\text{Int}\Delta(X)} \times \overline{\text{Int}\Delta(Y)}$$

- deterministic response to messages

\[\omega^2 = \omega_L\]

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\[\omega^2 = \omega_H\]

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</table>
Proof of Claim 4

- **Step 2:** For $\mu$-almost all $\phi = (\phi_1, \phi_2)$, IC for A2 requires that

  \[
  \Pr(x_3, y_1|\omega_L; \phi, \lambda) = 1 - \Pr(x_4, y_2|\omega_L; \phi, \lambda) = \frac{2}{3}
  \]

  \[
  \Pr(x_2, y_1|\omega_H; \phi, \lambda) = 1 - \Pr(x_1, y_2|\omega_H; \phi, \lambda) = \frac{2}{3}
  \]

<table>
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</table>
**Proof of Claim 4**

**Step 3:** For $\mu$-almost all $\phi$, there exists no pair of behavioral strategies inducing

\[
\Pr(x_3, y_1|\omega_L; \phi, \lambda) = 1 - \Pr(x_4, y_2|\omega_L; \phi, \lambda) = \frac{2}{3}
\]

\[
\Pr(x_2, y_1|\omega_H; \phi, \lambda) = 1 - \Pr(x_1, y_2|\omega_H; \phi, \lambda) = \frac{2}{3}
\]

- messages $A_2$ sends in state $\omega_H$ must have no bite
  - else $\omega_L$ can pick distribution induced by $\omega_H$, invert correlation while preserving marginals and do strictly better
  - ...but then $A_1$ has profitable deviation

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Indispensability of signals

Proposition 2

There exist eq. allocations in games with private disclosures that cannot be supported as eq. allocations in any game with standard mechanisms.

No matter

- richness of $M$
- availability of public randomizing devices
Indispensability of signals

- Result extends to
  - correlation in choice of mechanisms
  - reciprocal mechanisms
  - correlation in agents’ messages

- It does not extend to
  - direct (but private) communication between principals
Role of private disclosures in Example 2

- Correlate principals’ decisions with agents’ private information, respecting IC
- Supplement absence of direct communication among principals
- Different from action recommendations
- Disclosures may but need not change agents’ beliefs
  - encrypted “keys”
Conclusions

- Private disclosures
  - irrelevant in
    - single principal
    - competing principals with single agent (common agency)
  - important role when multiple principals contract w. multiple agents

- Non-robustness of equilibria with standard mechanisms

- Anti-folk theorems: payoffs above min-max-min not sustainable in eq.

- Non-universality of standard mechanisms
Ongoing Work

- Canonical mechanisms?
  - canonical signal spaces?
  - canonical message spaces?
  - canonical extensive form?

- $\phi : M \to \Delta(A)$
- $\phi : S \times M \to \Delta(A)$
- $\phi : S \times M \times S \to \Delta(A)$
- $\phi : S \times M \times S \cdots M \to \Delta(A)$
THANKS!

\[ \phi : S \times M \rightarrow \Delta(A) \]