Price Customization and Targeting in Matching Markets

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Abstract

We introduce a model of (platform-mediated) many-to-many matching in which agents' preferences are both vertically and horizontally differentiated. We first show how the model can be used to derive the profit-maximizing matching plans under customized pricing. We then investigate the implications for targeting and welfare of uniform pricing (be it explicitly mandated or induced by privacy regulation), preventing the platform from conditioning prices on agents' observable characteristics. The model can be applied to study ad exchanges, online retailing, and media markets.

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1 Introduction

Over the last two decades, new technologies have permitted the development of matching intermediaries of unprecedented scale engaging in unparalleled level of targeting. Notable examples include ad exchanges, matching publishers with advertisers, business-to-business platforms, matching firms with mutually beneficial commercial interests, and dating websites, matching agents with common passions. The same advances in technology that favored high levels of targeting also enabled greater price customization, whereby the price of a match finely depends on characteristics of the matching partners.

In advertising exchanges, for example, the assignment of, and payments from, advertisers depend on scores that summarize the compatibility of the ads with each publisher's content.[1] A similar trend can be found in other markets, not traditionally analyzed through the lens of matching. In online shopping, for example, it is common practice among retailers to use customers' personal data to set personalized prices. In one of the most publicized cases, Orbitz, an online travel agency, reportedly used information about customers' demographics to charge targeted customers higher hotel fees.[2] Similarly, Safeway, an online grocery chain, often proposes individualized price offers and quantity discounts to customers with certain profiles.[3] The retailers' knowledge about consumers' characteristics typically comes from data brokers, who collect and sell personal information (in the form of demographics, geolocation, and browsing history).[4]

In the markets mentioned above, price customization is easy to enforce, as the agents' "horizontal" characteristics are observable (for instance, in ad exchanges, the advertisers' profile is often revealed by the ads' content) or can be learnt from third parties (for example, in online retailing, information about consumers can often be obtained from data brokers or affiliated websites). In other markets, instead, the agents' horizontal characteristics that are relevant for price customization have to be indirectly elicited, and this may require bundling.[5] A case in point is that of media markets (for instance, satellite/cable TV providers) where sophisticated pricing strategies are used to condition payments on the entire bundle of channels selected by the subscribers.[6]

[1]See, for example, https://support.google.com/adxseller/answer/2913506?hl=en&ref_topic=3376095. Moreover, ad exchanges use advertiser-specific reservation prices which are easily automated using proxy-bidding tools. Ad exchanges also price discriminate on the publisher side, by making the payments to the publishers depend on the publishers' profile and on the volume of impressions.

[2]See the article "On Orbitz, Mac Users Steered to Pricier Hotels," the *Wall Street Journal*, August 23, 2012.

[3]See https://www.bloomberg.com/news/articles/2013-11-14/2014-outlook-supermarkets-offer-personalized-pricing.

[4]According to *The New York Times*, the data broker industry's revenue reached $156 billion in 2013 (see the article "The Dark Market for Personal Data," August 16, 2014). See also Montes et al. (2018) for a discussion of the value of privacy in online markets.

[5]For instance, ad exchanges have recently developed new contractual arrangements that allow them to bundle different ads as a way of screening the publishers' unobservable preferences (see, Mirrokni and Nazerzadeh (2017)).

[6]Most satellite/cable TV providers price discriminate on the viewer side by offering viewers packages of channels whereby the baseline configuration can be customized by adding channels at a cost that depends on the baseline configuration originally selected (see, among others, Crawford (2000), and Crawford and Yurukoglu (2012)). For

While having a long history in the policy debate, price customization has attracted renewed attention in the last decade due to the two-sided nature of matching intermediaries and the amount of information now available for customized pricing.[7] The concern is that, by leveraging the platforms' market power, price customization hinders the efficiency gains permitted by better targeting technologies. Recent regulations speak directly to these issues. In the European Union, for example, the General Data Protection Regulation (GDPR) and the ePrivacy Regulation (ePR) mandate that businesses ask for consumers' consent prior to collecting and transmitting personal data. Such regulations hamper price customization based on data from third parties.[8,9]

It is however challenging to assess the impact of customized pricing or, alternatively, of policies constraining it. Part of the difficulty lies in having an analytically amenable model of matching design that is rich enough to accommodate for both *horizontal differentiation* across consumers (capturing disagreements over the most desirable matches) and *vertical differentiation* (i.e., allowing for elastic demands). The main contribution of the present paper is to introduce a tractable model featuring these two dimensions of differentiation. We use the model to show how price customization shapes the matching opportunities offered by a profit-maximizing platform, and to study the impact on targeting and consumer welfare of uniform-price obligations (whereby payments to the platforms do not depend on the "horizontal characteristics" of the agents' profiles).

Specifically, we capture vertical and horizontal differentiation by letting the agents' types be located on a cylinder, where the height represents the vertical dimension, whereas the radial position determines the horizontal dimension (see Figure 1). Each agent's utility from interacting with any other agent from the opposite side increases with the agent's vertical dimension. Fixing the vertical dimension, each agent's utility is single-peaked with respect to the horizontal dimension. More specifically, we identify each agent's radial position with his "ideal match" on the opposite side. Accordingly, each agent's utility for interacting with any other agent from the opposite side decreases with the circular distance between the agent's ideal match (his radial position) and the partner's location (the partner's radial position). Such preference structure, in addition to its analytical convenience, mirrors the one in the "ideal-point" models used in the empirical literature on media

example, in the US, Direct TV offers various vertically differentiated (i.e., nested) packages (both in English and in Spanish). It then allows viewers to add to these packages various (horizontally differentiated) premium packages, which bundle together channels specialized in movies, sports, news, and games. In addition, viewers can further customize the packages by adding individual sports, news, and movie channels.

[7] In the case of media markets, see, for example, the Federal Communications Commission 2004 and 2006 reports on the potential harm of price customization through bundling. In the case of online retailing, see the UK Office of Fair Trading 2010 eponymous report on online targeting in advertising and pricing.

[8] See Regulation (EU) 2016/679 of the European Parliament and of the Council of 27 April 2016 on the processing of personal data and on the free movement of such data.

[9] In the US, the Federal Trade Commission (FTC) recommended in 2014 legislation increasing the transparency of the data broker industry and giving consumers greater control over their personal information. See https://www.ftc.gov/news-events/press-releases/2014/05/ftc-recommends-congress-require-data-broker-industry-be-more.

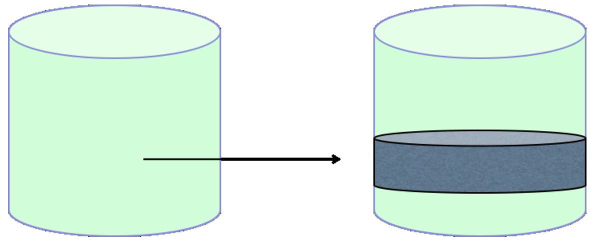

Figure 1: The cylinder model

and advertising markets (see, for example, Goettler and Shachar 2001).

A key element of our analysis is the focus on *matching tariffs*, which describe how the payments asked by the platform vary with the matching sets demanded by the agents. A tariff exhibits *uniform pricing* if all agents from a given side face the same price schedule for different quantities of the matches with agents at a given location on the opposite side. Formally, uniform tariffs are tariffs that do not condition an agent's payment to the platform on the agent's *own radial position* (i.e, the horizontal dimension of the agent's preferences). A particularly simple type of uniform pricing often proposed as a potential regulatory remedy to the market power enjoyed by media platforms is stand-alone pricing for TV channels (for a discussion, see Crawford and Yurukoglu (2012)). Stringent privacy policies that limit the use of consumers' browsing histories also induce uniform pricing, by limiting online retailers' and market places' ability to condition their offers on the characteristics of potential buyers.

Our first main result shows that, absent any regulation, platforms offer *customized* tariffs on both sides, which discriminate according to the agents' horizontal characteristics (third-degree price discrimination). Crucially, the marginal prices for matches with agents on the opposite side vary both with an agent's own location *and* with his partner's location. As marginal prices are not constant across matches, customized tariffs involve location-specific quantity premia (second-degree price discrimination). In online advertising markets, for instance, this corresponds to advertisers being charged differential marginal prices for access to each consumer of a given horizontal profile. The complex pricing algorithms used by ad exchanges, combining user- and advertiser-specific scores with nonlinear prices, are similar in spirit to the customized tariffs predicted by our model.

An alternative way of achieving this customization consists in offering agents menus of *matching plans*. Each plan is defined by its baseline configuration (i.e., a baseline set of partners from the opposite side), a baseline price, and a collection of prices describing the nonlinear cost to the subscriber of customizing the plan by adding extra matches. In the market for cable TV, for instance, most providers price discriminate on the viewer side by offering viewers packages of channels whereby the baseline configuration can be customized by adding channels at a cost that depends on the baseline configuration originally selected (see, among others, Crawford (2000)).

On technical grounds, the tractability of the model favors a convenient representation of the profit-maximizing tariffs linking location- and volume-specific prices to the various local elasticities of the demands on the two sides of the market. The representation constitutes the analog in a matching market of the familiar Lerner-Wilson formula of optimal non-linear pricing (see, for instance, Wilson (1993)) for standard goods.

Our second set of results provides a characterization of the effects on prices, targeting, and welfare of uniform-price obligations (be them explicitly mandated or induced by privacy regulation). Analogously to the generalized Lerner-Wilson formula discussed above, we provide a novel representation of the optimal price schedules that uses local elasticities to describe the prices agents on each side have to pay per quantity of matches from each location on the opposite side. Relative to the case of customized pricing, this new pricing formula identifies the relevant *aggregate elasticities* in environments where location-specific pricing is not possible. The typical marginal revenue and marginal cost terms (which determine the optimal cross-subsidization pattern) are now averages that take into account not only the uniformity of prices, but also how the procurement costs of matches are affected by the horizontal component of the agents' preferences.

From a theoretical perspective, the characterization contributes to the mechanism design literature by developing a novel technique to handle *constraints on the transfer rule* employed by the principal (as opposed to the familiar constraints on quantities, which are typically easier to analyze using standard techniques).

We then use such a characterization to study how uniform pricing affects targeting and welfare. Intuition might suggest that uniform pricing should increase targeting by preventing platforms from charging higher prices for the matches involving the most preferred partners. This simple intuition, however, fails to account for the fact that platforms re-optimize their entire price schedules to respond to aggregate elasticities. Perhaps surprisingly, uniform pricing can either decrease or increase the equilibrium level of targeting, depending on how match-demand elasticities vary with locations. To give empirical content to this finding, we relate elasticities to match values and type distributions. For an illustration, consider the case of an ad exchange. Under natural conditions on the payoffs of the advertisers and publishers, we show that price customization leads to less targeting than uniform pricing if the distribution of profits per sale of advertisers has thin tails (in the sense of an increasing hazard rate). Accordingly, anonymous pricing for advertising slots (e.g., as a result of regulation banning the use of scores) results in the advertisers being more often matched (relative to laissez-faire) to those publishers whose profile is closer to their ideal audience. That is, uniform pricing leads to more targeting in this case.

We conclude by looking into the welfare effects of uniform pricing. Exploiting a novel connection between uniform pricing in matching markets and the literature on third-degree price discrimination, we show how to adapt the analysis in Aguirre et al. (2010) to the matching markets under examination. The results identify sufficient conditions (both in terms of elasticities and in terms of match utilities and type distributions) for uniform pricing to increase surplus (in the side where prices are uniform). For instance, in the ad exchange example, advertisers' profits are higher under uniform pricing if the distribution of profits per sale of advertisers satisfies a (testable) convexity property.

We believe the model could be used more broadly to study the design of regulatory interventions in markets in which platforms enjoy significant power and price customization is a concern.

Outline of the Paper. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 identifies properties of profit-maximizing tariffs and of the induced matching demands, under customized pricing. Section 4 studies the effects of uniform-price obligations. Section 5 briefly reviews the pertinent literature. Section 6 concludes. All proofs are in the Appendix at the end of the document.

2 The Cylinder Model

A monopolistic platform matches agents from two sides of a market. Each side $k \in \{a, b\}$ is populated by a unit-mass continuum of agents. Each agent from each side k has a bi-dimensional type $\theta_k = (v_k, x_k) \in \Theta_k \equiv V_k \times X_k$ which parametrizes both the agent's preferences and the agent's attractiveness.

The parameter $v_k \in V_k \equiv [\underline{v}_k, \overline{v}_k] \subseteq \mathbb{R} \cup \{+\infty\}$ is a shifter that captures heterogeneity in preferences along a *vertical* dimension. It controls for the overall utility the agent derives from interacting with a generic agent from the opposite side, before doing any profiling. The parameter $x_k \in X_k \equiv [0, 1]$, instead, describes the agent's location and captures heterogeneity in preferences along a *horizontal* dimension. It controls for the agent's relative preferences over any two agents from the opposite side. Figure 1 depicts the above structure. The cylinder on each side represents the population on that side of the market. Each individual is located on the external surface of the cylinder, with the height of the cylinder measuring the vertical type and the position on the circle measuring the horizontal type.

Agents derive higher utility from being matched to agents whose locations are closer to their own. Their utility also increases, over all locations, with their vertical type. We assume the utility that an agent from side k with type $\theta_k = (v_k, x_k)$ derives from being matched to an agent from side $l \neq k$ with type $\theta_l = (v_l, x_l)$ is represented by the function

$$u_k(v_k, |x_k - x_l|),$$

where $|x_k - x_l|$ is the circular (minimal) distance between the two agents' locations. The function u_k is Lipschitz continuous, bounded, strictly increasing in v_k, and weakly decreasing in $|x_k - x_l|$. To make things interesting, we assume u_k is strictly decreasing in $|x_k - x_l|$ on at least one side.

Each agent's type $\theta_k = (v_k, x_k)$ is an independent draw from the absolutely continuous distribution function F_k with support Θ_k. The total payoff that type $\theta_k = (v_k, x_k)$ obtains from being matched, at a price p, to a set of types $\mathbf{s}_k \subseteq \Theta_l$ from side $l \neq k$ is given by

$$\pi_k(\mathbf{s}_k, p; \theta_k) = \int_{\mathbf{s}_k} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - p. \tag{1}$$

Accordingly, matches are non-rival, in that agents always benefit from having "access" to more agents from the other side of the market.[10] The payoff that the same agent obtains outside of the platform is equal to zero.[11]

We assume that the vertical dimensions v_k are the agents' private information. As for the horizontal dimensions, x_k, to ease the exposition, in the main text, we assume they are publicly observable. In the Appendix, however, we explain how the analysis can accommodate for the possibility that locations are also private information, under additional assumptions on the distributions F_k that guarantee that the platform can screen the agents' locations without leaving extra informational rents to the agents (see the proof of Lemma 1).

Let F_k^v (alternatively, F_k^x) denote the marginal distribution of F_k with respect to v_k (alternatively, x_k), and $F_k^{v|x}$ the distribution of v_k conditional on x_k. Then let f_k^v be the density of F_k^v and $\lambda_k^v \equiv f_k^v/[1-F_k^v]$ its hazard rate. An analogous notation applies to the densities and hazard rates of the conditional distributions $F_k^{v|x}$, which we also assume to be absolutely continuous. Finally, let $\text{Int}[V_k]$ denote the interior of the set V_k and $\Sigma(\Theta_l)$ the collection of all F_l-measurable subsets of Θ_l. Hereafter, we assume that the "virtual values"

$$\varphi_k\left(\theta_k, \theta_l\right) \equiv u_k\left(v_k, |x_k - x_l|\right) - \frac{1 - F_k^{v|x}(v_k|x_k)}{f_k^{v|x}(v_k|x_k)} \cdot \frac{\partial u_k}{\partial v}\left(v_k, |x_k - x_l|\right)$$

respect the same rankings as the true values, which is the natural analog of standard regularity conditions (e.g., Myerson (1981)) in matching environments.[12] The next two examples illustrate the type of markets the analysis can be applied to.

Example 1. ***(ad exchange)*** The platform is an ad exchange matching advertisers from side a to publishers from side b. The expected profit that an advertiser of type $\theta_a = (v_a, x_a)$ obtains from an impression at the website of a publisher of type $\theta_b = (v_b, x_b)$ is given by

$$u_a(v_a, |x_a - x_b|) = v_a \phi\left(|x_a - x_b|\right),$$

where v_a is the advertiser's profit per sale and where the strictly decreasing function $\phi : [0, \frac{1}{2}] \to [0, 1]$ describes how the probability of a conversion (i.e., the probability the ad view turns into a sale) varies

[10]The utilities u_k and u_l should be interpreted as ex-ante expected payoffs. Ex-post, the agents may learn that the match is unattractive (to one or both agents) and refrain from interacting.

[11]The representation in (1) assumes the agent is matched to all agents from side $l \neq k$ whose type is in $\mathbf{s}_k$. That matching sets are described by the agents' types, as opposed to their identities, reflects the property that, under both the welfare- and the profit-maximizing tariffs, each agent from each side k who decides to include in his matching set some agent from side $l \neq k$ whose type is θ_l optimally chooses to include in his matching set *all* agents from side l whose type is θ_l. The specification in (1) also implies that the utility that agent i from side k derives from being matched to agent j from side $l \neq k$ is invariant to who else the agent is matched with, as well as who else from the agent's own side is matched to agent j. In a previous version, we considered a more general setting where such assumptions are relaxed. We opted here for the representation in (1) because it permits us to simplify the exposition and favors sharper conclusions. See also Valenzuela-Stookey (2020) for a related model with congestion effects.

[12]By this we mean the following. Take any pair $(\theta_a, \theta_b), (\theta'_a, \theta'_b) \in \Theta_a \times \Theta_b$. Then $\varphi_k(\theta_a, \theta_b) \geq \varphi_k(\theta'_a, \theta'_b)$ if and only if $u_k(v_k, |x_k - x_l|) \geq u_k(v'_k, |x'_k - x'_l|)$, $k, l \in \{a, b\}$, $l \neq k$.

with the distance between the publisher's profile, x_b, and the advertiser's ideal audience, x_a. By contrast, publishers can be viewed (to a first approximation) as indifferent with respect to the kind of advertisement displayed at their websites. The matching (dis)utility of a publisher reflects the opportunity cost of not using the advertisement space to sell its own products, or from not selling the ad slot outside of the platform. Accordingly, the profit that a publisher of type $\theta_b = (v_b, x_b)$ derives from displaying the ad of an advertiser of type $\theta_a = (v_a, x_a)$ is given by $u_b(v_b, |x_a - x_b|) = v_b \leq 0$, all $x_a, x_b \in [0, 1]$. Both the advertisers' ideal type of audience, x_a, and each content provider's profile, x_b, are observable by the platform. $\diamondsuit$

Example 2. ***(media platform)*** The platform is a media outlet matching viewers from side a with content providers from side b. The utility $u_a(v_a, |x_a - x_b|)$ that a viewer of type $\theta_a = (v_a, x_a)$ derives from the content of a provider of type $\theta_b = (v_b, x_b)$ is increasing in the overall importance that the viewer assigns to having access to content, captured by the parameter $v_a \in V_a \subset \mathbb{R}_+$, and decreasing in the distance between the viewer's ideal type of content, x_a, and the provider's content, x_b. The matching (dis)utility $u_b(v_b, |x_b - x_a|))$ of the content provider may reflect the extra revenue from advertisers (which may depend on the type of viewers reached, as advertisers typically pay more to content providers with a higher exposure to viewers of certain characteristics), or the expenses from broadcasting rights paid to third parties (which are typically invariant to the type of audience reached). While each content provider's profile (the type of content provided) is observable, each viewer's ideal type of content may be known to the media outlet (for example, when the latter has access to data about viewers' characteristics), or be the viewer's private information (for example, as the result of privacy-protecting regulations).[13] $\diamondsuit$

Another example that shares the preference structure of Example 2 is that of an online intermediary matching consumers from side a with sellers from side b. Each seller's location, x_b, identifies the seller's product variety, whereas each buyer's location, x_a, identifies the buyer's most preferred variety. In turn, the vertical parameters $v_a > 0$ and $v_b < 0$ capture heterogeneity in consumers' willingness to pay and in sellers' marginal costs, respectively.

Tariffs and Matching Demands

The platform offers matching tariffs on each side of the market. A *matching tariff* T_k specifies the (possibly negative) total payment $T_k(\mathbf{s}_k|x_k)$ that each agent from each location $x_k \in X_k$ is asked to pay to be matched to each set of types $\mathbf{s}_k$ from the opposite side of the market.[14] To guarantee participation by all agents, we require that, for all x_k, $T_k(\mathbf{s}_k|x_k) = 0$ if $\mathbf{s}_k = \emptyset$.

[13] The structure of this example follows closely the one typically assumed in the empirical literature on media markets (see, e.g., Goettler and Shachar 2001).

[14] In their most general form, matching tariffs might depend on locations (which are observable by the platform), but not on the agents' vertical dimensions, which are the agents' private information.

Given the tariff T_k, we say that the function $\mathbf{s}_k : \Theta_k \to \Sigma(\Theta_l)$ is a *matching demand* consistent with the tariff T_k if, for any $\theta_k = (v_k, x_k) \in \Theta_k$,

$$\mathbf{s}_k(\theta_k) \in \arg \max_{\mathbf{s}_k \in \Sigma(\Theta_l)} \left\{ \int_{\Theta_l} u_k \left(v_k, |x_k - x_l| \right) dF_l(\theta_l) - T_k \left(\mathbf{s}_k | x_k \right) \right\}. \tag{2}$$

Definition 1. The tariff profile $(T_k)_{k=a,b}$ is *feasible* if there exists a pair of matching demands $(\mathbf{s}_k)_{k=a,b}$ consistent with $(T_k)_{k=a,b}$ satisfying the following *reciprocity condition*, for all $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l \in \{a, b\}$, $l \neq k$:

$$\theta_l \in \mathbf{s}_k(\theta_k) \Longleftrightarrow \theta_k \in \mathbf{s}_l(\theta_l). \tag{3}$$

That is, if an agent from side k with type θ_k finds it optimal to be matched to all agents from side $l \neq k$ with type θ_l, then all agents from side l with type θ_l find it optimal to be matched to all agents from side k with type θ_k.

The platform's problem consists of choosing a pair of feasible tariffs $(T_k)_{k=a,b}$, along with a pair of matching demands $(\mathbf{s}_k)_{k=a,b}$ consistent with the selected tariffs, that jointly maximize the platform's profits, which are given by

$$\sum_{k=a,b} \int_{\Theta_k} T_k(\mathbf{s}_k(\theta_k)|x_k) dF_k(\theta_k). \tag{4}$$

A pair of tariffs $(T_k^*)_{k=a,b}$ is *profit-maximizing* if there exist matching demands $(\mathbf{s}_k^*)_{k=a,b}$ consistent with $(T_k^*)_{k=a,b}$ such that the platform's profits under $(T_k^*, \mathbf{s}_k^*)_{k=a,b}$ are at least as high as under any other quadruple $(T_k, \mathbf{s}_k)_{k=a,b}$, where $(T_k)_{k=a,b}$ is a pair of feasible tariffs and $(\mathbf{s}_k)_{k=a,b}$ are demands consistent with $(T_k)_{k=a,b}$. Hereafter we denote by $(T_k^*)_{k=a,b}$ a pair of profit-maximizing tariffs, and by $(\mathbf{s}_k^*)_{k=a,b}$ the matching demands that, together with $(T_k^*)_{k=a,b}$, maximize the platform's profits.

3 Customized Pricing

We now introduce a class of tariffs that plays an important role in the analysis below. Under such tariffs, which we call *customized*, the platform offers to each side-k agent a baseline matching set at a baseline price, along with a collection of personalized prices that the agent can use to customize his matching set. The total price of the customization is *separable* in the locations of the agents added to the baseline configuration, but may vary non-linearly with the amount of agents from each location (a form of second-degree price discrimination). Importantly, the personalized prices the agents pay for the customizations depend on the agents' own locations (a form of third degree price discrimination). Customized tariffs capture important features of the matching plans offered by platforms such as cable TV providers, ad exchanges, and online retailers. Before proceeding to the definition, we need to introduce the following piece of notation: Given any matching set $\mathbf{s}_k$, and any location x_l, we let $q_{x_l}(\mathbf{s}_k)$ denote the "mass" of side-l agents located at x_l included in the matching set $\mathbf{s}_k$.[15]

[15] Hereafter, we abuse of terminology by referring to the density of agents of a certain type as the "mass" of agents of that type.

Definition 2. The tariff T_k is *customized* if there exists a collection of triples

$$\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k)) : x_k \in X_k\},$$

one for each location $x_k \in X_k$, such that each side-k agent located at x_k choosing the matching set $\mathbf{s}_k \in \Sigma(\Theta_l)$ is asked to make a total payment equal to

$$T_k(\mathbf{s}_k|x_k) = \underline{T}_k(x_k) + \int_0^1 \rho_k(q_{x_l}(\mathbf{s}_k)|x_l; x_k)dx_l, \tag{5}$$

with $\rho_k\left(q_{x_l}\left(\underline{\mathbf{s}}_k(x_k)\right)|x_l; x_k\right) = 0$ for all $x_l \in X_l$.

A customized tariff can thus be thought of as a collection of *matching plans*, one for each location x_k. Each plan comes with a baseline configuration, given by the default set of types $\underline{\mathbf{s}}_k(x_k)$ from side $l \neq k$ included in the package, and a baseline price $\underline{T}_k(x_k)$. Each agent selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k))$ can then customize his matching set by adding extra matches. The cost of the customization is separable in the type of matches added to the baseline configuration, with each schedule $\rho_k(q_{x_l}|x_l; x_k)$ describing the non-linear fee for adjusting the amount of x_l-agents from the default level $q_{x_l}\left(\underline{\mathbf{s}}_k(x_k)\right)$ to q_{x_l}.[16] Importantly, each price $\rho_k(q_{x_l}|x_l; x_k)$ depends on the baseline plan, which is conveniently indexed by the location of the agents targeted by the plan. The dependence of the price $\rho_k(q_{x_l}|x_l; x_k)$ on the plan x_k is a manifestation of a particular form of *bundling*. In particular, note that a customized tariff combines elements of *second-degree* price discrimination (each price function $\rho_k\left(q_{x_l}|x_l; x_k\right)$ is possibly non-linear in q_{x_l}) with elements of *third-degree* price discrimination (each non-linear price function $\rho_k\left(q_{x_l}|x_l; x_k\right)$ depends on the plan, and hence on the location of the side-k agents). The baseline configurations are meant for those agents with the lowest vertical type, $\underline{v}_k$, whereas the customizations are meant for those agents with higher vertical types.

Clearly, not all tariffs are customized, in the sense of Definition 2. For instance, tariffs that condition the price for the x_l-matches on the demand for the x_l'-matches are not customized.

The following result shows that the platform can attain maximal profits by offering a pair of customized tariffs.

Lemma 1. ***(properties of the optimum)*** *The following are true:*

1. *there exists a pair of customized tariffs $(T_k^*)_{k=a,b}$ that are profit-maximizing;*
2. *the matching demands $(\mathbf{s}_k^*)_{k=a,b}$ consistent with the profit-maximizing customized tariffs $(T_k^*)_{k=a,b}$ are described by threshold functions $t_k^* : \Theta_k \times X_l \to V_l$ such that*

$$\mathbf{s}_k^*(\theta_k) = \{(v_l, x_l) \in \Theta_l : \; v_l \geq t_k^*(\theta_k, x_l)\},$$

with the function t_k^* non-increasing in v_k and non-decreasing in $|x_k - x_l|$.

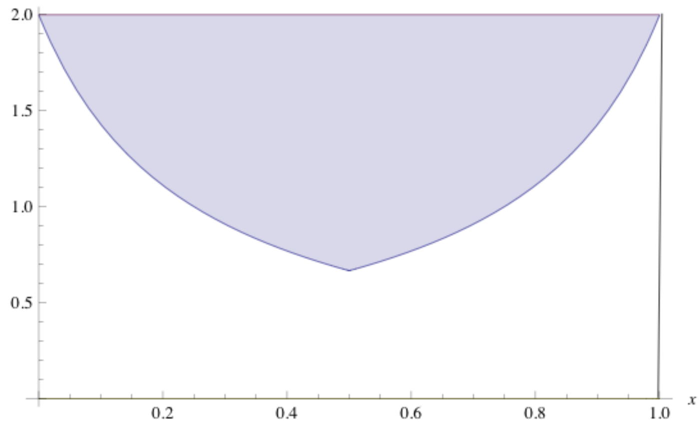

Figure 2: Matching sets under profit-maximizing tariffs. The shaded area in the figure describes the matching set for an agent from side a located at $x_a = 1/2$.

Under the profit-maximizing tariffs, for any given location x_k, the matching sets demanded by those agents with higher vertical types are supersets of those demanded by agents with lower vertical types. Moreover, side-l agents located at x_l with a low vertical type v_l are included in the matching sets of the side-k agents located at x_k only if the latter's vertical types v_k are large enough. Finally, the range of vertical types $[t_k(\theta_k, x_l), \bar{v}_l]$ of side-l agents located at x_l that each side-k agent of type $\theta_k = (v_k, x_k)$ is matched with is smaller, the larger is the distance $|x_k - x_l|$ between the agents' locations. Figure 2 illustrates these properties.

To gain intuition, note that the marginal profits the platform obtains by matching type $\theta_l = (v_l, x_l)$ from side l with type $\theta_k = (v_k, x_k)$ from side k are positive if, and only if,

$$\varphi_k(\theta_k, \theta_l) + \varphi_l(\theta_l, \theta_k) \geq 0. \tag{6}$$

Echoing Bulow and Roberts (1989), the above condition can be interpreted as stating that two agents are matched if, and only if, their *joint* marginal revenue to the platform is weakly positive (we elaborate on this point further in the next subsection). That virtual values respect the same rankings as the true values implies existence of a threshold $t_k^*(\theta_k, x_l)$ such that Condition (6) is satisfied if, and only if, $v_l \geq t_k^*(\theta_k, x_l)$, with the threshold $t_k^*(\theta_k, x_l)$ non-increasing in v_k and non-decreasing in $|x_k - x_l|$. Jointly, these properties imply that, as v_k increases, the matching set of type θ_k expands to include new agents with lower vertical types. Moreover, as v_k increases, the vertical type of the marginal agents located at x_l added to the matching set $\mathbf{s}_k^*(v_k, x_k)$ are higher the "farther" away the location x_l is from x_k. The properties described are the analogs of those in Gomes and Pavan (2016) in a setting with horizontally-differentiated preferences.[17]

[16]The schedules $\rho_k(q_{x_l}|x_l; x_k)$ also specify the price for reducing the amount of x_l-agents below the default level. However, as we show in the Appendix, in equilibrium, under both the profit-maximizing and the welfare-maximizing tariffs, the induced demands are such that $\mathbf{s}_k(\theta_k) \supset \underline{\mathbf{s}}_k(x_k)$ for all $\theta_k = (v_k, x_k)$, $k = a, b$, meaning that no agent asks to reduce the number of matches below the level specified in the baseline configuration.

[17]In a setting with purely vertically-differentiated preferences, Gomes and Pavan (2016) identify conditions under

3.1 Lerner-Wilson formula for matching schedules

We now derive further properties of the customized tariffs that maximize the platform's profits. Consider first the problem of a side-k agent of type $\theta_k = (v_k, x_k)$ under the plan x_k (recall that this is the plan designed for all side-k agents located at x_k). The mass of agents located at x_l demanded by type θ_k is given by[18]

$$\hat{q}_{x_l}(\theta_k) \in \arg \max_{q \in [0, f_l^x(x_l)]} \left\{ u_k(v_k, |x_k - x_l|) \cdot q - \rho_k(q|x_l; x_k) \right\}.$$

Assuming the price schedule $\rho_k(\cdot|x_l; x_k)$ is convex and differentiable in q, with derivative $\rho_k'(\cdot|x_l; x_k)$, we have that $\hat{q}_{x_l}(\theta_k)$ is a solution to the following first-order condition[19]

$$u_k(v_k, |x_k - x_l|) = \rho_k' \left(\hat{q}_{x_l}(\theta_k)|x_l; x_k\right) \tag{7}$$

whenever $\hat{q}_{x_l}(\theta_k)$ is interior, i.e., whenever $\hat{q}_{x_l}(\theta_k) \in (0, f_l^x(x_l))$.

Next, for any pair of locations $x_k, x_l \in [0, 1]$, and any "interior" marginal price

$$\rho_k' \in \left[u_k(\underline{v}_k, |x_k - x_l|), u_k(\bar{v}_k, |x_k - x_l|)\right],$$

let $\hat{v}_{x_l}\left(\rho_k'|x_k\right)$ denote the value of v_k that makes each agent from side k located at x_k indifferent between adding the extra q-th unit of the x_l-agents or not, given the marginal price ρ_k'. Note that, irrespective of q, $\hat{v}_{x_l}\left(\rho_k'|x_k\right)$ is implicitly defined by

$$u_k(v_k, |x_k - x_l|) = \rho_k'. \tag{8}$$

If, instead, $\rho_k' \notin [u_k(\underline{v}_k, |x_k - x_l|), u_k(\bar{v}_k, |x_k - x_l|)]$, let $\hat{v}_{x_l}\left(\rho_k'|x_k\right) = \underline{v}_k$ for all $\rho_k' < u_k(\underline{v}_k, |x_k - x_l|)$, and $\hat{v}_{x_l}\left(\rho_k'|x_k\right) = \bar{v}_k$ for all $\rho_k' > u_k(\bar{v}_k, |x_k - x_l|)$.

Note that, because the price function $\rho_k(\cdot|x_l; x_k)$ is strictly convex over the range of quantities purchased in equilibrium, the marginal price ρ_k' uniquely identifies the quantity q. Furthermore, because agents with higher vertical types demand larger matching sets, the *demand* for the q-th unit of the x_l-agents by the x_k-agents, at the *marginal price* ρ_k', is given by[20]

$$D_k\left(\rho_k'|x_l; x_k\right) \equiv \left[1 - F_k^{v|x}\left(\hat{v}_{x_l}\left(\rho_k'|x_k\right)|x_k\right)\right] f_k^x(x_k), \tag{9}$$

where, as above, we dropped the arguments $(q|x_l; x)$ of the marginal price to lighten the notation. Accordingly, $D_k\left(\rho_k'|x_l; x_k\right)$ coincides with the mass of agents from side k located at x_k whose vertical type exceeds $\hat{v}_{x_l}\left(\rho_k'|x_k\right)$.

which the profit-maximizing tariffs induce demands with a threshold structure. Valenzuela-Stookey (2020) shows that the result in Gomes and Pavan (2016) is robust to the possibility of congestion effects.

[18]Recall that the maximal amount of side-l agents that are located at x_l is $f_l^x(x_l)$.

[19]The strict convexity of the price function $\rho_k(\cdot|x_l; x_k)$ over the set of quantities purchased in equilibrium is a direct implication of the supermodularity of the agents' payoffs $u_k(v_k, |x_k - x_l|) \cdot q$ in (v_k, q).

[20]By the *demand* for the q-th unit of the x_l-agents by the x_k-agents, we mean the mass of agents from side k located at x_k who demand at least q matches with the x_l-agents.

Using (9), we then define the elasticity of the demand by the side-k agents located at x_k (in short, the x_k-demand) for the q-th unit of the x_l-agents with respect to its marginal price ρ'_k by (once again, the arguments of the marginal price ρ'_k are dropped to ease the notation)

$$\varepsilon_k\left(\rho'_k|x_l;x_k\right) \equiv -\frac{\partial D_k\left(\rho'_k|x_l;x_k\right)}{\partial\left(\rho'_k\right)}\cdot\frac{\rho'_k}{D_k\left(\rho'_k|x_l;x_k\right)}. \tag{10}$$

The next proposition characterizes the price schedules associated with the profit-maximizing customized tariffs of Lemma 1 in terms of the location-specific elasticities of the demands on both sides of the market. To ease the exposition, the dependence of the marginal prices, $\rho^{*\prime}_k$, the demands D_k, and the elasticities, ε_k, on the locations (x_a, x_b) is dropped from all the formulas in the proposition.

Proposition 1. ***(Lerner-Wilson price schedules)*** *The price schedules ρ^*_k associated with the profit-maximizing customized tariffs T^*_k are differentiable and convex over the equilibrium range.*[21] *Moreover, for all pair of locations (x_a, x_b), and all interior pairs of demands (q_a, q_b) such that $q_a = D_b\left(\rho^{*\prime}_b(q_b)\right)$ and $q_b = D_a\left(\rho^{*\prime}_a(q_a)\right)$,*[22] *the marginal prices $\rho^{*\prime}_a(q_a)$ and $\rho^{*\prime}_b(q_b)$ jointly satisfy the following Lerner-Wilson formulas*

$$\underbrace{\rho^{*\prime}_a(q_a)\left(1-\frac{1}{\varepsilon_a\left(\rho^{*\prime}_a(q_a)\right)}\right)}_{\textit{net effect on side-a profits}}+\underbrace{\rho^{*\prime}_b(q_b)\left(1-\frac{1}{\varepsilon_b\left(\rho^{*\prime}_b(q_b)\right)}\right)}_{\textit{net effect on side-b profits}}=0. \tag{11}$$

The Lerner-Wilson formulas (11) jointly determine the price schedules on both sides of the market. Intuitively, these formulas require that the marginal contribution to profits from adding to the matching sets of the x_k-agents the q_k-th unit of the x_l-agents coincide with the marginal contribution to profits from adding to the matching sets of the x_l-agents the q_l-th unit of the x_k-agents, where q_k and q_l are jointly related by the reciprocity condition in the proposition.

As for the standard Lerner-Wilson formula for monopoly/monopsony pricing, on each side, the marginal contribution to profits of such an adjustment has two terms: the term $\rho^{*\prime}_k$ captures the marginal benefit from adding the extra agents, whereas the semi-inverse-elasticity term $\rho^{*\prime}_k\left[\varepsilon_k\left(\rho^{*\prime}_k\right)\right]^{-1}$ capture its associated infra-marginal losses.

Importantly, as anticipated above, the quantities q_k and q_l at which the conditional price schedules are evaluated have to clear the market, as required by the reciprocity condition (3). The result in the proposition uses the fact that the demands under the optimal tariffs satisfy the threshold structure in Lemma 1 to establish that the mass of x_k-agents that, at the marginal price $\rho^{*\prime}_k(q_k)$, demand q_k agents or more of type x_l coincide with the mass $D_k\left(\rho^{*\prime}_k(q_k)\right)$ of x_k-agents with vertical type above $\hat{v}_{x_l}\left(\rho^{*\prime}_k|x_k\right)$, where recall that $\hat{v}_{x_l}\left(\rho^{*\prime}_k|x_k\right)$ is the threshold type for whom the utility of interacting with the x_l-agents equals the marginal price ρ^*_k, as defined in (8). Together with reciprocity, Lemma 1 then also implies that the mass q_k of x_l-agents that, at the marginal price $\rho^{*\prime}_l(q_l)$, demand $q_l = D_k\left(\rho^{*\prime}_k(q_k)\right)$ or more of the x_k-agents coincides with the mass of x_l-agents with vertical type above $\hat{v}_{x_k}\left(\rho^{*\prime}_l|x_l\right)$.

[21] Namely, at any $q_l \in [q_{x_l}(\mathbf{s}_k(\underline{v}_k, x_l + .5)), q_{x_l}(\mathbf{s}_k(\overline{v}_k, x_l))]$, $k, l = a, b$, $l \neq k$.

[22] The pair (q_a, q_b) is interior for a location pair (x_a, x_b) if $q_k \in (0, f^x_l(x_l))$ for all $k, l \in \{a, b\}$.

Finally, that the price schedules $\rho_k^*(q_k)$ are convex in q_k reflects the fact that the matching demands of the x_k-agents for the x_l-agents are increasing in the vertical types v_k. As a result, the marginal price $\rho_k^{*\prime}(q_k)$ for the q_k-unit of the x_l-agents has to increase with q_k.

The formulas in (11) also reveal how profit-maximizing platforms optimally cross-subsidize interactions among agents from multiple sides of the market while accounting for heterogeneity in preferences along both vertical and horizontal dimensions. In particular, the price schedules offered at any two locations x_k and x_l are a function of the *location-specific demand elasticities* $\varepsilon_k(\cdot|x_l; x_k)$ at these locations. This reflects the fact that, at the optimum, platforms make use of information about horizontal preferences to offer matching tariffs that extract as much surplus as possible from both sides. As we show below, the ability to tailor price schedules to locations (a form of third-degree price discrimination) has important implications for the composition of the demands prevailing under optimal tariffs.

The formulas in (11) define a system of structural equations that relate the cut-off types on both sides of the market.[23] The spirit of these formulas is the same as in the reduced-form approach pioneered by Saez (2001) in the context of optimal taxation. Under the *assumption* that a platform prices matches optimally, Proposition 1 can be used by the econometrician to estimate the distribution of the agents' preferences from data on price schedules and match volumes.

Alternatively, Proposition 1 can be employed to assess the optimality of mechanisms *currently* used. In online advertising markets, for instance, a complex system of prices is employed by ad exchanges to match publishers with advertisers. These prices employ user- and advertiser-specific scores, and are non-linear in the number of impressions, which is consistent with what predicted by Proposition 1. The Lerner-Wilson formulas (11) can serve as a test the optimality of this "indirect" mechanism.

4 Uniform Pricing

Stringent regulations on the transfer of personal data together with restrictions on bundling imposed on certain platforms are expected to hinder the customization of prices and favor instead uniform pricing. In this section, we study platforms' behavior when subject to uniform-price obligations.

[23] To see this, fix (x_a, x_b) and drop it to ease the notation. For any $q_a \in [0, f_b^x(x_b)]$, the result in Lemma 1 implies that the most economical way of giving the x_a-agents access to q_a agents located at x_b is to match them to all x_b-agents whose vertical type is above $\tilde{v}_b$, with $\tilde{v}_b$ defined by $[1 - F_b^{v|x}(\tilde{v}_b|x_b)]f_b^x(x_b) = q_a$. For any $q_b \in [0, f_a^x(x_a)]$, the marginal price $\rho_b^{*\prime}(q_b)$ is then equal to $u_b(\tilde{v}_b, |x_a - x_b|)$. Given q_a and $\rho_b^{*\prime}(q_b)$, the marginal price $\rho_a^{*\prime}(q_a)$ is then given by equation (11). Once $\rho_a^{*\prime}(q_a)$ is identified, the threshold $v_a = t_b^*((\tilde{v}_b, x_b), x_a)$ is given by the unique solution to $u_a(v_a, |x_a - x_b|) = \rho_a^{*\prime}(q_a)$.

Uniform Pricing and Aggregate Demand Elasticities

Definition 3. The tariff T_k is consistent with *uniform pricing* if there exists a collection of (possibly non-linear) *price schedules* $p_k(q|x_l)$, one for each location $x_l \in X_l$, such that the total payment asked by the platform to the side-k agents for each matching set $\mathbf{s}_k \in \Sigma(\Theta_l)$ is given by

$$T_k(\mathbf{s}_k) = \int_0^1 p_k(q_{x_l}(\mathbf{s}_k)|x_l)dx_l. \tag{12}$$

Hence, under uniform pricing, the tariff offered by the platform to the side-k agents consists of a collection of non-linear price schedules, $(p_k(\cdot|x_l))_{x_l\in[0,1]}$, one for each location of the side-l agents, with each schedule $p_k(q|x_l)$ specifying the total price each side-k agent has to pay to be matched to q agents from side $l \neq k$ located at $x_l \in [0,1]$. Importantly, contrary to the case of price customization, the price $p_k(q|x_l)$ is independent of the agent's own location, x_k.

Suppose the platform is forced to adopt a uniform-price schedule $p_a(\cdot|x_b)$ on side a (with marginal schedule $p'_a(\cdot|x_b)$). Recall that, for each location $x_b \in [0,1]$, and each quantity $q \in [0, f_b^x(x_b)]$, such a schedule specifies the price that the side-a agents have to pay to be matched to q agents from side b located at x_b. Under such a schedule, the aggregate demand (over all locations x_a) for the q-th unit of the x_b-agents at the marginal price $p'_a\left(q|x_b\right)$ is equal to

$$\bar{D}_a\left(p'_a|x_b\right) \equiv \int_0^1 D_a\left(p'_a|x_b;x_a\right)dx_a = \int_0^1 \left\{1 - F_a^{v|x}\left(\hat{v}_{x_b}\left(p'_a|x_a\right)|x_a\right)\right\} f_a^x(x_a)dx_a,$$

where, as in the previous section, $D_a\left(p'_a|x_b;x_a\right)$ denotes the mass of agents located at x_a that demand q units or more of the x_b-agents, and where, as in the previous section, the arguments $(q|x_b)$ of the marginal prices $p'_a\left(q|x_b\right)$ have been dropped, to ease the exposition.

The elasticity of the side-a *aggregate demand* for the q-th unit of the x_b-agents with respect to the marginal price p'_a is then equal to

$$\bar{\varepsilon}_a\left(p'_a|x_b\right) \equiv -\frac{\partial \bar{D}_a\left(p'_a|x_b\right)}{\partial\left(p'_a\right)} \cdot \frac{p'_a}{\bar{D}_a\left(p'_a|x_b\right)} = \mathbb{E}_{\bar{H}(\tilde{x}_a|x_b,p'_a)}\left[\varepsilon_a\left(p'_a|x_b;\tilde{x}_a\right)\right],$$

where $\varepsilon_a\left(p'_a|x_b;x_a\right)$ is the elasticity of the demand by the x_a-agents, as defined in (10), and where the expectation is over $X_a = [0,1]$, under the distribution $\bar{H}\left(\cdot|x_b,p'_a\right)$ whose density is equal to

$$\bar{h}\left(x_a|x_b,p'_a\right) \equiv \frac{D_a\left(p'_a|x_b;x_a\right)}{\int_0^1 D_a\left(p'_a|x_b;x'_a\right)dx'_a}.$$

Hereafter, we refer to $\bar{\varepsilon}_a\left(\cdot|x_b\right)$ as the *aggregate elasticity* of the side-a demand for the q-th unit of the x_b-matches. This elasticity measures the percentage variation in the mass of agents from side a that demand at least q matches with the side-b agents located at x_b in response to a percentage change in the marginal price for the q-th unit of the x_b-agents. It is also equal to the average (over the side-a locations) elasticity of the x_a-demands for the q-th unit of the x_b-agents with respect to the marginal price p'_a, where the average is under a distribution that assigns to each location x_a a weight proportional to the mass of agents $D_a\left(p'_a|x_b, x_a\right)$ located at x_a demanding q units, or more, of the x_b-agents.

The next proposition derives properties of the profit-maximizing tariffs T_a^u and T_b^u offered by a platform that is constrained to price uniformly on side a. To ease the exposition, the dependence of the marginal price $p_a^{u\prime}$ and of the aggregate elasticity $\bar{\varepsilon}_a$ on x_b, as well as the dependence of the marginal price $\hat{\rho}_b^{u\prime}$ and of the local elasticity $\hat{\varepsilon}_b$ on (x_a, x_b), are dropped from all the formulas in the proposition.

Proposition 2. ***(uniform pricing)*** *Suppose the platform is constrained to price uniformly on side a, but is free to offer any tariff on side b. The profit-maximizing tariffs $(T_k^u)_{k=a,b}$ are such that T_b^u is customized. The price schedules p_a^u and ρ_b^u associated with the profit-maximizing tariffs $(T_k^u)_{k=a,b}$ are differentiable and convex over the equilibrium ranges.*[24] *Moreover, for all locations $x_b \in X_b$, and all interior quantity pairs $(q_a, q_b(\tilde{x}_a))$, $\tilde{x}_a \in X_a$, such that*

$$q_a = D_b\left(\rho_b^{u\prime}\left(q_b|x_b; x_a\right)|x_b; x_a\right) \quad \textit{and} \quad q_b(\tilde{x}_a) \equiv D_a\left(p_a^{u\prime}(q_a)|x_b; x_a\right),$$

the marginal prices schedules $p_a^{u\prime}$ and $\rho_b^{u\prime}$ jointly satisfy the following optimality condition:

$$\underbrace{p_a^{u\prime}(q_a)\left(1 - \frac{1}{\bar{\varepsilon}_a\left(p_a^{u\prime}(q_a)\right)}\right)}_{\textit{net effect on side-a profits}} + \mathbb{E}_{H(\tilde{x}_a|x_b, p_a^{u\prime}(q_a))}\left[\underbrace{\rho_b^{u\prime}(q_b(\tilde{x}_a))\left(1 - \frac{1}{\varepsilon_b\left(\rho_b^{u\prime}(q_b(\tilde{x}_a))\right)}\right)}_{\textit{net effect on side-b profits}}\right] = 0, \tag{13}$$

where $H\left(x_a|x_b, p_a^{u\prime}\right)$ is the distribution over X_a whose density is given by

$$h\left(x_a|x_b, p_a^{u\prime}\right) \equiv \frac{\partial D_a\left(p_a^{u\prime}|x_b; x_a\right)}{\partial\left(p_a^{u\prime}\right)}\left(\frac{\partial \bar{D}_a\left(p_a^{u\prime}|x_b\right)}{\partial\left(p_a^{u\prime}\right)}\right)^{-1}.$$

The result in the proposition provides structural equations similar to those in Proposition 1, but adapted to account for the imposition of uniform pricing on side a. Such structural conditions jointly determine the price schedules on both sides of the market. Under uniform pricing, the price schedule on side a for the sale of the x_b-agents cannot condition on the location of the side-a agents. As a result, the markup for the sale of the q-th unit of the x_b-matches is constant across all side-a locations x_a. The relevant elasticity for determining this markup is then the aggregate elasticity $\bar{\varepsilon}_a(\cdot|x_b)$, rather than the location-specific elasticities $\varepsilon_a\left(\cdot|x_b; x_a\right)$ in the Lerner-Wilson formula (11). Interestingly, even if the platform can price discriminate on side b by offering different price schedules $\rho_b^u(q|x_a; x_b)$ to the side-b agents as a function of their locations, x_b, when it is constrained to price uniformly on side a, the cost of procuring the x_b-agents is the average (mark-up augmented) price

$$\mathbb{E}_{H(\tilde{x}_a|x_b, p_a^{u\prime}(q_a))}\left[\rho_b^{u\prime}(q_b(\tilde{x}_a))\left(1 - \frac{1}{\varepsilon_b\left(\rho_b^{u\prime}(q_b(\tilde{x}_a))\right)}\right)\right]$$

[24] Namely, at any $q_a \in [q_{x_b}(\mathbf{s}_a(\underline{v}_a, x_b + .5)), q_{x_b}(\mathbf{s}_a(\overline{v}_a, x_b))]$ and $q_b \in [q_{x_a}(\mathbf{s}_b(\underline{v}_b, x_b)), q_{x_a}(\mathbf{s}_a(\overline{v}_b, x_b))]$.

charged to the x_b-agents for their interactions with the various x_a-agents demanding q_a, or more, of the x_b-matches.

Also note that, by virtue of the reciprocity condition (3), the quantities q_a and q_b at which the price schedules are evaluated have to clear the market for any pair of locations (x_a, x_b). For this to be possible, it is important that the platform be able to price discriminate on side b, as this ensures that the platform has enough price instruments to procure the side-b matches demanded by the side-a agents, while respecting reciprocity.

Finally, as in the case where price discrimination is allowed on both sides, the convexity of the price schedules $p_a^u(\cdot|x_b)$ and $\rho_b^u(\cdot|x_a; x_b)$ in q_a and q_b, respectively, reflects the fact that the matching demands of those agents with a higher vertical type are supersets of those with a lower vertical type.

As revealed by the pricing formulas (11) and (13), the effects of the imposition of uniform pricing on side a on the composition of the matching sets on both sides hinge on the comparison between the aggregate inverse-elasticity $1/\bar{\varepsilon}_a(\cdot|\cdot)$ and the location-specific inverse-elasticities $1/\varepsilon_a(\cdot|\cdot; x_a)$ on side a, as well as the comparison between the average inverse-elasticity $\mathbb{E}_{H(\tilde{x}_a|p'_a)}\left[1/\varepsilon_b(\cdot|\tilde{x}_a; \cdot)\right]$ of the x_b-demands for the various x_a-matches and the inverse-elasticities $1/\varepsilon_b(\cdot|\tilde{x}_a; \cdot)$ of the same demands for the specific matches. In turn, such comparisons naturally reflect how the average virtual valuations on both sides compare to their location-specific counterparts.

4.1 Targeting under Uniform and Customized Pricing

Digital technology is often praised for its ability to increase match precision (or targeting) in a variety of markets. Yet, technology alone is no guarantee of large targeting gains, as the matches enjoyed by the agents obviously depend on the pricing practices followed by the platforms. Price customization allows a platform to charge the agents prices that depend on the agents' own horizontal characteristics (either directly, when the latter are observable, as assumed here, or indirectly, through bundling, as discussed in the Appendix). One might expect price-customization to hinder targeting, as it permits platforms to set higher prices for those matches the agents like the most. Without further inquiry, this observation seems to lend support to policies that impose uniform-price obligations. Indeed, recent proposals requiring stringent protection of consumer privacy (de facto banning price customization), stand-alone pricing for media content (thus banning bundling), or anonymous pricing for advertising slots, appear broadly consistent with this line of reasoning. This intuition, however, is incomplete, as it ignores the (endogenous) changes in prices that platforms undertake in response to uniform-price obligations. The analysis below provides some guidelines as to the effects of uniform and customized pricing on targeting.

Definition 4. (targeting) Customized pricing (on both sides) leads to more targeting than uniform pricing (on side a) if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^*(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} < 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ > 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

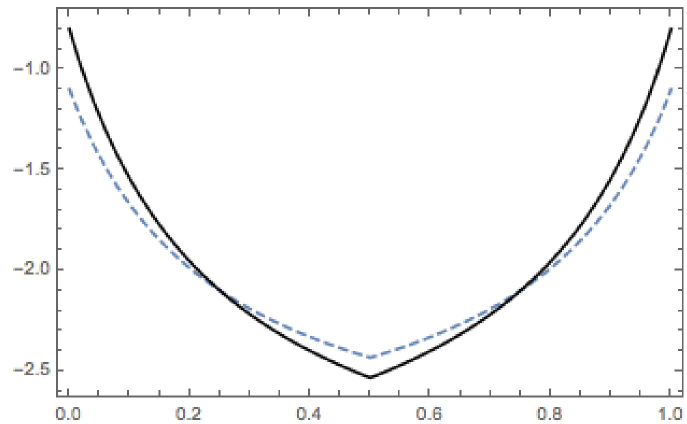

Figure 3: The threshold function $t_a^*(\theta_a, x_b)$ under customized pricing (solid black curve) and uniform pricing $t_a^u(\theta_a, x_b)$ (dashed blue curve) when customized pricing (on both sides) leads to more targeting than uniform pricing (on side a).

Conversely, uniform pricing on side a leads to more targeting than customized pricing on both sides if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^*(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} > 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ < 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

Intuitively, customized pricing (on both sides) leads to more targeting than uniform pricing (on side a) if, under the profit-maximizing customized tariffs, agents demand more matches close to their ideal points, and less matches far from their ideal points, relative to what they do under uniform pricing. Accordingly, the threshold function $t_a^*(\theta_a, x_b)$ under customized pricing is below the corresponding threshold function $t_a^u(\theta_a, x_b)$ for nearby matches (i.e., for locations x_b such that $|x_a - x_b| < d(\theta_a)$), and above $t_a^u(\theta_a, x_b)$ for more distant matches (for which $|x_a - x_b| > d(\theta_a)$). Figure 3 illustrates the situation captured by the above definition.

Note that, because matching is reciprocal, the above definition has an analogous implication for side b. Namely, when customized pricing (on both sides) leads to more targeting than uniform pricing (on side a), then the side-b threshold function under customized pricing, $t_b^*(\theta_b, x_a)$, also single-crosses its counterpart under uniform pricing, $t_b^u(\theta_b, x_a)$, only once, and from below, as a function of the distance $|x_a - x_b|$.

The next proposition identifies conditions under which uniform pricing on side a (for short, uniform pricing) leads to more targeting than customized pricing on both sides (for short, customized pricing). For simplicity, the result in Proposition 3 is for a market in which preferences on side b are location-invariant. The general case is in the Appendix (proof of Proposition 3).

Proposition 3. ***(comparison: targeting)*** *Suppose preferences on side b are location-invariant.*

1. Uniform pricing (on side a) leads to more (alternatively, less) targeting than customized pricing (on both sides) when the side-a semi-elasticities are increasing (alternatively, decreasing) in both distance and price.

2. The side-a semi-elasticities are increasing in both distance and price when x_a and v_a are independent, the hazard rate for F_a^v is increasing in v_a, and u_a is submodular and concave in v_a (alternatively, they are decreasing in both both distance and price when x_a and v_a are independent, the hazard rate for F_a^v is decreasing in v_a, and u_a is supermodular and convex in v_a).

Consider Part 1 and fix the side-b location x_b. Under uniform pricing, the elasticity of the aggregate demand by the side-a agents for the q-th unit of the x_b-agents is invariant to the distance $|x_b - x_a|$, as the marginal price for the q-th unit of x_b-matches is the same for all x_a-locations. As a consequence, when the semi-elasticities of the side-a demands increase (alternatively, decrease) with distance, the marginal price for the q-th unit of x_b-matches charged to the x_a-agents under customized pricing is lower than the corresponding price under uniform pricing when locations are far apart, whereas the opposite is true at nearby locations. Accordingly, there is more targeting under uniform pricing than under customized pricing. The result in Part 2 then uses the characterization of the matching demands in the previous section to translate the result in Part 1 in terms of conditions on match values and type distributions.

Example 3. (***ad exchange* - continued**) Consider the ad exchange market of Example 1. Price customization leads to less targeting than uniform pricing (as the match function u_a is submodular and linear in v_a) when F_a^v has an increasing hazard rate (e.g., a uniform or exponential cdf). Under this condition, anonymous pricing for advertising slots (e.g., as a result of regulation banning the use of scores) makes advertisers be more often matched (relative to laissez-faire) to those publishers whose profile is closer to their ideal audience. $\diamondsuit$

Accordingly, anonymous pricing for advertising slots (e.g., as a result of regulation banning the use of scores) makes advertisers be more often matched (relative to laissez-faire) to those publishers whose profile is closer to their ideal audience if the distribution F_a^v has thin tails. This condition is testable. Analogous testable conditions can be derived fro other applications of our model.

The matching rule that maximizes utilitarian welfare is such that, for any two agents with types $\theta_k = (v_k, x_k)$ and $\theta_l = (v_l, x_l)$,

$$\theta_l \in s_k(\theta_k) \quad \Longleftrightarrow \quad u_k\left(v_k, |x_k - x_l|\right) + u_l\left(v_l, |x_k - x_l|\right) \geq 0.$$

By standard arguments, it is easy to show that under either uniform or customized tariffs, the profit-maximizing matching rule inefficiently excludes too many agents (who get no matches) and matches each participating agent to a subset of her efficient matching set. In light of this observation, Proposition 3 reveals that the profit-maximizing matching rule under uniform pricing is closer to the efficient rule (than the rule under customized pricing) at closer locations (resp., far-away locations) if and only uniform pricing leads to more targeting than customized pricing.

4.2 Welfare under Uniform and Customized Pricing

The result in Proposition 3 can also be used to study the welfare implications of uniform-price obligations. To see this, suppose that targeting is higher under uniform pricing than under customized pricing. Then, under uniform pricing on side-a, the side-a agents face lower marginal prices $p_a'(q|x_b)$ for the x_b-agents they like the most and higher marginal prices for those side-b agents whose location is far from their ideal match.

The above findings permit us to adapt results from the third-degree price discrimination literature to the matching environment under consideration here to identify conditions under which welfare of the side-a agents increases with the imposition of uniform pricing on side a. Formally, recall that, under uniform pricing, the demand by the x_a-agents for each q-th unit of the x_b-matches at the marginal price p_a' is given by

$$D_a\left(p_a'|x_b;x_a\right) = \left[1 - F_a^{v|x}\left(\hat{v}_{x_b}\left(p_a'|x_a\right)|x_a\right)\right] f_a^x(x_a)$$

where, to ease the notation, we dropped $(q|x_b)$ from the arguments of the marginal price $p_a'(q|x_b)$. Now let

$$CD_a\left(p_a'|x_b;x_a\right) = -\frac{\partial^2 D_a\left(p_a'|x_b;x_a\right)}{\partial\left(p_a'\right)^2}\left(\frac{\partial D_a\left(p_a'|x_b;x_a\right)}{\partial\left(p_a'\right)}\right)^{-1} p_a'$$

denote the convexity of the demand by the x_a-agents for the q-th unit of the x_b-agents with respect to the marginal price p_a'.[25] Before proceeding, we impose the following regularity condition.

Condition 1. [NDR] Nondecreasing Ratio: For any $(x_a, x_b) \in X_a \times X_b$, any $q \in [0, f_b^x(x_b)]$, the function

$$z_a\left(p_a'|x_b;x_a\right) \equiv \frac{p_a'}{2 - CD_a\left(p_a'|x_b;x_a\right)}$$

is nondecreasing in the marginal price p_a' for the q-th unit of the x_b-agents.

We then have the following result:

Proposition 4. ***(comparison: welfare)*** *Suppose Condition NDR holds and either one of the following alternatives is satisfied:*

1. targeting is higher under uniform pricing than under customized pricing and, for any p_a' and x_b, $CD_a\left(p_a'|x_b;x_a\right)$ declines with $|x_a - x_b|$.

2. targeting is higher under customized pricing than under uniform pricing and, for any p_a' and x_b, $CD_a\left(p_a'|x_b;x_a\right)$ increases with $|x_a - x_b|$.

Then welfare of the side-a agents is higher under uniform pricing on side a than under customized pricing on both sides.

The next example applies Proposition 4 to assess the welfare effects of uniform pricing in the ad exchange application. We assume that Condition NDR holds.

[25] Note that $CD_a(p_a'|x_b;x_a)$ is also the elasticity of the marginal demand $\partial D_a(p_a'|x_b;x_a)/\partial p_a'$ with respect to the marginal price p_a'.

Example 4. (***ad exchange*** **- continued**) Consider the ad exchange market of Example 1. Advertisers' profits are higher under uniform pricing, and so is the level of targeting, when F_a^v has an increasing hazard rate and its convexity function

$$CF_a^v(v_a) \equiv -\frac{d^2 F_a^v}{dv_a^2}\left(\frac{dF_a^v}{dv_a}\right)^{-1} v_a = -\frac{f_a^{v\prime}(v_a)}{f_a^v(v_a)} v_a$$

is decreasing. The latter condition is weaker than the requirement that the density $f_a(v_a)$ is log-convex. $\diamondsuit$

Condition NDR, as well as the convexity properties of the demand functions in Proposition 4, parallel those in Aguirre et al (2010). The value of the proposition is in showing how our results about the connection between targeting and customized pricing also permit us to apply to the environment under examination here the welfare results from the third-degree price discrimination literature. Note that Proposition 3 is key to the result in Proposition 4. It permits us to identify "stronger markets," in the sense of Aguirre et al. (2010), with those for matches involving agents from closer locations (Part 1) or more distant locations (Part 2). Once the connection between targeting and price customization is at hand, the welfare implications of customized pricing naturally parallel those in the third-degree price discrimination literature.

Also note that the result in Proposition 4 is just an illustration of the type of welfare results that Proposition 3 permits. Paralleling the analysis in Proposition 2 in Aguirre et al. (2010), for example, one can also identify primitive conditions under which welfare is higher under price customization than under uniform pricing, as well as conditions under which price customization impacts negatively one side of the market and positively the other.

5 Related Literature

This paper studies many-to-many matching (with monetary transfers) in markets in which the agents' preferences are both vertically and horizontally differentiated. Related are Jeon et al. (2021) and Gomes and Pavan (2016). The first paper studies the provision of quality by a platform in a setting where quality provision enhances match values. The second paper studies the inefficiencies of the matching allocations under profit maximization. Both papers assume that preferences are vertically differentiated, thus ignoring the issues of targeting and price customization that emerge when preferences are horizontally differentiated and that are the focus of the present paper. Importantly, neither of the above works studies the implications of uniform-price obligations, which is one of the key contributions of the present paper.

Fershtman and Pavan (2020) also studies many-to-many matching in a model in which preferences are both vertically and horizontally differentiated. The focus of that paper, however, is bidding in dynamic matching markets in which agents arrive over time, experience shocks to their match values, and are repeatedly re-matched. The present paper, instead, abstracts from dynamics and focuses on how uniform-price obligations impact targeting and welfare.

Related are also Jullien and Pavan (2019), and Tan and Zhou (2020). The former paper studies platform competition in markets where agents' preferences for the products of different platforms are heterogenous but where all agents have the same preferences for interacting with agents from the opposite side of the market. The latter paper studies price competition in a model where multiple platforms compete by offering differentiated services to the various sides of the market, and where agents' preferences are heterogenous and exhibit both within-side and across-sides network effects. These papers, however, abstract from (second and third-degree) price discrimination, which is the focus of the present paper. Price discrimination in matching markets is examined in Damiano and Li (2007) and Johnson (2013). Contrary to this paper, these works consider markets where matching is one-to-one and where agents' preferences are differentiated only along a vertical dimension.[26]

The present paper considers a many-to-many matching market where agents might disagree on the relative attractiveness of any two agents from the other side (horizontal differentiation). Similar preference structures are examined in the matching literature surveyed in Roth (2018) (see also Kojima (2017) and Pathak (2017) for a detailed discussion of some of the recent contributions). This literature is methodologically distinct from the current paper, in that it focus on solution concepts such as stability and typically does not allow for transfer.[27]

More broadly, markets where agents purchase access to other agents are the focus of the literature on two-sided markets (see Belleflamme and Peitz (2017, 2021) and Jullien et al. (2021) for some recent overviews). Most of this literature, however, restricts attention to a single network, or to mutually exclusive networks. Ambrus et al. (2016) relax this structure by proposing a model of competing media platforms with overlapping viewerships (i.e., multi-homing). By contrast, we stick to a monopolistic market, but introduce a richer preferences structure (allowing for horizontal tastes for matches), which enables us to study targeting and price customization in such markets.[28]

The study of price customization is related to the literature on price discrimination. In the case of second-degree price discrimination, Mussa and Rosen (1978), Maskin and Riley (1983), and Wilson (1993) study the provision of quality/quantity in markets where agents possess private information about a vertical dimension of their preference. Our analysis differs from this literature in two important dimensions. First, the platform's feasibility constraint (namely, the reciprocity of the matches) has no equivalent in standard markets for commodities. Second, agents' preferences are differentiated along both a vertical and a horizontal dimension. This richer preferences structure

[26]See also Belleflamme and Peitz (2020) for a recent study of price discrimination in platform market with network effects.

[27]See the book "Market Design" by Haeringer (2018) and the forthcoming book "Online and Matching-Based Market Design" for a connection between the two literatures.

[28]Most of the literature on two-sided markets assumes that platforms price discriminate across sides but not within side. For models of within-side price discrimination, see also Halaburda and Yehezkel (2013), Reisinger (2014), Gomes and Pavan (2016), Belleflamme and Peitz (2020), and Jeon et al. (2021).

calls for a combination of second- and third-degree price discrimination and leads to novel cross-subsidization patterns.[29]

The paper also contributes to the literature on third-degree price discrimination. In addition to the paper by Aguirre et al. (2010) mentioned above, see Bergemann, Brooks, and Morris (2015) for an excellent overview of this literature and for recent developments. The latter paper characterizes all combinations of producer and consumer surplus that arise from different information structures about the buyers' willingness-to-pay (alternatively, from different market segmentations). The present paper differs from the above two papers in the assumed preferences structure and in the two-sidedness of the platform's problem.

Related is also the literature on bundling (see, among others, Armstrong (2013), Hart and Reny (2015), and the references therein). The present paper differs from that literature in two important aspects. First, in our setting, preferences can be decomposed into a vertical and a horizontal dimension. The bundling literature, instead, assumes a more general preference structure, which, however, hinders the characterization of the optimal price schedules, except in certain special cases. Second, reflecting the practices of many-to-many matching intermediaries, we assume that sales are monitored, so that prices can condition on the entire matching set of each agent. The bundling literature, by contrast, typically assumes that purchases are anonymous.

Lastly, the paper contributes to the literature on targeting in advertising markets (see, for example, Bergemann and Bonatti (2011, 2015) and Kox et al. (2017) and the references therein). Our work contributes to this literature by introducing a richer class of (non-linear) pricing strategies and by focusing on the matching outcomes that emerge in platform markets where the matching between the advertises and the publishers (or content providers) is mediated. Contrary to some of the papers in this literature, however, we abstain from platform competition. Importantly, we also assume that agents can perfectly communicate their preferences and face no informational frictions regarding the desirability of the matches. Eliaz and Spiegler (2016) relax these assumptions and consider the mechanism design problem of a platform that wants to allocate firms into search pools created in response to noisy preference signals provided by the consumers. Relatedly, Eliaz and Spiegler (2019) consider the problem of a profit-maximizing advertising platform eliciting the advertisers' profiles so as to match them to consumers with preferences for diversity. These papers do not investigate the effects of uniform pricing, but rather focus on the incentives of firms to truthfully reveal their "ideal audiences."

[29] Related is also Balestrieri and Izmalkov (2015). That paper studies price discrimination in a market with horizontally differentiated preferences by an informed seller who possesses private information about its product's quality (equivalently, about the "position" of its good in the horizontal spectrum of agents' preferences). The focus of that paper is information disclosure, whereas the focus of the present paper is matching, targeting, and price customization.

6 Concluding Remarks

The paper proposes a new model of (platform-mediated) many-to-many matching for markets in which agents' preferences are both vertically and horizontally differentiated. The model can be used to study the effects on prices, the composition of the matching sets, and welfare, of uniform-price obligations that hinder platforms' possibility to condition prices on agents' "locations," as in the case of privacy regulations preventing online retailers from conditioning prices on buyers' age, gender, of physical location.

We believe the results have useful implications for various markets. Consider, for example, online shopping. As mentioned in the Introduction, recent regulations requiring consumers' consent for the diffusion of personal information are expected to hinder price customization when third-party data are needed. Perhaps surprisingly, our analysis shows that this may either increase or decrease targeting, depending on testable characteristics of consumer demand. Related conditions can also be used to evaluate whether or not the imposition of uniform-price obligations increase consumer welfare.

Another natural application of our framework is the market for online advertising (see, among others, Bergemann and Bonatti (2011) for an overview of such a market). Ad exchanges such as AppNexus, AOL's Marketplace, Microsoft Ad Exchange, OpenX, Rubicon Project Exchange, and Smaato, use sophisticated pricing algorithms whereby prices depend not only on volumes but also on advertisers' and publishers' profiles. Such algorithms thus enable price-customization practices that appear similar, at least in spirit, to those studied in the present paper. While such algorithms have initially been praised for the customization possibilities they offer, more recently they have been associated with targeting and price-discriminatory practices often seen with suspicion by consumers and regulators. The policy debate about the desirability of regulations imposing uniform pricing lacks a formal model shedding light on how matching demands and welfare are affected by such changes. Our paper contributes to such a debate by offering a stylized, yet quite flexible, framework to analyze market outcomes under uniform pricing.

We conclude by discussing a few limitations of our analysis and venues for future research. First, our analysis abstracts from platform competition. Second, and related, it assumes platform have the power to set prices on both sides of the market. While these assumptions are a plausible starting point, there are many markets where multiple platforms compete on multiple sides and their ability to set prices is hindered by their lack of bargaining power. For example, the market for cable TV is populated by multiple providers. Furthermore, as indicated in Crawford and Yurukoglu (2012), large channel conglomerates enjoy nontrivial bargaining power vis-a-vis cable TV providers, which suggests that prices are likely to be negotiated on the channel side instead of being set directly by the platforms. Extending the analysis to accommodate for platform competition and limited bargaining power on one, or multiple, sides of the market is an important step for future research.

Furthermore, certain platforms, most notably B2B platforms, have recently expanded their services to include e-billing and supply-management. These additional services open the door to more sophisticated price-discriminatory practices that use instruments other than the composition of the matching sets. Extending the analysis to accommodate for such richer instruments is another interesting direction for future work (see Jeon et al. (2021) for related issues).

Lastly, in future work, it would be desirable to extend the analysis to accommodate for "within-side" network effects (e.g., congestion and limited attention) and dynamics (whereby agents gradually learn the attractiveness of the partners and platforms indirectly control the speed of such learning with their pricing strategies).[30] All the above extensions are challenging yet worth exploring.

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[30] As mentioned in the discussion of the related literature, congestion is examined in a recent paper by Valenzuela-Stookey (2020), whereas dynamics is examined in Fershtman and Pavan (2020). These papers, however, do not consider the effects of uniform-price obligations on targeting and welfare, which is the focus of the analysis in the present paper.

7 Appendix

In this Appendix, we provide the proofs for all the results in the main text. The proofs are established for more general environments in which, in addition to possessing private information about the vertical dimensions, v_k, the agents may possess private information also about their horizontal dimensions, i.e., about their locations x_k. In particular, we consider the following four scenarios:

- *Scenario (i)*: Locations are public on both sides;
- *Scenario (ii)*: Locations are private on side a and public on side b;
- *Scenario (iii)*: Locations are public on side a and private on side b;
- *Scenario (iv)*: locations are private on both sides.

All the results in the main body are for Scenario (i). Below, we discuss how the results extend to Scenarios (ii)-(iv) provided a certain combination of the following two conditions holds.

Condition 2. [I$_k$] Independence on side k: for any $\theta_k = (v_k, x_k) \in \Theta_k$, $F_k(\theta_k) = F_k^x(x_k)F_k^v(v_k)$.

Condition I_k requires that the vertical dimensions v_k be drawn independently from the locations x_k. This condition implies that knowing an agent's ideal match carries no information about the overall importance the agent assigns to matching with agents from the opposite side.

Condition 3. [Sy$_k$] Symmetry on side k: for any $\theta_k = (v_k, x_k) \in \Theta_k$, $F_k(\theta_k) = x_k F_k^v(v_k)$.

Condition Sy_k strengthens the independence condition by further requiring that locations be uniformly distributed over $X_k = [0, 1]$, as typically assumed in models of horizontal differentiation.[31]

To accommodate the possibility that locations are private information, we need to generalize the notion of customized tariffs, as follows:

Definition 5. The tariff T_k is *customized* if there exists a collection of quadruples

$$\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k)) : x_k \in X_k\},$$

one for each location $x_k \in X_k$, with $\mathcal{S}_k(x_k) \subseteq \Sigma(\Theta_l)$ denoting a set of *permissible customizations*, such that each side-k agent selecting the plan indexed by x_k and then choosing the customization $\mathbf{s}_k \in \mathcal{S}_k(x_k)$ from the set of permissible customizations $\mathcal{S}_k(x_k)$, is asked to make a total payment equal to[32]

$$T_k(\mathbf{s}_k|x_k) = \underline{T}_k(x_k) + \int_0^1 \rho_k(q_{x_l}(\mathbf{s}_k)|x_l; x_k)dx_l, \tag{14}$$

where $\rho_k\left(q_{x_l}\left(\underline{\mathbf{s}}_k(x_k)\right)|x_l; x_k\right) = 0$ for all $x_l \in [0, 1]$.

[31] Similar assumptions are typically made also in the targeting literature; see, for example, Bergemann and Bonatti (2011, 2015), and Kox et al. (2017).

[32] The payment specified by the tariff for any non-permissible customization $\mathbf{s}_k \notin \{\cup \mathcal{S}_k(x_k) : x_k \in X_k\}$ can be taken to be arbitrarily large to guarantee that no type finds it optimal to select any such customization. The existence of such payments is guaranteed by the assumption that u_k is bounded, $k = a, b$. Furthermore, in case loca-

Relative to Definition 2 in the main text, Definition 5 adds the requirement that a customization must be permissible, that is, it has to belong to the collection of possible customizations $\mathcal{S}_k(x_k) \subseteq \Sigma(\Theta_l)$. As we show below, when locations are public on side k, without loss of optimality the platform can set $\mathcal{S}_k(x_k) \equiv \Sigma(\Theta_l)$, in which case Definitions 2 and 5 coincide.

Proof of Lemma 1. The proof below establishes the following result, for which the claim in the main text is a special case. *Suppose the environment satisfies the properties of one of the following four cases: Scenario (i); Scenario (ii) along with Conditions I_a and Sy_b; Scenario (iii) along with Conditions Sy_a and I_b; Scenario (iv) along with Conditions Sy_a and Sy_b. Then,*

1. *there exists a pair of customized tariffs $(T_k^*)_{k=a,b}$ that is profit-maximizing;*

2. *the matching demands $(\mathbf{s}_k^*)_{k=a,b}$ consistent with the profit-maximizing customized tariffs $(T_k^*)_{k=a,b}$ are described by threshold functions $t_k^* : \Theta_k \times X_l \to V_l$ such that*

$$\mathbf{s}_k^*(\theta_k) = \{(v_l, x_l) \in \Theta_l : \; v_l > t_k^*(\theta_k, x_l)\},$$

   with the threshold function t_k^* non-increasing in v_k and non-decreasing in $|x_k - x_l|$.

3. *When locations are public on side $k \in \{a, b\}$, $\mathcal{S}_k(x_k) = \Sigma(\Theta_l)$,* for all $x_k \in X_k$.

Conditions I_k and Sy_k guarantee that the platform can price discriminate along the agents' locations, without leaving the agents rents for the private information the agents may possess regarding their locations. That is, in Scenarios (ii)-(iv), these conditions guarantee that the platform obtains the same profits as when locations are public on both sides, as in Scenario (i). To gain some intuition, consider first Scenario (ii). Under Conditions I_a and Sy_b, the platform's pricing problem on side a is symmetric across any two locations. This is because of two reasons. First, the location of any agent from side a provides no information about the agent's vertical preferences (this is guaranteed by Condition I_a). Second, when the platform offers the same tariffs as in Scenario (i), the gross utility that each type $\theta_k = (v_a, x_a)$ obtains from the matching set $\mathbf{s}_a^*(\theta_k)$ coincides with the gross utility obtained by type $(v_a, x_a + d)$ from choosing the matching set $\mathbf{s}_a^*(v_a, x_a + d)$, $d \in [0, 1/2]$. This occurs because the matching set $\mathbf{s}_a^*(v_a, x_a + d)$ is a *parallel translation* of the matching set $\mathbf{s}_a^*(v_a, x_a)$ by d units of distance, along the horizontal dimension (this is guaranteed by Condition Sy_b). As a result, when, in Scenario (ii), the platform offers the same matching plans and tariffs as in Scenario (i), the matching sets demanded by any two agents with types (v_a, x_a) and $(v_a, x_a + d)$ are parallel translations of one another, and are priced identically. Note that, to guarantee that agents

tions are private information on side k, the collection of matching plans is required to have the property that for any set $\mathbf{s}_k \in \mathcal{S}_k(x_k) \cap \mathcal{S}_k(x_k')$, the total payment associated with $\mathbf{s}_k$ is the same no matter whether the set is obtained by selecting the plan x_k or the plan x_k'. When, instead, locations are public, the collection of matching plans $\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k)) : x_k \in X_k\}$ may entail multiple prices for the same matching set $\mathbf{s}_k$. However, because, in this case, each agent can be constrained to choosing the plan designed for his location, de facto each agent faces a tariff specifying a single price for each set.

reveal their locations when the latter are the agents' private information, the platform may need to restrict the set of possible customizations at each location, $\mathcal{S}_k(x_k)$, to coincide with the matching sets demanded under Scenario (i).[33] The above properties, together with a judicious restriction on the set of possible customizations $\mathcal{S}_k(x_k)$, imply that the matching demands and payments induced in Scenario (i) are implementable also under Scenario (ii). A symmetric situation applies to Scenario (iii). Arguments similar to the ones above for Scenarios (ii) and (iii) imply that, in Scenario (iv), where locations are private on both sides, when the platform offers the same matching plans and tariffs as in Scenario (i) to each side, agents continue to choose the same matching sets as in Scenario (i), provided that Condition Sy_k holds on both sides of the market.

We establish the above results using mechanism design techniques. Let

$$(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$$

denote a direct revelation mechanism, where agents are asked to report their types and where $(\mathbf{s}_k(\theta_k), p_k(\theta_k))$ denotes the allocation (matching set and total transfer) specified by the mechanism for each side-k agent reporting θ_k.

By familiar envelope arguments, a necessary condition for each type $\theta_k = (v_k, x_k) \in \Theta_k$, $k = a, b$, to prefer reporting truthfully to lying with respect to the vertical dimension v_k while reporting truthfully the horizontal dimension x_k is that transfers satisfy the envelope conditions

$$\begin{aligned} p_k(\theta_k) = & \int_{\mathbf{s}_k(\theta_k)} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - \int_{\underline{v}_k}^{v_k} \int_{\mathbf{s}_k(y, x_k)} \frac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dF_l(\theta_l) dy, \\ & - U_k(\underline{v}_k, x_k), \end{aligned} \tag{15}$$

where $U_k(\underline{v}_k, x_k)$ is the payoff of a side-k agent with type $(\underline{v}_k, x_k)$.

Using (15), the platform's profits under any incentive-compatible mechanism can then be written as

$$\sum_{k=a,b} \int_{\Theta_k} \left\{ \begin{array}{c} \int_{\mathbf{s}_k(\theta_k)} \left[u_k\left(v_k, |x_k - x_l|\right) - \frac{1 - F_k^{v|x}(v_k|x_k)}{f_k^{v|x}(v_k|x_k)} \cdot \frac{\partial u_k}{\partial v}\left(v_k, |x_k - x_l|\right) \right] dF_l(\theta_l) \\ -U_k(x_k, \underline{v}_k) \end{array} \right\} dF_k(\theta_k).$$

Using the definition of the virtual-value functions $\varphi_k(\theta_k, \theta_l)$ in the main text, we then have that the platform's profits are maximal when $U_k(\underline{v}_k, x_k) = 0$ for all $x_k \in X_k$, $k = a, b$, and when the matching sets are chosen so as to maximize

$$\sum_{k=a,b} \int_{\Theta_k} \left\{ \int_{\mathbf{s}_k(\theta_k)} \varphi_k(\theta_k, \theta_l)\, dF_l(\theta_l) \right\} dF_k(\theta_k) \tag{16}$$

subject to the reciprocity condition

$$\theta_l \in \mathbf{s}_k(\theta_k) \Longleftrightarrow \theta_k \in \mathbf{s}_l(\theta_l), \quad l, k \in \{a, b\},\ k \neq l. \tag{17}$$

[33] In the absence of such restrictions, an agent of type $\theta_k = (v_k, x_k)$ misrepresenting his location to be $x_k' \neq x_k$ may find it optimal to select a matching set that no agent located at x_k' would have demanded under Scenario (i).

Hereafter, we first describe the matching sets that maximize (16) subject to the above reciprocity condition and then show that, under the assumptions in the lemma, the platform can implement the allocations $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k\in\Theta_k}^{k=a,b}$, where the functions $s_k(\cdot)$ are those that maximize (16) subject to (17), and where the functions $p_k(\cdot)$ are as in (15), with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$.

Define the indicator function $m_k(\theta_k, \theta_l) \in \{0, 1\}$ taking value one if and only if $\theta_l \in \mathbf{s}_k(\theta_k)$, that is, if and only if the two types θ_k and θ_l are matched. Then define the following measure on the Borel sigma-algebra over $\Theta_k \times \Theta_l$:

$$\nu_k(E) \equiv \int_E m_k(\theta_k, \theta_l)dF_k(\theta_k)dF_l(\theta_l). \tag{18}$$

Reciprocity implies that $m_k(\theta_k, \theta_l) = m_l(\theta_l, \theta_k)$. As a consequence, the measures ν_k and ν_l satisfy $d\nu_k(\theta_k, \theta_l) = d\nu_l(\theta_l, \theta_k)$. Equipped with this notation, the expression in (16) can be rewritten as

$$\begin{aligned}&\sum_{k,l=a,b,\, l\neq k} \int_{\Theta_k\times\Theta_l} \varphi_k\left(\theta_k, \theta_l\right) d\nu_k(\theta_k, \theta_l)\\ &= \int_{\Theta_k\times\Theta_l} \triangle_k(\theta_k, \theta_l)m_k(\theta_k, \theta_l)dF_k(\theta_k)dF_l(\theta_l),\end{aligned} \tag{19}$$

where, for $k, l = a, b$, $l \neq k$,

$$\triangle_k(\theta_k, \theta_l) \equiv \varphi_k\left(\theta_k, \theta_l\right) + \varphi_l\left(\theta_l, \theta_k\right).$$

Note that the functions $\triangle_a(\theta_a, \theta_b) = \triangle_b(\theta_b, \theta_a)$ represent the marginal effects on the platform's profits of matching types θ_a and θ_b. It is then immediate that the rule $(m_k(\cdot))_{k=a,b}$ that maximizes the expression in (19) is such that, for any $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l = a, b$, $l \neq k$, $m_k(\theta_k, \theta_l) = 1$ if and only if

$$\triangle_k(\theta_k, \theta_l) \geq 0.$$

That the virtual values $\varphi_k\left(\theta_k, \theta_l\right)$ are strictly increasing in v_k, $k, l = a, b$, $l \neq k$, then implies that the matching rule that maximizes (16) subject to the reciprocity condition (17) can be described by means of a collection of threshold functions $t_k^* : \Theta_k \times X_l \to V_l$, $k, l = a, b$, $l \neq k$, such that, for any $\theta_k = (v_k, x_k)$, any $\theta_l = (v_l, x_l)$, $\theta_l \in \mathbf{s}_k(\theta_k)$ if, and only if, $v_l \geq t_k^*\left(\theta_k, x_l\right)$. The threshold functions $t_k^*\left(\cdot\right)$ are such that, for any $\theta_k \in \Theta_k$, any $x_l \in [0, 1]$, $t_k^*\left(\theta_k, x_l\right) = \underline{v}_l$ if $\triangle_k(\theta_k, (\underline{v}_l, x_l)) > 0$, $t_k^*\left(\theta_k, x_l\right) = \bar{v}_l$ if $\triangle_k(\theta_k, (\bar{v}_l, x_l)) < 0$, and $t_k^*\left(\theta_k, x_l\right)$ is the unique solution to $\triangle_k\left(\theta_k, \left(t_k^*\left(\theta_k, x_l\right), x_l\right)\right) = 0$ if

$$\triangle_k(\theta_k, (\underline{v}_l, x_l)) \leq 0 \leq \triangle_k(\theta_k, (\bar{v}_l, x_l)).$$

That the virtual values $\varphi_k\left(\theta_k, \theta_l\right)$ are increasing in v_k and decreasing in $|x_k - x_l|$ also implies that, for ay $x_k, x_l \in [0, 1]^2$, the threshold $t_k^*\left(\theta_k, x_l\right)$ is decreasing in v_k, and that, for any v_k, $t_k^*\left(\theta_k, x_l\right)$ is non-decreasing in $|x_l - x_k|$.

Equipped with the above result, we now show that, in each of the environments stated in the generalized version of the lemma reported above, the platform can implement the allocations $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k\in\Theta_k}^{k=a,b}$, where $\mathbf{s}_k(\theta_k)$ are the matching sets described by the above threshold rule, and where the payment functions $p_k(\theta_k)$ are the ones in (15), with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$.

First observe that the payoff that each type θ_k obtains in the above direct revelation mechanism when reporting truthfully is equal to

$$U_k(\theta_k) = \int_{\underline{v}_k}^{v_k} \int_{\mathbf{s}_k(y,x_k)} \frac{\partial u_k}{\partial v} (y, |x_k - x_l|) \, dF_l(\theta_l) dy.$$

That $U_k(\theta_k) \geq 0$ follows directly from the fact that u_k is non-decreasing in v_k. This means that the mechanism is individually rational (meaning that each type θ_k prefers participating in the mechanism and receiving the allocation $(\mathbf{s}_k(\theta_k), p_k(\theta_k))$ to refusing to participate and receiving the allocation $(\emptyset, 0)$ yielding a payoff equal to zero).

Below we show that either the above direct mechanism is also incentive-compatible (meaning that each type θ_k prefers the allocation $(\mathbf{s}_k(\theta_k), p_k(\theta_k))$ designed for him to the allocation $(\mathbf{s}_k(\theta_k'), p_k(\theta_k'))$ designed for any other type θ_k'), or it can be turned, at no cost to the platform, into a mechanism implementing the same allocations as the above ones which is both incentive compatible and individually rational.

Definition 6. (nested matching) A matching rule $\mathbf{s}_k(\theta_k)$ is nested if, for any pair $\theta_k = (v_k, x_k)$ and $\hat{\theta}_k = (\hat{v}_k, \hat{x}_k)$ such that $x_k = \hat{x}_k$, either $\mathbf{s}_k(\theta_k) \subseteq \mathbf{s}_k(\hat{\theta}_k)$, or $\mathbf{s}_k(\theta_k) \supseteq \mathbf{s}_k(\hat{\theta}_k)$. A direct revelation mechanism is nested if its matching rule is nested.

Clearly, the direct mechanism defined above where the matching rule is described by the threshold function $t_k^* (\theta_k, x_l)$ is nested. Now let $\Pi_k(\theta_k; \hat{\theta}_k)$ denote the payoff that type θ_k obtains in a direct revelation mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ by mimicking type $\hat{\theta}_k$.

Definition 7. (ICV) A direct revelation mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ satisfies incentive compatibility along the v dimension (ICV) if, for any $\theta_k = (v_k, x_k)$ and $\hat{\theta}_k = (\hat{v}_k, \hat{x}_k)$ with $x_k = \hat{x}_k$, $U_k(\theta_k) \geq \Pi_k(\theta_k; \hat{\theta}_k)$.

The following property is then true (the proof is standard and hence omitted):

Property 1. *A nested direct revelation mechanism* $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ *satisfies ICV if, and only if, the following conditions jointly hold:*

1. *for any* $\theta_k = (v_k, x_k)$ *and* $\hat{\theta}_k = (\hat{v}_k, \hat{x}_k)$ *such that* $x_k = \hat{x}_k$, $v_k > \hat{v}_k$ *implies that* $\mathbf{s}_k(\theta_k) \supseteq \mathbf{s}_k(\hat{\theta}_k)$;
2. *the payment functions* $p_k(\theta_k)$ *satisfy the envelope formula (15).*

Clearly, the direct revelation mechanism where the matching rule is the one corresponding to the threshold functions $t_k^*(\cdot)$ described above and where the payment functions $p_k(\theta_k)$ are the ones in (15), with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$, is not only nested but satisfies the two conditions in the lemma. It follows that such a mechanism satisfies ICV.

Equipped with the above results, we now show that, in each of the environments corresponding to the combination of conditions described in the general version of the lemma presented above, the above direct revelation mechanism is either incentive-compatible, or it can be augmented to implement the same allocations prescribed by $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ at no extra cost to the platform.

Consider first Scenario (i). Recall that, in this case, locations are public on both sides. That the mechanism is ICV implies that any deviation along the vertical dimension is unprofitable. Furthermore, because locations are public on both sides, any deviation along the horizontal dimension is detectable. It is then immediate that the platform can augment the above direct revelation mechanism by adding to it punishments (in the form of large fines) for those agents lying along the horizontal dimension. The augmented mechanism is both individually rational and incentive compatible and implements the same allocations as the original mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$, at no extra cost to the platform.

Next suppose the environment satisfies the properties of Scenario (ii) and, in addition, Conditions I_a and Sy_b hold. Again, because locations are public on side b, incentive compatibility on side b can be guaranteed by augmenting the mechanism as described above for Scenario (i). Thus consider incentive compatibility on side a. The latter requires that

$$U_a(v_a, x_a) \geq \Pi_a((v_a, x_a); (\hat{v}_a, \hat{x}_a)),$$

for all $(x_a, \hat{x}_a, v_a, v_a) \in X_a^2 \times V_a^2$. The above inequality is equivalent to

$$\begin{aligned}\int_{\underline{v}_a}^{v_a} \int_{\mathbf{s}_a(y,x_a)} \frac{\partial u_a}{\partial v}(y, |x_a - x_b|)\, dF_b(\theta_b) dy \geq \int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,\hat{x}_a)} \frac{\partial u_a}{\partial v}(y, |\hat{x}_a - x_b|)\, dF_b(\theta_b) dy \\ + \int_{\mathbf{s}_a(\hat{v}_a,\hat{x}_a)} [u_a(v_a, |x_a - x_b|) - u_k(\hat{v}_a, |\hat{x}_a - x_b|)] dF_b(b).\end{aligned} \tag{20}$$

It is easy to see that, for any $\theta_a = (v_a, x_a) \in \Theta_a$,

$$\int_{\mathbf{s}_a(v_a,x_a)} \frac{\partial u_a}{\partial v}(v_a, |x_a - x_b|)\, dF_b(\theta_b) = \int_{d \in [0,1/2]} \frac{\partial u_a(v_a, d)}{\partial v} dW(d; \theta_a), \tag{21}$$

where $W(d; \theta_a)$ is the measure of agents whose distance from x_a is at most d included in the matching set $\mathbf{s}_a(v_a, x_a)$ of type θ_a under the proposed mechanism. It is also easy to see that, under Conditions I_a and Sy_b, the expression in (21) is invariant in x_a. That is, $W(d; \theta_a) = W(d; \theta'_a)$ for any $d \in [0, 1/2]$, any $\theta_a, \theta'_a \in \Theta_a$ with $v_a = v'_a$.[34] This means that

$$\int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,\hat{x}_a)} \frac{\partial u_a}{\partial v}(y, |\hat{x}_a - x_b|)\, dF_b(\theta_b) dy = \int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,x_a)} \frac{\partial u_a}{\partial v}(y, |x_a - x_b|)\, dF_b(\theta_b) dy.$$

By the same arguments,

$$\int_{\mathbf{s}_a(\hat{v}_a,\hat{x}_a)} u_a(\hat{v}_a, |\hat{x}_a - x_b|)\, dF_b(\theta_b) = \int_{\mathbf{s}_a(\hat{v}_a,x_a)} u_a(\hat{v}_a, |x_a - x_b|)\, dF_b(\theta_b).$$

Furthermore, because virtual values respect the same ranking as the true values, the threshold functions $t_k^*(\theta_k, x_l)$ are non-decreasing in the distance $|x_l - x_k|$. In turn, this implies that

$$\int_{\mathbf{s}_a(\hat{v}_a,\hat{x}_a)} u_a(v_a, |x_a - x_b|)\, dF_b(\theta_b) \leq \int_{\mathbf{s}_a(\hat{v}_a,x_b)} u_a(v_a, |x_b - x_a|)\, dF_b(\theta_b).$$

[34] Conditions I_k, $k = a, b$, suffice to guarantee that the function $\Delta_k(\theta_k, \theta_l)$ depends only on v_k, v_l, and $|x_l - x_k|$. The strengthening of Condition I_b to Sy_b is, however, necessary to guarantee that the mass of agents of a given distance d included in the matching sets of any pair of types $\theta_a, \theta'_a \in \Theta_a$ with $v_a = v'_a$ is the same.

It follows that the right hand side of (20) is smaller than

$$\int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,x_a)} \frac{\partial u_k}{\partial v}\left(y, |x_a - x_b|\right) dF_b(\theta_b) dy$$
$$+ \int_{\mathbf{s}_a(\hat{v}_a,x_a)} [u_a\left(v_a, |x_a - x_b|\right) - u_a\left(\hat{v}_a, |x_a - x_b|\right)] dF_b(\theta_b),$$

which is the payoff that type $\theta_a = (v_a, x_a)$ obtains by announcing $(\hat{v}_a, x_a)$ (that is, by lying about the vertical dimension but reporting truthfully the horizontal one). That the inequality in (20) holds then follows from the fact that the direct revelation mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ satisfies ICV.

The arguments for an environment satisfying the properties of Scenario (iii) along with Conditions I_b and Sy_a are symmetric to those for an environment satisfying the properties of Scenario (ii) along with Conditions I_a and Sy_b, and hence the proof is omitted.

Finally, consider an environment satisfying the properties of Scenario (iv) along with Conditions Sy_a and Sy_b. That the proposed mechanism is incentive compatible follows from the same arguments as for Scenario (ii) above, now applied to both sides of the market.

We conclude that, in each of the environments considered in the general version of the lemma reported above, the allocations $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$, where the matching sets $\mathbf{s}_k(\theta_k)$ are the ones specified by the threshold functions $t_k^*(\cdot)$ described above, and where the payments are the ones in (15) with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$ can be sustained in a mechanism that is both individually rational and incentive compatible. The result we wanted to establish then follows from the fact that (a) such allocations are profit-maximizing among those consistent with the rationality of the agents (i.e., satisfying the IC and IR constraints), and (b) can be induced by offering customized tariffs

$$\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k)) : x_k \in [0, 1]\}$$

satisfying the properties described below. For each plan $x_k \in [0, 1]$, the baseline configuration is given by

$$\underline{\mathbf{s}}_k(x_k) = \mathbf{s}_k(\underline{v}_k, x_k),$$

the baseline price is given by

$$\underline{T}_k(x_k) = p_k(\underline{v}_k, x_k) = \int_{\mathbf{s}_k(\underline{v}_k, x_k)} u_k\left(\underline{v}_k, |x_k - x_l|\right) dF_l(\theta_l),$$

the set of possible customizations is given by

$$\mathcal{S}_k(x_k) = \{\mathbf{s}_k(v_k, x_k) : v_k \in V_k\},$$

and the price schedules $\rho_k(q|x_l; x_k)$ are such that, for $q = q_{x_l}\left(\mathbf{s}_k(\underline{v}_k, x_k)\right)$, $\rho_k(q|x_l; x_k) = 0$, while for $q \in \left(q_{x_l}\left(\mathbf{s}_k(\underline{v}_k, x_k)\right), q_{x_l}\left(\mathbf{s}_k(\bar{v}_k, x_k)\right)\right]$,

$$\rho_k(q|x_l; x_k) = q u_k\left(v_k(q; x_k, x_l), |x_k - x_l|\right) - \int_{\underline{v}_k}^{v_k(q; x_k, x_l)} q_{x_l}\left(\mathbf{s}_k(y, x_k)\right) \frac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dy - \underline{T}_k(x_k) \tag{22}$$

where

$$v_k(q; x_k, x_l) \equiv \inf \left\{ v_k \in V_k : q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q \right\}.$$

Any agent selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k))$ and then choosing a matching set $\mathbf{s}_k \notin \mathcal{S}_k(x_k)$ is charged a fine large enough to make the utility of such a set, net of the payment, negative for all types. Likewise, when locations are public on side k, any side-k agent selecting a plan other than $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k))$ is charged a large enough fine to make the choice unprofitable for any type. Note that the existence of such fines is guaranteed by the assumption that u_k is bounded, $k = a, b$.

That the above customized tariff implements the same allocations as the direct mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ then follows from the following considerations. Each type $\theta_k = (v_k, x_k)$, by selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k))$ designed for agents with the same location as type θ_k and then choosing the customization $\mathbf{s}_k(v_k, x_k)$ specified by the direct mechanism for type θ_k is charged a total payment equal to

$$\underline{T}_k(x_k) + \int_0^1 \left[q_{x_l}(\mathbf{s}_k(v_k, x_k)) u_k\left(v_k, |x_k - x_l|\right) - \int_{\underline{v}_k}^{v_k} q_{x_l}(\mathbf{s}_k(y, x_k)) \tfrac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dy \right] dx_l - \underline{T}_k(x_k)$$

$$= \int_{\mathbf{s}_k(\theta_k)} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - \int_{\underline{v}_k}^{v_k} \int_{\mathbf{s}_k(y, x_k)} \tfrac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dF_l(\theta_l) dy$$

$$= p_k(\theta_k),$$

exactly as in the direct mechanism. That each type θ_k maximizes his payoff by selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k))$ and then choosing the customization $\mathbf{s}_k(v_k, x_k)$ specified for him by the direct mechanism then follows from the fact that (a) the direct mechanism is incentive compatible, (b) the payment associated with any other plan $(\underline{\mathbf{s}}_k(\hat{x}_k), \underline{T}_k(\hat{x}_k), \rho_k(\cdot|\cdot; \hat{x}_k), \mathcal{S}_k(\hat{x}_k))$ followed by the selection of a set $\mathbf{s}_k$ is either equal to the payment specified by the direct mechanism for some report $(\hat{v}_k, \hat{x}_k)$, or is so large to make the net payoff of such selection negative.

Finally, to see that, when locations are public on side k, without loss of optimality, the side-k customized tariff does not need to restrict the agents' ability to customize their matching sets (that is, $\mathcal{S}_k(x_k) = \Sigma(\Theta_l)$, all x_k) recall that, in this case, each side-k agent located at x_k can be induced to select the matching plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k))$ designed for agents located at x_k by setting the fee associated with the selection of any other plan sufficiently high. The separability of the agents' preferences then implies that, once the plan $\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathcal{S}_k(x_k))$ is selected, even if $\mathcal{S}_k(x_k) = \Sigma(\Theta_l)$, because the price schedules $\rho_k(\cdot|\cdot; x_k)$ satisfy (22), type θ_k prefers to select $q_{x_l}(\mathbf{s}_k(v_k, x_k))$ agents from each location x_l to any other mass of agents from the same location x_l, irrespective of the mass of agents from other locations type θ_k includes in his matching set. Q.E.D.

Proof of Proposition 1. Fix a pair of locations $x_a, x_b \in [0, 1]$. From Lemma 1, the profit-maximizing tariffs are customized and induce agents to select matching sets satisfying the threshold property of Lemma 1. Furthermore, from the proof of Lemma 1, for any $\theta_k = (v_k, x_k)$, any

$x_l \in [0,1]$, the threshold t_k^* is such that $t_k^*(\theta_k, x_l) = \underline{v}_l$ if $\triangle_k(\theta_k, (\underline{v}_l, x_l)) > 0$, $t_k^*(\theta_k, x_l) = \bar{v}_l$ if $\triangle_k(\theta_k, (\bar{v}_l, x_l)) < 0$, and $t_k^*(\theta_k, x_l)$ is the unique solution to $\triangle_k(\theta_k, (t_k^*(\theta_k, x_l), x_l)) = 0$ if

$$\triangle_k(\theta_k, (\underline{v}_l, x_l)) \leq 0 \leq \triangle_k(\theta_k, (\bar{v}_l, x_l)).$$

This means that, for any $q_k \in (0, f_l^x(x_l))$, either there exists no $v_k \in V_k$ such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$, or there exists a unique $v_k \in V_k$ such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$. Now take any $q_k \in (0, f_l^x(x_l))$ for which there exists $v_k \in V_k$ such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$. As explained in the main text, for any such q_k, the unique value of v_k such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$ is also the unique value of v_k that solves

$$u_k(v_k, |x_k - x_l|) = \rho'_k(q_k|x_l; x_k). \tag{23}$$

Now let $\hat{v}_{x_l}(\rho'_k|x_k)$ be the unique solution to (23) and $v'_l(q_k; x_l)$ be the unique solution to

$$\left[1 - F_l^{v|x}\left(v'_l(q_k; x_l)|x_l\right)\right] f_l^x(x_l) = q_k.$$

That the demands under the profit-maximizing tariffs satisfy the threshold structure of Lemma 1 implies that

$$t_k^*\left(\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), x_k\right), x_l\right) = v'_l(q_k; x_l)$$

and that

$$\varphi_k\left(\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), x_k\right), \left(v'_l(q_k; x_l), x_l\right)\right) + \varphi_l\left(v'_l(q_k; x_l), x_l), \left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), x_k\right)\right) = 0. \tag{24}$$

Lastly, observe that, for any such q_k,

$$\frac{\rho'_k(q_k|x_l; x_k)}{\varepsilon_k\left(\rho'_k|x_l; x_k\right)} = \frac{1 - F_k^{v|x}(\hat{v}_{x_l}\left(\rho'_k|x_k\right)|x_k)}{f_k^{v|x}(\hat{v}_{x_l}\left(\rho'_k|x_k\right)|x_k)} \frac{\partial u_k}{\partial v}\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), |x_k - x_l|\right). \tag{25}$$

Using the definition of φ_k from the main text together with (23) and (25), we then have that, for any such q_k,

$$\varphi_k\left(\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), x_k\right), \left(v'_l(q_k; x_l), x_l\right)\right) = \rho'_k(q_k|x_l; x_k)\left[1 - \frac{1}{\varepsilon_k\left(\rho'_k|x_l; x_k\right)}\right]. \tag{26}$$

Likewise, when $q_l = \left[1 - F_k^{v|x}(\hat{v}_{x_l}(\rho'_k|x_k)|x_k)\right] f_k^x(x_k)$,

$$\varphi_l\left(\left(v'_l(q_k; x_l), x_l\right), \left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), x_k\right)\right) = \rho'_l(q_l|x_k; x_l)\left[1 - \frac{1}{\varepsilon_l\left(\rho'_l|x_k; x_l\right)}\right]. \tag{27}$$

Combining (26) and (27) with (24), we obtain the result in the proposition. Q.E.D.

Proof of Proposition 2. The platform's problem consists in choosing a collection of side-a uniform price schedules $p_a(\cdot|x_b)$, one for each side-b location $x_b \in [0,1]$, along with a collection of side-b price schedules $\rho_b(\cdot|x_a;x_b)$, one for each pair $(x_a,x_b)\in[0,1]^2$, that jointly maximize its profits, which can be conveniently expressed as

$$\int_0^1\int_0^{f_b^x(x_b)}\bar{D}_a\left(p_a'(q|x_b)|x_b\right)p_a'(q|x_b)dqdx_b$$

$$+\int_0^1\int_0^1\int_0^{f_a^x(x_a)}D_b\left(\rho_b'(q|x_a;x_b)|x_a;x_b\right)\rho_b'(q|x_a;x_b)dqdx_adx_b,$$

subject to the feasibility constraint (3).

For any x_b, $q\le f_b^x(x_b)$, and $p_a'(q|x_b)$, let

$$\hat{v}_{x_b}\left(p_a'|x_a\right)=\begin{cases} v_a\ s.t.\ u_a(v_a,|x_a-x_b|)=p_a' & \text{if } p_a'\in[u_a(\underline{v}_a,|x_a-x_b|),u_a(\bar{v}_a,|x_a-x_b|)] \\ \\ \underline{v}_a & \text{if } p_a'<u_a(\underline{v}_a,|x_a-x_b|) \\ \\ \bar{v}_a & \text{if } p_a'>u_a(\bar{v}_a,|x_a-x_b|). \end{cases} \tag{28}$$

Given the above definition, we have that the demand by the x_a-agents for the q-th unit of the x_b-agents at the marginal price $p_a'(q|x_b)$ is equal to

$$D_a\left(p_a'(q|x_b)|x_b;x_a\right)=\left[1-F_a^{v|x}\left(\hat{v}_{x_b}\left(p_a'|x_a\right)|x_a\right)\right]f_a^x(x_a).$$

Also, for any $q\le f_b^x(x_b)$, recall that $v_b'(q;x_b)$ is the unique solution to $\left[1-F_b^{v|x}\left(v_b'(q;x_b)|x_b\right)\right]f_b^x(x_b)=q$. Reciprocity, along with optimality, implies that the most profitable way to deliver q units of x_b-agents to each x_a-agent demanding to be matched to q units of x_b-agents is to match the x_a-agent to every x_b-agent whose vertical type exceeds $v_b'(q;x_b)$. In other words, the optimal tariffs induce matching demands with a threshold structure, as in the case where tariffs are customized on both sides of the market (cfr Lemma 1). Now for each $x_a,x_b\in[0,1]$, each $q\le f_b^x(x_b)$, let

$$\hat{q}_b(q;x_a;x_b)\equiv D_a\left(p_a'(q|x_b)|x_b;x_a\right).$$

Given $p_a'(q|x_b)$, the platform thus optimally selects customized prices for the x_b-agents for each quantity $\hat{q}_b(q;x_a;x_b)$ of the x_a-agents equal to

$$\rho_b'(\hat{q}_b(q;x_a;x_b)|x_a;x_b)=u_b(v_b'(q;x_b),|x_b-x_a|). \tag{29}$$

Such prices guarantee that, for each $x_a\in[0,1]$, $D_b\left(\rho_b'(\hat{q}_b(q;x_a;x_b)|x_a;x_b)|x_a;x_b\right)=q$, thus clearing the market.

The function $p_a'(q|x_b):\mathbb{R}\times[0,1]\to\mathbb{R}$ thus uniquely defines the matching sets on both sides of the market. Now, from the arguments in the proof of Lemma 1, we know that the maximal revenue the platform receives from the side-b agents when each x_b-agent with vertical type v_b is assigned a matching set equal to $\mathbf{s}_b(v_b,x_b)$ is given by

$$\int_{\Theta_b}\left\{\int_0^1\left\{u_b\left(v_b,|x_b-x_a|\right)-\frac{1-F_b^{v|x}(v_b|x_b)}{f_b^{v|x}(v_b|x_b)}\cdot\frac{\partial u_b}{\partial v}\left(v_b,|x_b-x_a|\right)\right\}q_{x_a}(\mathbf{s}_b(v_b,x_b))dx_a\right\}dF_b(\theta_b).$$

In turn, this means that the platform's problem can be re-casted as choosing a function $\frac{dp_a}{dq}(q|x_b) : \mathbb{R} \times [0,1] \to \mathbb{R}$ that maximizes

$$\int_0^1 \int_0^{f_b^x(x_b)} \left\{ \bar{D}_a \left(p_a'(q|x_b)|x_b \right) p_a'(q|x_b) - \mathcal{C}\left[p_a'(q|x_b) \right] \right\} dq dx_b$$

where, for each $x_b \in [0,1]$, each $q \le f_b^x(x_b)$, the function

$$\mathcal{C}\left[p_a'(q|x_b)\right] \equiv$$

$$-\int_0^1 \left\{ u_b\left(v_b'(q;x_b), |x_b - x_a|\right) - \frac{1-F_b^{v|x}(v_b'(q;x_b)|x_b)}{f_b^{v|x}(v_b'(q;x_b)|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v_b'(q;x_b), |x_b - x_a|\right) \right\} D_a\left(p_a'(q|x_b)|x_b; x_a\right) dx_a$$

captures the "procurement costs" of clearing the matching demands of all side-a agents that demand at least q matches with the x_b-agents. This problem can be solved by point-wise maximization of the above objective function, i.e., by selecting for each $x_b \in [0,1]$, $q \le f_b^x(x_b)$ (equivalently, for each $(x_b, v_b) \in [0,1] \times V_b$), $p_a'(q|x_b)$ so as to maximize

$$\bar{D}_a \left(p_a'(q|x_b)|x_b \right) p_a'(q|x_l) - \mathcal{C}\left[p_a'(q|x_b) \right].$$

The first-order conditions for such a problem are given by

$$p_a'(q|x_b) \frac{\partial \bar{D}_a \left(p_a'(q|x_b)|x_b \right)}{\partial \left(p_a' \right)} \left[1 - \frac{1}{\bar{\varepsilon}_a \left(p_a'(q|x_b)|x_b \right)} \right] - \mathcal{C}'\left[p_a'(q_a|x_b) \right] = 0,$$

where

$$\mathcal{C}'\left[p_a'(q_a|x_b)\right]$$

$$= -\int_0^1 \left\{ u_b\left(v_b'(q;x_b), |x_b - x_a|\right) - \frac{1-F_b^{v|x}(v_b'(q;x_b)|x_b)}{f_b^{v|x}(v_b'(q;x_b)|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v_b'(q;x_b), |x_b - x_a|\right) \right\} \frac{\partial D_a(p_a'(q|x_b)|x_b;x_a)}{\partial(p_a')} dx_a.$$

Now observe that (29) implies that

$$u_b\left(v_b'(q;x_b), |x_b - x_a|\right) - \frac{1-F_b^{v|x}(v_b'(q;x_b)|x_b)}{f_b^{v|x}(v_b'(q;x_b)|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v_b'(q;x_b), |x_b - x_a|\right)$$

$$= \rho_b'(\hat{q}_b(q;x_a;x_b)|x_a;x_b) \left(1 - \frac{1}{\varepsilon_b\left(\rho_b'(\hat{q}_b(q;x_a;x_b)|x_a;x_b)|x_a;x_b\right)} \right).$$

This means that the above first-order conditions can be rewritten as

$$p_a'(q|x_b) \left[1 - \frac{1}{\bar{\varepsilon}_a(p_a'(q|x_b)|x_b)} \right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b, p_a'(q|x_b))} \left[\rho_b'(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b) \left(1 - \frac{1}{\varepsilon_b\left(\rho_b'(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)|\tilde{x}_a;x_b\right)} \right) \right] = 0,$$

where $H(x_a|x_b,q)$ is the distribution over $X_a = [0,1]$ whose density is given by

$$h_a\left(x_a|x_b, p_a'(q|x_b)\right) \equiv \frac{\frac{\partial D_a(p_a'(q|x_b)|x_b;x_a)}{\partial(p_a')}}{\frac{\partial \bar{D}_a(p_a'(q|x_b)|x_b)}{\partial(p_a')}}.$$

The above properties imply the result in the proposition. Q.E.D.

Proof of Proposition 3. The proof below is for the more general case in which the side-b preferences may depend on the locations.

Fix $\theta_b = (v_b, x_b)$ and let $q = f_b^x(x_b)\,[1 - F_b^v(v_b)]$. The result in Proposition 2 implies that, under uniform pricing on side a and customized pricing on side b, for any $x_a \in X_a$ such that $t_b^u(\theta_b, x_a) \in \text{Int}[V_a]$, $t_b^u(\theta_b, x_a)$ is such that

$$u_a(t_b^u(\theta_b, x_a), |x_b - x_a|) - \mathbb{E}_{H(\tilde{x}_a|x_b, p_a^{u\prime})}\left[\frac{1-F_a^{v|x}\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}{f_a^{v|x}\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right), |\tilde{x}_a - x_b|\right)\right]$$
$$+\mathbb{E}_{H(\tilde{x}_a|x_b, p_a^{u\prime})}\left[\varphi_b\left(\theta_b, \left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right), \tilde{x}_a\right)\right)\right] = 0, \tag{30}$$

where $H\left(x_a|x_b, p_a^{u\prime}\right)$ is the distribution over $X_a = [0, 1]$ whose density is given by

$$h\left(x_a|x_b, p_a^{u\prime}\right) \equiv \frac{\frac{\partial D_a(p_a^{u\prime}|x_b; x_a)}{\partial (p_a^{u\prime})}}{\frac{\partial \bar{D}_a(p_a^{u\prime}|x_b)}{\partial (p_a^{u\prime})}},$$

and where $p_a^{u\prime}$ is a shortcut for $p_a^{u\prime}(q|x_b)$ with the latter equal to $p_a^{u\prime}(q|x_b) = u_a(t_b^u(\theta_b, x_a), |x_a - x_b|)$. Note that, to arrive at (30), we used the result in Proposition 2 along with the property in (**??**) and the fact that, for any x_a such that $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right) \notin Int[V_a]$, $h\left(x_a|x_b, p_a^{u\prime}\right) = 0$, whereas for any x_a such that $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right) \in Int[V_a]$,

$$\frac{p_a^{u\prime}}{\varepsilon_a\left(p_a^{u\prime}|x_b; x_a\right)} = \frac{1 - F_a^{v|x}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\right)}{f_a^{v|x}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\right)} \cdot \frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right), |x_a - x_b|\right).$$

We also used the fact that, for any x_a such that $h\left(x_a|x_b, p_a^{u\prime}\right) > 0$ (equivalently, $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right) \in Int[V_a]$),

$$\rho_b'(\hat{q}_b(q; x_a; x_b)|x_a; x_b)\left(1 - \frac{1}{\varepsilon_b\left(\rho_b'(\hat{q}_b(q; x_a; x_b)|x_a; x_b)|x_a; x_b\right)}\right)$$
$$= \varphi_b\left(\theta_b, \left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right), x_a\right)\right),$$

as shown in the proof of Proposition 2.

On the other hand, under customized pricing on both sides, for any such $\theta_b = (v_b, x_b)$, any $x_a \in X_a$ such that $t_b^*(\theta_b, x_a) \in Int[V_a]$, the threshold $t_b^*(\theta_b, x_a)$ is such that

$$u_a(t_b^*(\theta_b, x_a), |x_b - x_a|) - \frac{1-F_a^{v|x}\left(t_b^*(\theta_b, x_a)\right)}{f_a^{v|x}\left(t_b^*(\theta_b, x_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b, x_a), |x_a - x_b|\right) + \varphi_b\left(\theta_b, \left(t_b^*(\theta_b, x_a), x_a\right)\right) = 0.$$

It is then immediate that, for any x_a such that

$$-\mathbb{E}_{H(\tilde{x}_a|x_b, p_a^{u\prime})}\left[\frac{1-F_a^{v|x}\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}{f_a^{v|x}\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right), |x_b - \tilde{x}_a|\right)\right]$$
$$+\mathbb{E}_{H(\tilde{x}_a|x_b, p_a^{u\prime})}\left[\varphi_b\left(\theta_b, \left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right), \tilde{x}_a\right)\right)\right]$$
$$\leq -\frac{1-F_a^{v|x}\left(t_b^*(\theta_b, x_a)\right)}{f_a^{v|x}\left(t_b^*(\theta_b, x_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b, x_a), |x_b - x_a|\right) + \varphi_b\left(\theta_b, \left(t_b^*(\theta_b, x_a), x_a\right)\right)$$

we have that $t_b^u(\theta_b, x_a) \geq t_b^*(\theta_b, x_a)$, whereas, for any x_a such that

$$-\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^u{}')}\left[\frac{1-F_a^{v|x}\left(\hat{v}_{x_b}(p_a^u{}'|\tilde{x}_a)\right)}{f_a^{v|x}\left(\hat{v}_{x_b}(p_a^u{}'|\tilde{x}_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^u{}'|\tilde{x}_a\right), |x_b - \tilde{x}_a|\right)\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^u{}')}\left[\varphi_b\left(\theta_b, \left(\hat{v}_{x_b}\left(p_a^u{}'|\tilde{x}_a\right), \tilde{x}_a\right)\right)\right]$$

$$\geq -\frac{1-F_a^{v|x}\left(t_b^*(\theta_b,x_a)\right)}{f_a^{v|x}\left(t_b^*(\theta_b,x_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b, x_a), |x_b - x_a|\right) + \varphi_b\left(\theta_b, \left(t_b^*(\theta_b, x_a), x_a\right)\right)$$

we have that $t_b^u(\theta_b, x_a) \leq t_b^*(\theta_b, x_a)$.

Also note that, by virtue of reciprocity, $t_b^u(\theta_b, x_a) \leq t_b^*(\theta_b, x_a)$ if and only if

$$t_a^u((t_b^*(\theta_b, x_a), x_a), x_b) \leq t_a^*((t_b^*(\theta_b, x_a), x_a), x_b)$$

and, likewise, $t_b^u(\theta_b, x_a) \geq t_b^*(\theta_b, x_a)$ if and only if

$$t_a^u((t_b^*(\theta_b, x_a), x_a), x_b) \geq t_a^*((t_b^*(\theta_b, x_a), x_a), x_b).$$

The above properties imply that uniform pricing (on side a) leads to more (alternatively, less) targeting than customized pricing (on both sides), if, for any θ_b, the function

$$L(x_a|\theta_b) \equiv \varphi_b\left(\theta_b, \left(t_b^*(\theta_b, x_a), x_a\right)\right) - \frac{1-F_a^{v|x}\left(t_b^*(\theta_b,x_a)|x_a\right)}{f_a^{v|x}\left(t_b^*(\theta_b,x_a)|x_a\right)} \cdot \frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b, x_a), |x_a - x_b|\right)$$

$$= \rho_b'\left(1 - \frac{1}{\varepsilon_b\left(\rho_b'|x_a;x_b\right)}\right)\Bigg|_{\rho_b'=u_b(v_b,|x_a-x_b|)} - \frac{\rho_a'}{\varepsilon_a(\rho_a'|x_b;x_a)}\Bigg|_{\rho_a'=u_a(t_b^*(\theta_b,x_a),|x_a-x_b|)}$$

is non-decreasing (alternatively, non-increasing) in the distance $|x_a - x_b|$.

Fixing θ_b, the function $L(x_a|\theta_b)$ is nondecreasing in $|x_a - x_b|$ when the side-a inverse-semi-elasticities are decreasing in distance and in price and the side-b preferences are invariant to distance. It is non-increasing in $|x_a - x_b|$ when the side-a inverse-semi-elasticities are increasing in distance and in price and the side-b preferences are invariant to distance. These properties establish the result in Part 1 in the proposition. The result in Part 2 then follows from the result in Part 1 along with the fact that the side-a inverse-semi-elasticities are decreasing (alternatively, increasing) in both distance and price when x_a and v_a are independent, the hazard rate for F_a^v is increasing in v_a, and u_a is submodular and concave in v_a (alternatively, x_a and v_a are independent, the hazard rate for F_a^v is decreasing in v_a, and u_a is supermodular and convex in v_a). Q.E.D.

Proof of Proposition 4. The proof follows from the combination of the results in Proposition 3 with the results in Proposition 1 in Aguirre et al (2010). When the environment satisfies the conditions in Part 1 of Proposition 3, starting from uniform pricing on side a, the introduction of customized pricing on side a leads to an increase in prices for nearby locations and a reduction in prices for distant locations. Proposition 1 in Aguirre et al (2010), along with the fact that the environment satisfies Condition NDR and that, for any x_b and p'_a, the convexity $CD_a\left(p'_a|x_b;x_a\right)$ of the demands by the x_a-agents for the q-th unit of the x_b-agents declines with the distance $|x_a - x_b|$, then implies that welfare of the side-a agents is higher under uniform pricing. Likewise, under the conditions in Part 2 of Proposition 3, that welfare of the side-a agents is higher under uniform pricing follows from the fact that, starting from uniform pricing on side a, the introduction of customized pricing on side a leads to an increase in prices for distant locations and a reduction in prices for nearby locations. The welfare implications of such price adjustments then follow again from Proposition 1 in Aguirre et al (2010), along with the fact that Condition NDR holds and that, for any x_b and p'_a, the convexity $CD_a\left(p'_a|x_b;x_a\right)$ of the demands by the x_a-agents for the q-th unit of the x_b-agents increases with the distance $|x_b - x_a|$. Q.E.D.

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