Two-sided Markets, Pricing, and Network Effects *

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Abstract

The chapter has 9 sections. It starts with the theory of two-sided markets and then moves to empirics. Section 1 introduces the reader to the literature on two-sided markets. Section 2 covers the case of markets dominated by a single monopolistic firm. Section 3 discusses the theoretical literature on competition for the market, focusing on pricing strategies that firms may follow to prevent entry. Section 4 discusses pricing in markets in which multiple platforms are active and serve both sides. Section 5 presents alternative models of platform competition. Section 6 discusses richer matching protocols whereby platforms price-discriminate by granting access only to a subset of the participating agents from the other side and discusses the related literature on matching design. Section 7 discusses identification in empirical work. Section 8 discusses estimation in empirical work. Finally Section 9 concludes.

1 Introduction

Consider a consumer that starts the day by searching for news on a tablet computer and then orders a car ride from a ride-sharing service over a mobile telephone. At each of these points, an intermediary connects the consumer to various providers. The consumer’s search terms for news are processed by a web search engine that mediates between content consumers and content providers. Simultaneously with this search, a separate intermediary system connects the consumer (and the consumer’s search behavior) to advertisers, leading to advertisements. The ride-sharing service is an intermediary between the consumer and potential ride-providers. Payment for the ride-sharing service is processed through a payment network, an intermediary that facilitates payments between buyers and sellers. These interactions are processed through an internet service provider that is an intermediary between consumers and a complex market of telecommunication systems. In addition, the

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devices in this story, the tablet and phone, can be regarded as intermediaries themselves, connecting consumers to software applications that are enabled by the devices’ operating systems. And most likely, at least one of them was purchased through an online marketplace that connects consumers to sellers.

Economics regards these intermediaries as platforms. Platforms are firms, or services of firms, that connect market participants and allow them to interact or transact. Platforms are prevalent on the internet and in many other markets: health insurance companies, for example, mediate between consumers and care providers, stock exchanges mediate between buyers and sellers (or liquidity providers and liquidity takers) and cable television companies mediate between content providers and viewers. The rise of platforms has changed the way vast parts of the economy are organized, and has led to intense reexamination of the nature of market power, competition policy, and regulation. Platform economics is central in understanding a wide variety of recent policy debates, such as net neutrality, financial market reforms, antitrust policy, as well as related topics such as privacy, consumer protection and media diversity.

A central aspect of platform economics is the role of network effects, which apply when a product is valued based on the extent to which other market participants adopt or use the same product. Of particular interest are indirect network effects, which emerge when the adoption and use of a product leads to increased provision of complementary products and services, with the value of adopting the original product increasing with the provision of such complementary goods. For instance, as more consumers adopt a video game console such as the Sony Playstation, more game-developers invest in developing games for that platform, raising the value of the console to consumers. In this sense, indirect network effects lead the platform firm to take into account the various interdependencies between the two sides of the market, and the pertinent literature studying such interdependencies is often termed the study of two-sided markets. Naturally, one can also consider multi-sided markets consisting of more than two sides. For instance, in producing the Windows operating system, Microsoft manages a three-sided market between hardware providers, software providers, and consumers. One might even distinguish between different types of hardware (CPUs, printers, screen devices, etc.) and types of software (productivity apps, games, etc.), in which case Windows mediates a large many-sided market.

As a product with network effects diffuses into the market, it becomes more valuable and drives further adoption. Indirect network effects thus lead to a feedback loop as more participants on each side of the platform find it more valuable to adopt and use the platform when they expect the other side to attract more users. This phenomenon leads to efficiencies as more market participants are able to interact with each other but also, in some circumstances, market power, as network effects can protect platform owners from entry. In markets with low marginal costs, as is the case for many digital markets, platforms with strong network effects can grow to be enormous and eventually dominate the market.

The most obvious tool that a platform has to manage and expand its use is price. Pricing decisions in the face of indirect network effects are complex because raising the price on one side of the market reduces demand not only on that side of the market but also on the other, as each side responds to changes in the participation on the other side. Finding the correct approach to pricing is often the difference between success and failure for a platform. For policy-makers attempting to evaluate the performance of a market, understanding the
complex determinants of prices is crucial.

In this chapter, we provide an overview of the literature on platform economics, focusing on indirect network effects and pricing. We first provide a unified theoretical treatment of a wide variety of approaches to these issues. We derive equilibrium and welfare-maximizing prices and compare the factors that lead to deviations from optimality. Standard intuition about how prices should respond to costs and competition often do not hold, and we lay out the intuition required to understand these markets. We discuss different treatments of how network effects may enter into consumer decision-making. For instance, whether agents have heterogeneous valuations of the number of agents on the platform turns out to have important consequences. In addition, the ability of platforms to price-discriminate is critical, leading to so-called divide-and-conquer strategies. Modeling platform firms can require new equilibrium concepts as these models are rife with multiple equilibria and one wishes to focus on the most relevant ones.

We contrast monopoly platforms with oligopoly platform competition. Whether each agent multi-homes across several platforms or single-homes on one platform turns out to be surprisingly important. We say surprising because one may think that if agents are divided into two sides of the market, such as buyers and sellers, and one set of agents multi-homes, then irrespective of whether this set comprises buyers or sellers, all agents can interact with all agents on the other side and this guarantees a level of efficiency. However, a robust result in the literature is that which side multi-homes has important implications for pricing and the ensuing allocation. When only one side multi-homes, platforms compete on the single-homing side but become like monopolists on the side that multi-homes, selling access to their single-homing agents. Total network benefits increase, but prices on each side may increase or decrease as a result of multi-homing.

Finally, we take on some more challenging concepts to model, such as consumer information and beliefs, and dynamics. Furthermore, in many contexts, a platform plays the central role of a matchmaker. Matching markets, and more generally market design, play an important role in platform economics. We provide an overview of the matching-design literature, once again focusing on pricing. Although we show that prices respond to this wide variety of modeling assumptions, we are able to draw out some consistent themes about pricing in these markets.

Next, we turn to empirical work in this area. A starting point and a central issue in this literature is estimating the magnitude of network effects. We view network effects as a form of “social spillover” or “neighborhood effect”; the choice of an agent is influenced by the choices of the agent’s peers. Identifying network effects is fraught with problems of simultaneity and omitted variables. These problems are similar to those in the literature on peer effects in social economics, such as identifying how the performance of one student affects the performance of classmates, or how out-of-wedlock childbirth and criminal arrest are influenced by the behavior and outcomes of peers. The social economics literature has made great strides in understanding the identification of these peer effects, exemplified by the work of Manski (1993, 1995) on the reflection problem and the literature that followed. We view the relationship between the social economics and network effects literature as underexplored. In this chapter, we provide a review of empirical approaches to estimate network effects from the perspective of the literature on social economics. Doing so provides a valuable connection between the network effects literature and the wider economics
literature and provides new insights into the identification and estimation of network effects.

With the estimation of network effects in hand, we turn to a review of the estimation of pricing in settings with network effects. We argue that empirical papers have mostly taken one of two approaches. The first approach is to obtain data on many platforms, often separated by geography, and to analyze how endogenous variables such as prices respond to exogenous market characteristics. These papers often use reduced-form econometrics and attempt to use theory to evaluate the effects that are expected to be found. The other approach is to obtain data on a single platform and study the interaction between agents on the platform, taking the platform pricing as exogenous. These papers are often quite structural and utilize detailed models of agent behavior on the platform. These papers then calculate various counterfactual outcomes associated with different price changes, and evaluate why firms choose the prices they do and what would happen if prices were to change. These methods provide different sorts of insights into pricing by platforms, but we are able to draw out some general lessons and connect them to the theoretical results in the first part of the chapter. For instance, our theoretical section discussed the seesaw effect that factors that depress price on one side of the market tend to raise price on the other side and our empirical section discusses papers that support the existence of the seesaw effect across a variety of industries.

1.1 Terminology and background


In this section, we define what we mean by a platform. Hereafter, we use the terms “two-sided market” and “platform market” interchangeably. Several papers offer definitions of twosidedness, such as Rochet and Tirole (2006), Weyl (2010), and Hagiu and Wright (2015a,b). We do not wish to adjudicate among these different definitions, but rather offer some sense of what economists normally mean with these terms. We generally consider a two-sided market to be one in which at least two distinct sets of agents (or sides) interact through an intermediary—a two-sided platform (hereafter platform)— and in which the behavior of each set of agents directly impacts the utility, or the profit, of the other set of agents. This impact of one set of agents on the other, and the resulting feedback to the first set of agents, is an indirect network effect, and to the extent that the effect is not perfectly internalized by the platform’s pricing, it represents a network externality.
For example, suppose buyers and sellers appear at an open marketplace where they may engage in trade. The marketplace organizer is the intermediary, or platform, and the sellers naturally care about how successful the intermediary is in attracting buyers, and vice versa, so the firm is two-sided. In contrast, suppose the marketplace organizer instead decides to purchase the sellers’ products wholesale and then retail the inventory to buyers. The sellers receive the wholesale price whether buyers materialize or not, and so the sellers no longer have an interest in the buyers’ behavior. In this case, we refer to the firm as “traditional,” or one-sided.\(^1\)

Network effects are often an important element of platform markets. However, platforms are not restricted to markets with network effects. Consider, for example, a single consumer and a single merchant navigating whether to use a payment card or not. Rochet and Tirole (2011) study a market like this. In this case, there is no sense in which more agents on one side of the payment card platform make the platform more valuable to agents on the other side. In practice, however, most papers that self-identify into the platform literature consider issues that involve network effects.

In reality, almost every real-world firm has some elements of two-sidedness to it, and it makes little sense to classify firms with some binary distinction as being a platform or not. Rather, one should see the platform nature of a firm as a continuous dimension.\(^2\) Platform economics may be more important for understanding some phenomena than others, even within a given firm or market. In antitrust, or in a regulatory setting, it may be necessary to categorize firms as one- or two-sided. In this chapter, instead, we do not attempt to do so, and we instead favor the continuous nature of two-sidedness.

It is common in overview articles on platforms to list the largest firms in the world and identify which are platforms and which are not, highlighting the growth and importance of platform firms in the organization of modern business activity. However, we eschew this because we do not view firms as inherently platform firms or not, but rather on a continuum. For example, it is typical to identify Amazon as a platform firm and automobile manufacturers as traditional one-sided firms. However, Amazon has always sold books in a one-sided manner (subject to local regulation) and, to the extent that it is expanding the sales of its own branded products, it is moving further in that direction. A substantial source of operating profits at Amazon currently comes from its cloud computing services, and it appears that a substantial share of that comes from services provided directly by Amazon.\(^3\) In contrast, Ford Motors can be seen as a platform mediating the interactions

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\(^1\)This distinction is similar to the one in Hagiu and Wright (2015b), who locate the definition of a platform market in the control the platform exerts over the transactions. The market is one-sided if the intermediary controls all of the features of trade, particularly the price that the buyer pays to the seller. In practice things are however more nuanced as we discuss below.

\(^2\)To push this point further, consider again the example of a marketplace organizer buying a seller’s product wholesale and retailing the inventory to consumers. We argued above that such instances are typically associated with traditional one-sided markets, due to the fact that the seller does not care about the success of the organizer. In practice, however, we often observe buy-back clauses whereby the marketplace organizer can force the seller to buy inventory back if the organizer sells poorly. Even in the absence of such formal clauses, if the seller has a long-term relationship with the organizer, the seller may have an interest in the success of the organizer with consumers, and this breaks down the strict one-sided nature of the example.

\(^3\)In fact, Amazon also has an AWS Marketplace that connects cloud users to third-party sellers through Amazon’s cloud services offerings. To the extent that the marketplace is driving Amazon’s cloud services
between consumers and dealerships. Ford cannot attract dealers to sell its product unless it has interested consumers and consumers cannot purchase Ford products unless dealers offer Ford products. In addition to setting a wholesale price to dealers, Ford manages a system of promotions to consumers and dealers to achieve successful outcomes. In practice, whether a firm is best viewed as a platform or a traditional firm can depend on which questions one seeks to answer about the firm’s behavior.

Taking a broader perspective, whether a firm is two-sided (or multi-sided) is often an endogenous choice of a firm, and firms can always integrate into one side of the market to reduce the platform nature of the firm. For example, as pointed out above, Microsoft produces the Windows operating system and thus relies on a three-sided market between consumers, hardware manufacturers, and software developers. Adjacent to that, Microsoft produces both the operating system and the hardware for the Xbox gaming system, leaving only a two-sided market between consumers and software developers. In this sense, it may be best to talk about two-sided strategies rather than two-sided markets to emphasize the role of a firm’s choice. The example above about Amazon expanding its own brand of products is an example of a firm endogenously choosing to be less two-sided in specific product markets, which ties together these ideas about two-sidedness being endogenous and continuous. Similarly, internet search is thought of as a canonical two-sided market that connects content consumers with content providers. However, to the extent that Google integrates into content (i.e., videos, maps, shopping services, music, etc.), it appears to be choosing a less two-sided approach.

An idea related to the endogeneity of two-sidedness is that two-sidedness describes firms, not markets. There are many examples of platform firms competing against one-sided firm in the same market. For instance, brokers that connect shippers with trucks compete in the same market against shippers that own their own trucks and may not need for-hire trucks. Media that is supported by advertising (and so is two-sided) competes against media that does not rely on advertising. Examples of media that does not use advertising are Consumer Reports, Wirecutter.com and Le Canard Enchainé. We refer the reader to Hagiu and Wright (2015b) for an overview of the recent literature addressing the endogeneity of two-sidedness.

Thus, from our perspective, calling this literature “two-sided markets” is not ideal. Two-sidedness is not a binary outcome endowed by a market but is rather a choice made by firms in what ways to be two-sided. Having clarified all of this, we follow in this chapter the common practice of referring to many of the issues we study as pertaining to two-sided and platform markets, as is clear from the chapter’s title. After all, this is the way this literature is referred to, and, given our focus on pricing (which is the most well-studied phenomenon

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4A famous definition of two-sidedness appears in Rochet and Tirole (2006), in which they define a market as two-sided if the price structure of the transactions matters beyond the price level. As they point out, this often hinges on whether the platform charges participation fees or not. However, we often think of a firm’s price structure as endogenous and not as a natural way to define a market type. One could argue that their definition is a useful way to determine whether a firm adopts a two-sided strategy or not, rather than whether the market is two-sided or not.

5Some readers will recall that America Online provided internet search in the early days of the World Wide Web and for some consumers, essentially all of their internet content. To the extent that America Online adopted a one-sided approach to internet search, it is an example of how markets are not inherently two-sided or not.
in this literature), there is little lost by treating two-sidedness as a market concept.

Our focus on pricing leads us to leave out large and important swaths of the literature, that we briefly review here. While the concepts we describe can be applied to all varieties of platform markets, we do not attempt to address the details of specific industries, such as finance and advertising-supported media. For overviews of the literature on two-sided media, see Anderson and Jullien (2015) and Peitz and Reisinger (2015). Rysman and Wright (2014) review the two-sided markets literature in the context of payment cards. Many industry-specific applications of this literature exist, such as voluntary standard setting organizations (see Lerner and Tirole, 2006).

Further, while we discuss deviations of pricing from social optimality and the effects of competition on outcomes, we do not review here the competition policy literature. See, for example, Jullien and Sand-Zantman (2020) and Evans and Schmalensee (2015) for an overview of this literature. The effects of platform economics on competition policy are profound, with numerous commissions around the world addressing issues related to platforms and laying out strategies for approaching digital platforms.\(^6\) Platform economics has pointedly affected policy-making. For instance, in the United States, in the context of a monopolization case, the Supreme Court has recently integrated two-sided markets into its approach to market definition, and the Securities and Exchange Commission explicitly recognizes the role of platform economics in regulating securities exchanges.\(^7\) Indeed, platform economics appears to be leading to new forms of regulation, such as the recently proposed Digital Markets Act in the EU and the Digital Markets Unit in the United Kingdom.\(^8\) Further, platform economics has implications for privacy regulation.\(^9\) A good starting place on digital markets is Goldfarb and Tucker (2019).

Finally, we stop short of covering platform design. A successful platform requires more than just appropriate pricing. Platforms also invest in quality that affects one side or the other, and choose the rules and nature of engagement between market participants, as well as the information that each side gets to observe. Ratings systems are a familiar feature of many digital platforms, and they bring up many design questions, as described in Tadelis (2016). The question of whether to be two-sided or not, as in Hagiu and Wright (2015b) is also largely an example of platform design. While these issues come up in the chapter, particularly when we discuss matching markets, we focus on them only to the extent that they interact with our questions about pricing. Along these lines, we restrict ourselves to matching markets with payments, although there is a substantial literature on matching

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\(^6\) Examples include (a) the report of the Stigler Committee on Digital Platforms from the Stigler Center for the Study of the Economy and the State, University of Chicago Booth School of Business, (b) “Unlocking Digital Competition” from the Digital Competition Expert Panel for the United Kingdom, and (c) “Competition policy for the digital area” by Jacques Crémèr, Yves-Alexandre de Montjoye and Heike Schweitzer, for the European Commission, all released in 2019.

\(^7\) The SEC released its “Staff Guidance on SRO Rule Filings Relating to Fees” in May 2019 which states “The platform theory of competition ("a total platform theory") also provides a potential pathway to demonstrating a competitive environment. Total platform theory generally asserts that when a business offers facilities that bring together two or more distinct types of customers, it is the overall return of the platform, rather than the return of any particular fees charged to a type of customer, that should be used to assess the competitiveness of the platform’s market.” See also Ohio vs. American Express Co., 585 US 2018.

\(^8\) See European Commission (2020) and United Kingdom (2020).

\(^9\) See Acquisti, Taylor, and Wagman (2016) for an overview of some of the privacy literature.
without payments (see, e.g., Roth, 2018).

In peer-to-peer networks, such as WhatsApp and initially Facebook and LinkedIn, content creators and content consumers are one and the same. Their dynamics are governed by a direct network effect rather than indirect network effects, and so their analysis falls outside of the scope of this chapter. Einav, Farronato, and Levin (2016) reviews peer-to-peer markets. To the extent that these peer-to-peer networks also sell advertising that reaches network users, these networks fall within our scope of analysis, although our focus is not on media. In particular, we do not discuss situations in which spillovers run in only one direction and we largely ignore negative spillovers. For instance, advertisers value positively newspaper readers but readers may not value, or negatively value, advertisers. The articles on media that we cite above cover the case of negative spillovers in detail.

It is also worth keeping in mind that terms such as “platforms,” “networks,” “network effects,” and “network externalities,” are used in many other areas in economics and across different literatures. For instance, in the strategic management literature, Cusumano, Gawer, and Yoffie (2019) distinguish between innovation platforms and transaction platforms. Innovation platforms are systems or general-purpose tools that facilitate new products. In this view, Java is a platform for creating internet applications and cloud computing is a platform for delivering services or infrastructure over the internet. Transaction platforms facilitate transactions. From our perspective, there is nothing particularly two-sided or connective about what this literature calls innovation platforms and our concept of platform and two-sidedness fits within their concept of transaction platforms. Similarly, there is a large literature on social networks and network formation that models exactly which agent is connected to which other agent (see, for example, Jackson, 2008). We discuss how platforms use payments to influence specific matches and hence shape the network of the interactions. Furthermore, some of the issues we review when discussing the empirical literature on two-sided markets also appear relevant for the broader literature on network formation. However, we do not discuss the connection between these two literatures at length in this chapter.

2 Monopoly

In this section we review the literature that examines pricing in two-sided markets where the interactions between the sides are mediated by a single platform. We first introduce the setting and some useful notation and review the key results. We then discuss distortions compared to welfare maximization. Finally we discuss various issues such as the chicken & egg problem, zero prices, congestion, transaction fees or dynamic pricing.

2.1 Basic framework and notation

A single firm (hereafter the platform) matches agents from two different sides of the market, denoted by $i = 1, 2$. Each side is populated by a unit-mass continuum of agents. Each agent derives utility from the product offered by the platform (e.g., Amazon Retail or the iPhone) but also by interacting with agents from the other side of the market who also join the platform. Formally, each side-$i$ agent has a type $\theta_i = (v_i, \gamma_i) \in \Theta_i \subset \mathbb{R}^2$ and derives a gross
payoff

\[ u_i(\theta_i, q_j) = v_i + \gamma_i q_j \]

from interacting with a mass of agents of size \( q_j \) from side \( j \neq i \). These gross payoffs are net of any total payment \( P_i \) that the agents make to (or receive from) the platform. Preferences are quasi-linear in payments so that the net payoff is equal to \( u_i(\theta_i, q_j) - P_i \). Each agent who does not join the platform obtains a reservation payoff normalized to zero. Each type \( \theta_i \) is an independent draw from some distribution \( F_i \) and is the agent’s private information. The component \( v_i \) in the agent’s payoff is a proxy for the utility that the agent derives from the platform’s product or service, whereas the component \( \gamma_i \) is a proxy for the importance the individual assigns to interacting with agents from the other side of the market.\(^{10}\) Hereafter, we refer to \( v_i \) as the agent’s “stand-alone value” and to the term \( \gamma_i \) as the agent’s “interaction benefit.” Following the pertinent literature, interactions with the other side will also be referred to as “usage.”

Notice that while different agents from the same side may assign different value to interacting with agents from the opposite side, each agent values all agents from the other side equally.\(^{11}\) Consistently with the rest of the literature surveyed in the first few sections of this chapter, we also assume that platforms do not engage in price discrimination within sides. Any pair of agents from the same side who joins the platform is granted access to all agents on-board from the opposite side and is charged the same total payment by the platform. We relax these assumptions in Section 6 where we consider richer preference specifications and allow for discriminatory prices.

The total payment \( P_i \) to the platform may have two components, an access fee \( p_i \) and a transaction fee proportional to the number of interactions with agents from side \( j \neq i \). Assuming that the number of interactions is proportional to the mass of agent on side \( j \) and letting \( t_i \) denote the expected transaction fee per agent on side \( j \), the total payment by each agent joining from side \( i \) is equal to \( P_i = p_i + t_i q_j \). This formulation implicitly assumes that every agent who joins on one side is willing to interact with any agent participating from the opposite side. The above representation also assumes that agents do not exchange money directly with agents from the other side or do so at a fixed price. Lastly, the only relevant margin is the extensive one: agents choose whether or not to join the platform but do not control the intensity of their interactions with agents from the other side of the market [We discuss the implications of relaxing some of these assumptions later in the chapter].

Fixing the measure of agents \( q_j \) from side \( j \), we then have that the total demand from side \( i \) when the side-\( i \) total payment is equal to \( P_i \) is given by

\[ q_i = D_i(P_i; q_j) \equiv \Pr \{(v_i, \gamma_i) \in \Theta_i : v_i + \gamma_i q_j \geq P_i\}. \tag{1} \]

The measure of agents \( q_1 \) and \( q_2 \) from the two sides of the market is then the solution to the two conditions \( q_i = D_i(P_i; q_j), j \in \{1, 2\}, j \neq i \). This formulation highlights that from

\(^{10}\)Given our emphasis on pricing, in most of the chapter we treat \( v_i \) and \( \gamma_i \) as exogenous parameters, but they may be influenced by non-price decisions that we do not consider. As most of the literature, we also assume that network benefits are linear in the other side’s participation for conciseness. Linearity is relaxed in Section 4.1 and 6. Sections 2.5 and 6 endogenize network benefits through taxation and matching.

\(^{11}\)This assumption needs only hold before the participation decision, reflecting lack of information about each participant on the other side at the contracting stage. This will be relaxed when considering price discrimination in more details
side-\(i\) perspective, \(q_j\) is a quality dimension that is determined endogenously through prices. The platform incurs a cost \(c_i\) for each side-\(i\) agent it brings on board and a cost \(\sigma\) for each interaction between the two sides. The platform’s profits are thus given by

\[
\Pi = \sum_{i,j=1,2, j\neq i} (P_i - c_i) q_i - \sigma q_1 q_2.
\]

Hereafter, we refer to the term

\[
\eta_i(P_i; q_j) \equiv -\frac{D_i(P_i; q_j)}{\frac{\partial D_i(P_i; q_j)}{\partial P_i}} = \frac{P_i}{\epsilon_i(P_i; q_j)}
\]

as the side-\(i\) inverse semi–price elasticity of the demand, where

\[
\epsilon_i(P_i; q_j) \equiv -\frac{\partial D_i(P_i; q_j)}{\partial P_i} \frac{P_i}{D_i(P_i; q_j)}
\]

is the standard elasticity of the side-\(i\) demand with respect to its own price.

Note that the sale of the platform’s stand-alone product is bundled with the interaction with the opposite side.

The environment described above encompasses most of the models considered in the literature. In particular, it admits the following specifications as special cases.

- Caillaud and Jullien (2001, 2003): \(F_i\) is a Dirac measure at some \((v_0^i, \gamma_0^i) \in \mathbb{R}^2\), \(i = 1, 2\), meaning that agents’ preferences are homogeneous within sides;

- Rochet and Tirole (2003): for each \(i = 1, 2\), \(c_i = p_i = 0\); furthermore, \(F_i\) assigns measure one to the set \(\{(v_i, \gamma_i) \in \Theta_i : v_i = 0\}\), meaning that all agents derive utility only from interacting with other agents from the opposite side of the market (zero stand-alone values);

- Armstrong (2006): \(\sigma \equiv 0\) and \(t_i = 0\), \(i = 1, 2\); furthermore, for each \(i = 1, 2\) there exists \(\gamma_0^i\) such that \(F_i\) assigns measure one to the set \(\{(v_i, \gamma_i) \in \Theta_i : \gamma_i = \gamma_0^i\}\), meaning that all agents assign the same value to interacting with agents from the opposite side of the market (homogeneous interaction benefits);

- Rochet and Tirole (2006) and Weyl (2010): \(F_i\) is absolutely continuous over \(\mathbb{R}^2\), meaning that agents are heterogeneous in both their stand-alone values and their interaction benefits.

### 2.2 Profit-maximizing prices

Assuming a smooth demand and no coordination problems\(^\text{12}\), the profit-maximizing prices \((P_1^*, P_2^*)\), along with the demand/participation \((q_1^*, q_2^*)\) they induce, solve\(^\text{13}\)

\[
P_i^* - \left[ c_i + q_i^* (\sigma - \gamma_j(P_1^*, P_2^*)) \right] = \eta_i(P_i^*; q_j^*)
\]

\(^\text{12}\)There may be multiple demand configurations compatible with the same prices. When this is the case, we assume here that the monopolist can coordinate the market on the allocation most favorable to her.

\(^\text{13}\)See Weyl (2010). This first-order condition corresponds to the optimal choice of price \(P_i\) holding the quantity \(q_j\) constant on the other side, which requires adjusting the price \(P_j\) accordingly.
for $i, j = a, b, j \neq i$, with $q_i^* = D_j(P^*_j; q_i^*)$, and with

$$
\tilde{\gamma}_j(P_a, P_b) \equiv -\frac{\partial D_j(P_j; q_j)}{\partial q_i} = \mathbb{E} [\gamma_j | v_j + \gamma_j q_i = P_j].
$$

The above formula is the two-sided analog of the familiar monopoly price formula in one-sided markets. The right-hand side is the familiar effect by which a monopolist’s ability to price above marginal cost is inversely related to the elasticity of the demand. The only novelty relative to one-sided markets is that such elasticity is now computed by accounting for the fact that it depends on the size of the side-$j$’s participation, $q_j^*$. The term in square parenthesis on the left-hand-side is the cost of bringing a marginal agent on board, $c_i$, augmented by the cost of matching the agent to all agents on board from the opposite side, $\sigma q_j^*$, and reduced by the product between the average interaction benefit $\tilde{\gamma}_j(P_a, P_b)$ experienced by the side-$j$ marginal agents and the size of the side-$j$ demand $q_j^*$. To understand this last term, note that when the platform brings on board an extra agent from side $i$, it can then increase its side-$j$ price by $\tilde{\gamma}_j(P_a, P_b)$ while keeping constant the demand on side $j$ at $q_j^*$. Holding the side $j$’s participation constant, the marginal effect on the side $j$’s profits is thus equal to $\tilde{\gamma}_j(P_a, P_b)q_j^*$. When the average network effect $\tilde{\gamma}_j(P_a, P_b)$ among the side-$j$ marginal agents is positive, this last effect thus contributes to a reduction in the marginal cost of bringing new agents on board from side $i$.

### 2.2.1 Homogeneous interaction benefits (Armstrong, 2006)

Consider the case in which all agents from the same side have the same interaction benefit, with the latter equal to $\gamma_1$ on side 1 and $\gamma_2$ on side 2. Further assume that the platform incurs no cost in matching agents, with the only cost being the cost of getting agents on board: $\sigma = 0$ and $c_1, c_2 > 0$. In this case, the profit-maximizing prices are given by

$$
P^*_i = c_i - \gamma_j q_j^* + \eta_i (P^*_i; q_j^*)
$$

with $\eta_i (P^*_i; q_j^*) = [1 - F_i^v(P^*_i - \gamma_i q_j^*)]/f_i^v(P^*_i - \gamma_i q_j^*)$ and $q_j^* = 1 - F_j^v(P^*_j - \gamma_j q_i^*)$, $i = 1, 2$, $j \neq i$, where $F_i^v$ and $f_i^v$ are the cumulative distribution function and the density of the marginal distribution of $F_i$ over the $v$-dimension, respectively.

The novel term relative to the formula for the optimal price in a one-sided market is the term $-\gamma_j q_j^*$. This term can be interpreted as an opportunity cost of excluding a marginal consumer on side $i$. Indeed suppose the platform increases the side-$i$ participation by $\varepsilon_i > 0$. It can then maintain the participation on side $j$ constant at $q_j^*$ by increasing its side-$j$ price by $\gamma_j \varepsilon_i$, thereby generating an additional revenue of $\gamma_j q_j^* \varepsilon_i$.

### 2.2.2 Homogeneous stand-alone benefits (Rochet and Tirole, 2003)

Next, consider the case where agents are homogeneous in their stand-alone values, with the latter normalized to zero, but have heterogeneous interaction benefits, on both sides. Further assume that the platform incurs no cost to bring an agent on board, but incurs a cost for each agent it matches: $c_1 = c_2 = 0$ and $\sigma > 0$. Finally assume that the platform
charges each side-\(i\) agent \(t_i\) per transaction, i.e., per each match with the other side. The total payment collected from each participating agent from side \(i\) is thus equal to \(P_i = t_i q_j\), \(i, j = 1, 2, j \neq i\). Let \(F_i^\gamma\) and \(f_i^\gamma\) denote the cumulative distribution function and the density of the marginal distribution of \(F_i\) over the \(\gamma\)-dimension, respectively. Hence, in this case, \(q_i^* = 1 - F_i^\gamma(P_i^*/q_j^*)\), which implies that
\[
\eta_i(P_i^*; q_j^*) = \frac{1 - F_i^\gamma(P_i^*/q_j^*)}{f_i^\gamma(P_i^*/q_j^*)}.
\]
The profit-maximizing payments can then be obtained from (2) which in this case yields
\[
P_i^* - \sigma q_j^* + \frac{P_j^*}{q_i^*} q_j^* = \eta_i(P_i^*; q_j^*).
\]
Letting \(t_i^* = P_i^*/q_j^*\), \(i = 1, 2, j \neq i\), be the transaction fees associated with the profit-maximizing total payments, we then have that the optimal transaction fees \(t_1^*\) and \(t_2^*\) are given by the solution to
\[
t_1^* + t_2^* - \sigma = \frac{1 - F_1^\gamma(t_1^*)}{f_1^\gamma(t_1^*)} = \frac{1 - F_2^\gamma(t_2^*)}{f_2^\gamma(t_2^*)},
\]
which is the price formula identified by Rochet and Tirole (2003). Again, to interpret this price formula, note that \([1 - F_i^\gamma(t_i^*)]/f_i^\gamma(t_i^*)\) is the inverse semi-transaction fee-elasticity of the side-\(i\) demand (only agents with interaction benefit above \(t_i^*\) participate). The term \(\sigma - t_j^*\), instead, is the opportunity cost (per agent joining from side \(j\)) of getting on board one extra agent from side \(i\). The latter combines the cost to the platform of matching the agent with any of the agents joining from side \(j\) along with the fact that any side-\(j\) agent pays \(t_j^*\) per each match with any of the side-\(i\) agents. The optimal transaction fees thus reflect the general cross-subsidization pattern identified by the optimality conditions in (2).

### 2.3 Welfare-maximizing prices

Consider next the problem of a planner maximizing the sum of all agents’ utilities and the platform’s profit. For any vector of prices \(P_1\) and \(P_2\), total welfare is given by
\[
W = \sum_{i=1,2} \int_{\{\theta_i; v_i + \gamma_i q_j \geq P_i\}} (v_i + \gamma_i q_j - c_i) dF_i(\theta_i) - \sigma q_1 q_2.
\]
One can then show that the welfare-maximizing prices \((P_1^*, P_2^*)\), along with the efficient participation profile \((q_1^*, q_2^*)\) they induce, solve
\[
P_i^* = c_i + q_j^* (\sigma - \bar{\gamma}_j(P_1^*, P_2^*))
\]
for \(i, j = 1, 2, j \neq i\), with \(q_j^* = D_i(P_i^*; q_j^*)\), where the demand functions \(D_i\) are given by the same functions as in (1), and where
\[
\bar{\gamma}_j(P_1^*, P_2^*) \equiv E[\gamma_j v_j + \gamma_j q_i^* \geq P_j^*]
\]
is the average interaction benefit of the participating agents from side $j$.

The efficient price on each side is thus equal to the marginal cost $c_i + \sigma q^e_j$ that the platform incurs to get a marginal agent on board from side $i$ and then match him to all participating agents from side $j$, discounted by the network externality that the marginal agent exerts on all the participating agents from the other side of the market. The key difference with respect to profit-maximization is that the externality accounts for the benefit that all agents from side $j$ derive from interacting with the marginal agent from side $i$, whereas, in case of profit maximization, the only relevant externality accounted for by the monopolist is the one exerted on the marginal agent from the opposite side of the market (as in Spence, 1981, in a one-sided context with quality).

### 2.4 Distortions

Using (2) and (5), one can express the difference between the profit-maximizing and the efficient prices as follows (see Tan and Wright, 2018a and Gomes and Pavan, 2019):

$$P^*_i - P^e_i = \eta_i(P^*_i; q^*_j) + q^e_j \left( \tilde{\gamma}_j(P^*_1, P^*_2) - \tilde{\gamma}_j(P^e_1, P^e_2) \right)$$

$$+ q^e_j \left( \tilde{\gamma}_j(P^e_1, P^e_2) - \tilde{\gamma}_j(P^*_1, P^*_2) - \sigma \right)$$

(7)

The first term in the right-hand side of (7) is the usual distortion originating in the platform’s market power, according to which the firm reduces output to extract more surplus from the infra-marginal agents. The only novelty relative to one-sided markets is the dependence of the markup on the measure of the participating agents from the opposite side, $q^*_j$.

The second term in the right-hand side of (7) is a Spencian distortion. It originates in the fact that a profit-maximizing monopolist internalizes only the effect of expanding the side-$i$ participation on the side-$j$ marginal agents, whereas a welfare-maximizing platform internalizes the effect that such an expansion has on all participating agents from side $j$.

The third term in the right-hand side of (7) is a distortion that accounts for the fact that the marginal agents under the profit-maximizing prices are not the same as under the welfare-maximizing ones and hence may have different interaction benefits.

Lastly, the fourth term in the right-hand side of (7) is a distortion that originates in the different size of the participation by side $j$ under profit and welfare maximization: the average benefit $\tilde{\gamma}_j(P^*_1, P^*_2)$ enjoyed by the side-$j$ marginal agents when the platform expands the participation on side $i$ (net of the platform’s matching cost $\sigma$) applies to a measure of agents equal to $q^e_j$ under welfare maximization, whereas it applies to a measure of agents of size $q^*_j$ under profit maximization.

In general, signing the net effect of the interaction among the above four distortions is difficult. One can identify markets in which the profit-maximizing prices are higher than their efficient counterparts on one side but lower on the other, as well as markets in which they are higher on both sides (see Tan and Wright, 2018a for examples).

To obtain further insights, Tan and Wright (2018b) suggest to express prices on a per-unit basis, i.e., by normalizing them by the size of the participating population on the
opposite side of the market. To see this, assume that the costs of getting more agents on
board on each side is equal to zero, that is, $c_1 = c_2 = 0$. Furthermore, and without loss of
optimality (both in the case of profit and in the case of welfare maximization), assume that
the participation fees are equal to zero (i.e., $p^*_i = p^*_j$, $i = 1, 2$) so that the total payments
can be expressed entirely in terms of transaction fees, with $t^*_i \equiv P^*_i / q^*_j$ and $t^*_j \equiv P^*_j / q^*_j$,
\(i, j = 1, 2, j \neq i\).\(^{14}\) We then have that

$$
t^*_i - t^*_j = \frac{1}{q_j} \eta_i(P^*_i, q^*_j) + (\bar{\gamma}_j(P^*_1, P^*_2) - \tilde{\gamma}_j(P^*_1, P^*_2))
$$

\text{markup} + \frac{\tilde{\gamma}_j(P^*_1, P^*_2) - \tilde{\gamma}_j(P^*_1, P^*_2))}{q_j} \text{Spence distortion}

$$
+ \left(\bar{\gamma}_j(P^*_1, P^*_2) - \tilde{\gamma}_j(P^*_1, P^*_2)\right). \quad (8)
$$

displacement distortion

Note that both the displacement and the Spence distortions are equal to zero in the
Armstrong (2006) model (due to the fact that the interaction benefits are homogeneous,
on both sides). In this case, the efficient transaction fees are unambiguously lower under
welfare maximization than under profit maximization, on both sides of the market: $t^*_i < t^*_j$,
\(i = 1, 2\).

When, instead, preferences are as in the Rochet and Tirole (2003) model, one can
show that the Spence distortion is always positive on both sides, whereas the displacement
distortion is negative on at least one side. While, in general, transaction fees can be either
smaller or larger under welfare maximization than under profit maximization on either
side of the market, the sum (across sides) of the transaction fees is always higher under
profit maximization than under welfare maximization: $t^*_1 + t^*_2 > t^*_1 + t^*_2$. The possibility
that prices are lower under profit maximization than under welfare maximization, though,
is a consequence of the impossibility for the platform to engage in second-degree price
discrimination. As we show in Section 6, when platforms discriminate among agents from
the same side of the market, prices are always higher under profit than under welfare
maximization (see also Gomes and Pavan, 2016).

2.5 Miscellaneous: chicken & egg problem, non-negative prices, distortionary taxation

2.5.1 Chicken & egg

The analysis so far assumes that either there exists a unique demand configuration compat-
ible with the proposed prices, or the monopoly can choose its preferred price-quantity pair
on the demand schedule. This last assumption may not be appropriate in many markets.
When the monopolist cannot pick its preferred allocation, a network may fail to launch
despite the long-run efficiency of the activity (Rohlf, 1974). In a nutshell, if all consumers
have a negative stand-alone utility (joining is costly), then for any positive price, there is an
equilibrium market allocation where no consumer joins, despite the fact that some positive
participation would be efficient and profitable. In the case of platforms, this possibility

\(^{14}\)Clearly, the results below extend to arbitrary payments $P_i$ by interpreting $t_i$ as the “per-unit” payment
(i.e., the payment per transaction).
translates into the following chicken & egg problem (Caillaud and Jullien, 2003).\textsuperscript{15} Consider a new platform with no users on board and suppose that the stand-alone utility is negative for all users on both sides, that is, $v_i \leq 0$ for all consumers. To induce the side-$i$ agents to join, the platform must convince them that the side-$j$ agents will join as well. In the worst-case scenario (see the discussion of consumers’ pessimistic beliefs in Section 3), given any pair of positive prices $p_1$ and $p_2$, no consumer joins from either side. To overcome such pessimistic beliefs and successfully launch, the platform can subsidize one side, through monetary payments or by offering additional services that raise the stand-alone values and make them positive.

Notice that the ability to charge different prices on the two sides is key to overcoming the chicken & egg problem, an important difference with respect to standard one-sided markets with network effects. To illustrate this, suppose that sides are symmetric, with demand equal to $D(p_i; q_j)$ on each side and let $\tilde{v}$ be the smallest price such that $0 = D(\tilde{v}; 0)$. Consider a firm selling at a uniform price a good with one-sided network effects such that demand $Q$ verifies $Q = 2D(p; Q/2)$ — the demand thus coincides with the total symmetric demand $2q$ of the platform at symmetric prices $p$. Then because for all $p \geq \tilde{v}$, the demand vanishes, under the worst-case scenario, the one-sided platform cannot be active if its cost is above $\tilde{v}$. By contrast, the platform can set prices such that $p_1 < \tilde{v}$ and $\tilde{v} < p_2 < \tilde{v} + \gamma D(p_1; 0)$ which ensures positive sales on both sides (because $q_1 \geq D(p_1; 0) > 0$). The profit may then be positive if the margin on side 2 is large enough. Notice that a similar strategy could be adopted by the one-sided platform if it could price-discriminate.

\subsection*{2.5.2 Vertical integration}

When subsidies are not possible, an alternative way to overcome the chicken & egg problem and convince consumers on one side of the market to join is for the platform to integrate vertically.\textsuperscript{16} Hagiu and Spulber (2013) argue that a platform mediating the interactions between content suppliers and consumers may offer some integrated content to appear more attractive to consumers.\textsuperscript{17} Offering content in addition to matching services allows the platform to raise the value for buyers and secure their participation. However, it comes at the cost of creating congestion on the seller side if integration reduces the expected profit per consumer of non-integrated content suppliers.

Miao (2009) and De Cornière and Taylor (2014) study integration in complementary services used as input for the interactions between the two sides and point to the risk of foreclosure of competing services when the platform cannot appropriate all profits from interactions.\textsuperscript{18}

A recent empirical literature studies integration of platforms into the provision of complementary services, such as Google producing apps for Android (Wen and Zhu, 2019) and Amazon selling products under its own brand name (Zhu and Liu, 2016). They find that

\textsuperscript{15}See Evans and Schmalensee (2010) for a dynamic version emphasizing the need to reach a critical mass to succeed.

\textsuperscript{16}Like any firm, platforms may have other efficiency motivations for vertical integration, such as eliminating double marginalization, creating investment incentives, and promoting diversity and stability of supply (see, for example, Perry, 1989, and Rey and Tirole, 2007).

\textsuperscript{17}See also Nocke, Peitz, and Stahl (2007).

\textsuperscript{18}Arguments in these papers require models going beyond this chapter.
the platform is attracted to product categories with relatively high sales and that the threat of entry leads to reduced innovation and investment on the part of producers of complementary products. These results suggest that platforms are not entirely driven by the goal of ensuring the consumer experience, at least in these cases. In contrast, Lee (2013) finds that producers of video game consoles sign exclusive deals with game developers in order to attract consumers and this practice particularly contributes to the success of smaller console producers.

2.5.3 Non-negative prices

When marginal cost is small (which is often the case for digital platforms), the interaction benefits on the other side of the market are large, and/or the demand is very elastic, the platform may find it optimal to set a negative price. Several contributions discuss why negative prices may attract opportunistic demand from users that are of no value to the platform, who join without any serious intent to interact with the other side. When this is the case, the lowest price that a platform may set is often zero.\(^{19}\) Alternatively, a platform may optimally set the price on one side to zero if it faces transaction costs in collecting monetary payments from that side. Examples are free-to-the-air TV and street markets.\(^{20}\)

Amelio and Jullien (2012) point out that, when the unconstrained monopoly price is negative, constraining the price to be non-negative has a detrimental effect not only on the platform but also on consumers. The reason is that the motivation for setting a negative price on side \(i\) is to boost the side-\(i\) participation so as to raise the value perceived by the side-\(j\) users. As for any quality, as long as the pass-through of the interaction benefits into prices is less than one, the net utility of the side-\(j\) consumers increases when the platform is allowed to set a negative price on side \(i\) and so does the side-\(j\) participation. Consumers on side \(i\) then benefit from a larger participation on side \(j\) and a larger subsidy. By contrast, when the impossibility to set negative prices is due to transaction costs, the welfare effect on consumer surplus of a zero price on one side is ambiguous.

Consider the optimal subscription price on side 2 when the platform is constrained to set a non-negative price on side 1 and the constraint binds. Profits in this case can be written as

\[
\Pi_2 = (p_2 - c_2)D_2(p_2; q_1) - \sigma q_1 D_2(p_2; q_1) - c_1 q_1
\]

with \(q_1 = D_1(0; D_2(p_2; q_1))\). To ensure uniqueness and positivity of \(q_1\), assume that \(\Delta \equiv \frac{\partial D_1}{\partial q_2} \frac{\partial D_2}{\partial q_1} < 1\) and that \(D_1(0, 0) > 0\). Assuming also positive profits, the first-order condition for the optimal price \(p_2^0\) is given by (Here we use superscripts “0” to highlight the zero-lower-bound on the side-1 prices; We also use \(\Delta^0\) as a shortcut for \(\Delta\) evaluated at \(p_2^0\) and \(q_1^0\)):

\[
p_2^0 - \left[ c_2 + \sigma q_1^0 + (\sigma q_2^0 + c_1) \frac{\partial D_1}{\partial q_2}(0; q_2^0) \right] = (1 - \Delta^0) \eta_2(p_2^0; q_1^0).
\]

\(^{19}\)Gans (2019) provides a formal justification of non-negative prices, based on a kinked demand curve at a zero price due to free disposal of the platform’s good.

\(^{20}\)These examples were proposed by M. Peitz.
Comparing these conditions to their counterparts in the absence of the zero-lower-bound (see Condition (2) above), we can highlight two differences (in addition to the change in the quantity $q_1$). First, as the price $p_1$ is not adjusted to compensate for the change in $q_2$ and maintain the side-1 demand constant, the relevant opportunity cost of getting more agents on board from side 2 is larger: the extra benefit $\tilde{\gamma}_1 q_1$ on side 1 is now replaced by a cost $(\sigma q_2^* + c_1) \frac{\partial D_1}{\partial q_2}$ corresponding to the increase in $q_1$. Second, the demand on side 2 is more elastic at the constant price $p_1$ than at the constant sales $q_1$, which is reflected in the term $1 - \Delta$. The global effect on the side-2 price of imposing a zero-lower bound on the side-1 price is thus a priori ambiguous.

For an illustration, consider a market with linear interior demands $D_i (p_i; q_j) = a_i - p_i + \gamma_i q_j$ and zero interaction costs, $\sigma = 0$. In this case, the optimal prices with and without the zero lower bound are related by $p_2^0 = p_2^* - \gamma_2 p_1^*$, $q_2^0 = q_2^*$ and $q_1^0 = q_1^* + p_1^*$. Hence all prices increase and quantities decrease (weakly) when $p_1^* < 0$ whereas the reverse is true when $p_1^* > 0$.

Welfare with a zero-lower-bound on the side-1 price is maximal when

$$p_2 - \left[ c_2 + \sigma q_1 + (\sigma q_2 + c_1) \frac{\partial D_1 (0; q_2)}{\partial q_2} \right] = - \left[ \tilde{\gamma}_1 (0; p_2) q_1 + \tilde{\gamma}_2 (0; p_2) q_2 \frac{\partial D_1 (0; q_2)}{\partial q_2} \right],$$

where $\tilde{\gamma}_j (0; p_2)$ is the average interaction benefit experienced by the side-$j$ agents, as defined in (6). Because the right hand side is negative whereas it is positive in (9), this suggests that the profit-maximizing price $p_2^0$ is excessively high compared to its welfare-maximizing level.

### 2.5.4 Distortionary taxation and two-part tariffs

The demand representation in (1) assumes that the value that each agent from each side $i = 1, 2$ assigns to interacting with each agent from the other side does not depend on prices. As a result, tariffs affect “participation” but not “usage” conditional on participation. When this is the case, the structure of the tariffs is not important for the monopoly, so that the first-order conditions in (2) apply to any two-part tariff $P_t = p_t + t_i q_j$, $i, j = 1, 2$, $i \neq j$, irrespective of how the latter is decomposed into a participation fee $p_t$ and a transaction fee

\footnote{Note that a similar condition holds for optimal prices in the absence of the zero-lower-bound. Such a condition can be obtained by maximizing profits by means of $p_2$, holding $p_1^*$ constant:

$$p_2^* - \left[ c_2 + \sigma q_1^* + (\sigma q_2^* + c_1 - p_1^*) \frac{\partial D_1 (p_1^*; q_2^*)}{\partial q_2} \right] = (1 - \Delta^*) \eta_2 (p_2^*; q_1^*).$$

\footnote{From the previous footnote, the condition

$$p_2 - [c_2 + (c_1 - p_1) \gamma_2] = (1 - \gamma_1 \gamma_2) q_2$$

holds both in the presence of the zero-lower-bound and in its absence. Treating $p_1$ as a parameter varying between 0 and $p_1^*$ and differentiating yields

$$\frac{\partial p_2}{\partial p_1} = -\gamma_2; \quad \frac{\partial q_2}{\partial p_1} = 0; \quad \frac{\partial q_1}{\partial p_1} = -1.$$}
which is not affected by $t_i$. Rochet and Tirole (2006) also show that their analysis of optimal transaction fees can be adjusted to the case of two-part tariffs.\footnote{They show that their formula (4) applies to normalized transaction fees $t_i = (p_i - c_i)/q_j + t_i$, by replacing $1 - F_{s_1}(p_i, q_j)$ with $\eta_i(P_i, q_j)/q_j$, where $\eta_i(P_i, q_j)$ is a generic expression for the inverse-semi-price elasticity.}

A key assumption is that transaction fees have no distortionary effect on usage. This is the case if agents have perfect information about interaction benefits when deciding on participation, they value equally all users on the other side, and there is no intensive margin in interactions. However, in general, taxation of usage has a distortionary effect and in this case the choice of transaction fees matters for efficiency. To see this, suppose that any pair of agents from different sides is a match with probability 1 and can decide to "trade". Trade involves transfer of quantity $x$ from the side-2 agent to the side-1 agent in exchange of a payment $\tau$. Hence, one may think of the side-1 agents as buyers and of the side-2 agents as sellers. Upon trade, each party receives a surplus $s_1 = u_1(x) - \tau x$ and $s_2 = u_2(x) + \tau x$, respectively. No trade generates a zero surplus for both parties. The analysis above assumes that $x$ and $\tau$ are exogenous (e.g., $x = 1$, $\tau = 0$), agents know their surplus when choosing on participation (i.e., $s_i = \gamma_i$), and $s_i$ does not depend on the identity of the trading partner. In this case, interaction occurs as long as $t_i \leq s_i$ on both sides. But taxes $t_1$ and $t_2$ are likely to be distortionary in many cases of interest. For instance, consider the following situations.

- The interaction benefit $s_i$ for a side-$i$ consumer is random and learned only after the agent chooses to participate. In this case, a type $\theta_i$ participating trades only if the realization of $s_i$ is above $t_i$. Assuming that $s_i$ is independent of $v_i$, a consumer on side $i$ expects interaction benefits equal to

$$\gamma_i q_j = \mathbb{E}(s_i | s_i \geq t_i) \Pr(s_1 \geq t_1) \Pr(s_2 \geq t_2) q_j$$

and the expected payment to the platform is equal to

$$p_i + t_i \Pr(s_1 \geq t_1) \Pr(s_2 \geq t_2) q_j.$$

- The quantity $x$ is variable and both $x$ and $\tau$ are determined ex-post (i.e., after joining the platform) through efficient Nash bargaining. In this case, for total per-unit tax is $t = t_1 + t_2$,\footnote{This is the total payment to the platform per unit of output transferred from side 2 to side 1, within each match.} the total level of trade surplus is equal to $s(t) = \max_{x} \left( u_1(x) + u_2(x) - tx \right)$ and is achieved at $x(t)$. Each agent on side $i$ expects interaction benefits equal to $\gamma_i q_j = \frac{s(t)}{2} q_j$ and the payment to the platform by each side-$i$ agent is equal to $p_i + t_i x(t) q_j$.

Under these circumstances, the composition of the tariffs in terms of participation fees $p_i$ and transaction fees $t_i$ matters for optimality. For instance, Bedre-Defolie and Calvano (2013) extend the credit-card model of Rochet and Tirole (2002) by considering two-part tariffs on both sides with random interaction benefits. In their model, participating merchants do not decide on the payment mode, so that their interaction benefits are $\mathbb{E}(s_2) \Pr(s_1 > t_1) q_1$, which is not affected by $t_2$, and a merchant participates if $\mathbb{E}(s_2) \geq t_2$ (assuming, as they do,
that $v_2 = p_2 = 0$). However, the tax $t_1$ on consumers is distortionary and the decomposition of the tariff into a participation and a transaction fee on the consumer side matters. As a consequence of the asymmetry between merchants and consumers, they find that the transaction fees are inefficiently distorted (from a total-welfare perspective) toward lower fees for consumers.\footnote{In their credit card model, this means that the interchange fee (the fee paid by the merchant's bank to the consumer's bank) is too large.}

Notice that in both examples above, and more generally when interaction benefits are ex-ante homogeneous, the model can be rewritten in reduced form by expressing interaction benefits as a function of transaction fees, $\gamma_i \equiv \gamma_i(t_1, t_2)$, and introducing a measure of the expected fee paid by the side-$i$ agent per interaction with each side-$j$ agent, $\beta_i(t_1, t_2)$. Then the gross utility of each consumer joining from side $i$ becomes

$$v_i + \gamma_i(t) q_j,$$

where $t \equiv (t_1, t_2)$. The expected payment by each side-$i$ agent to the platform is $p_i + \beta_i(t) q_j$. With this notation at hand, the demand on side $i$ (i.e., the measure of side-$i$ agents who join the platform) can be written as

$$q_i = 1 - F_i^v(p_i + (\beta_i(t) - \gamma_i(t)) q_j).$$

The expected interaction cost to the platform $\sigma(t)$ must also account for the effect of $t$ on the probability of trade. One may then invert the demand to write the platform’s total profits, using $p_i + \beta_i(t) q_j = (F_i^v)^{-1}(1 - q_i) + \gamma_i(t) q_j$, as

$$\sum_{i,j=1,2,j \neq i} \left( (F_i^v)^{-1}(1 - q_i) - c_i \right) q_i + (\gamma_1(t) + \gamma_2(t) - \sigma(t)) q_1 q_2.$$

The first part is the profit net of trade surplus and takes a standard form. The second part is the total trade surplus generated by the platform. With this formulation we see that if the platform is free to set any participation fees $p = (p_1, p_2)$ and any transaction fees $t = (t_1, t_2)$, then it chooses the transaction fees that maximize the total interaction benefits between participants $\gamma_1(t) + \gamma_2(t) - \sigma(t)$. Participation fees can then be adjusted to induce the desired participation levels $q_1$ and $q_2$.

The conclusion that transaction fees should aim at maximizing the surplus generated by the activity of all agents on board is in line with similar conclusions on two-part tariffs in other domains of IO (vertical wholesale contracts for instance). It hinges on the platform’s ability to use participation fees $p_i$ to extract the surplus. As pointed out in Jullien (2012), the conclusion also relies on the fact that consumers are risk neutral. With risk aversion, and uncertainty of future interaction benefits, transaction fees affect risk sharing between the platform and consumers, and efficiency cannot be guaranteed.

Moreover, with heterogeneous agents, a platform must account for the screening effect of different tariffs. In particular, if the trade benefits are ex-post heterogeneous and the agents have private information, then the platform should optimally use the transaction fees to implement a screening mechanism (Myerson and Satterthwaite, 1983). In this context, an analysis by Halaburda and Yehezkel (2013) points to the interaction between ex-post screening and the chicken & egg problem, arguing that solving the chicken & egg problem implies distortions in the trade mechanism. In their model, the platform’s users learn the trade benefits only after participating and, absent any chicken & egg issue, the platform
would implement efficient trade. However, to solve the chicken & egg problem, the platform needs to subsidize participation of one side and charge only the other side. If buyers do not pay for participation, it becomes profitable to bias the trade mechanism in favor of sellers by reducing trade volumes, so as to raise seller surplus and capture it through the participation fee. By contrast, if sellers do not pay, the trade mechanism implemented is distorted toward excessive trade. The paper then shows that competition may exacerbate the trade distortion.

Hagiu (2006) argues that transaction fees matter when the platform cannot fully commit. In his model of a video-game platform, there are two stages. In a first stage, transaction fees are negotiated with independent game developers, who then invest in developing games. In a second stage, the platform sells the game console while developers sell their games. Retail prices of the console and of the games are not contractible, and as the console and the games are complements, prices tend to be too high (Cournot, 1838). It would be efficient from an interaction surplus perspective—an interaction occurs when a consumer buys a game—to set low transaction fees, so as to avoid double marginalization on games. But then the platform would exploit its monopoly power by setting an excessively high price for the console and restricting sales. Increasing the transaction fees permits the platform to increase the expected revenue to the platform per gamer and this reduces its incentives to restrict sales. For a given mass of games available, a higher transaction fee implies more sales of consoles, which may attract more developers. Hence a high transaction fee works as a commitment device that helps the platform convincing the developers of its willingness to set low prices and high marketing efforts.

2.6 Dynamic pricing

Cabral (2019) offers a dynamic analysis of a monopoly platform with switching costs. The model assumes that, at random times, consumers outside the platform decide whether to pay a subscription fee to join, and consumers inside the platform decide whether to renew their subscription to the platform. The platform is thus treated as a durable good with two-sided network externalities and random obsolescence. Consumers have fixed homogeneous interaction benefits, $\gamma_i$, heterogeneous and random stand-alone valuations (on each side, $\tilde{v}_{it}$ is i.i.d. across agents and time $t$) and consumers are myopic—meaning that when deciding to pay the subscription fee, they assess the value of joining/staying at the platform based on current utility given prices and participation. Hence demand at time $t$ depends on subscription prices ($p_{1t}, p_{2t}$) and the volume of consumers with active subscription ($q_{1t}, q_{2t}$). The platform price-discriminates between renewing insiders and joining outsiders and follows a Markov strategy with state variable ($q_{1t}, q_{2t}$).

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26 As sellers choose prices of games, interaction benefits are endogenous in this model.
27 Peitz, Rady, and Trepper (2017) consider a continuous-time model of monopoly with no switching costs but imperfect information about demand, where the intertemporal leakage follows from learning and experimentation by the platform.
29 Each consumer thus also ignores the effect of her current decision on future participation levels. The analysis also rules out within-period coordination issues by assuming that, in any period, only one consumer is given the chance to move (as in Cabral, 2011, discussed in Section 4).
Cabral (2019) finds that, for small discount factors, the price on side \( i \) is increasing in \( q_{jt} \) and, under some regularity conditions, decreasing in \( q_{it} \). At any time, the platform must find a balance between exploiting current market power (with high prices) and building the customer base (with low prices) so as to "harvest" in the future with higher prices. When \( q_{jt} \) increases (and thus side-\( i \) interaction benefits increase), and the firm is impatient, the market power effect dominates and the firm raises its side-\( i \) price. The effect of \( q_{it} \) on \( p_{it} \) is more specific to the dynamic setting, as in a static framework there would be no reason to link the price to the same-side participation, in the absence of within-side network externalities. The dynamic effect of a consumer on the participation of the other side, along with the complementarity between sides due to the two-sided network effects, induces a dynamic indirect network externality between consumers on the same side. Another interesting point is that, when consumers are homogeneous with non-random stand-alone valuations (\( \bar{v}_{it} \equiv 0 \)), the trade-off between exploitation and building customer base disappears. In this case, the firm achieves monopoly profit with full participation by setting a price \( p_{it} = \gamma_i q_{jt} \) at any date, which is the maximal price that any myopic consumer on side \( i \) is willing to pay to join at date \( t \).

Simulations show that when stand-alone values are often negative, dynamics may have two absorbing states, one with no participation and one with full participation (this is in line with insights from Evans and Schmalensee, 2010, that a platform may fail to launch if it does not reach a critical mass). Results also suggest that heterogeneity in stand-alone values may help a successful launch of the platform (reaching the large participation absorbing state): as heterogeneity increases, more agents may be willing to join even if they expect little interactions on the platform, which helps initiating the process of participation.

The paper shows also that, in a pure intermediation model—where \( v_{it} < 0 \) with probability 1—the initial subscription fees charged by a platform starting with zero participation are negative. Here, myopia plays a role similar to expectations in Caillaud and Jullien (2001, 2003)'s analysis of the role of subsidies to overcome consumers' coordination problems. In both cases, the consumers expecting or contemplating no participation on the other side refuse to subscribe unless the fee is negative.\(^{30}\) The difference is that, in a dynamic setting, the subsidy vanishes and prices become positive as the customer base increases.

Peitz et al. (2017)'s analysis of learning by experimentation also shows that a platform may benefit from lowering its prices at an early stage of development, relative to short-run profit maximization, but for a different reason. Lower prices speed up the learning process by raising the informativeness of the market signal. This effect is however countervailed by the fact that reducing the price on one side raises the marginal benefit of increasing the price on the other side (a "seesaw" effect that will be discussed in Section 4.3). Hence, in their model, early monopoly prices are below myopic level on one side but may be below or above the myopic level on the other side, depending on the nature of the uncertainty.

\(^{30}\)Note that, for an infant platform, consumers' myopic beliefs are disadvantageous. Indeed a forward-looking consumer expecting positive participation in the future may be willing to pay a positive price to join in the current period, if she fears that a long time will elapse before she is given again the opportunity to join. Indeed, in this case, not joining would result in foregone interaction benefits.
3 Competition for the Market

We now consider markets in which multiple firms (the platforms) compete to match agents from various sides of the market. Specifically, suppose there are now two platforms $k = A, B$ and that each side-$i$ agent has a type $\theta_i = (v_i^A, v_i^B, \gamma_i)$, where $v_i^k$ is the agent’s stand-alone valuation for firm $k$’s product. By joining platform $k$, the agent obtains a utility equal to

$$u_i^k(\theta_i, q_j^k) = v_i^k + \gamma_i q_j^k$$

(10)

where $q_j^k$ is the measure of agents from side $j$ joining platform $k$. Let $p_i = (p_i^A, p_i^B)$ denote the side-$i$ prices set by the two platforms and $q_i = (q_i^A, q_i^B)$ the measure of agents joining each of the two platforms on side $i$. The side-$i$ demand for platform $A$ is equal to

$$D_i^A(p_i; q_i) = \Pr (v_i^A + \gamma_i q_j^A - p_i^A \geq \max \{v_i^B + \gamma_i q_j^B - p_i^B, 0\}) .$$

To simplify the formulas, in this section we ignore the interaction costs by assuming that $\sigma^k = 0$, $k = A, B$, and then denote by $c_i^k$ the cost that firm $k$ incurs to bring agents on-board on side $i = 1, 2$.

A key driver of the nature of competition in such markets is the magnitude of the interaction benefits vis-a-vis the degree of horizontal differentiation between the two platforms’ products, as measured by the differential in agents’ stand-alone values. When the interaction benefits are small relative to the degree of differentiation between the two platforms’ products, the two platforms can share the market. When, instead, they are large, a single platform is likely to dominate the entire market. In this section, we focus on the case of large interaction benefits. The case where interaction benefits are relatively small is examined in the next section.

3.1 Divide-and-conquer strategies

To understand how competition works in markets with large interaction benefits, Caillaud and Jullien (2001, 2003) analyze a model in which an incumbent platform (firm $A$) and an entrant (firm $B$) set prices to protect and conquer the market, respectively. Their model is one in which platforms are pure matching intermediaries (agents derive no value from the platforms’ products) and where the agents’ interaction benefits are homogeneous within sides. That is, there exists $\gamma_1$ and $\gamma_2$ such that $F_i$ is a Dirac assigning measure one to $(0, \gamma_i)$, $i = 1, 2$.

Caillaud and Jullien (2001, 2003) analyze a simultaneous pricing game and characterize the full set of equilibria (a continuum) under a mild regularity assumption on demand (referred to as monotonicity). In particular they do not assume any incumbency advantage. As the equilibrium outcome yielding the highest profit to the incumbent firm coincides with the equilibrium outcome of a Stackelberg game and arguments are easier to expose in this set-up, we consider instead a Stackelberg timing whereby firm $A$ sets prices before firm $B$ does.

When all agents single-home (i.e., choose at most one of the two platforms), Caillaud and Jullien (2001) show that an incumbent platform may monopolize the market and make
positive profits when facing an equally efficient competitor. This possibility is related to the multiplicity of equilibria in the subgame among the agents when they choose which platform to join, a classical feature of markets with strong network effects. In this simple setup, when the price differential $|p^A_i - p^B_i|$ between platforms is small on each side $i = 1, 2$, both tipping on firm $A$ (that is, all agents joining platform $A$) and tipping on firm $B$ (all agents joining platform $B$) can be sustained in equilibrium. The multiplicity of equilibria in the network-formation game (that is, in the continuation game among the agents, for given access prices set by the firms) translates into a multiplicity of equilibria in the price-posting game. Whether or not the incumbent firm can have positive profits in equilibrium then depends on the extent of the coordination failure between the agents. The latter in turn depends on both equilibrium selection and the nature of the platforms’ tariffs. If agents coordinate efficiently on joining the platform that can deliver them the highest surplus, then a standard undercutting logic implies that no platform can enjoy strictly positive profits in equilibrium. In equilibrium, all consumers join the same platform because this is efficient, but this platform makes zero profit. By contrast, if firm $B$ faces “unfavorable beliefs,” the incumbent may benefit from the consumers’ coordination failure and enjoy strictly positive profits. The notion of “unfavorable beliefs” in Caillaud and Jullien (2003) captures precisely the idea that consumers’ coordination failure disadvantages a firm.

**Unfavorable beliefs** A firm faces unfavorable beliefs at prices $(p_1, p_2)$ if consumers coordinate on the equilibrium allocation that minimizes the firm’s market share on both sides. When $B$ faces unfavorable beliefs, we say that platform $A$ is focal. Suppose that initially each agent expects all other agents to join platform $A$. Then an equally efficient platform $B$ facing unfavorable beliefs would have no demand if it sets positive prices. This is because any agent expects no match on platform $B$ and such beliefs are self-enforcing. However, firm $B$ could convince one of the two sides to join with a subsidy and then charge a positive price on the other side. For instance, when total payments take the form of subscription fees, so that $t^k_i = 0$ and $P^k_i = p^k_i$, $i = 1, 2$, $k = A, B$, then given platform $A$’s side-$i$ price $p^A_i \leq \gamma_i$, platform $B$ can serve the market by setting a negative price on side $i$ equal to $p^B_i = p^A_i - \gamma_i \leq 0$ on side $i$ (equivalently, offering a subsidy for joining) and then setting a positive price $p^B_j$ on side $j$. Such strategies are referred to in the literature as divide-and-conquer strategies.

**Divide-and-conquer strategies:** Strategies that set prices below cost for some consumers compensated with prices above cost on other consumers.

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31 This is not unique, another equilibrium exists where the competitor wins.
32 In the chapter, we say that there is tipping when all consumers join the same platform.
33 That is, no agent joins platform $k$ when all agents joining platform $-k$ (from both sides) yields a Pareto improvement vis-a-vis the situation where all agents (from both sides) join platform $k$.
34 See also Hagiu (2006), Halaburda and Yehezkel (2016) and the discussion in Halaburda, Jullien, and Yehezkel (2020).
35 Such an allocation exists because the network formation game between consumers is of a super-modular nature. Unfavorable beliefs amounts to assuming that consumers coordinate on a “minimal” equilibrium of the network-formation game, which always exists for supermodular games (Topkis, 1998). Unfortunately, this existence result does not immediately extend to markets with more than 2 networks.
Divide-and-conquer strategies have been shown to be an effective instrument to steal consumers in markets with increasing return to scale (Innes and Sexton, 1994). When the entrant is expected to use such strategies, the incumbent cannot corner the market at a positive price if the entrant is equally or more efficient. However it can protect itself, preventing entry, by subsidizing one side.

Divide-and-conquer strategies in two-sided markets mirror introductory pricing in dynamic models of competition with interaction benefits, where a firm subsidizes early purchases to induce a bandwagon effect and recover the subsidy on late-comers (Farrell and Saloner, 1986, Katz and Shapiro, 1986, 1992). With introductory pricing, a firm subsidizes demand in one period and recovers the loss in future periods. In two-sided markets, the loss on one side is recovered on the other side within the same period. In some markets, there is a natural ordering, with one side joining after the other (see for instance Hagiu, 2006, on video games). In other cases, instead, there is no natural order and either side can be targeted with a subsidy. In this respect, divide-and-conquer strategies are more flexible than sponsoring strategies in dynamic setting.

When the entrant faces unfavorable beliefs, Caillaud and Jullien (2001, 2003) show that the incumbent can indeed obtain positive profits. As an illustration, continue to assume that stand-alone values are equal to zero on both sides \( v_i^k = 0, i = 1, 2 \) and that interaction benefits are homogenous within each side and equal to \( \gamma_1 \) on side 1 and \( \gamma_2 \) on side 2. Further assume that the two platforms are equally efficient (meaning that they face the same cost of bringing agents on board on each side, that is, \( c_i^k = c_i, i = 1, 2, k = A, B \)). Finally assume that \( c_1 + c_2 < \gamma_2 - \gamma_1 < \gamma_1 \). Then in a sequential-pricing game, the incumbent firm A can sell at prices \( p_2^A = 2(\gamma_2 - \gamma_1) > 0 \) and \( p_1^A = c_1 + c_2 + \gamma_1 - \gamma_2 < 0 \) with profit equal to \( \gamma_2 - \gamma_1 > 0 \). To see this, consider the possible reactions by firm B when facing unfavorable beliefs. The default for consumers is to join firm A with utility \( 2\gamma_1 - \gamma_2 \) for the side-2 agents and utility \( \gamma_2 - c_1 - c_2 \) for the side-1 agents. The assumption that B faces unfavorable beliefs implies that, to get agents on board, the platform must make it iteratively dominant for the agents to join.\(^{36}\) This means that platform B must set a price weakly below \( p_2^B = p_2^A - \gamma_2 \) on side 2 to get any participation by that side. The highest price it can get on side 1 is then equal to \( p_1^B = p_1^A + \gamma_1 \). Total profits for platform B under such prices are equal to zero. Alternatively, firm B can make it dominant for the side-1 agents to join by setting a price \( p_1^B = p_1^A - \gamma_1 \). In this case, no agent from side 2 would join platform A at the above prices given that \( p_2^A > 0 \) and that no agent from side 1 is expected to join platform A. The maximal price platform B can charge to the side-2 agent is then equal to \( p_2^B = \gamma_2 \). Under such prices, firm 2 obtains again zero profits. Thus, once the incumbent firm A sets the above prices, there is no way for firm B to enter the market and make positive profits.

Note that the above strategy permits the incumbent firm to obtain positive profit only if there is an asymmetry in interaction benefits between the two sides and if the differential \( \gamma_2 - \gamma_1 \) is large compared to the cost. When, instead, \( \gamma_2 = \gamma_1 = \gamma \) with \( \gamma > c_1 + c_2 \), the most profitable strategy for the incumbent firm A is to set \( p_1^A = p_2^A = c_1 + c_2 \). Faced with such prices and with unfavorable beliefs, the best that the entrant firm B can do

\(^{36}\)The result is similar to the one in the literature on contracting with externalities (e.g., Segal, 1999 and Winter, 2004).
is to undercut by \( \gamma \) the price set by the incumbent platform \( A \) on one side (that is, set \( p^B_i = p^A_i - \gamma \)) thus inducing all the side-\( i \) agents to join platform \( B \), and then set a price equal to \( p^B_j = \gamma \) on side \( j \), which brings zero profit to the entrant firm. Thus, with this strategy, platform \( A \) can guarantee that firm \( B \) stays out of the market yielding profits equal to \( p^A_1 + p^A_2 - c_1 - c_2 = c_1 + c_2 \) to firm \( A \).

The above results rely on several modeling assumptions that were already discussed in the monopoly case.

**Negative prices:** If subsidies are limited then an efficient firm facing unfavorable beliefs may not be able to enter the market. To see this, suppose that firm \( A \) sets prices \( 0 \leq p^A_i \leq \gamma_i \) on both sides. Then firm \( B \) cannot sell when it faces unfavorable beliefs and negative prices are not feasible. Indeed, for any pair of non-negative prices by firm \( B \), the network-formation game admits an equilibrium in which all consumers join only firm \( A \). In this case, an equilibrium exists where firm \( A \) obtains monopoly profits, unless firm \( B \) finds a way to turn the agents’ coordination in its favor.

**Transaction fees:** The positive-profit result discussed above relies on the firms offering simple prices that are not contingent on the participation of the other side. Firms may develop static and dynamic sophisticated pricing strategies to overcome consumers’ adversarial coordination associated with unfavorable beliefs (See the discussion in White and Weyl, 2016). For instance, if the incumbent firm \( A \) posts prices yielding strictly positive profit, then the entrant platform \( B \) could set non-distortionary transaction fees equal to \( t^B_i = \gamma_i \) on each side, subsidize participation up to an amount equal to \( \gamma_1 + \gamma_2 \), and win the entire market.\(^{37}\) In this case, platform \( A \) cannot deter entry of more efficient competitors or generate positive profits when faced with an equally efficient potential entrant. Caillaud and Jullien (2003) then show that the equilibrium where the two firms offer the same quality of service is efficient—that is, all consumers buy from the same firm— with the latter making zero profits. Business models that rely extensively on the taxation of exchanges are commonly observed in markets with platforms.\(^{38}\) However, as discussed below, in many markets, transaction fees are distortionary so the scope for the above mechanism is limited. More generally, it is unlikely that platforms have access to pricing instruments that allow full internalization of the interaction benefits in which case coordination failures may be difficult to overcome.  [See Section 5.1]

The analysis in Caillaud and Jullien (2001, 2003) focuses on pure matching intermediaries, where agents derive value only from the platforms’ matching services. Jullien (2011) points to the importance of the relative magnitude of interaction benefits and stand-alone values for the nature of platform competition. The paper contrasts the case of pure intermediation with no stand-alone values and large interaction benefits to the case of platforms offering services generating large stand-alone values and relatively small interaction benefits. In the latter case, an incumbent firm cannot secure positive profits when faced with

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\(^{37}\)This is because the total surplus (from both sides) that consumers obtain by joining platform \( A \) is strictly less than \( \gamma_1 + \gamma_2 \) if platform \( A \) makes strictly positive profits.

\(^{38}\)In addition to helping overcome coordination problems, these pricing strategies also allow risk sharing with the platform.
an equally efficient competitor. To see that, suppose that agents’ stand-alone values are homogeneous within sides and across platforms with $v_i^k = v_i > \gamma_i$, $i = 1, 2, k = A, B$. Suppose that firm $A$ sets prices $p^A_1 = -\gamma_1$ and $p^A_2 = \gamma_2$. Then firm $B$ can set prices $p^B_1 = p^A_1 - \gamma_1 = -2\gamma_1$ and $p^B_2 = p^A_2 + \gamma_2 = 2\gamma_2$, generating revenues $2(\gamma_2 - \gamma_1)$ that are twice as large as those obtained by $A$ in the absence of firm $B$. More generally, for any pair of prices by firm $A$ satisfying $p^A_1 < v_i$, firm $B$ can attract both sides and generate revenues equal to $p^A_1 + p^A_2 + \gamma_2 - \gamma_1$ by undercutting firm $A$ on side 1 with a price equal to $p^B_1 = p^A_1 - \gamma_1$ and then charging a premium on side 2 by asking a price equal to $p^B_2 = p^A_2 + \gamma_2$. The paper in fact shows that, in the presence of multi-sided network effects, a Stackelberg leader looses the market when faced with an equally efficient or even slightly less efficient competitor. When, instead, the leader is more efficient on some side, it may be able to win the market by choosing an adequate pattern of subsidies and taxes to the various sides.

This provides insights into the nature of barriers to entry with large network effects. Suppose that $v_i = 0$ and assume that firm $A$ charges $p^A_1 < 0$ and (as above) $p^A_2 = \gamma_2$. Then, firm $B$ can still undercut by charging $p^B_1 = p^A_1 - \gamma_1$ and secure participation of side 1. However, the maximal price it can charge on side 2 is now $p^A_2 = \gamma_2$ because it doesn’t offer a positive stand-alone value. As the maximal price $p^B_2$ is strictly less than $p^A_2 + \gamma_2$, entry is less profitable. This allows the incumbent to raise its price on side 1 and secure more profits. Hence, in both cases, the divide part of $B$’s strategy is the same but the conquer part is less profitable with no stand-alone values.

### 3.2 Congestion within sides

The canonical model assumes that there are no network externalities within sides. Such externalities may originate from congestion effects or competition between agents on the same side. Belleflamme and Toulemonde (2009) shows that intra-group negative externalities expand the set of divide-and-conquer strategies, by allowing platforms to target a fraction of agents from the congested side. This is because migration from $A$ to $B$ becomes less attractive as congestion is reduced at $A$. To illustrate that, consider again a focal platform $A$ and an entrant $B$. Assume that $v_i^k = 0, i = 1, 2, k = A, B$, and let the interaction benefits be homogeneous at $\gamma_1$ on side 1. Suppose that agents from side 2 are exposed to congestion. This can be captured by assuming that the side-2 interaction benefits depend on the measure $q_2$ of the side-2 participating agents, with the function $\hat{\gamma}_2(q_2)$ decreasing in $q_2$. When all agents join $A$, the side-1 consumers experience an utility equal to $\gamma_1 - p^A_1$ whereas the side-2 agents experience utility $\hat{\gamma}_2(1) - p^A_1$. Faced with prices $p^A_1 < 0$ and $p^A_2 > 0$, platform $B$ can now attract all consumers from both sides with the following strategy: choose $m_2 \in [0, 1]$ and set prices (slightly below) $p^B_2 = p^A_2 - \hat{\gamma}_2(m_2)$ and $p^B_1 = p^A_1 + \gamma_1(1 - 2m_2)$. First notice that, at these prices, in any equilibrium demand configuration, at least $1 - m_2$ consumers from side 2 join $B$, because the (negative) price $p^B_2$ makes $B$ more attractive than $A$ to the side-2 agents if congestion on platform $A$ exceeds $m_2$, irrespective of side-1 participation. This in turn implies that all consumers on side 1 join $B$ because $p^B_1$ is such that they strictly prefer joining $B$ than joining $A$ when the former platform attracts strictly more than $1 - m_2$ side-2 agents and the latter platform attracts strictly less than $m_2$ side-2 agents. Finally, given that all side-1 agents join $B$, it must also be the case that all side-2 agents join $B$ because no consumer on side 2 is willing to pay the positive price $p^A_2$ for no
interaction with the side-1 agents. Hence platform $B$ has some flexibility in the choice of the subsidy it offers to the side-2 agents. A higher subsidy on the congested side 2 allows platform $B$ to charge a higher price on the non-congested side 1.\footnote{The other divide-and-conquer strategy, subsidizing side 1, is unchanged if $B$ wants to sell to all agents, with $\gamma_2(1)$ being the relevant interaction benefit on side 2.} The analysis shows that negative within-side network externalities may or may not facilitate entry.

Negative within-side network externalities may also prevent tipping, as agents may prefer to avoid negative externalities by joining a smaller platform.\footnote{See Ellison and Fudenberg (2003) and Ellison, Fudenberg, and Möbius (2004) for an analysis of tipping and market fragmentation in markets with indirect and direct network externalities but no prices charged by the platforms.} For instance, Karle, Peitz, and Reisinger (2020) develop a model of e-commerce platforms where sellers can multi-home but competition with other sellers reduces their profit, inducing negative externalities between sellers on the same platform. When seller competition is intense, sellers have an incentive to join different platforms to avoid head-to-head competition. In this case, market fragmentation emerges even if platforms offer identical services and efficiency would require all trade to take place on the same platform.

These effects can be important empirically. Augereau, Greenstein, and Rysman (2006) study the adoption of of 56K modems in which the market was originally served by two essentially identical but incompatible technologies. Despite the benefits to consumers from having the market coordinate on a single technology, Augereau et al. (2006) show that ISPs in the same local market adopted different technologies, presumably in order to differentiate themselves. Augereau et al. (2006) argue that this led not only to market fragmentation, but to overall adoption failure of 56K modems until a standard setting organization intervened.

Halaburda, Jan Piskorski, and Yıldırım (2018) argue that platforms may choose to restrict agents’ choices in order to limit within-side negative externalities (they study restrictions on each agent’s available choice set under inefficient multi-lateral bargaining in a one-to-one matching process).\footnote{Casadesus-Masanell and Halaburda (2014) also argue that a platform may limit supply if the goods exchanged on the platform exhibit network effects.} The degree of restriction is then a competitive tool. With heterogeneous populations on both sides, offering different levels of restrictions allows platforms to differentiate their services, which prevents tipping equilibria to emerge. (See also the discussion of Ambrus and Argenziano, 2009, is Section 4.1).

### 3.3 Multi-homing

The analysis so far assumes that agents who join one platform do not join the other (single-homing). The opposite situation where agents may opt to join more than one firm (multi-homing) leads to slightly different results. For the discussion, assume that each agent cares only about the mass of agents she can interact with on the other side and not about the number of channels for interactions. The conclusion that firm $A$ can corner the market and make strictly positive profit still holds but the nature of the divide-and-conquer strategies is different. When agents can multi-home at no cost, it is easier to convince a consumer to try a new product—the divide part of the strategy. This is because a consumer who decides to join firm $B$ does not have to forgo interacting with those agents from the other
side who join only firm $A$. However, inducing a consumer to join firm $B$ does not generate the same competitive effects as in the case of single-homing. In particular, it does not trigger the same strong bandwagon effect as in the case of single-homing. This is because attracting users to firm $B$ does not necessarily imply poaching them from firm $A$ and hence does not necessarily make firm $A$ weaker. This makes the conquest of the side where prices are positive less profitable. In the context of the simple example discussed above, when $p^A_j > 0$, firm $B$ attract both sides by asking a price slightly below zero on side $i$ and then setting a price sightly below $p^A_j$ on side $j$. The maximal profit for firm $B$ is thus equal to $p^A_j - c_1 - c_2$. Firm $A$ can however prevent firm $B$ from profiting from such a strategy by setting prices $p^A_1 = p^A_2 = c_1 + c_2$ and still make positive profits equal to $c_1 + c_2$.\footnote{Note that the same prices also prevent firm $B$ from attracting agents under single-homing.} Thus incumbent firms still enjoy an incumbency advantage under multi-homing but their profits are smaller than under single-homing. In particular, Caillaud and Jullien (2003) show that the maximal incumbent’s profit is bounded from above by the total cost $c_1 + c_2$. The incumbency advantage is thus small when costs are low.

Multi-homing also alters the way complex tariffs, involving taxation of interactions, can be used to conquer a market. The role of taxation of usage is analyzed by Caillaud and Jullien (2003) who allows agents to multi-home and firms to charge fees for both participation and usage. In the case of single-homing, an optimal strategy for an entrant facing unfavorable beliefs is to subsidize participation and heavily tax interactions when taxes are not distortionary. This strategy relies on the fact that, once one side is attracted, agents from the other side must also join if they want to benefit from the network effects. However, under multi-homing, even if a subsidized side is attracted, agents from the other side can continue to enjoy interaction benefits by staying with the incumbent firm. Hence if the entrant taxes interactions too heavily, all users may stay with the incumbent firm and benefit from lower transaction fees. Moreover, multi-homing creates scope for a new type of entry strategies. The entrant does not need to convince consumers to stop buying the incumbent’s product. It needs only to convince them to stop interacting with the other side using the incumbent’s platform and instead use its own platform. The aim is to induce all agents to multi-home and then compete on transaction fees. As a result, when agents can multi-home, divide-and-conquer strategies typically involve low transaction fees. By setting low transaction fees, an incumbent firm can also raise its protection against entry. The paper shows that, in the equilibrium maximizing the incumbent’s profits, the incumbent sets zero transaction fees and relies solely on participation fees. Moreover, the incumbent makes zero profits when faced with an equally efficient entrant, unless the volume of interactions increases when all agents multi-home compared with the case where they single-home on the same platform.\footnote{Their model allows for an imperfect matching technology such that a pair of agents on each side using two platforms obtains higher interaction benefits than if they use only one platform. In this case, entry with all agents multi-homing is efficient if costs are low enough (this is referred to as global multi-homing). For larger costs, instead, the possibility of global multi-homing makes entry less profitable by increasing the cost of strategies that rely on one side single-homing.}
3.4 Dynamic competition

Most of the existing literature on the dynamics of competition is related to the irreversibility of the choice of a network by consumers, which may be due to the durability of equipment or to switching costs (adoption cost, learning, data transfer). It has been known since Arthur (1989) that in the presence of network externalities and irreversibility, firms with a large installed base benefit from a competitive hedge that may prevent entry of more efficient competitors. As discussed in Section 7.2.4, a number of empirical papers use installed base as an explanatory variable to explain technology diffusion in network effects contexts. An implication of the strategic value of an installed base is that firms supplying network goods gain from quickly building clientele (see Farrell and Saloner, 1986, and Katz and Shapiro, 1986, 1992). This allows them to induce a bandwagon effect that facilitates sales to future consumers. The dynamics of competition is thus shaped by the intensity of network effects, with incentives to preempt and intense competition at the earlier stages of competition.

Under imperfect competition, firms competing with network effects have to find a balance between maximizing short-run profits and investing in building their network. In this context, an important question for policy is whether there is increasing dominance of the largest network eventually resulting in tipping. Almost all contributions on the dynamics of competition focus on one-side network effects and uniform pricing by firms. We highlight here some recent contributions that are particularly relevant for two-sided markets.

In a computational model of product differentiation featuring no coordination failures within each period, Cabral (2011) offers a new perspective on tipping by showing that the market may exhibit long periods of dominance of one firm, alternating with transition periods of intense competition. The general idea is that a firm that has acquired a large customer base will be tempted to exploit its position by setting a high price. This strategy permits a firm with fewer customers to survive with a low price on a niche and eventually challenge the incumbent if the aggregate taste of the population changes. This possibility is shown in the context of a market with one-sided direct network effects instead of a two-sided market, but the insight seems robust. The model assumes dynamic competition between two firms in the presence of a constant flow of arrival and departure of consumers. The goods are durable so that a consumer pays only once at purchase and remains on the same network until leaving the market. Consumers derive a stand-alone utility from the firm's product but also enjoy a flow utility each period from network effects. In every period, one new consumer arrives which avoids coordination issues within periods. The consumer is forward-looking and his stand-alone utility differential between the two firms is randomly drawn from a stationary distribution. With small network effects, the market converges to a stable situation where the two firms share the market. When network effects are large, instead, the equilibrium is stochastic and features relatively long periods with one large

46 See also Chen, Doraszelski, and Harrington (2009).
47 All contributions described in this section assume that firms set uniform prices (that is, the price they set is the same on both sides), but the insights are clearly relevant for the dynamics of two-sided markets.
firm and a small firm that ends when a sufficiently large shift in the market demand favors the small firm. Once the small firm succeeds in growing, a new cycle starts converging stochastically to dominance by one firm, at which point a new cycle starts. Therefore, the market dynamics exhibit the following properties. Total monopolization never occurs and dominance is temporary (although it can last for long periods). This is due to the balance between two effects at work under dynamic competition. First, the large firm gains more from attracting new customers than the small firm. This is because acquiring one more customer allows a firm to raise the value for future customers (due to the network externality). This effect is more profitable for the firm with the largest future demand, (that is, the current large firm) because the total value of interaction benefits generated by one consumer is proportional to the size of the network he joins. Second, a firm that has succeeded in building a large customer base is tempted to exploit its market power by charging a high price. To see why the small firm can survive and take over when demand changes, suppose that firm A captured many more consumers than firm B. At equal prices, most new consumers would buy from firm A and the latter would grow further. Firm B needs to set a low price to accommodate its lower quality. By contrast, firm A may prefer to “harvest” with a high price and accommodate the presence of the small cheap competitor. The second effect grows with size and when firm A is very large, this may result in higher growth rate of firm B’s customer base and prevent full tipping.

A key point in the analysis of Cabral (2011) is that the expected delay before the small firm reaches a position where it becomes a credible challenger is long enough that the incumbent does not find it worthy to eliminate its competitor. Things are different when markets switch fast from one dominant firm to another. Fudenberg and Tirole (2000) point out that a monopoly firm may inflate its sales by reducing prices to deter entry and preserve its dominance. They build a model with overlapping generations of consumers with switching costs and show that, in equilibrium, when firms are patient, an incumbent facing the threat of entry in every period may sustain permanent low prices so as to maintain a large installed base that dissuades entry. Indeed, an entrant’s profitability decreases if it has to compete against a firm with a large installed base. The outcome is that of a contestable market where monopoly is persistent but prices remain low. In this case, the price reflects the shadow value of sales which includes entry deterrence.

Halaburda et al. (2020) and Biglaiser and Crémer (2020) point out that dynamic leakage may not be related to switching costs but to consumer inertia. Halaburda et al. (2020) relies on the concept of pessimistic beliefs and a focal platform. They consider two competing platforms offering a stand-alone good with the same cost c but different quality, with \( v^A < v^B \). In addition, buying a platform’s product brings one-sided interaction benefits that are homogeneous between platforms and equal to \( \gamma \). The paper assumes that the network effects are larger than the quality differential, that is, \( \gamma > v^B - v^A \).

In a static framework, if platform A is focal (that is, benefits from favorable beliefs), it wins the market at price \( p^A_1 = c + \gamma + v^A - v^B > c \). This corresponds to the maximal mark-up that a consumer is willing to pay for firm A’s product if firm B’s product is sold at cost, \( p^B = c \), and she expects all other consumers to buy firm A’s product. In a dynamic context, instead, the paper argues that “focality” is endogenous and history-dependent. More precisely, the firm that wins the market in one period gains an incumbency advantage
(see also Biglaiser and Crémer, 2020) by becoming focal in the next period. The dynamic model involves no switching costs but history dependency through consumers expectations. At any date, consumers buy from focal firm $k$ if the price differential $p_{1k} - p_{i}^k$ is less than $v^k - v^{-k} + \gamma$. The prospect of winning focality affects incentives in a somewhat similar way as the prospect of building an installed based in the case of switching costs: firms are willing to subsidize sales in early periods to benefit from the incumbency advantage in later periods (Farrell and Saloner, 1986, Katz and Shapiro, 1986, 1992). The paper shows that, with a finite but long horizon, the more efficient platform $B$ wins the market in the first and all subsequent periods. The reason is that it is willing to pay more for gaining focality than firm $A$ is willing to pay for preserving focality. To see that consider two periods. Then if firm $B$ wins the market in the period before the last one, it becomes focal and its profit in the last period is $\gamma + v_B - v_A$. This profit is larger than the profit $\gamma + v_A - v_B$ of a focal firm $A$ because $B$ offers higher stand-alone value. But then, for a discount factor $\delta$, firm $B$ is willing to sacrifice $\delta (\gamma + v_B - v_A)$ to win in the first period which is larger than the maximal sacrifice $\delta (\gamma + v_A - v_B)$ that firm $A$ is willing to sacrifice. As a consequence the non-focal platform $B$ wins in the first period if:

$$v_B - c + \delta (\gamma + v_B - v_A) > v_A - c + \gamma + \delta (\gamma + v_A - v_B).$$

The LHS represents the largest surplus that firm $B$ is willing/able to give to consumers expecting no interaction benefits at $B$ while the RHS is the maximal surplus that $A$ is willing/able to give to consumers expecting maximal interaction benefits at $A$. Firm $B$ wins in both periods in a two-period model if $(1 + 2\delta) (v_B - v_A) > \gamma > v_B - v_A$. More generally, the longer the horizon, the larger the difference between the value of winning incumbency for the most efficient firm and for the the least efficient firm. With long horizons, this difference becomes positive and the most efficient firm wins in all periods. Hence, dynamic considerations weaken concerns that an incumbency advantage creates barriers to entry, because prospective future benefits of a superior-quality entrant are larger than the prospective benefits of the lower-quality incumbent. However, this requires that the entrant is forward looking and willing and able to sustain large losses in current competition with the incumbent.

This conclusion, however, must be qualified when the horizon is infinite. Although efficient equilibria always exist for large enough discount factors, inefficient equilibria also exist for large discount factors. This reflects the fact that, in a dynamic competition setting, not only consumers’ beliefs matter but also firms’ beliefs about future play of the game. In particular, a firm’s aggressiveness or softness in current competition can be self-enforcing in the presence of network effect. In other words, a firm may be aggressive because it expects its competitor to be soft in the future and vice versa. This point is worth developing. In an efficient equilibrium, firms expect the efficient firm to win in subsequent periods and there is no reason for an inefficient firm to be aggressive. But consider a situation where the least efficient firm $A$ is expected to win the market in the future, irrespective of which firm will be the incumbent, due to aggressive behavior. Intuitively, in this case, $A$ wins the current period if it is focal because firm $B$ has no reason to make a sacrifice. If firm $A$ is not focal, it must accept a negative profit to win, and somewhat counter-intuitively given that it wins anyway in the future, firm $A$ is willing to make such a sacrifice. The reason is that, by doing so, it becomes the incumbent next period and enjoys positive profits due to the
incumbency advantage, while not doing so would imply incurring the sacrifice next period or never selling in the future. When firm A is patient enough, it may prefer winning the market through aggressive pricing. In this case, the other firm becomes a weak competitor that refuses to sacrifice because it doesn’t foresee future benefits from current market share. In this equilibrium, firm A is willing to suffer a sacrifice to restore its incumbency advantage after firm B has succeeded in winning the market, which discourages firm B from competing for the incumbency position.\footnote{In this equilibrium, firm B prices at cost, \( p^B = c \), in every period while firm A sells at price \( p^A = c + v^A - v^B + \gamma \) when it is focal and \( p^A = c + v^A - v^B - \gamma \) when firm B is focal. Firm B is not willing to undercut because it would need to set a price below cost to sell even when it is focal. This is an equilibrium if selling when firm B is focal is more profitable for firm A than waiting and discounted profit is non-negative, which is the case when \((2\delta - 1) \gamma > v^B - v^A\).} Backward unraveling would kill the credibility of such an equilibrium in finite horizon but is self-enforcing in an infinite-horizon setting. In simple words, a firm’s reputation of being aggressive in defending and restoring its incumbency position after a downturn may be credible and dissuade other firms from challenging it, if firms are patient.

Biglaiser and Crémer (2020) analyze the effect of demand heterogeneity on market fragmentation in a dynamic model with no switching costs but an incumbency advantage. Their modeling of the incumbency advantage is based on a fictitious network formation game. Their pseudo-dynamics start from the allocation of consumers of the last period and assumes that consumers change their demand sequentially to a strictly preferred (myopic) choice, where the first to move are those with the highest benefit to do so. In the case of homogeneous network effects, the dynamics are the same as in Halaburda et al. (2020). Their concept applies to multiple platforms, which they exploit to study market fragmentation. In a model of price competition between firms offering homogeneous (one-sided) network goods to an heterogeneous population, they show that, for small discount factors and large heterogeneity of interaction benefits, an inefficient equilibrium with two active networks exists (where tipping would be efficient). This conclusion follows from the same logic that prevents full tipping in models with switching costs such as Cabral (2011), except that, as in Halaburda et al. (2020), an installed base doesn’t raise value but affects the consumer coordination process. An incumbent with a large legacy network, and thus a large incumbency advantage, prefers to “harvest” its position by charging a high price and keep only consumers with high interaction benefits, which allows a second firm to attract unsatisfied customers with low interaction benefits. Obviously such an equilibrium is sustainable only if the second network is small.

4 Competition on the Market

Four forces that can prevent tipping in two-sided markets are: platform differentiation, multi-homing, compatibility\footnote{We use the term compatibility for technologies that allow users of one platform to benefit from interactions with users of other platforms. Examples are the connectivity of telecommunication networks or email.}, and as discussed in Section 3.2, congestion.\footnote{Another force against tipping may be dynamic pricing. Mitchell and Skrzypacz (2006) study a dynamic duopoly model of direct network effects (so not a two-sided market) and show settings in which a firm with} Many plat-
forms offer differentiated products and services in addition to their intermediation services. For example, smartphones provide communication services, integrated apps, and photo and video recording, in addition to permitting users to download the apps that have been developed for their operating system and which are available “on-demand” (the other side of the market). By differentiating their stand-alone offering, platforms alleviate the competition with other platforms. If the stand-alone service provided by a smaller platform is valuable enough, then the platform may be able to survive even if it attracts only a few consumers. In this section, we discuss some of the key results in the literature on competition between platforms, when the market has stabilized with more than one firm serving both sides of the market.

4.1 Single-homing

We first consider the case where all users single-home on both sides. This may result from some physical constraint or this may result from the price level or from contractual restrictions imposed by platforms (Armstrong and Wright 2007). In this part we treat single-homing as a technical constraint.

4.1.1 Homogeneous interaction benefits

Armstrong (2006) and Rochet and Tirole (2006) study platform competition under high degrees of product differentiation. In particular, Armstrong (2006) considers a duopoly in which platforms sell a horizontally-differentiated consumption good delivering different stand-alone values to consumers in addition to two-sided interaction benefits (through access to the other side). The analysis assumes that (i) consumers on both sides single-home, (ii) the market is covered (meaning that each agent from each side \(i = 1, 2\) who does not join platform \(k\) joins platform \(-k\)) and (iii) interaction benefits are homogeneous within each side.

Preferences take the additive form in (10) with \(\gamma_i\) identical for any pair of agents from the same side. Interaction costs are zero (i.e., \(\sigma_i = 0\), \(i = 1, 2\)) and prices take the form of access fees so that \(P_k^i = p_k^i\), \(i = 1, 2\), \(k = A, B\).

Platform differentiation on each side \(i = 1, 2\) is captured by the dispersion of the stand-alone value differential \(v_{Bi} - v_{Ai}\). When there is little differentiation, coordination failures may arise, and multiple allocations of consumers over the two platforms can be sustained in equilibrium. In this case, insights similar to those from the analysis of competition for the market apply. When, instead, the dispersion of stand-alone values is sufficiently large, a unique “stable” equilibrium allocation exists for any price vector.\(^{51}\) In this case, the demand for firm \(A\) is defined by

\[
D_i^A(p; q_j) = 1 - D_i^B(p; q_j) = \Phi_i \left( p_i^B - p_i^A - \gamma_i (q_j^B - q_j^A) \right),
\]

where \(\Phi_i\) is the side-\(i\) cumulative distribution function of the differential in stand-alone values \(v_{Bi} - v_{Ai}\), assumed to be smooth on a compact support, \(p_i = (p_i^A, p_i^B)\) is the vector

\(^{51}\)The assumption of large differentiation bears some similarity with the role of heterogeneous beliefs in global games (see Jullien and Pavan, 2019 for a discussion).
of side-\(i\) prices, and \(q_j = (q_j^A, q_j^B)\) is the vector of side-\(j\) quantities (equivalently, of side-\(j\) participation).

Specifically, Armstrong (2006) assumes that stand-alone values take the Hotelling form \(v_i^A = v - \tau_i x_i\) and \(v_i^B = v - \tau_i (1 - x_i)\) where \(v\) is large enough to guarantee that the entire market is served in equilibrium, and where \(x_i\) is drawn from a uniform distribution over \([0, 1]\). In this case, the condition for uniqueness of the allocation of agents over the two platforms for all price vectors reduces to \(\gamma_1 \gamma_2 < \tau_1 \tau_2\). To see this, consider an arbitrary allocation of consumers over the two platforms covering the entire market \((q_i^k)_{i=1,2,k=A,B}\) and assume that a small mass \(\varepsilon\) of side-\(i\) consumers moves from platform \(A\) to \(B\). Then, the value of platform \(B\) relative to that of platform \(A\) for the side-\(j\) agents increases by \(2\gamma_j \varepsilon\). This induces \(\Phi_j'2\gamma_j \varepsilon\) side-\(j\) agents to switch from \(A\) to \(B\). By the same reasoning, this change in the side-\(j\) participation in turn induces an additional mass of agents from side \(i\) of measure \(\Phi_i'2\gamma_i (\Phi_j'2\gamma_j) \varepsilon\) to migrate from \(A\) to \(B\). The adjustment process converges if \(4\Phi_i'\Phi_j'\gamma_i \gamma_j < 1\). In the Hotelling model, this condition reduces to \(\gamma_1 \gamma_2 < \tau_1 \tau_2\). In this case, a small change in prices induce a small change in demand. On the contrary, when \(\gamma_1 \gamma_2 > \tau_1 \tau_2\) the above adjustment process diverges so that a small change in prices can induce tipping. Under the above condition, given any collection of prices \(p_1 = (p_1^A, p_1^B)\) and \(p_2 = (p_2^A, p_2^B)\), each platform’s profits are equal to

\[
\Pi^k = \sum_{i=1,2} \left( p_i^k - c_i^k \right) q_i^k
\]

where

\[
q_i^k = D_i^k (p_i; q_j).
\]

It is known from the earlier literature on one-sided markets that network effects can raise the elasticity of the demand by magnifying the demand response to a change in prices, as consumers that would not react to a price increase alone may react to the induced reduction of interaction benefits. The same logic applies to the elasticity of the residual demand of each platform in a two-sided market. Moreover, a higher elastic demand on one side means that increasing price on this side induces a higher loss of sales on both sides of the market. The equilibrium pricing formulas thus reflect the enhanced price elasticity as discussed in the monopoly case, but accounting for the presence of a competing firm. To derive intuition, consider a small price change by firm \(k\) consisting in a reduction of its side-\(i\) price along with an increase in the firm’s side-\(j\) price that maintains the platform’s side-\(j\) demand constant at \(q_j^k\). When the platform adds \(\varepsilon\) new consumers on side \(i\), it can increase its side-\(j\) price \(p_j^k\) by \(2\gamma_j \varepsilon\). This is because, in a covered market, the \(\varepsilon\) consumers that join the platform come from the competing firm. Attracting such consumers thus also means reducing the attractiveness of the other platform. That is, stealing a consumer from the competitor on side \(i\) not only does it raise the platform’s value for those agents on the other side of the market who join the same platform but also reduces the value of joining the competing firm. By attracting \(\varepsilon\) more consumers from side \(i\) the platform can thus increases its side-\(j\) revenues by \(2\gamma_j \varepsilon q_j^k\) without expanding the side-\(j\) participation and hence without incurring any extra cost. Holding constant the participation on side \(j\) at \(q_j^k\), this means that the total opportunity cost of attracting \(\varepsilon\) more consumers on side \(i\) is equal to \((c_i^k - 2\gamma_j q_j^k) \varepsilon\). Once
such opportunity costs are incorporated into the analysis, the optimality condition for for equilibrium prices can be derived as in standard oligopoly games.

Under the above specification, equilibrium prices of an interior equilibrium take the form

\[ p_i^k - c_i^k - 2\gamma_j q_j^k = \mu_i^k (p_i; q_j) \]

(12)

where

\[ \mu_i^k (p_i; q_j) = - \frac{D_i^k (p_i; q_j)}{\partial D_i^k (p_i; q_j)} \]

(13)

is the inverse semi-elasticity of firm \( k \)'s demand on side \( i \), holding the participation of side \( j \) fixed at the level specified by \( q_j = (q_j^A, q_j^B) \) and fixing the other firm's prices at the level specified in \( p_i = (p_i^A, p_i^B) \). In the case of the symmetric Hotelling model of Armstrong (2006), the equilibrium quantities on the other side are equal to \( q_j^A = q_j^B = 1/2 \) and the inverse-semi-elasticity of each firm's residual demand at the symmetric allocation is equal to \( \mu_i^k (p_i; q_j) = \tau_i \). The equilibrium prices on each side are thus equal to \( p_i^A = p_i^B = c_i + \tau_i - \gamma_j \).

Extending these pricing formula to general oligopoly games turns out to be complex for two reasons. First, computing the opportunity cost is complicated if the intensity of the interaction benefits \( \gamma_j \) is heterogeneous across agents and/or if there are more than 2 firms. Second, computing the price elasticity of the residual demands at constant opportunity cost may also be challenging, for neutralizing the effects of a change in the side-\( i \) demand on all the remaining sides is more involved. However, the idea of deriving the equilibrium prices on each side by holding quantities constant on the other sides has proved useful in richer settings too. The key assumption in the derivation of the equilibrium prices is that the market is fully covered.

For instance, when the demand on each side takes the general form \( q_i^k = D_i^k \left( p_i, p_i^{-k}, q_j^k, q_j^{-k} \right) \), changing price on side \( i \) while adjusting the price on side \( j \) so as to maintain the side-\( j \) participation constant and using the full-market-coverage condition \( q_j^{-k} = 1 - q_j^k \) yields the following pricing formula on each side of the market and for each platform

\[ p_i^k - c_i^k - q_j^k \left( \frac{\partial D_i^k}{\partial q_j^k} - \frac{\partial D_i^k}{\partial q_k^j} \right) = \mu_i^k (p_i; q_j) \].

(14)

The above formula applies to arbitrary (asymmetric) duopoly markets.

In a recent paper, Tan and Zhou (2020) generalize the pricing formula derived by Armstrong (2006) to the case where there are more than two sides and more than two platforms,

\[ \mu_i^A (p_i; q_j) = \frac{\Phi_i (\sigma_i^2, \rho_i, \gamma_i, \gamma_i^2 - \gamma_i)}{\Phi_i (\sigma_i^2, \rho_i, \gamma_i, (\gamma_i^2 - \gamma_i))} \quad \text{and} \quad \mu_i^B (p_i; q_j) = \frac{1 - \Phi_i (\sigma_i^2, \rho_i, \gamma_i, \gamma_i^2 - \gamma_i)}{\Phi_i (\sigma_i^2, \rho_i, \gamma_i, (\gamma_i^2 - \gamma_i))} \].

52 More precisely \( \mu_i^A (p_i; q_j) = \frac{\Phi_i (\sigma_i^2, \rho_i, \gamma_i, \gamma_i^2 - \gamma_i)}{\Phi_i (\sigma_i^2, \rho_i, \gamma_i, (\gamma_i^2 - \gamma_i))} \) and \( \mu_i^B (p_i; q_j) = \frac{1 - \Phi_i (\sigma_i^2, \rho_i, \gamma_i, \gamma_i^2 - \gamma_i)}{\Phi_i (\sigma_i^2, \rho_i, \gamma_i, (\gamma_i^2 - \gamma_i))} \).

53 Existence of the equilibrium requires that \( \gamma_1 \gamma_2 > \gamma_1 \gamma_2 + \frac{1}{2} (\gamma_1 - \gamma_2)^2 \) which is stronger than the previous condition for demand stability. Weyl and White (2016) show that, in the symmetric case with \( \gamma_i = \gamma \) and \( \tau_i = \tau \), there is a unique symmetric equilibrium if \( \gamma < \tau \) while only tipping equilibria exist if \( \gamma > \tau \).
assuming a symmetric discrete choice model. The paper shows that, with two sides, no interaction costs (i.e., $\sigma^k = 0$, all $k$) and $K$ firms covering the market, the first-order conditions for a symmetric equilibrium $(p^k_i, q^k_i) = (p_i, 1/K)$ all $k$ is equal to

$$p_i - \left[ c_i - \frac{\gamma_j}{K - 1} \right] = \mu^k_i,$$

where $\mu^k_i$ is again the inverse semi-price elasticity of the side-$i$ demand of firm $k$, holding constant the participation of side $j$ and the other firms’ prices. The interpretation is similar to the one discussed above. Attracting a small mass $\varepsilon$ of consumers on side $i$ allows firm $k$ to raise its side-$j$ price by $\gamma_j \left( \varepsilon + \frac{\varepsilon}{K - 1} \right)$. The first term in parenthesis is the direct benefit of increasing the attractiveness of platform $k$ for the side-$j$ agents. The second term is the extra benefit of weakening the attractiveness of firm $k$’s competitors, again in the eyes of the side-$j$ agents. In a symmetric equilibrium, the $\varepsilon$ new agents the platform brings on board on side $i$ come from a uniform reduction (by $q^k_i = \varepsilon/(K - 1)$) in the side-$i$ participation to each of the other $K - 1$ firms. As in Armstrong (2006), the above conditions are valid only in markets with a sufficiently high degree of horizontal differentiation. With more than 2 sides of mass 1 each, the same reasoning shows that $\gamma_j$ should be replaced by $\sum_{j \neq i} \gamma_{ij}$, where $\gamma_{ij}$ is the intensity of the interaction benefits from side $i$ to side $j$ (that is, the extent to which side $j$ benefits from a marginal variation in the side-$i$ participation).

The above pricing formulas can also be adjusted to accommodate for interaction costs. For example, in a symmetric market with $K$ firms and two sides, it suffices to replace the side-$i$ cost $c_i$ in the above pricing formulas with $c_i + \sigma/K$, where the term $\sigma/K$ is the cost of matching any new agent brought on board on side $i$ to all the agents the platform attracts on the other side of the market (in a symmetric equilibrium, the measure of such agents is equal to $1/K$).

While it brings important new insights, the literature still lacks a tractable model of platform competition in asymmetric and/or partially-covered markets. As anticipated above, the difficulty is the complexity of identifying the price adjustments necessary to maintain participation on other sides constant when the participation of one side changes, as well as the complexity of identifying the elasticity of the relevant residual demands when maintaining the prices of the competing firms fixed. Section 8.3 presents several papers that structurally estimate models of asymmetry and partial coverage. As these papers are naturally computational in nature, they can utilize models that are intractable from a theoretical perspective.

### 4.1.2 Dispersed information

The possibility for multiple platforms to share the market hinges on the presence of heterogeneous stand-alone values, as interaction benefits alone tend to generate tipping, as explained

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54 They also present first-order equilibrium condition for the case where the market is not covered. In this case, reducing a price expands the total demand and the simple additivity of the formulation is lost.

55 An exception is Belleflamme, Peitz, and Toulemonde (2020) who extend Armstrong duopoly model by allowing different network externalities on each platform and argue that the platform with the largest market shares need not be the most profitable.

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above. However, in most markets of interest, it is difficult for an agent to predict whether many agents prefer one platform to another and hence how the platforms’ prices will lead to different participation to each platform. This raises the question of how platforms should price access to each side when agents face idiosyncratic uncertainty about the distribution of preferences and hence about the ability of each platform to attract participation. This issue has been studied recently by Jullien and Pavan (2019) in a model that can be thought of as the extension of Armstrong (2006) to a market with dispersed information. The analysis accommodates for the possibility that each agent is uncertain about the distribution of preferences on each side of the market and uses his own appreciation for each platform’s product to form beliefs about other agents’ participation to each platform.

Formally, consider a two-sided market populated by two platforms and assume that interaction benefits are homogeneous within each side and given by γ_1 and γ_2, respectively. The distributions from which the agents’ stand-alone values v_i^A and v_i^B for the two platforms are drawn is unknown both to the platforms and to the individual agents. Under suitable assumptions, the market is fully covered and each agent from each side i = 1, 2 uses the differential v_i = v_i^B - v_i^A in his stand-alone values for the two platforms to form beliefs over the distributions of preferences on each side.

Assume that the relevant “aggregate state” which is unknown to both the agents and the platforms is the pair of marginal distributions F_v = (F_1, F_2) from which the agents’ stand-alone differentials v_i are drawn. Given F_v, each side-i agent’s stand-alone differential v_i is drawn from F_i independently across agents. Further assume that each agent’s stand-alone differential v_i is the only information the agent has about the platforms, which is expected by the platforms, which is drawn

\[ v_i^A = \text{Pr}(\tilde{v}_i \leq \hat{v}_i | \tilde{v}_i = \hat{v}_i) \]

Importantly, such participation need not coincide with the participation to platform A expected by the platforms, which is given by q_j = Pr (\tilde{v}_j \leq \hat{v}_j). This happens even if the platforms and all the agents share a common prior over the distribution from which the “aggregate state” F_v is drawn.\(^{56}\)

Let \( \hat{q}_{ji}^A = \text{Pr}(\tilde{v}_j \leq \hat{v}_j | \tilde{v}_i = \hat{v}_i) \) denote the side-j participation to firm A expected by the “marginal” agent on side i, i.e., by the side-i agent with type \( \hat{v}_i \) implicitly defined by

\[ \hat{v}_i = p_i^B - p_i^A - \gamma_i \left[ \text{Pr}(\tilde{v}_j \geq \hat{v}_j | \tilde{v}_i = \hat{v}_i) - \text{Pr}(\tilde{v}_j \leq \hat{v}_j | \tilde{v}_i = \hat{v}_i) \right] \]  

who is just indifferent between joining platform A and joining platform B.

Continue to assume that interaction costs are zero and that prices on each side take the form of participation fees so that \( P^k_i = p_i^k, i = 1, 2, k = A, B \). Jullien and Pavan (2019) then show that platform A’s equilibrium prices satisfy the following optimality conditions (similar conditions apply to platform B)

\(^{56}\)The analysis in Jullien and Pavan (2019) does not assume that the agents’ and the platforms’ beliefs be derived from a common prior. We assume a common prior here just to facilitate the exposition.
affect the slope of the platform’s side-$i$ turn, such changes in the beliefs of the side-$i$ perspective, the side-$j$ platform expands the side-$i$ participation, it expects from side $i$. 

Private information affects pricing in two important ways. First, the opportunity cost of expanding the side-$i$ participation (the term in square brackets in (16)) must now account for the fact that when the platform expects a change in the side-$i$ price to bring on board $\varepsilon$ more agents from side $i$, the marginal agent on side $j$ expects the side-$i$ demand to change by $\varepsilon \cdot \partial \hat{q}_{ij}^A / \partial q_i^A$. The difference in the side-$i$ participation expected by the platform and by the side-$j$ marginal agent reflects the difference in their beliefs over the “aggregate state” (i.e., over the cross sectional distribution of stand-alone differentials).\textsuperscript{57}

Second, the inverse-semi-price elasticity must now account for the fact that, when the platform expands the side-$i$ participation, it also changes the beliefs of the side-$i$ marginal agent over the side-$j$ participation. This change in beliefs occurs even if, from the platform’s perspective, the side-$j$ demand remains constant (that is, even if $\hat{v}_j$ does not change). In turn, such changes in the beliefs of the side-$i$ marginal agent over the side-$j$’s participation affect the slope of the platform’s side-$i$’s inverse residual demand which is now given by

$$
\frac{\partial p_i^A}{\partial q_i^A} = \frac{\partial p_i^A}{\partial q_i^A} \bigg|_{q_i^A = \text{cst}} + 2 \gamma_i \frac{\partial \hat{q}_{ij}^A}{\partial q_i^A}.
$$

The sign of the second term depends on whether preferences are “aligned” or “misaligned” between the two sides (that is, on the affiliation between the stand-alone differentials between the two sides).\textsuperscript{58} When preferences are aligned, agents with a higher appreciation for a platform’s product also expect a higher appreciation by agents from the opposite side, whereas the opposite is true when preferences are misaligned.

When preferences are aligned, the new term is negative, thus contributing to a steeper inverse residual demand, whereas the opposite is true when preferences are misaligned. This effect has no counterpart under complete information. To understand this new effect, notice that increasing the side-$i$ demand implies increasing $\hat{v}_i$ which in turn means attracting marginal consumers with a lower appreciation for platform $A$’s product vis-a-vis platform $B$’s product. When consumers form beliefs over the distribution of preferences on the other side of the market using their own appreciation for the platforms’ products, and preferences

\textsuperscript{57}The platform and the side-$j$ marginal agent agree that a decrease in the side-$i$ price $p_i^A$ by $\delta$, when paired with a variation in the side-$j$ price $p_j^A$ that leaves the identity of the side-$j$ marginal agent $\hat{v}_j$ (and hence the side-$j$ demand expected by the platform) unchanged triggers a change in the identity of the side-$i$ marginal agent by $\frac{dp_i^A}{dp_j^A} \delta$, with $\frac{dp_i^A}{dp_j^A}$ obtained from (15). However, they disagree over the effect that such a change in the identity of the side-$i$ marginal agent has on the side-$i$ demand. In particular, while the platform expects the side-$i$ demand to change at a rate $d\Pr (\tilde{v}_i \leq \tilde{v}_i) / d\tilde{v}_i$, the side-$j$ marginal agent expects it to change at a rate $\partial \Pr (\tilde{v}_j \leq \tilde{v}_j | \tilde{v}_i = \tilde{v}_i) / d\tilde{v}_i$. The term $\partial \hat{q}_{ij}^A / \partial q_i^A$ captures this discrepancy.

\textsuperscript{58}Formally, preferences are aligned if, for all $v_j$, $\Pr (\tilde{v}_j \leq v_j | \tilde{v}_i = v_i)$ is decreasing in $v_i$. They are misaligned if $\Pr (\tilde{v}_j \leq v_j | \tilde{v}_i = v_i)$ is increasing in $v_i$. 

\[38\]
are aligned between sides, the new marginal agent from side \( i \) is thus less optimistic about the side-\( j \) participation to platform \( A \) than the old marginal agent (the one with a lower \( \hat{v}_i \)). Other things equal, this new effect tends to reduce the elasticity of the residual demand and thus contribute to higher prices when preferences are aligned, and to increase the elasticity of the residual demands and hence reduce the equilibrium prices when preferences are misaligned.

This finding reflects the fact that, when preferences are aligned, the new marginal agent is less optimistic than the infra-marginal agents about the participation of the other side. Therefore, the platform must cut the price more to trigger the same expansion in the side-\( i \) demand compared to the case where preferences are independent across sides (note that preferences are always independent when the cross-sectional distribution of stand-alone differentials on each side is common knowledge).

The above observations have various implications for the structure of the equilibrium prices. For example, the equilibrium price on each side depends directly on the magnitude of the interaction benefits on both sides whereas, with complete information, it depends only on the intensity of the interaction benefits on the opposite side, as shown in Armstrong (2006). More importantly, platforms may alter the intensity of competition by engaging in marketing activities and various other information-management policies geared at influencing the ability of each side to predict the other side’s participation (through forums, showrooms, the dissemination of information about early adoption decisions and the like). Jullien and Pavan (2019) study the effects of the aforementioned policies on equilibrium prices, profits, and consumer surplus restricting attention primarily to symmetric markets that are fully covered. Extending the analysis to richer configurations remains an interesting line for future research.

\subsection{Heterogeneous interaction benefits}

Consider now a two-sided market in which preferences are as in (10), but where the interaction benefits are heterogeneous across agents (that is, \( F_i \) is not restricted to assign probability one to types \( \theta_i = (v^A_i, v^B_i, \gamma_i) \) with the same \( \gamma_i \)).

To make things simple assume that types can be parametrized by a uni-dimensional variable \( \omega_i \in \Omega_i \subset \mathbb{R} \), drawn from a distribution \( F_i \), \( i = 1, 2 \). That is, there exists a function \( g_i : \Omega_i \rightarrow \Theta_i \) such that when \( \omega_i \) is drawn from \( \Omega_i \) according to \( F_i \), independently across agents, then the distribution of \( g_i(\omega_i) \) coincides with the type distribution \( F_i \) on \( \Theta_i \). For any \( \omega_i \) then let \( \gamma_i(\omega_j) \) be the intensity of the interaction benefit for each side-\( i \) agent whose type \( \theta_i \) is parametrized by \( \omega_i \) (in short, for any \( \omega_i \)-agent). Assuming for conciseness that the differential in stand-alone values \( v_i(\omega_i) \equiv v_i^B(\omega_i) - v_i^A(\omega_i) \) is increasing in \( \omega_i \), under appropriate conditions, we then have that, for each price vector, there exist thresholds \( \hat{\omega}_1 \) and \( \hat{\omega}_2 \) such that each side-\( i \) agents joins firm \( A \) if \( \omega_i < \hat{\omega}_i \) whereas he joins firm \( B \) if \( \omega_i > \hat{\omega}_i \), \( i = 1, 2 \).\(^{59}\) Paralleling the analysis above, we then have that the price adjustment required to maintain the participation of side \( j \) constant when expanding the side-\( i \) participation can be identified by preserving the utility differential of the side-\( j \) marginal agent (the one whose type is indexed by \( \hat{\omega}_j \)). The equilibrium prices then solve the first-order conditions

\(^{59}\)This is the case for instance if \( \gamma_i(\omega_i) \) are small, for all \( \omega_i, i = 1, 2 \), and if the distribution of \( v_i(\omega_i) \) is sufficiently diffused.
\[ p_i^A - \left[ c_i^A - 2\gamma_j (\hat{\omega}_j) q_j^A \right] = \mu_i^A (p_i; q_j), \]

where the term \( \mu_i^A (p_i; q_j) \) continues to denote the inverse-semi-price elasticity of platform \( A \)'s side-\( i \) demand with respect to the price \( p_i^A \), for given prices by form \( B \) and given participation \( q_j \) on side \( j \). As in the monopoly case, a Spence distortion then arises due to the fact that the firm accounts only for the effect of a variation in the side-\( i \) demand on the interaction benefit \( \gamma_j (\hat{\omega}_j) \) of the side-\( j \)'s marginal agent, instead of all side-\( j \) agents on board (see Weyl, 2010). This distortion is in addition to the one originating in the firm’s market power, as reflected in the inverse-semi-price elasticity term \( \mu_i^A \).

When the heterogeneity of the interaction benefits \( \gamma_i \) is large, asymmetric equilibria may arise. An alternative to product differentiation is for platforms to differentiate their business models. Introducing different price skewness to court different market segments on each side of the market is one way to alleviate competition. An illustration is provided in Ambrus and Argenziano (2009) who investigate the conditions for the existence of asymmetric networks, as for example, in the market for online job search in the US or in the credit-card industry. They assume that, on each side, there are two types of consumers, the first with large interaction benefits \( \gamma_i \), the second with small interaction benefits. Assuming that the measure of the second type is large relative to that of the first type, Ambrus and Argenziano (2009) then show that each platform’s optimal strategy is to set a low price on one side (say side 1), in order to attract the large mass of consumers with low interaction benefits and set a high price on the other side (side 2) targeted to those agents with large interaction benefits. When platform \( A \) sets low prices on side 1 and high prices on side 2, the best response for platform \( B \) is to do the opposite (set a low price on side 2 and a high price on side 1). This is because many agents are not served by platform \( A \) on side 2 and hence are easily attractable by firm \( B \) by setting a low price. Conversely, on side 1 those agents with low interaction benefits are already attracted by platform \( A \) in which case it is more profitable for platform \( B \) to target those side-1 agents with high interaction benefits who value being connected to many agents from the other side. In the case of online job search, the result implies that a natural equilibrium configuration is one where two platforms asymmetrically split the market, with one platform attracting many job posters and few job seekers and the other platforms attracting many job seekers and few job posters.\(^{60}\)

\section*{4.2 Multi-homing}

The previous contributions share the important assumption that each agent joins at most one platform. How robust are the ideas developed above to the possibility of multi-homing? As long as there are two active firms, and some agents choose to join only one platform, the agents on the other side will have incentives to subscribe to both platforms. Multi-homing allows agents to benefit from larger interaction benefits but also enjoy the differentiated products supplied by both firms.\(^{61}\)

\(^{60}\)In a media context, Calvano and Polo (2020) study how platforms may achieve product differentiation by choosing different business models, arguing that when consumers multi-home, an ad-financed business model becomes less attractive if the other platform adopts it.

\(^{61}\)There is a link between single-homing/multi-homing and substitutability/complementarity. But since the features of a platform depend on the number/type of agents joining the platform, the notion of substi-
Note that, for given horizontal differentiation in the stand-alone values, the incentives for each side-\(i\) agent to multi-home are inversely related to the measure of side-\(j\) agents who multi-home. Let \(m_1\) and \(m_2\) denote the measure of agents who multi-home on side 1 and 2, respectively. The incremental value of multi-homing relative to single-homing depends on the additional stand-alone value that each agent obtains by joining a second platform and the incremental interaction benefit that the agent obtains by reaching those agents from the opposite side that single-home. Specifically, consider a side-\(i\) agent of type \(\theta_i = (v_i^A, v_i^B, \gamma_i)\) buying from firm \(A\). The agent prefers to multi-home if

\[
v_i^B - \kappa_i + \gamma_i(q_j^B - m_j) \geq p_i^B
\]

where \(\kappa_i\) can be interpreted as the cost of multi-homing, which can be either a direct adoption cost or the utility loss from purchasing a product that is not fundamentally different from that provided by firm \(A\). In other words, \(v_i^B - \kappa_i\) is the incremental value from consuming the good provided by platform \(B\) when consuming already the good provided by firm \(A\). The term \(\gamma_i(q_j^B - m_j)\), instead, is the interaction benefit of reaching those side-\(j\) agents who are present on firm \(B\) and who do not multi-home. Everything else equal, it is easy to see that the mass of consumers willing to multi-home on side \(i\) decreases with the mass \(m_j\) of side-\(j\) consumers who multi-home. In particular, when interaction benefits are an important driver of the demand on each side (relative to stand-alone values), if all consumers multi-home on side-\(j\), few consumers multi-home on side \(i\). This led researchers to investigate asymmetric models with multi-homing on one side only.

4.2.1 Competitive bottleneck

As an example of a model where agents multi-home on one side only, consider again the setting in Armstrong (2006) but assume now that the side-2 agents multi-home whereas the side-1 agents continue to single-home (this is the structure considered in Armstrong and Wright, 2007\(^{62}\)).

The multi-homing behavior of the side-2 agents dramatically changes the competition between the two platforms. The side-2 agents de facto choose whether or not to join each platform as if it were a monopoly: given that choosing a platform does not preclude joining the other, there is no direct competition between the two platforms to attract the side-2 agents.

The lack of competition on side 2 de facto changes the bargaining power between the agents and the platform. Indeed, because the side-1 agents single-home, they are a scarce resource for which the platforms are ready to fight. If any of the side-2 agents wants to be connected to a specific agent from the other side, he has to join the platform this specific agent has joined. This situation is often referred to as a competitive bottleneck. Because of the lack of competition on side 2, platforms raise their side-2 price, inducing too few side-2 agents to join. The effects of multi-homing on the equilibrium side-1 prices are ambiguous.

tutability/complementarity is endogenous in a two-sided context. If one side multi-homes, the other sides views the competing platforms are providing imperfectly substitutable goods. But if there is single-homing on the first side, the competing platforms are seen as providing complementary goods by the agents on the other side of the market.

\(^{62}\)In this paper the side multi-homing is endogenously derived
On the one hand, because side 2 multi-homes, the value of stealing a side-1 agent as a way to weaken the competitor is not present anymore. On the other hand, the value of attracting a side-1 agent may be larger under multi-homing because each platform may attract more agents on side 2 under multi-homing and hence may match the additional side-1 agent it attracts to a larger set of side-2 agents.

To illustrate the above effects assume that consumers on side 2 multi-home at no cost ($\kappa_2 = 0$), that all side-1 consumers single-home ($m_1 = 0$) and that interaction benefits are homogeneous within each side and equal to $\gamma_1$ and $\gamma_2$ for side 1 and side 2, respectively. Then for each consumer on side 2, the decision of whether or not to join platform $k$ doesn’t depend on the other platform’s price or sales. The side-2 demand for each platform $k = A, B$ is given by

$$q^k_2 = \Pr \left( v^k_2 + \gamma_2 q^k_1 - p^k_2 \geq 0 \right) \geq D^k_2 (p_2; q_1).$$

Many applications of the competitive bottleneck model assume that firms set quantities on the multi-homing side instead of prices. In this case, each firm $k$ chooses a price $p^k_1$ for the single-homers and the quantity $q^k_2$ of multi-homers, with the quantity $q^k_1$ and the price $p^k_2$ obtained as residuals by market-clearing conditions. In this case, most of the intuitions developed in the previous sections extend but multi-homing has two novel implications for the determination of the opportunity costs. First, as anticipated above, the value of stealing a side-1 user as a way to weaken the competitor on the other side of the market vanishes, as the two platforms no longer compete on the multi-homing side. Hence, in computing the opportunity cost of getting a marginal side-1 consumer on board, each firm internalizes only the value of increasing the attractiveness of its network for its side-2 agents. As a consequence of this effect, the optimal prices on the single-homing side are now determined by the first-order condition

$$p^k_1 - \left[ c^k_1 - \gamma_2 q^k_2 \right] = \mu^k_1 (p_1; q_2)$$

where $\mu^k_1 (p_1; q_2)$ continues to denote the inverse-semi-price elasticity of the residual demand on the single-homing side.

Second, on the multi-homing side, attracting a marginal consumer does not imply stealing him from the competitor, if the marginal consumer multi-homes. This is because the attracted consumer continues to buy from the competing firm. Attracting $dq^k_2$ new side-2 consumers while adjusting the side-1 price $p^k_1$ by $\gamma_1 dq^k_2$ would leave $q_1$ unchanged. The first-order optimality condition on the multi-homing side then becomes

$$p^k_2 - \left[ c^k_2 - \gamma_1 q^k_1 \right] = \eta^k_2 (p_2^k; q_1^k)$$

where $\eta^k_2 (p_2^k; q_1^k)$ is the inverse-semi-price elasticity of the monopoly demand.

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63 This is for instance the case for most models of advertising. See Anderson and Coate (2005), Armstrong (2006), and Crampes, Haritchabalet, and Jullien (2009).

64 This condition is computed assuming that platforms choose quantity on side 2. If, instead, they choose prices, the interaction benefits between sides induce a 2-sided feedback loop on the competing firm’s demand $q^k_2$ that requires tracking the change of $q^k_2$ in the computation of the inverse-semi-price elasticity $\mu^k_1$. This happens even if the market is fully covered. See Armstrong (2006) and Crampes et al. (2009).
Hence each firm de facto acts as a two-sided monopolist on the multi-homing side, with the opportunity cost adjusted to account for the effects of expanding the side-2 participation on the side-1 price (recall the discussion in Section 2). Notice that despite each firm acting as a monopoly on the multi-homing side, there is still an indirect price externality between the firms on this side. Any reduction of the side-2 price $p_A^2$ induces a shift of the side-1 demand toward $A$ and therefore a reduction of the demand for firm $B$ on the single-homing side.

As pointed out in Armstrong (2006), the formula in (17) implies that the platforms choose their prices on the multi-homing side so as to maximize the joint surplus of the firm and of the single-homing side at constant volume of participation. In other words, $p_B^2$ maximizes $(p_B^2 - c_B^2) q_B^2 + \gamma_1 q_1^B q_2^B$ holding $q_1^B$ constant. General comparisons between the multi-homing and the single-homing prices is not easy because multi-homing changes the level and the elasticity of the demands. For given participation on side 2 and elasticity on side 1, the price each platform sets on side 1 is higher because the firm does not value stealing consumers on side 1 as a way of weakening its competitor on side 2. In this respect, multi-homing reduces the impact of network effects on prices by reducing the competitive edge generated by higher interaction benefits. On the single-homing side, this effect, however, must be balanced by the change in sales on side 2, as anticipated above. When multi-homing increases the side-2 demands, the value of attracting a marginal consumer on the single-homing side is higher because the firm may then match the consumer to a larger user base on the other side of the market. Thus, there are conflicting effects. For instance, in a model where all consumers on side 2 multi-home and have inelastic demands, Armstrong (2006) shows that the equilibrium price on the single-homing side is the same as when both sides single-home.

The implications of multi-homing for consumer welfare are not straightforward either. First, ceteris paribus, allowing some agents to multi-home raises their options and thus their utility. However, prices may increase or decrease due to multi-homing, thus making the analysis of the effects of the latter on consumer surplus ambiguous. Moreover, the elasticity $\eta_2^k$ of the monopoly demand differs in a non-trivial way from the elasticity of the residual demand $\mu_2^k$ in the duopoly case, which also makes the comparison of prices and hence of consumer welfare tedious. Still, intuition suggests that the single-homing side should benefit if its participation is very valuable to the other side ($\gamma_2$ large) and if multi-homing raises the volume of sales significantly. Similarly, the multi-homing side should benefit if the monopoly demand is elastic enough. Belleflamme and Peitz (2019a) confirm these insights in an extension of Armstrong (2006)’s Hotelling model, varying the cost $\kappa_2$ of multi-homing.

### 4.2.2 Multi-homing on both sides

The competitive bottleneck model assumes that agents multi-home only on one side. For example, when platforms are the owners of smartphone operating systems, it makes sense to expect most users to buy a single smartphone. This is because the cost of purchasing multiple handsets typically offsets the benefit of being able to use the apps that are developed...
for one smartphone operating system but not the others. On the other hand, developers
typically develop apps for multiple operating systems.

In other markets, though, multi-homing may occur on both sides and this possibility
brings new effects. There are three types of reasons for multi-homing to occur.

1. Incremental stand-alone values: the two stand-alone products need not be perfect
substitutes so that buying the second good may provide additional utility irrespective
of whether it also provides access to more agents from the other side.

2. Incremental volume of interactions: when some agents single-home on the other side,
multi-homing allows an agent to raise the volume of interactions.

3. Incremental value of interactions: interacting with the same agents on multiple plat-
forms may be more valuable than interacting with the same agents only on one plat-
form. In the context of advertising, multiple impressions may be more valuable than
single impressions. More generally, interacting over multiple platforms may increase
the likelihood that a final match is formed and/or its quality, as pointed out in Cail-
laud and Jullien (2003).

To see the implications of multi-homing on both sides on equilibrium prices, assume
that, on each side, a consumer who multi-homes obtains a total stand-alone utility equal to
\[ w_i = v_A^i + v_B^i - \kappa, \]
where, as before, \( \kappa \) denotes the cost of combining the two stand-alone
products. Further assume that each multi-homing consumer obtains a additional interaction
benefit equal to \( \chi_i \) from interacting with an agent from the other side who multi-homes.
Continue to let \( q_k^j \) denote the total participation to firm \( k \) on side \( j \) and \( m_j \) the mass of
side-\( j \) multi-homers. The total participation on side \( j \) is then equal to \( q_A^j + q_B^j - m_j \). The
utility of a multi-homing consumer on side \( i \) is thus equal to
\[ w_i + \gamma_i (q_A^j + q_B^j - m_j) + \chi_i m_j - p_A^i - p_B^i. \]

Assuming full participation (i.e., \( q_A^j + q_B^j - m_j = 1 \)), the consumer thus chooses to multi-
home if
\[ w_i - v_i^{-k} + \gamma_i (q_j^k - m_j) + \chi_i m_j > p_k \text{ for } k = A, B. \] (18)

The term \( w_i - v_i^{-k} \) captures the degree of substitutability between the two firms’ stand-
alone products, ranging from 0 in case of perfect substitutes to \( v_i^k \) in case of independent
products. The second term captures the increase in the volume of interactions with the other
side (recall that \( q_j^k - m_j \) is the measure of side-\( j \) agents who are present only on platform
\( k \)). The third term \( \chi_i m_j \) captures the increase/decrease in interaction efficiency (i.e., the
change in benefits originating from interacting with the same agents on both platforms as
opposed to a single platform).

For our concerns, the interaction terms are the most interesting. Typically, the addi-
tional benefit of interacting with the same agent on multiple platforms is less than the
benefit of interacting with the same agent on a single platform, that is, \( \chi_i < \gamma_i \). This implies
that, at fixed prices, increasing the volume of agents multi-homing on one side decreases the

\[ ^{65} \text{See Bakos and Halaburda (2020) and Jeitschko and Tremblay (2020) recent analysis.} \]
volume of agents who multi-home on the other side. Hence, when products are substitutes and the incremental gain $\chi_i$ of interacting with agents from the opposite side on multiple platforms is small, we may expect one side to single-home if the number of agents who multi-home on the other side is large. Gabszewicz and Wauthy (2014) illustrate this point. They find that only one side multi-homes in a model with heterogeneous interaction benefits. They suggest a link between multi-homing in two-sided markets and multi-purchases in vertical differentiation models. Interaction benefits constitute a quality dimension and multi-homing is similar to buying a bundle that raises quality compared to each single product in the bundle. Interaction benefits thus introduce a vertical dimension in the competition between the firms. As in models of standard vertical differentiation (Mussa and Rosen, 1978 and Shaked and Sutton, 1983), only consumers with a high value for quality (high interaction benefits) buy the high quality product (here multi-home).

Along this line, Doganoglu and Wright (2006) build a model of platforms where consumers are heterogeneous in their interaction benefits and both sides multi-home.\(^{66}\) Suppose there are two types of consumers on each side: half of the consumers have a high interaction benefit $\bar{\gamma}_i$ and the other half has a low interaction benefit $\gamma_i < \bar{\gamma}_i$. Products are independent and there is no efficiency from multi-homing ($\chi_i = 0$) so the sole motive for multi-homing is to expand the network interactions. Then a consumer multi-homes if $v_i^k + \gamma_i (q_j^k - m_j) - p_i^k > 0$. Assume that in equilibrium the market is covered on each side and prices are at intermediate levels so that low types single-home whereas high types multi-home. The demand from low types is then as in the baseline model $D_k^j(p_i; q_j) / 2$ (accounting for the mass 1/2) while the demand from high types is as in the competitive bottleneck model and given by

$$D_k^j(p_i; q_j - m_j) = \frac{1}{2} \Pr\left(v_i^k + \bar{\gamma}_i (q_j^k - m_j) - p_i^k > 0\right),$$

with the volume of multi-homers given by\(^{67}\)

$$m_j = \tilde{D}_j^A(p_j^A; q_i^A - m_i) + \tilde{D}_j^B(p_j^B; q_i^B - m_i) - \frac{1}{2}.$$  

Compared to the competitive bottleneck model, there are two differences. First, each firm faces both single-homers and multi-homers and marginal consumers are spread over the two categories. Hence the situation is a hybrid between the single-homing case and the competitive bottleneck. Second, the value offered to marginal multi-homing consumers on one side decreases with the volume of multi-homing agents on the other side. This is the consequence of a more general principle that governs competition with multi-homing. When the marginal consumer is already buying from the other firm, a firm can only charge for the additional interaction benefits it brings to the consumer given the level of interactions at the other firm. This has been referred to as the “incremental pricing principle” in the advertising literature (see Anderson, Foros, and Kind, 2018, Ambrus and Reisinger, 2006, and Ambrus, Calvano, and Reisinger, 2016).

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\(^{66}\) See also Kim and Serfes (2006).

\(^{67}\) As the market is covered, every agent on side $j$ is either single-homer or multi-homer. If $x_j^A$ and $x_j^B$ are the mass of high type single-homers then $\tilde{D}_j^k(p_i^j; q_i^j - m_j) = x_j^k + m_j$ and $x_j^A + x_j^B + m_j = 1/2$. 

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When applied to each firm, this principle implies that each firm is constrained in its ability to extract rents from the consumers. For instance, if all consumers multi-home on side \( j \) (that is, \( q_j^A = q_j^B = m_j \)) and all marginal consumers multi-home on side \( i \), then no firm can charge a mark-up for interaction benefits on side \( i \). This incremental-pricing principle has been used to explain the decline of aggregate advertising revenue following the rise of the Internet and the declining ability by large advertising platforms to charge higher prices (Athey, Calvano, and Gans, 2018). Rochet and Tirole (2003) point to another aspect of competition with multi-homing that they refer to as “steering”. In their setup, firms are credit cards setting transaction fees. Sellers must decide whether to accept transactions on both platforms (multi-home) or only one.\(^{68}\) In case a seller accepts to trade on both platforms, the buyer chooses which platform to use. A seller choosing to trade only at the platform setting the lowest transaction fee reduces its volume of sales but increases its average margin by forcing some buyers to trade on their least-preferred platform. Reducing transaction fees in this context may not raise acceptance by sellers but convince some sellers to stop accepting intermediation of transaction by the other more expensive platform, thereby increasing usage.

4.3 Concentration, merger and collusion

4.3.1 Platform Entry

It is well known that there may be excessive or insufficient entry in industries absent network effects, the same holds true for goods involving network externalities (see Katz and Shapiro, 1994, for a discussion). The literature based on one-period models confirms this insight for two-sided markets. For instance Jullien (2011) and Vasconcelos (2015) argue that both excess inertia or excess fragmentation may occur if network externalities are not too large. This is because, despite an incumbency advantage, it may be too costly for an incumbent to try to deter all divide-and-conquer strategies by an entrant rather that accommodating some market fragmentation (which may be inefficient). Amelio, Giardino-Karlinger, and Valletti (2020) characterize limit-pricing equilibria with inefficient entry deterrence for some intermediate range of entry costs.

4.3.2 Horizontal mergers and concentration

Unlike markets where firms sell substitutes without network effects nor scale efficiencies, increasing concentration (through exit or mergers) need not hurt consumers in markets with network externalities. First, reduced market fragmentation may raise the interaction benefits on each platform, which directly benefits consumers and affects each firm’s internalization of network externalities. Second, following a merger, the owner of two platforms can raise compatibility between platforms and consequently boost the interaction benefits on the merged platforms (Leonello, 2010). Because of two-sidedness, increasing concentration may not lead to higher prices or consumer surplus on both sides.\(^{69}\) This was first

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\(^{68}\) Liu, Teh, Wright, and Zhou (2020) extend the model to allow consumers to multi-home as well, showing that it induces some re-balancing of transaction fees in favor of consumers.

\(^{69}\) The possibility that increasing concentration may reduce prices also exists in one-sided setting with product differentiation (Chen and Riordan, 2008). We focus on platform-specific effects.
pointed out by Chandra and Collard-Wexler (2009). Considering two newspapers selling content to readers and space to advertisers, they notice that, due to imperfect screening, the reader subscription price may be excessive from the industry-profit perspective. Newspapers cannot discriminate between readers so that advertising revenue per reader is equal to the average value of a reader for advertisers. In order to sell to the readers most valuable to advertisers, a newspaper needs also to sell to readers not valuable to advertisers, who may contribute negatively to profit. Then suppose that when a newspaper raises its subscription price, the marginal readers diverted toward the competing newspaper are those contributing negatively to profit, while the most attractive marginal consumers stop buying at all. In this case, raising one newspaper price has a negative externality on the other. Thus a monopoly owner of two newspapers may reduce subscription prices.

Tan and Zhou (2019) analyze the effect of concentration in the discrete-choice model with \( K \) differentiated platforms and single-homing consumers on both sides discussed in Section 4.1.1. Reducing market fragmentation—reducing \( K \)—raises average participation in each platform on both sides and thus raises total interaction benefits. However, lower elasticity of residual demand tend to increase price-cost margins, due to the familiar market-power effect. The market-power effect dominates if network effects are small. They show that, depending on the specification of the demand function, increasing \( K \) may reduce or increase prices on any side of the market. In particular, prices may increase on one side and decrease on the other side. This property is a manifestation of the “seesaw” principle of Rochet and Tirole (2006), according to which “a factor that is conducive to a higher price on one side, to the extent that it raises the platform’s margin on that side, tends also to call for a lower price on the other side, as attracting members on that other side becomes more profitable.”

Correia-da Silva, Jullien, Lefouili, and Pinho (2019) study the effect of concentration in a Cournot model with \( K \) homogeneous platforms (see Section 5.2 for a discussion of the Cournot competition). They find that reducing \( K \) reduces (respectively, increases) quality-adjusted prices, \( p^k_i - \gamma_j q^k_j \), \( i \neq j \), on both sides \( i = 1, 2 \) if pre-merger quality-adjusted prices are below (respectively, above) mean marginal costs on both sides, which occurs when interactions benefits are small (respectively, large). As higher quality-adjusted prices translate directly into lower consumer surplus in their model, these conclusions imply that consumer surplus increases on both sides with concentration if network effects are important. They also identify an intermediate range of interaction benefits such that increasing concentration harms consumers on one side and benefits consumers on the other side—a seesaw effect. Anderson and Peitz (2020) also analyze the seesaw effect in an aggregative-game formulation of a media market. They focus on a competitive bottleneck

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70 This occurs when the sum of the subscription price and the value of the consumer for advertisers is smaller than the newspaper’s marginal cost.

71 They also extend conditions in Anderson, de Palma, and Nesterov (1995) for excessive entry (from a total-welfare perspective) to occur in their multi-sided market model.

72 For linear demand, they show that only total interaction benefits \( \gamma_1 + \gamma_2 \) matter for equilibrium quality-adjusted prices.

73 Aggregative games simplify the equilibrium analysis by using an aggregate of players’ actions to reduce the dimensionality of strategic interactions (Selten, 1973). See Acemoglu and Jensen (2013) and Anderson, Erkal, and Piccinin (2020). This allows Anderson and Peitz (2020) to consider asymmetric platforms and partial market coverage.
model and argue that the seesaw effect is more likely to occur when advertising is a nuisance and content is offered free of charge to consumers.\textsuperscript{74}

Similar conclusions apply to the coordination effect of a merger that keeps both platforms active. For instance, Tan and Zhou (2020) show, in a model with three platforms and $\gamma_2 = 0$, that if $\gamma_1$ is intermediate then a merger is profitable and results in lower prices and higher consumer surplus on side 2. Intuitively if the pass-through of interaction benefits from side $i$ to side $j$ increases, it may be worthy boosting participation on side $i$ by reducing the prices on that sides. Higher participation on side $i$—which is a quality dimension for side-$j$ consumers—may then benefit side $j$.

Baranes, Cortade, and Cosnita-Langlais (2019) consider a model where four firms are located on the Salop circle so that competition is localized. When two adjacent firms merge without synergy, the market power effect dominates and all prices increase. They then show the possibility of a seesaw effect of cost synergies, with consumer surplus increasing on one side but decreasing on the other side when cost saving increases.

Finally, the merged entity may reduce some price when the merger raises the interaction benefits by increasing compatibility between the two platforms. In this case, the interaction benefits of a side-$j$ consumer joining platform $k$ that is merged with platform $l$ becomes $\gamma_j (q^k_i + \xi q^l_i)$. As the merged entity coordinates prices, it internalizes the effect of side $i$’s participation to platform $k$ on the side-$j$’s participation to platform $k'$ and may decrease the price $p^k_i$ for $\xi > 0$ while it would not do it for $\xi = 0$ (Leonello, 2010). The analysis of Correia-da Silva et al. (2019) applies to mergers as well, by noticing that, in their model, reducing $K$ by one unit is equivalent to a merger with full compatibility, that is, with $\xi = 1$.

4.3.3 Collusion

A seesaw effect is identified in Dewenter, Haucap, and Wenzel (2011)’s analysis of price coordination within a competitive bottleneck model: coordination reduces participation on the multi-homing side but may increase participation and consumer welfare on the single-homing side. Dewenter et al. (2011) also introduce the possibility of price coordination on one side only (with competition on the other side), referred to as “semi-collusion,” and show that from a total-welfare perspective, semi-collusion in the multi-homing side dominates a merger and may dominate competition.\textsuperscript{75}

Lefouili and Pinho (2020) present a full-fledged analysis of tacit collusion in grim-trigger strategies (with Nash reversal) in Armstrong’s model. At the most profitable collusive equilibrium, incentive compatibility implies additional distortions of the price structure compared to competition, raising the price differential between the side with the larger degree of differentiation and the other side. In this model, the prices maximizing total profit are above competitive prices on both sides, but incentive compatibility may call for reducing the price below the competitive level on the least differentiated side, so as

\textsuperscript{74}In a competitive bottleneck setup with ad nuisance, the attention to ads is the price to access content. A merger relaxes competition for consumers so that it raises the volume of advertising and hurts consumers (Anderson and Coate, 2005). This result may be reversed if consumers multi-home, because multi-homing soften competition for consumers (Anderson, Foros, and Kind, 2019). See also Foros, Kind, and Sørgard (2015) for a discussion of the impact of mergers on two-sided media markets.

\textsuperscript{75}See Argentesi and Filistrucchi (2007) in Section 8.3 for empirical evidence.
to reduce deviation profits. The paper then finds a seesaw effect of a different nature than in the merger analysis. A more standard seesaw effect occurs under semi-collusion, although the price may decrease on any side depending on the level of network effects. Under semi-collusion, firms have two possibilities: they may agree to raise margins with the consequence of reducing prices on the competitive side; or they may agree to reduce margins so as to boost demand on the collusive side, and raise interaction benefits and prices on the competitive side.\textsuperscript{76}

4.4 Exclusivity and bundling)

4.4.1 Exclusivity

Platforms use exclusive dealing as a way to prevent multi-homing.\textsuperscript{77} Requiring exclusivity on a multi-homing side $j$ allows platforms to raise the price charged on side $i$ for a given mass of side-$j$ users on the platform. In a duopoly context, imposing exclusivity amounts to imposing that agents single-home. When there is little differentiation between platforms, exclusivity permits platforms to monopolize the side where exclusivity is imposed and gain a competitive hedge on the other side of the market. Hence exclusive contracts may destabilize equilibria where agents multi-home. Armstrong and Wright (2007) show that this is the case in a model of competitive bottleneck and characterize the resulting tipping equilibrium (discussed in Section 3) where all agents use the same platform that emerges when exclusive contracts are allowed and platforms are homogeneous. The tipping equilibrium is efficient but with lower consumer surplus than the competitive bottleneck without exclusive contracts.

The emergence of tipping equilibria when platforms can require exclusivity raises the question of strategic barriers to entry. The fact that exclusivity may be used by an incumbent platform to raise barriers to entry is confirmed by Doganoglu and Wright (2010). They first extend the naked-exclusion principle (Rasmusen, Ramseyer, and Wiley Jr, 1991, Segal and Whinston, 2000)\textsuperscript{78} to markets with network effects.\textsuperscript{79} They then focus on a two-sided market and assume that an incumbent platform $A$ cannot discriminate within each side but may offer an exclusive contract to all agents on one side before an entrant $B$ steps in and competes. Agents are assumed to be homogeneous on both sides with no stand-alone values ($v_{ik}^k = 0$, $i = 1, 2$, $k = A, B$) and multi-homing. They assume that the entrant offers higher interaction benefits than the incumbent (i.e., $\gamma_i^B > \gamma_i^A$, $i = 1, 2$) and faces no chicken & egg problem (consumers coordinate on Pareto efficient allocations). Under these assumptions, absent exclusive contracts, the equilibrium is efficient and $B$ wins the market.

\textsuperscript{76}The paper also analyzes semi-collusion in a competitive bottleneck with similar insights (full collusion in their competitive bottleneck model has no effect on quantities and welfare because the market is fully covered on both sides).

\textsuperscript{77}There are of course other motives for exclusivity not specific to platforms, such as protecting specific investments or preventing free riding.

\textsuperscript{78}This principle states that, with increasing returns to scale, a monopoly can prevent efficient entry by offering discriminatory exclusive contracts, subsidizing a subset of consumers large enough to make trading with the entrant inefficient for other consumers.

\textsuperscript{79}In this case, subsidizing a subset of consumers for exclusive participation raises the value differential with the entrant for other consumers. See also Karlinger and Motta (2012).
When exclusive contracts are feasible, the incumbent can however prevent entry by offering exclusivity on one side of the market. To see that, suppose that all agents on side 1 grants exclusivity to platform A before B can make any offer. Then platform B cannot generate any value as it can only trade with one side. This allows A to charge a monopoly price $\gamma_2^A$ to the non-exclusive side 2. Platform A thus offers exclusivity if there is a price $p_1^A > -\gamma_2^A$ that is accepted by the side-1 agents for their exclusive participation. Given that the surplus on side 1 is $\gamma_1^A - p_1^A$, this is the case if $\gamma_1^A + \gamma_2^A$—the total value of A—is larger than the surplus accruing to side 1 under no exclusivity and competition between A and B.

But without exclusivity, competition implies that consumers surplus, and a fortiori side-1 surplus, cannot exceed the maximal surplus that platform A can offer, which is precisely $\gamma_1^A + \gamma_2^A$. As a consequence, the platform can profitably sell exclusive contract and exclude its competitor. Notice that the argument requires no incumbency advantage but the ability to obtain exclusivity prior to the competition stage with the entrant. Clearly, if the entrant can also make such offers, there would be competition to sell exclusivity and the entrant would win the market.

The above argument reflects the view that exclusive dealing may protect incumbents. An alternative view is that exclusive contracts may help the entrant when the incumbent benefits from an incumbency advantage (such as favorable beliefs). In this case, exclusive offers may help a new platform to overcome the chicken & egg problem by targeting key strategic players (Markovich and Yehezkel, 2021). Indeed one issue faced by an entrant using a divide-and-conquer strategy when consumers can multi-home is that, if it subsidizes the participation of side $i$, it needs to prevent the possibility that side $j$ stays with the incumbent and side $i$ just multi-homes (collecting the subsidy but interacting with the other side on the incumbent’s platform). This requires low prices on side $j$. Offering exclusivity to side $i$ would then amount to switching to the single-homing divide-and-conquer strategy, which allows a platform to charge higher prices to side $j$ but requires a larger subsidy to side $i$. When introductory exclusive contracts can be targeted in a discriminatory way, the entrant may then selectively target some agents on side $i$ with a large subsidy for exclusive participation. This is in particular the case if some agents generate large interaction benefits, or if a subset of agents are part of an institution coordinating their choices (a user-group). By convincing these key agents to join exclusively, the entrant raises its relative value and the maximal price inducing the other side to join.

The view that exclusive deals can help smaller firms has some support empirically. As mentioned above, Lee (2013) studies exclusive deals between producers of video game consoles and video game developers. In his empirical results, without exclusivity, consumers and developers (who are allowed to multi-home) flow to a single console but exclusive deals provide a way for smaller consoles to differentiate themselves and maintain market share.

An interesting feature pointed out in Carroni, Madio, and Shekhar (2020) is that exclusivity offered by a platform $k$ to some key strategic agents on side $i$ not only boosts participation on the other side $j$ but may also help convincing other agents of the same side $i$ to stop multi-homing and join exclusively platform $k$. The reason is that increasing platform $k$'s demand on side $j$ to the expense of the other platform raises the value of platform $k$ for agents of side $i$ and reduces the incremental value of the other platform. To

\footnote{This argument relies on efficient coordination of consumers.}
see that, recall from the analysis in the previous sections that the (gross) incremental value of platform \(-k\) is the extra utility that a user of platform \(k\) obtains by joining platform \(-k\) (in addition to joining platform \(k\)) and is given by

\[
\begin{align*}
    w_i - v_i^k + \gamma_i \left( q_j^{-k} - m_j \right) + \chi_im_j &= w_i - v_i^k + \left( \gamma_i - \chi_i \right) \left( q_j^{-k} - m_j \right) + \chi_i q_j^{-k},
\end{align*}
\]

where \(q_j^{-k}\) is total participation to platform \(-k\) on side \(j\) and \(q_j^{-k} - m_j\) is the exclusive participation to platform \(-k\). Assume that \(0 \leq \chi_i < \gamma_i\). Then given that both total and exclusive participation to platform \(-k\) decrease when platform \(k\) becomes relatively more attractive, the incremental value of platform \(-k\) decreases and more agents single-home. Hence, gaining exclusivity of some agents brings additional exclusive participation at no cost on the same side.\(^\text{81}\) However, gaining exclusivity of these strategic agents is costly as they need to be compensated for giving up some interaction benefits. In the analysis of Carroni et al. (2020) of a competitive bottleneck, exclusivity arises when competition on the single-homing side is intense. As in Armstrong and Wright (2007), the equilibrium may be more efficient due to larger interaction benefits when concentration increases.

Hence, exclusivity raises equilibrium concentration when network effects are large, and depending on the context, it may raise or reduce barriers to entry.\(^\text{82}\)

### 4.4.2 Bundling

Bundling is a widespread practice by platforms (which stands a bit outside models discussed so far) that is motivated by specific features of platforms and leads to new economic insights. Choi (2010), and its corrigendum Choi, Jullien, and Lefouili (2017), argue that bundling may help a firm coordinating agents on its platform. They consider a situation where the platforms are pure complements of a monopoly base product. The monopoly provider \(A\) of the base product owns one platform, but cannot capture the revenue from an independent complementary platform—because it cannot directly charge the platform. When platform \(A\) is not tied to the base product, partial multi-homing on both sides of the market dissipates profits. By tying the platform to the base product, platform \(A\) then convinces both sides to single-home on \(A\), excluding the competing platform and raising total revenue.\(^\text{83}\)

Amelio and Jullien (2012) point out that non-negative price constraints create new incentives for bundling that are specific to platforms.\(^\text{84}\) Indeed, as discussed in Section 2.2.3, a

\(^{81}\)Ishihara and Oki (2020) point out that, when both sides multi-home, increasing exclusive participation on one side raises the incremental value of each platform and thus the mass of multi-homing consumers on the other side. Put it differently, exclusivity on one side increases platform differentiation on the other side and may relax competition.

\(^{82}\)See also the literature on exclusivity in media, mostly focusing on premium content. For instance, Chowdhury and Martin (2017) focus on exclusive licensing. D’Annunzio (2017) and Stennek (2014) study the relation between exclusivity and quality of content. Also related is Kourandi, Krämer, and Valletti (2015), focusing on Internet fragmentation and net-neutrality.

\(^{83}\)De Cornière and Taylor (2019) argue that a seller of several platform goods that are distributed through another platform may use bundling as a commitment device to raise its bargaining power vis-a-vis the distributing platform.

\(^{84}\)Traditional motives for bundling include price-discrimination and exclusion of competitors (see Fumagalli, Motta, and Calcagno, 2018, for an overview).

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zero price in some contexts reveals that the platform would be willing to set negative prices to attract consumers but cannot do so. Bundling may then be used to boost adoption when the service is free. In a competitive context, relaxing the non-negative price constraint by bundling participation with goodies has two effects. First, it raises the attractiveness of the platform on the other side. Second, it relaxes the incentives to compete with other platforms as consumers on the profitable side become more costly (due to the feedback effect on subsidies). Bundling has thus both a direct effect on the attractiveness of the platform and a strategic effect that reduces the intensity of competition. While in the monopoly case, where bundling is welfare improving, the effects of bundling in a duopoly are ambiguous. In particular, in the symmetric model of Armstrong (2006), allowing symmetric bundling possibilities reduces consumer surplus. Choi and Jeon (2021) clarify that a key factor beyond new theories of bundling in two-sided markets is that a platform’s ability to cope with aggressive competition is limited by the non-negative price constraint. Hence, if one platform has exclusive access to some way of relaxing the non-negative price constraint and subsidizing the participation of one side (with bundling for instance), it can win the competition even if it is less efficient than its competitor and there are no consumer coordination failures.

Bundling is of great interest in the context of cable television. Cable systems are platforms between channels and consumers, and cable systems offer most channels in sets of large bundles. Crawford and Yurukoglu (2012) estimate a structural model of consumer demand, pricing, and bargaining between cable systems and channels. They find that consumers would benefit from à la carte pricing if fees from cable systems to channels remained constant. However, the switch to à la carte pricing increases pricing power for channels and enables them to bargain for higher fees because these will be passed through to consumers more directly. When accounting for this effect on input prices, à la carte pricing hurts consumers relative to bundled pricing.

5 Alternative modeling of competition, coordination and beliefs

Equilibria in games of platform competition with network externalities typically depend on the assumptions involving (i) the set of feasible price structures, and (ii) the demand configuration given the firms’ prices. In this section, we discuss in more detail each of the above aspects.

5.1 Richer price structures

The standard Bertrand oligopoly model discussed above combines several assumptions typically encountered in the literature on competing platforms. First, platforms set a single price on each side that can take the form of a participation fee or a transaction fee, without transferability.\(^\text{85}\) Second, demand on each side adjusts to changes in prices to always yield

\(^{85}\)We define transferability as in the matching literature as the possibility for a pair of agents interacting on a platform to negotiate a transfer between them to redistribute the fees.
an equilibrium demand configuration in the continuation game that starts after the firms announce their prices.\footnote{See Correia-da Silva et al. (2019) for a discussion of this assumption and a few issues related to it.}

Many platforms can monitor both participation and usage. In this case, the choice of the price instruments is endogenous and the platforms can either charge fixed participation fees or individual transaction fees. As mentioned in Section 2, in the case of monopoly pricing, Rochet and Tirole (2006) allow for general two-part tariffs by focusing on the average price per transaction, assuming away the issue of multiplicity of continuation equilibria in the agents’ choice of whether or not to join the platform. The generalization to oligopoly raises challenging problems and it seems difficult to find a single representation capturing the diversity and richness of oligopoly interactions with multi-sided network effects.

The difficulty in the oligopoly case is that the elasticity of the residual demand for any firm on any side depends on the structure of the tariffs used by the competing firms on all sides. Indeed, even if a firm can coordinate participation on both sides with its tariffs, the value of the agents’ outside option (buying from the other firms) depends on the other firm’s prices (assuming it faces no chicken & egg problem).

Suppose agents single-home on both sides and firm $B$ charges a total price $P_i^B = p_i^B + t_i^B q_i^B$ on each side $i = 1, 2$, where $p_i^B$ is a fixed fee and $t_i^B$ is a fee per interaction. A deviation from the equilibrium tariffs by firm $A$ that raises participation on side $i$ and leaves the side-$j$ participation constant requires an adjustment of the side-$j$ price $P_j^A$ equal to $\gamma_j + \left( \gamma_j - t_j^B \right) = 2\gamma_j - t_j^B$ per consumer attracted on side $i$, where $\gamma_j$ is the extra value a side-$j$ user of platform $A$ derives from the increased participation to platform $A$ on side $i$ whereas the term $\gamma_j - t_j^B$ is the lost interaction value for a side-$j$ user of platform $B$. The first-order condition for platform $A$’s prices (assuming it faces no chicken & egg problem) can be written as

$$P_i^A - \left[ c_i^A + \sigma_A q_j^A - 2\gamma_j q_j^A + t_j^B q_j^A \right] = \mu_i^A (P_i; q_j). \tag{19}$$

where again $\mu_i^A$ is the inverse-semi-price elasticity of the residual demand of platform $A$ on side $i$, holding $q_j^A, q_j^B$ and $P_i^B$ constant. In this approach to pricing, the tariff offered by the competing platform $B$ determines the relevant opportunity cost of bringing a marginal consumer on board. This is another illustration of the point made by Rochet and Tirole (2003) that, in two-sided markets, the price structure matters as well as the price level.

The equilibrium allocations then depend on whether the firms set flat prices or two-part tariffs with a positive slope. For instance, when platforms compete in tariffs, Armstrong (2006) shows the existence of a continuum of equilibria indexed by the level of the interaction fees $t_i^k$. To see that, suppose that demand is as in (11) and focus on the case of a symmetric equilibrium where quantities are $q_i^k = 1/2$, $k = A, B$, $i = 1, 2$. Consider a symmetric tariff in which the participation fees are $p_i^A = p_i^B = p_i$ and the interaction fees are $t_i^A = t_i^B = t_i$, with $t_i$ not too large. In a symmetric equilibrium with full coverage, the total price paid by each side-$i$ consumer is $P_i = p_i + t_i/2$. Under suitable concavity conditions, any symmetric tariff such that

$$p_i - c_i^A - \sigma_A/2 + \gamma_j + t_i/2 - t_j/2 = \frac{\Phi_i(0)}{\Phi_i’(0)}$$

\footnote{The complexity here is similar to the one noticed in the competing-principals literature – see, e.g., Martimort and Stole (2009).}
can be sustained in a symmetric equilibrium, where $\Phi_i$ is the cdf of the side-$i$ distribution of the stand-alone value differential between the two platforms (Armstrong, 2006).

Reisinger (2014) shows however that the multiplicity disappears if there is enough heterogeneity in the interaction benefits $\gamma_i$ as well as in the stand-alone values. In his model, different types of consumers have different trade volume and thus pay a different total price for the same transaction fee. Each part of a two-part tariff then induces a different demand reaction, reflecting the mix of the different types of marginal consumers. Optimal subscription fees and transaction fees are tailored to these different elasticities, leading to unique equilibrium tariffs.

Weyl (2010) and White and Weyl (2016) argue that platforms have enough pricing instruments to overcome any coordination problem. They define the concept of Insulated Equilibrium, based on the idea that prices can be designed so that participation on each side is independent of the participation decisions on the other side. More specifically, they consider an insulating tariff system whereby the tariffs set by all firms make the choice of whether or not to join a platform invariant in the agents’ beliefs about the participation of the other side. In other words, given the equilibrium tariffs, each agent’s participation decision is determined by a dominant strategy. Formally, the notion of dominance applies to each agent in the case of homogeneous interaction benefits or to a representative consumer on each side in the case of heterogeneous interaction benefits. In either case, the insulating tariff system is given by $P_i(q_j) = (P^A_i(q_j), P^B_i(q_j))$ with

$$\frac{\partial D^k_i(P_i(q_j); q_j)}{\partial q^l_j} = 0 \text{ for } k, l = A, B.$$

The concept of an insulating equilibrium however implicitly assumes coordination between the firms. Indeed whether the tariff of a firm is insulating or not depends on the tariff set by other firms. In general, a firm may not find it profitable to offer an insulating tariff if it expects the other firms to offer tariffs other than the insulating ones, which raises the usual question of equilibrium selection. Lastly, while insulating tariffs are quite simple in the case of homogeneous interaction benefits, these tariffs become highly complex in the case of heterogeneous interaction benefits. This is because an insulating tariff system must preserve for the marginal consumer the value differential between the platform and the consumer’s outside opportunities. This requires controlling the value offered by the platform to the representative consumer in a manner that compensates for any change in outside options.\(^{88}\)

In the duopoly model with homogeneous interaction benefits discussed above, things are simpler and an insulated equilibrium arises when each firm selects a two-part tariff of the form $p^k_i + \gamma_i q^k_j$. When both firms offer such tariffs, all externalities are eliminated through

\(^{88}\)Hence, the firm must have more instruments than needed to implement optimal prices and these instruments must be flexible enough (see the discussion in Veiga, Weyl, and White, 2017). White and Weyl (2016) and Veiga et al. (2017) state that contingencies can be achieved with dynamic pricing, with firms adjusting their price to market conditions overtime. As mentioned in Section 2, for the case of a monopoly platform, Cabral (2019) finds one particular instance (i.e. homogeneous consumers with $v_i = 0$) where optimal dynamic tariffs mimic insulating tariffs, although the tariffs in his analysis reflect exploitation motives rather than insurance motives. In the case of competition, dynamic strategic considerations would need to be accounted for, which casts some doubts about the dynamic justification of insulated equilibria.
taxation and the characterization of the insulated tariffs is given by the following first-order conditions:

\[ P_k^i - \left[ c_k^i + \sigma_k^i q_j^k - \gamma_j^k q_j^k \right] = \mu_i^k (P_i; q_j). \] (20)

The difference with Condition (12) is that the term \( \gamma_j^k q_j^k \) is counted only once instead of twice. Insulated tariffs eliminate the feedback loop between sides discussed above. When a firm attracts one more consumer on side \( i \), the competitor’s tariff responds by reducing the price on side \( j \) so as to preserve the competitor’s value, which reduces the platform’s ability to raise its own price on side \( j \). Because the competitor’s tariff taxes interaction benefits, demand is less elastic and prices are higher than under flat access fees. Hence, tariffs that solve the consumers’ coordination problem may not benefit consumers. On the contrary, coordination failures may protect consumers against excessive use of market power.\(^{89}\)

Earlier versions of insulating tariffs appear in Caillaud and Jullien (2003). The paper shows that competition in two-part tariffs between homogeneous platforms leads to an efficient outcome with zero profits (see also Armstrong (2006) for a similar point). With complex tariffs, a platform can tax all the surplus it creates for its members and redistribute it through flat registration fees. By doing so, the firm ensures its customers against the risk that participation does not meet expectations. When all firms offers such tariffs, network effects are internalized and coordination failures vanish.

The concept of insulating tariffs relies on the existence of multiple best-replies. In other words, it assumes that, given other firms’ tariffs, each firm has several tariffs that would achieve the same revenue. The concept then implicitly assumes that each firm breaks indifference by choosing the tariff that minimizes the risk of consumers’ mis-coordination. However, whether this multiplicity of best replies remains in cases involving more realistic assumptions than the base model is questionable. For instance, in Reisinger (2014) (discussed above), the platforms prefer to exploit the richness of tariffs to achieve second-degree price discrimination and there is a unique optimal tariff that may not be insulating.\(^{90}\)

5.2 Quantity competition

As an alternative to the model of price competition, consider a Cournot model in which platforms choose quantities instead of prices. Indeed, the assumption in Rochet and Tirole (2006) that the monopoly can select the desired demand configuration among all allocations consistent with the posted prices amounts to assuming that the monopoly chooses the participation on each side, which then uniquely determines the prices. This observation suggests that having platforms choosing quantity (i.e., the participation) on each side instead of prices may bypass some of the complexity of the pricing games discussed above. To our knowledge, Katz and Shapiro (1985) are the first to introduce network effects in the Cournot model of quantity competition. The Cournot equilibrium concept has also been used in a

\(^{89}\)To illustrate this point, consider a monopoly platform facing “unfavorable beliefs”. Consumers benefit if the platform must subsidize participation on one side to overcome the unfavorable beliefs. When, instead, consumers coordinate efficiently, the platform can extract the full surplus from the consumers by charging high prices on both sides.

\(^{90}\)See also Halaburda and Yehezkel (2013) who show how to use tariffs to screen agents with adequate choices of transaction fees.
limited number of papers on two-sided markets. An early example is Schiff (2003) who finds, in a setting with homogeneous interaction benefits, that a monopoly is socially preferable to a duopoly because it permits each side to interact with a larger number of agents on the opposite side. Gabszewicz and Wauthy (2014) show that heterogenous interaction benefits may lead to asymmetric market shares, despite the platforms being symmetric ex ante. They also find that the equilibrium participation and welfare are higher under the Cournot equilibrium than under price competition with passive beliefs (see also the discussion below). Correia-da Silva et al. (2019) analyze the impact of platform mergers in a Cournot setting with homogeneous interaction benefits within each side.

Finally, many papers in the media literature assume that media platforms set quantities on the advertiser side. The literature is surveyed by Anderson and Jullien (2015) and Peitz and Reisinger (2015). For instance, Kind, Nilssen, and Sørgard (2007) build a model in which TV stations offer advertising space to advertisers and (free) content to viewers. They show that there is too little advertising when the channels’ programs are close substitutes and that the more viewers dislike ads, the more likely it is that social welfare is increasing in the number of channels. Crampes et al. (2009) and Peitz and Valletti (2008) consider similar models but investigate also the case in which platforms charge viewers. This allows them to compare the free-to-air and pay-tv business models (see also Calvano and Polo, 2020). Rysman (2004) utilizes a quantity setting assumption in an empirical study of Yellow Pages directories in order to avoid the multiple equilibria arise under price setting. A common feature of these papers is that consumers are assumed to single-home. Anderson et al. (2019) relax this assumption and show that a number of puzzles identified in the previous literature on media economics can be resolved by allowing for multi-homing consumers.

The Cournot model should be interpreted as imposing a specific form of firm’s conduct. Its key assumption is that each firm anticipates that its rivals will maintain their quantities at a given level when the platform under consideration changes its practices and that equilibrium prices will clear the market at targeted quantities.

Models of Cournot competition can be more tractable than models of price competition in the presence of cost asymmetries or if there are more than two platforms. In particular, the Cournot model generates predictions for externality-adjusted prices that make it possible to determine the welfare effects of a merger. To see that, consider a market with $K$ firms and suppose that demand on each side depends only on the hedonic prices on that side, $p_i - \gamma_i q_j$. Invert the demand to obtain each platform’s price on side $i$ as a function of participation to all platforms: $p_i^k - \gamma_i q_j^k = Z_i^k(q_i)$. Then each platform’s profits can be written as

$$\sum_i \left( Z_i^k(q_i) + \gamma_i q_j^k - c_i^k \right) q_i^k - \sigma^k q_1^k q_2^k.$$

---

91 The logic is similar to the one behind the emergence of vertical quality differentiation as a way of relaxing competition. The quantity chosen on one side is the quality perceived on the other side. As price competition would erode profit for equal quality, platforms are better off differentiating their offers, with one firm offering a high quantity catering to consumers with high interaction benefits and the other a low quantity catering to consumers with low interaction benefits.

92 In a competitive bottleneck model, this is equivalent to setting an ad price per consumer.

93 As mentioned, in media markets this may be due to pricing per user. In some two-sided markets, capacity is a key strategic variable (e.g. ad-finance media platforms committing to ad-space and auctioning ad inventories, other examples are nightclubs, shopping centers and exhibition halls). In such cases, Cournot competition is a natural way of modeling platforms’ behavior, at least on one side.
In the Cournot model of platform competition, each firm $k$ chooses quantities $(q^k_1, q^k_2)$ and prices adjust so that demand equals supply on each side and for each firm, $p^k_i = Z^k_i(q_i) + \gamma_i q^k_j$. It is then easy to see that equilibrium quantities and prices must satisfy

$$p^k_i - \left[ c^k_i + \sigma^k j^k q^j - \gamma_j q^k_j \right] = \mu^k_i(p_i, q_j)$$

(21)

where $\mu^k_i(p_i, q_j)$ is as defined above. Comparing the above formula with the first-order conditions for insulated equilibria discussed above, we see that, under homogeneous interaction benefits and regularity conditions ensuring equilibrium existence and uniqueness, the outcome of Cournot equilibrium and Insulated Equilibrium coincide. However, this coincidence should not be expected to hold in more general situations.

The concepts of Insulated Equilibrium and of Cournot equilibrium are related. They both reduce the effect of two-sidedness on prices by mitigating the internalization of demand feedback effects by the platforms. However, the two concepts differ when interaction benefits are heterogeneous, because the insulating tariffs insure only a “representative” consumer against the risk related to the other side’s participation, whereas the Cournot equilibrium entails insurance for all users.\(^{94}\)

### 5.3 Passive Beliefs

Coordination plays an important role in platform competition. As coordination largely depends on agents’ beliefs, the way beliefs are formed and modeled plays a crucial role in the analysis of platform markets.

All contributions using static models de facto assume that consumers’ expectations about participation decisions are fulfilled in equilibrium. The equilibrium concept must then include a conduct assumption for firms, as discussed above, and an assumption about the way consumers coordinate when firms act out of equilibrium. The most prominent view (Caillaud and Jullien (2003), Rochet and Tirole (2003), Armstrong (2006), Weyl (2010)) is that, when prices change, consumers adjust their beliefs and correctly anticipate the adjustment in the participation level on the other side. This property is sometimes referred to as rational expectations, or reactive beliefs, although neither expression seems ideal. Some contributions take the alternative view that consumers hold fixed beliefs about the other side’s participation, that do not vary when prices change.\(^ {95}\) These beliefs are referred to as “passive” and their interaction with network effects was first examined in Katz and Shapiro (1985). With passive beliefs, because the side-$j$ agents’ expectations of the side-$i$ demand do not vary when a platform changes its side-$i$ prices, then opportunity costs no longer incorporate network effects. Indeed profits can now be written as

$$\Pi^k = \sum_i \left( p^k_i - c^k_i \right) D^k_i(p_i, q^k_j)$$

---

\(^{94}\) As pointed out informally by Glen Weyl, this property impedes the existence of a Cournot equilibrium with multi-dimensional heterogeneity, which is one motivation for the introduction of the concept of insulated equilibria.

\(^{95}\) See Gabszewicz and Wauthy (2014) and Griva and Vettas (2011). Hurkens and López (2014) study the role of expectations in the context of mobile telephony and show that passive beliefs provide better modeling of actual behaviors.
where $q^j_i$ is the side-$j$ participation expected by the side-$i$ users and cannot be affect by the firm’s strategy. It follows that the first-order condition for the side-$i$ prices under passive beliefs reduces to the standard oligopoly pricing rule

$$p^k_i - c^k_i = \mu^k_i \left(p_i; q^j_j\right).$$

In this case, two-sidedness affects prices only through the effect of interaction benefits on the elasticity of the residual demands, not through the firms’ incentives.

Hagiu and Halaburda (2014) show that monopoly profits are lower under passive beliefs than under reactive beliefs. This is because, with passive beliefs, platforms de facto cannot credibly commit to the prices they set on the other side. Things, however, are different in a duopoly context. Duopoly profits in their competitive bottleneck model are higher under passive beliefs than under reactive beliefs because residual demands are less elastic in the absence of feedback effects (see also Belleflamme and Peitz, 2019b, for a model with single-homing). While interesting, a concern with the assumption of passive beliefs is that because platforms take users’ expectations about participation as given, they do not account for the value that a user creates for the other side(s) of the market when setting prices or quantities. Thus, this approach ignores a key driver of the differences between one-sided and multi-sided markets (see the discussion in Rochet and Tirole, 2003, 2006): the (imperfect) internalization of network externalities.\footnote{Belleflamme and Peitz (2019b) obtain partial internalization by considering a mix of passive and reactive beliefs reflecting different information of consumers on prices.}

\section{Matching Design}

\subsection{Second-degree Price Discrimination and Matching Design}

Over the last few years the literature on platform markets has started evolving from models in which all agents on board are matched to all participating agents from the opposite side of the market (the case considered in the sections above) to models in which platforms engage in more sophisticated design, matching participating agents in a customized manner. This change in modeling reflects the observation that many platforms engage in discriminatory practices whereby they match different agents from the same side to different sets of agents from the opposite side of the market. For example, ad-exchanges match advertisers to only a subset of the content providers they have access to. The packages offered by most Cable TV providers are designed with the similar intent of price discriminating among the viewers by granting them access to different subsets of the channels. Likewise, Business-to-Business platforms typically match vendors to a subset of the procurers in their network. Similar discriminatory practices are used by video-game consoles when mediating the interactions between gamers and video-game developers, and by employment agencies when mediating the interactions between employers and job seekers.\footnote{Most of the literature on two-sided markets assumes that platforms price discriminate across sides but not within side. Price discrimination within the same side has been considered in the networks literature. See, for example, Bloch and Quérou (2013), Candogan, Bimpikis, and Ozdaglar (2012), Chen, Zenou, and Zhou (2018) and Fainmesser and Galeotti (2016). For models of price discrimination in two-sided markets see Halaburda and Yehezkel (2013), Reisinger (2014), Gomes and Pavan (2016, 2019), Belleflamme and Peitz (2020), and Jeon, Kim, and Menicucci (2021).}
The possibility of offering differential access to the other side of the market opens the door to matching design. In this subsection, we review some of the results in this literature, focusing on existing work where matching allocations are supported by payments. In addition to accommodating for price discrimination, the literature on matching design also allows for richer preference structures and for the possibility that agents differ not only in their preferences but also in their attractiveness, that is, in the utility they bring to those agents they are matched with.

Consider the baseline model of Section 2, where a single platform mediates the interactions between two sides of a market, but now assume that the interaction benefit that a side-\(i\) agent of type \(\theta_i\) derives from interacting with a side-\(j\) agent of type \(\theta_j\) depends on the latter agent’s type.

Think of the platform as offering to each side of the market a sophisticated tariff specifying the price asked to each agent for each possible set of matches with the agents from the other side of the market. The equilibrium outcomes induced by any pair of such tariffs can be conveniently described by a pair of matching rules \(s_i(\cdot))_{i=1,2}\) and a pair of payment rules \(p_i(\cdot))_{i=1,2}\) one for each side. The matching rule \(s_i\) specifies, for each type \(\theta_i \in \Theta_i\), the set of types \(s_i(\theta_i) \subseteq \Theta_j\) from the opposite side of the market that each side-\(i\) agent of type \(\theta_i\) is matched to. The payment rule \(p_i\) specifies for each side-\(i\) type \(\theta_i\) the total payment \(P_i(\theta_i)\) that type \(\theta_i\) makes to the platform, where the payment can be positive for some types and negative for others. As in other mechanism-design problems, such functions should be interpreted as describing the equilibrium allocations generated by the agents’ choices on each side of the market.

A feasible pair of matching rules \(\{s_i(\cdot))\}_{i=1,2}\) must satisfy the reciprocity condition according to which type \(\theta_j\) from side \(j\) is matched to type \(\theta_i\) from side \(i\) if and only if type \(\theta_i\) is in type \(\theta_j\)’s matching set, \(i, j = 1, 2, j \neq i\):

\[
\theta_j \in s_i(\theta_i) \implies \theta_i \in s_j(\theta_j). \tag{22}
\]

The utility that each side-\(i\) agent of type \(\theta_i\) derives from paying \(P_i(\theta_i)\) to the platform and being matched to a set of types \(s_i(\theta_i)\) from the opposite side is equal to \(u_i(s_i(\theta_i)|\theta_i) - P_i(\theta_i)\), with the real-valued function \(u_i\) describing the agent’s gross payoff.

As types are the agents’ private information, the matching and the payment rules must be individually rational and incentive compatible, meaning that each type \(\theta_i\) must prefer the allocation \((s_i(\theta_i), P_i(\theta_i))\) to his outside option (assumed to yield a payoff equal to zero) and to any other available allocation, in particular to the equilibrium allocation \((s_i(\theta'_i), p_i(\theta'_i))\) of any other type \(\theta'_i\).

To simplify things, we assume here that the costs to the platform of getting agents on board are equal to zero, so that \(c_i = 0, i = 1, 2\).

\[98\] This is in contrast to the rich literature examining the stability of matching allocations in environments without payments. See Roth and Sotomayor (1992) and Roth (2008) for a review of the earlier literature, and Kojima (2017) and Pathak (2017) for more recent developments. See also the book “Market Design” by Haeringer (2018) and the forthcoming book “Online and Matching-Based Market Design” [Marc: Do we need a cite here?] for a connection between the two literatures.

\[99\] Importantly, the analysis accommodates for the case where the matching rule is stochastic, in which case \(s_i(\theta_i)\) is a lottery over the collection of feasible subsets of \(\Theta_j\). We describe the rule as a deterministic here to facilitate the exposition.
6.1.1 One-to-One Matching

Consider first markets in which matching is one-to-one, meaning that each agent can be matched up to at most one agent from the opposite side. Also assume that the type of each agent is unidimensional and drawn from $\Theta_i = [\underline{\theta}_i, \bar{\theta}_i] \subset \mathbb{R}$ according to an atom-less distribution $F_i$, independently across agents.

The gross utility that each side-$i$ agent of type $\theta_i$ derives from interacting with a side-$j$ agent of type $\theta_j$ is equal to
\[
\phi_i(\theta_i, \theta_j),
\] (23)
where the function $\phi_i$ is strictly increasing in both arguments and supermodular. An example often considered in applications is
\[\phi_i(\theta_i, \theta_j) = \theta_i \theta_j.\]

To make things simple (but interesting), assume that the cost $\sigma$ that the platform incurs for each match it creates is such that it is inefficient to match pairs of agents with the lowest types, but it is efficient to match those with the highest types:
\[\sum_i \phi_i(\theta_1, \theta_2) < \sigma < \sum_i \phi_i(\bar{\theta}_1, \bar{\theta}_2).\]

A deterministic matching rule of special interest is the (truncated) positive assortative one. Formally, for any $U \in [0, 1]$, let $\mathcal{F}^{-1}_i(U) \equiv \inf \{\theta_i \in \Theta_i : F_i(\theta_i) \geq U \}$ be the generalized inverse cdf of the type distribution on side $i$. Say that the pair of matching rules $\{s_i(\cdot)\}_{i=1,2}$ is truncated positive assortative if, for each $i = 1, 2$, there exists a threshold $\hat{\theta}_i \in \Theta_i$ such that
\[
s_i(\theta_i) = \begin{cases} (\mathcal{F}_j)^{-1}(F_i(\theta_i)) & \text{if } \theta_i \geq \hat{\theta}_i \\ 0 & \text{if } \theta_i < \hat{\theta}_i. \end{cases}
\]

Hence, any agent from side $i$ with a type below $\hat{\theta}_i$ is excluded (that is, he is matched to nobody), whereas any side-$i$ agent with a type above $\hat{\theta}_i$ is matched in a positively assortative manner. That is, higher types from each side are matched to agents from the other side with higher types. The optimality of matching agents assortatively is a direct consequence of the gross payoffs $\phi_i$ being supermodular. Given the feasibility constraint imposed by one-to-one matching, positive assortativeness requires that an agent whose type $\theta_i$ occupies the $n$th quantile on one side be matched with an agent from the opposite side whose type occupies the same quantile.

Next, let
\[
\hat{\phi}_i(\theta_i, \theta_j) \equiv \phi_i(\theta_i, \theta_j) - \left(1 - F_i(\theta_i) \right) \frac{\partial \phi_i}{\partial \theta_i}(\theta_i, \theta_j)
\]
denote the “virtual” utility that type $\theta_i$ derives from being matched to type $\theta_j$. This is the true utility $\phi_i(\theta_i, \theta_j)$, adjusted by a term that controls for the cost to the platform of matching the pair $\theta_i$ and $\theta_j$, due to the informational rents that the platform must leave to those agents from side $i$ with a type above $\theta_i$.

When the virtual utility functions $\hat{\phi}_i$ are also supermodular, on both sides, one can show that both the welfare-maximizing and the profit-maximizing matching rules are truncated positive assortative (variants of this result appear in Damiano and Li (2007), Johnson

\[\text{In addition, the function is typically assumed to be differentiable and equi-Lipschitz continuous which guarantees that the the equilibrium payoffs are well behaved.}\]

\[\text{Feasibility also requires that } \hat{\theta}_j = F_j^{-1}(F_i(\hat{\theta}_i)).\]
(2013), and Galichon (2018) – see also Gomes and Pavan (2019) for an alternative and unifying proof). That is, when the virtual utility functions \( \hat{\phi}_i \) satisfy the same supermodularity properties of the true utility functions \( \phi_i \), the matching partner of any participating agent under profit maximization is the same as under welfare maximization. The set of participating agents under profit maximization though need not coincide with the efficient set: A profit-maximizing platform excludes an inefficiently large set of agents from both sides of the market: \( \hat{\theta}_i^* \geq \theta_i^e, \ i = 1, 2 \), where \( \hat{\theta}_i^* \) is the threshold type under profit maximization and \( \theta_i^e \) is the threshold type under welfare maximization.\(^{102}\) The above result, however, hinges on the virtual utility functions \( \hat{\phi}_i \) being supermodular.\(^{103}\) When the true utility functions are supermodular but their virtual analogs are not, profit-maximization, in addition to excluding too many agents, may lead to inefficient matching of the participating agents.

Finally, one can show that, both in the case of profit maximization as well as in the case of welfare maximization, the total payment made by each side-\( i \) agent of type \( \theta_i \) to the platform is equal to

\[
P_i(\theta_i) = \phi_i(\theta_i, s_i(\theta_i)) - \int_{\hat{\theta}_i}^{\theta_i} \frac{\partial \phi_i}{\partial \theta_i}(\hat{\theta}_i, s_i(\hat{\theta}_i))d\hat{\theta}_i,
\]

all \( \theta_i \in \theta_i, \ i = 1, 2. \(^{104}\)

### 6.1.2 Many-to-many matching

Now suppose that matching is many-to-many, meaning that each agent from each side may be matched to many agents from the other side of the market, with the only constraint being that the matching be reciprocal (that is, the resulting matching rules satisfy Condition (22)). As in the previous subsection, let \( \Theta_i = [\theta_i, \overline{\theta}_i] \subseteq \mathbb{R} \). Assume that agents with a positive type like interacting with agents from the opposite side, whereas the opposite is true for agents with a negative type. To avoid trivial cases, assume that \( \overline{\theta}_i > 0 \) for some \( i \in \{a, b\} \).

In addition to parametrizing an agent’s preferences, suppose that each agent’s type also parametrizes the utility the agent brings to the other side. To capture such a possibility in the simplest possible way, suppose that there exists a function \( \sigma_i : \Theta_i \rightarrow \mathbb{R}_+ \) describing the “salience” of each side-\( i \) agent of type \( \theta_i \). An agent of higher salience contributes more to the utility of those agents from the other side who like interacting with the opposite side and generates higher disutility to those who dislike interacting with the other side. For example, in the case of advertising, an ad of higher salience is one delivering a higher utility to those viewers or readers who like advertisement and a larger disutility to those who dislike it.

For any set of type \( s \subset \Theta_j \), let

\[
|s|_j \equiv \int_{\theta_j \in s} \sigma_j(\theta_j)dF_j(\theta_j)
\]

\(^{102}\)These threshold solve \( \hat{\phi}_1(\hat{\theta}_1^*, \hat{\theta}_2^*) + \hat{\phi}_2(\hat{\theta}_1^*, \hat{\theta}_2^*) = \sigma \) and \( \phi_1(\bar{\theta}_1^*, \bar{\theta}_2^*) + \phi_2(\bar{\theta}_1^*, \bar{\theta}_2^*) = \sigma \), respectively.

\(^{103}\)When \( \phi_i(\theta_i, \theta_j) = \theta_i \theta_j \), supermodularity of \( \hat{\phi}_i(\theta_i, \theta_j) \) is equivalent to the virtual values \( \theta_i - \left( \frac{1-F_k(\theta_i)}{f_k(\theta_i)} \right) \) being strictly increasing (see, e.g., Myerson, 1981).

\(^{104}\)The result follows from usual envelope-theorem-type of arguments standard in mechanism design.
denote the “total salience” of the set \( s \). Assume that the gross utility \( u_i(s|\theta_i) \) that any side-\( i \) agent of type \( \theta_i \) obtains from being matched to all agents from the opposite side whose type is in \( s \subset \Theta_j \) depends only on the agent’s type and on the set’s total salience and takes the form

\[
u_i(s|\theta_i) = \theta_i \cdot g_i \left( |s|_j \right).
\]

(25)

The function \( g_i(\cdot) \) is positive, strictly increasing, continuously differentiable, and satisfies \( g_i(0) = 0 \). Note that the above specification admits as a special case the additive preference structure considered in most of the discussion in the previous sections, which is nested as \( \theta_i = \gamma_i \), \( \sigma_j(\theta_j) = 1 \) all \( \theta_j \), and \( g_i(x) = x, i, j = 1, 2, j \neq i \). In other words, the structure considered in the previous sections corresponds to a market in which salience is constant across types (in which case agents care only about the measure of agents they interact with and not the type of such agents) and where preferences are linear in the size of the matching set. The more flexible specification in (25) accommodates for the possibility that different agents contribute differently to the utility they bring to the other side, and that the incremental value of adding a marginal agent to an agent’s matching set depends on the salience (or size) of the latter agent’s matching set. The case where the functions \( g_i(\cdot), i = 1, 2 \), are concave corresponds to the case where agents experience decreasing returns to scale to match quality, whereas the case where these functions are convex corresponds to the case of increasing returns to scale.

However, even when the functions \( g_i \) are linear and salience is constant across types, an important point of departure from the analysis in the previous sections is that the platform is not restricted to matching each agent on board to all agents on board from the other side. Instead, the platform may engage in second-degree price discrimination by charging different payments for different matching sets (when preferences take the special structure as in the previous sections, this means that total payments need not be linear in the number of interactions with the other side).

Finally, note that the multiplicative specification in (25) permits one to retain the usual vertical-differentiation structure from the price-discrimination literature (e.g., Mussa and Rosen, 1978) and conveniently interpret an agent’s type as the (positive or negative) value the agent assigns to interacting with agents from the opposite side.

To make things simple, assume that the virtual values \( \theta_i - [1 - F_i(\theta_i)]/f_i(\theta_i) \) are nondecreasing in \( \theta_i \), as typically assumed in the monopolistic screening literature. Finally, assume that \( c_i = 0 = \sigma \) so that there are no costs to getting agents on board and/or to match any pair of agents.

Say that a matching rule is a threshold rule if there exits a pair of non-increasing functions \( t_i : \theta_i \rightarrow \Theta_j \), along with threshold types \( \tilde{\theta}_i \in \Theta_i \), such that, for any \( \theta_i \in \Theta_i, i = 1, 2 \), the equilibrium matching set takes the form

\[
s_i(\theta_i) = \begin{cases} 
[t_i(\theta_i), \overline{\theta}_j] & \text{if } \theta_i \geq \tilde{\theta}_k \\
\emptyset & \text{otherwise},
\end{cases}
\]

with \( t_i(\theta_i) = \min\{\theta_j : t_j(\theta_j) \leq \theta_i\} \) for all \( \theta_i \in [\tilde{\theta}_i, \overline{\theta}_i] \).

Thus, under a threshold rule, any type below \( \tilde{\theta}_i \) is excluded, while any type \( \theta_i > \tilde{\theta}_k \) is matched to any agent from the other side whose type is above the threshold \( t_i(\theta_i) \). Finally,
the condition \( t_i(\theta_i) = \min\{\theta_j : t_j(\theta_j) \leq \theta_i\} \) means that the pair of rules \( s_i(\cdot), i = 1, 2, \)
satisfies the reciprocity condition (22).

Gomes and Pavan (2016) show that, both in the case of profit maximization and in the
case of welfare maximization, a threshold rule is optimal provided one of the following two
sets of conditions holds:\(^{105}\)

(a) the functions \( g_i(\cdot) \) are weakly concave and the functions \( \sigma_i(\cdot) \) are weakly increasing, \( i = 1, 2; \)

(b) the functions \( g_i(\cdot) \) are weakly convex and the functions \( \sigma_i(\cdot) \) are weakly decreasing, \( i = 1, 2. \)

In markets satisfying one of the above two sets of conditions, the tariffs that maximize
profits (alternatively, welfare) thus induce a form of negative assortativeness at the margin:
those agents with a lower value for interacting with the opposite side are matched only to
to those agents from the opposite side whose value for matching is large enough. Furthermore,
the matching sets of any pair of agents from the same side are ranked, in the sense that
one is a superset of the other. The above properties reflect the optimal way platforms
subsidize interactions in markets in which they can engage in price discrimination (across
and within sides). Importantly, note that the above conditions are trivially satisfied by the
linear specification considered in the previous sections. Gomes and Pavan (2016) also show
that the above conditions are “almost necessary” in the sense that, when violated, one can
identify cases in which either the profit-maximizing or the welfare-maximizing tariffs fail to
induce matching sets with a threshold structure.\(^{106}\)

The intuition for the above result is the following. Consider a welfare-maximizing platform
(the problem for a profit-maximizing platform is similar, once values are replaced with
their “virtual” analogs). Suppose that the market satisfies the conditions in scenario (a).
Clearly, efficiency requires that any agent from side \( i \) whose type \( \theta_i \) is positive be matched
to any agent from the opposite side whose type is non-negative. However, efficiency may
also require some cross-subsidization. That is, the platform may want to assign to a side-\( i \) agent with type \( \theta_i > 0 \) a matching set of total intensity

\[
|s_i(\theta_i)|_j > \int_{[0, \bar{\theta}_j]} \sigma_j(\theta_j) dF_j(\theta_j).
\]

This requires matching the agent also to some of the side-\( j \) agents who dislikes interacting
with the side-\( i \) agents. When \( g_i(\cdot) \) are weakly concave and \( \sigma_i(\cdot) \) are weakly increasing,
\( i = 1, 2, \) and the agents’ types are the agents’ private information (all these assumptions
are important), the least costly way to deliver such matching intensity is to match type
\( \theta_i \) to all agents from the opposite side whose type is large enough (i.e., above a threshold
\( t_i(\theta_i) \) satisfying \( \int_{[t_i(\theta_i), \bar{\theta}_j]} \sigma_j(\theta_j) dF_j(\theta_j) = |s_i(\theta_i)|_j \)). This is because the side-\( j \) agents with
the highest type are the most attractive ones (by virtue of the monotonicity of the salience
functions \( \sigma_j \)) and because using the same side-\( j \) agents intensively to deliver high mach

\(^{105}\)The role of the convexity/concavity of the \( g_i \) functions is to guarantee the optimality of threshold rules.
It has nothing to do with the optimality/suboptimality of stochastic rules.

\(^{106}\)The analysis in Gomes and Pavan (2016) is however more general and accommodates for the possibility
that the agents’ salience \( \sigma_i \) is a stochastic function of the agents’ match values \( \theta_i \) and that the agents have
private information about both \( \theta_i \) and \( \sigma_i. \)
quality to the side-$i$ agents who ask for high match quality is less costly than using different agents from side $j$ (by virtue of the $g_{j}\cdot\cdot$ functions being concave).

A similar logic applies to markets satisfying the conditions in scenario (b). That $g_{i}\cdot\cdot$ are weakly convex, and $\sigma_{i}\cdot\cdot$ weakly decreasing, on both sides, along with the fact that types are private information, implies that the most profitable way of using any agent from side $i$ with a negative type $\theta_{i} < 0$ is to match the agent to those agents from side $j$ with the highest positive type. This is because such side-$j$ agents are those that benefit the most from interacting with the side-$i$ agent and because such agents are the least salient ones and hence exert the smallest negative externalities on type $\theta_{i}$.\textsuperscript{107}

We conclude by discussing possible distortions in the provision of matching services due to market power. For this purpose, let $\nu_{j}(\theta_{i}) \equiv g'_{j}\left([|\theta_{i}, \bar{\theta}_{j}|_{j}\right) \cdot \sigma_{i}(\theta_{i})$ be type $\theta_{i}$’s marginal contribution to the total salience of a matching set that contains all side-$i$ types above $\theta_{i}$. That is, for any side-$j$ type $\theta_{j}$, $\theta_{j}\nu_{j}(\theta_{i})$ is the marginal value of expanding the matching set of type $\theta_{j}$ starting from $[\theta_{i}, \bar{\theta}_{i}]$. Assume that the functions

\[
\psi_{i}^{W}(\theta_{i}) \equiv \frac{\theta_{i}}{\nu_{j}(\theta_{i})} \quad \text{and} \quad \psi_{i}^{P}(\theta_{i}) \equiv \frac{\theta_{i} - \frac{1 - F_{i}(\theta_{i})}{f_{i}(\theta_{i})}}{\nu_{j}(\theta_{i})}
\]

are strictly increasing, $i = 1, 2$. Gomes and Pavan (2016) refer to this property as “matching regularity”. To interpret the condition, take the case of profit-maximization. The numerator in $\psi_{i}^{P}(\theta_{i})$ is type $\theta_{i}$’s “virtual type”. This term captures the effect on the platform’s profits of expanding the salience of type $\theta_{i}$’s matching set (accounting for the extra surplus that the platform must leave to all side-$i$ agents with type above $\theta_{i}$, as required by incentive compatibility). The denominator of $\psi_{i}^{P}(\theta_{i})$, instead, captures the effect on the platform’s profit of adding type $\theta_{i}$ to the matching set of any side-$j$ agent $\theta_{j}$ whose matching set is $[\theta_{i}, \bar{\theta}_{i}]$ (i.e., whose threshold is given by $t_{j}(\theta_{j}) = \theta_{i}$). The condition then requires that the contribution of each side-$i$ agent to the platform as a “consumer” increases faster with the agent’s type $\theta_{i}$ than his contribution as an “input.” This condition is thus the analog of Myerson (1981)’s regularity condition in a two-sided many-to-many matching environment. As in standard screening problems, the role of the match regularity condition is to rule out bunching. Under such a condition, in a matching environment, bunching occurs only at the bottom of the distribution where it takes the form of exclusion, or at the very top of the distribution where agents are matched to all agents on board from the other side of the market.

Gomes and Pavan (2016) then show that when, in addition to the conditions guaranteeing the optimality of a threshold rule, the agents’ utilities satisfy the above match regularity condition, then, relative to the welfare-maximizing rule, the profit-maximizing rule (a) always excludes a larger group of agents and (b) matches each agent who is on board to a subset of his efficient matching set.

The result follows directly from the fact that a profit-maximizing platform internalizes the effects of cross-subsidization on marginal revenues (which are proportional to virtual

\textsuperscript{107} That the side-$j$ agents with the highest type are those that benefit the most from interacting with the side-$i$ agent follows from the fact that such agents necessarily have matching sets with the highest matching intensity – as required by incentive compatibility. Because of the convexity of the functions $g_{j}\cdot\cdot$, such agents thus have the highest marginal utility from interacting with the side-$i$ agents.
values \( \theta_i - [1 - F_i(\theta_i)] / f_i(\theta_i) \), rather than their effects on marginal welfare (which are proportional to the true values \( \theta_i \)). Because the former values are always smaller than the latter (reflecting the extra costs due to informational rents), the matching sets of all agents are a subset of the efficient sets.

Note that, contrary to other mechanism design problems, the matching sets of all types may be distorted, including those types \( \bar{\theta}_i \) at the “top” of the distribution. The reason is that the cost of cross-subsidizing such types is strictly larger under profit maximization than under welfare maximization, due to the infra-marginal losses imposed by reciprocity on the opposite side.\(^{108}\)

Strong market power in such markets may thus result in the complete exclusion of many agents from either side of the market, and/or to inefficient matching, whereby those agents on board are either matched to the wrong partners (in the case of one-to-one matching) or to a subset of their efficient matching set (in the case of many-to-many matching). Such distortions call for regulation and government interventions, an area that is receiving growing attention in recent years.

### 6.2 Targeting and Third-Degree Price Discrimination

The result about the optimality of threshold rules in Gomes and Pavan (2016) assumes that any pair of agents from side \( i \) may disagree on the relative attractiveness of any two agents from side \( j \) only when the first agent from side \( i \) likes interacting with the side-\( j \) agents (formally, has a positive type \( \theta_i \)) whereas the second agent dislikes it (formally, has a negative type \( \theta_i \)). This is a direct consequence of the assumed multiplicative structure. The above property is a good representation for markets in which preferences are differentiated only along a vertical dimension. In a recent paper, Gomes and Pavan (2019) show that the optimality of threshold rules extends to various models where agents’ preferences are both vertically and horizontally differentiated, under appropriate regularity conditions.\(^{109}\)

To see this, suppose each agent’s type is now bi-dimensional \( \theta_i = (x_i, v_i) \). The first component, \( x_i \), is the agent’s “location,” whereas the second component, \( v_i \), is the agent’s “value for matching,” that is, the overall importance the agent assigns to interacting with agents from the opposite side of the market. The component \( x_i \) thus parametrizes heterogeneity along a horizontal dimension, whereas the component \( v_i \) parametrizes heterogeneity along a vertical dimension.

For example, in the case of cable TV, the vertical dimension may capture the overall importance that a viewer assigns to cable TV, or the overall importance that a channel assigns to reaching viewers, reflecting the channel’s expected advertising revenue as well as possible costs stemming from broadcasting rights. In turn, a viewer’s location captures the viewer’s tastes for different types of programming, whereas a channel’s location proxies the type of content broadcasted by the channel.

Gomes and Pavan (2019) assume that the vertical dimensions are the agents’ private information. As for the locations, they consider cases where they are publicly observable as

\(^{108}\) A similar result appears in Jeon et al. (2021).

\(^{109}\) See also Valenzuela-Stookey (2020) where it is shown that such rules are also optimal in certain markets combining across-side network effects with within-side network effects (e.g., congestion or, more generally, within-side agents’ competition).
well as cases where they are the agents’ private information. In the cable TV application, for example, each viewer’s ideal type of broadcasting is likely to be his own private information, whereas each channel’s broadcasting profile is publicly observable. The analysis in Gomes and Pavan (2019) then shows that, under appropriate conditions, the optimal matching rule continues to have a threshold structure, but with a threshold \( t_i(v_i; x_i, x_j) \) that depends on the involved agents’ locations (a form of third-degree price discrimination). The paper then uses the result to study the effects on matching allocations and welfare of various regulations prohibiting platforms from engaging in third-degree price discrimination (see also Belleflamme and Peitz, 2020, for an alternative model of price discrimination in platform markets).\(^{110}\)

Finally, the paper shows how the analysis of the effects of such regulations can also shed light on the effects of a transition from a centralized structure whereby matching is mediated by platforms to a decentralized structure whereby one side (typically the seller side) sets prices and the other side then chooses the composition of the matching sets. The equivalence between such a decentralized structure and a centralized one subject to the aforementioned non-discriminatory restrictions follows from the fact that, in a decentralized market, sellers cannot bundle their products with other sellers’ products. Because bundling is what permits platforms to condition the composition of the matching sets on locations, when the latter are private information, the matching allocations induced by a profit-maximizing platform when third-degree price discrimination (alternatively, bundling) is banned are similar to those in a decentralized market.

6.3 Dynamic arrivals and evolving private information

Another question that is receiving growing attention in the matching-design literature is how platforms respond to the dynamic arrival of information and/or of new agents by adjusting the matching allocations they propose to agents from different sides of the market.

In a couple of recent papers, Fershtman and Pavan (2017, 2020) consider dynamic matching markets in which agents arrive stochastically over time and experience shocks to their match values for specific agents from the other side of the market. Such shocks can be interpreted as reflecting the arrival of new information or variations in the environment that alter the attractiveness of the potential interactions. As a result of such changes, agents on both sides of the market are frequently re-matched. Both the agents’ arrivals and the shocks the agents experience to their matching valuations are the agents’ private information. Agents face capacity constraints that limit the number of interactions they can entertain in each period. In addition, the platform may also face constraints on the total number of interactions it can accommodate within each period. Such limits may reflect time, resource, or facility constraints, but also capture certain non-separabilities and decreasing returns to scale in the agents’ preferences.

The combination of the agents’ private information with the aforementioned capacity constraints suggests that platforms could benefit from using auctions to determine the

\(^{110}\) The paper develops tools that one can use to derive the optimal matching plans in the presence of such regulatory restrictions. The difficulty is that these restrictions do not allow one to use standard results from mechanism design to represent the payments as a function of the matching allocations. The reason is that such regulations impose specific restrictions directly on the structure to the implementing tariffs.
relevant matches (e.g., Martens, 2016, and Pinker, Seidmann, and Vakrat, 2003, argue that search rankings and price auctions will soon become the main tools to facilitate online matching). However, standard auction formats used for the sale of physical goods or services (e.g., first-price, second-price, English, clock, and double auctions) do not appear appropriate for such matching markets in which agents play the double role of buyers and inputs and where the match values are expected to evolve frequently over time and, as a result, agents need to be often re-matched.

Fershtman and Pavan (2017, 2020) show that, in many such markets, the profit-maximizing matches can be induced by using appropriate “matching auctions,” specifically designed for such markets. Upon arrival, agents are asked to select a “membership status” which is then used to determine the weight assigned to the agents’ bids. In each subsequent period, each agent is then asked to submit a bid for each possible agent on board from the other side of the market, with the identity of such agents disclosed by the platform. Each bilateral match is then assigned a “score” that depends on the involved agents’ reciprocal bids, their membership status, and the platform’s cost of implementing the match. The matches maximizing the sum of the bilateral scores are implemented in each period, and each agent is charged a total payment that reflects the “externality” the agent imposes on others (by altering the composition of other agents’ matching sets). Importantly, such externalities are adjusted to account for the platforms’ cost of leaving information rents to the agents.

When the arrival of new information is endogenous (as is the case when agents learn the attractiveness of their partners by interacting with them), the score assigned to each match resembles a Gittins “index”, and accounts for the option value of generating new information to be used in subsequent periods. Despite the complexity of the environment and the discrepancy between the agents’ and the platform’s value for experimentation, the proposed matching auctions admit simple profit-maximizing equilibria in which bidding is straightforward and myopic in each period.

The paper then uses the model to contrast matching dynamics under profit maximization to their counterparts under welfare maximization. It shows that, when all agents like interacting with all other agents from the opposite side of the market and only the aggregate capacity constraints possibly bind, profit maximization involves fewer and shorter interactions than welfare maximization. This conclusion, however, need not extend to environments where some agents dislike certain interactions and/or where agents’ individual capacity constraints limit the number of matches that each agent can be involved in at each period.

111 Online ad exchanges such as Google’s DoubleClick and Microsoft’s Exchange already use auctions to match advertisers with content providers. Advertisers bid repeatedly over time to place their ads on the website of multiple content providers and, over time, content providers ask different ad-specific prices to display the ads (see, e.g., Mansour, Muthukrishnan, and Nisan, 2012. See also the book “Market Design: Auctions and Matching” by Haeringer, 2018).

112 These scores play a role similar to that played by the “compatibility scores” used by ad exchanges to select matches between advertisers and content providers (see, e.g., Moghaddam and Nof, 2017).
7 Empirical analysis of two-sided markets: Identification

The previous sections built up theoretical models of indirect network effects to study pricing equilibria and the implications for profit and welfare. In this section, we focus on empirical analysis of these market phenomena. We start with the question of how to estimate the strength of a direct network effect. The preceding sections focused on indirect network effects almost exclusively. However, from an empirical perspective, it will be useful to begin with direct network effects. In some ways, it will be an easier context in which establish ideas about identification. Also, some empirical papers that study indirect network effects settings actually model them as direct network effects, often because of lack of data. For instance, consumers may value an operating system based on how many applications it has but a researcher that lacks data on the application market may model consumers as making choices based on the installed base of the operating system. The researcher relies on the close association between installed base and application availability.

Another reason to study direct network effects is that direct network effects are more easily comparable to other fields of economics. A central goal of this section is discuss identification of direct network effects from the perspective of the social economics literature, as in Durlauf and Ioannides (2010), Benhabib, Bisin, and Jackson (2011), Angrist (2014) and Graham (2018). This line of research has made substantial progress on the identification and estimation of neighborhood effects and social spillovers, which are modeled in similar ways to direct network effects. Our goal is to utilize the results from the social economics literature to gain insight into the estimation of network effects. Doing so provides perspective on estimating network effects and, in cases where the social economics literature has used approaches that the empirical network effects literature has not, provides new avenues for research. We are particularly interested in the reflection problem of Manski (1993). We discuss the identification of direct and indirect network effects from this perspective.\footnote{An early version of these ideas appears in Rysman (2019), which covered only Manski (1993) and Manski (1995). This chapter covers the large literature on the reflection problem that has followed, as well as a wider set of empirical papers.}

The starting point for the empirical social economics literature is a linear-in-means model that is best represented by continuous choices. The theoretical treatment in the previous sections discussed primarily discrete choices by consumers, such as whether to adopt a technology or not. We return to this question below, but for now, we adjust the presentation of the theoretical model to generate a continuous choice.

7.1 Direct Network Effects

In this section, we present a simple model of network effects and discuss issues with identifying the network effect in empirical settings. We start with a model with no consumer heterogeneity. We add consumer heterogeneity to the model and then add a contextual effect, further discussed below.

7.1.1 Base Model

We start with the case of direct network effects. Consider a partition of agents into markets $m = 1, \ldots, M$. Each market has one network and agents choose how much to use the
network in their market. Markets play the role of neighborhoods and determine which agents interact, so network effects operate only within a market. Market can be thought of as distinct networks. Each market $m$ has an integer-valued number of agents (which differs from the continuum of agents assumed in the models in the previous sections). Each agent makes a continuous choice $a \subset \mathbb{R}$ that incurs a constant marginal price of $P_m$. The optimal choice for each agent is:

$$a = \beta_0 - \beta_1 P_m + \beta_2 q_m + \gamma_m + u$$

(26)

The variable $q_m = \bar{a}$ is the average choice of agents in the same market. In the parlance of the reflection problem, $\beta_2$ represents the endogenous effect, or what we refer to as the direct network effect. We may observe many markets, each with a different $P_m$ and thus a different $q_m$. We assume that $E[u] = 0$ in each market $m$. As there are a finite number of agents, draws of $u$ will realize some mean $\bar{u}_m \neq 0$. We suppress that here, but we can think of it as part of $\gamma_m$.

A primary concern in both network effects papers and social economics papers is addressing omitted variables or correlation across agents in the same market in unobserved terms. These may arise for a variety of reasons, such as unobserved shocks to a market or sorting by agents with similar unobserved terms into the same markets. These omitted variables are captured by $\gamma_m$, and Manski (1993) terms $\gamma_m$ as correlated effects. These terms are typically treated as a nuisance to be controlled for or to be eliminated by experimental design. We argue in Section 7.3 that while this makes sense for direct network effects, common approaches to indirect network effects turn the focus of interest onto the correlated effects rather than the endogenous effect.

An equally difficult challenge is addressing simultaneity in choices. Because $u$ affects the agent’s choice, and the agent’s choice affects neighbors’ choices, $u$ is correlated with $q_m$. However, in this simple model, a more significant problem arises than omitted variables or simultaneity. In particular, we recognize that $q_m$ can be derived by taking the mean of Equation 26:

$$q_m = \beta_0 - \beta_1 P_m + \beta_2 q_m + \gamma_m.$$  

(27)

Because the left-hand side equals $q_m$ by definition, we rewrite this as:

$$q_m = \frac{\beta_0 - \beta_1 P_m + \gamma_m}{1 - \beta_2}.$$

Plugging back into Equation 26 leads to the reduced-form of our model:

$$a = \frac{\beta_0 - \beta_1 P_m + \gamma_m}{1 - \beta_2} + u$$

We have more parameters than regressors and thus, the parameters are unidentified. As is well-known, estimating the regression in Equation 26 is guaranteed to find $\beta_2 = 1$.\textsuperscript{114} That negative result holds even if we eliminated correlated effects by assuming $\gamma_m = 0$ for all $m$.

\textsuperscript{114}Angrist (2014) argues that because we can estimate Equation 26 and find a parameter for $\beta_2$, the parameter is identified (in some sense), but the estimand is not informative about the underlying model.
What is the economic problem? Intuitively, suppose that we observe the price decrease on a platform. The reduced price leads agents to use the platform more as governed by $\beta_1$, a standard demand effect. The increased use makes the platform more valuable to agents and thus leads to increased usage as governed by $\beta_2$, the network effect. But all we observe in data is that price went down and quantity went up, and we cannot distinguish how much of that increase was due to shifts along the demand curve ($\beta_1$) versus the demand curve shifting out ($\beta_2$).

One might think that this relationship could be broken by calculating the mean of neighbors’ choice only among other neighbors, so that the it varies across each agent. However, note that the equation already includes $a$ and $q$, which are sufficient (along with the number of agents) to construct the leave-one-out mean. In this sense, the leave-one-out mean does not introduce any new information. Angrist (2014) is particularly critical of using the leave-one-out mean.

7.1.2 Consumer heterogeneity

If we observe individual-level explanatory variables, the situation is somewhat improved. Suppose that agents have a type $\nu$ that describes some demographic variables such as income or perhaps an individual-specific platform tax or subsidy. Our new version of Equation 26 is:

$$a = \beta_0 - \beta_1 P_m + \beta_2 q_m + \beta_3 \nu + \gamma_m + u.$$

(28)

Then, $q_m$ is:

$$q_m = \frac{\beta_0 - \beta_1 P_m + \beta_3 \nu_m + \gamma_m}{1 - \beta_2}.$$

(29)

Thus, the reduced-form is:

$$a = \frac{\beta_0 - \beta_1 P_m + \beta_2 \beta_3 \nu_m + \gamma_m}{1 - \beta_2} + \beta_3 \nu + u.$$

Because $\beta_3$ appears by itself in front of $\nu$, it could be identified, which gives the key to separating out the elements in the fraction. This is at least somewhat more promising than the result without $\nu$.

However, the regression in Equation 28 still faces the simultaneity problem. If possible, we would replace $q_m$ in Equation 28 with $E[a|m]$, that is, by computing the expected average choice for the market by integrating out $u$ rather than using the average of the observed choices. Equation 29 provides a path to doing so. Interpreting the equation as a first-stage prediction of the endogenous variable in a two-stage least squares estimator, $\nu_m$ predicts the endogenous variable but is otherwise excluded from the agent’s decision. That is, exogenous characteristics of other agents in the market provide instruments for their choices that can be used for identification.

This regression still faces the problem that $P_m$ and $\nu_m$ must be distinguished from the correlated effects $\gamma_m$. If $\gamma_m$ are addressed with market fixed effects, we have a collinearity

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115Note that doing so in the problem without $\nu$ implies that $q_m$ is perfectly collinear with $P_m$, and so does not lead to identification.
problem leading to a lack of identification. If we were willing to restrict the flexibility of these effects, identification follows. For instance, the presentation of Manski (1993) assumes that $\gamma_m$ is a linear function of observable market characteristics. Intuitively, unobserved market-level shocks make interpreting the effect of $q_m$ impossible, and so one approach is to restrict those shocks in some way.

7.1.3 Contextual effects

Relative to the social economics literature, it is important to keep in mind that we have so far ruled out one important effect. In the presentation of Manski (1993), $\nu_m$ enters directly into Equation 28 as a variable of interest, i.e.:

$$a = \beta_0 - \beta_1 P_m + \beta_2 q_m + \beta_3 \nu + \beta_4 \nu_m + \gamma_m + u.$$  

The parameter $\beta_4$ measures the contextual effect and represents the response of an agent directly to the characteristics of others in the market separately from the effect of their choices $a$. The contextual effect is often of major interest in social economics studies. For instance, researchers might be interested in the effect on the performance of students in a classroom from having a higher proportion of students from an underrepresented minority population, or from having very wealthy classmates. Research on racial segregation in friendship networks or in housing sorting sometimes focuses almost entirely on the contextual effect and largely ignores the endogenous effect. Graham (2018) discusses this line of research.

In contrast, we are not aware of network effects papers that allow for a contextual effect or attempt to address it. It certainly could be an issue. For instance, we might think that technology adoption by a household depends not only on whether neighborhood households adopt (Goolsbee and Klenow, 2002, has a structure like this) but also on whether neighbors have high income or white-collar jobs. Neighbor demographics might affect the likelihood of hearing about a new technology or the value of the technology if it allowed the household to sell items to neighbors.

It is clear from the modeling above that adding the contextual effect will create significant problems. Going through the steps above leads to:

$$a = \frac{\beta_0 - \beta_1 P_m + (\beta_2 \beta_3 + \beta_4) \nu_m + \gamma_m}{1 - \beta_2} + \beta_3 \nu + u.$$  

Again, this equation allows us to learn $\beta_3$ but that is not sufficient to unpack the parameters in the fraction because $\beta_4$ also sits in front of $\nu_m$. Thus, we cannot distinguish between the endogenous effect and the contextual effect. This identification problem is the heart of the reflection problem, which Manski (1993) describes as the difficulty in determining causality when the behavior of agents is reflected in their neighbors.\footnote{Manski (1993) writes (pg. 532): “This paper examines the ‘reflection’ problem that arises when a researcher observing the distribution of behaviour in a population tries to infer whether the average behaviour in some group influences the behaviour of the individuals that comprise the group. The term reflection is appropriate because the problem is similar to that of interpreting the almost simultaneous movements of a person and his reflection in a mirror. Does the mirror image cause the person’s movements or reflect them? An observer who does not understand something of optics and human behaviour would not be able to tell.”}
Although we cannot generally separate between the contextual and endogenous effects, we can check if the composite term is equal to zero.\footnote{Blume, Brock, Durlauf, and Ioannides (2011) discuss how the contextual effect could be some alternative function of group characteristics other than the mean, which can lead to identification.} If we find that it is not, we can conclude that there is some social effect, either endogenous or contextual. That is the approach of a number of social economics papers. For instance, Angrist and Lang (2004) measure the effect of the number of minority students in a classroom on other students without distinguishing whether the effect is driven by the presence of those students or by the performance of those students.

Because network effects papers do not seem to have considered the contextual effect, we do not allow for a contextual effect for the rest of this chapter, but it is worth considering whether the contextual effect might be important in some circumstances. The focus on contextual effects in the social economics literature suggests it might be relevant in the network effects literature.

### 7.2 Some solutions for direct network effects

The social economics literature has made progress on finding settings that lead to identification, and we review several approaches here. We also consider whether these approaches have been considered in the network effects literature and how that might be extended.

#### 7.2.1 Random assignment

Perhaps the most substantial focus has been on random assignment to markets. Reviews of these approaches are in Sacerdote (2014) and Graham (2018). A well-known example is Sacerdote (2001), which relies on random matching of college roommates to study the effects of roommate characteristics on GPA. Another example is Angrist and Lang (2004), which studies the effect of a program that assigns students from urban schools to suburban schools. The implementation of the program generates randomness in the number of urban students assigned to each class.

The basic idea of random assignment is that it should eliminate self-selection into the market. If selection is the only source of variation in $\gamma_m$, random assignment can eliminate $\gamma_m$. Note that if we are considering a contextual effect, eliminating $\gamma_m$ does not allow us to separately identify $\beta_2$ and $\beta_4$ in Equation 30. Sacerdote (2001) estimates the overall effect of student characteristics on roommate outcomes. Thus, Sacerdote estimates a reduced-form coefficient that could be positive either because of the endogenous effect or the contextual effect ($\beta_2$ or $\beta_4$), both of which represent social effects.

Unfortunately, random assignment does not obviously set $\gamma_m = 0$. Recall that $\gamma_m$ can represent not only sorting and self-selection but also any common market shocks. For instance, college roommates may be jointly affected by their residence counselor or room location. This causes Sacerdote to focus only on the effect of characteristics that are determined before the students start at college. For example, even in the case of random roommate matching, regressing GPA on the fraternity membership status of a roommate may be subject to omitted variable bias because GPA and fraternity membership for roommates may be affected by common shocks. However, regressing GPA on a roommate’s...
parent income or distance to their household can be regarded as exogenous due to random roommate matching.

Similarly, Angrist and Lang (2004) focus on the effect of the number of students assigned to a class (a predetermined number) rather than the effect of student performance (an outcome) on one and another. We are not aware of papers in the network effects literature that have been implemented in the same way as Sacerdote (2001) or Angrist and Lang (2004). Given that contextual effects are not of primary interest in network effects models, the focus on contextual effects due to pre-determined heterogeneity is likely not particularly attractive. That is, randomly assigning agents to markets does not allow us to be sure an endogenous effect exists because of common market shocks, and in this sense, is of limited value for identifying a network effect.

Unlike typical social economics problems, network effects often operate through the size of the network rather than just the average behavior of the people in the network. As a result, even when randomly matching agents to markets of the same size (like the roommates or classroom examples) may be of limited value, randomly matching agents to networks of different sizes could be fruitful. Weiergräber (2019) is an example of this approach. He estimates the network effect between adopters of a given mobile carrier. A cell phone is valuable in part because of how many phones it can call and at what price. He studies a period when carriers eliminated fees for calling users who subscribed to a different service. Eliminating fees led consumers to exogenously experience a wider and more accessible network, with the extent of this depending on the consumer’s carrier and market shares of other carriers. In this sense, consumers experience random assignment to a new network size. Weiergräber exploits this exogenous variation to find a significant network effect.

7.2.2 Heterogeneous network effects

The linear-in-means model implies that each agent responds to every other agent in the market in an symmetric manner. An alternative is to allow agents to respond to only a subset of agents in the same market. This approach leads us to consider which agents are linked to which.

Our presentation follows Bramoullé, Djebbari, and Fortin (2009). Let the number of agents be an integer and let \( \mathbf{a} \) with length equal to the number of agents be the vector of actions \( a \), and similarly for \( \mathbf{v}, \mathbf{\gamma} \) and \( \mathbf{u} \). Let \( \mathbf{\Lambda} \) be a square matrix with dimension equal to the number of agents. Let element \( \{i,j\} = 1 \) if \( i \) is affected by \( j \) and \( \{i,j\} = 0 \) otherwise.\(^\text{118}\) There is no need to distinguish between markets in this approach, as the existence of separate markets could be embedded in \( \mathbf{\Lambda} \). Also, \( \mathbf{\Lambda} \) need not be symmetric, so some agents may respond to agents that do not respond to them.

In matrix notation, the equivalent to Equation 26 is:

\[
\mathbf{a} = \beta_0 - \beta_1 \mathbf{P} + \beta_2 \mathbf{\Lambda a} + \beta_3 \mathbf{v} + \mathbf{\gamma} + \mathbf{u}.
\]

Here, the endogenous effect \( q \) from Equation 26 is replaced by \( \mathbf{\Lambda a} \).

Solving for \( \mathbf{a} \) finds:

\(^{118}\)Alternatively, rows of \( \mathbf{\Lambda} \) could be scaled so that the agent responded to the mean of actions by connected agents rather than the sum.
\[ a = (1 - \beta_2 \Lambda)^{-1} (\beta_0 - \beta_1 P + \beta_3 \nu + \gamma + u). \]

Multiplying each side by \( \Lambda \) and taking an expectation over \( u \):

\[ E[\Lambda a \mid P, \nu] = (1 - \beta_2 \Lambda)^{-1} \Lambda (\beta_0 - \beta_1 P + \beta_3 \nu + \gamma). \]

We can interpret this equation as a reduced-form for the endogenous effect and, as in Equation 29, it provides a potential instrument for \( \Lambda a: \Lambda \nu \). In this model, an agent’s choice is instrumented by exogenous characteristics of connected agents, as these characteristics affect the choices of neighbors but are otherwise excluded from the agents’ decision problem.

Augereau et al. (2006) follow an approach like this. As described in Section 3.2, they study the adoption decision of internet-service providers (ISPs) in the face of competing incompatible 56K modems. ISPs may agglomerate on the same standard due to indirect network effects with consumer adoption or ISPs may choose to differentiate from each other in order to avoid competition even if that means serving a smaller market. Because ISPs have overlapping service areas, the ISP market exhibits a network structure. In addition to a two-stage least squares specification as suggested here, the main specification is a structural model of adoption in a Bayesian Nash equilibrium where firms take expectations over the choices of rivals, as suggested by Seim (2006). This structural model exploits exogenous characteristics of connected agents as an instrument.

Another example is Tucker (2008), who studies the adoption of a video-messaging technology within an international investment bank. She allows different employees to have different sets of connections and allows connections to be asymmetric; for instance, the choice of a manager may affect the choice of an employee differently than an employee affects a manager. The technology could had alternative uses, such as for viewing the World Cup soccer tournament. Thus, whether an employee is interested in soccer (perhaps by explained the country of the employee) is an exogenous shifter that plays the role of an instrument for the effect of the employee’s choice on others in its network. Ryan and Tucker (2012) provide dynamic analysis of this setting in a structural model.

An even more straightforward approach arises if some agents are affected by agents that are not affected by any other agents, that is element \( i, j \) of \( \Lambda \) equals one for \( j \) such that element \( j, k = 0 \) for all \( k \). We can take these agents that are not affected by other agents as exogenous actors. There is no need to instrument for the choices of exogenous actors. We can regress the choices of agents on the choices of exogenous actors directly.

In this vein, Gowrisankaran and Stavins (2004) study the adoption of clearinghouse services by banks. The clearinghouse provides a very efficient method of moving funds across banks, so Gowrisankaran and Stavins (2004) expect a network effect as more local banks (the ones a bank is most likely to trade with) adopt. Gowrisankaran and Stavins (2004) take several approaches including assuming that large banks are exogenous to small banks, so for instance, Citibank’s choice can affect the Bank of Cape Cod but not vice versa.

Interestingly, the approach of Bramoullé et al. (2009) can be used to distinguish between the endogenous and contextual effect. Letting the contextual effect have the same network structure as the endogenous effect, we can write the choice function as:

\[ a = \beta_0 - \beta_1 P + \beta_2 \Lambda a + \beta_3 \nu + \beta_4 \Lambda \nu + \gamma + u. \]
Then, the function for the endogenous term is:

\[ E [\Lambda a \mid P, \nu] = (1 - \beta_2 \Lambda)^{-1} \Lambda (\beta_0 - \beta_1 P + \beta_3 \nu + \beta_4 \Lambda \nu + \gamma). \]

Thus, \( \Lambda^2 \nu \) provides a potential instrument for \( \Lambda a \).\(^{119}\) Instruments for the actions of connected agents are exogenous features of the agents that are connected to them. Blume, Brock, Durlauf, and Jayaraman (2015) extend this point in several ways, such as allowing different network structures to determine the endogenous and contextual effect. Bramoullé, Djebbari, and Fortin (2020) review recent developments in the literature on using network structure to identify peer effects.

With all of these approaches based on network structure, it is possible to find spurious results if error terms have the same correlation structure. That would be the case if the correlation of unobserved terms across agents were based on network structure. Angrist (2014) constructs an example like this when discussing Bramoullé et al. (2009). It could be reasonable to consider in some situations. For instance, we might normally assume that error terms have a market-level component as in Equation 26, but if we are studying firms with a network of interaction across different markets, we should ask why firms exhibit their connection pattern, which might lead us to consider unobserved terms with a parallel structure.

### 7.2.3 Nonlinear models

Many of these problems in identification follow from the linear-in-means set-up. However, many if not most network effects are better thought of as binary or discrete choices, such as the choice to adopt or not. Binary outcome models are often nonlinear, and that can lead to new identification results. Important contributions on this are Brock and Durlauf (2001, 2007).

Consider the following model. Let \( a \in \{-1, 1\} \) be the choice of a given agent. For instance, we can interpret \( a = 1 \) as adoption and \( a = -1 \) as non-adoption. As before, let \( q_m \) be the average choice in a market, so naturally, \( q_m \in [-1, 1] \). Let \( a^* \) be the latent utility parameter defined as:

\[ a^* = \beta_0 - \beta_1 P_m + \beta_2 q_m + u. \]

Brock and Durlauf (2001) consider parametric assumptions on \( u \) such that this model exhibits multiple equilibria. For instance, there may be one equilibrium with very high adoption and another with very low adoption.\(^{120}\) Brock and Durlauf (2001) argue that this nonlinearity generates identification:

Why is there this difference between the binary choice and the linear-in-means frameworks? The answer is that the binary choice framework imposes a nonlinear relationship between group characteristics and group behaviours whereas the

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\(^{119}\) A requirement is that \( \Lambda \) and \( \Lambda^2 \) are not perfectly collinear. If agents are broken up into markets and interact with everyone in their market, then \( \Lambda \) is a block-diagonal matrix of ones, which is perfectly collinear with \( \Lambda^2 \). Thus, this approach requires some asymmetry in who is connected with who.

\(^{120}\) Brock and Durlauf (2001) interpret \( u \) as private information to the agent, and so the agents take expectations over other agents’ choices and play a Bayesian Nash equilibrium. We suppress that notation here, as well as the market effect \( \gamma_m \).
linear-in-means model (by definition, of course) does not. So, for example, if one moves across a sequence of richer and richer communities, the percentage of high school graduates cannot always increase proportionately with income. (p. 254)

Brock and Durlauf (2007) extend these ideas to consider minimal assumptions on the functional forms for $u$ and $q_m$. However, the existence of the identification problem in linear-in-means models means that this approach cannot be extended to a fully nonparametric estimator. All of the papers mentioned in this section up to now (such as Gowrisankaran and Stavins, 2004; Augereau et al., 2006; Tucker, 2008; Goolsbee and Klenow, 2002; Weiergraeber, 2019) and many more use discrete models of technology adoption. However, they tend to focus on sources of identification other than nonlinearity. Whether the network effects literature also provides applications for the ideas about nonlinearity as a source of identification for social interactions remains an interesting question.

7.2.4 Dynamics

In general, installed base is a key determinant of future consumer decision-making in models of product diffusion under network effects. Section 3.4 discussed the importance of installed base in the theory of dynamic adoption of technologies characterized by network effects. Although this approach does not seem to have been widely explored in the social economics literature, a number of empirical papers on network effects adopt estimation strategies in which current decisions depend on accumulated past decisions of nearby agents. Past choices can sometimes be treated as exogenous to current choices, which makes this particularly attractive. Even in a model in which agents’ utility depends on current adoption rather than past adoption, focusing on installed base might be justified by a model in which agents have imperfect information over each other’s current choices and use installed base as a predictor of current choices.

A number of papers have adopted strategies like this. For instance, Ohashi (2003) and Park (2004) study adoption of VCRs. In this case, indirect network effects between console standards (i.e. VHS vs. Beta) and pre-recorded movies are likely at play but for data reasons, they treat the problem as one of direct network effects between purchasers of the same standard. They use installed base at the end of the previous period as the measure for the size of the network. It is the key variable for measuring the network effect. Ohashi (2003) experiments with different lags of the installed base such as the one year lead (which might capture expectations) and the two-year lag.\textsuperscript{121} Naturally, relying on installed base or lagged sales as an exogenous variable requires a strong assumption on the independence of the error term over time. This may be supported with first-differencing strategies well-known in time series economics.\textsuperscript{122}

\textsuperscript{121}Park (2004) uses time-format dummies for the main specification which captures all elements of the network effect, such as installed base, available complements (like the number of compatible pre-recorded videos), and expectations about the future. He uses installed based to decompose the elements of the network effect captures by the dummy variable.

\textsuperscript{122}Weiergraeber (2019), mentioned above in the context of random assignment, considers an alternative identification scheme utilizing lagged values of other consumers in the market.
7.2.5 Variance

Graham (2008) focuses on conditional variance rather than conditional means. Network effects are a social multiplier that magnify the response of a population of agents to choices of any one agent. Intuitively, if there is no network effect or social spillover, the variance of outcomes $a$ will be the same regardless of how many agents are in the market. With network effects, the choices $a$ respond to the average shock in the market $\bar{u}_m$, and if the variance of $\bar{u}_m$ increases, so should the variance of $a$. As the number of agents rises, we average over more shocks so the variance of $\bar{u}_m$ decreases. If the variance of outcomes $a$ differs across markets with different numbers of agents holding all else equal, it must be because of the existence of a social multiplier such as a network effect.

To see this, we return to the model in Section 7.1. Recall that we allowed the realized mean of $u$ in market $m$ to be part of $\gamma_m$. We break those up now, so let $\gamma_m = \gamma'_m + \bar{u}_m$. Then, Equation 28 is:

$$a = \beta_0 - \beta_1 P_m + \beta_2 q_m + \gamma'_m + u$$

where we recognize that $u$ is no longer necessarily of mean zero in small samples. Thus:

$$q_m = \beta_0 - \beta_1 P_m + \beta_2 q_m + \gamma'_m + \bar{u}_m$$

Solving for $q_m$ and plugging into the previous equation:

$$a = \frac{\beta_0 - \beta_1 P_m + \gamma'_m}{1 - \beta_2} + \frac{\beta_2}{1 - \beta_2} \bar{u}_m + u$$

We see that $\bar{u}_m$ affects $a$ only if the network effect parameter $\beta_2$ is non-zero. The variable $\bar{u}_m$ is not observed we cannot estimate this equation to learn about $\beta_2$. However, Graham (2008) recognizes that we can work with the conditional variance rather than the conditional mean. The variance of $\bar{u}_m$ should fall as the number of agents in a market grows. If we assume that the variance of the first term and the variance of $u$ stay the same as the number of agents grows, then we can take reductions in the variance of $a$ as the number of agents increases as evidence of the existence of a network effect.

Graham (2008) outlines the assumptions that are necessary for this approach to work and allows for market characteristics to differ in a number of dimensions. He applies his method to an education setting where he can use institutional knowledge to argue that classes with different numbers of students are comparable. For instance, he needs to argue that the variance of teacher quality does not differ with classroom size.

We are not aware of an application of this method to a network effects setting. The requirements of the conditional variance method for the orthogonality of potentially confounding factors are significant for a typical industrial organization application, but it seems feasible. For instance, Tucker (2008) and Gowrisankaran and Stavins (2004) study markets (or offices in the case of Tucker) with different numbers of agents making simultaneous adoption decisions, which would seem to fit into the general approach of Graham (2008).

7.2.6 Other approaches

Naturally, not every paper fits into one of the categories we describe so far. Particularly in industry-specific settings, specialized arguments may be feasible. For instance, an early
empirical paper on network effects is Gandal (1994), which shows that spreadsheet software that was compatible with the most popular brand at the time (Lotus 1-2-3) was more highly valued, presumably because it allowed users to more easily trade files. In addition to the approach described above, Gowrisankaran and Stavins (2004) also test whether more concentrated markets are more likely to adopt ACH banking, as firms in these markets are better able to internalize network effects.

7.3 Indirect Network Effects

Many of the techniques we discuss for estimating direct network effects can be applied to indirect network effects, but indirect network effects open up new possibilities. Suppose there are two sets of agents, $i = 1, 2$ in each market $m$. Let $q_{im}$ be the average choice of agents on side $i$ in the market, and let $P_{im}$ be the price of the platform to the set of agents $i$. We assume the agent chooses so that:

$$a_i = \beta_0 - \beta_1 P_{im} + \gamma_i q_{jm} + u.$$  

Thus, we have dropped $\beta_2$ (the direct network effect) and replaced it with the value of activity on the other side of the market. The function captures agents that are indifferent to how many agents on their side of the market use the product but care about how many agents on the other side do so. It is unusual to study models with both direct and indirect network effects, although it could happen. For instance, consumers might value Windows operating system both because there is a variety of software available (an indirect network effect) and because it is easier to share files with neighborhood agents (a direct network effect).

From the perspective of the reflection problem, $\gamma_i q_{jm}$ represents a correlated effect, a factor that affects all agents in a market. Whereas in the network effect setting, correlated effects were something to be controlled for in order to estimate the endogenous effect, in the case of indirect network effects, the correlated effect becomes the primary interest. In this case, the realization of $q_{im}$ does not create the same sort of problem as before as it does not show up on both sides of the equation:

$$q_{im} = \beta_0 - \beta_1 P_{im} + \gamma_i q_{jm} + \bar{u}_{im}$$  \hspace{1cm} (31)

There is still a simultaneity problem between $q_{im}$ and $q_{jm}$ because the equation holds for both sides of the market. Realizations of $u$ on side $i$ affect $q_{im}$ which affects $q_{jm}$, leading to correlation between the error term and a regressor.

However, this case provides natural instruments, which are variables that affect one side but not the other. In Equation 31, $P_{im}$ affects $q_{im}$ but does not otherwise affect $q_{jm}$. Similarly, $P_{jm}$ affects $q_{jm}$ but does not otherwise enter into Equation 31 and thus can play the role of an instrumental variable in estimating that equation. In practice, we will often think of prices as endogenous, so then we require some other variable that affects one side of the market but not the other, similar to the way $P_{im}$ is written here. We turn to specific examples below, but the key point is that this appears quite feasible.

In the direct network effects case, we could not distinguish between shifts along a demand curve and shifts of a demand curve resulting from a change in price. In the indirect network,
effects case, we have separate prices for each side of the market. Price changes on the other side shift out the demand curve on the first side and do not lead to shifts along the demand curve, leading to identification.

For intuition, consider Figure 7.3. In this figure, we distinguish between the demand curve and the willingness-to-pay curve. The figure shows the willingness-to-pay at price $P_i'$. The distribution of willingness-to-pay is provided by $D(P_i, q_j')$ where $q_j' = q_j(P_i')$. That is, we graph willingness-to-pay by varying price $P_i$ but holding quantity on the other side fixed at the level induced by $P_i'$. The demand curve is labeled as $D(P_i, q_j(P_i))$. Writing the argument as $q_j(P_i)$ rather than $q_j$ emphasizes the dependence of $q_j$ on $P_i$ through $q_i$. This is demand as observed by the firm; it maps out how $q_i$ changes as the firm adjusts $P_i$.

Demand is less than willingness-to-pay above $P_i'$. A consumer on side $i$ may be willing to pay more than $P_i'$ when price is at $P_i'$ but if price actually rose to that level, the reduction in $q_i$ reduces $q_j$ and so the consumer is no longer willing to pay that amount. The slope of willingness-to-pay is $\beta_1$, whereas the slope of demand also involves $\gamma_i$, which causes the slope to be less steep. For instance, the graph shows that when price drops from $P_i'$ to $P_i''$. As econometricians, we can never actually observe a change in price $P_i$ with quantity on the other side $q_j$ held constant. But instrumental variables approaches allow us to shift $q_j$ while holding $P_i$ constant and thus separately identify these different parameters.

The instrumental variables idea is not restricted to indirect network effects. If we can find instruments that affect some agents and not others, we can use instrumental variables in the case of direct network effects. Moffitt (2001) presents a social-spillover model of two agents that highlights this point, and we could see the use of pre-existing heterogeneity in Equation 29 along these lines. However, in practice, this approach is much more prevalent in the case of indirect network effects and the actual instruments used tend to be quite different. We take that up now.

A large number of papers have attempted to estimate indirect network effects in practice. Most have used variants of the instrumental variables approach laid out Section 7.3, although some have used techniques more similar to what we described in Section 7.2 and some approaches fall outside of these frameworks. In this part, we describe several different approaches.

8.1 Dynamics

Lee (2013) studies consumers buying video game consoles and video games as well as game developers releasing consoles. His model captures that as more video games become available for a given console, consumers value that console more. Consumers are forward-looking both in their choices over games and consoles. He estimates his model in first-differences which allows him to use lagged variables as instruments. This is in part reliant on assumptions of how unobserved terms are not correlated. Lee (2013) argues that because of the long lag between when a game developer commits to producing a video game and when the game is released, these assumptions are likely to hold. Zhou (2017) also studies video games in a fully dynamic game between consumers and firms and relies on similar timing assumptions in a Bayesian econometric framework.

8.2 Exclusions in two-sided markets

A number of papers have used exclusion restrictions by assuming that some variables affect outcomes on one side of the market but do not otherwise affect outcomes on the other side. Early papers in this vein are Gandal, Kende, and Rob (2000) and Rysman (2004). Gandal et al. (2000) study consumer demand for compact disk players and compact disks. We might worry about simultaneity in the determination of these outcomes and the paper uses the evolving cost of building a plant to produce disks as a shifter for the availability of titles. Dranove and Gandal (2003) analyze sales of DVD players during its competition with the DIVX standard. They allow sales of DVD players to depend on the number of titles available. Rather than use installed base of DVD players as an instrument, they use installed base of related technologies such as CDs and camcorders, which should better satisfy exogeneity requirements.

Rysman (2004) studies Yellow Pages directories and models consumer demand for Yellow Pages advertising and advertiser demand for readership as a simultaneous equations problem that leads to excluded variables. For instance, the number of people covered by a directory affects advertiser demand but does not enter into the consumer choice, and some consumer demographics are assumed to affect whether consumers use a directory but not affect the value that advertisers place on those consumers conditional on usage. Similarly, Clements and Ohashi (2005) study video games and use software age as an instrument for software demand.

123Both of these papers are far pre-dated by Rosse (1970), which studies the interaction of readers and advertisers at newspapers although the focus is on the cost function rather than this feedback loop.
A more recent application of these ideas appears in Caoui (2020), which studies the adoption of digital projection technology by movie theaters and the production of movies in the digital format. The paper focuses on France and uses the digital production of movies in the United States as an exogenous shifter of digital movie availability in France, as many US movies are released in France. A related idea is in Gowrisankaran, Park, and Rysman (2014) which studies the adoption of DVD players in response to DVD titles and uses box office outcomes as a shifter of the attractiveness of titles. Like Lee (2013) and Caoui (2020), Gowrisankaran et al. (2014) use a dynamic model of consumer demand. Sokullu (2016) studies the interaction of advertising and readership in newspapers relying on characteristics from each side of the market as excluded variables to identify the indirect network effect. In particular, she allows the network effect to be non-monotonic and utilizes nonparametric instrumental variables in this context.

These ideas are being applied in a wide variety of industries. For instance, Li, Tong, Xing, and Zhou (2017) and Li (2019) study the diffusion of electric vehicles and charging stations. Li et al. (2017) assumes that gas prices affect only consumer demand and identify the effect of consumers on charging stations. Li (2019) uses, among other instruments, federal and state subsidies as instruments. As subsidies exist on both sides of the market (subsidies to vehicles and subsidies to charging stations), these can identify the network effect in both directions. Rysman (2007) studies correlation between consumer usage of payment card networks and merchant acceptance, although does not attempt to establish causality. Ackerberg and Gowrisankaran (2006) study ACH banking adoption as a two-sided market, analyzing both consumer adoption and bank adoption. They use adoption behavior in other markets as a proxy for costs, which provides identification. They quantify the importance of network effects and the extent of multiple equilibria.

Farronato, Fong, and Fradkin (2020) study digital platforms that match pet owners to pet sitters. From the perspective of a pet owner, a merger between two platforms creates a larger set of potential pet sitters to interact with on the remaining platform than either platform provided before the merger. As platform market shares differ across markets, this provides an opportunity to evaluate the presence of network effects. In a sense, this is a version of random assignment discussed in Section 7.2.1 adapted to a two-sided setting. Pet owners are randomly assigned different increases in the size of the network of pet sitters with which they may interact, with assignment depending on what city the pet owner lives in. Farronato et al. (2020) find that in markets where the acquiring platform was dominant, there is little change after the merger. Network effects are apparent in markets where the two platforms were similar in size, although pet sitters of the acquired platform often did not participate after the merger. This evidence of switching costs provides a potential downside to network effects realized through merger.

Similarly, Reshef (2020) exploits a partnership between Yelp and Grubhub. Because market shares differ for these firms across cities, their partnership generates plausibly exogenous variation across cities in the size of the network expansion. Obviously, this version of random assignment is different from the social economics literature in which random assignment typically takes the form of literal randomness imposed at an administrative level, such as random assignment of college roommates. In the platform economics setting, establishing randomness (or more realistically, exogeneity) requires specialized arguments, but conditional on that, the thought experiment is very similar.
8.3 Pricing studies

The emphasis in this chapter is on network effects and pricing in two-sided markets, and with our discussion of the identification of network effects completed, we can now turn to approaches to study pricing in these markets. We distinguish between two approaches, which loosely correspond to treating price as endogenous or exogenous. In the first, the researcher observes many platforms setting prices and studies how observed prices respond to factors such as market structure. The researcher can use hypothesis testing to understand whether platform behavior is consistent with theory. In this approach, we find papers that are both structural and reduced-form in their econometric techniques. In the second approach, the researcher observes one platform or set of platforms with relatively few sets of prices such as a single pricing schedule used in many geographic markets. The researcher typically models how agents interact on this platform, and studies pricing by analyzing how agents would behave under some counterfactual pricing regime. Modeling agent interactions in ways that make the counterfactual compelling can often lead to very involved models and this approach tends to be structural. Naturally, the two approaches answer different questions and are complementary for understanding platform markets.

Early papers in the first approach focus on media. Rysman (2004), discussed earlier in the context of identification of network effects, also fits here. He observes many markets, each populated by several Yellow Pages directories, which are the platforms, and he models quantity (and by extension price) as the choice variable. Rysman (2004) assumes that consumers single-home, which he captures with a discrete choice model. He derives a log-linear demand for advertising from a model of advertiser multi-homing. A pricing first-order condition provides a third set of estimation equations. This general approach has proven popular for media studies, such as Fan (2013). Rysman (2004) simulates outcomes as the number of competing directories change. More directories reduce market power but also dissipate network effects. He finds that the former effect dominates, and the market efficiency is enhanced as the number of directory increases, at least within the range of the data.

Kaiser and Wright (2006) study German magazines, again studying the interaction of readers and advertisers. They estimate a structural model of both consumer and advertiser demand, in this case, assuming that both sides single-home. They use several instrumental variables, in particular, prices of publishers in related markets, which can be interpreted as related to cost-shifters in the spirit of Hausman (1996). They find that profits are collected largely from the advertiser side rather than the consumer side. Higher demand on the advertiser side actually reduces consumer prices as magazines seek to attract these advertisers, similar to the seesaw effect described in Section 4.3.2. Another paper on media is Argentesi and Filistrucchi (2007), which provides a structural model of Italian newspapers. They extend the approach of Kaiser and Wright (2006) to include a publisher first-order condition, as in Rysman (2004), which allows them to study market power. Chandra and Collard-Wexler (2009), mentioned in Section 4.3.2, study mergers between newspapers and show theoretically and empirically that the logic of two-sided markets implies that prices do not necessarily increase. Jeziorski (2014) embeds a related model of radio stations in a dynamic model of mergers. He finds that merger results in market power that allows radio stations to lower the quantity and increase the price of advertisements. However, because
consumers dislike advertising, this outcome enhances consumer welfare.

Reduced-form approaches often provide compelling tests of whether theoretical models make accurate predictions. Empirical papers confirm the existence of the seesaw effect of Section 4.3.2 that a shock that tends to raise price on one side often reduces prices on the other side, in a variety of industries. In media, in addition to Kaiser and Wright (2006) mentioned above, Seamans and Zhu (2014) study how newspaper prices and outcomes respond to the entry of Craigslist, which cannibalizes classified advertisement revenue, an important source of newspaper revenue up until that point. The response is complex. For instance, consumer prices increase and advertiser prices decrease. Their interpretation is that the reduction in demand for classified advertisements reduces the value of consumers leading to an increase in consumer price, and the resulting reduction in readers lowers value, and thus price, to advertisers.

In telecom, Boik (2016) studies cable systems, which are platforms between content providers, such as local television stations, and consumers. Boik confirms the two-sided intuition that local television stations in markets with high advertisement rates set lower fees to cable systems as the advertising rates incentivize the stations to seek more consumers. Similarly, Genakos and Valletti (2011) find that pricing reductions imposed by regulators in how mobile carriers price in-coming calls lead to increases in price to consumers, as the consumers become less valuable. Genakos and Valletti (2011) show how this effect differs across different types of consumers.

In payments, Kay, Manuszak, and Vojtech (2018) study the debit card market in the US. Under the Financial Reform Act of 2010, the Federal Reserve Bank capped the interchange fee on debit cards for banks above a size threshold, which effectively reduced the amount that these banks charge to merchants. Kay et al. (2018) compare banks around the threshold to document that the regulation led to more fees to consumers. Manuszak and Wozniak (2017) take a similar approach to analyze the nature of fees added to checking accounts and similar products. Carbo-Valverde, Chakravorti, and Rodriguez-Fernandez (2015) study the Spanish payment card market during a period of declining interchange fees, which reduced effective payments from merchants to consumers. Although this likely led to higher consumer fees, the paper shows that it also led to increased merchant acceptance. Overall, Carbo-Valverde et al. (2015) find that consumer adoption increased.

It would seem unintuitive that competition between firms would not lead to better prices for consumers but that is natural in a two-sided context. As described above, competition between platforms can lead to better terms for one side but worse terms for the other. The study of local television fees of Boik (2016) finds that in markets with more substantial competition between television delivered by cable companies and telephone companies, local television charge lower rather than higher fees. An explanation consistent with a number of theoretical models is that local television stations multi-home and the benefits of competition between platforms (cable and telephone) go to the single-homers, the consumers.124

Similarly, Jin and Rysman (2015) analyze sports card conventions, which bring together buyers and sellers of sports memorabilia. Taking the convention organizer as the platform,

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124Boik (2016) finds fees to local television stations increase when there is more significant competition from satellite, but he points out that it is likely driven by the advertisement market. Cable competes with local television for local advertisements whereas satellite does not, giving the local television stations incentives to direct consumers to satellite service.
they observe prices charged to each side at thousands of conventions. They find that pricing responds to competition between platforms much more on the single-homing side. We discussed prices that face a lower bound at zero in Section 2.5.3, and about 50% of conventions offer free admission to consumers. Jin and Rysman (2015) find that when price on the single-homing side hits zero, pricing on the other side shows a strong response to competition, as predicted by Armstrong and Wright (2007).

All of these studies use covariation in prices and other variables. The second approach studies a platform with fixed prices and studies how agents interact on the platform. Typically using structural estimation, these papers study counterfactual outcomes under alternative pricing structures. Several papers study markets with many competing intermediaries, such as brokers. An example is Gavazza (2016) who studies intermediaries in the market for trading used aircraft and looks at outcomes without intermediaries. Barwick and Pathak (2015) study the effects of the commission rate for agents in the Boston real estate market, which is typically fixed across brokers. They find that allowing brokers to compete on commission rates leads to a reallocation of market share to more efficient brokers and substantially more efficient outcomes. Similarly, Hendel, Nevo, and Ortalo-Magné (2009) show in a reduced-form comparison between outcomes on an internet platform for selling homes and the traditional broker market that high commissions drive sellers to the on-line platform, which creates important selection effects in which sellers participate on each platform. Robles-Garcia (2020) models the relationship between mortgage lenders and borrowers with a focus on mortgage brokers that connect them. She finds that caps on commissions to brokers increase welfare in part by aligning incentives with consumers rather than lenders. A common theme across these papers appear to be that if market-wide commission structure is not efficient, competition between brokers does not necessarily correct for that.

Some recent papers apply this method to online markets. Marra (2020) studies an online platform for trading wine. Echoing the selection effects of Hendel et al. (2009), she shows how fees on one side of the platform affect not only the number of participants but also the quality of those participants, which has implications for agents on the other side. Castillo (2020) provides a detailed model of participants in the ride-sharing platform Uber. In ride-sharing, the value to riders from participating in the platform depends on the number of drivers participating, and vice versa. Castillo considers the effect of eliminating surge pricing, the increase in fees from riders to drivers during high demand periods. He finds that riders in particular benefit from surge pricing, contrary to some of the popular discussion, as they benefit from increased service levels. Rosaia (2021) studies a model of drivers choosing not only whether to drive but also choosing between Uber and Lyft. He studies pricing power of the two companies and finds that merger leads to efficiencies due to the management of traffic congestion. These papers highlight the complex interactions between agents in a platform and the sometimes surprising outcomes that arise from policy interventions that would be more straightforward in a one-sided market.

9 Conclusion

Platforms are ubiquitous in modern society and play a key role in the organization of economic activity. Platforms connect different sets of market participants, such as buyers and sellers, or advertisers and readers. When the choices of one set of participants (such as
whether to join the platform) affect the value of the platform to other sets of participants, indirect network effects emerge and the literature refers to the market as “two-sided.” Platform firms thrive in these settings and, in particular in digital markets with low marginal costs and large scale economies, can grow very large. As a result, platform economics is central to some of the most closely watched debates in competition policy.

Platforms set prices on each side of the market accounting for the complex interactions between the various platform’s users. Pricing strategies respond to the existence of network effects but also contribute to the realization of such network effects. This chapter focuses on pricing in two-sided markets, the most well-studied phenomenon in the growing literature on platform economics.

We start with a theoretical treatment of monopoly platforms, compare profit-maximizing to welfare-maximizing solutions, and analyze different ways in which network effects shape prices and market outcomes.

Next, we turn to competition between platforms, which has been the subject of a recent research. We emphasize the (somewhat fuzzy) difference between “competition for the market” and “competition on the market,” and review the different approaches to modeling platform competition. We point to the importance of accounting for the various ways that agents may interact with the platforms, such as by single-homing or multi-homing. In addition, we discuss the nascent literature on the closely-related topic of matching design, opening the “black box” of network effects and allowing for individualized matching and within-side price discrimination.

Despite the fact that many issues related to platforms are dynamic in nature, the literature on platforms is largely static; very few works study ignition and dynamic platform competition. This gap has also influenced the policy debate. Furthermore, most of the existing contributions focus on one-sided network effects and more progress is needed to understand the evolution of platform markets.

Finally, we discuss empirical approaches to the study of these markets. We discuss identification issues in the estimation of network effects. We argue that connecting this literature to the well-developed literature on the estimation of social spillovers provides valuable insights and opens the door to promising new directions for future research. We review a number of empirical papers that rely on a variety of strategies to identify network effects. We review the study of pricing in these markets and show that support for the several theoretical results.

The chapter provides a jumping-off point for the wide literature on two-sided markets. Topics such as platform design, competition policy, and specialized industry studies in areas such as media, finance, transportation, housing, and health insurance, are covered only tangentially in the chapter but represent important directions for current and future research.
References


Li, J. (2019). Compatibility and investment in the U.S. electric vehicle market. Unpublished manuscript, MIT.


Selten, R. (1973). A simple model of imperfect competition, where 4 are few and 6 are many. *International Journal of Game Theory* 2(1), 141–149.


