Persuasion in Global Games with Application to Stress Testing

Supplementary Material

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Abstract

This document contains proofs and additional results for the manuscript "Persuasion in Global Games with Application to Stress Testing". All numbered items (i.e., sections, subsections, lemmas, conditions, propositions, and equations) in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text. Please refer to the main text for notation and definitions. The notation and definitions here are the same as in the main text.

Section S1 contains the full proof of Theorem 3, accommodating for the possibility that the prior from which the fundamentals are drawn as well as the distribution from which the agents' signals are drawn have bounded support (the proof in the main text confines attention to the unbounded case). Section S2 contains the proof of Example 2 in the main text, whereas Section S3 contains the proof of Example 3 (these examples establish sub-optimality of monotone rules when Condition M in the main text is violated). Section S4 discusses the role played by the multiplicity of the receivers and by their exogenous private information. Section S5 extends Theorem 1 in the main text to a class of economies in which (a) agents' prior beliefs need not be consistent with a common prior, nor be generated by signals drawn independently across agents, conditionally on θ, (b) the number of agents is arbitrary (in particular, finitely many agents), (c) payoffs can be heterogenous across agents, (d) agents have a level-K degree of sophistication, (e) the policy maker may possess imperfect information about the payoff state and/or the agents' beliefs, (f) the policy maker may disclose different information to different agents. Finally, Section S5 discusses the benefits to discriminatory disclosures, when the latter are feasible.

Section S1: Proof of Theorem 3 in the Main Text

We restate the result here for convenience.

Theorem 3. Suppose Condition M holds. Given any policy Γ, there exists a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ satisfying the perfect-coordination property and yielding the policy maker a payoff weakly higher than Γ. The policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ is defined by a threshold $\hat{\theta} \in [0,1]$ such that, for any $\theta \leq \hat{\theta}$, $\pi^{\hat{\theta}}(\theta)$ assigns probability one to $s = 0$, whereas for any $\theta > \hat{\theta}$, $\pi^{\hat{\theta}}(\theta)$ assigns probability one to $s = 1$.

Proof of Theorem 3. Without loss of generality, assume that the policy $\Gamma = (\mathcal{S}, \pi)$ (a) is a (possibly stochastic) "pass/fail"policy (i.e., $\mathcal{S} = \{0,1\}$, with $\pi(1|\theta) = 1 - \pi(0|\theta)$ denoting the probability that signal $s = 1$ is disclosed when the fundamentals are θ), (b) is such that $\pi(1|\theta) = 0$ for all $\theta \leq 0$ and $\pi(1|\theta) = 1$ for all $\theta > 1$, and (c) satisfies the perfect-coordination property. Theorems 1 and 2 imply that, if Γ does not satisfy these properties, there exists another policy Γ' that satisfies these properties and yields the policy maker a payoff weakly higher than Γ. The proof then follows from applying the arguments below to Γ' instead of Γ.

Suppose that Γ is such that there exists no $\hat{\theta}$ such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$.[1] We establish the result by showing that there exists a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ satisfying the perfect-coordination property that yields the policy maker a payoff strictly higher than Γ.

Recall that, for the policy Γ to satisfy the perfect-coordination property, it must be that, when the policy discloses the signal $s = 1$, $U^{\Gamma}(x, 1|x) > 0$ for all x such that $(x, 1)$ are mutually consistent, where $U^{\Gamma}(x, 1|x)$ is the expected payoff of an agent with signal x who hears that $s = 1$ and who expects all other agents to follow a cut-off policy with cut-off x.

Now let $\mathbb{G}$ denote the set of policies $\Gamma' = (\mathcal{S}, \pi')$ that, in addition to properties (a) and (b) above, are such that $U^{\Gamma'}(x, 1|x) \geq 0$ for all x such that $(x, 1)$ are mutually consistent. Observe that some policies Γ' in $\mathbb{G}$ need not satisfy the perfect-coordination property (namely, those for which there exists x such that $(x, 1)$ are mutually consistent and $U^{\Gamma'}(x, 1|x) = 0$). For any Γ, let $\mathcal{U}^P[\Gamma]$ denote the policy maker's ex-ante expected payoff under MARP consistent with the policy Γ. Denote by $\arg\max_{\tilde{\Gamma} \in \mathbb{G}} \left\{ \mathcal{U}^P[\tilde{\Gamma}] \right\}$ the set of policies that maximize the policy maker's payoff over the set $\mathbb{G}$.[2]

Step 1 below shows that any $\Gamma \in \arg\max_{\tilde{\Gamma} \in \mathbb{G}} \left\{ \mathcal{U}^P[\tilde{\Gamma}] \right\}$ is such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$

[1] Clearly, if the policy $\Gamma = (\{0,1\}, \pi)$ is such that there does exists $\hat{\theta} \in [0,1]$ such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi(1|\theta) = 1$ for F-almost all $\theta \geq \hat{\theta}$, then the deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ with cut-off $\hat{\theta}$ (that is, the policy such that $\pi^{\hat{\theta}}(1|\theta) = \mathbb{I}(\theta > \hat{\theta})$ for all θ) also satisfies the perfect-coordination property and yields the policy maker the same payoff as Γ, in which case the result trivially holds.

[2] That $\arg\max_{\tilde{\Gamma} \in \mathbb{G}} \left\{ \mathcal{U}^P[\tilde{\Gamma}] \right\} \neq \emptyset$ follows from the compactness of $\mathbb{G}$ and the upper hemi-continuity of $\mathcal{U}^P$ over $\mathbb{G}$.

and $\pi(1|\theta) = 1$ for F-almost all $\theta > \theta^*$, with

$$\theta^* \equiv \inf \left\{ \hat{\theta} \geq 0 : \int_{\hat{\theta}}^{\infty} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) dF(\tilde{\theta}) \geq 0 \text{ for all } \theta \in \left[\hat{\theta}, 1\right) \right\}.$$

We establish the result by showing that, given any policy $\Gamma' \in \mathbb{G}$ for which there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'. This property, together with the fact that any policy $\Gamma' = (\{0,1\}, \pi')$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$, for some $\hat{\theta}$, belongs to $\mathbb{G}$ only if $\hat{\theta} \in [\theta^*, 1]$ then gives the result.

Step 2 then shows that the policy maker's payoff under the optimal deterministic monotone policy $\Gamma^{\theta^*} = (\{0,1\}, \pi^{\theta^*})$ with cut-off θ^* can be approximated arbitrarily well by a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}}) \in \mathbb{G}$ that satisfies the perfect-coordination property (i.e., such that $U^{\Gamma^{\hat{\theta}}}(x, 1|x) > 0$ for all x such that $(x, 1)$ are mutually consistent), thus establishing the result in the theorem.

Step 1. Take any policy $\Gamma' \in \mathbb{G}$ for which there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$. Let

$$X^{\Gamma'} \equiv \left\{ x : (x, 1)\ \Gamma'\text{-mutually consistent and } U^{\Gamma'}(x, 1|x) = 0 \right\}.$$

Clearly, if $X^{\Gamma'} = \emptyset$, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'.[3] Thus, assume that $X^{\Gamma'} \neq \emptyset$, and let[4]

$$\bar{x} \equiv \sup X^{\Gamma'}.$$

The proof below is based on 3 claims. Claim shows that, given any policy $\Gamma' \in \mathbb{G}$ for which $X^{\Gamma'} \neq \emptyset$, the set $\{\theta \in \Theta(\bar{x}) : \pi'(1|\theta) < 1\}$ has strict positive F-measure. Claim then shows that, given any $\Gamma' \in \mathbb{G}$ for which the posterior beliefs of the marginal agent with signal $\bar{x}$ differ from those obtained by Bayes rule conditioning on the event that fundamentals are above some threshold $\hat{\theta}$, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'. Finally, Claim shows that, under the properties in Condition M, the only policies $\Gamma' \in \mathbb{G}$ that generate posterior beliefs for the marginal agent with signal $\bar{x}$ equal to those obtained from Bayes rule by conditioning on the event that fundamentals are above some threshold $\hat{\theta}$ are such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 0$ for F-almost all $\theta > \hat{\theta}$.

Given any x, let $\theta_0(x)$ be the fundamental threshold below which the agents' expected payoff differential is negative and above which it is positive, when all agents follow a cut-off strategy with

[3] To see this, note that, because there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$, and because $X^{\Gamma'} = \emptyset$, there must exists a set $(\theta', \theta'') \subseteq [0, 1]$ of F-positive measure over which $\pi'(1|\theta) < 1$. The policy Γ'' can then be obtained from Γ' by increasing $\pi'(1|\theta)$ over such a set. Provided the increase is small, the new policy is such that $U^{\Gamma''}(x, 1|x) \geq 0$ for all x such that $(x, 1)$ are mutually (consistent under Γ''), and hence $\Gamma'' \in \mathbb{G}$. Because $U^P(\theta, 1) > U^P(\theta, 0)$ over $[0, 1]$, the new policy improves over the original one.

[4] Clearly, $\bar{x}$ depends on the policy Γ'. We do not highlight the dependence to ease the notation.

cut-off x.[5] For any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}$, let $p^{\Gamma}(x,1) \equiv \int_{-\infty}^{+\infty} \pi(1|\theta)p(x|\theta)dF(\theta)$ denote the joint probability density of the exogenous signal x and the endogenous signal $s = 1$.

Claim S1-A. *For any $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}$ such that $X^{\Gamma'} \neq \emptyset$, $\{\theta \in \Theta(\bar{x}) : \pi'(1|\theta) < 1\}$ has strict positive F-measure.*

Proof of Claim S1-A. Suppose that $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$. Property 1 in Condition M then implies that $\bar{x} > \bar{x}_G$, where

$$\bar{x}_G \equiv \sup\left\{x \in \mathbb{R} : \int_{\Theta} u(\theta, 1 - P(x|\theta))\, \mathbb{I}(\theta \geq 0) p(x|\theta)\, dF(\theta) \leq 0\right\}. \tag{S3}$$

In fact, if this was not the case, the monotonicity of $\Theta(\cdot)$ would imply that

$$\inf \Theta(\bar{x}) \leq \inf \Theta(\bar{x}_G) < 0.$$

That $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$ would then imply that $\pi'(1|\theta) = 1$ for a set of fundamentals $\theta < 0$ of strictly positive F-measure, which is inconsistent with the assumption that $\Gamma' \in \mathbb{G}$. Thus, necessarily, $\bar{x} > \bar{x}_G$.

Now suppose that $\inf \Theta(\bar{x}) \geq 0$. That $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$ means that, from the perspective of an agent with signal $\bar{x}$, the information conveyed by the announcement that $s = 1$ under the policy Γ' is the same as under the monotone deterministic policy $\Gamma^0 = \left(\{0,1\}, \pi^0\right)$ with cut-off $\hat{\theta} = 0$. As a result, $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = U^{\Gamma^0}(\bar{x}, 1|\bar{x})$. Because $\bar{x} > \bar{x}_G$, and because, by definition of $\bar{x}_G$, $U^{\Gamma^0}(x, 1|x) > 0$ for all $x > \bar{x}_G$, it must be that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) > 0$, which contradicts the assumption that $\bar{x} \in X^{\Gamma'}$. Hence, it must be that $\inf \Theta(\bar{x}) < 0$. But then it cannot be that $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$, for, as explained above, this would imply that $\pi'(1|\theta) = 1$ for a set of fundamentals $\theta < 0$ of strictly positive F-measure, which is inconsistent with the assumption that $\Gamma' \in \mathbb{G}$. $\square$ Next, for any $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}$, let

$$\theta_H \equiv \sup\left\{\theta \in \Theta : \exists \delta > 0 \,\text{s.t.}\, \pi'(1|\theta') < 1 \,\text{for}\, F\text{-almost all}\, \theta' \in [\theta - \delta, \theta)\right\}.$$

That $\{\theta \in \Theta(\bar{x}) : \pi'(1|\theta) < 1\}$ has strict positive F-measure guarantees that θ_H is such that $\theta_H > \inf \Theta(\bar{x})$.

Claim S1-B. *Take any $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}$ such that $X^{\Gamma'} \neq \emptyset$. Suppose that*

$$\left\{\theta \in (\inf \Theta(\bar{x}), \theta_H) : \pi'(1|\theta) > 0\right\} \text{ has strict positive } F\text{-measure.} \tag{S1}$$

Then, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'.

Proof of Claim S1-B. The proof below distinguishes two cases.

Case 1: $\inf \Theta(\bar{x}) < \theta_0(\bar{x}) < \theta_H$.

Consider the policy $\Gamma^{\epsilon,\delta} = (\{0,1\}, \pi^{\epsilon,\delta})$ defined by $\pi^{\epsilon,\delta}(1|\theta) = \pi'(1|\theta)$ for all $\theta \leq \theta_0(\bar{x} + \delta)$, with $\delta > 0$ small so that $\theta_0(\bar{x} + \delta) < \theta_H$, and $\pi^{\epsilon,\delta}(1|\theta) = \min\{\pi'(1|\theta) + \epsilon, 1\}$ for all $\theta > \theta_0(\bar{x} + \delta)$, with

[5] When the default outcome is a function of A and θ only, $\theta_0(x)$ coincides with the threshold below which default occurs and above which it does not occur, when agents follow a cut-off strategy with cut-off x.

$\epsilon > 0$ also small. To see that, when ϵ and δ are small, $\Gamma^{\epsilon,\delta} \in \mathbb{G}$, note that, by definition of $\theta_0(\cdot)$, for any x, and any $\theta > \theta_0(x)$, $u(\theta, 1 - P(x|\theta)) > 0$. This property, together with the monotonicity of $\theta_0(\cdot)$, jointly imply that, for any $x \leq \bar{x} + \delta$,

$$\begin{aligned}
&\int_{-\infty}^{\theta_0(\bar{x}+\delta)} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta)+ \\
&+ \int_{\theta_0(\bar{x}+\delta)}^{+\infty} u(\theta, 1 - P(x|\theta)) \min\{\pi'(1|\theta) + \epsilon, 1\})p(x|\theta)dF(\theta) \\
&\geq \int_{-\infty}^{\theta_0(\bar{x}+\delta)} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta)+ \\
&+ \int_{\theta_0(\bar{x}+\delta)}^{+\infty} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta).
\end{aligned} \tag{S2}$$

The inequality follows from the fact that, for any $\theta > \theta_0(\bar{x} + \delta)$, when $x \leq \bar{x} + \delta$, $u(\theta, 1 - P(x|\theta)) > 0$. Because $\Gamma' \in \mathbb{G}$, the right-hand side of (S2) is non-negative.[6] Hence, for any $x \leq \bar{x} + \delta$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\delta}$, because the left-hand side of (S2) is equal to $U^{\Gamma^{\epsilon,\delta}}(x, 1|x)p^{\Gamma^{\epsilon,\delta}}(x, 1)$ and because, for such x, $p^{\Gamma^{\epsilon,\delta}}(x, 1) > 0$, we have that $U^{\Gamma^{\epsilon,\delta}}(x, 1|x) \geq 0$. That $U^{\Gamma^{\epsilon,\delta}}(x, 1|x) \geq 0$ also for all $x > \bar{x} + \delta$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\delta}$ follows from the fact that, by definition of $\bar{x}$, for any $x \geq \bar{x} + \delta$, the function

$$J(x) \equiv \int_{-\infty}^{+\infty} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta)$$

is bounded away from 0, along with the fact that, for any $\delta > 0$, the function family $\left(J^{\epsilon,\delta}(\cdot)\right)_\epsilon$ whose elements $J^{\epsilon,\delta}(\cdot)$ are given by

$$J^{\epsilon,\delta}(x) \equiv \int_{-\infty}^{+\infty} u(\theta, 1 - P(x|\theta))\pi^{\epsilon,\delta}(1|\theta)p(x|\theta)dF(\theta),$$

is continuous in ϵ in the sup-norm in a neighborhood of 0.[7]

Because the new policy $\Gamma^{\epsilon,\delta} \in \mathbb{G}$ so constructed is such that $\pi^{\epsilon,\delta}(1|\theta) \geq \pi'(1|\theta)$ for all θ with the inequality strict over a set of fundamentals $\theta < 1$ of strict F-positive measure, the policy maker's payoff under $\Gamma^{\epsilon,\delta}$ is strictly higher than under Γ', as claimed.

Case 2: $\inf \Theta(\bar{x}) < \theta_H \leq \theta_0(\bar{x})$.

Consider the monotone deterministic policy $\Gamma^0 = \left\{\{0, 1\}, \pi^0\right\}$ with cut-off $\hat{\theta} = 0$ (i.e., such that

[6] To see this, note that either $(x, 1)$ are not mutually consistent under Γ', in which case the right-hand side of (S2) is zero, or they are mutually consistent, in which case the right-hand side of (S2) is equal to $U^{\Gamma'}(x, 1|x)p^{\Gamma'}(x, 1)$, which is non-negative because $p^{\Gamma'}(x, 1) > 0$ and $U^{\Gamma'}(x, 1|x) \geq 0$.

[7] This means means that, for any $z > 0$, there exists $\Delta > 0$ such that, for any $0 < \epsilon < \Delta$, $|J^{\epsilon,\delta}(x) - J(x)| \leq z$ for all $x \geq \bar{x} + \delta$.

$\pi^0(1|\theta) \equiv \mathbb{I}(\theta > 0))$. Note that, for any $x \geq \bar{x}$,

$$\begin{aligned}
& \int_0^{\theta_0(x)} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta) + \\
& + \int_{\theta_0(x)}^{+\infty} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta) \\
\geq \; & \int_0^{\theta_0(x)} u(\theta, 1 - P(x|\theta))p(x|\theta)dF(\theta) + \\
& + \int_{\theta_0(x)}^{+\infty} u(\theta, 1 - P(x|\theta))p(x|\theta)dF(\theta)
\end{aligned}$$

where the inequality follows from (i) the fact that, for any $x \geq \bar{x}$ and any $\theta \leq \theta_0(\bar{x})$, $u(\theta, 1 - P(x|\theta)) < 0$, along with (ii) the fact that, by definition of θ_H, $\pi'(1|\theta) = 1$ for F-almost all $\theta \geq \theta_0(x) \geq \theta_0(\bar{x}) \geq \theta_H$. Furthermore, when $x = \bar{x}$, the above inequality is strict and, because $p^{\Gamma^0}(\bar{x}, 1) > p^{\Gamma'}(\bar{x}, 1) > 0$, it implies that

$$U^{\Gamma^0}(\bar{x}, 1|\bar{x}) \quad < \quad U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0.$$

By continuity of $U^{\Gamma^0}(x, 1|x)$ in x, we thus have that $\bar{x} < \bar{x}_G$. This property in turn permits us to apply part (3) of Condition M to $\bar{x}$ in the arguments below.

Next, let

$$\theta_L \equiv \inf\{\theta \in \Theta : \exists \delta > 0 \, \text{s.t.} \, \pi'(1|\theta') > 0 \, \text{for} \, F\text{-almost all} \, \theta' \in [\theta, \theta + \delta)\}.$$

That $\theta_L < \theta_H$ follows from the assumption that $\{\theta \in (\inf \Theta(\bar{x}), \theta_H) : \pi'(1|\theta) > 0\}$ has strict positive F-measure. Furthermore, $u(\theta_L, 1 - P(\bar{x}|\theta_L)) < 0$.[8] Also observe that $\inf \Theta(\bar{x}) < \theta_L$. This follows from the fact that, as shown above, $\bar{x} < \bar{x}_G$, which, together with Property 1 in Condition M, implies that $\inf \Theta(\bar{x}) < 0$. Because $\theta_L \geq 0$, we thus have that $\inf \Theta(\bar{x}) < \theta_L$.

For any $\gamma > 0$, let $\theta_L^\gamma \equiv \theta_L + \gamma$ and $\theta_H^\gamma \equiv \theta_H - \gamma$. Pick $\gamma, e_L, e_H > 0$ small such that (i) $\pi'(1|\theta_L^\gamma) > 0$ and $\pi'(1|\theta) > 0$ for $F-$almost $\theta \in (\theta_L^\gamma, \theta_L^\gamma + e_L)$, (ii) $\pi'(1|\theta_H^\gamma) < 1$ and $\pi'(1|\theta) < 1$ for $F-$almost all $\theta \in (\theta_H^\gamma - e_H, \theta_H^\gamma)$, and (iii) $\theta_L^\gamma + e_L < \theta_H^\gamma - e_H$.[9] Next, pick $\eta \in (0, \bar{x}_G - \bar{x})$ small such that $U^{\Gamma'}(x, 1|x) > \eta$ for all $x \geq \bar{x} + \eta$. Pick $\epsilon > 0$ also small and let $\delta(\epsilon, \eta)$ be implicitly defined by

$$\int_{\theta_L^\gamma}^{\theta_L^\gamma + \epsilon} u(\theta, 1 - P(\bar{x} + \eta|\theta))\pi'(1|\theta)p(\bar{x} + \eta|\theta)dF(\theta) = \int_{\theta_H^\gamma - \delta(\epsilon, \eta)}^{\theta_H^\gamma} u(\theta, 1 - P(\bar{x} + \eta|\theta))(1 - \pi'(1|\theta))p(\bar{x} + \eta|\theta)dF(\theta) \tag{S4}$$

Note that, for $\epsilon > 0$ small, $\theta_L^\gamma + \epsilon < \theta_H^\gamma - \delta(\varepsilon, \eta)$. Consider the policy $\Gamma^{\epsilon,\gamma,\eta} = \{\{0, 1\}, \pi^{\epsilon,\gamma,\eta}\}$ defined by the following properties: (a) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = \pi'(1|\theta)$ for all $\theta \notin \{[\theta_L^\gamma, \theta_L^\gamma + \epsilon] \cup [\theta_H^\gamma - \delta(\epsilon, \eta), \theta_H^\gamma]\}$; (b)

[8] That $u(\theta_L, 1 - P(\bar{x}|\theta_L)) < 0$ follows from the fact that, by definition of $\bar{x}$ and θ_L, $\int_{\theta_L}^{+\infty} u(\theta, 1 - P(\bar{x}|\theta))\pi'(1|\theta)p(\bar{x}|\theta)dF(\theta) = 0$, together with the single-crossing property of $u(\theta, 1 - P(\bar{x}|\theta))$ in θ.

[9] If a single γ satisfying properties (i)-(iii) does not exist, let $\gamma = (\gamma_L, \gamma_H) \in \mathbb{R}^2_{++}$. The arguments below then apply verbatim by letting $\theta_L^\gamma = \theta_L + \gamma_L$ and $\theta_H^\gamma = \theta_H + \gamma_H$ and noting that a $\gamma = (\gamma_L, \gamma_H)$ satisfying properties (i)-(iii) always exists.

$\pi^{\epsilon,\gamma,\eta}(1|\theta) = 0$ for all $\theta \in [\theta_L^\gamma, \theta_L^\gamma + \epsilon]$; and (c) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = 1$ for all $\theta \in [\theta_H^\gamma - \delta(\epsilon,\eta), \theta_H^\gamma]$. Note that Condition (S4) implies that $U^{\Gamma^{\epsilon,\gamma,\eta}}(\bar{x}+\eta, 1|\bar{x}+\eta) = U^{\Gamma'}(\bar{x}+\eta, 1|\bar{x}+\eta) > 0$.

We now show that, under the new policy, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \geq 0$ for any x such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$.

Clearly, for any (ϵ, γ, η), and any $x \leq x^*(\theta_L)$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$ (alternatively, under Γ') $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) > 0$ (alternatively, $U^{\Gamma'}(x, 1|x) > 0$). This is because, for any such x, $\theta_0(x) < \theta_L$ implying that $u(\theta, 1 - P(x|\theta)) > 0$ for all $\theta > \theta_L$. The result then follows from the fact that, under both Γ' and $\Gamma^{\epsilon,\gamma,\eta}$,

$$\int_{-\infty}^{\theta_L} \pi^{\epsilon,\gamma,\eta}(1|\theta)\, dF(\theta) = \int_{-\infty}^{\theta_L} \pi'(1|\theta)\, dF(\theta) = 0,$$

meaning that all agents assign probability one to the event that $\theta \geq \theta_L$.

Furthermore, the continuity of

$$\int_{\theta_L}^{+\infty} u(\theta, 1 - P(x|\theta))\, p(x|\theta)\, \pi'(1|\theta)\, dF(\theta)$$

in x, along with the fact that

$$\int_{\theta_L}^{+\infty} u(\theta, 1 - P(x|\theta))\, p(x|\theta)\, \pi'(1|\theta)\, dF(\theta) > \eta$$

for all $x \geq \bar{x} + \eta$ such that $(x, 1)$ are mutually consistent under Γ' imply that there exists $\xi > 0$ such that, for any $x \in [x^*(\theta_L), x^*(\theta_L) + \xi] \cup [\bar{x} + \eta, +\infty)$, if $(x, 1)$ are mutually consistent under Γ', then $U^{\Gamma'}(x, 1|x)p^{\Gamma'}(x, 1) > \xi$.

Now let $S^{\Gamma^{\epsilon,\gamma,\eta}}(\cdot)$ be the function defined by

$$S^{\Gamma^{\epsilon,\gamma,\eta}}(x) \equiv \int_{\theta_L}^{+\infty} u(\theta, 1 - P(x|\theta))\, p(x|\theta)\, \pi^{\epsilon,\gamma,\eta}(1|\theta)\, dF(\theta).$$

Note that, for any η, the function family $\left(S^{\Gamma^{\epsilon,\gamma,\eta}}(\cdot)\right)_{\epsilon,\gamma}$ is continuous in (γ, ϵ) in the sup-norm, in a neighborhood of $(0, 0)$[10] and $x^*(\theta)$ is continuous in θ. Hence, there exist $\bar{\gamma}, \bar{\epsilon} > 0$ such that, when $\gamma \leq \bar{\gamma}$ and $\epsilon \leq \bar{\epsilon}$, for any $x \in (-\infty, x^*(\theta_L^\gamma + \epsilon)] \cup [\bar{x} + \eta, +\infty)$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \geq 0$.

Next observe that, for any $x \in \left(x^*(\theta_L^\gamma + \epsilon), x^*\left(\theta_H^\gamma - \delta(\epsilon,\eta)\right)\right]$,

$$\begin{aligned} &- \textstyle\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u(\theta, 1 - P(x|\theta))\, p(x|\theta)\, f(\theta)\pi'(1|\theta)\, d\theta \\ &+ \textstyle\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u(\theta, 1 - P(x|\theta))\, p(x|\theta)\, f(\theta)\,(1 - \pi'(1|\theta))\, d\theta \geq 0, \end{aligned}$$

where the inequality follows from the fact that the integrand in the first integral is non-positive, whereas that in the second integral is non-negative. Hence, for any such x, if $(x, 1)$ are mutually

[10] This means that, for any $z > 0$, there exists $\Delta > 0$ such that, for any (ϵ, γ) with $0 < \epsilon < \Delta$ and $0 < \gamma < \Delta$, and all x, $|S^{\Gamma^{\epsilon,\gamma,\eta}}(x) - S^{\Gamma^{0,0,\eta}}(x)| \leq z$, where $\Gamma^{0,0,\eta} = \Gamma'$.

consistent under Γ', meaning that

$$p^{\Gamma'}(x,1)=\int_{\theta_L}^{+\infty}p\left(x|\theta\right)\pi'\left(1|\theta\right)dF(\theta)>0,$$

and are also mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, meaning that

$$p^{\Gamma^{\epsilon,\gamma,\eta}}(x,1)=\int_{\theta_L}^{+\infty}p\left(x|\theta\right)\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)dF(\theta)>0,$$

it must be that $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)\geq 0$. This is because, for any such x,

$$U^{\Gamma'}(x,1|x)p^{\Gamma'}(x,1)=\int_{\theta_L}^{+\infty}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)\pi'\left(1|\theta\right)dF(\theta)\geq 0$$

and

$$\begin{aligned}&U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)p^{\Gamma^{\epsilon,\gamma,\eta}}(x,1)\\=&U^{\Gamma'}(x,1|x)p^{\Gamma'}(x,1)-\textstyle\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)f(\theta)\pi'\left(1|\theta\right)d\theta+\\&+\textstyle\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)f(\theta)\left(1-\pi'\left(1|\theta\right)\right)d\theta.\end{aligned}$$

If, instead, for any such x, $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$ but not under Γ', then

$$\begin{aligned}U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)p^{\Gamma^{\epsilon,\gamma,\eta}}(x,1)&=\textstyle\int_{\theta_L}^{+\infty}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)dF(\theta)\\&=\textstyle\int_{\theta_L}^{+\infty}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)\pi'\left(1|\theta\right)dF(\theta)\\&\quad-\textstyle\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)\pi'\left(1|\theta\right)dF\left(\theta\right)\\&\quad+\textstyle\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)\left(1-\pi'\left(1|\theta\right)\right)dF\left(\theta\right)\\&=\textstyle\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma}u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)\left(1-\pi'\left(1|\theta\right)\right)dF\left(\theta\right)\geq 0\end{aligned}$$

where the first equality follows from the fact $p^{\Gamma^{\epsilon,\gamma,\eta}}(x,1)>0$ along with the definition of $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)$, the second equality is by the construction of the policy $\Gamma^{\epsilon,\gamma,\eta}$, the third equality follows from the fact that

$$p^{\Gamma'}(x,1)=\int_{\theta_L}^{+\infty}p\left(x|\theta\right)\pi'\left(1|\theta\right)dF(\theta)=0$$

and the last inequality follows again from the fact that, when $x\in\left(x^*(\theta_L^\gamma+\epsilon),x^*\left(\theta_H^\gamma-\delta\left(\epsilon,\eta\right)\right)\right]$, the integrand function is non-negative. We thus conclude that, under the new policy, for any such x, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)\geq 0$.

Next, consider $x\in\left(x^*\left(\theta_H^\gamma-\delta\left(\epsilon,\eta\right)\right),x^*(\theta_H^\gamma)\right)$. For any (x,θ), let

$$\Delta S(x)\equiv\int_{\theta_L}^{+\infty}u\left(\tilde\theta,1-P\left(x|\tilde\theta\right)\right)p\left(x|\tilde\theta\right)\left(\pi^{\epsilon,\gamma,\eta}\left(1|\tilde\theta\right)-\pi'\left(1|\tilde\theta\right)\right)dF(\tilde\theta)$$

and

$$q\left(\theta,x\right)\equiv\left|u\left(\theta,1-P\left(x|\theta\right)\right)\right|p\left(x|\theta\right).$$

Note that, for any $x\in\left(x^*\left(\theta_H^\gamma-\delta\left(\epsilon,\eta\right)\right),x^*(\theta_H^\gamma)\right)$,

$$
\begin{aligned}
\Delta S(x) &= \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\eta)} -u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&\quad+\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_0(x)} -u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&\quad+\int_{\theta_0(x)}^{\theta_H^\gamma} -u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&\geq \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\eta)} \frac{q\left(\theta,x\right)}{q\left(\theta,\bar{x}+\eta\right)}q\left(\theta,\bar{x}+\eta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&\quad+\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_0(x)} \frac{q\left(\theta,x\right)}{q\left(\theta,\bar{x}+\eta\right)}q\left(\theta,\bar{x}+\eta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&\quad+\frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)}\int_{\theta_0(x)}^{\theta_H^\gamma} q\left(\theta,\bar{x}+\eta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&\geq \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)}\int_{\theta_L^\gamma}^{\theta_H^\gamma} q\left(\theta,\bar{x}+\eta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
&= \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)}\Delta S(\bar{x}+\eta) \\
&= 0.
\end{aligned}
$$

The first equality is by definition. The first inequality follows from the fact that (i) for any $\theta\leq\theta_0(x)$, $u\left(\theta,1-P\left(x|\theta\right)\right)<0$, whereas, for any $\theta>\theta_0(x)$, $u\left(\theta,1-P\left(x|\theta\right)\right)>0$, along with the fact that, (ii) for $\theta\in[\theta_0(x),\theta_H^\gamma]$, $\pi'\left(1|\theta\right)\leq\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)$. Together, these two properties imply that

$$
\begin{aligned}
&\int_{\theta_0(x)}^{\theta_H^\gamma} -u\left(\theta,1-P\left(x|\theta\right)\right)p\left(x|\theta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta \\
\geq 0 \geq\ &\frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)}\int_{\theta_0(x)}^{\theta_H^\gamma} q\left(\theta,\bar{x}+\eta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta.
\end{aligned}
$$

The second inequality follows from the fact that, $\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)$ turns from positive to negative at $\theta=\theta_H^\gamma-\delta(\epsilon,\eta)\leq\theta_0(x)$, along with the fact that, for any $\theta\in\left[\theta_L^\gamma,\theta_0\left(x\right)\right]$, the function $q(\theta,x)/q(\theta,\bar{x}+\eta)$ is non-increasing in θ as implied by the log-supermodularity of $\left|u\left(\theta,1-P\left(x|\theta\right)\right)\right|p\left(x|\theta\right)$ over

$$
\left\{(\theta,x)\in[0,1]\times\mathbb{R}:u(\theta,1-P(x|\theta))\leq 0\right\}.
$$

Finally, the last two equalities follow from the fact that $\theta_0(\bar{x}+\eta)>\theta_0(\bar{x})>\theta_H\geq\theta_H^\gamma$, which implies that $u(\theta,1-P(\bar{x}+\eta|\theta))\leq 0$ for all $\theta\leq\theta_H^\gamma$, and hence that

$$
\int_{\theta_L^\gamma}^{\theta_H^\gamma} q\left(\theta,\bar{x}+\eta\right)f(\theta)\left(\pi'\left(1|\theta\right)-\pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right)d\theta=\Delta S(\bar{x}+\eta)
$$

along with the fact that, by construction of the policy $\Gamma^{\epsilon,\gamma,\eta}$, $\Delta S(\bar{x}+\eta)=0$. Hence, for any $x\in\left(x^*\left(\theta_H^\gamma-\delta\left(\epsilon,\eta\right)\right),x^*(\theta_H^\gamma)\right)$, $\Delta S(x)\geq 0$, which implies that, for any x such that $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)\geq 0$.

Similar arguments imply that, for any $x \in \left[x^*(\theta_H^\gamma), \overline{x}+\eta\right]$,

$$\begin{aligned}
\Delta S(x) &= \int_{\theta_L^\gamma}^{\theta_H^\gamma} -u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) f(\theta)\left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\
&= \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\gamma)} \frac{q\left(\theta, x\right)}{q\left(\theta, \bar{x}+\eta\right)} q\left(\theta, \bar{x}+\eta\right) f(\theta)\left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\
&\quad + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} \frac{q\left(\theta, x\right)}{q\left(\theta, \bar{x}+\eta\right)} q\left(\theta, \bar{x}+\eta\right) f(\theta)\left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\
&\geq \frac{q(\theta_H^\gamma - \delta(\epsilon,\eta), x)}{q(\theta_H^\gamma - \delta(\epsilon,\eta), \bar{x}+\eta)} \Delta S(\bar{x}+\eta) = 0,
\end{aligned}$$

which implies that, for such x too, if $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, then $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \geq 0$.

Together, the results above thus imply that, when ϵ, γ, η are small, the new policy $\Gamma^{\epsilon,\gamma,\eta} \in \mathbb{G}$.

We now show that, when property 3 in Condition M holds, the new policy yields the policy maker an expected payoff strictly higher than Γ'. To see this, observe that, fixing (γ,η), for any $\epsilon > 0$, the policy maker's payoff under the policy $\Gamma^{\epsilon,\gamma,\eta}$ is equal to

$$\begin{aligned}
\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}] &= \int_{-\infty}^{\theta_L^\gamma+\epsilon} U^P(\theta,0)dF(\theta) + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} U^P(\theta,1)dF(\theta) \\
&+ \int_{(\theta_L^\gamma+\epsilon,\theta_H^\gamma-\delta(\epsilon,\eta))\cup(\theta_H^\gamma,+\infty)} \left\{\pi'(1|\theta)U^P(\theta,1) + (1-\pi'(1|\theta))U^P(\theta,0)\right\} dF(\theta).
\end{aligned}$$

Differentiating $\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]$ with respect to ϵ, and taking the limit as $\epsilon \to 0^+$, we have that

$$\begin{aligned}
\lim_{\epsilon\to 0^+} \tfrac{d\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]}{d\epsilon} &= f(\theta_H^\gamma)(1-\pi'(1|\theta_H^\gamma))\left[U^P(\theta_H^\gamma,1) - U^P(\theta_H^\gamma,0)\right]\left(\lim_{\epsilon\to 0^+} \tfrac{\partial\delta(\epsilon,\eta)}{\partial\epsilon}\right) \\
&\quad - f(\theta_L^\gamma)\pi'(1|\theta_L^\gamma)\left[U^P(\theta_L^\gamma,1) - U^P(\theta_L^\gamma,0)\right] \\
= f(\theta_L^\gamma)\pi'(1|\theta_L^\gamma) &\left(\left[U^P(\theta_H^\gamma,1) - U^P(\theta_H^\gamma,0)\right] \tfrac{p(\bar{x}+\eta|\theta_L^\gamma)u(\theta_L^\gamma,1-P(\bar{x}+\eta|\theta_L^\gamma))}{p(\bar{x}+\eta|\theta_H^\gamma)u(\theta_H^\gamma,1-P(\bar{x}+\eta|\theta_H^\gamma))} - \left[U^P(\theta_L^\gamma,1) - U^P(\theta_L^\gamma,0)\right]\right).
\end{aligned}$$

Therefore, $\lim_{\epsilon\to 0^+} \frac{d\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]}{d\epsilon} > 0$ if and only if

$$\frac{U^P\left(\theta_H^\gamma,1\right) - U^P\left(\theta_H^\gamma,0\right)}{U^P\left(\theta_L^\gamma,1\right) - U^P\left(\theta_L^\gamma,0\right)} > \frac{p\left(\bar{x}+\eta|\theta_H^\gamma\right) u\left(\theta_H^\gamma, 1-P\left(\bar{x}+\eta|\theta_H^\gamma\right)\right)}{p\left(\bar{x}+\eta|\theta_L^\gamma\right) u\left(\theta_L^\gamma, 1-P\left(\bar{x}+\eta|\theta_L^\gamma\right)\right)}.$$

Property 3 in Condition M, together with the fact that $\bar{x} \leq \bar{x}_G$ (as proved above), guarantee this is the case. We conclude that the policy $\Gamma^{\epsilon,\gamma,\eta} \in \mathbb{G}$ yields the policy maker a payoff strictly higher than Γ'. This completes the proof of Claim S1-B. □

Claim S1-C. *Suppose that Condition M holds and that* $\Gamma' \in \mathbb{G}$ *is such that*

$$\left\{\theta \in \left(\inf \mathit{\Theta}\left(\bar{x}\right), \theta_H\right) : \pi'(1|\theta) > 0\right\} \text{ has zero } F\text{-measure.} \tag{S5}$$

Then $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$.

Proof of Claim S1-C. First observe that Condition (S5), together with the definition of θ_H and the fact that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$, jointly imply that $\theta_H < \sup \Theta(\bar{x})$ and that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = U^{\Gamma^{\theta_H}}(\bar{x}, 1|\bar{x})$, where $\Gamma^{\theta_H} = \left\{\{0,1\}, \pi^{\theta_H}\right\}$ is the monotone deterministic policy with cut-off θ_H, that is, such that $\pi^{\theta_H}(1|\theta) = \mathbb{I}\{\theta > \theta_H\}$ for all θ.[11] In other words, from the perspective of an agent with signal $\bar{x}$, the information learned, under the policy Γ', by the announcement that $s = 1$ is the same as the one learnt under the deterministic monotone policy with cut-off θ_H.

Now suppose that $\theta_H > \theta^*$. For any deterministic monotone policy $\Gamma^{\hat{\theta}} = \left(\{0,1\}, \pi^{\hat{\theta}}\right)$, any any $\tilde{\theta} \geq \hat{\theta}$ let

$$\varphi\left(\tilde{\theta}; \hat{\theta}\right) \equiv \int_{\hat{\theta}}^{\sup \Theta\left(x^*(\tilde{\theta})\right)} u\left(\theta, 1 - P\left(x^*\left(\tilde{\theta}\right)|\theta\right)\right) p\left(x^*\left(\tilde{\theta}\right)|\theta\right) dF(\theta)$$

and

$$\bar{\varphi}\left(\hat{\theta}\right) \equiv \inf_{\tilde{\theta} \geq \hat{\theta}} \varphi\left(\tilde{\theta}; \hat{\theta}\right).$$

Note that, for any $\tilde{\theta}$ such that $(x^*(\tilde{\theta}), 1)$ are mutually consistent under the policy $\Gamma^{\hat{\theta}}$, then

$$\varphi\left(\tilde{\theta}; \hat{\theta}\right) = U^{\Gamma^{\hat{\theta}}}\left(x^*\left(\tilde{\theta}\right), 1|x^*\left(\tilde{\theta}\right)\right) p^{\Gamma^{\hat{\theta}}}\left(x^*\left(\tilde{\theta}\right), 1\right)$$

We claim that, for any $\hat{\theta} > \theta^*$, $\bar{\varphi}\left(\hat{\theta}\right) > 0$. To see this, consider first the case where $\hat{\theta} \in \arg\min_{\tilde{\theta} \geq \hat{\theta}} \varphi\left(\tilde{\theta}; \hat{\theta}\right)$. Observe that, if each agent follows a threshold strategy with cut-off $x^*\left(\hat{\theta}\right)$, then default occurs only for fundamentals weakly below $\hat{\theta}$. Because $u\left(\theta, 1 - P\left(x^*\left(\hat{\theta}\right)|\theta\right)\right) > 0$ for all $\theta > \hat{\theta}$ and because $p\left(x^*\left(\hat{\theta}\right)|\theta\right) > 0$ in a right-neighborhood of $\hat{\theta}$, then necessarily $\bar{\varphi}\left(\hat{\theta}\right) = \varphi\left(\hat{\theta}; \hat{\theta}\right) > 0$. Next, suppose that $\hat{\theta} \notin \arg\min_{\tilde{\theta} \geq \hat{\theta}} \varphi\left(\tilde{\theta}; \hat{\theta}\right)$. Then observe that, for almost any $\hat{\theta} \geq \theta^*$, and any $\tilde{\theta}_m \in \arg\min_{\tilde{\theta} \geq \hat{\theta}} \varphi\left(\tilde{\theta}; \hat{\theta}\right)$ with $\tilde{\theta}_m > \hat{\theta}$,[12]

$$\frac{\partial \varphi\left(\tilde{\theta}_m; \hat{\theta}\right)}{\partial \hat{\theta}} = -u\left(\hat{\theta}, 1 - P\left(x^*\left(\tilde{\theta}_m\right)|\hat{\theta}\right)\right) p\left(x^*\left(\tilde{\theta}_m\right)|\hat{\theta}\right) f\left(\hat{\theta}\right) \geq 0,$$

where the inequality follows from the fact that $u\left(\hat{\theta}, 1 - P\left(x^*\left(\tilde{\theta}_m\right)|\hat{\theta}\right)\right) < 0$ which, in turn, is a consequence of (i) the definition of $x^*(\cdot)$ and (ii) the fact that $\tilde{\theta}_m > \hat{\theta}$.

By the definition of θ^*, $\bar{\varphi}(\theta^*) = 0$, and $d\bar{\varphi}(\theta^*)/d\hat{\theta} > 0$. The above properties thus imply that, for any $\hat{\theta} > \theta^*$, $\bar{\varphi}\left(\hat{\theta}\right) > 0$, as claimed.

[11] If $\theta_H \geq \sup \Theta(\bar{x})$ then $p^{\Gamma'}(\bar{x}, 1) \equiv \int p(\bar{x}|\theta)\pi'(1|\theta)dF(\theta) = 0$ contradicting the assumption that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$ which requires that $(x, 1)$ are mutually consistent under Γ'.

[12] Note that $\varphi\left(\tilde{\theta}; \hat{\theta}\right)$ is absolutely continuous in $\hat{\theta}$, and therefore is differentiable in $\hat{\theta}$ almost everywhere.

Now recall that, by the definition of $\bar{x}$, $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$. Under Condition (S5), this implies that, when agents pledge for $x > \bar{x}$ and refrain from pledging for $x < \bar{x}$, the default outcome $\theta_0(\bar{x})$ must necessarily satisfy $\theta_0(\bar{x}) > \theta_H$, for, otherwise, an agent with signal $\bar{x}$ would strictly prefer pledging to not pledging. Because $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = U^{\Gamma^{\theta_H}}(\bar{x}, 1|\bar{x})$, that $\theta_0(\bar{x}) > \theta_H > \theta^*$, along with the fact that $\varphi(\theta_0(\bar{x}); \theta_H) > 0$, however, implies that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) > 0$, a contradiction.

Hence, it must be that $\theta_H \leq \theta^*$. However, by definition of θ^*, if $\theta_H < \theta^*$, then there exists $\theta > \theta_H$ such that $(x^*(\theta), 1)$ are mutually consistent under Γ^{θ_H} (that is, $p^{\Gamma^{\theta_H}}(x^*(\theta), 1) > 0$) and such that

$$U^{\Gamma^{\theta_H}}(x^*(\theta), 1|x^*(\theta)) = \frac{\int_{\theta_H}^{\sup \Theta(x^*(\theta))} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right)}{p^{\Gamma^{\theta_H}}(x^*(\theta), 1)} < 0.$$

Now note that

$$U^{\Gamma'}(x^*(\theta), 1|x^*(\theta)) = \frac{\int_{\inf \Theta(x^*(\theta))}^{\theta_H} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right)\pi'(1|\tilde{\theta}) p\left(x^*(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right)}{p^{\Gamma'}(x^*(\theta), 1)}$$

$$+ \frac{\int_{\theta_H}^{\sup \Theta(x^*(\theta))} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right)}{p^{\Gamma'}(x^*(\theta), 1)}$$

with

$$\begin{aligned} p^{\Gamma'}(x^*(\theta), 1) = \int_{\inf \Theta(x^*(\theta))}^{\theta_H} \pi'(1|\tilde{\theta}) p\left(x^*(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right) + \\ + \int_{\theta_H}^{\sup \Theta(x^*(\theta))} p\left(x^*(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right) \end{aligned}$$

$$= \int_{\inf \Theta(x^*(\theta))}^{\theta_H} \pi'(1|\tilde{\theta}) p\left(x^*(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right) + p^{\Gamma^{\theta_H}}(x^*(\theta), 1) > 0.$$

Because, for any $\tilde{\theta} < \theta_H$, $u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) < 0$, we thus have that

$$U^{\Gamma'}(x^*(\theta), 1|x^*(\theta)) < 0.$$

But this contradict the assumption that $\Gamma' \in \mathbb{G}$.

We thus conclude that necessarily $\theta_H = \theta^*$. Furthermore, because $\{\theta \in (\inf \Theta(\bar{x}), \theta_H) : \pi'(1|\theta) > 0\}$ has 0 F-measure, it must be that

$$U^{\Gamma'}(\bar{x}, 1|\bar{x}) = U^{\Gamma^{\theta^*}}(\bar{x}, 1|\bar{x}).$$

Furthermore, because $\theta_0(\bar{x}) > \theta^*$, we also have that $U^{\Gamma^0}(\bar{x}, 1|\bar{x}) \leq U^{\Gamma^{\theta^*}}(\bar{x}, 1|\bar{x}) = 0$. Hence, $\bar{x} \leq \bar{x}_G$, which by virtue of Property 1 in Condition M implies that $\inf \Theta(\bar{x}) \leq 0$. Condition (S5), along with the fact that $\pi'(1|\theta) = 0$ for all $\theta \leq 0$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta_H = \theta^*$, thus imply that Γ' is such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$. This completes the proof of Claim S1-C. □

Step 2. The results in Step 1 imply that $\arg\max_{\tilde{\Gamma} \in \mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\} \neq \emptyset$ and that any $\Gamma' \in \arg\max_{\tilde{\Gamma} \in \mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\}$ is such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$. The result in the theorem then follows from observing that, given any $\Gamma' \in \arg\max_{\tilde{\Gamma}\in\mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\}$, there exists a nearby deterministic monotone policy $\Gamma^{\hat{\theta}} \in \mathbb{G}$ with cut-off $\hat{\theta} = \theta^* + \tilde{\varepsilon}$, for $\tilde{\varepsilon} > 0$ but small, such that $\Gamma^{\hat{\theta}}$ satisfies the perfect-coordination property (i.e., $U^{\Gamma^{\hat{\theta}}}(x, 1|x) > 0$ all x such that $(x, 1)$ are mutually consistent under $\Gamma^{\hat{\theta}}$) and yields the policy maker a payoff arbitrarily close to that under Γ'. This completes the proof of the theorem. Q.E.D.

Section S2: Proof of Example 2 in the main text

We restate the result here for convenience.

Example 2. Suppose that there exist scalars $g, b, W, L \in \mathbb{R}$, with $g > 0 > b$ and $W > L$ such that, for any θ, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$, and $L(\theta) = L$. Suppose that θ is drawn from a uniform distribution on $[-K, 1+K]$. Finally, assume that the agents' exogenous signals are given by $x_i = \theta + \sigma\epsilon_i$, with $\sigma \in (0, K/2)$ and with each ϵ_i drawn independently across agents from a uniform distribution over $[-1, 1]$. Let

$$\theta^*_\sigma \equiv \inf\left\{\hat{\theta} : \int_{\hat{\theta}}^{\infty} u\left(\tilde{\theta}, 1 - P_\sigma\left(x^*_\sigma(\theta)|\tilde{\theta}\right)\right) p_\sigma\left(x^*_\sigma(\theta)|\tilde{\theta}\right) dF\left(\tilde{\theta}\right) \geq 0 \text{ for all } \theta \in \left[\hat{\theta}, 1\right)\right\}.$$

[13] There exists $\sigma^\# \in (0, K/2)$ such that, for all $\sigma \in (0, \sigma^\#)$, when the quality of the agents' exogenous signals is parametrized by σ, starting from the optimal deterministic monotone policy with cut-off θ^*_σ, one can construct a *non-monotone* policy that satisfies PCP and saves banks over a set of fundamentals of strictly larger Lebesgue measure[14] (and hence yields the policy maker a payoff strictly higher than under the optimal deterministic monotone rule).[15]

Proof Example 2. The proof is in two steps. Step 1 characterizes the threshold θ^*_σ defining the optimal deterministic monotone rule, whereas Step 2 constructs the non-monotone policy that strictly improves over the optimal deterministic monotone one.

Step 1. Observe that the primitives in this example satisfy the conditions in Theorem 2 in the main text. This means that, given any signal s disclosed by any policy Γ, MARP is in threshold strategies, as shown in the proof of Theorem 2, which in turn implies that the default outcome is monotone in θ.

[13] $P_\sigma(\cdot|\theta)$ is the cdf of the agents' signals when the fundamentals are θ, $p_\sigma(\cdot|\theta)$ is the pdf associated with P_σ, F is the cdf of the common prior, $u(\theta, A)$ is the agents' payoff differential between pledging and not pledging when the fundamentals are θ and the size of the pledge is A, and $x^*_\sigma(\theta)$ is the threshold on the agents' exogenous signals such that, when each agent pledges when $x > x^*_\sigma(\theta)$ and does not pledge when $x < x^*_\sigma(\theta)$, default occurs if fundamentals are below θ and does not occur if fundamentals are above θ. We highlight here the dependence of P, p, θ^*, and x^* on σ to avoid confusion in the derivations below.

[14] Because the prior F is uniform, that the new policy saves banks over a set of fundamentals of strictly larger Lebesgue measure than the optimal deterministic monotone rule implies that it saves banks over a set of fundamentals of larger ex-ante probability under F.

[15] Recall that the monotone policy with cut-off θ^*_σ is the one saving banks over the largest set of fundamentals over all deterministic monotone rules.

Next recall that, for any default threshold $\theta \in [0,1]$, the signal threshold $x_\sigma^*(\theta)$ is such that, when each agent pledges when $x > x_\sigma^*(\theta)$ and does not pledge when $x < x_\sigma^*(\theta)$, default occurs if fundamentals are below θ and does not occur if fundamentals are above θ. The value of $x_\sigma^*(\theta)$ is implicitly defined by

$$P_\sigma\left(x_\sigma^*(\theta)\,|\theta\right) = \theta.$$

Using the fact that, for any $\theta \in [-K, 1+K]$ and $x \in [\theta-\sigma, \theta+\sigma]$, $P_\sigma(x|\theta) = (x-\theta+\sigma)/2\sigma$, we have that

$$x_\sigma^*(\theta) = (1+2\sigma)\,\theta - \sigma.$$

For any threshold $\hat{\theta} \in [0,1]$, let $\Gamma^{\hat{\theta}} \equiv \left\{\{0,1\}, \pi^{\hat{\theta}}\right\}$ be the *deterministic monotone policy* with cutoff $\hat{\theta}$. That is, for any $\theta \in [-K, 1+K]$, $\pi^{\hat{\theta}}(1|\theta)$ is given by

$$\pi^{\hat{\theta}}(1|\theta) \equiv \mathbb{I}\left(\theta \geq \hat{\theta}\right),$$

where $\mathbb{I}\left(\theta \geq \hat{\theta}\right)$ is the indicator function taking value 1 if $\theta \geq \hat{\theta}$ and taking value 0 if $\theta < \hat{\theta}$.

Next, for any $\theta \in [\hat{\theta}/(1+2\sigma), 1]$, let $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) \equiv U_\sigma^{\Gamma^{\hat{\theta}}}\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)$ be the expected payoff differential between pledging and not pledging of the marginal agent with signal $x_\sigma^*(\theta)$, when all agents with signal above $x_\sigma^*(\theta)$ pledge and all agents with signal below $x_\sigma^*(\theta)$ refrain from pledging (and hence default occurs for fundamentals below θ and does not occur for fundamentals above θ), the quality of the agents' signal is σ, and the the policy $\Gamma^{\hat{\theta}}$ announces that $s=1$, thus revealing that $\theta \geq \hat{\theta}$. Note that, for any $0 \leq \theta < \hat{\theta}/(1+2\sigma)$, $x_\sigma^*(\theta) + \sigma < \hat{\theta}$, which implies that the signal $x_\sigma^*(\theta)$ is not consistent with the event that fundamentals are above $\hat{\theta}$. Equivalently, when $\theta \geq \hat{\theta}$, the lowest possible signal that an individual may receive is $\hat{\theta} - \sigma$. When each agent pledges for $x > \hat{\theta} - \sigma$ and does not pledge for $x < \hat{\theta} - \sigma$, default occurs for $\theta \leq \hat{\theta}/(1+2\sigma)$ and does not occur for $\theta > \hat{\theta}/(1+2\sigma)$. Hence, the lowest default threshold that is consistent with the policy $\Gamma^{\hat{\theta}}$ is $\hat{\theta}/(1+2\sigma)$. The function $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta)$ is thus defined only for $\theta \in [\hat{\theta}/(1+2\sigma), 1]$.

Equipped with this notation, note that the cutoff θ_σ^* characterizing the optimal deterministic monotone policy is given by

$$\theta_\sigma^* = \inf\left\{\hat{\theta} \in [0,1] : V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) \geq 0 \;\text{ for all } \theta \in [\hat{\theta}/(1+2\sigma), 1]\right\}. \tag{S6}$$

Claim S2. *For any threshold $\hat{\theta} \in [0,1]$, $V_\sigma^{\Gamma^{\hat{\theta}}}(\cdot)$ has a unique minimizer. Letting $\theta_\sigma^{\min}\left(\hat{\theta}\right) \equiv \arg\min_{\theta \in [\hat{\theta}/(1+2\sigma), 1]} V_\sigma^{\Gamma^{\hat{\theta}}}(\theta)$, we have that $\theta_\sigma^{\min}\left(\hat{\theta}\right)$ satisfies $x_\sigma^*\left(\theta_\sigma^{\min}\left(\hat{\theta}\right)\right) - \sigma = \hat{\theta}$.*

Proof of Claim S2. Clearly, for any $\theta \in [\hat{\theta}/(1+2\sigma), \hat{\theta}]$, $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) = g$. This is because when each agent pledges when $x > x_\sigma^*(\theta)$ and does not pledge when $x < x_\sigma^*(\theta)$ default occurs only for fundamentals below θ. Hence the announcement that fundamentals are above $\hat{\theta}$ reveals to the marginal agent with signal $x_\sigma^*(\theta)$ that default will not occur.

Next, observe that for any $\theta \in \left(\hat{\theta}, \left(\hat{\theta}+2\sigma\right)/(1+2\sigma)\right]$, $x_\sigma^*(\theta) - \sigma < \hat{\theta}$, implying that[16]

$$\begin{aligned} V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \geq \hat{\theta}; x_\sigma^*(\theta)\right\} \\ &= g - (g+|b|)\left(\frac{\theta - \hat{\theta}}{x_\sigma^*(\theta) + \sigma - \hat{\theta}}\right) \\ &= g - (g+|b|)\left(\frac{\theta - \hat{\theta}}{(1+2\sigma)\,\theta - \hat{\theta}}\right), \end{aligned}$$

which is strictly decreasing in θ.

Finally, note that, for any $\theta \in \left(\left(\hat{\theta}+2\sigma\right)/(1+2\sigma), 1\right]$, $x_\sigma^*(\theta) - \sigma > \hat{\theta}$, implying that

$$\begin{aligned} V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \geq \hat{\theta}; x_\sigma^*(\theta)\right\} \\ &= g - (g+|b|)\left(\frac{\theta - (x_\sigma^*(\theta) - \sigma)}{2\sigma}\right) \\ &= g + (g+|b|)(\theta - 1), \end{aligned}$$

which is strictly increasing in θ. Hence, $V_\sigma^{\Gamma^{\hat{\theta}}}(\cdot)$ has a single minimizer over $[\hat{\theta}/(1+2\sigma), 1]$. The latter is equal to $\theta_\sigma^{min}\left(\hat{\theta}\right) = \left(\hat{\theta}+2\sigma\right)/(1+2\sigma)$ and is such that $x_\sigma^*\left(\theta_\sigma^{min}\left(\hat{\theta}\right)\right) - \sigma = \hat{\theta}$. □

Next, let $\Gamma^{\theta_\sigma^*} \equiv \left\{\{0,1\}, \pi^{\theta_\sigma^*}\right\}$ be the optimal deterministic monotone policy (the one with cut-off $\hat{\theta} = \theta_\sigma^*$). Using the characterization of θ_σ^* in (S6), we thus have that, under the policy $\Gamma^{\theta_\sigma^*}$, at the point $\theta_\sigma^{min}(\theta_\sigma^*)$ at which $V_\sigma^{\Gamma^{\theta_\sigma^*}}$ reaches its minimum, $V_\sigma^{\Gamma^{\theta_\sigma^*}}\left(\theta_\sigma^{min}(\theta_\sigma^*)\right) = 0$.

Using the fact that

$$V_\sigma^{\Gamma^{\theta_\sigma^*}}\left(\theta_\sigma^{min}(\theta_\sigma^*)\right) = g - (g+|b|)\left(\frac{\theta_\sigma^{min}(\theta_\sigma^*) - \theta_\sigma^*}{(1+2\sigma)\,\theta_\sigma^{min}(\theta_\sigma^*) - \theta_\sigma^*}\right)$$

we then have that

$$\theta_\sigma^* = (1+2\sigma)\frac{|b|}{g+|b|} - 2\sigma.$$

Next, let $\Gamma_\emptyset$ be the no-disclosure policy and note that, under such a policy, for any $\theta \in [0,1]$,

$$\begin{aligned} V_\sigma^{\Gamma_\emptyset}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | x_\sigma^*(\theta)\right\} \\ &= g - (g+|b|)\left(\frac{\theta - (x_\sigma^*(\theta) - \sigma)}{2\sigma}\right) \\ &= g + (g+|b|)(\theta - 1), \end{aligned}$$

which is increasing in θ and has a unique zero at

$$\theta = \frac{|b|}{g+|b|} \equiv \theta^{MS}.$$

[16] The notation $\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \geq \hat{\theta}; x\right\}$ stands for the probability that an agent with signal x assigns to the event that $\tilde{\theta} \leq \theta$ when the quality of his exogenous signal is parametrized by σ and the policy reveals that $\tilde{\theta} \geq \hat{\theta}$.

This means that, in the absence of any disclosure, under the unique rationalizable strategy profile (and hence under MARP), each agent pledges if $x > x^*_\sigma\left(\theta^{MS}\right)$ and does not pledge if $x < x^*_\sigma\left(\theta^{MS}\right)$, and default occurs if fundamentals are below θ^{MS} and does not occur if fundamentals are above θ^{MS}. The results above then imply that the optimal deterministic policy $\Gamma^{\theta^*_\sigma}$ is defined by a threshold

$$\theta^*_\sigma = (1+2\sigma)\,\theta^{MS} - 2\sigma = x^*_\sigma\left(\theta^{MS}\right) - \sigma$$

that coincides with the left end-point of the support of the posterior beliefs of each agent with signal $x^*_\sigma\left(\theta^{MS}\right)$. In fact, for any truncation point $\hat{\theta} < x^*_\sigma\left(\theta^{MS}\right) - \sigma$, there exists θ close to θ^{MS} such that $V^{\Gamma^{\hat{\theta}}}_\sigma(\theta) < 0$ implying that refraining from pledging for all $x < x^*_\sigma\left(\theta^{MS}\right)$ is rationalizable in the continuation game following the announcement that $\theta \geq \hat{\theta}$, implying that the policy $\Gamma^{\hat{\theta}}$ fails to satisfy PCP. To see this, note that the effects of the announcement that fundamentals are above $\hat{\theta}$ on the payoff of the marginal agent with signal $x^*_\sigma(\theta)$ are nil for any $\theta > \left(\hat{\theta} + 2\sigma\right)/(1+2\sigma)$. This is because, for any such θ, $x^*_\sigma(\theta) - \sigma > \hat{\theta}$, meaning that the marginal agent with signal $x^*_\sigma(\theta)$ already knows that fundamentals are above $x^*_\sigma(\theta) - \sigma > \hat{\theta}$ in the absence of any policy announcement and hence learns nothing new by hearing that the fundamentals are above $\hat{\theta}$. Because, in the absence of any public announcement, the payoff $V^{\Gamma_\emptyset}_\sigma(\theta)$ of the marginal agent with signal $x^*_\sigma(\theta)$ is negative for any $\theta < \theta^{MS}$, any deterministic policy with threshold $\hat{\theta} < x^*_\sigma\left(\theta^{MS}\right) - \sigma$ fails to guarantee that all agents pledge in the continuation game after the policy announces that fundamentals are above $\hat{\theta}$. Similarly, for any truncation point $\hat{\theta} > x^*_\sigma\left(\theta^{MS}\right) - \sigma$, $V^{\Gamma^{\hat{\theta}}}_\sigma(\theta)$ reaches its minimum at $\theta^{min}_\sigma\left(\hat{\theta}\right) > \theta^{MS}$ and is such that

$$V^{\Gamma^{\hat{\theta}}}_\sigma\left(\theta^{min}_\sigma\left(\hat{\theta}\right)\right) = V^{\Gamma_\emptyset}_\sigma\left(\theta^{min}_\sigma\left(\hat{\theta}\right)\right) > V^{\Gamma_\emptyset}_\sigma\left(\theta^{MS}\right) = 0,$$

where the inequality follows from the monotonicity of $V^{\Gamma_\emptyset}_\sigma(\cdot)$. Hence, $\theta^*_\sigma = x^*_\sigma\left(\theta^{MS}\right) - \sigma$.

Step 2. Having characterized the optimal deterministic monotone policy $\Gamma^{\theta^*_\sigma}$, we now show that, when σ is small, there exists another policy Γ that also satisfies PCP and guarantees that no default for a set of fundamentals that is a strict superset of those for which default does not occur under $\Gamma^{\theta^*_\sigma}$.

Let

$$\sigma^{\#} \equiv \frac{\theta^{MS}}{2(1-\theta^{MS})} > 0$$

and note that, for any $\sigma \in (0, \sigma^{\#})$, $\theta^*_\sigma = (1+2\sigma)\,\theta^{MS} - 2\sigma > 0$. For any $\sigma, \delta, \gamma > 0$ small, let

$$\theta''_\sigma(\delta,\gamma) \equiv x^*_\sigma\left(\theta^{MS} - \delta\right) - \sigma = (1+2\sigma)(\theta^{MS} - \delta) - 2\sigma$$

and

$$\theta'_\sigma(\delta,\gamma) \equiv \theta''_\sigma(\delta,\gamma) - \gamma.$$

Note that, for any $\sigma \in (0, \sigma^{\#})$, $\delta > 0$ and $\gamma > 0$ can be chosen so that $0 < \theta'{}_\sigma(\delta,\gamma) < \theta''_\sigma(\delta,\gamma) < \theta^*_\sigma$.

Consider the non-monotone deterministic policy $\Gamma_{\delta,\gamma} \equiv \{\{0,1\}, \pi_{\delta,\gamma}\}$ given by

$$\pi_{\delta,\gamma}(1|\theta) \equiv \mathbb{I}\left\{\theta \in \left[\theta'{}_\sigma(\delta,\gamma), \theta''_\sigma(\delta,\gamma)\right] \cup [\theta^*_\sigma, \infty)\right\}.$$

In what follows we show that, for any $\sigma \in (0, \sigma^{\#})$, there exit $\delta, \gamma > 0$ such that (i) $0 \leq \theta'_\sigma(\delta, \gamma) < \theta''_\sigma(\delta, \gamma) < \theta^*_\sigma$, and (ii) $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) \geq 0$ for all $\theta > \theta'_\sigma(\delta, \gamma)/(1 + 2\sigma)$, with $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) = 0$ only for $\theta = \theta^{MS}$.[17][18]

First observe that, for any $\sigma \in (0, \sigma^{\#})$,

$$\delta \in \left(0, \theta^{MS} - \frac{2\sigma}{1 + 2\sigma}\right)$$

and

$$0 < \gamma \leq \underbrace{(1 + 2\sigma)\left(\theta^{MS} - \delta\right) - 2\sigma}_{\equiv R_0(\delta, \theta^{MS}, \sigma)}$$

guarantee that $0 \leq \theta'_\sigma(\delta, \gamma) < \theta''_\sigma(\delta, \gamma) < \theta^*_\sigma$.[19]

Next note that, for any (σ, δ, γ) with $\sigma \in (0, \sigma^{\#})$, $\delta \in \left(0, \theta^{MS} - 2\sigma/(1 + 2\sigma)\right)$ and $0 < \gamma \leq R_0\left(\delta, \theta^{MS}, \sigma\right)$, $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) = V_\sigma^{\Gamma^{\theta^*_\sigma}}(\theta)$ for all $\theta \in \left[\theta^{MS} - \delta, 1\right]$. Indeed, for any $\theta \in \left[\theta^{MS} - \delta, 1\right]$, $x^*_\sigma(\theta) - \sigma > \theta''_\sigma(\delta, \gamma)$ implying that the the posterior beliefs of the marginal agent with signal $x^*_\sigma(\theta)$ under the policy $\Gamma_{\delta,\gamma}$ coincide with those under the policy $\Gamma^{\theta^*_\sigma}$. Formally,

$$\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \geq \theta^*_\sigma; x^*_\sigma(\theta)\right\} = \mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \in \left[\theta'_\sigma(\delta, \gamma), \theta''_\sigma(\delta, \gamma)\right] \cup [\theta^*_\sigma, \infty); x^*_\sigma(\theta)\right\}.$$

Let $\theta^\sharp_\sigma(\delta, \gamma)$ be such that $x^*_\sigma\left(\theta^\sharp_\sigma(\delta, \gamma)\right) - \sigma = \theta'_\sigma(\delta, \gamma)$. Dropping the arguments of $\theta^\sharp_\sigma(\delta, \gamma)$, $\theta'_\sigma(\delta, \gamma)$ and $\theta''_\sigma(\delta, \gamma)$ to ease the notation, we have that

$$\begin{aligned} \theta' &= \theta'' - \gamma \\ &= x^*_\sigma\left(\theta^{MS} - \delta\right) - \sigma - \gamma \\ &= (1 + 2\sigma)\left(\theta^{MS} - \delta\right) - 2\sigma - \gamma. \end{aligned}$$

From the definition of $\hat{\theta}$ we have that

$$x^*_\sigma\left(\hat{\theta}\right) - \sigma = (1 + 2\sigma)\,\theta^\sharp - 2\sigma = \theta'.$$

Combining the above two results we obtain that

$$\theta^\sharp = \theta^{MS} - \delta - \frac{\gamma}{1 + 2\sigma}. \tag{S7}$$

[17] Consistently with the notation above, $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta)$ is the expected payoff of the marginal agent with signal $x^*_\sigma(\theta)$ when the policy $\Gamma_{\delta,\gamma}$ announces that $s = 1$ and the quality of the agents' exogenous signals is parametrized by σ.

[18] For any $\theta < \theta'_\sigma(\delta, \gamma)/(1+2\sigma)$, $x^*_\sigma(\theta) + \sigma < \theta'$, which implies that the signal $x^*_\sigma(\theta)$ is not consistent with the event that fundamentals are above $\theta'_\sigma(\delta, \gamma)$. Equivalently, because the lowest signal that is consistent with $\theta \in [\theta'_\sigma(\delta, \gamma), \theta''_\sigma(\delta, \gamma)] \cup [\theta^*_\sigma, \infty)$ is $\theta'_\sigma(\delta, \gamma) - \sigma$, the lowest default threshold is $\theta'_\sigma(\delta, \gamma)/(1 + 2\sigma)$.

[19] Observe that $\sigma \in (0, \sigma^{\#})$ implies that $\theta^{MS} - 2\sigma/(1 + 2\sigma) > 0$. In turn, $\delta \in \left(0, \theta^{MS} - 2\sigma/(1 + 2\sigma)\right)$ implies that $0 < \theta''_\sigma(\delta, \gamma) < \theta^*_\sigma$ and that $R_0\left(\delta, \theta^{MS}, \sigma\right) > 0$. Finally, that $0 < \gamma \leq R_0\left(\delta, \theta^{MS}, \sigma\right)$ implies that $0 \leq \theta'_\sigma(\delta, \gamma) < \theta''_\sigma(\delta, \gamma)$.

Fixing $\sigma \in (0, \sigma^{\#})$, note that, for $\delta, \gamma > 0$ small, $\theta^{\sharp} \geq \theta^*_\sigma$. Specifically, for any $\sigma \in (0, \sigma^{\#})$ and any $0 < \delta < 2\sigma\left(1 - \theta^{MS}\right)$, $\theta^{\sharp} \geq \theta^*_\sigma$ if and only if

$$\gamma \leq \underbrace{(1+2\sigma)\left(2\sigma\left(1-\theta^{MS}\right)-\delta\right)}_{\equiv R_1(\delta, \theta^{MS}, \sigma)}.$$

Next, observe that, for any $\theta \in \left[\theta^{\sharp}, \theta^{MS} - \delta\right)$,

$$\begin{aligned} V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \in \left[x^*_\sigma(\theta) - \sigma, \theta''\right] \cup [\theta^*_\sigma, \infty); x^*_\sigma(\theta)\right\} \\ &= g - (g+|b|)\frac{\theta - \theta^*_\sigma + \theta'' - (x^*_\sigma(\theta) - \sigma)}{x^*_\sigma(\theta) + \sigma - \theta^*_\sigma + \theta'' - (x^*_\sigma(\theta) - \sigma)} \\ &= g - (g+|b|)\frac{\theta - \theta^*_\sigma + \theta'' - ((1+2\sigma)\theta - 2\sigma)}{(1+2\sigma)\theta - \theta^*_\sigma + \theta'' - ((1+2\sigma)\theta - 2\sigma)} \\ &= g - (g+|b|)\frac{\theta'' - \theta^*_\sigma + 2\sigma(1-\theta)}{\theta'' - \theta^*_\sigma + 2\sigma}, \end{aligned}$$

which is strictly increasing in θ.

Similarly, for any $\theta \in \left[\theta^*_\sigma, \theta^{\sharp}\right)$,

$$\begin{aligned} V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \in \left[\theta', \theta''\right] \cup [\theta^*_\sigma, \infty); x^*_\sigma(\theta)\right\} \\ &= g - (g+|b|)\frac{\theta - \theta^*_\sigma + \gamma}{x^*_\sigma(\theta) + \sigma - \theta^*_\sigma + \gamma} \\ &= g - (g+|b|)\frac{\theta - \theta^*_\sigma + \gamma}{(1+2\sigma)\theta - \theta^*_\sigma + \gamma}, \end{aligned}$$

which is strictly deceasing for any $\gamma \leq \theta^*_\sigma$. Note that $\theta' \geq 0$ requires that $\gamma \leq \theta^*_\sigma$.

Next, note that, for $\theta \in [\theta'', \theta^*_\sigma)$,

$$\begin{aligned} V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \in \left[\theta', \theta''\right] \cup [\theta^*_\sigma, \infty); x^*_\sigma(\theta)\right\} \\ &= g - (g+|b|)\frac{\gamma}{x^*_\sigma(\theta) + \sigma - \theta^*_\sigma + \gamma} \\ &= g - (g+|b|)\frac{\gamma}{(1+2\sigma)\theta - \theta^*_\sigma + \gamma}, \end{aligned}$$

and, therefore, $U_\sigma^{\Gamma_{\delta,\gamma}}(\cdot)$ is increasing over this range.

Finally, for $\theta \in [\theta', \theta'')$, we have that

$$\begin{aligned} V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) &= g - (g+|b|)\,\mathbb{P}_\sigma\left\{\tilde{\theta} \leq \theta | \tilde{\theta} \in \left[\theta', \theta''\right] \cup [\theta^*_\sigma, \infty); x^*_\sigma(\theta)\right\} \\ &= g - (g+|b|)\frac{\theta - \theta'}{x^*_\sigma(\theta) + \sigma - \theta^*_\sigma + \gamma} \\ &= g - (g+|b|)\frac{\theta - \theta'}{(1+2\sigma)\theta - \theta^*_\sigma + \gamma}. \end{aligned}$$

This implies that

$$\begin{aligned} \frac{d}{d\theta}V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) &= -(g+|b|)\frac{(1+2\sigma)\theta - \theta^*_\sigma + \gamma - (1+2\sigma)(\theta - \theta')}{\left((1+2\sigma)\theta - \theta^*_\sigma + \gamma\right)^2} \\ &= -(g+|b|)\frac{(1+2\sigma)\theta' - \theta^*_\sigma + \gamma}{\left((1+2\sigma)\theta - \theta^*_\sigma + \gamma\right)^2}. \end{aligned}$$

Hence $V_\sigma^{\Gamma_{\delta,\gamma}}(\cdot)$ is decreasing over $[\theta',\theta'')$ if

$$(1+2\sigma)\,\theta' = x_\sigma^*\left(\theta'\right)+\sigma > \theta_\sigma^*.$$

Using the fact that $\theta' = \theta'' - \gamma$, together with the fact that $\theta'' = x_\sigma^*\left(\theta^{MS}-\delta\right)-\sigma$ and $\theta_\sigma^* = (1+2\sigma)\,\theta^{MS}-2\sigma$, we have that $(1+2\sigma)\,\theta' > \theta_\sigma^*$ if

$$\begin{aligned}\gamma &< \frac{2\sigma\theta_\sigma^*}{(1+2\sigma)} - (1+2\sigma)\,\delta \\ &= \underbrace{\frac{2\sigma\left[(1+2\sigma)\,\theta^{MS}-2\sigma\right]}{(1+2\sigma)} - (1+2\sigma)\,\delta}_{\equiv R_2(\delta,\theta^{MS},\sigma)}.\end{aligned}$$

Lastly, observe that for any $\theta \in [\theta'/(1+2\sigma),\theta']$, $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) = g$.

We thus have that $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) \geq 0$ for all $\theta \geq \theta'/(1+2\sigma)$, with $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta) = 0$ only if $\theta = \theta^{MS}$, if and only if (a) $V_\sigma^{\Gamma_{\delta,\gamma}}\left(\theta^\sharp\right) > 0$, and (b) $V_\sigma^{\Gamma_{\delta,\gamma}}(\theta'') > 0$.

Requiring that $V_\sigma^{\Gamma_{\delta,\gamma}}\left(\theta^\sharp\right) > 0$ is equivalent to requiring that

$$g - (g+|b|)\,\frac{\theta^\sharp - \theta_\sigma^* + \gamma}{x_\sigma^*\left(\theta^\sharp\right)+\sigma-\theta_\sigma^*+\gamma} > 0.$$

This, in turn, is equivalent to

$$\begin{aligned}\frac{\theta^\sharp-\theta_\sigma^*+\gamma}{(1+2\sigma)\,\theta^\sharp-\theta_\sigma^*+\gamma} &< \frac{g}{g+|b|} = 1-\theta^{MS} \\ \Leftrightarrow \theta^\sharp-\theta_\sigma^*+\gamma &< \left(1-\theta^{MS}\right)\left((1+2\sigma)\,\theta^\sharp-\theta_\sigma^*+\gamma\right) \\ \Leftrightarrow 0 &< 2\sigma\theta^\sharp - \theta^{MS}\left((1+2\sigma)\,\theta^\sharp-\theta_\sigma^*+\gamma\right) \\ \Leftrightarrow \theta^{MS}\left((1+2\sigma)\,\theta^\sharp-\theta_\sigma^*+\gamma\right) &< 2\sigma\theta^\sharp \\ \Leftrightarrow \left((1+2\sigma)\,\theta^{MS}-2\sigma\right)\theta^\sharp &< \theta^{MS}\left(\theta_\sigma^*-\gamma\right).\end{aligned}$$

Recall that

$$\theta_\sigma^* = (1+2\sigma)\,\theta^{MS}-2\sigma.$$

Using the fact that $\theta^\sharp = \theta^{MS}-\delta-\frac{\gamma}{1+2\sigma}$, we conclude that a sufficient condition for $V_\sigma^{\Gamma_{\delta,\gamma}}\left(\theta^\sharp\right) > 0$ is that

$$\begin{aligned}\left(\theta^{MS}-\delta-\gamma/(1+2\sigma)\right)\theta_\sigma^* &< \theta^{MS}\left(\theta_\sigma^*-\gamma\right) \\ \Leftrightarrow \gamma\theta^{MS} &< \left(\delta+\gamma/(1+2\sigma)\right)\theta_\sigma^* \\ \Leftrightarrow \gamma\left(\theta^{MS}-\theta_\sigma^*/(1+2\sigma)\right) &< \delta\theta_\sigma^* \\ \Leftrightarrow \gamma &< \frac{\delta\theta_\sigma^*}{\left(\theta^{MS}-\theta_\sigma^*/(1+2\sigma)\right)} \\ \Leftrightarrow \gamma &< \frac{\delta\theta_\sigma^*(1+2\sigma)}{\left(\theta^{MS}(1+2\sigma)-\theta_\sigma^*\right)} \\ \Leftrightarrow \gamma &< \underbrace{\frac{\delta(1+2\sigma)\left((1+2\sigma)\,\theta^{MS}-2\sigma\right)}{2\sigma}}_{\equiv R_3(\delta,\theta^{MS},\sigma)}.\end{aligned}$$

Next, observe that $V_{\sigma}^{\Gamma_{\delta,\gamma}}(\theta'') > 0$ is equivalent to

$$\begin{aligned}
\frac{\gamma}{(1+2\sigma)\,\theta''-\theta_{\sigma}^{*}+\gamma} &< \frac{g}{g+|b|}=1-\theta^{MS} \\
\Leftrightarrow \frac{\gamma}{1-\theta^{MS}} &< (1+2\sigma)\,\theta''-\theta_{\sigma}^{*}+\gamma \\
\Leftrightarrow \frac{\gamma}{1-\theta^{MS}} &< (1+2\sigma)\left((1+2\sigma)\left(\theta^{MS}-\delta\right)-2\sigma\right)-(1+2\sigma)\,\theta^{MS}+2\sigma+\gamma \\
\Leftrightarrow \gamma &< \underbrace{\left(\frac{1-\theta^{MS}}{\theta^{MS}}\right)\left((1+2\sigma)\left[(1+2\sigma)\left(\theta^{MS}-\delta\right)-2\sigma\right]-(1+2\sigma)\,\theta^{MS}+2\sigma\right)}_{\equiv R_4(\delta,\theta^{MS},\sigma)}.
\end{aligned}$$

We conclude that, for any $\sigma\in(0,\sigma^{\#})$, (i) $0\le\theta'_{\sigma}(\delta,\gamma)<\theta''_{\sigma}(\delta,\gamma)<\theta_{\sigma}^{*}$, and (ii) $V_{\sigma}^{\Gamma_{\delta,\gamma}}(\theta)\ge 0$ for all $\theta>\theta'_{\sigma}(\delta,\gamma)/(1+2\sigma)$, with $V_{\sigma}^{\Gamma_{\delta,\gamma}}(\theta)=0$ only for $\theta=\theta^{MS}$, if

$$0<\delta<\min\left\{\theta^{MS}-\frac{2\sigma}{1+2\sigma},2\sigma\left(1-\theta^{MS}\right),\frac{2\sigma\left[(1+2\sigma)\,\theta^{MS}-2\sigma\right]}{(1+2\sigma)^{2}},\frac{2\sigma}{1+2\sigma}\left[\theta^{MS}-\frac{2\sigma}{1+2\sigma}\right]\right\}$$

and

$$0<\gamma<\min_{i\in\{0,1,2,3,4\}}R_{i}\left(\delta,\theta^{MS},\sigma\right).$$

Note that $\sigma<\sigma^{\#}$ implies that

$$\min\left\{\theta^{MS}-\frac{2\sigma}{1+2\sigma},2\sigma\left(1-\theta^{MS}\right),\frac{2\sigma\left[(1+2\sigma)\,\theta^{MS}-2\sigma\right]}{(1+2\sigma)^{2}},\frac{2\sigma}{1+2\sigma}\left[\theta^{MS}-\frac{2\sigma}{1+2\sigma}\right]\right\}>0$$

whereas

$$\delta<\min\left\{\theta^{MS}-\frac{2\sigma}{1+2\sigma},2\sigma\left(1-\theta^{MS}\right),\frac{2\sigma\left[(1+2\sigma)\,\theta^{MS}-2\sigma\right]}{(1+2\sigma)^{2}},\frac{2\sigma}{1+2\sigma}\left[\theta^{MS}-\frac{2\sigma}{1+2\sigma}\right]\right\}$$

implies that

$$\min_{i\in\{0,1,2,3,4\}}R_{i}\left(\delta,\theta^{MS},\sigma\right)>0.$$

Finally note that, for any $\sigma\in(0,\sigma^{\#})$, and any $\theta\ge\theta'_{\sigma}(\delta,\gamma)$, the payoff $V_{\sigma}^{\Gamma_{\delta,\gamma}}(\theta)$ is continuous in the threshold θ_{σ}^{*}. Hence there exists a policy Γ whose rule π is given by

$$\pi(1|\theta)\equiv\mathbb{I}\left\{\theta\in\left[\theta'_{\sigma}(\delta,\gamma),\theta''_{\sigma}(\delta,\gamma)\right]\cup[\theta_{\sigma}^{*}+\varepsilon,\infty)\right\}$$

with $\varepsilon>0$ arbitrarily small, such that Γ strictly improves over $\Gamma^{\theta_{\sigma}^{*}}$ and is such that $V_{\sigma}^{\Gamma}(\theta)>0$ for all $\theta>\theta'_{\sigma}(\delta,\gamma)/(1+2\sigma)$, implying that Γ satisfies PCP.[20]Q.E.D.

[20]That is, pledging is the unique rationalizable action when Γ announces that $s=1$ and not-pledging is rationalizable, for any x, when Γ announces that $s=0$.

Section S3: Proof of Example 3 in Main Text

In this section we provide the proof for Example 3 in the main text. We re-state the result for convenience. Let $\pi(\theta) = 1$ denote the distribution over $\{0, 1\}$ assigning probability one to $s = 1$ and $\pi(\theta) = 0$ the distribution assigning probability one one $s = 0$. Given any deterministic policy Γ (that is, any policy such that, for any θ, either $\pi(\theta) = 1$ or $\pi(\theta) = 0$), let $d \in D^{\Gamma}$ denote a generic element of the partition D^{Γ} of the interval $[0, \theta^{MS}]$, and, for any $\theta \in [\underline{\theta}, \theta^{MS}]$, denote by $d^{\Gamma}(\theta) \in D^{\Gamma}$ the partition cell that contains θ. Let $M(\Gamma) \equiv \max_{i=1,...,N} |\bar{\theta}_i - \underline{\theta}_i|$ denote the *mesh* of D^{Γ}, that is, the Lebesgue measure of the cell of D^{Γ} of maximal Lebesgue measure. Finally, because the policy is deterministic, we abuse notation and let $\pi(\theta) \in \{0, 1\}$ denote the Dirac measure assigning probability 1 to the signal $s = 0$ (alternatively, to the signal $s = 1$).

Example 3. *There exists a scalar $\bar{\sigma} > 0$ and a function $\mathcal{E} : (0, \bar{\sigma}] \to \mathbb{R}_+$, with $lim_{\sigma \to 0^+} \mathcal{E}(\sigma) = 0$, such that, for any $\sigma \in (0, \bar{\sigma}]$, in the game in which the noise in the agents' information is σ, the following is true: given any binary policy Γ satisfying the perfect-coordination property and such that $M(\Gamma) > \mathcal{E}(\sigma)$, there exists another binary policy Γ^* with $M(\Gamma^*) < \mathcal{E}(\sigma)$ that also satisfies the perfect-coordination property and such that the ex-ante probability of default under Γ^* is smaller than under Γ.*

Proof of Example 3. For any $\theta \in (0, 1)$, any $\sigma \in \mathbb{R}_+$, let $x^*_\sigma(\theta) \equiv \theta + \sigma\Phi^{-1}(\theta)$ denote the value of the private signal such that, when every agent $i \in [0, 1]$ pledges for $x_i > x^*_\sigma(\theta)$ and does not pledge for $x_i < x^*_\sigma(\theta)$, default occurs when the fundamentals fall below θ and does not occur when they are above θ.[21] Also let $x^*_\sigma(0) \equiv -\infty$ and $x^*_\sigma(1) \equiv +\infty$.

For any $(\theta_0, \hat{\theta}, \sigma) \in (0, 1) \times \mathbb{R} \times \mathbb{R}_+$, let $\psi(\theta_0, \hat{\theta}, \sigma)$ denote the payoff from pledging of an agent with private signal $x^*_\sigma(\theta_0)$, when default occurs for $\theta \leq \theta_0$ and does not occur for $\theta > \theta_0$, the policy reveals that $\theta \geq \hat{\theta}$, and the precision of private information is σ^{-2}. Then let

$$\hat{\sigma} \equiv \inf \{\sigma \in \mathbb{R}_+ : \ \psi(\theta_0, 0, \sigma) > 0 \ \text{ all } \ \theta_0 \in (0, 1)\}$$

if $\{\sigma \in \mathbb{R}_+ : \ \psi(\theta_0, 0, \sigma) > 0 \ \text{ all } \ \theta_0 \in (0, 1)\} \neq \emptyset$ and else $\hat{\sigma} = +\infty$.[22] Then let

$$\Psi(\sigma) \equiv \inf_{\theta_0 \in (0,1)} \psi(\theta_0, 0, \sigma)$$

and note that $\lim_{\sigma \to 0^+} \Psi(\sigma) < 0$, implying that $\hat{\sigma} > 0$. For any $\sigma \in \mathbb{R}_+$ for which $\psi(\theta_0, 0, \sigma) > 0$ for all $\theta_0 \in (0, 1)$, the policy-maker can avoid default for every $\theta \geq 0$ by using the monotone rule $\pi(\theta) = \mathbb{I}\{\theta > 0\}$ that fails all institutions with fundamentals $\theta \leq 0$ and passes the rest. This case

[21] Given that default occurs if and only if $A \leq 1 - \theta$, $x^*_\sigma(\theta)$ is implicitly defined by the solution to the equation $\Phi\left(\frac{x-\theta}{\sigma}\right) = \theta$. Hence, at θ, the measure of agents pledging (which coincides with the measure of agents receiving signals above $x^*_\sigma(\theta)$) is exactly equal to $1 - \theta$.

[22] Recall that, when the announcement that $s = 1$ reveals to the market that $\theta \geq 0$, the unique rationalizable profile features all agents pledging, irrespective of their private information, if and only if $\psi(\theta_0, 0, \sigma) > 0$ for all $\theta_0 \in (0, 1)$. This follows directly from Lemma 1 in the main text.

is uninteresting. Hereafter, we confine attention to $\sigma < \hat{\sigma}$, which guarantees that the policy maker's problem is not trivial.

Let $U_\sigma^\Gamma(x, 1|x)$ denote the payoff from pledging of an agent with signal x who expects all other agents to pledge if and only if their signal exceeds x , when the precision of private information is σ^{-2}, and the policy Γ announces that $s = 1$. Also let $U_\sigma^\Gamma\left(x_\sigma^*(0), 1|x_\sigma^*(0)\right) \equiv \lim_{x\to-\infty} U_\sigma^\Gamma(x, 1|x)$ and $U_\sigma^\Gamma\left(x_\sigma^*(1), 1|x_\sigma^*(1)\right) \equiv \lim_{x\to+\infty} U_\sigma^\Gamma(x, 1|x)$.

From the proofs of Theorems 1 and 2 in the main text, recall that a policy $\Gamma = (\{0, 1\}, \pi)$ satisfies the perfect coordination property only if, after signal $s = 1$ is disclosed, the unique rationalizable profile features all agents pledging, which is the case if and only if $U_\sigma^\Gamma(x, 1|x) > 0$ for all x.

Now let $\mathbb{G}_\sigma$ denote the set of binary policies $\Gamma = (\{0, 1\}, \pi)$ such that (a) $\pi(\theta) = 0$ for all $\theta \leq 0$, $\pi(\theta) = 1$ for all $\theta > 1$, and (b) for all $x \in \mathbb{R}$, $U_\sigma^\Gamma(x, 1|x) \geq 0$. From the proofs of Theorems 1 and 2 in the main text, given any σ, and any binary policy Γ' satisfying the perfect coordination property, there exists a binary policy $\Gamma \in \mathbb{G}_\sigma$ that also satisfies the perfect coordination property and such that the probability of default under Γ is weakly smaller than under Γ'. Hence, without loss of generality, hereafter we restrict attention to policies $\Gamma \in \mathbb{G}_\sigma$. However, note that the set $\mathbb{G}_\sigma$ contains also policies that do not satisfy the perfect coordination property.[23]

Proof Structure. The proof proceeds in four steps. Step 1 establishes that, when σ is small, any policy $\Gamma = (\{0, 1\}, \pi) \in \mathbb{G}_\sigma$ must have the property that any interval $(\theta', \theta''] \subset \left(0, \theta^{MS}\right]$ receiving a pass grade (i.e., such that $\pi(\theta) = 1$ for all $\theta \in (\theta', \theta'']$ has a sufficiently small Lebesgue measure, with the measure vanishing when $\sigma \to 0^+$. If this were not the case, for some $\theta \in (\theta', \theta'']$, we would have that $U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) < 0$, contradicting the fact that $\Gamma \in \mathbb{G}_\sigma$.

Step 2 then considers an *auxiliary game* G_σ in which the agents play less aggressively than in the original game. Namely, G_σ is the game in which (i) the policy maker's choice set is $\mathbb{G}_\sigma$ and (ii) given *any* policy $\Gamma \in \mathbb{G}_\sigma$, all agents pledge after receiving the signal $s = 1$ and refrain from pledging after receiving the signal $s = 0$, irrespective of their private information. By the definition of $\mathbb{G}_\sigma$, the agents' behavior is rationalizable. However, the above action profile is MARP only for those $\Gamma \in \mathbb{G}_\sigma$ for which, for all x, $U_\sigma^\Gamma(x, 1|x) > 0$. For those $\Gamma \in \mathbb{G}_\sigma$ for which there exists x such that $U_\sigma^\Gamma(x, 1|x) = 0$, instead, the above action profile is less aggressive than MARP. We show that, when σ is small, for any given any policy $\Gamma = (\{0, 1\}, \pi) \in \mathbb{G}_\sigma$ that gives a fail grade to an interval $(\theta', \theta''] \subseteq (\underline{\theta}, \theta^{MS}]$ of large Lebesgue measure, there exists another policy $\Gamma^\# \in \mathbb{G}_\sigma$ that gives a pass grade to a F-positive measure subset of $(\theta', \theta'']$, has a mesh smaller than Γ, and is such that, when agents play as in G_σ (that is, pledge irrespective of x when hearing that $s = 1$), the probability of default under $\Gamma^\#$ is strictly smaller than under Γ.

Step 3 then combines the results from Steps 1 and 2 to shows that, when σ is small, given any

[23]These are policies Γ for which there exists x such that $U_\sigma^\Gamma(x, 1|x) = 0$; when this is the case, in the continuation game that starts after the policy Γ announces $s = 1$, in addition to the rationalizable profile under which all agents pledge irrespectively of their signal, there also exists a rationalizable profile where each agent pledges if and only if his private signal exceeds x.

policy $\Gamma \in \mathbb{G}_\sigma$ for which the mesh $M(\Gamma)$ of $(0, \theta^{MS}]$ is larger than ε, there exists another policy $\Gamma' \in \mathbb{G}_\sigma$ with a mesh $M(\Gamma')$ smaller than ε such that, when agents play as in the auxiliary game G_σ, the probability of default is strictly smaller under Γ' than under Γ. Starting from $\Gamma' \in \mathbb{G}_\sigma$ one can then construct a "nearby" policy $\Gamma^* \in \mathbb{G}_\sigma$ such that the probability of default under Γ^* is arbitrarily close to that under Γ' (and hence strictly smaller than under Γ) and such that $U_\sigma^{\Gamma^*}(x, 1|x) > 0$ for all x. As shown in the proof of Theorem 2 in the main text, the last property implies that Γ^* satisfies the perfect coordination property: when Γ^* discloses the signal $s = 1$, the unique rationalizable profile features all agents pledging, irrespective of their private signals. The policy Γ^* thus improves upon Γ also in the original game, as claimed in the theorem.

Finally, step 4 closes the proof by showing how to construct the function $\mathcal{E}$ in the theorem relating the noise σ in the agents' exogenous private information to the bound $\mathcal{E}(\sigma)$ on the mesh of the policies.

Step 1. We start with the following result:

Lemma S3-A. *For any $\varepsilon \in \mathbb{R}_{++}$, there exists $\sigma(\varepsilon) \in \mathbb{R}_{++}$ such that, for any $\sigma \in (0, \sigma(\varepsilon)]$, the following is true: for any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$ and any $(\theta', \theta''] \in D^\Gamma$ with $|\theta'' - \theta'| > \varepsilon$, necessarily $\pi(\theta) = 0$ for some strictly positive F-measure subset of $(\theta', \theta'']$.*[24]

Proof of Lemma S3-A. While the intuition for the result (in the main text) is simple, the formal proof is quite tedious. Below, we first shows that, for any $\sigma > 0$, any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$, and any cell $(\theta', \theta''] \in D^\Gamma$ such that $\pi(\theta) = 1$ for all $\theta \in (\theta', \theta'']$, if the policy maker were to replace Γ with a cutoff policy $\Gamma^{\theta'}$ that fails with certainty all types below θ' and passes with certainty all types above θ', then for any $\theta \leq \theta''$, the payoff of the marginal agent with signal $x_\sigma^*(\theta) \equiv \theta + \sigma\Phi^{-1}(\theta)$ implementing default for all fundamentals below θ would be higher than under the original policy Γ: that is, $U_\sigma^{\Gamma^{\theta'}}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) \geq U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta))$ for any $\theta \leq \theta''$. Starting from this result, the rest of the proof then shows that for any interval $(\theta', \theta''] \subset (0, \theta^{MS})$ of Lebesgue measure $|\theta'' - \theta'| > \epsilon$, there exists $\sigma(\varepsilon) \in \mathbb{R}_{++}$ such that for all $\sigma \in (0, \sigma(\varepsilon)]$, $U_\sigma^{\Gamma^{\theta'}}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) < 0$ for some $\theta \in (\theta', \theta'']$. Together the two results then imply that, for small σ, $U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta) < 0$ for some $\theta \in (\theta', \theta'']$, thus implying that $\Gamma \notin \mathbb{G}_\sigma$ (recall that, for Γ to be in $\mathbb{G}_\sigma$, it must be that $U_\sigma^\Gamma(x, 1|x) \geq 0$ for all x).

Formal proof. As explain in the main text, the idea is the following. By the definition of θ^{MS}, when σ is small, in the absence of any disclosure, the unique rationalizable profile features all agents refraining from pledging when receiving signals $x < \theta^{MS}$. If the policy Γ were to assign the same pass grade to all $\theta \in (\theta', \theta'']$, when σ is small, no matter the shape of the policy outside the interval $(\theta', \theta'']$, it would continue to be rationalizable for the agents to refrain from pledging when hearing that $s = 1$ and receiving signals $x \in (\theta', \theta'']$. This implies that the policy is not in $\mathbb{G}_\sigma$.

The formal proof below first shows that, for any $\sigma > 0$, any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$, and any cell $(\theta', \theta''] \in D^\Gamma$ such that $\pi(\theta) = 1$ for all $\theta \in (\theta', \theta'']$, if the policy maker were to replace Γ with a

[24] Recall that $D^\Gamma \equiv \{d_i = (\underline{\theta}_i, \bar{\theta}_i] : i = 1, ..., N\}$ is the partition of $[\underline{\theta}, \theta^{MS}]$ induced by the policy Γ.

cutoff policy $\Gamma^{\theta'}$ that fails with certainty all types below θ' and passes with certainty all types above θ', then for any $\theta \leq \theta''$, the payoff of the marginal agent implementing default for all fundamentals below θ would be higher than under the original policy Γ: that is, $U_\sigma^{\Gamma^{\theta'}}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) \geq U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta))$ for any $\theta \leq \theta''$. This property corresponds to Property S1 below.

The rest of the proof then shows that for any interval $(\theta', \theta''] \subset (0, \theta^{MS})$ of Lebesgue measure $|\theta'' - \theta'| > \epsilon$, there exists $\sigma(\varepsilon) \in \mathbb{R}_{++}$ such that for all $\sigma \in (0, \sigma(\varepsilon)]$, $U_\sigma^{\Gamma^{\theta'}}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) < 0$ for some $\theta \in (\theta', \theta'']$. This is established by showing that, given any monotone rule with cut-off θ', for any $\theta > \theta'$, as σ goes to zero, the payoff $U_\sigma^{\Gamma^{\theta'}}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta))$ converges uniformly to the limit payoff $\int_0^1 u(\theta, l)dl$ in the absence of any disclosure. This property corresponds to Property S3 below. Because $\int_0^1 u(\theta, l)dl < 0$ for $\theta < \theta^{MS}$, the above two properties together imply that, for small σ, $U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta) < 0$ for some $\theta \in (\theta', \theta'']$, thus implying that $\Gamma \notin \mathbb{G}_\sigma$ (recall that, for Γ to be in $\mathbb{G}_\sigma$, it must be that $U_\sigma^\Gamma(x, 1|x) \geq 0$ for all x). The result in the lemma then follows by contrapositive.

For any $\hat{\theta} \in [0,1]$, Let $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ be the monotone policy with threshold $\hat{\theta}$ (i.e., whose rule is given by $\pi^{\hat{\theta}}(\theta) = \mathbb{I}\left\{\theta \geq \hat{\theta}\right\}$).

Property S3-A. *For any $\sigma > 0$, any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$, and any cell $(\theta', \theta''] \in D^\Gamma$ such that $\pi(\theta) = 1$ for all $\theta \in (\theta', \theta'']$, the following is true: for any $\theta \leq \theta''$, $U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) \leq U_\sigma^{\Gamma^{\theta'}}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta))$.*

Proof of Property S3-A. The proof of this property follows from Results S3-A-1 and S3-A-2 below.

Result S3-A-1. *Fix $\sigma > 0$ and pick any policy $\Gamma = \{\{0,1\}, \pi\} \in \mathbb{G}_\sigma$. Given the partition $D^\Gamma \equiv \left\{d_i = \left(\underline{\theta}_i, \bar{\theta}_i\right] : i = 1, ..., N\right\}$ induced by Γ, take any $i \geq 2$ for which $\pi(\theta) = 1$ for all $\theta \in d_i$. Let $\Gamma_L^i = \left\{\{0,1\}, \pi_L^i\right\} \in \mathbb{G}_\sigma$ be the policy constructed from Γ as follows: (a) $\pi_L^i(\theta) = 0$ for all $\theta \leq \underline{\theta}_i$; and (b) $\pi_L^i(\theta) = \pi(\theta)$ for all $\theta > \underline{\theta}_i$. Then, for all $\theta \in [0,1]$, $U_\sigma^{\Gamma_L^i}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) \geq U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta))$.*

Proof of Result S3-A-1. The idea is that, under the new policy, the endogenous signal $s = 1$ carries the same information about θ as the signal $s = 1$ under the original policy, along with the extra information that $\theta > \underline{\theta}_i$. To see this, note that, under the new policy, $\pi_L^i(\theta) = \pi(\theta) \times \mathbb{I}\{\theta > \underline{\theta}_i\}$. The posterior beliefs $\Lambda_\sigma^{\Gamma_L^i}(\cdot|x, 1)$ about θ of an agent with exogenous signal x (of precision σ^{-2}) and endogenous signal $s = 1$ under the new policy Γ_L^i thus dominate, in the FOSD sense, the analogous beliefs $\Lambda_\sigma^\Gamma(\cdot|x, 1)$ under the original policy Γ.[25] The result then follows from the fact that, given any default threshold θ^*, the payoff from pledging when the fundamentals are equal to θ and default occurs if and only if the fundamental are below θ^* is nondecreasing in the fundamentals θ. Along with the fact that $\Lambda_\sigma^{\Gamma_L^i}(\cdot|x, 1) \succ_{FOSD} \Lambda_\sigma^\Gamma(\cdot|x, 1)$, this implies that $U_\sigma^{\Gamma_L^i}(x, 1|x) \geq U_\sigma^\Gamma(x, 1|x)$ for all x. Because the result holds for all x, it also holds for $x = x_\sigma^*(\theta)$, any $\theta \in [0,1]$. This completes the proof of Result S3-A-1. $\square$

[25]No matter the shape of the beliefs $\Lambda_\sigma^\Gamma(\cdot|x, 1)$, the announcement that $\theta > \underline{\theta}_i$ is always "good news" in the sense of Milgrom (1981) and hence $\Lambda_\sigma^{\Gamma_L^i}(\cdot|x, 1) \succ_{FOSD} \Lambda_\sigma^\Gamma(\cdot|x, 1)$.

Result S3-A-2. *Fix $\sigma > 0$ and pick any policy $\Gamma = \{\{0,1\},\pi\} \in \mathbb{G}_\sigma$. Given the partition $D^\Gamma \equiv \left\{d_i = \left(\underline{\theta}_i, \bar{\theta}_i\right] : i = 1, ..., N\right\}$ induced by Γ, pick any $i \geq 1$ for which $\pi(\theta) = 1$ for all $\theta \in d_i$. Let $\Theta^1 \equiv \{\theta \in \Theta : \pi(\theta) = 1\}$ be the collection of fundamentals for which the policy Γ gives a pass grade (i.e., sends the message $s = 1$). Similarly, let $\Theta_i^0 \equiv \{\theta \in (\underline{\theta}_i, 1] : \pi(\theta) = 0\}$ be the collection of fundamentals above $\underline{\theta}_i$ for which the policy gives a fail grade (i.e., sends the signal $s = 0$), where $\underline{\theta}_i$ is the lower end-point of the partition cell d_i. Let $\Gamma_R^i = \left\{\{0,1\}, \pi_R^i\right\} \in \mathbb{G}_\sigma$ be the policy constructed from Γ as follows: (a) $\pi_R^i(\theta) = \pi(\theta)$ for all $\theta \leq \underline{\theta}_i$; and (b) $\pi_R^i(\theta) = 1$ for all $\theta > \underline{\theta}_i$. Then, for all $\theta \leq \bar{\theta}_i$, $U_\sigma^{\Gamma_R^i}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) \geq U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta))$.*

Proof of Result S3-A-2. Observe that, for any $\theta^\# \leq \bar{\theta}_i$, and any x, the following is true:

$$\begin{aligned}
\Lambda_\sigma^{\Gamma_R^i}\left(\theta^\# | x, 1\right) &= Pr\left\{\theta \leq \theta^\# | x, \theta \in \left(\Theta^1 \cup \Theta_i^0\right)\right\} \\
&= \frac{Pr\left\{\theta \leq \theta^\# \wedge \theta \in \left(\Theta^1 \cup \Theta_i^0\right) | x\right\}}{Pr\left\{\theta \in \left(\Theta^1 \cup \Theta_i^0\right) | x\right\}} \\
&= \frac{Pr\left\{\theta \leq \theta^\# \wedge \theta \in \Theta^1 | x\right\}}{Pr\left\{\theta \in \left(\Theta^1 \cup \Theta_i^0\right) | x\right\}} + \frac{Pr\left\{\theta \leq \theta^\# \wedge \theta \in \Theta_i^0 | x\right\}}{Pr\left\{\theta \in \left(\Theta^1 \cup \Theta_i^0\right) | x\right\}} \\
&= \frac{Pr\left\{\theta \leq \theta^\# \wedge \theta \in \Theta^1 | x\right\}}{Pr\left\{\theta \in \left(\Theta^1 \cup \Theta_i^0\right) | x\right\}} \\
&\leq Pr\left\{\theta \leq \theta^\# | x, \theta \in \Theta^1\right\} \\
&= \Lambda_\sigma^\Gamma\left(\theta^\# | x, 1\right).
\end{aligned}$$

The first equality follows from the definition of the posterior beliefs along with the fact that, under the new policy Γ_R^i, the endogenous signal $s = 1$ carries the same information as the announcement that $\theta \in \left(\Theta^1 \cup \Theta_i^0\right)$. The second equality follows from the definition of conditional probability. The third equality follows from the fact that $\Theta^1 \cap \Theta_i^0 = \emptyset$. The forth equality follows from the fact that Θ_i^0 contains only fundamentals above $\bar{\theta}_i$ (recall that $\pi(\theta) = 1$ for all $\theta \in \left(\underline{\theta}_i, \bar{\theta}_i\right]$) and that $\theta^\# \leq \bar{\theta}_i$. The inequality follows from the fact that $Pr\left\{\theta \in \left(\Theta^1 \cup \Theta_i^0\right) | x\right\} \geq Pr\left\{\theta \in \Theta^1 | x\right\}$ along with the definition of conditional probability. The last equality follows from the definition of the posterior beliefs along with the fact that, under the original policy Γ, the endogenous signal $s = 1$ carries the same information as the announcement that $\theta \in \Theta^1$.

Given the above inequality, and the fact that $b < 0 < g$, we then have that, for any $\theta \leq \bar{\theta}_i$,

$$\begin{aligned}
U_\sigma^\Gamma(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)) &= b \cdot \Lambda_\sigma^\Gamma(\theta|x_\sigma^*(\theta), 1) + g \cdot \left[1 - \Lambda_\sigma^\Gamma(\theta|x_\sigma^*(\theta), 1)\right] \\
&\leq b \cdot \Lambda_\sigma^{\Gamma_R^i}(\theta|x_\sigma^*(\theta), 1) + g \cdot \left[1 - \Lambda_\sigma^{\Gamma_R^i}(\theta|x_\sigma^*(\theta), 1)\right] \\
&= U_\sigma^{\Gamma_R^i}(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)).
\end{aligned}$$

This completes the proof of Result S3-A-2. □

Property S3-A then follows from Results S3-A-1 and S3-A-2, by taking the cell $d_i = (\theta', \theta'']$.

Now, fix $\varepsilon \in (0, \theta^{MS})$. For any $\theta^* \in [0, \theta^{MS} - \varepsilon]$, let Γ^{θ^*} be the monotone rule with cut-off equal to θ^*. For any $\theta^* \in [0, \theta^{MS} - \varepsilon]$, any $\sigma \in \mathbb{R}_{++}$, let

$$H_\sigma(\theta^*; \varepsilon) \equiv \inf_{\theta \in [\theta^*, \theta^* + \varepsilon]} U_\sigma^{\Gamma^{\theta^*}}(x_\sigma^*(\theta), 1 | x_\sigma^*(\theta)).$$

Note that $U_\sigma^{\Gamma^{\theta^*}}(x_\sigma^*(\theta), 1 | x_\sigma^*(\theta))$ is continuous in $(\theta^*, \theta, \sigma)$ over $[0, 1]^2 \times (0, \hat{\sigma}]$. From Berge's Maximum Theorem, $H_\sigma(\theta^*; \varepsilon)$ is thus continuous in (θ^*, σ) over $[0, \theta^{MS} - \varepsilon] \times (0, \hat{\sigma}]$.

Next, observe that, for all $\theta^* \in [0, \theta^{MS} - \varepsilon]$, all $\theta \in (\theta^*, \theta^* + \varepsilon]$,

$$\lim_{\sigma \to 0^+} U_\sigma^{\Gamma^{\theta^*}}(x_\sigma^*(\theta), 1 | x_\sigma^*(\theta)) = \int_0^1 u(\theta, l) dl.$$

Because $\int_0^1 u(\theta, l) dl$ is strictly increasing in θ and $\int_0^1 u(\theta^{MS}, l) dl = 0$, then for any $\theta^* \in [0, \theta^{MS} - \varepsilon]$,

$$H_{0^+}(\theta^*; \varepsilon) \equiv \lim_{\sigma \to 0^+} H_\sigma(\theta^*; \varepsilon) = \lim_{\sigma \to 0^+} \lim_{\theta \to \theta^{*+}} U_\sigma^{\Gamma^{\theta^*}}(x_\sigma^*(\theta), 1 | x_\sigma^*(\theta)) = \int_0^1 u(\theta^*, l) dl.$$

We show next that $H_\sigma(\cdot; \varepsilon)$ converges uniformly to the limit function $H_{0^+}(\cdot; \varepsilon)$ over $[0, \theta^{MS} - \varepsilon]$.

Property S3-B. *Fix $\varepsilon \in (0, \theta^{MS})$. For any $\epsilon < \varepsilon$, there exists $\sigma^\#(\epsilon) > 0$ such that, for any $\sigma \leq \sigma^\#(\epsilon)$, and any $\theta^* \in [0, \theta^{MS} - \varepsilon]$, $|H_\sigma(\theta^*; \varepsilon) - H_{0^+}(\theta^*; \varepsilon)| < \epsilon$.*

Proof of Property S3-B. The limit function $H_{0^+}(\cdot; \varepsilon)$ is *uniformly* continuous over $[0, \theta^{MS} - \varepsilon]$. As a consequence, there exists $\delta > 0$ such that for any $\theta, \tilde{\theta} \in [0, \theta^{MS} - \varepsilon]$, with $\left|\tilde{\theta} - \theta\right| \leq \delta$, necessarily $\left|H_{0^+}\left(\tilde{\theta}; \varepsilon\right) - H_{0^+}(\theta; \varepsilon)\right| < \frac{\epsilon}{2}$. Next, let $D_\delta \equiv \left\{\left(\underline{\theta}_i, \bar{\theta}_i\right] : i = 1, ..., N\right\}$, $N \in \mathbb{N}$, be any interval partition of $(0, \theta^{MS} - \varepsilon)$ with the property that every cell $\left(\underline{\theta}_i, \bar{\theta}_i\right] \in D_\delta$ satisfies that $\left|\bar{\theta}_i - \underline{\theta}_i\right| \leq \delta$. For any $i = 1, ..., N$, any $\sigma > 0$, let

$$\hat{\theta}_\sigma^i \equiv \sup\left\{\arg\max_{\theta \in \left[\underline{\theta}_i, \bar{\theta}_i\right]} H_\sigma(\theta; \varepsilon)\right\}.$$

That $H_\sigma(\theta; \varepsilon)$ is continuous in (σ, θ) implies that the hypothesis of Berge's Maximum Theorem hold and, hence, the correspondence $\arg\max_{\theta \in \left[\underline{\theta}_i, \bar{\theta}_i\right]} H_\sigma(\theta; \varepsilon)$ is compact valued and upper semi-continuous in σ. As a result, for any $\sigma > 0$,

$$\hat{\theta}_\sigma^i = \max\left\{\arg\max_{\theta \in \left[\underline{\theta}_i, \bar{\theta}_i\right]} H_\sigma(\theta; \varepsilon)\right\}.$$

Moreover, $\lim_{\sigma \to 0^+} H_\sigma\left(\hat{\theta}_\sigma^i; \varepsilon\right) = H_{0^+}\left(\hat{\theta}_{0^+}^i; \varepsilon\right)$, where $\hat{\theta}_{0^+}^i \equiv \lim_{\sigma \to 0^+} \hat{\theta}_\sigma^i$.

Next, for any $\theta^* \in [0, \theta^{MS} - \varepsilon]$, let $\left(\underline{\theta}_j, \bar{\theta}_j\right] \in D_\delta$ be the partition cell containing θ^*. The following is then true:

$$
\begin{aligned}
H_\sigma(\theta^*;\varepsilon) - H_{0^+}(\theta^*;\varepsilon) &\leq H_\sigma\left(\hat{\theta}^j_\sigma;\varepsilon\right) - H_{0^+}(\theta^*;\varepsilon) \\
&= \left(H_\sigma\left(\hat{\theta}^j_\sigma;\varepsilon\right) - H_{0^+}\left(\hat{\theta}^j_{0^+};\varepsilon\right)\right) + \left(H_{0^+}\left(\hat{\theta}^j_{0^+};\varepsilon\right) - H_{0^+}(\theta^*;\varepsilon)\right) \\
&< \left(H_\sigma\left(\hat{\theta}^j_\sigma;\varepsilon\right) - H_{0^+}\left(\hat{\theta}^j_{0^+};\varepsilon\right)\right) + \frac{\epsilon}{2} \\
&< \epsilon, \quad \text{for all } \sigma < \bar{\sigma}_j(\epsilon), \text{ for some } \bar{\sigma}_j(\epsilon) > 0.
\end{aligned}
$$

The first inequality follows from the definition of $\hat{\theta}^i_\sigma$. The second inequality follows from the fact that $\left|\hat{\theta}^j_{0^+} - \theta^*\right| < \delta$. The last inequality follows from the fact that $\lim_{\sigma\to 0^+} H_\sigma\left(\hat{\theta}^j_\sigma\right) = H_{0^+}\left(\hat{\theta}^j_{0^+}\right)$.

Similar arguments imply that

$$
H_\sigma(\theta^*;\varepsilon) - H_{0^+}(\theta^*;\varepsilon) > -\epsilon, \quad \text{for all } \sigma < \underline{\sigma}_j(\epsilon), \text{ for some } \underline{\sigma}_j(\epsilon) > 0.
$$

Now let $\sigma^\#(\epsilon) \equiv \min\left\{\min_{i\in N}\{\bar{\sigma}_i(\epsilon)\}, \min_{i\in N}\{\underline{\sigma}_i(\epsilon)\}\right\}$. For any $\sigma \leq \sigma^\#(\epsilon)$, and any $\theta^* \in [0, \theta^{MS} - \varepsilon]$, we thus have that

$$
|H_\sigma(\theta^{**};\varepsilon) - H_{0^+}(\theta^{**};\varepsilon)| < \epsilon,
$$

thus proving that $H_\sigma(\cdot;\varepsilon)$ converges uniformly to $H_{0^+}(\cdot;\varepsilon)$ as $\sigma \to 0^+$. This completes the proof of Property S3-B. □

Next, given $\varepsilon \in (0, \theta^{MS})$, pick an arbitrary $\eta \in \left(\int_0^1 u(\theta^{MS} - \varepsilon, l)dl, 0\right)$. Because $H_{0^+}(\theta^*;\varepsilon) \leq \eta$ for all $\theta^* \in [0, \theta^{MS} - \varepsilon]$, and because $H_\sigma(\cdot;\varepsilon)$ converges uniformly to $H_{0^+}(\cdot;\varepsilon)$, there exists $\sigma(\varepsilon) > 0$ such that, for any $\sigma < \sigma(\varepsilon)$, and any $\theta^* \in [0, \theta^{MS} - \varepsilon]$, $H_\sigma(\theta^*;\varepsilon) \leq \eta < 0$. Therefore, for any $\sigma < \sigma(\varepsilon)$, and any monotone policy Γ^{θ^*} with cut-off $\theta^* \in [0, \theta^{MS} - \varepsilon]$, there exists $\theta \in [\theta^*, \theta^* + \varepsilon]$ such that $U^{\Gamma^{\theta^*}}_\sigma(x^*_\sigma(\theta), 1|x^*_\sigma(\theta)) \leq \eta$.

Together, Properties S3-A and S3-B then imply that, for any $\sigma < \sigma(\varepsilon)$, and any policy Γ such that $\pi(\theta) = 1$ for all $\theta \in (\theta', \theta'')$ for some $(\theta', \theta'') \in D^\Gamma$ with $|\theta'' - \theta'| > \varepsilon$, necessarily $U^\Gamma_\sigma(x^*_\sigma(\theta), 1|x^*_\sigma(\theta)) < 0$ for some $\theta \in (\theta', \theta'')$. Hence $\Gamma \notin \mathbb{G}_\sigma$. The claim in Lemma S3-A then follows by contrapositive. This completes the proof of Lemma S3-A. ■

Step 2. Next, we show that, for any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$ that gives a fail grade to an interval $(\theta', \theta''] \subseteq (0, \theta^{MS}]$ of large Lebesgue measure, there exists another policy $\Gamma^\# \in \mathbb{G}_\sigma$ with a mesh $M(\Gamma^\#) < M(\Gamma)$ such that, when agents play as in G_σ, the probability of default under $\Gamma^\#$ is strictly smaller than under Γ.

We start with the following result:

Lemma S3-B. *Fix* $\sigma > 0$. *For any* $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$ *such that* $\inf_{\theta\in[0,1]} U^\Gamma_\sigma(x^*_\sigma(\theta), 1|x^*_\sigma(\theta)) > 0$, *there exists another policy* $\hat{\Gamma} = (\{0,1\}, \hat{\pi}) \in \mathbb{G}_\sigma$ *such that, in the auxiliary game* G_σ, *the policy maker's payoff under* $\hat{\Gamma}$ *is strictly higher than under* Γ.

Proof of Lemma S3-B. That $\inf_{\theta\in[0,1]} U^\Gamma_\sigma(x^*_\sigma(\theta), 1|x^*_\sigma(\theta)) > 0$ implies that, starting from $\Gamma = (\{0,1\}, \pi)$, one can construct another policy $\hat{\Gamma} = (\{0,1\}, \hat{\pi})$, sufficiently close to Γ (in the L_1

norm), and such that $\hat{\pi}(\theta) \geq \pi(\theta)$ for all θ, with the inequality strict over some positive F-measure set $(\tilde{\theta}', \tilde{\theta}'') \subseteq (0,1]$, and such that (a) $\hat{\pi}(\theta) = 0$ for all $\theta \leq 0$, (b) $\hat{\pi}(\theta) = 1$ for all $\theta > 1$, and (c) $U_\sigma^{\hat{\Gamma}}(x, 1|x) \geq 0$ all x. By definition of $\mathbb{G}_\sigma$, $\hat{\Gamma} \in \mathbb{G}_\sigma$. That, in the auxiliary game G_σ, $\hat{\Gamma}$ strictly improves over Γ then follows from the fact that the probability of default under $\hat{\Gamma}$ is strictly smaller than under Γ. This completes the proof of Lemma S3-B. ■

Next observe that, for any $\sigma > 0$, and any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$, $U_\sigma^\Gamma\left(x_\sigma^*(\cdot), 1|x_\sigma^*(\cdot)\right)$ is continuous in θ over $[0,1]$. Hence

$$\inf_{\theta \in [0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) = \min_{\theta \in [0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right).$$

Lemma S3-C. *Fix $\sigma \in (0, \hat{\sigma})$. Take any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$ such that*

$$\min_{\theta \in [0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) = 0.$$

For any $\theta_\sigma^\sharp \in \arg\min_{\theta \in [0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)$, there exists $\gamma_\sigma^\Gamma > 0$ such that

$$\pi(\theta) = 1 \quad F\text{-almost all } \theta \in \left(\theta_\sigma^\sharp - \gamma_\sigma^\Gamma, \theta_\sigma^\sharp\right).$$

Proof of Lemma S3-C. The proof is by contraposition. Suppose there exists $\delta > 0$ such that $\pi(\theta) = 0$ for F-almost all $\theta \in \left(\theta_\sigma^\sharp - \delta, \theta_\sigma^\sharp\right)$. Observe that

$$\begin{aligned}
\operatorname{sign}\left\{U_\sigma^\Gamma\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right), 1|x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right)\right)\right\} &= \operatorname{sign}\left\{b\int_{-\infty}^{\theta_\sigma^\sharp - \delta} \phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta) + \right. \\
&\left. + g\int_{\theta_\sigma^\sharp - \delta}^{+\infty} \phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta)\right\}.
\end{aligned}$$

Next observe that

$$\begin{aligned}
0 &= U_\sigma^\Gamma\left(x_\sigma^*\left(\theta_\sigma^\sharp\right), 1|x_\sigma^*\left(\theta_\sigma^\sharp\right)\right)\int_{-\infty}^{+\infty} \phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta) \\
&= \int_{-\infty}^{\infty} \left(b\mathbb{I}\left\{\theta \leq \theta_\sigma^\sharp\right\} + g\mathbb{I}\left\{\theta > \theta_\sigma^\sharp\right\}\right)\phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta) \\
&> \int_{-\infty}^{\infty} \left(b\mathbb{I}\left\{\theta \leq \theta_\sigma^\sharp\right\} + g\mathbb{I}\left\{\theta > \theta_\sigma^\sharp\right\}\right)\phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta) \\
&= \int_{-\infty}^{\infty} \left(b\mathbb{I}\left\{\theta \leq \theta_\sigma^\sharp - \delta\right\} + g\mathbb{I}\left\{\theta > \theta_\sigma^\sharp - \delta\right\}\right)\phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta) \\
&= U_\sigma^\Gamma\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right), 1|x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right)\right)\left[\int_{-\infty}^{+\infty} \phi\left(\left(x_\sigma^*\left(\theta_\sigma^\sharp - \delta\right) - \theta\right)/\sigma\right)\pi(\theta)\,dF(\theta)\right]
\end{aligned}$$

The first equality follows from the assumptions of the lemma. The second equality follows from the definition of the function $U_\sigma^\Gamma\left(x_\sigma^*\left(\theta_\sigma^\sharp\right), 1|x_\sigma^*\left(\theta_\sigma^\sharp\right)\right)$. The inequality follows from the monotonicity of $x_\sigma^*(\cdot)$, the fact that $\phi\left(\frac{x-\theta}{\sigma}\right)$ is log-supermodular in (x,θ), and Property SC in the proof of Lemma 1 in Section 1 in this online supplement. The third equality follows from the fact that $\pi(\theta)=0$ for F-almost all $\theta \in \left(\theta_\sigma^\sharp-\delta, \theta_\sigma^\sharp\right)$. The last equality follows from the definition of the function $U_\sigma^\Gamma\left(x_\sigma^*\left(\theta_\sigma^\sharp-\delta\right), 1|x_\sigma^*\left(\theta_\sigma^\sharp-\delta\right)\right)$.

Hence, $U_\sigma^\Gamma\left(x_\sigma^*\left(\theta_\sigma^\sharp-\delta\right), 1|x_\sigma^*\left(\theta_\sigma^\sharp-\delta\right)\right) < 0$, thus contradicting the assumption that $\Gamma \in \mathbb{G}_\sigma$. This completes the proof of Lemma S3-C. ■

The result in Lemma S3-C in turn implies that, given any policy $\Gamma \in \mathbb{G}_\sigma$ such that

$$\min_{\theta\in[0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)=0,$$

for any $\theta_\sigma^\sharp \in \arg\min_{\theta\in[0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)$, and any $\theta \in d^\Gamma\left(\theta_\sigma^\sharp\right)$, $\pi(\theta)=1$, where $d^\Gamma\left(\theta_\sigma^\sharp\right)$ is the cell of the partition of $[0,1]$ corresponding to the policy Γ containing $\theta_\sigma^\sharp$. Conversely, for any $d \in D^\Gamma$ such that $\pi(\theta)=0$ for all $\theta\in d$, necessarily $U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)>0$ for all $\theta \in d$.

Next, for any $\sigma\in(0,\hat{\sigma})$, and any policy $\Gamma \in \mathbb{G}_\sigma$, let $T_\sigma^\Gamma \equiv \arg\min_{\theta\in[0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 0|x_\sigma^*(\theta)\right)$.

Lemma S3-D. *For any $\varepsilon>0$, there exists $\sigma^{\#}(\varepsilon)\in(0,\hat{\sigma})$ such that, for any $\sigma\in(0,\sigma^{\#}(\varepsilon)]$, and any policy $\Gamma=(\{0,1\},\pi)\in\mathbb{G}_\sigma$ for which there exists $(\theta',\theta'']\in D^\Gamma$ such that (a) $|\theta''-\theta'|>\varepsilon$ and (b) $\pi(\theta)=0$ for all $\theta\in(\theta',\theta'']$, there exists another policy $\Gamma^{\#}=\left(\{0,1\},\pi^{\#}\right)\in\mathbb{G}_\sigma$, with $M(\Gamma^{\#})<M(\Gamma)$, such that, in the auxiliary game G_σ, the probability of default under $\Gamma^{\#}$ is strictly smaller than under Γ.*

Proof of Lemma S3-D. For any $\theta\in(0,\theta^{MS}]$, $\lim_{\sigma\to0^+} x_\sigma^*(\theta)\equiv x_{0^+}^*(\theta)=\theta$. Furthermore, for any $\varepsilon\in(0,\theta^{MS})$, the function $x_{0^+}^*:\left[\frac{\varepsilon}{4},\theta^{MS}\right]\to\mathbb{R}$ is uniformly continuous. Hence, for any $\delta<\varepsilon/4$, there exists $\tilde{\sigma}(\delta)>0$ such that, for any $\sigma\in(0,\tilde{\sigma}(\delta)]$, and any $\theta\in[\frac{\varepsilon}{4},\theta^{MS}]$, we have that $|x_\sigma^*(\theta)-\theta|\le\delta$.[26] In turn, this implies that, for any $\varepsilon>0$, there exists $\sigma^{\#}(\varepsilon)\in(0,\hat{\sigma}]$ such that, for any $\sigma\in(0,\sigma^{\#}(\varepsilon)]$, and any $(\theta',\theta'']\in D^\Gamma$ such that $|\theta''-\theta'|>\varepsilon$, we have that, for any $\theta\ge\theta''$, $|\theta-x_\sigma^*(\theta)|<|(\theta'+\theta'')/2-x_\sigma^*(\theta)|$. Likewise, for any $\theta\le\theta'$, and any $\hat{\theta}\ge\theta''$, we have that $|\theta-x_\sigma^*(\theta)|<\left|x_\sigma^*(\theta)-\hat{\theta}\right|$ when $\sigma\in(0,\sigma^{\#}(\varepsilon)]$.

Next, pick any policy $\Gamma=(\{0,1\},\pi)\in\mathbb{G}_\sigma$ for which there exists $d\equiv(\theta',\theta'']\in D^\Gamma$ such that (a) $|\theta''-\theta'|>\varepsilon$ and (b) $\pi(\theta)=0$ for all $\theta\in(\theta',\theta'']$. If $\min_{\theta\in[0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)>0$, the result follows directly from Lemma S3-B. Thus assume that $\min_{\theta\in[0,1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)=0$.

Suppose first that $\min_{\theta\in[\theta'',1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta), 0|x_\sigma^*(\theta)\right)>0$. From Lemma S3-C, necessarily

$$U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)>0$$

[26] The proof for the existence of a sequence $\left\{x_{\sigma_n}^*(\cdot)\right\}_n$ with domain $\left[\frac{\varepsilon}{4},\theta^{MS}\right]$ converging uniformly to its limit function $x_{0^+}^*(\cdot)$ follows from the same arguments that establish the uniform convergence of $\left\{H_{\sigma_n}(\cdot)\right\}_n$ to $H_{0^+}(\cdot)$ in the proof of Lemma S2.

for all $\theta \in (\theta', \theta'']$. Hence, $\min_{\theta\in(\theta',1]} U_{\sigma}^{\Gamma}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right) > 0$. Below we show that, starting from Γ, we can then construct a policy Γ^{η} that continues to satisfy the property that $U_{\sigma}^{\Gamma^{\eta}}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right) \geq 0$ for all $\theta \in [0, 1]$ and such that the probability of default under Γ^{η} is smaller than under Γ. Intuitively, Γ^{η} is obtained from Γ by giving a pass grade to a positive-measure interval of types in the middle of $(\theta', \theta'']$. Formally, take $\eta \in (0, (\theta'' - \theta')/2)$ and let $\Gamma^{\eta} = (\{0, 1\}, \pi^{\eta})$ be the policy whose rule π^{η} is given by (a) $\pi^{\eta}(\theta) = \pi(\theta)$ for all $\theta \notin [(\theta' + \theta'')/2, (\theta' + \theta'')/2 + \eta]$, and (b) $\pi^{\eta}(\theta) = 1$ for all $\theta \in [(\theta' + \theta'')/2, (\theta' + \theta'')/2 + \eta]$. Below we show that $U^{\Gamma^{\eta}}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right) \geq 0$ for all $\theta \in [0, 1]$.[27] To see this, let $\Theta^{1} \equiv \{\theta \in \Theta : \pi(\theta) = 1\}$ be the collection of fundamentals for which the original policy Γ gives a pass grade (i.e., sends the message $s = 1$). Observe that, for any $\theta^{\#} \leq \theta'$, and any x, the following is true:

$$\begin{aligned}
\Lambda_{\sigma}^{\Gamma^{\eta}}\left(\theta^{\#}|x, 1\right) &= Pr\left\{\theta \leq \theta^{\#}|x, \theta \in \left(\Theta^{1} \cup [(\theta' + \theta'')/2, (\theta' + \theta'')/2 + \eta]\right)\right\} \\
&= \frac{Pr\left\{\theta \leq \theta^{\#} \wedge \theta \in \Theta^{1}|x\right\}}{Pr\left\{\theta \in \left(\Theta^{1} \cup [(\theta' + \theta'')/2, (\theta' + \theta'')/2 + \eta]\right)|x\right\}} \\
&< Pr\left\{\theta \leq \theta^{\#}|x, \theta \in \Theta^{1}\right\} \\
&= \Lambda_{\sigma}^{\Gamma}\left(\theta^{\#}|x, 1\right)
\end{aligned}$$

The first equality is by the definition of the posterior beliefs along with the fact that, under the policy Γ^{η}, the endogenous signal $s = 1$ carries the same information as the announcement that $\theta \in \left(\Theta^{1} \cup [(\theta' + \theta'')/2, (\theta' + \theta'')/2 + \eta]\right)$. The second equality follows from the definition of conditional probability. The inequality follows from the fact that $Pr\left\{\theta \in \left(\Theta^{1} \cup [(\theta' + \theta'')/2, (\theta' + \theta'')/2 + \eta]\right)|x\right\} \geq Pr\left\{\theta \in \Theta^{1}|x\right\}$ along with the definition of conditional probability. The last equality follows from the definition of the posterior beliefs along with the fact that, under the original policy Γ, the endogenous signal $s = 1$ carries the same information as the announcement that $\theta \in \Theta^{1}$.

Given the above inequality, and the fact that, $b < 0 < g$, we then have that, for any $\theta \leq \theta'$,

$$\begin{aligned}
U_{\sigma}^{\Gamma}(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)) &= b \cdot \Lambda_{\sigma}^{\Gamma}\left(\theta|x_{\sigma}^{*}(\theta), 1\right) + g \cdot \left[1 - \Lambda_{\sigma}^{\Gamma}\left(\theta|x_{\sigma}^{*}(\theta), 1\right)\right] \\
&< b \cdot \Lambda_{\sigma}^{\Gamma^{\eta}}\left(\theta|x_{\sigma}^{*}(\theta), 1\right) + g \cdot \left[1 - \Lambda_{\sigma}^{\Gamma^{\eta}}\left(\theta|x_{\sigma}^{*}(\theta), 1\right)\right] \\
&= U_{\sigma}^{\Gamma^{\eta}}(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)).
\end{aligned}$$

Hence $U^{\Gamma^{\eta}}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right) > 0$ for all $\theta \leq \theta'$. That $\min_{\theta\in[\theta',1]} U_{\sigma}^{\Gamma}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right) > 0$, along with the fact that $U_{\sigma}^{\Gamma^{\eta}}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right)$ is continuous in η implies that $\min_{\theta\in[0,1]} U_{\sigma}^{\Gamma^{\eta}}\left(x_{\sigma}^{*}(\theta), 1|x_{\sigma}^{*}(\theta)\right) > 0$ for η small. Hence $\Gamma^{\eta} \in \mathbb{G}_{\sigma}$. That, when agents play according to G_{σ}, the probability of default is strictly smaller under Γ^{η} than under Γ follows directly from the fact that the set of fundamentals that receive a pass grade under Γ^{η} is a strict superset of the set of fundamentals that receive a pass grade under Γ.

[27] The proof follows from arguments similar to those establishing Result 2 in the proof of Lemma S2.

Thus consider the more interesting case in which $\min_{\theta\in[\theta'',1]} U_\sigma^\Gamma\left(x_\sigma^*(\theta),0|x_\sigma^*(\theta)\right)=0$. Let $\theta_\sigma^\#=\inf\left\{\theta\in T_\sigma^\Gamma:\theta\geq\theta''\right\}$. An implication of Lemma S3-C is that that $\theta_\sigma^\#>\theta''$. Also observe that, by definition of D^Γ, $\pi(\theta)=1$ in a right neighborhood of θ''. Then let $d^\Gamma(\theta'')=(\theta'',\theta''']$ denote the interval to the immediate right of the interval $(\theta',\theta'']$ in the partition induced by the policy Γ and let $\hat{\theta}=\min\left\{\theta''',\theta_\sigma^\#\right\}$.

Now, pick $\xi>0$ small and let $\delta(\xi)$ be implicitly defined by

$$F((\theta''+\theta')/2+\xi)-F((\theta''+\theta')/2)=F((\theta''+\hat{\theta})/2+\delta(\xi))-F((\theta''+\hat{\theta})/2). \tag{S8}$$

Consider the policy $\Gamma^\xi=(\{0,1\},\pi^\xi)$ defined by (a) $\pi^\xi(\theta)=\pi(\theta)$ for all $\theta\notin[(\theta''+\theta')/2,(\theta''+\theta')/2+\xi]\cup[(\theta''+\hat{\theta})/2,(\theta''+\hat{\theta})/2+\delta(\xi)]$, (b) $\pi^\xi(\theta)=1$ for all $\theta\in[(\theta''+\theta')/2,(\theta''+\theta')/2+\xi]$, and (c) $\pi^\xi(\theta)=0$ for all $\theta\in[(\theta''+\hat{\theta})/2,(\theta''+\hat{\theta})/2+\delta(\xi)]$. Below we establish that, when $\xi>0$ is small, such a policy is such that $\min_{\theta\in[0,1]} U^{\Gamma^\xi}\left(x_\sigma^*(\theta),1|x_\sigma^*(\theta)\right)>0$ and hence $\Gamma^\xi\in\mathbb{G}_\sigma$. To see this, for any arbitrary policy $\tilde{\Gamma}=\{\{0,1\},\tilde{\pi}\}$, any $\theta\in[0,1]$, let

$$V_\sigma^{\tilde{\Gamma}}(\theta)\equiv U_\sigma^{\tilde{\Gamma}}\left(x_\sigma^*(\theta),1|x_\sigma^*(\theta)\right)p_\sigma^{\tilde{\Gamma}}\left(x_\sigma^*(\theta),1\right),$$

where, for any x, $p_\sigma^{\tilde{\Gamma}}(x,1)\equiv\int_\Theta\tilde{\pi}(\theta)p_\sigma(x|\theta)dF(\theta)$ and where $p_\sigma(x|\theta)=\frac{1}{\sigma}\phi((x-\theta)/\sigma)$ is the conditional density of x given θ and σ.

By definition of $\theta_\sigma^\#$, we must have that, for all θ, $0=V_\sigma^\Gamma\left(\theta_\sigma^\#\right)\leq V_\sigma^\Gamma(\theta)$. Next, for any $\xi>0$, define

$$\varphi_R(\xi)\equiv\min_{\theta\in[\theta'',1]} V_\sigma^{\Gamma^\xi}(\theta). \tag{S9}$$

Observe that, for any θ,

$$\begin{aligned} V_\sigma^{\Gamma^\xi}(\theta) &= V_\sigma^\Gamma(\theta)+\int_{(\theta''+\theta')/2}^{(\theta''+\theta')/2+\xi}\overline{u}\left(\tilde{\theta},\theta\right)p_\sigma\left(x_\sigma^*(\theta)|\tilde{\theta}\right)dF\left(\tilde{\theta}\right)+ \\ &\quad -\int_{(\theta''+\hat{\theta})/2}^{(\theta''+\hat{\theta})/2+\delta(\xi)}\overline{u}\left(\tilde{\theta},\theta\right)p_\sigma\left(x_\sigma^*(\theta)|\tilde{\theta}\right)dF\left(\tilde{\theta}\right), \end{aligned}$$

where $\overline{u}\left(\tilde{\theta},\theta\right)\equiv g\mathbb{I}\left\{\tilde{\theta}>\theta\right\}+b\mathbb{I}\left\{\tilde{\theta}\leq\theta\right\}$. Using the envelope theorem and (S9), we have that, for any $\theta_\sigma^\xi\in\arg\min_{\theta\in[\theta'',1]} V_\sigma^{\Gamma^\xi}(\theta)$,

$$\begin{aligned} \varphi_R'(\xi) &= f\left((\theta''+\theta')/2+\xi\right)\overline{u}\left((\theta''+\theta')/2+\xi,\theta_\sigma^\xi\right)p_\sigma\left(x_\sigma^*\left(\theta_\sigma^\xi\right)|(\theta''+\theta')/2+\xi\right) \\ &\quad -f\left((\theta''+\hat{\theta})/2+\delta(\xi)\right)\overline{u}\left((\theta''+\hat{\theta})/2+\delta(\xi),\theta_\sigma^\xi\right)p_\sigma\left(x_\sigma^*\left(\theta_\sigma^\xi\right)|(\theta''+\hat{\theta})/2+\delta(\xi)\right)\delta'(\xi) \\ &= f\left((\theta''+\theta')/2+\xi\right)\left[\overline{u}\left((\theta''+\theta')/2+\xi,\theta_\sigma^\xi\right)p_\sigma\left(x_\sigma^*\left(\theta_\sigma^\xi\right)|(\theta''+\theta')/2+\xi\right)\right. \\ &\quad \left.-\overline{u}\left((\theta''+\hat{\theta})/2+\delta(\xi),\theta_\sigma^\xi\right)p_\sigma\left(x_\sigma^*\left(\theta_\sigma^\xi\right)|(\theta''+\hat{\theta})/2+\delta(\xi)\right)\right] \end{aligned}$$

where the second equality uses the implicit function theorem applied to (S8) to obtain that $\delta'(\xi)=f\left((\theta''+\theta')/2+\xi\right)/f\left((\theta''+\hat{\theta})/2+\delta(\xi)\right)$.

As a consequence, in the limit as $\xi \to 0^+$,

$$\lim_{\xi \to 0^+} \varphi_R'(\xi) = f\left((\theta'' + \theta')/2\right) [\overline{u}\left((\theta'' + \theta')/2, \theta_\sigma^{\#}\right) p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#}\right) |(\theta'' + \theta')/2\right) \tag{S10}$$
$$- \overline{u}\left((\theta'' + \hat{\theta})/2, \theta_\sigma^{\#}\right) p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#}\right) |(\theta'' + \hat{\theta})/2\right)]$$

That $\sigma < \sigma^{\#}(\varepsilon)$ implies that $|x_\sigma^*\left(\theta_\sigma^\sharp\right) - (\theta'' + \hat{\theta})/2| < |x_\sigma^*\left(\theta_\sigma^\sharp\right) - (\theta'' + \theta')/2|$. That the noise distribution $p_\sigma(x|\theta)$ is single-peaked in turn implies that

$$p_\sigma \left(x_\sigma^* \left(\theta_\sigma^\sharp\right) |(\theta'' + \theta')/2\right) < p_\sigma \left(x_\sigma^* \left(\theta_\sigma^\sharp\right) |(\theta'' + \hat{\theta})/2\right)$$

and hence that

$$\overline{u}\left((\theta'' + \theta')/2, \theta_\sigma^{\#}\right) p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#}\right) |(\theta'' + \theta')/2\right) - \overline{u}\left((\theta'' + \hat{\theta})/2', \theta_\sigma^{\#}\right) p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#}\right) |(\theta'' + \hat{\theta})/2\right)$$

$$= b \times \left(p_\sigma \left(x_\sigma^* \left(\theta_\sigma^\sharp\right) |(\theta'' + \theta')/2\right) - p_\sigma \left(x_\sigma^* \left(\theta_\sigma^\sharp\right) |(\theta'' + \hat{\theta})/2\right)\right) > 0.$$

Thus, $\lim_{\xi \to 0^+} \varphi_R'(\xi) > 0$. By continuity of $U_\sigma^{\Gamma^\xi}\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right)$ in ξ, we then have that, for $\xi > 0$ small, $\min_{\theta \in [\theta'', 1]'} U_\sigma^{\Gamma^\xi}\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) > 0$.

Next, we prove that, under the policy Γ^ξ, $\min_{\theta \in [0, \theta'']} U_\sigma^{\Gamma^\xi}\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) > 0$. For any $\xi > 0$, define

$$\varphi_L(\xi) \equiv \min_{\theta \in [0, \theta']} V_\sigma^{\Gamma^\xi}(\theta). \tag{S11}$$

Arguments similar to those used above to compute $\lim_{\xi \to 0} \varphi_R'(\xi)$ imply that, for any $\theta_\sigma^{\#\#} \in \arg\min_{\theta \in [0, \theta']} V_\sigma^\Gamma(\theta)$, when $\sigma \leq \sigma^{\#}(\varepsilon)$,

$$\begin{aligned}
\lim_{\xi \to 0^+} \varphi_L'(\xi) &= f\left((\theta'' + \theta')/2\right) [\overline{u}\left((\theta'' + \theta')/2, \theta_\sigma^{\#\#}\right) p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#\#}\right) |(\theta'' + \theta')/2\right) \\
&\quad - \overline{u}\left((\theta'' + \hat{\theta})/2, \theta_\sigma^{\#\#}\right) p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#\#}\right) |(\theta'' + \hat{\theta})/2\right)] \\
&= f\left((\theta'' + \theta')/2\right) g \left[p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#\#}\right) |(\theta'' + \theta')/2\right) - p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#\#}\right) |(\theta'' + \hat{\theta})/2\right)\right] > 0
\end{aligned}$$

The first equality follows from steps analogous to those used to establish (S10). The second equality follows from the fact that, by assumption $\theta_\sigma^{\#\#} \leq \theta'$. The inequality is a consequence of the fact that, for $\sigma \leq \sigma^{\#}(\varepsilon)$, $|x_\sigma^*\left(\theta_\sigma^{\#\#}\right) - (\theta'' + \theta')/2| < |x_\sigma^*\left(\theta_\sigma^{\#\#}\right) - \theta''|$, which, together with the fact that the noise distribution is single-peaked, implies that

$$p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#\#}\right) |(\theta'' + \theta')/2\right) > p_\sigma \left(x_\sigma^* \left(\theta_\sigma^{\#\#}\right) |(\theta'' + \hat{\theta})/2\right).$$

Because $U_\sigma^\Gamma\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) > 0$ for all $\theta \in (\theta', \theta'']$, we then have that, for $\xi > 0$ small, under the new policy Γ^ξ, $\min_{\theta \in [0,1]} U_\sigma^{\Gamma^\xi}\left(x_\sigma^*(\theta), 1|x_\sigma^*(\theta)\right) > 0$. Hence $\Gamma^\xi \in \mathbb{G}_\sigma$.

By construction, $M(\Gamma^{\xi}) < M(\Gamma)$. Furthermore, when agents play according to G_σ, the probability of default under Γ^{ξ} is the same as under Γ. Lemma S3-B then implies that, starting from Γ^{ξ}, one can construct a policy $\Gamma^{\#} \in \mathbb{G}_\sigma$, sufficiently close to Γ^{ξ} in the L_1 norm, such that (1) $M(\Gamma^{\#}) < M(\Gamma)$ and (2), when the agents play according to G_σ, the probability of default under $\Gamma^{\#}$ is strictly smaller than under Γ. This completes the proof of Lemma S3-D. ■

Step 3. Steps 1 and 2 imply that there exists a function $\bar{\sigma} : (0, \theta^{MS}) \to \mathbb{R}_{++}$, with $\bar{\sigma}(\varepsilon) \leq \min\{\sigma(\varepsilon), \sigma^{\#}(\varepsilon)\}$ for all $\varepsilon \in (0, \theta^{MS})$ and with $\bar{\sigma}(\varepsilon) \to 0^+$ as $\varepsilon \to 0^+$, such that the following is true: For any $\varepsilon \in (0, \theta^{MS})$, any $\sigma \in (0, \bar{\sigma}(\varepsilon)]$, and any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}_\sigma$ with $M(\Gamma) > \varepsilon$, there exists another policy $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}_\sigma$ with $M(\Gamma') \leq \varepsilon$ such that, when the agents play as in the auxiliary game G_σ, the probability of default under Γ' is strictly smaller than under Γ.[28]

Arguments similar to those establishing Lemma S3-B then imply that, starting from Γ', one can construct a nearby policy $\Gamma^* = (\{0,1\}, \pi^*) \in \mathbb{G}_\sigma$ such that the probability of default under Γ^* is arbitrarily close to that under Γ' (and hence strictly smaller than under Γ) and such that $U_\sigma^{\Gamma^*}(x, 1|x) > 0$ for all x. The last property implies that Γ^* satisfies the perfect-coordination property.

Step 4. We now complete the proof by showing how to construct the function $\mathcal{E}$ in the theorem relating the noise σ in the agents' exogenous private information to the bound $\mathcal{E}(\sigma)$ on the mesh of the policies. Let (ε_n) be a non-increasing sequence satisfying $\lim_{n\to\infty} \varepsilon_n = 0$. For each $n \in \mathbb{N}$, then let $\sigma_n = \overline{\sigma}(\varepsilon_n)$, with the function $\overline{\sigma}(\cdot)$ as defined in Step 3. The results in Steps 1-3 above imply that, given $(\varepsilon_n, \sigma_n)$, there exist strictly decreasing subsequences $(\tilde{\varepsilon}_n)$ and $(\tilde{\sigma}_n)$ satisfying $\lim_{n\to\infty} \tilde{\varepsilon}_n = \lim_{n\to\infty} \tilde{\sigma}_n = 0$ such that, for any $n \in \mathbb{N}$, the conclusions in Step 3 hold for $\varepsilon = \tilde{\varepsilon}_n$ and $\bar{\sigma}(\varepsilon_n) = \tilde{\sigma}_n$. Then let $\bar{\sigma} = \tilde{\sigma}_0 > 0$ and $\mathcal{E} : (0, \bar{\sigma}] \to \mathbb{R}_+$ be the function defined by $\mathcal{E}(\sigma) = \varepsilon_n$ for all $\sigma \in (\sigma_{n+1}, \sigma_n]$. The result in the theorem then follows from Steps 1-3, by letting $\mathcal{E}(\cdot)$ be the function constructed above.

This completes the proof of the example. Q.E.D.

Section S4: Role of Multiplicity of Receivers and of Exogenous Private Information

Subsection S4.1. Multiple Receivers vs Single Receiver

To appreciate the role that the multiplicity of the receivers plays for our results, consider the following variant of the economy of Section 2.

Timing. At $t = 0$, the policy maker chooses a disclosure policy $\{\pi, S\}$ that, for each fundamental θ, sends a signal s from a distribution $\pi(\theta) \in \Delta(S)$.[29]At $t = 1$, a single receiver with signal x drawn

[28]Observe that the thresholds $\sigma(\varepsilon)$ and $\sigma^{\#}(\varepsilon)$ identified in Steps 1 and 2 above are invariant to the initial policy Γ. The same arguments used to identify a policy $\Gamma^{\#}$ with mesh $M(\Gamma^{\#}) < M(\Gamma)$ can then be iterated till one arrives at a policy Γ' with mesh $M(\Gamma') \leq \varepsilon$.

[29]Here we accommodate for stochastic disclosure rules, although this is not essential to the results, as explained

from a log-supermodular distribution $p(x|\theta)$ has to decide whether to take a "friendly" action, $a = 1$, or an adversarial" action, $a = 0$.

Payoffs. The policy maker's payoff is equal to $W > 0$ in case of no default and $L < 0$ in case of default. Default occurs if and only if $\theta \leq 1 - a$. Hence,

$$U^{PM}(\theta, a) \equiv W \times \mathbb{I}\{\theta > 1 - a\} + L \times \mathbb{I}\{\theta \leq 1 - a\}.$$

As for the receiver's payoff, we consider two cases. The first one corresponds to a market where the receiver's payoff is aligned with the policy maker's payoff, as in the baseline model of Section 2. The second case, instead, corresponds to a market where the receiver's payoff is misaligned with the policy maker's payoff, over the critical region of fundamentals $(0, 1)$ where the fate of the bank depends on the receiver's behavior.

Case 1: Aligned Preferences

The receiver's payoff differential between taking the friendly action (interpreted as "pledging" to the bank) and the adversarial action (interpreted as "refraining from pledging") is given by $u^I(\theta, 1) - u^I(\theta, 0) = b < 0$ in case of default, and by $u^I(\theta, 1) - u^I(\theta, 0) = g > 0$ in case of no default, with $g > 0 > b$. In this case, it is immediate to see that the following pass/fail policy is optimal: the policy maker gives a pass to all banks with fundamentals $\theta > 0$ and a fail to all banks with fundamentals $\theta \leq 0$.

Case 2: Misaligned Preferences

The receiver's payoff differential between taking the friendly action (interpreted as "refraining from speculating" against the bank) and the adversarial action (interpreted as "speculating" against the bank) is given by $u^I(\theta, 1) - u^I(\theta, 0) = -g < 0$ for any $\theta \leq 1$, and by $u^I(\theta, 1) - u^I(\theta, 0) = -b > 0$ for any $\theta > 1$, with $g > 0 > b$. That is, the receiver obtains a payoff equal to 0 when she abstains from speculating against the bank (the friendly action). When, instead, she speculates against the bank (the adversarial action), she obtains a payoff equal to $g > 0$ in case speculation is successful (i.e., in case of default) and a payoff equal to $b < 0$ in case the bank survives the attack.

We start by showing that, in this case, Assumptions 2 and 3 in Guo & Shmaya (2019) are satisfied. In fact, note that, for any realization $x \in \mathbb{R}$ of the receiver's signal, the ratio between the receiver's and the sender's payoff differential is equal to

$$\varphi(\theta) \equiv \frac{u^I(\theta, 1) - u^I(\theta, 0)}{U^{PM}(\theta, 1) - U^{PM}(\theta, 0)} = \begin{cases} -\infty & \theta \leq 0 \\ \frac{-g}{W-L} & \theta \in (0, 1] \\ +\infty & \theta > 1 \end{cases}$$

and is increasing in θ, which implies that Assumption 2 in Guo & Shmaya (2019) holds. That Assumption 3 also holds follows from noting that the receiver's payoff differential changes from negative to positive at $\theta = 1$, for any $x \in \mathbb{R}$. By virtue of Theorem 3.1 in Guo & Shmaya (2019),

below.

the optimal policy is thus a deterministic cutoff mechanism that recommends to take action $a = 1$ on intervals $(\underline{\pi}(x), \bar{\pi}(x)) \subset \Theta$, with $\underline{\pi}(x)$ decreasing in x, and $\bar{\pi}(x)$ increasing in x.

Next observe, when there is a continuum of receivers with the same payoffs as the representative speculator above, under an adversarial/robust design, the optimal policy satisfies the same properties as in Theorems 1-3 in the baseline model of Section 2 in the main body, despite the misalignment in payoffs. This is because, under MARP, independently of whether the payoffs are aligned or misaligned, all agents play the adversarial action unless it is iteratively dominant for them to play the friendly action, exactly as in the baseline model.

The optimal policy with a single receiver is thus fundamentally different from the optimal policy with multiple receivers. First, with a single receiver, the optimal policy cannot be implemented with a simple pass/fail announcement. It requires sending multiple (in fact a continuum) of grades. Each grade is associated with a different cut-off $x^*(s)$ such that, given the announced grade s, the receiver plays the friendly action only if $x > x^*(s)$. With multiple receivers, instead, when $p(x|\theta)$ is log-supermodular, as assumed here, the optimal policy is a simple pass/fail test (Theorems 2 and 4 in the main body).

Second, observe that, with a single receiver, the optimal policy has the *interval structure.* That is, for any x, the optimal policy induces the receiver to play the friendly action over an interval $(\underline{\pi}(x), \bar{\pi}(x)) \subset \Theta$ of states. With multiple receivers, instead, the optimal policy has the interval structure only when it is monotone (in this case, the interval is $(\underline{\pi}(x), \bar{\pi}(x)) = (\theta^*, +\infty)$ for all x).

We conclude that the structure of the optimal policy with a single (privately informed) receiver is fundamentally different from the one with multiple (privately informed) receivers.

Subsection S4.2. Multiple Receivers with No Exogenous Private Information

Next, consider an economy with a continuum of investors/receivers, of measure 1, but assume that they do not possess any exogenous private information. All investors share the policy maker's prior F about θ. As in the main body, denote by $A \in [0, 1]$ the measure of investors taking the friendly action and let $u^I(\theta, A)$ denote the representative investor's payoff differential between taking the friendly and the adversarial action when fundamentals are θ and the fraction of investors choosing the friendly action is A.

When investors do not possess exogenous private information, the optimal policy is a monotone binary policy, irrespective of whether the agents' payoffs are aligned or not with the policy maker's payoff. To see this, denote by $\mu_s^\pi \in \Delta(\Theta)$ the common posterior generated by the observation of signal realization $s \in S$ under the policy π. When investors play according to MARP, the only way the policy maker can induce an investor to play the friendly action is to convince him that the friendly action is strictly dominant for him. That is, each investor plays the friendly action at s if, and only if,[30]

$$\int_{\Theta} u^I(\theta, 0)\, \mu_s^{\pi}(d\theta) > 0.$$

As a result, under the adversarial/robust design, the game with multiple receivers who possess no exogenous private information is isomorphic to a game with a single receiver with payoff differential equal to $u^I(\theta, 0)$. That the optimal policy in such a case is monotone follows from Mensch (2018) and Inostroza (2019).

The optimal policy is thus again fundamentally different from the optimal policy for the economy with multiple receivers possessing heterogenous private information.

Section S5: Generalization of Perfect Coordination Property

The model in Section 2 in the main text is modified as follows.

Agents and exogenous information. Let N denote the set of agents; N is assumed to be measurable and can be finite or infinite. For each $i \in N$, let X_i denote a measurable set and define $\mathcal{X} \equiv \prod_{i \in N} X_i$. The set $\mathcal{X}$ is endowed with the product topology. For each $i \in N$, let $\Lambda_i : X_i \to \Delta(\Theta \times \mathcal{X})$ be a measurable function (with respect to the Borel sigma-algebra associated with X_i). The profile $\mathbf{x} = (x_i)_{i \in N} \in \mathcal{X}$ indexes the hierarchy of the agents' exogenous beliefs about θ and the beliefs of other agents.

The state of Nature in this environment is denoted by $\omega = (\theta, \mathbf{x}) \in \Omega \equiv \Theta \times \mathcal{X}$ and comprises the realization of the payoff fundamental θ and the exogenous profile of the agents' beliefs $\mathbf{x}$. Note that no restriction on the agents' belief profile $\mathbf{x}$ is imposed. In particular, the agents' beliefs need not be consistent with a common prior, nor be generated by signals drawn independently conditionally on θ.

Payoffs. Each agent's expected payoff differential (between pledging and not pledging) is given by

$$u_i(\theta, A) = \begin{cases} g_i(\theta, A) & \text{if } r = 1 \\ b_i(\theta, A) & \text{if } r = 0, \end{cases}$$

$i \in N$, where A denotes the aggregate size of the pledge (in case of finitely many agents, A coincides with the number of agents pledging). The functions g_i and b_i are continuously differentiable and satisfy the same monotonicity assumptions as in the main text. In other words, for any $i \in N$, any (θ, A): (a) $\frac{\partial}{\partial \theta} g_i(\theta, A), \frac{\partial}{\partial \theta} b_i(\theta, A) \geq 0$, (b) $\frac{\partial}{\partial A} g_i(\theta, A), \frac{\partial}{\partial A} b_i(\theta, A) \geq 0$; and (c) $g_i(\theta, A) > 0 > b_i(\theta, A)$. Default occurs if and only if $R(\theta, A) \leq 0$, where R is increasing in (θ, A).

For simplicity, and to better highlight the novel effects, we abstract from the possibility that the regime outcome (i.e., default), as well as the agents' payoffs, may depend on variables z only

[30] To see this, note that, because the game is supermodular, when the above inequality is reversed, playing the aggressive action becomes a best response to the conjecture that everyone else plays the aggressive action.

imperfectly correlated with θ. As explained in the main text (see the discussion around Theorem 4), the possibility of increasing the agents' expected payoffs while coordinating them on the same course of action extends to economies in which the regime outcome is a stochastic function of (θ, A). The optimality of policies satisfying the perfect coordination property also extends to these more general economies provided the distribution from which the agents' signals are drawn satisfies the MLRP property and the planner's payoff satisfies condition PC in the main text.

Disclosure Policies. Let $\mathcal{S}$ be a compact metric space defining the set of possible disclosures to the agents. Let $m : N \to \mathcal{S}$ denote a *message function,* specifying, for each individual $i \in N$, the endogenous signal $m_i \in \mathcal{S}$ disclosed to the individual. Let $M(\mathcal{S})$ denote the set of all possible message functions with codomain $\mathcal{S}$. Let $\mathcal{P}$ be a partition of Ω and $h(\omega)$ the information set (equivalently, the cell) in $\mathcal{P}$ containing the state $\omega \in \Omega$. A *disclosure policy* $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ consists of a set $\mathcal{S}$ along with a mapping $\pi : \Omega \to \Delta(M(\mathcal{S}))$ measurable with respect to the σ-algebra defined by the partition $\mathcal{P}$.[31] For each ω, $\pi(\omega)$ denotes the lottery whose realization yields the message function used by the policy maker to communicate with the agents. The case in which the partition $\mathcal{P}$ coincides with Ω corresponds to the case in which the policy maker is able to distinguish any two states in Ω (in this case the σ-algebra associated with Ω is the Borel σ-algebra).

Solution Concept. Agents have a level-K degree of sophistication. The policy maker adopts a conservative approach and evaluates the performance of any given policy on the basis of the "worse outcome" consistent with the agents playing (interim correlated) level-K rationalizable strategies. That is, for any given selected policy Γ, the policy maker expects the market to play according to the "most aggressive level-K rationalizable profile" defined as follows:

Definition S1. Given any policy Γ, any $K \in \mathbb{N} \cup \{\infty\}$, the most aggressive level-K rationalizable profile (MARP-K) associated with Γ is the strategy profile $a^{\Gamma}_{(K)} \equiv (a^{\Gamma}_{(K),i})_{i \in [0,1]}$ that minimizes the policy maker's ex-ante expected payoff, among all profiles surviving K rounds of *iterated deletion of interim strictly dominated strategies.*

Hereafter we use IDISDS to refer to the process of iterated deletion of interim strictly dominated strategies.

Definition S2. A policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ satisfies the **perfect-coordination property (PCP)** if, for any $\omega \in \Omega$, any message function $m \in supp[\pi(\omega)]$, any $i, j \in N$, $a^{\Gamma}_{(K),i}(x_i, m_i) = a^{\Gamma}_{(K),j}(x_j, m_j)$.

Fix an arbitrary policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$. For any $\omega \in \Omega$, any message function $m \in supp[\pi(\omega)]$, let $r(\omega, m; a^{\Gamma}_{(K)}) \in \{0, 1\}$ denote the regime outcome that prevails at ω when the distribution of endogenous signals is m, and agents play according to the strategy profile $a^{\Gamma}_{(K)}$.

[31] That is, by the collection of $\mathcal{P}$-saturated sets. Let $\mathcal{B}$ be the standard Borel σ-algebra associated with the primitive set Ω. A set $A \in \mathcal{B}$ is $\mathcal{P}$-saturated if $\omega \in A$ implies $h(\omega) \subseteq A$. Thus $A = \cup_{\omega \in A} h(\omega)$.

Definition S3. The disclosure policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ is **regular** if for any $\omega', \omega'' \in \Omega$ for which $h(\omega') = h(\omega'')$ and any $m \in supp\ [\pi(\omega')] = supp\ [\pi(\omega'')]$, $r(\omega', m; a^{\Gamma}_{(K)}) = r(\omega'', m; a^{\Gamma}_{(K)})$.

A disclosure policy is thus regular if the default outcome induced by MARP-K compatible with Γ is measurable with respect to the policy maker's information (as captured by the partition $\mathcal{P}$).[32] With an abuse of notation, when we find it convenient to highlight the measurability restriction implied by the regularity of the policy, we will denote by $r(h(\omega), m; a^{\Gamma}_{(K)}) \in \{0, 1\}$ the regime outcome that prevails *at any state in* $h(\omega)$ under the message function m. Observe that, when the policy maker can perfectly distinguish between any two states, then any policy is regular.

Theorem S1. *For any regular policy* Γ*, there exists another regular policy* Γ^* *satisfying* ***PCP*** *and such that, for any* ω*, the probability of default under* Γ^* *is the same as under* Γ*.*

Proof of Theorem S1. Let $\mathcal{A}^{\Gamma} \equiv \{(a_i(\cdot) : X_i \times \mathcal{S} \to [0, 1])_{i \in N}\}$ denote the entire set of strategy profiles in the continuation game among the agents that starts with the policy maker announcing the policy Γ. For any $n \in \mathbb{N}$, let $T^{\Gamma}_{(n)}$ denote the set of strategies surviving n rounds of IDISDS under the original policy Γ, with $T^{\Gamma}_{(0)} = \mathcal{A}^{\Gamma}$. Denote by $\bar{a}^{\Gamma}_{(n)} \equiv (\bar{a}^{\Gamma}_{(n),i}(\cdot))_{i \in [0,1]} \in T^{\Gamma}_{(n)}$ the profile in $T^{\Gamma}_{(n)}$ that minimizes the policy maker's ex-ante payoff. Such a profile also minimizes the policy maker's interim payoff, as it will become clear from the arguments below. Hereafter, we refer to the profile $\bar{a}^{\Gamma}_{(n)}$ as the most aggressive profile surviving n rounds of IDISDS. The profiles $(\bar{a}^{\Gamma}_{(n)})_{n \in \mathbb{N}}$ can be constructed inductively as follows. The profile $\bar{a}^{\Gamma}_{(0)} \equiv (\bar{a}^{\Gamma}_{(0),i}(\cdot))_{i \in [0,1]})$ prescribes that all agents refrain from pledging irrespective of their exogenous and endogenous signals; that is, each $\bar{a}^{\Gamma}_{(0),i}(\cdot)$ is such that $\bar{a}^{\Gamma}_{(0),i}(x_i, s) = 0$, for all $(x_i, s) \in X_i \times \mathcal{S}$.[33] Given any strategy profile $a \in \mathcal{A}^{\Gamma}$, any $i \in N$, let $U^{\Gamma}_i(x_i, m_i; a)$ denote the payoff that agent i with exogenous signal x_i and endogenous signal m_i obtains from pledging, when all other agents follow the behavior specified by the strategy profile a. For any $n \geq 1$, the most aggressive strategy profile surviving n rounds of IDISDS is the one specifying, for each agent i, each $(x_i, m_i) \in X_i \times \mathcal{S}$, $\bar{a}^{\Gamma}_{(n),i}(x_i, m_i) = 1$ if $U^{\Gamma}_i(x_i, m_i; \bar{a}^{\Gamma}_{(n-1)}) > 0$ and $\bar{a}^{\Gamma}_{(n),i}(x_i, m_i) = 0$ if $U^{\Gamma}_i(x_i, m_i; \bar{a}^{\Gamma}_{(n-1)}) \leq 0$. The most aggressive level-K rationalizable strategy profile (MARP-K) consistent with the policy Γ is thus the profile $\bar{a}_{(K)}{}^{\Gamma} = (\bar{a}^{\Gamma}_{(K),i}(\cdot))_{i \in N}) \in T^{\Gamma}_{K}$. The case of fully rational agents in the main text corresponds to the limit in which $K \to \infty$. To be consistent with the notation in the main text, we denote MARP consistent with Γ by dropping the subscript K and denoting such profile by $\bar{a}^{\Gamma} \equiv ((\bar{a}^{\Gamma}_i(\cdot))_{i \in N}$, with $\bar{a}^{\Gamma}_i(\cdot) \equiv \lim_{K \to \infty} \bar{a}^{\Gamma}_{(K),i}(\cdot)$, all $i \in N$.

Now, consider the policy $\Gamma^+ = (\mathcal{S}^+, \mathcal{P}, \pi^+)$, $\mathcal{S}^+ = \mathcal{S} \times \{0, 1\}$, obtained from the original policy Γ by replacing each message function $m : N \to \mathcal{S}$ in the support of each $\pi(\omega)$ with the message function $m^+ : N \to \mathcal{S}^+$ that discloses to each agent $i \in N$ the same message m_i disclosed by the original policy m, along with the regime outcome $r(\omega, m; \bar{a}^{\Gamma}_{(K)})$ that would have prevailed at

[32] Note that regularity is violated in the two-state-two-receiver model in Alonso and Zachariadis (2021)

[33] Note that, to ease the notation, we let each individual strategy prescribe an action for all $(x_i, m_i) \in \mathbb{R} \times \mathcal{S}$, including those that may be inconsistent with the policy Γ.

(ω, m) under Γ when all agents play according to the most aggressive level-K rationalizable strategy profile $\bar{a}^{\Gamma}_{(K)}$ consistent with the original policy Γ. That is, for each $\omega \in \Omega$, each $m \in supp[\pi(\omega)]$, the policy Γ^+ selects the message function m^+ obtained from the original message function m by adding to its codomain the regime outcome $r(\omega, m; \bar{a}^{\Gamma}_{(K)})$ that would have prevailed at (ω, m) under MARP-K $\bar{a}^{\Gamma}_{(K)}$, with the same probability that Γ would have selected the original message function m. Hereafter, we denote by $m_i^+ = (m_i, r(\omega, m; \bar{a}^{\Gamma}_{(K)}))$ the message sent to agent i under the new policy Γ^+ when the exogenous state is ω and the message function selected under the original policy Γ is m. Note that the assumption that Γ is regular implies that Γ^+ is measurable with respect to the σ-algebra generated by $\mathcal{P}$ and hence also regular.

Let $\mathcal{A}^{\Gamma^+} \equiv \{(a_i(\cdot) : X_i \times \mathcal{S} \times \{0, 1\}\} \to [0, 1])_{i \in N}\}$ denote the set of strategy profiles in the continuation game among the agents that starts with the policy maker announcing the new policy Γ^+. For any $n \in \mathbb{N}$, let $T^{\Gamma^+}_{(n)} \subset \mathcal{A}^{\Gamma^+}$ denote the set of strategies surviving n rounds of IDISDS under the new policy Γ^+, with $T^{\Gamma^+}_{(0)} = \mathcal{A}^{\Gamma^+}$. Denote by $\bar{a}^{\Gamma^+}_{(n)} \equiv (\bar{a}^{\Gamma^+}_{(n),i}(\cdot))_{i \in N} \in T^{\Gamma^+}_{(n)}$ the profile in $T^{\Gamma^+}_{(n)}$ that minimizes the policy maker's ex-ante payoff, and observe that $\bar{a}^{\Gamma^+}_{(0)} \equiv (\bar{a}^{\Gamma^+}_{(0),i}(\cdot))_{i \in N}$ prescribes that all agents refrain from pledging, irrespective of their exogenous and endogenous signals.

Step 1. First, we prove that, for any $i \in N$,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \forall a \in \mathcal{A}^{\Gamma}\}$$

$$\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 1); a) > 0 \ \forall a \in \mathcal{A}^{\Gamma^+}\}.$$

That is, any agent who finds it dominant to pledge under Γ after receiving information (x_i, m_i) also finds it dominant to pledge under Γ^+ after receiving information $(x_i, (m_i, 1))$. To see this, first use the fact that the game is supermodular to observe that, given any policy Γ,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \forall a \in \mathcal{A}^{\Gamma}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) > 0\}.$$

Likewise,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 0); a) > 0 \ \forall a \in \mathcal{A}^{\Gamma^+}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 0); \bar{a}^{\Gamma^+}_{(0)}) > 0\}.$$

Next, observe that, because under both $\bar{a}^{\Gamma}_{(0)}$ and $\bar{a}^{\Gamma^+}_{(0)}$ all agents refrain from pledging, regardless of their exogenous and endogenous information, under both $\bar{a}^{\Gamma}_{(0)}$ and $\bar{a}^{\Gamma^+}_{(0)}$, default occurs if, and only if, $\theta \leq \bar{\theta}$ (with $\bar{\theta}$ defined by $R(\bar{\theta}, 0) = 0$). Then, note that, under Γ^+, for any $i \in N$, any $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\partial \Lambda_i^{\Gamma^+}(\omega, m | x_i, (m_i, 1)) = \frac{\mathbb{I}_{\{r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}}}{\Lambda_i^{\Gamma}(1 | x_i, m_i)} \partial \Lambda^{\Gamma}(\omega, m | x_i, m_i), \tag{S12}$$

where

$$\Lambda_i^{\Gamma}(1 | x_i, m_i) \equiv \int_{\{(\omega, m) : r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}} d\Lambda^{\Gamma}(\omega, m | x_i, m_i)$$

is the total probability that, under the policy Γ, agent i with information (x_i, m_i) assigns to the event $\{(\omega, m) \in \Omega \times M(\mathcal{S}) : r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}$. Under Bayesian learning, the agents' beliefs under the new policy policy Γ^+ thus correspond to "truncations" of their beliefs under the original policy Γ. In turn, this property of Bayesian updating implies that, for any $(x_i, m_i) \in X_i \times \mathcal{S}$ such that

$$U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) = \int_{(\omega,m)} \left(b_i(\theta, 1)\mathbb{I}_{\{\theta \leq \bar{\theta}\}} + g_i(\theta, 1)\mathbb{I}_{\{\theta > \bar{\theta}\}} \right) d\Lambda_i^{\Gamma}(\omega, m|x_i, m_i) > 0,$$

it must be that

$$\begin{aligned}
U_i^{\Gamma^+}(x_i, (m_i, 1); \bar{a}^{\Gamma^+}_{(0)}) &= \frac{1}{\Lambda_i^{\Gamma}(1|x_i, m_i)} \int_{(\omega,m)} \left(b_i(\theta, 1)\mathbb{I}_{\{\theta \leq \bar{\theta}\}} + g_i(\theta, 1)\mathbb{I}_{\{\theta > \bar{\theta}\}} \right) \times \\
&\quad \times \mathbb{I}_{\{r(\omega, m; \bar{a}^{\Gamma}_{(K)})=1\}} d\Lambda_i^{\Gamma}(\omega, m|x_i, m_i) \\
&> \frac{1}{\Lambda_i^{\Gamma}(1|x_i, m_i)} \int_{(\omega,m)} \left(b_i(\theta, 1)\mathbb{I}_{\{\theta \leq \bar{\theta}\}} + g_i(\theta, 1)\mathbb{I}_{\{\theta > \bar{\theta}\}} \right) d\Lambda_i^{\Gamma}(\omega, m|x_i, m_i) \\
&= \frac{1}{\Lambda_i^{\Gamma}(1|x_i, m_i)} U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) \\
&> 0,
\end{aligned}$$

where the first equality follows from the truncation property of Bayesian updating, the first inequality from the fact that, for all $(\omega, m) \in \Omega \times M(\mathcal{S})$ such that $r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 0$, $\theta \leq \bar{\theta}$, and hence $r(\omega, m; \bar{a}^{\Gamma}_{(0)}) = 0$, implying that

$$b_i(\theta, 1)\mathbb{I}_{\{\theta \leq \bar{\theta}\}} + g_i(\theta, 1)\mathbb{I}_{\{\theta > \bar{\theta}\}} = b_i(\theta, 1) < 0,$$

the second equality from the definition of $U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)})$, and the second inequality from the fact that $U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) > 0$.

The above result implies that any an agent who, under Γ, finds it dominant to pledge after receiving information (x_i, m_i) also finds it dominant to pledge under Γ^+ after receiving information $(x_i, (m_i, 1))$, as claimed.

Step 2. We now show that a property analogous to the one established in Step 1 applies to any other round of the IDISDS procedure. The result is established by induction. Take any round $n \in \{1, 2, ..., K\}$ and assume that, for any $0 \leq k \leq n - 1$, any $i \in [0, 1]$,

$$\begin{aligned}
&\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \;\; \forall a \in T^{\Gamma}_{(k-1)}\} \\
&\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 1); a) > 0, \;\; \forall a \in T^{\Gamma^+}_{(k-1)}\}.
\end{aligned} \tag{S13}$$

Recall that this means that any agent who, under Γ, finds it optimal to pledge when his opponents play *any* strategy surviving k rounds of IDISDS under Γ continues to find it optimal to pledge when expecting his opponents to play *any* strategy surviving k rounds of IDISDS under Γ^+. Below we show that that the same property extends to strategies surviving n rounds of IDISDS. That is,

$$\begin{aligned}
&\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \;\; \forall a \in T^{\Gamma}_{(n-1)}\} \\
&\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 1); a) > 0, \;\; \forall a \in T^{\Gamma^+}_{(n-1)}\}.
\end{aligned} \tag{S14}$$

To see this, use again the fact that the game is supermodular to observe that

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \ \forall a \in T_{(n-1)}^{\Gamma}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; \bar{a}_{(n-1)}^{\Gamma}) > 0\} \tag{S15}$$

and, likewise,

$$\begin{aligned} &\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 1); a) > 0, \ \ \forall a \in T_{(n-1)}^{\Gamma^+}\} \\ &= \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, m_i; \bar{a}_{(n-1)}^{\Gamma^+}) > 0\}, \end{aligned} \tag{S16}$$

where recall that $\bar{a}_{(n-1)}^{\Gamma}$ (alternatively, $\bar{a}_{(n-1)}^{\Gamma^+}$) is the most aggressive profile surviving $n-1 < K$ rounds of IDSIDS under Γ (alternatively, Γ^+).

Now let $A(\omega, m; a)$ denote the aggregate size of the pledge that, under Γ, prevails at (ω, m), when agents play according to $a \in \mathcal{A}^{\Gamma}$. Then take any $i \in N$ and any $(x_i, m_i) \in X_i \times \mathcal{S}$ such that

$$U_i^{\Gamma}(x_i, m_i; \bar{a}_{(n-1)}^{\Gamma}) = \int_{(\omega,m)} u_i(\theta, A(\omega, m; \bar{a}_{(n-1)}^{\Gamma})) d\Lambda_i^{\Gamma}(\omega, m | x_i, m_i) > 0.$$

Because $\bar{a}_{(n-1)}^{\Gamma}$ is more aggressive than $\bar{a}_{(K)}^{\Gamma}$, in the sense that, for any $i \in N$, any $(x_i, m_i) \in X_i \times \mathcal{S}$, $\bar{a}_{(n-1),i}^{\Gamma}(x_i, m_i) \leq \bar{a}_{K,i}^{\Gamma}(x_i, m_i)$, then for all (ω, m),

$$r(\omega, m; \bar{a}_{(K)}^{\Gamma}) = 0 \Rightarrow r(\omega, m; \bar{a}_{(n-1)}^{\Gamma}) = 0.$$

This implies that

$$\begin{aligned} \int_{(\omega,m)} u_i(\theta, A(\omega, m; \bar{a}_{(n-1)}^{\Gamma})) \mathbb{I}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=0\}} d\Lambda_i^{\Gamma}(\omega, m | x_i, m_i) &= \\ \int_{(\omega,m)} b_i(\theta, A(\omega, m; \bar{a}_{(n-1)}^{\Gamma})) \mathbb{I}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=0\}} d\Lambda_i^{\Gamma}(\omega, m | x_i, m_i) &< 0 \end{aligned} \tag{S17}$$

This observation, together with the truncation property in (S12), implies that, for any $i \in N$, any $(x_i, m_i) \in X_i \times \mathcal{S}$ such that $U_i^{\Gamma}(x_i, m_i; \bar{a}_{(n-1)}^{\Gamma}) > 0$,

$$\begin{aligned} U_i^{\Gamma^+}(x_i, (m_i, 1); \bar{a}_{(n-1)}^{\Gamma}) &= \int_{(\omega,m)} u_i(\theta, A(\omega, m; \bar{a}_{(n-1)}^{\Gamma})) d\Lambda_i^{\Gamma^+}(\omega, m | x_i, m_i) \\ &= \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)} \int_{(\omega,m)} u(\theta, A(\omega, m; \bar{a}_{(n-1)}^{\Gamma})) \mathbb{I}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=1\}} d\Lambda_i^{\Gamma}(\omega, m | x_i, m_i) \\ &> \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)} \int_{(\omega,m)} u(\theta, A(\omega, m; \bar{a}_{(n-1)}^{\Gamma})) d\Lambda_i^{\Gamma}(\omega, m | x_i, m_i) \\ &= \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)} U_i^{\Gamma}((x_i, m_i); \bar{a}_{(n-1)}^{\Gamma}) \\ &> 0 \end{aligned} \tag{S18}$$

where the first and third equalities are by definition, the second equality follows from (S12), the first inequality follows from (S17), and the last inequality from the fact that $U_i^{\Gamma}(x_i, m_i; \bar{a}_{(n-1)}^{\Gamma}) > 0$, by assumption.

Next, note that $\bar{a}_{(n-1)}^{\Gamma}$ and $\bar{a}_{(n-1)}^{\Gamma^+}$ are such that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$, $\bar{a}_{(n-1),i}^{\Gamma}(x_i, m_i)$, $\bar{a}_{(n-1),i}^{\Gamma^+}(x_i, (m_i, 0)) \in \{0, 1\}$ and

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : \bar{a}_{(n-1),i}^{\Gamma}(x_i, m_i) = 1\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; \bar{a}_{(n-2)}^{\Gamma}) > 0\}$$

and, likewise,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : \bar{a}^{\Gamma+}_{(n-1),i}(x_i, (m_i, 1)) = 1\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U^{\Gamma+}_i(x_i, (m_i, 1); \bar{a}^{\Gamma+}_{(n-2)}) > 0\}.$$

Together properties (S13), (S15) and (S16) imply that $\bar{a}^{\Gamma}_{(n-1)}$ and $\bar{a}^{\Gamma+}_{(n-1)}$ are such that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma}_{(n-1),i}(x_i, m_i) = 1 \Rightarrow \bar{a}^{\Gamma+}_{(n-1),i}(x_i, (m_i, 1)) = 1. \tag{S19}$$

Condition (S19), along with the fact that the game is supermodular, implies that

$$U^{\Gamma+}_i(x_i, (m_i, 1); \bar{a}^{\Gamma}_{(n-1)}) > 0 \Rightarrow U^{\Gamma+}_i(x_i, (m_i, 1); \bar{a}^{\Gamma+}_{(n-1)}) > 0. \tag{S20}$$

Together (S18) and (S20) imply the property in (S14).

Step 3. Equipped with the results in steps 1 and 2 above, we now prove that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma+}_{(K),i}(x_i, (m_i, 1)) = 1.$$

This follows directly from the fact that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma}_{(K),i}(x_i, m_i) = 1 \Rightarrow \bar{a}^{\Gamma+}_{(K),i}(x_i, (m_i, 1)) = 1, \tag{S21}$$

which, in turn implies that, for any (ω, m),

$$r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1 \Rightarrow r(\omega, m; \bar{a}^{\Gamma+}_{(K)}) = 1.$$

Under Γ^+, the announcement that $r = 1$ thus reveals to the agents that (ω, m) is such that $r(\omega, m; \bar{a}^{\Gamma+}_{(K)}) = 1$. Because the payoff from pledging is strictly positive when the bank survives, any agent i receiving a signal $(m_i, 1)$ thus necessarily pledges. Under MARP-K consistent with the new policy Γ^+ thus all agents pledge, regardless of their exogenous and endogenous private signals, when the policy publicly announces $r = 1$. That they all refrain from pledging, irrespective of (x_i, m_i), when the policy announces $r = 0$ follows from the fact that $r = 0$ makes it common certainty among the agents that (ω, m) is such that $r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 0$ and hence that $\theta \leq \bar{\theta}$. But then, irrespective of (x_i, m_i), any agent $i \in N$ receiving exogenous information x_i and endogenous information $m^+_i = (m_i, 0)$ finds it optimal to refrain from pledging when expecting all other agents to abstain from pledging no matter their exogenous and endogenous information. This implies that under MARP-K consistent with the new policy Γ^+, all agents refrain from pledging when hearing that $r = 0$.

We conclude that the new policy Γ^+ satisfies the perfect coordination property and is such that, for any $(\theta, \mathbf{x}) \in \Theta \times \mathcal{X}$, the probability of default under Γ^+ is the same as under Γ. This completes the proof of Theorem S1. Q.E.D.

Section S5: Discriminatory Disclosures

In this section, we consider an extension in which the policy maker can disclose different information to different market participants. The purpose of the section is to illustrate the possible benefits of discriminatory disclosures, when the latter are feasible. To maintain the analysis as simple as possible, we assume that the environment satisfies the conditions in Theorem 5, implying that, if the policy maker were to restrict attention to non-discriminatory policies, the optimal policy would be a simple monotone pass/fail test failing with certainty all institutions with fundamentals below a cut-off θ^* and passing with certainty all the others.

We start by explaining that the benefits of discriminatory disclosures stem from the possibility of increasing the uncertainty each agent faces about the beliefs that rationalize other agents' behavior. We then consider a parametric setting in which the policy maker can engineer any public disclosure of her choice, but is constrained to use Gaussian signals when communicating privately with the agents. The advantage of such a parametric approach is that the combination of the exogenous and the endogenous private information can be conveniently summarized in a uni-dimensional sufficient statistics. This in turn permits us to relate the benefits of discriminatory disclosures to the type of securities issued by the banks (more generally, to the sensitivity of the agents' payoffs to the underlying fundamentals).[34]

In this section, to simplify the exposition, we assume away the shocks z imperfectly correlated with θ.

Subsection S5.1: Benefits of Discriminatory Disclosures

Perhaps surprisingly, the reason why discriminatory disclosures may improve upon non-discriminatory ones has little to do with the possibility of tailoring the information disclosed to the agents to their prior beliefs. Discriminatory disclosures may outperform non-discriminatory ones because, by enhancing the dispersion of posterior beliefs, they make it harder for the agents to refrain from pledging, thus permitting the policy maker to save a larger set of institutions.

To illustrate the point in the simplest possible way, consider an economy in which the agents' prior beliefs are homogenous (formally, this amounts to assuming the exogenous private signals x are completely uninformative). Next let $u(\theta, A)$ denote the payoff from pledging when the fundamentals are θ and the aggregate size of the pledge is A. Notice that, for any $\hat{\theta}$ such that

$$\int u(\theta, 0) dF(\theta | \theta > \hat{\theta}) > 0,$$

the most aggressive rationalizable strategy profile following the public announcement that $\theta > \hat{\theta}$ is

[34] See also Li et al (2020) and Morris et al (2020) for the characterization of the optimal *discriminatory* policy when agents do not possess any exogenous private information.

such that every agent pledges.[35] Under the assumptions of Theorem 5 in the main text, the optimal non-discriminatory policy is then a threshold rule with cut-off equal to[36]

$$\hat{\theta}^* = \inf\{\hat{\theta} \in \mathbb{R} \; s.t. \int u(\theta, 0) dF(\theta|\theta > \hat{\theta}) > 0\}. \tag{S22}$$

Suppose now the policy maker, instead of announcing whether θ is above or below some threshold $\hat{\theta}$, sends to each individual a private signal of the form $m_i = \theta + \sigma\xi_i$, where $\sigma \in \mathbb{R}_+$ is a scalar, and where the idiosyncratic terms ξ_i are drawn from a smooth distribution over the entire real line (e.g., a standard Normal distribution), independently across agents, and independently from θ. From standard results in the global games literature, we know that, as the private messages become highly precise (formally, as $\sigma \to 0^+$), in the absence of any public disclosure, under the most aggressive rationalizable profile, each agent pledges if, and only if, his endogenous private signal falls above the threshold $\theta^{MS} \in (\underline{\theta}, \bar{\theta})$ implicitly defined by the unique solution to

$$\int_0^1 u(\theta^{MS}, A) dA = 0. \tag{S23}$$

As explained in the main text, the threshold θ^{MS} corresponds to the value of the fundamentals at which an agent who knows θ and holds *Laplacian* beliefs with respect to the size of the pledge[37] is indifferent between pledging and not pledging. Importantly, θ^{MS} *is independent of the initial common prior* and of the distribution of the noise terms ξ in the agents' signals. The above result thus implies that, with discriminatory disclosures, the policy maker can always guarantee that default never occurs for any $\theta > \theta^{MS}$. We then have the following result:[38]

Proposition S1. *Assume the agents possess no exogenous private information about the underlying fundamentals. Let $\hat{\theta}^*$ be the threshold in (S22) and θ^{MS} be the threshold in (S23). Whenever $\theta^{MS} < \hat{\theta}^*$, discriminatory disclosures strictly improve upon non-discriminatory ones.*

The result follows directly from the arguments preceding the proposition. Because $\hat{\theta}^*$ can be arbitrarily close to $\overline{\theta}$ for particular prior distributions, and because θ^{MS} is invariant in the prior distribution from which θ is drawn, the result in Proposition S1 is relevant in many cases of interest.

As anticipated above, the reason why discriminatory disclosures can improve upon non-discriminatory ones is that they permit the policy maker to enhance the dispersion of the agents higher-order beliefs. A higher dispersion in turn makes it more difficult for the agents to play adversarially to the policy

[35]The notation $F(\theta|\theta > \hat{\theta})$ stands for the common posterior obtained from the prior F by conditioning on the event that $\theta > \hat{\theta}$.

[36]Here we follow the same abuse as in the main text and refer to the optimal non-discriminatory policy as the monotone policy whose threshold is given by $\hat{\theta}^*$.

[37]This means that the agent believes that the size of the aggregate pledge is uniformly distributed over $[0, 1]$.

[38]The proposition shows that the condition $\theta^{MS} < \hat{\theta}^*$ is sufficient for discriminatory policies to strictly improve upon non-discriminatory ones. When, instead, $\theta^{MS} \geq \hat{\theta}^*$, whether or not the optimal policy is discriminatory depends on the prior F and on the sensitivity of the agents' payoffs to θ. See Li et al (2020) and Morris et al (2020) for a characterization of the optimal discriminatory policy when payoffs are constant in θ.

maker (i.e, to refrain from pledging). Formally speaking, when beliefs are sufficiently dispersed, an agent receiving a private signal indicating that the bank may collapse under a sufficiently large attack (i.e, in case few agents pledge) may nonetheless pledge if he expects many other agents to have received extreme signals indicating that the fundamentals are strong enough for the bank not to collapse, no matter the size of the attack. In this case, pledging may become *iteratively dominant* for this individual. The optimality of discriminatory policies thus follows from a "divide-and-conquer" logic reminiscent of the one in the vertical contracting literature (see, e.g., Segal, 2003 and the references therein). Importantly, when discriminatory policies outperform non-discriminatory ones, this is not because they mis-coordinate the response by the market (recall that, by virtue of Theorem 4, the optimal policy always satisfies the perfect-coordination property, irrespectively of whether or not it is discriminatory), but because, by enhancing the heterogeneity in structural uncertainty, they make it difficult for market participants to coordinate on an adversarial course of action when the planner recommends that they pledge.

Subsection S5.2: Payoff Sensitivity and the Optimality of Discriminatory Policies

We conclude by showing how the optimality of discriminatory policies may depend to the sensitivity of the agents' payoffs to the underlying fundamentals and relate such sensitivity to the type of securities issued by the banks under scrutiny. To gain on tractability, we consider an environment in which the prior distribution F from which θ is drawn is an improper uniform distribution over the entire real line and where the agents' exogenous private signals are given by $x_i = \theta + \sigma_\eta \eta_i$, with $\eta_i \sim \mathcal{N}(0,1)$.[39] Furthermore, to facilitate the aggregation of the agents' exogenous and endogenous signals into a uni-dimensional statistics, we restrict attention to the following parametric structure. The policy maker can engineer any public disclosure of her choice but is constrained to sending signals of the Gaussian form $\widetilde{m}_i = \theta + \sigma_\xi \xi_i$, with $\xi_i \sim \mathcal{N}(0,1)$, when communicating privately with the agents. The restriction to Gaussian private applies only to the information the policy maker discloses *privately* to the agents, over and above the information conveyed by the public test. In each state θ, the endogenous information $m_i = (\widetilde{s}, \widetilde{m}_i)$ disclosed to each agent i thus comprises a public signal $\widetilde{s}$, along with a private signal $\widetilde{m}_i$. Under such a structure, the quality of the private signals is then conveniently parametrized by the variance $\sigma_\xi^2 > 0$ of the endogenous noise terms.

We also assume the agents' payoff from pledging depends on the aggregate size of the pledge A only through the effects of the latter on the default outcome. In other words, we assume that there exist strictly increasing functions $\bar{g}(\theta)$ and $\bar{b}(\theta)$ such that the payoff of each agent pledging to the bank is equal to $\bar{g}(\theta)$ in case the bank does not default and equal to $\bar{b}(\theta)$, in case the bank default. The payoff from not pledging is equal to zero. Finally, we assume that the function R determining

[39] The assumption that F is an improper uniform distribution is standard in the global-game literature. It simplifies the formulas below, without any serious effect on the results. Also observe that the entire hierarchy of the agents' beliefs is well defined, despite the prior being improper.

the default outcome takes the same linear form $R(\theta, A) = \theta - 1 + A$ as in the baseline model.[40]

Then observe that the information contained in each pair $(x_i, \widetilde{m}_i)$ of exogenous and endogenous private signals is the same as the information contained in the sufficient statistics

$$z_i \equiv \frac{\sigma_\xi^2 x_i + \sigma_\eta^2 \tilde{m}_i}{\sigma_\eta^2 + \sigma_\xi^2},$$

which, given θ, is normally distributed with mean θ and variance $\sigma_z^2 \equiv \left(\sigma_\eta^2 \sigma_\xi^2\right) / \left(\sigma_\eta^2 + \sigma_\xi^2\right)$. Hence, the policy maker's choice of the discriminatory component of her disclosure policy can be conveniently reduced to the choice of the standard deviation σ_z of the above sufficient statistics, with $\sigma_z \in (0, \sigma_\eta]$.

Arguments analogous to those establishing Lemma 1 in the Appendix in the main document then imply that, for any realization $\widetilde{s}$ of the endogenous public signal, the most aggressive rationalizable strategy profile a^Γ is characterized by a unique cut-off $\bar{z}(\widetilde{s})$ (whose value depends on the distribution from which the public signal is drawn) such that, for all $i \in [0, 1]$, $a_i^\Gamma(x_i, (\widetilde{s}, \tilde{m}_i)) = \mathbb{I}\{z_i > \bar{z}(\tilde{s})\}$. Moreover, arguments similar to those establishing Theorem 2 in the main text imply that, for any given choice of σ_z^2, the optimal public announcement is binary with $\tilde{s} \in \{0, 1\}$ — that is, the public test has a pass/fail structure. Finally, from Theorem 5, the optimal policy has the perfect-coordination property which means that, given σ_z^2, $\bar{z}(0) = +\infty$, and $\bar{z}(1) = -\infty$. That is, all agents pledge when $\tilde{s} = 1$, and they all refrain from pledging when $\widetilde{s} = 0$.

Next, let Φ denotes the cdf of the standard Normal distribution, and, for any $\theta \in [0, 1]$, define

$$z_{\sigma_z}^*(\theta) \equiv \theta + \sigma_z \Phi^{-1}(\theta),$$

to be the private statistics threshold such that, when all agents refrain from pledging when $z_i < z_{\sigma_z}^*(\theta)$ and pledge when $z_i > z_{\sigma_z}^*(\theta)$, default occurs when the fundamentals fall below θ and does not occur when they are above θ.[41]

For any $(\theta_0, \hat{\theta}, \sigma_z)$, let $\psi(\theta_0, \hat{\theta}, \sigma_z)$ denote the payoff from pledging of an agent with private statistics $z_{\sigma_z}^*(\theta_0)$, when regime change occurs for all $\theta \leq \theta_0 \in [0, 1]$, public information reveals that $\theta \geq \hat{\theta}$, and the total precision of private information is σ_z^{-2}. Then let

$$\theta_{\sigma_z}^{inf} \equiv \inf\left\{\hat{\theta} : \ \psi(\theta_0, \hat{\theta}, \sigma_z) > 0 \ \text{ all } \ \theta_0 \in [0, 1]\right\}.$$

Note that, for any $\hat{\theta} > \theta_{\sigma_z}^{inf}$, under the most aggressive rationalizable strategy profile, all agents pledge after the public signal reveals that $\theta \geq \hat{\theta}$. Hereafter, we assume that all agents pledge also when public disclosures reveal that $\theta \geq \theta_{\sigma_z}^{inf}$. This simplifies the exposition below by permitting us to talk about the "optimal policy." As discussed in the main body, the latter does not formally exist when agents are expected to play according to the most aggressive rationalizable profile. However,

[40]The results below extend to more general payoff functions, as long as the agents' exogenous signals x are sufficiently precise.

[41]Given that $R(\theta, A) = \theta - 1 + A$, $z_{\sigma_z}^*(\theta)$ is implicitly defined by the equation $\Phi\left(\frac{z^* - \theta}{\sigma_z}\right) = \theta$.

because the policy maker can always guarantee that, no matter the selection of the rationalizable strategy profile, each agent pledges for any $\theta > \theta^{inf}_{\sigma_z}$, we find the abuse justified.

Proposition S2. *Suppose the policy maker is constrained to using Gaussian signals when communicating privately with the agents. Let*

$$\sigma_z^* \equiv \arg\min_{\sigma_z \in (0,\sigma_\eta]} \theta^{inf}_{\sigma_z}.$$

The optimal disclosure policy has the following structure. The policy maker publicly announces whether $\theta < \theta^{inf}_{\sigma_z^*}$, or whether $\theta \geq \theta^{inf}_{\sigma_z^*}$. In addition, when $\theta \geq \theta^{inf}_{\sigma_z^*}$, the policy maker sends a Gaussian private signal to each agent of precision $\sigma_\xi^{-2} = [\sigma_\eta^2 - (\sigma_z^*)^2]/(\sigma_z^*)^2\sigma_\eta^2$.

The result follows from the arguments preceding the proposition – note that the precision of the endogenous private information σ_ξ^{-2} in the proposition is the one that, together with the precision of the exogenous signals σ_η^{-2} yields a total precision σ_z^{-2} for the sufficient statistics z_i that minimizes the threshold $\theta^{inf}_{\sigma_z}$ defining the default outcome.

Equipped with the result in Proposition S2, we can then identify primitive conditions under which the optimal policy is non-discriminatory. By virtue of Proposition S2, discriminatory disclosures strictly dominate non-discriminatory ones if, and only if, $\sigma_z^* < \sigma_\eta$ (equivalently, if, and only if, there exists $\sigma_z < \sigma_\eta$ such that $\theta^{inf}_{\sigma_z} < \theta^{inf}_{\sigma_\eta}$). For any precision σ_z^{-2} of the agents' private statistics, let $\theta^{\#}_{\sigma_z}$ denote the unique solution to the equation $\psi(\theta^{\#}_{\sigma_z}, \theta^{inf}_{\sigma_z}, \sigma_z) = 0$. Note that, under MARP, $\theta^{\#}_{\sigma_z}$ identifies the fundamental threshold below which regime change occurs when the total precision of the agents' private information is σ_z^{-2} , and the endogenous disclosure of public information reveals that $\theta \geq \theta^{inf}_{\sigma_z}$. Let[42]

$$D(\theta, \theta^{\#}_{\sigma_z}) \equiv \begin{cases} \bar{b}'(\theta) \text{ if } \theta < \theta^{\#}_{\sigma_z} \\ \bar{g}'(\theta) \text{ if } \theta \geq \theta^{\#}_{\sigma_z}. \end{cases}$$

Proposition S3. *Suppose that, for any $\sigma_z \in (0, \sigma_\eta]$,*

$$\mathbb{E}[D(\theta, \theta^{\#}_{\sigma_z})(\theta - \theta^{\#}_{\sigma_z})|z^*(\theta^{\#}_{\sigma_z}), \theta \geq \theta^{inf}_{\sigma_z}; \sigma_z] \geq 0. \tag{S24}$$

Then the optimal policy is non-discriminatory.

The formal proof is below. Here we first discuss the intuition behind the result and its implications. The condition in Proposition S3 is a measure of the sensitivity of the marginal agent's net payoff from pledging to the underlying fundamentals.[43] To see this, note that the condition is equivalent to[44]

$$\frac{\mathbb{E}[\bar{g}'(\theta)(\theta - \theta^{\#}_{\sigma_z})|z^*(\theta^{\#}_{\sigma_z}), \theta \geq \theta^{\#}_{\sigma_z}; \sigma_z]}{\mathbb{E}[\bar{g}(\theta)|z^*(\theta^{\#}_{\sigma_z}), \theta \geq \theta^{\#}_{\sigma_z}; \sigma_z]} \geq \frac{\mathbb{E}[\bar{b}'(\theta)(\theta^{\#}_{\sigma_z} - \theta)|z^*(\theta^{\#}_{\sigma_z}), \theta \in (\theta^{inf}_{\sigma_z}, \theta^{\#}_{\sigma_z}); \sigma_z]}{\mathbb{E}[|\bar{b}(\theta)||z^*(\theta^{\#}_{\sigma_z}), \theta \in (\theta^{inf}_{\sigma_z}, \theta^{\#}_{\sigma_z}); \sigma_z]}.$$

[42]Here $\bar{b}'$ and $\bar{g}'$ denote the derivatives of the $\bar{b}$ and $\bar{g}$ functions, respectively.

[43]The marginal agent is the one with signal $z^*_{\sigma_z}(\theta^{\#}_{\sigma_z})$.

[44]See also Iachan and Nenov (2015) for a similar condition in a related class of games of regime change.

The left-hand side is the elasticity of the marginal agent's expected net payoff from pledging with respect to the underlying fundamentals, in case of no default. The right-hand side is the corresponding elasticity in case of default.[45]

To gather some intuition, consider the case in which, when default occurs, the agents' payoff differential between pledging and not pledging is constant in the underlying fundamentals (i.e., $\bar{b}'(\theta) = 0$ for all θ). In this case, the marginal agent faces only *upside risk*. Hence, when the quality of private information decreases (which amounts to a mean-preserving increase in risk), the agent's expected net payoff from pledging increases. Starting from any policy that discloses private information to the agents (i.e., for which $\sigma_z < \sigma_\eta$), the policy maker can then do better by reducing the precision of the agents' private information. In this case, the optimal policy is non-discriminatory.

The value of Proposition S3 is in indicating how the optimality of discriminatory disclosures relates to the sensitivity of the agents' payoffs to the underlying fundamentals. In turn, such sensitivity typically depends on the type of security issued by the banks. For example, the above condition is more likely to hold when investors are *equity holders*. In this case, when the bank defaults, their claims are junior (i.e., subordinated) with respect to those from other stake holders with higher seniority (e.g., bond holders). In case of default, the agents' payoff then amount to a liquidation value that is typically little sensitive to the exact amount of the bank's performing loans (the bank's fundamentals). On the contrary, when the bank does not default (i.e., when the government succeeds in persuading the bank's equity holders to stay put), the value of the equity-holders' claims reflect the bank's long-term profitability, which is sensitive to the amount of the bank's performing loans. The result in Proposition S3 thus indicates that discriminatory disclosures are more likely to be beneficial when the banks are seeking external funding by issuing bonds than when they do so by issuing equity.

Proof of Proposition S3. We establish the result by showing that, when Condition (S24) holds, for any fixed $\hat{\theta}$, the function $\Psi(\hat{\theta}, \sigma_z) \equiv \min_{\theta_0 \in [0,1]} \psi(\theta_0, \hat{\theta}, \sigma_z)$ is increasing in σ_z. Moreover, in this case, the regime threshold in the absence of any public disclosure, $\theta^*_{\sigma_z}$, implicitly defined by $\psi(\theta^*_{\sigma_z}, -\infty, \sigma_z) = 0$, is decreasing in σ_z, with $\lim_{\sigma_z \to 0^+} \theta^*_{\sigma_z} = \theta^{MS}$.

To ease the notation, let $\sigma = \sigma_z$. By the envelope theorem, we have that $\frac{\partial}{\partial \sigma}\Psi(\hat{\theta}, \sigma) = \frac{\partial}{\partial \sigma}\psi(\bar{\theta}_\sigma, \hat{\theta}, \sigma)$,

[45] Observe that, for the marginal agent with signal $z^*_{\sigma_z}(\theta^\#_{\sigma_z})$,

$$Pr(\theta \geq \theta^\#_{\sigma_z} | z^*_{\sigma_z}(\theta^\#_{\sigma_z}), \theta \geq \theta^{inf}_{\sigma_z}; \sigma_z)\mathbb{E}[\bar{g}(\theta)|z^*_{\sigma_z}(\theta^\#_{\sigma_z}), \theta \geq \theta^\#_{\sigma_z}; \sigma_z] =$$
$$Pr(\theta \in (\theta^{inf}_{\sigma_z}, \theta^\#_{\sigma_z})|z^*_{\sigma_z}(\theta^\#_{\sigma_z}), \theta \geq \theta^{inf}_{\sigma_z}; \sigma_z)\mathbb{E}[|\bar{b}(\theta)||z^*_{\sigma_z}(\theta^\#_{\sigma_z}), \theta \in (\theta^{inf}_{\sigma_z}, \theta^\#_{\sigma_z}); \sigma_z].$$

with $\bar{\theta}_\sigma \in \arg\min_{\theta_0 \in [0,1]} \psi(\theta_0, \hat{\theta}, \sigma)$. Note that, for any $\theta_0 > \hat{\theta}$, any σ,

$$\frac{\partial}{\partial\sigma}\psi(\theta_0, \hat{\theta}, \sigma) = \frac{\partial}{\partial\sigma}\int_{\hat{\theta}}^{\infty}\left(\bar{b}(\theta)\mathbb{I}_{\theta<\theta_0} + \bar{g}(\theta)\mathbb{I}_{\theta\geq\theta_0}\right)\frac{\phi\left(\frac{z^*_\sigma(\theta_0)-\theta}{\sigma}\right)}{\sigma\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}d\theta$$

$$= \frac{\partial}{\partial\sigma}\frac{\int_{1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}^{1}\left(\bar{b}(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbb{I}_{A<1-\theta_0}+\bar{g}(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbb{I}_{A>1-\theta_0}\right)dA}{\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

$$= \frac{\int_{1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}^{1}\left(\bar{b}'(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbb{I}_{A<1-\theta_0}+\bar{g}'(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbb{I}_{A>1-\theta_0}\right)(\Phi^{-1}(\theta_0)-\Phi^{-1}(1-A))dA}{\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

$$+\frac{(\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)(\theta_0-\hat{\theta})}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

where the second equality follows from the change of variables $A = 1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\theta}{\sigma}\right)$ along with the fact that, by definition, $1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\theta_0}{\sigma}\right) = 1-\theta_0$, while the third equality from using $z^*_\sigma(\theta) = \theta+\sigma\Phi^{-1}(\theta)$. Lastly, by reverting the change of variables, and letting

$$D(\theta,\theta_0) \equiv \begin{cases} \bar{b}'(\theta) \text{ if } \theta < \theta_0 \\ \bar{g}'(\theta) \text{ if } \theta \geq \theta_0, \end{cases}$$

we have that

$$\begin{aligned}
\frac{\partial}{\partial\sigma}\psi(\theta_0,\hat{\theta},\sigma) &= \frac{\int_{\hat{\theta}}^{\infty} D(\theta,\theta_0)(\theta-\theta_0)\phi\left(\frac{z^*_\sigma(\theta_0)-\theta}{\sigma}\right)d\theta + (\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)\left(\theta_0-\hat{\theta}\right)}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)} \\
&= \sigma^{-1}\mathbb{E}[D(\theta,\theta_0)(\theta-\theta_0)|z^*_\sigma(\theta_0),\theta\geq\hat{\theta}] + \frac{(\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)\left(\theta_0-\hat{\theta}\right)}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}.
\end{aligned}$$

When evaluated at $\hat{\theta} = \theta^{inf}_\sigma$ and $\theta_0 = \theta^\#_\sigma$, because $\psi(\theta^\#_\sigma,\theta^{inf}_\sigma,\sigma) = 0$, we have that the above expression becomes

$$\frac{\partial}{\partial\sigma}\psi(\theta^\#_\sigma,\theta^{inf}_\sigma,\sigma) = \sigma^{-1}\mathbb{E}[D(\theta,\theta^\#_\sigma)(\theta-\theta^\#_\sigma)|z^*_\sigma(\theta^\#_\sigma),\theta\geq\theta^{inf}_\sigma] + \frac{|b(\theta^{inf}_\sigma)|\phi\left(\frac{z^*_\sigma(\theta^\#_\sigma)-\theta^{inf}_\sigma}{\sigma}\right)\left(\theta^\#_\sigma-\theta^{inf}_\sigma\right)}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta^\#_\sigma)-\theta^{inf}_\sigma}{\sigma}\right)}. \tag{S6}$$

It is now easy to see that Condition (S24) implies that $\frac{\partial}{\partial\sigma}\psi(\theta^\#_\sigma,\theta^{inf}_\sigma,\sigma) > 0$.

The above property implies that, fixing $\theta^{inf}_{\sigma_z}$, a marginal increase in the standard deviation of the agents' private information at σ_z increases $\Psi(\theta^{inf}_{\sigma_z},\sigma_z)$. Furthermore, because the threshold $\theta^\#_{\sigma_z}$

solves $\psi(\theta^{\#}_{\sigma_z}, \theta^{inf}_{\sigma_z}, \sigma_z) = 0$, we have that, by increasing σ_z while keeping the threshold $\theta^{inf}_{\sigma_z}$ fixed, the policy maker guarantees that, for any $\theta > \theta^{inf}_{\sigma_z}$, $\psi(\theta, \theta^{inf}_{\sigma_z}, \sigma_z) > 0$. Next, note that $\theta^{inf}_{\sigma_z}$ is decreasing in σ_z. This follows from the fact that, for any σ_z, any $\theta > \hat{\theta}$, $\psi(\theta, \hat{\theta}, \sigma_z)$ is strictly increasing in $\hat{\theta}$ (this last property in turn follows from Lemma 2 in Angeletos at al. (2007)). From the above results, we thus have that, starting from any discriminatory policy, a reduction in the precision of the agents' private information (i.e., a marginal increase in σ_z) lowers the fundamental threshold $\theta^{inf}_{\sigma_z}$ below which regime default occurs, thus improving the policy maker's payoff. This completes the proof of Proposition S3. Q.E.D.

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