Persuasion in Global Games with Application to Stress Testing*

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Abstract

We study robust/adversarial information design in global games, with an application to stress testing. We show that the optimal policy coordinates all market participants on the same course of action. Importantly, while it removes any "strategic uncertainty," it preserves heterogeneity in "structural uncertainty" (that is, in beliefs over payoff fundamentals). We identify conditions under which the optimal policy is a "pass/fail" test, show that the optimal test need not be monotone in fundamentals, but also identify conditions under which it is monotone. Finally, we show how the effects of an increase in market uncertainty on the toughness of the optimal stress test depend on the securities issued by the banks.

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1 Introduction

Differing opinions on how stress tests should be undertaken are welcome and important... We need to move away from simple pass/fail policies (Piers Haben, Director, European Banking Authority, Financial Times, August 1, 2016).

Coordination plays a major role in many socio-economic environments. The damages to society of mis-coordination can be severe and often call for government intervention. Think of the situation faced in 2016 by Monte dei Paschi di Siena, the oldest bank on the planet and the Italian third largest, trying to raise capital from multiple investors (mutual funds, creditors, and other major financial institutions), despite concerns about the size of the bank's non-performing loans. A default by an institution such as MPS can trigger a collapse in financial markets, and ultimately a deep recession in the Eurozone and beyond (The Economist, July 7, 2016).

Confronted with such prospects, governments and supervising authorities have incentives to intervene. However, a government's ability to calm the market by injecting liquidity into a troubled bank can be limited. For example, in Europe, legislation passed in 2015 prevents Eurozone member states from rescuing banks by purchasing assets or, more generally, by acting on the banks' balance sheets. In such situations, interventions aimed at influencing market beliefs, for example through the design of stress tests, or other targeted information policies, play a fundamental role. The questions policy makers face in designing such information policies are the following: (a) What disclosures minimize the risk of default? (b) Should all the information collected through the stress tests be passed on to the market, or should the supervising authorities commit to coarser policies, for example, a simple announcement of whether or not a bank under scrutiny passed the tests? (c) Should stress tests pass institutions with strong fundamentals and fail the rest, or are there benefits to non-monotone rules? (d) What are the effects of an increase in market uncertainty on the structure of the optimal tests, and how do they depend on the banks' recapitalization strategies, i.e., on the type of security issued by the banks?

In this paper, we develop a theoretical framework that permits us to investigate the above questions. We study the design of optimal information policies in markets in which a large number of receivers (e.g., market investors) must choose whether to play an action favorable to the designer (e.g., pledging to a bank, or refraining from speculating against it), or an "adversarial" action (e.g., refraining from pledging, or engaging in predatory trading, for example by short-selling securities linked to the bank's assets, or buying credit-default swaps, which are known to put strain on illiquid banks). Market participants are endowed with heterogenous private information about relevant economic fundamentals, such as a bank's non-performing loans, the long-term profitability of its assets, or other elements of the bank's balance sheet not in the public domain. A cash-constrained policy maker (e.g., a benevolent government, or a supervising authority such as the European Banking Authority, or the Federal Reserve Bank) can influence the market's beliefs (for example, by designing a stress test), but is constrained in its ability to use financial instruments to shape directly the market outcome.[1]

While motivated by the design of stress tests, the analysis delivers results that are relevant also for many other applications, including currency crises, technology and standards adoption, and political change.[2] We explicitly account for the role that coordination plays among multiple, heterogeneously informed, receivers. Coordination plays a key role in the funding of solvent but illiquid banks (see, among others, Diamond and Dybvig (1983) and Goldstein and Pauzner (2015) for runs on deposits, Copeland et al. (2014) and Gorton and Metrick (2012) for runs on repos, Covitz et al. (2013) for runs on asset-backed commercial paper, and Pérignon et al. (2018) for dry-ups on certificates of deposit).

The backbone of our analysis is a general *global game of regime change* in which, prior to receiving information from the information designer (the policy maker), each agent is endowed with an exogenous private signal about the strength of the underlying fundamentals. In the absence of additional information, such a game admits a unique rationalizable strategy profile, whereby agents play the action favorable to the policy maker (i.e., pledge to the bank) if, and only if, they assign sufficiently high probability to the underlying fundamentals being strong, and whereby regime change (i.e., default) occurs only for sufficiently weak fundamentals. In such settings, the design of the optimal persuasion strategy must account for the effects of information disclosure not just on the agents' first-order beliefs, but also on their *higher-order beliefs* (that is, the agents' beliefs about other agents' beliefs, their beliefs about other agents' beliefs about their own beliefs, and so on). Equivalently, the optimal policy must be derived by accounting for how different information disclosures affect both the agents' *structural uncertainty* (i.e., their beliefs about the underlying economic fundamentals), and the agents' *strategic uncertainty* (i.e., the agents' beliefs about other agents' behavior).

We take a "robust approach" to the design of the optimal information policy. We assume that, when multiple rationalizable strategy profiles are consistent with the information disclosed, the policy maker expects the agents to play according to the "most aggressive" strategy profile (the one that minimizes the policy maker's payoff over the entire set of rationalizable profiles). This is an important departure from both the mechanism design and the persuasion literature, where the designer is typically assumed to be able to coordinate the market on the course of action most favorable to her (among those consistent with the assumed solution concept). Given the type of applications the analysis is meant for, such "robust approach" appears more appropriate.[3]

[1]For an account of the key institutional details of the stress tests conducted in Europe, see, for example, Henry and Christoffer (2013) and Homar et al. (2016).

[2]For example, in the context of currency crises, the policy maker may represent a central bank attempting to convince speculators to refrain from short-selling the domestic currency by releasing information about the bank's reserves and/or about domestic economic fundamentals. Alternatively, the policy maker may represent the owners of an intellectual property, or more broadly the sponsors of an idea, choosing among different certifiers in the attempt to persuade heterogenous market users (buyers, developers, or other technology adopters) of the merits of a new product, as in Lerner and Tirole (2006)'s analysis of forum shopping.

[3]If the designer trusted the market to coordinate on the course of action most favorable to her, she would fully

Our first result shows that the optimal policy has the "*perfect coordination property.*" It induces all market participants to take the same action, irrespective of the heterogeneity in the agents' first- and higher-order posterior beliefs. In other words, the optimal policy completely removes any strategic uncertainty, while retaining heterogeneity in structural uncertainty. Under the optimal policy, each agent is able to predict the actions of any other agent, but not the beliefs that rationalize such actions. In the context of our application, an investor who is induced to pledge need not be able to predict whether other investors pledge because they expect the bank's fundamentals to be so strong that the bank will never collapse, irrespective of what over investors do, or because they expect other investors to pledge.

The optimality of policies satisfying the perfect coordination property should not be taken for granted given the robustness requirement. When the designer trusts the receivers to follow her recommendations, the optimality of the perfect coordination property is straightforward and follows from arguments similar to those establishing the Revelation Principle. This is not the case under adversarial design, for information policies that facilitate perfect coordination among the Receivers may also open the door to rationalizable profiles in which some of the agents play adversarially to the designer (in the stress testing application, refrain from pledging).

Our second result identifies primitive conditions under which the optimal policy takes the form of a simple "pass/fail" test, with no further information disclosed to the market. We show that the optimality of such simple policies hinges on a certain co-movement between fundamentals and beliefs, namely on the property that states of Nature in which the fundamentals are strong are also states in which most agents expect the fundamentals to be strong, expect other agents to expect the fundamentals to be strong, and so on.[4] This property is consistent with what is typically assumed in the literature on coordination under incomplete information. Importantly, we show by means of an example that, when such a property is not satisfied, the policy maker may be strictly better off disclosing information to the agents in addition to whether or not the bank passed the test.[5]

The above two results contribute to the debate about the (sub)optimality of European stress tests. Such tests have been criticized for not disclosing the details of the simulations (see, e.g., "Stress tests do little to restore faith in European banks," Financial Times, August 1, 2017). Our results indicate that simple pass/fail policies might actually be optimal. Importantly, optimal stress tests should be *transparent*, in the sense of facilitating coordination among the relevant actors, but should

disclose the state and then recommend that all agents pledge, unless the bank is doomed to collapse irrespective of the agents' behavior. This appears both uninteresting and unrealistic.

[4]Formally, when the agents' beliefs are parametrized by a uni-dimensional signal, this amounts to assuming that the distribution from which the signals are drawn is log-supermodular or, equivalently, satisfies the *monotone likelihood ratio property.*

[5]This is another point of departure with respect to the pertinent literature. When the designer trusts her ability to coordinate the receivers on the course of action most favorable to her, optimal policies always take the form of action recommendations (and hence pass/fail policies are optimal, irrespective of the agents' primitive beliefs). This is not the case under adversarial/robust design.

not generate consensus among market participants about the soundness of the financial institutions under scrutiny. Preserving heterogenous beliefs over a bank's fundamentals is instrumental to the minimization of default risk.

Our third result is about the optimality of monotone rules that pass with certainty institutions whose fundamentals are strong and fail with certainty those whose fundamentals are weak. We identify conditions under which such policies are optimal. These conditions relate the policy maker's preferences over the fundamentals of the banks saved to the distribution of the agents' primitive beliefs and the agents' payoffs. We show that these conditions are fairly sharp in the sense that, when violated, non-monotone rules may strictly outperform monotone ones. We also explain that the conditions guaranteeing the optimality of monotone rules are more stringent when the policy maker faces multiple privately-informed receivers than when she faces either a single (possibly privately-informed) receiver, or multiple receivers who possess no exogenous private information.

The reason why, under adversarial design, non-monotone policies may outperform monotone ones is that they make it more difficult for the agents to commonly learn the precise fundamentals when hearing that a bank passed the test and hence help reduce the risk of the market responding adversarially to the disclosed information. In turn, this permits the policy maker to give a pass grade to more banks, while guaranteeing that, after a pass grade is announced, the unique rationalizable strategy profile features all agents pledging.

We also show how the results extend to settings in which the policy maker faces uncertainty about the fate of the financial institutions under scrutiny, for example because default may be determined also by variables orthogonal to, or imperfectly correlated with, those measurable by the policy maker (e.g., by the behavior of noisy/liquidity traders, or by macroeconomic events only imperfectly correlated with the banks' fundamentals). Lastly, we show how the model favors micro-foundations in which the banks under scrutiny issue equity or debt to fund their short-term liquidity obligations, and where the (market-clearing) price of the securities is endogenous and depends on the information revealed through the stress tests. We use such micro-foundations to show how the model can be used for comparative statics. As an example, we investigate the effects of an increase in market uncertainty on the toughness of the optimal stress tests and show how the latter depends on the type of security issued by the banks.

Throughout the analysis, we restrict attention to situations in which the agents possess primitive private information before hearing from the policy maker and where the latter is constrained to disclose the same information to all market participants, which is the relevant case in practice. In the online Supplement, however, we also discuss why, when feasible, discriminatory disclosures may improve upon non-discriminatory ones.

Organization. The rest of the paper is organized as follows. Below, we wrap up the introduction with a brief review of the most pertinent literature. Section 2 presents the model. Section 3 contains all the results about properties of optimal policies (perfect-coordination, pass/fail, monotonicity).

Section 4 discusses the robustness of the results with respect to a few extensions relevant for applications (namely, richer payoff specifications and aggregate uncertainty imperfectly correlated with the policy maker's and the market's information). Section 5 discusses comparative statics of the optimal stress test in a fully micro-founded model in which banks issue debt or equity to fund their obligations. Section 6 concludes. All proofs are either in the Appendix at the end of the document or in the online Supplement.

(Most) pertinent literature. The paper is related to different strands of the literature.

The first strand is the literature on *information design* (see Bergemann and Morris (2019) and Kamenica (2019) for overviews). This literature traces back to Myerson (1986), who introduced the idea that, in a general class of multi-stage games of incomplete information, the designer can restrict attention to private incentive-compatible action recommendations to the agents. Recent developments include Rayo and Segal (2010), Kamenica and Gentzkow (2011), Gentzkow and Kamenica (2016), Ely (2017), and Dworczak and Martini (2019). These papers consider persuasion with a single receiver. The case of multiple receivers is less studied. Calzolari and Pavan (2006a) consider an auction setting in which the sender is the initial owner of a good and where the different receivers are privately-informed bidders in an upstream market who then resell in a downstream market (see also Dworczak (2020) for an analysis of persuasion in other mechanism design environments with aftermarkets).[6] More recent papers with multiple receivers include Alonso and Camara (2016a), Bardhi and Guo (2017), Basak and Zhou (2019), Che and Hörner (2018), Doval and Ely (2020), Galperti and Perego (2020), Li et al. (2020), Mathevet et al. (2019), Morris et al. (2020) and Taneva (2019). In particular, Li et al. (2020) and Morris et al. (2020) consider adversarial design in a coordination setting similar to the one in the present paper. These papers, which are subsequent to ours, assume that (a) the receivers possess no exogenous private information prior to receiving the information from the designer, and (b) the designer can inform the receivers asymmetrically, that is, she can engage in discriminatory disclosures. In contrast, we assume that the receivers are endowed with exogenous private information and that the designer is constrained to disclose the same information to all the receivers, which appears the most relevant case for the type of applications the analysis is meant for (e.g., stress testing). Persuasion with privately-informed receivers has been examined primarily in settings with a single receiver (see, among others, Kolotilin et al. (2017), Alonso and Camara (2016b), Chan et al. (2019), and Guo and Shmaya (2019)). See Laclau and Renou (2017), Gitmez and Molavi (2020), and Heese and Lauermann (2021) for recent papers with multiple privately-informed receivers. These papers though do not look at the implications of (adversarial) coordination for the structure of the optimal policy, which is the focus of the present paper.[7] In a coordination setting with two privately-informed receivers and two states, Alonso and Zachariadis

[6]Related is also Calzolari and Pavan (2006b). That paper studies information design in a model of sequential contracting with multiple principals.

[7]See also Gick and Pausch (2012), Shimoji (2017), and Arieli and Babichenko (2019). These papers, though, abstract from strategic interactions among the receivers.

(2021) show that, when the precision of the receivers' exogenous information is sufficiently high, private and public information are complements in that an increase in the precision of the agents' private information leads to the provision of more accurate public information. Goldstein and Huang (2016) study persuasion in a coordination setting similar to ours but restricting the designer to monotone pass/fail policies. Our results show that the optimal policy need not be monotone in their setting, but also identify primitive conditions under which, in richer settings, monotone policies are optimal. In a similar vein, Galvão and Shalders (2020) look at the design of policies in a global game similar to ours but restricting the designer to monotone partitional rules (whereby if two types receive the same grade then all types between these two also receive the same grade).

The present paper contributes to this strand of the literature by identifying properties of optimal (non-discriminatory) policies when the receivers are privately informed and play adversarially.

The second strand is the literature on *stress testing*. Bouvard et al. (2015) study a credit rollover setting where a policy maker must choose between transparency (full disclosure) and opacity (no disclosure) but cannot commit to a disclosure policy. In contrast, we assume the policy maker can fully commit to her disclosure policy and allow for flexible information structures. Alvarez and Barlevy (2015) study the incentives of banks to disclose balance sheet (hard) information in a setting where the market is not able to observe how banks are exposed to each others' risks.[8] See also Goldstein and Sapra (2014) for an overview of some of the early contributions and Morgan et al. (2014), Flannery et al. (2017), and Petrella and Resti (2013) for an empirical analysis of the information provided by stress tests conducted in the US and the EU. Goldstein and Leitner (2018) study the design of stress tests by a regulator facing a competitive market, where agents have homogeneous beliefs about the bank's balance sheet.[9] Orlov et al. (2018) and Inostroza (2021) consider the joint design of stress tests and capital requirements. The latter paper also considers the interplay between information disclosures and the policy maker's role as a lender of last resort.[10]

The present paper contributes to this literature along the following dimensions: (a) it shows that optimal stress tests should not create conformism in market beliefs about banks' fundamentals but should be sufficiently transparent to eliminate any ambiguity about the market response to the tests; (b) it identifies conditions under which simple pass/fail announcements are optimal; (c) it provides conditions for optimal tests to be monotone; and (d) it discusses how the toughness of optimal tests relates to the type of securities issued by the banks.

Finally, the paper is related to the literature on *global games with endogenous information*. Angeletos et al. (2006), and Angeletos and Pavan (2013) consider settings whereby a policy maker,

[8] See also Corona et al. (2017) for an analysis of how stress tests disclosures may favor banks' coordinated risk taking in the spirit of Farhi and Tirole (2012).

[9] See also Williams (2017) for a related analysis of stress test design in a bank-run model a' la Allen and Gale (1998), with homogenous investors.

[10] See also Faria-e Castro et al. (2016) and Garcia and Panetti (2017) for a joint analysis of stress tests and government bailouts.

endowed with private information, engages in costly actions to influence the agents' behavior. Edmond (2013) considers a similar setting but assumes the cost of policy interventions is zero and agents receive noisy signals of the policy maker's action. Angeletos et al. (2007) consider a dynamic model in which agents learn from the accumulation of private signals over time and from the (possibly noisy) observation of past outcomes. Cong et al. (2016) consider a dynamic setting similar to the one in Angeletos et al. (2007) but allowing for policy interventions. Denti (2020), Szkup and Trevino (2015), Yang (2015) and Morris and Yang (2019) consider global games where, prior to committing their actions, agents acquire private information about payoff-relevant variables at a small cost.

The key contribution of the present paper vis-a-vis this literature is the characterization of the optimal provision of public information.

2 Model

To illustrate the key ideas in the simplest possible terms, we consider a stylized global game of regime change in the spirit of Rochet and Vives (2004). The game abstracts from many institutional details but highlights the effects of (adversarial) coordination among privately-informed receivers on the design of the optimal policy. Motivated by the application to stress testing, the model features a policy maker persuading investors to pledge to a bank.[11] The analysis, however, can be adapted easily to many other games of regime change.

Players and Actions. A policy maker designs a *stress test*, i.e., an information policy that evaluates the profitability, the liquidity, and the solvency of a representative bank and communicates the results of such evaluations to the market. To meet its short-term liquidity obligations, the bank may need funding from the market. The latter is populated by a (measure-one) continuum of investors distributed uniformly over $[0, 1]$. Each investor may either take a "friendly" action, $a_i = 1$, or an "adversarial" action, $a_i = 0$. The friendly action is interpreted as the decision to pledge funds to the bank (alternatively, to abstain from speculating against the bank by short-selling its assets or by engaging in predatory trading, e.g., by purchasing credit-default swaps). The adversarial action is interpreted as the decision to not pledge (alternatively, to speculate against the bank). We denote by $A \in [0, 1]$ the size of the aggregate pledge.

Fundamentals and Exogenous Information. The bank's fundamentals are parameterized by $\theta \in \mathbb{R}$. Before the bank is scrutinized, it is commonly believed (by the policy maker and the investors alike) that θ is drawn from a distribution F, absolutely continuous over $\Theta \supsetneqq [0, 1]$, with a smooth density f strictly positive over Θ. In addition, each investor $i \in [0, 1]$ is endowed with private information summarized in a uni-dimensional signal $x_i \in \mathbb{R}$ drawn independently across agents (given θ) from an absolutely continuous cumulative distribution function $P(x|\theta)$ with smooth density $p(x|\theta)$

[11] Rochet and Vives (2004) consider a three-period economy a' la Diamond and Dybvig (1983) but with heterogenous investors, in which banks can liquidate assets to boost liquidity and may fail early or late. As shown in that paper, the full model admits a reduced-form version similar to the one considered here.

strictly positive over an (open) interval $\varrho_\theta \equiv (\underline{\varrho}_\theta, \bar{\varrho}_\theta)$ containing θ, with $\underline{\varrho}_\theta, \bar{\varrho}_\theta$ monotone in θ.[12] The bounds $\underline{\varrho}_\theta, \bar{\varrho}_\theta$ can be either finite or infinite. For example, when $x_i = \theta + \sigma\varepsilon_i$, with ε_i drawn from a uniform distribution over $[-1, +1]$, then, for any θ, $\underline{\varrho}_\theta = \theta - \sigma$ and $\bar{\varrho}_\theta = \theta + \sigma$. When, instead, $x_i = \theta + \sigma\varepsilon_i$ with ε_i drawn from a standard Normal distribution, then, for any θ, $\underline{\varrho}_\theta = -\infty$ and $\bar{\varrho}_\theta = +\infty$. Furthermore, in this latter case, $P(x|\theta) = \Phi((x - \theta)/\sigma)$, where Φ is the cumulative distribution function of the standard Normal distribution.[13] We denote by $\mathbf{x} \equiv (x_i)_{i\in[0,1]}$ a profile of private signals and by $\mathbf{X}(\theta)$ the collection of all $\mathbf{x} \in \mathbb{R}^{[0,1]}$ that are consistent with the fundamentals being equal to θ. As usual, we assume that any pair of signal realizations $\mathbf{x}, \mathbf{x}' \in \mathbf{X}(\theta)$ has the same cross-sectional distribution of signals, with the latter equal to $P(x|\theta)$.

Default. The bank's fundamentals θ parametrize the critical size of the aggregate pledge that is necessary for the bank to avoid default. If $A > 1 - \theta$, the bank meets all its short-term obligations and avoids default. If, instead, $A \leq 1 - \theta$, the bank ends up in distress and defaults. We denote by $r = 0$ the event that the bank defaults, and by $r = 1$ the complement event in which the bank avoids the default.

Dominance Regions. Clearly, for any $\theta \leq 0$, the bank defaults, whereas for any $\theta > 1$ the bank avoids default, irrespective of the size of the aggregate pledge. For $\theta \in (0, 1]$, instead, whether or not the bank defaults is determined by the behavior of the market.

Payoffs. Each investor's payoff differential between the friendly and the adversarial action is equal to $g(\theta) > 0$ in case the bank avoids default and $b(\theta) < 0$ otherwise. The policy maker's payoff is equal to $W(\theta)$ in case default is avoided and $L(\theta)$ in case of default, with $W(\theta) > L(\theta)$ for all θ.[14] When W and L are invariant to θ, the policy maker's objective reduces to minimizing the probability of default. The functions b, g, W, and L are all bounded.

Stress Tests. Let $\mathcal{S}$ be a compact metric space defining the set of possible signal realizations (think of these as grades or scores given to the bank under examination). A *stress test* $\Gamma = (\mathcal{S}, \pi)$ consists of the set $\mathcal{S}$ along with a mapping $\pi : \Theta \to \Delta(\mathcal{S})$ specifying, for each θ, a (probability distribution over the) score given to type θ.[15]

Timing. The sequence of events is the following:

1. The policy maker publicly announces the policy $\Gamma = (\mathcal{S}, \pi)$ and commits to it.
2. The fundamentals θ are drawn from the distribution F and the agents' exogenous signals $\mathbf{x} \in \mathbf{X}(\theta)$ are drawn from the distribution $P(x|\theta)$.
3. The score s is drawn from $\pi(\theta)$ and publicly announced.

[12] Formally, $\varrho_\theta = supp[P(\cdot|\theta)]$.

[13] The uniform and Gaussian distributions are the ones considered in most of the literature.

[14] The choice of this notation is meant to be mnemonic, with g and b standing for "good" and "bad" outcomes, and W and L for "win" and "lose" payoffs.

[15] Here we assume that, through the stress test, the policy maker learns all information that is relevant for the fate of the bank, for the policy maker's payoff, and for the payoffs of all market participants. We relax these assumptions in Section 4.

4. Agents simultaneously choose whether or not to pledge.

5. The fate of the bank is determined and payoffs are realized.

Adversarial Coordination and Robust Design. The policy maker does not trust the market to follow her recommendations and play according to the strategy profile that is most advantageous to her (i.e., pledge to the bank whenever the latter is solvent, i.e., whenever $\theta > 0$).[16] Instead, the policy maker adopts a robust approach to the design of the stress test. She evaluates any policy Γ under the "worst-case" scenario. That is, given any policy Γ, the policy maker expects the market to play according to the rationalizable profile most adversarial to her.

Definition 1. Given any policy Γ, the **most aggressive rationalizable profile** (MARP) consistent with Γ is the strategy profile $a^{\Gamma} \equiv (a_i^{\Gamma})_{i\in[0,1]}$ that minimizes the policy maker's ex-ante expected payoff over all profiles surviving *iterated deletion of interim strictly dominated strategies* (henceforth IDISDS).

In the IDISDS procedure leading to MARP, agents update their beliefs about the fundamentals θ and the other agents' exogenous signals $\mathbf{x} \in \mathbf{X}(\theta)$ using the common prior, F, the signal distribution, $P(x|\theta)$, the disclosure policy, Γ, and Bayes rule. Under MARP, given (x, s), each agent $i \in [0, 1]$, after receiving exogenous information x and endogenous information s, then refrains from pledging whenever there exists at least one conjecture over (θ, A) consistent with the above Bayesian updating and supported by all other agents playing strategies surviving IDISDS, under which refraining from pledging is a best response for the individual.

Remarks. Hereafter, we confine attention to policies Γ for which MARP exists. As it will become clear from the analysis below, because the game among the agents is supermodular, the strategy profile a^{Γ} is then a Bayes-Nash equilibrium (BNE) of the continuation game among the agents, and minimizes the policy maker's payoff state-by-state, and not just in expectation.

Furthermore, given a policy $\Gamma = (\mathcal{S}, \pi)$, when describing the agents' behavior, we do not distinguish between pairs (x, s) that are mutually consistent given Γ (meaning that the joint density of (x, s) is positive, i.e., $\int_{\theta: s\in supp(\pi(\theta))} p(x|\theta)dF(\theta) > 0$) and those that are not. Because the policy maker commits to the policy Γ, the abuse is legitimate and permits us to ease the exposition. Any claim about the optimality of the agents' behavior, however, should be interpreted to apply to pairs (x, s) that are mutually consistent given Γ.

3 Properties of Optimal Policies

We discuss three key properties of optimal policies.

[16] If she did, a simple monotone test revealing whether or not $\theta > 0$ would be optimal.

3.1 Perfect-coordination property

Definition 2. A policy $\Gamma = (\mathcal{S}, \pi)$ satisfies the **perfect-coordination property** (PCP) if, for any $\theta \in \Theta$, any exogenous information $\mathbf{x} \in \mathbf{X}(\theta)$, any $s \in supp(\pi(\theta))$, and any pair of individuals $i, j \in [0,1]$, $a_i^\Gamma(x_i, s) = a_j^\Gamma(x_j, s)$, where $a^\Gamma = (a_i^\Gamma)_{i\in[0,1]}$ is the most aggressive rationalizable profile (MARP) consistent with the policy Γ.

Hence, a disclosure policy has the perfect-coordination property if it induces all market participants to take the same action, after any information it discloses. For any $\theta \in \Theta$, any $s \in supp(\pi(\theta))$, let $r^\Gamma(\theta, s) \in \{0, 1\}$ denote the default outcome when investors play according to a^Γ (i.e., $r^\Gamma(\theta, s)$ is the fate of the bank that prevails at (θ, s), when the agents play according to MARP consistent with Γ).[17] Hereafter, we say that the policy Γ is *regular* if MARP under Γ is well defined and the default outcome under a^Γ is measurable in θ.[18]

Theorem 1. *Given any (regular) policy Γ, there exists another (regular) policy Γ^* satisfying the perfect coordination property and such that, for any θ, the default probability under Γ^* is the same as under Γ.*

Proof of Theorem 1: See the Appendix.

The policy Γ^* is obtained from the original policy Γ by disclosing, for each θ, in addition to the score $s \in supp(\pi(\theta))$ disclosed by the original policy Γ, the fate of the bank $r^\Gamma(\theta, s) \in \{0, 1\}$ under MARP consistent with the original policy Γ. That, under Γ^*, it is rationalizable for all agents to pledge when the policy discloses the information $(s, 1)$ and to refrain from pledging when the policy discloses the information $(s, 0)$ is straight-forward. In fact, the announcement of $(s, 1)$ (alternatively, of $(s, 0)$) makes it common certainty among the agents that $\theta > 0$ (alternatively, that $\theta \leq 1$).

The interesting part of the result is that, in the continuation game that starts after the policy Γ^* announces $(s, 1)$, pledging is the *unique* rationalizable action for any agent, irrespective of his signal. When, under the original policy Γ, given s, $r^\Gamma(\theta, s)$ is increasing in θ, the new piece of information that θ is such that $r^\Gamma(\theta, s) = 1$ is equivalent to the announcement that $\theta > \hat{\theta}(s)$, for some threshold $\hat{\theta}(s)$. In this case, agents update their first-order beliefs about θ upward when receiving the additional information that $r^\Gamma(\theta, s) = 1$. That each agent is more optimistic about the strength of the fundamentals, however, does not guarantee that MARP under the new policy is less aggressive than under the original policy. In fact, the new piece of information changes not only the agent's first-order beliefs about θ but also his higher-order beliefs and the latter matter for the determination of the most-aggressive rationalizable profile. Furthermore, in general, $r^\Gamma(\theta, s)$ need not be monotone in θ. This is because MARP under the original policy Γ need not entail strategies

[17] Because the cross-sectional distribution of signals is uniquely pinned down by $P(x|\theta)$, the fate of the bank under MARP is the same across any pair of signal profiles $\mathbf{x}, \mathbf{x}' \in \mathbf{X}(\theta)$ and hence depends only on Γ, θ, and s.

[18] Because the game has infinitely many states and players, these properties, while fairly natural, cannot be guaranteed for arbitrary policies.

that are monotone in x. As a result, the announcement that $r^{\Gamma}(\theta, s) = 1$ need not trigger an upward revision of the agents' beliefs.[19]

The result in Theorem 1 holds irrespectively of whether or not, given s, $r^{\Gamma}(\theta, s)$ is monotone in θ. It follows from the fact that, at any stage n of the IDISDS procedure, any agent who, under the original policy Γ would have pledged under the most aggressive strategy profile surviving $n-1$ rounds of deletion, continues to do the same under the new policy Γ^*. In the Appendix, we show that this last property in turn follows from the game being supermodular along with the fact that Bayesian updating preserves the likelihood ratio of any two states that are consistent with no default under the original policy Γ. Formally, for any $s \in supp(\pi(\Theta))$, and pair of states θ' and θ'' such that (a) $s \in supp(\pi(\theta'))$ and $s \in supp(\pi(\theta''))$, and (b) $r^{\Gamma}(\theta', s) = r^{\Gamma}(\theta'', s) = 1$, the likelihood ratio of such two states under Γ^* is the same as under the original policy Γ. This property, together with the announcement that default would have not occurred under MARP consistent with the original policy Γ, guarantees that, for any agent for whom pledging was optimal under MARP consistent with the original policy Γ, pledging is the unique rationalizable action under the new policy Γ^*.[20]

The policy Γ^* thus removes any strategic uncertainty. When $(s, 1)$ (alternatively, $(s, 0)$) is announced, each agent knows that all other agents will pledge (alternatively, will refrain from pledging), irrespective of their exogenous private information. Importantly, while the policy Γ^* removes any strategic uncertainty, it preserves structural uncertainty (i.e., heterogeneity in the agents' posterior beliefs about θ). To avoid default at certain fundamentals, it is essential that agents who pledge are uncertain as to whether other agents pledge because they find it dominant to do so, or because they expect other agents to pledge, which requires heterogeneity in posterior beliefs.

When it comes to stress testing, Theorem 1 implies that optimal tests should not be expected to create conformism in market beliefs about the soundness of a bank under scrutiny. On the other hand, there is no value in leaving any room to ambiguity as to whether or not the bank will succeed in raising the liquidity it need to continue operating.

[19] In richer settings, the fate of the bank may depend also on variables other than θ for which both the policy maker and the market have imperfect information about. As we discuss in Section 4, when this is the case, perfect coordination can still be attained but it involves disclosing information other than the predicted fate of the bank. We come back to this point in due course.

[20] Formally, as we show in the Appendix, the above two properties jointly imply that each agent's posterior beliefs after $r^{\Gamma}(\theta$,s$)$=1 is announced are a "truncation" of the agent's beliefs under the original policy Γ, with the truncation eliminating from the support of the agent's beliefs states θ at which, under the most aggressive profile surviving n rounds of IDISDS under the original policy Γ, the agent's payoff from pledging would have been negative. The truncation thus contributes to making the agent more willing to pledge.

3.2 Pass/Fail

Theorem 2. *Suppose that $p(x|\theta)$ is log-supermodular.*[21] *Given any policy Γ satisfying the perfect coordination property, there exists a binary policy $\Gamma^* = (\{0,1\}, \pi^*)$ that also satisfies the perfect coordination property and such that, for any θ, the probability of default under Γ^* is the same as under Γ.*

Proof of Theorem 2: See the Appendix.

Take any policy $\Gamma = (\mathcal{S}, \pi)$ satisfying the perfect coordination property. Given the result in Theorem 1, without loss of generality, assume that $\Gamma = (\mathcal{S}, \pi)$ is such that $\mathcal{S} = \{0,1\} \times S$, for some measurable set S, and is such that (a) when the policy discloses any signal $(s, 1)$, all agents pledge and default does not happen, whereas (b) when the policy discloses any signal $(s, 0)$, all agents refrain from pledging and default happens. Given the policy Γ, let $U^{\Gamma}(x, (s,1)|k)$ denote the expected payoff differential of an agent with exogenous information x who receives information $(s, 1)$ from the policy maker and who expects all other agents to pledge if and only if their exogenous signal exceeds a cut-off k. In the Appendix, we show that, no matter the shape of the policy Γ, because $p(x|\theta)$ has the *monotone likelihood ratio property* (in short, MLRP), MARP associated with the policy Γ is in cut-off strategies. Hence, each agent's expected payoff differential when all other agents play according to MARP can be written as $U^{\Gamma}(x, (s,1)|k)$ for some k that depends on s. That the original policy Γ satisfies the perfect-coordination policy in turn implies that, for any s and k such that $(k, (s,1))$ are mutually consistent,[22] $U^{\Gamma}(k, (s,1)|k) > 0$. That is, the expected payoff differential of any agent whose private signal x coincides with the cutoff k must be strictly positive. If this were not the case, the continuation game would also admit a rationalizable profile (in fact, a continuation equilibrium) in which some of the agents refrain from pledging, contradicting the fact that pledging irrespectively of x is the unique rationalizable profile following the announcement of $(s, 1)$.

Now consider a policy Γ^* that, for any θ, discloses the same outcome $r^{\Gamma}(\theta, s)$ as the original policy Γ but conceals the additional information s. By the law of iterated expectations, for all k such that $(k, (s,1))$ are mutually consistent, because $U^{\Gamma}(k, (s,1)|k) > 0$ then $U^{\Gamma^*}(k, 1|k) > 0$. The last property implies that the new policy Γ^* also satisfies the perfect-coordination property. The policy maker can thus drop the additional signals s from the original policy Γ and continue to guarantee that, after $r = 1$ is announced, pledging is the unique rationalizable action for all agents. The result in the theorem thus implies that simple pass/fail policies are optimal.

The property that justifies restricting attention to simple pass/fail policies is the log-supermodularity of the signal distribution $p(x|\theta)$. This property, which is formally equivalent to MLRP, is essential for the optimality of simple pass/fail policies.[23]

[21] The property that $p(x|\theta)$ is log-supermodular means that, for any $x', x'' \in \mathbb{R}$, with $x' < x''$, and any $\theta', \theta'' \in \Theta$, with $\theta'' > \theta'$, then $p(x''|\theta'')p(x'|\theta') \geq p(x''|\theta')p(x'|\theta'')$.

[22] Recall that the latter means that the set $\theta \in \Theta$ such that (a) $k \in \varrho_\theta$ and $(s, 1) \in supp(\pi(\theta))$ have positive measure.

[23] The example below features signals drawn from a distribution with finite support. This, however, is only for

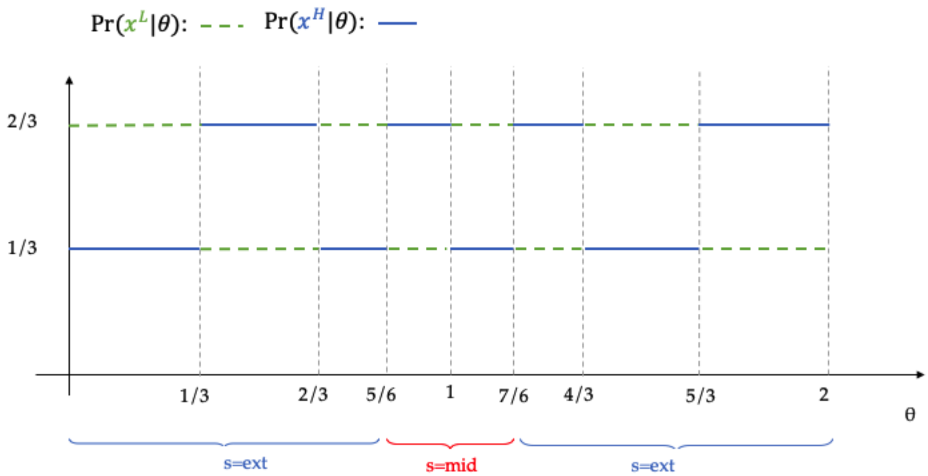

Figure 1: Suboptimality of simple pass/tail tests

Example 1. Suppose that θ is drawn from a uniform distribution over $[-1, 2]$. Given θ, each agent $i \in [0, 1]$ receives an exogenous signal $x_i \in \{x^L, x^H\}$, drawn independently across agents from a Bernoulli distribution with probability

$$Pr(x^L|\theta) = \begin{cases} 2/3 & \text{if } \theta \in (0, 1/3) \cup [2/3, 5/6) \cup [1, 7/6) \cup [4/3, 5/3) \\ 1/3 & \text{if } \theta \in [1/3, 2/3) \cup [5/6, 1) \cup [7/6, 4/3) \cup [5/3, 2). \end{cases}$$

The value of $Pr(x^L|\theta)$ for $\theta \in [-1, 0]$ plays no role in this example, so it can be taken arbitrarily. Suppose that agents' payoffs are such that $g(\theta) = 1 - c$ and $b(\theta) = -c$, for all θ, with $c \in (1/2, 8/15)$. There exits a deterministic policy that satisfies PCP and guarantees that no $\theta > 0$ defaults, but no deterministic pass/fail policy can spare all $\theta > 0$ from default.

Proof of Example 1. Figure 1 illustrates the signal structure considered in Example 1. The dash line depicts the probability of signal x^L whereas the solid line the complementary probability of signal x^H, as a function of θ. Note that the agents' posterior beliefs under the signal structure of Example 1 can be ranked according to FOSD. Each agent observing x^H has posterior beliefs that dominate those of each agent observing x^L. Nonetheless, the ratio $p(x^H|\theta)/p(x^L|\theta)$ is not increasing in θ over the entire domain, meaning that $p(x|\theta)$ is not log-supermodular. Also note that, under the payoff specification in the example, pledging is rationalizable if the probability of default is no greater than $1 - c$, whereas not pledging is rationalizable if such a probability is at least $1 - c$.

To see that there exists no pass/fail policy sparing all $\theta > 0$ from default, note that, by virtue of Theorem 1, if such a policy existed, there would also exist a binary policy satisfying PCP and such

simplicity. Conclusions similar to those in the example can be established by having the agents receive signals drawn from a continuous distribution. We thank Tommaso Denti for suggesting a related example with finite signals and Leifu Zhang for suggesting a related example with continuous signals.

that $\pi(1|\theta) = 0$ for all $\theta \leq 0$ and $\pi(1|\theta) = 1$ for all $\theta > 0$, with $\pi(1|\theta)$ denoting the probability that the policy discloses signal $s = 1$ when the fundamentals are θ. Under such a policy, after hearing that $s = 1$, no matter the private signal x, each agent assigns probability $1/2$ to $\theta \in [0, 1]$ and probability $1/2$ to $\theta \in [1, 2]$. Because $c > 1/2$, each agent expecting all other agents to refrain from pledging (and hence default to occur for all $\theta \in [0, 1]$) then finds it optimal to do the same. Hence, under MARP consistent with the above policy, after the signal $s = 1$ is announced, all agents refrain from pledging, meaning that the above policy fails to spare all $\theta > 0$ from default.

To see that all types $\theta > 0$ can be spared from default under richer policies, consider the policy $\Gamma = (\{0, (1, mid), (1, ext)\}, \pi)$ that, in addition to publicly announcing whether or not the bank passed the test, also announces whether fundamentals are extreme (i.e., $\theta \in (0, 5/6) \cup (7/6, 2]$), or intermediate (i.e., $\theta \in [5/6, 7/6]$). Formally, for any $\theta \in [-1, 0]$, $\pi(0|\theta) = 1$, meaning that the policy announces with certainty $s = 0$ meaning that the bank failed the test. For any $\theta \in [5/6, 7/6]$, $\pi(1, mid|\theta) = 1$, meaning that the the policy announces with certainty that the bank passed the test and that fundamentals are intermediate. Finally, for any $\theta \in (0, 5/6) \cup (7/6, 2]$, $\pi(1, ext|\theta) = 1$ meaning that the policy announces with certainty that the bank passed the test and that fundamentals are extreme. See again Figure 1 for a graphical representation of such a policy.

Under MARP associated with such a policy, all agents pledge when they hear that the bank passed the test, no matter whether the policy announces that fundamentals are extreme or intermediate, whereas all agents refrain from pledging when hearing that the bank failed the test.

Consider first the case in which the fundamentals are extreme, i.e., $\theta \in (0, 5/6) \cup (7/6, 2]$. All agents with exogenous signal x^H find it dominant to pledge when they hear that $s = (1, ext)$. In fact, even if all other agents were to refrain from pledging, the probability that each agent with signal x^H assigns to $\theta > 1$ (and hence to the bank surviving) is $Pr(\theta > 1|x^H, ext) = 8/15 > c$, making it dominant for the individual to pledge. As a consequence of this property, each agent receiving an exogenous signal x^L finds it *iteratively* dominant to pledge. This is because, for any $\theta \in [1/3, 5/6]$, even if all agents receiving a signal equal to x^L were to refrain from pledging, the aggregate size of the pledge from those agents receiving an x^H signal would suffice for the bank to survive. This means that the probability that each agent with signal x^L assigns to the bank surviving is at least equal to $Pr(\theta > 1/3|x^L, (1, ext)) = 11/15$, implying that it is optimal for the agent to pledge.

Next, consider the case in which fundamentals are intermediate, i.e., $\theta \in [5/6, 7/6]$. In this case, each agent with a signal equal to x^L assigns probability $2/3 > c$ to $\theta \geq 1$ and hence finds it dominant to pledge. Because, for any $\theta \in (5/6, 1)$, $1/3$ of the agents receives an x^L signal, the minimal size of the pledge that each agent with signal equal to x^H can expect at any $\theta \in (5/6, 1)$ is thus equal to $Pr(x^L|\theta) = 1/3 > 1 - \theta$, implying that even if all agents with signal equal to x^H were to refrain from pledging, the bank would survive. Because of the above properties, pledging is iteratively dominant for those agents receiving the x^H signal.

Hence, under the proposed policy, default does not occur for any $\theta > 0$. Because, under MARP, all agents pledge when they hear that the bank passed the test, no matter whether they hear that the fundamentals are extreme or intermediate, one may conjecture that the policy maker could refrain from specifying whether the fundamentals are extreme or intermediate and simply announce that the bank passed the test. However, as explained above, such a simple pass/fail policy would not induce all agents to pledge when playing according to MARP. □

The above example illustrates both the failure of the Revelation Principle (when the market is expected to play according to MARP, it is *with* loss of generality to confine attention to policies that take the form of action recommendations), as well as the sub-optimality of simple pass/fail tests, when beliefs and fundamentals do not co-move according to the monotone likelihood ratio property.

3.3 Monotone Rules

We now turn to the optimality of policies that fail with certainty institutions with weak fundamentals and pass with certainty those with strong fundamentals. For any $(\theta, A) \in \Theta \times [0,1]$, let

$$u(\theta, A) \equiv g(\theta)\mathbb{I}(A > 1-\theta) + b(\theta)\mathbb{I}(A \leq 1-\theta)$$

and

$$U^P(\theta, A) \equiv W(\theta)\mathbb{I}(A > 1-\theta) + L(\theta)\mathbb{I}(A \leq 1-\theta)$$

denote the payoffs of a representative agent and of the policy maker, respectively, when the fundamentals are θ and the aggregate size of the pledge is A. Let

$$\bar{x}_G \equiv \sup\left\{x \in \mathbb{R} : \int_\Theta u(\theta, 1-P(x|\theta))\, \mathbb{I}(\theta \geq 0) p(x|\theta)\, dF(\theta) \leq 0\right\} \tag{1}$$

denote the largest signal threshold x such that, when each agent pledges when receiving a signal above x and does not pledge when receiving a signal below x, then the expected payoff from pledging for the marginal agent with signal x is non-positive under the additional information that θ is non-negative. As we show in the Appendix, $\bar{x}_G$ corresponds to an upper bound for the set of cut-offs characterizing the strategies consistent with MARP across all disclosure policies Γ satisfying the perfect coordination property.

Next, for any $\theta \in (0,1)$, let $x^*(\theta)$ be the critical signal threshold such that, when agents follow a cut-off strategy with threshold $x^*(\theta)$ (that is, pledge for $x > x^*(\theta)$ and refrain from pledging for $x < x^*(\theta)$), then default occurs if and only if the fundamentals are below θ.[24] For any $\theta \in (0,1)$, the threshold $x^*(\theta)$ is implicitly defined by

$$P(x^*(\theta)|\theta) = \theta. \tag{2}$$

Let

$$\theta^* \equiv \inf\left\{\hat{\theta} \geq 0 : \int_{\hat{\theta}}^{\infty} u\left(\tilde{\theta}, 1-P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) dF(\tilde{\theta}) \geq 0 \text{ for all } \theta \in \left[\hat{\theta}, 1\right)\right\} \tag{3}$$

[24] When the noise in the agents' signals is bounded, the definition of $x^*(\theta)$ can be extended to $\theta = 0$ and $\theta = 1$. When the noise is unbounded, abusing notation, one can extend the definition to $\theta = 0$ and $\theta = 1$ by letting $x^*(0) = -\infty$ and $x^*(1) = +\infty$.

be the lowest truncation point $\hat{\theta}$ such that, when the policy reveals that fundamentals are above $\hat{\theta}$, then for any possible default threshold $\theta \in \left[\hat{\theta}, 1\right)$, if default were to occur for fundamentals below θ and not occur for fundamentals above θ, then the marginal agent with signal $x^*(\theta)$ would find it optimal to pledge. Finally, for any x, let $\Theta(x) \equiv \{\theta \in \Theta : x \in \varrho_\theta\}$ denote the set of fundamentals that are consistent with signal x.

Condition M. *The following properties hold*:

1. $\inf \Theta(\bar{x}_G) \leq 0$;

2. *the functions* $p\left(x|\theta\right)$ *and* $|u(\theta, 1 - P(x|\theta))|$ *are log-supermodular over*[25]

$$\{(\theta, x) \in [0, 1] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\}\,;$$

3. *for any* $\theta_0, \theta_1 \in [0, 1]$, *with* $\theta_0 < \theta_1$, *and any* $x \leq \bar{x}_G$ *such that (a)* $u(\theta_1, 1 - P(x|\theta_1)) \leq 0$ *and (b)* $x \in \varrho_{\theta_0}$,

$$\frac{U^P(\theta_1, 1) - U^P(\theta_1, 0)}{U^P(\theta_0, 1) - U^P(\theta_0, 0)} > \frac{p\left(x|\theta_1\right) u\left(\theta_1, 1 - P\left(x|\theta_1\right)\right)}{p\left(x|\theta_0\right) u\left(\theta_0, 1 - P\left(x|\theta_0\right)\right)}. \tag{4}$$

Property 1 in Condition M says that the lower bound of the support of the beliefs of the marginal agent with signal $\bar{x}_G$, where $\bar{x}_G$ is the threshold defined in (1), is not strictly positive. Clearly, this property trivially holds when, for any θ, the agents' signals are drawn from a distribution whose support is large enough (and hence, a fortiori, when the noise in the agents' signals is drawn from a distribution with unbounded support, e.g., a Normal distribution).

Property 2 says that signals are ordered according to MLRP and that the (percentage) reduction in the agents' loss from pledging due to higher fundamentals is larger when more agents pledge. Formally, for any $\theta' < \theta''$ and $x' < x''$ such that $u(\theta'', 1 - P(x'|\theta'')) < 0$,

$$\frac{u(\theta', 1 - P(x''|\theta')) - u(\theta'', 1 - P(x''|\theta''))}{u(\theta', 1 - P(x''|\theta'))} \leq \frac{u(\theta', 1 - P(x'|\theta')) - u(\theta'', 1 - P(x'|\theta''))}{u(\theta', 1 - P(x'|\theta'))}. \tag{5}$$

Note that $u(\theta'', 1-P(x'|\theta'')) < 0$ implies that $u(\theta', 1-P(x'|\theta')), u(\theta', 1-P(x''|\theta')), u(\theta'', 1-P(x''|\theta'')) < 0$. The left-hand side of (5) is thus the percentage reduction in the agents' payoff loss when fundamentals improve from θ' to θ'' and agents pledge when receiving a signal $x \geq x''$. The right-hand side of (5), instead, is the percentage reduction in the agents' payoff loss when fundamentals improve from θ' to θ'' and agents pledge for $x \geq x'$. Importantly, this property is required to hold only for fundamentals θ and thresholds x for which the agents' expected payoffs from pledging, $u(\theta, 1 - P(x|\theta))$, is non-positive. Also note that this property trivially holds in the baseline model considered so far,

[25] That $|u(\theta, 1 - P(x|\theta))|$ is log-supermodular over $\{(\theta, x) \in [0, 1] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\}$ means that, for any $x', x'' \in \mathbb{R}$, with $x' < x''$, and any $\theta', \theta'' \in \Theta$, with $\theta'' > \theta'$, such that $u(\theta'', 1 - P(x'|\theta'')) \leq 0$,

$$u(\theta'', 1 - P(x''|\theta''))u(\theta', 1 - P(x'|\theta')) \geq u(\theta'', 1 - P(x'|\theta''))u(\theta', 1 - P(x''|\theta')).$$

for payoffs $u(\theta, A)$ are invariant in A conditional on the fate of the bank. The reason for stating the condition in these more general terms is that, as we show in the next section, Condition M above plays a key role for the optimality of monotone policies also under richer payoff specifications in which $u(\theta, A)$ depends on A over and above the effect that the latter has on the default outcome.

Property 3 in turn says that the benefit that the policy maker derives from changing the agents' behavior (inducing all agents to pledge starting from a situation in which no agent pledges) increases with the fundamentals at a sufficiently high rate, with the critical rate determined by a combination of the agents' payoffs and beliefs (the right-hand-side of (4)). Such a property is required to hold only for fundamentals θ_0 and θ_1 in the critical region and for signal realizations $x \leq \bar{x}_G$ such that (a) $u(\theta_1, 1 - P(x|\theta_1)) \leq 0$ (meaning that the payoff from pledging is negative when agents pledge for signals above x and refrain from pledging for signals below x), and (b) $x \in \varrho_{\theta_0}$ (meaning that signal x is consistent with the fundamentals being θ_0). Also note that, in the baseline model considered so far, the right-hand side of (4) is equal to $p(x|\theta_1)\, b(\theta_1) / p(x|\theta_0)\, b(\theta_0)$. Once again, the reason for the more general condition is that the result in Theorem 3 below extends to richer payoff specifications, as we explain in the next section.

We then have the following result:

Theorem 3. *Suppose Condition M holds. Given any policy* Γ*, there exists a deterministic monotone policy* $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ *satisfying the perfect-coordination property and yielding the policy maker a payoff weakly higher than* Γ*. The policy* $\Gamma^{\hat{\theta}}$ *is such that there exists a threshold* $\hat{\theta} \in [0,1]$ *such that, for any* $\theta \leq \hat{\theta}$*,* $\pi^{\hat{\theta}}(\theta)$ *assigns probability one to* $s = 0$*, whereas for any* $\theta > \hat{\theta}$*,* $\pi^{\hat{\theta}}(\theta)$ *assigns probability one to* $s = 1$*.*

Proof of Theorem 3: See the Appendix.

When Condition M holds, the choice of the optimal policy reduces to the choice of the smallest threshold $\hat{\theta}$ such that, when agents commonly learn that $\theta > \hat{\theta}$, under the unique rationalizable profile, all agents pledge, irrespective of their signals. For this to be the case, it must be that, for any $x \in \mathbb{R}$, $\int_{\hat{\theta}}^{\infty} u(\theta, 1 - P(x|\theta))p(x|\theta)dF(\theta) > 0$. The above problem, however, does not have a formal solution, due to the lack of upper-hemicontinuity of the designer's payoff in $\hat{\theta}$.[26] Notwithstanding these complications, hereafter we follow the pertinent literature and refer to the monotone policy with cut-off $\hat{\theta} = \theta^*$, with θ^* as defined in (**??**), as the "optimal monotone policy".[27]

As we show in the Appendix, property 1 in Condition M guarantees that, starting from the optimal monotone policy (the one with cut-off θ^*), one cannot perturb the policy by assigning a pass

[26]This problem was first noticed in Goldstein and Huang (2016).

[27]The reason why this is an abuse is that, under the monotone policy with cut-off θ^*, in the continuation game that starts after the policy maker announces that the bank passed the test, there exists a rationalizable profile in which some of the agents refrain from pledging. However, there exists a monotone policy with cut-off $\hat{\theta}$ arbitrarily close to the threshold θ^* such that, after the policy maker announces that the bank passed the test, the unique rationalizable profile features all agents pledging. Because the policy maker's payoff under the latter policy is arbitrarily close to the one she obtains when all agents pledge for $\theta > \theta^*$ and refrain from pledging when $\theta \leq \theta^*$, the abuse appears justified.

grade also to a small interval $[\theta', \theta'']$ of fundamentals with $0 \le \theta' < \theta'' < \theta^*$, while guaranteeing that all agents necessarily pledge when hearing that the bank passed the test (i.e., when hearing that $s = 1$). This property trivially holds when the noise in the agents' signals is large (and hence, a fortiori, when noise is unbounded), but plays a key role when the noise is drawn from a bounded interval of small size (see Example 2 below for an illustration).

Properties 2 and 3 of Condition M in turn guarantee that, given a non-monotone rule, perturbations of the original policy that swap the probability of inducing all agents to pledge from low to high fundamentals in a way that preserves the agents' incentives to pledge (under MARP) when hearing that $s = 1$, increase the policy maker's payoff. These conditions guarantee that the higher value the policy maker derives from saving banks with stronger fundamentals compensates for the possibility that, from an ex-ante perspective, the probability of default may be larger under monotone policies than under non-monotone ones (see Example 3 for an illustration of why non-monotone rules may permit the policy maker to save a larger set of banks).

Condition M is fairly sharp in the sense that, when violated, one can construct examples where the optimal policy is non-monotone. We provide two such examples below. Example 2 illustrates the role of property 1 in Condition M, whereas Example 3 illustrates the role of properties 2 and 3 in Condition M.

Let $\theta^{MS} \in (0, 1)$ be implicitly defined by the unique solution to

$$\int_0^1 u(\theta^{MS}, A)dA = 0. \tag{6}$$

The threshold θ^{MS} corresponds to the value of the fundamentals at which an agent who knows θ and holds *Laplacian* beliefs with respect to the measure of agents pledging is indifferent between pledging and not pledging.[28] Importantly, θ^{MS} is independent of the initial common prior F and of the distribution of the agents' signals.

Example 2. Suppose that there exist scalars $g, b, W, L \in \mathbb{R}$, with $g > 0 > b$ and $W > L$, such that, for any θ, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$, and $L(\theta) = L$. Suppose that θ is drawn from a uniform distribution with support $[-K, 1 + K]$, for some $K \in \mathbb{R}_{++}$. Finally, assume that the agents' exogenous signals are given by $x_i = \theta + \sigma\epsilon_i$, with $\sigma \in \mathbb{R}_+$ and with each ϵ_i drawn independently across agents from a uniform distribution over $[-1, 1]$, with $\sigma < K/2$. Let θ^*_σ be the threshold defined in (3), applied to the primitives described in this example (with the subscript σ used to emphasize the dependence on the noise in the agents' exogenous signals). There exists $\sigma^\# \in (0, K/2)$ such that (a) $\inf \Theta(x^*_{\sigma^\#}(\theta^{MS})) > 0$, and (b) for all $\sigma \in (0, \sigma^\#)$, starting from the optimal monotone policy with cut-off θ^*_σ (the one saving the largest set of fundamentals over all monotone rules), there exists a deterministic *non-monotone* policy satisfying the perfect-coordination property and sparing the

[28]This means that the agent believes that the proportion of agents pledging is uniformly distributed over $[0, 1]$. See Morris and Shin (2006).

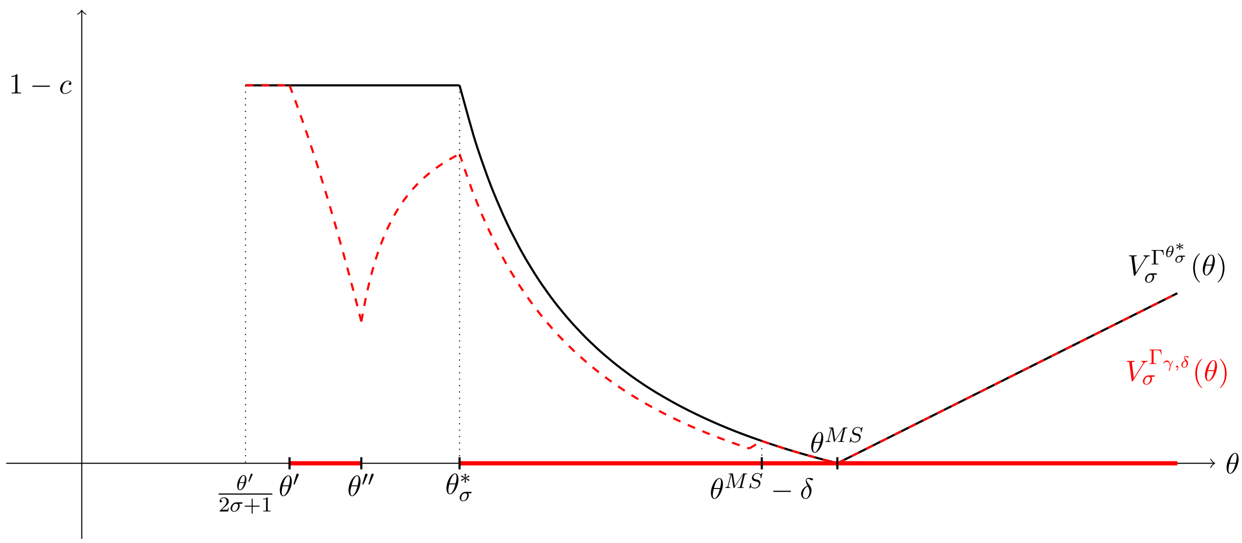

Figure 2: Suboptimality of deterministic binary monotone policies.

banks from default over a set of fundamentals of strictly larger probability measure than the optimal monotone policy (and hence yielding the policy maker a strictly higher payoff).

Proof of Example 2: The formal proof is in the online Supplement. Here we sketch the key arguments. To fix ideas, let $g = 1 - c$ and $b = -c$, with $c \in (0, 1)$, as in Example 1. For any $\theta \in [0, 1]$, let $x^*_\sigma(\theta)$ be the critical signal threshold such that, when the quality of the agents' exogenous information is σ and all agents pledge for $x > x^*_\sigma(\theta)$ and refrain from pledging for $x < x^*_\sigma(\theta)$, a bank survives if its fundamentals exceed θ and defaults otherwise, as defined in (2).[29] Note that, under the specification in this example $x^*_\sigma(\theta) = (1 + 2\sigma)\theta - \sigma$. For any binary policy $\Gamma = (\{0, 1\}, \pi)$, any quality of the agents' exogenous information σ, and any threshold $\theta \in [0, 1]$ such that $(x^*_\sigma(\theta), 1)$ are mutually consistent, let $V^\Gamma_\sigma(\theta)$ be the payoff of the marginal agent with signal $x^*_\sigma(\theta)$ when each agent with signal below $x^*_\sigma(\theta)$ refrains from pledging and each agent with signal above $x^*_\sigma(\theta)$ pledges, after the policy Γ announces that $s = 1$. That is,

$$V^\Gamma_\sigma(\theta) \equiv U^\Gamma_\sigma(x^*_\sigma(\theta), 1|x^*_\sigma(\theta)),$$

where U^Γ_σ is the function defined after Theorem 2.

Now, for any $\hat{\theta} \in \Theta$, let $\Gamma^{\hat{\theta}} = (\{0, 1\}, \pi^{\hat{\theta}})$ be the deterministic monotone policy with cut-off $\hat{\theta}$ (that is, the policy that discloses $s = 1$ with certainty when fundamentals are above $\hat{\theta}$ and discloses $s = 0$ with certainty when fundamentals are below $\hat{\theta}$). Note that the absence of any public disclosure is equivalent to a monotone policy with cut-off $\hat{\theta} = \min\Theta = -K$ and that, under such a policy, the default threshold is $\theta^{MS} = c$.[30]

[29]That is, for $\tilde{\theta} \le \theta$, $1 - P(x^*_\sigma(\theta)|\tilde{\theta}) \le 1 - \tilde{\theta}$, whereas, for $\tilde{\theta} > \theta$, $1 - P(x^*_\sigma(\theta)|\tilde{\theta}) > 1 - \tilde{\theta}$.

[30]To see this, observe that, for any $\theta \in [0, 1]$, $V^{\Gamma^{\min\Theta}}_\sigma(\theta) = Pr\left(\tilde{\theta} > \theta|x^*_\sigma(\theta)\right) - c = \theta - c$, which is strictly positive for

Compared to a situation in which the policy reveals no information, the knowledge that the fundamentals are above a threshold $\hat{\theta} \in [0,1]$, other things equal, increases the payoff of the marginal agent from pledging. However, because the noise in the agents' signals is bounded, the announcement that fundamentals are above $\hat{\theta}$ has a bite on the marginal agent's payoff only insofar $\theta \leq (\hat{\theta}+2\sigma)/(1+2\sigma)$. In fact, when $\theta > (\hat{\theta}+2\sigma)/(1+2\sigma)$, $x^*_\sigma(\theta) - \sigma > \hat{\theta}$ meaning that the marginal agent with signal $x^*_\sigma(\theta)$ already knows that fundamentals are above $\hat{\theta}$ from the observation of his own signal and thus learns nothing from learning that the bank passed the test (i.e., that fundamentals are above $\hat{\theta}$).

A necessary and sufficient condition for all agents to pledge under MARP associated with a monotone policy $\Gamma^{\hat{\theta}}$ is that, for any possible default threshold $\theta > \hat{\theta}$, $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) > 0$.[31] This implies that the cut-off θ^*_σ for the optimal deterministic monotone rule is $\theta^*_\sigma = x^*_\sigma(\theta^{MS}) - \sigma$.[32]

Now to see that the above monotone policy is improvable, assume that σ is small so that $\inf \Theta\left(x^*_\sigma\left(\theta^{MS}\right)\right) = x^*_\sigma\left(\theta^{MS}\right) - \sigma > 0$. Next, pick $\delta, \gamma > 0$ small and let $\theta'' \equiv x^*_\sigma(\theta^{MS} - \delta) - \sigma$ and $\theta' \equiv \theta'' - \gamma$. That $\inf \Theta\left(x^*_\sigma\left(\theta^{MS}\right)\right) > 0$ implies that it is possible to find $\delta, \gamma > 0$ small so that θ'' and θ' are non-negative. Next, consider a policy $\Gamma_{\gamma,\delta} = (\{0,1\}, \pi_{\gamma,d})$ that, in addition to passing all banks with fundamentals above θ^*_σ, also passes those with fundamentals $\theta \in [\theta', \theta'']$.[33] Let $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta)$ be the payoff of the marginal agent with signal $x^*_\sigma(\theta)$ when the policy $\Gamma_{\gamma,\delta}$ announces that $s = 1$, thus revealing that fundamentals belong to $[\theta', \theta''] \cup [\theta^*_\sigma, 1+K]$. This payoff is represented in Figure 2 along with the payoff $V_\sigma^{\Gamma^{\theta^*_\sigma}}(\theta)$ under the optimal monotone rule. Provided that δ and γ are small, $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta) \geq 0$ for all $\theta \in [\theta'/(1+2\sigma), 1]$, with $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta) = 0$ if and only if $\theta = \theta^{MS}$.[34] Starting from $\Gamma_{\gamma,\delta}$, one can then further perturb the policy $\Gamma_{\gamma,\delta}$ by giving a pass grade with certainty to banks with fundamentals in $[\theta', \theta''] \cup [\theta^*_\sigma + \varepsilon, +\infty)$, with $\varepsilon > 0$ but small, and failing with certainty the others. The new policy $\tilde{\Gamma}$ so constructed is such $V_\sigma^{\tilde{\Gamma}}(\theta) > 0$ for all $\theta \in [\theta'/(1+2\sigma), 1]$ meaning that, when $\tilde{\Gamma}$ announces that $s = 1$, under MARP all agents pledge, no matter their signal. Hence the policy $\tilde{\Gamma}$ so constructed satisfies the perfect-coordination property. That it strictly improves upon the original deterministic optimal monotone one follows directly from the fact that it spares a bank

$\theta > \theta^{MS} = c$ and strictly negative for $\theta < \theta^{MS}$. Hence, in the absence of any public disclosure, the unique rationalizable profile features all agents pledging for $x > x^*_\sigma(\theta^{MS})$ and all agents refraining from pledging for $x < x^*_\sigma(\theta^{MS})$.

[31] Indeed, if there exists $\theta > \hat{\theta}$ such that $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) \leq 0$, then not pledging when receiving a signal below $x^*_\sigma(\theta)$ is rationalizable. In this case, there must exist $\theta' > \hat{\theta}$ such that $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta') = 0$. In the continuation game that starts after the announcement that $s = 1$, refraining from pledging for any $x < x^*_\sigma(\theta')$ and pledging for any $x > x^*_\sigma(\theta')$ is thus a continuation equilibrium. Because $x^*_\sigma(\theta') > x^*_\sigma(\theta)$, it is then easy to see that not pledging for $x < x^*_\sigma(\theta)$ is rationalizable.

[32] As explained above, for any $\theta > (\hat{\theta}+2\sigma)/(1+2\sigma)$, the announcement that fundamentals are above $\hat{\theta}$ has no effect on the payoff of the marginal agent with signal $x^*_\sigma(\theta)$, meaning that $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) = V_\sigma^{\Gamma^{\min \Theta}}(\theta)$. Because $V_\sigma^{\Gamma^{\min \Theta}}(\theta) < 0$ for $\theta < \theta^{MS}$, we thus have that, for any $\hat{\theta} < x^*_\sigma(\theta^{MS}) - \sigma$, and any $\theta \in \left((\hat{\theta}+2\sigma)/(1+2\sigma), \theta^{MS}\right)$, $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) = \theta - c < 0$, meaning that not pledging is rationalizable for some agents.

[33] Formally, $\pi_{\gamma,\delta}$ discloses $s = 1$ with certainty when $\theta \in [\theta', \theta''] \cup [\theta^*_\sigma, 1+K]$ and discloses $s = 0$ with certainty when $\theta \in (-K, \theta') \cup (\theta'', \theta^*_\sigma)$.

[34] For $\theta < \theta'/(1+2\sigma)$, $x^*_\sigma(\theta) + \sigma < \theta'$, meaning that $(x^*_\sigma(\theta), 1)$ are not mutually consistent and hence the payoff function $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta)$ is not well defined.

from default over a set of fundamentals of strictly larger probability measure. □

The reason why the non-monotone policy $\tilde{\Gamma}$ constructed in the proof of Example 2 spares more banks from default than the optimal deterministic monotone policy with threshold θ^*_σ is that agents receiving signals around θ^{MS} are highly sensitive to the grade the policy gives to banks with fundamentals around θ^{MS} but not so much so to to the grade the policy gives to fundamentals far from θ^{MS}. In the above example with bounded noise, an agent receiving a signal $x^*_\sigma(\theta^{MS})$ is not sensitive at all to the grade the policy gives to a bank with fundamentals below $x^*_\sigma(\theta^{MS}) - \sigma$ given that his private signal informs him that the fundamentals are above $x^*_\sigma(\theta^{MS}) - \sigma$. Hence, while it is impossible to amend the optimal deterministic monotone policy (the one with cut-off $\theta^*_\sigma = x^*_\sigma(\theta^{MS}) - \sigma$) by giving a pass grade also to banks with fundamentals slightly below θ^*_σ (say, with fundamentals $\theta \in [\theta^*_\sigma - \varepsilon, \theta^*_\sigma]$), without inducing some of the agents to refrain from pledging, it is possible to amend the optimal deterministic monotone policy by extending the pass grade to a small interval $[\theta', \theta'']$ of fundamentals with $0 \leq \theta' < \theta'' < \theta^*_\sigma$, while continuing to induce all agents to pledge under MARP. Property 1 in Condition M implies that $x^*_\sigma(\theta^{MS}) - \sigma < 0$ thus making such perturbations unfeasible.

Next, consider the other two properties in Condition M. Given any policy $\Gamma = (\{0,1\}, \pi)$ in which π is deterministic (meaning that, for any θ, $\pi(\theta)$ assigns probability 1 either to $s = 1$ or to $s = 0$), let $D^\Gamma = \left\{(\underline{\theta}_i, \bar{\theta}_i] : i = 1, ..., N\right\}$ denote the partition of $\left(0, \theta^{MS}\right]$ induced by π, with $N \in \mathbb{N}$, $\theta_1 = 0$, and $\theta_N = \theta^{MS}$.[35] Let $d \in D^\Gamma$ denote a generic cell of the partition D^Γ and, for any $\theta \in [0, \theta^{MS}]$, denote by $d^\Gamma(\theta) \in D^\Gamma$ the cell that contains θ. Finally, let $M(\Gamma) \equiv \max_{i=1,...,N} |\bar{\theta}_i - \underline{\theta}_i|$ denote the *mesh* of D^Γ, that is, the Lebesgue measure of the cell of D^Γ of maximal Lebesgue measure.

The next example considers an economy in which the noise in the agents' exogenous signals is drawn from an unbounded distribution (in which case, property 1 in Condition M trivially holds), but where properties 2 and 3 in Condition M are violated. It shows that any deterministic policy giving the same grade to an interval of types to the left of θ^{MS} of measure larger than $\mathcal{E}(\sigma)$, with $\mathcal{E}(\sigma)$ going to zero as σ goes to zero, can be improved upon by a non-monotone deterministic policy with a smaller mesh. Optimal policies are thus highly non-monotone

Example 3. Suppose that θ is drawn from an improper uniform prior over $\mathbb{R}$ and that the agents' signals are given by $x_i = \theta + \sigma\varepsilon_i$ with ε_i drawn from a standard Normal distribution.[36] Further assume that there exist scalars $g, b, W, L \in \mathbb{R}$, with $g > 0 > b$ and $W > L$, such that, for any θ, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$ and $L(\theta) = L$. There exists a scalar $\bar{\sigma} > 0$ and a function $\mathcal{E} : (0, \bar{\sigma}] \to \mathbb{R}_+$, with $\lim_{\sigma \to 0^+} \mathcal{E}(\sigma) = 0$, such that, for any $\sigma \in (0, \bar{\sigma}]$, in the game in which the

[35] That is, letting $\pi(\theta) = 0$ denote the Dirac distribution assigning probability one to $s = 0$ and $\pi(\theta) = 1$ be the Dirac distribution assigning measure one to $s = 1$, we have that D^Γ is such that either (a) $\pi(\theta) = 0$ for all $\theta \in \cup_{i=2k, k=1,2,...,N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$ and $\pi(\theta) = 1$ for all $\theta \in \cup_{i=2k-1, k=1,2,...,N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$, or (b) $\pi(\theta) = 1$ for all $\theta \in \cup_{i=2k, k=1,2,...,N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$ and $\pi(\theta) = 0$ for all $\theta \in \cup_{i=2k-1, k=1,2,...,N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$.

[36] That the prior is improper simplifies the exposition but is not important. Also note that the agents' hierarchies of beliefs are well defined despite the improperness of the prior.

noise in the agents' information is σ, the following is true: given any deterministic pass/fail policy $\Gamma = (\{0,1\},\pi)$ satisfying the perfect-coordination property and such that $M(\Gamma) > \mathcal{E}(\sigma)$, there exists another deterministic pass/fail policy Γ^* with $M(\Gamma^*) < \mathcal{E}(\sigma)$ that also satisfies the perfect-coordination property and such that the ex-ante probability of default under Γ^* is strictly smaller than under Γ (and hence Γ^* yields the policy maker a payoff strictly higher than Γ).

Proof of Example 3: The formal proof is in the online Supplement. Here we sketch the key arguments. Heuristically, non-monotone policies permit the policy maker to save more banks than monotone policies by making it difficult for the agents to commonly learn the fundamentals when the latter are above 0 but below θ^{MS} and the policy maker announces that the bank passed the test. Intuitively, if the policy maker assigned a pass grade to an interval $[\theta',\theta''] \subset [0,\theta^{MS}]$ of large Lebesgue measure, when σ is small and $\theta \in [\theta',\theta'']$, most agents would receive signals $x_i \in [\theta',\theta'']$. No matter the grade assigned to fundamentals outside the interval $[\theta',\theta'']$, in the continuation game that starts after the policy maker announces that the bank passed the test, most agents receiving signals $x_i \in [\theta',\theta'']$ would then assign high probability to the joint event that $\theta \in [\theta',\theta'']$, that other agents assign high probability to $\theta \in [\theta',\theta'']$, and so on. When this is the case, it is rationalizable for such agents to refrain from pledging. Hence, when σ is small, the only way the policy maker can guarantee that, when $\theta \in [0,\theta^{MS}]$, the agents pledge after hearing that the bank passed the test is by dividing the subset of $[0,\theta^{MS}]$ receiving a pass grade into a collection of disjoint intervals, each of small Lebesgue measure.[37]

Next, suppose that the intervals $\left(\underline{\theta}_i,\bar{\theta}_i\right] \subset \left(0,\theta^{MS}\right]$, $i = 1,...,N$, receiving a pass grade were far apart, implying that the policy maker fails an interval $[\theta',\theta''] \subset (0,\theta^{MS}]$ of large Lebesgue measure (note that this is indeed the case under the optimal monotone deterministic rule with cut-off θ^*, as defined in (3)). The proof in the online Supplement then shows that, starting from Γ, the policy maker could assign a pass grade to some types in the middle of $[\theta',\theta'']$ and a fail grade to some types to the right of θ'', in such a way that (a) pledging continues to be the unique rationalizable action for all agents after hearing that the bank passed the test, and (b) the set of fundamentals receiving a pass grade under the new policy is strictly larger than under the original one. Formally, suppose that, starting from the original policy Γ, the policy maker assigns a pass grade to types $\theta \in [(\theta'+\theta'')/2, (\theta'+\theta'')/2+\xi]$ and a fail grade to types $\theta \in [\theta''+\delta/2, \theta''+\delta]$, with ξ and δ small and chosen so that the ex-ante probability of a pass grade is the same as under the original policy Γ. Now take any individual with signal $x < (\theta'+\theta'')/2$. Suppose that, under the original policy Γ, the individual pledges and rationalizes such behavior by expecting all individuals with signal above his to also pledge. When σ is small, the individual then expects default to occur only for $\theta < x$. Because the new policy assigns a pass grade to fundamentals $\theta > (\theta'+\theta'')/2$ closer to the individual's own signal than the original policy, and because such fundamentals are associated with no default, the

[37] Formally speaking, a highly non-monotone policy guarantees that the support of each agent's posterior beliefs after hearing that the bank passed the test is not connected. Connectedness of the supports facilitates rationalizable profiles where some agents refrain from pledging.

individual's incentives to pledge under the new policy are stronger than under the original one.

Next consider an individual with signal $x \geq \theta'' + \delta$. Suppose again that, under the original policy Γ, such an individual pledges and rationalizes his behavior by expecting all individuals with signal higher than his to also pledge. When σ is small, such individual expects the bank to default only for $\theta < x$. Because the new policy assigns a pass grade to types $\theta < x$ farther away from x than the original policy, and because such fundamentals are associated with default, the individual's incentives to pledge are again stronger under the new policy than under the original one.

In the online Supplement, we show that the above two properties in turn imply that, for those individuals with signals $x \notin [(\theta' + \theta'')/2, \theta'' + \delta]$, if pledging was the unique rationalizable action under the original policy Γ then pledging continues to be the unique rationalizable action under the new policy.

For those agents with signal $x \in [(\theta' + \theta'')/2, \theta'' + \delta]$, instead, the incentives to pledge may be smaller under the new policy than under the original one. However, as we show in the online Supplement, for such individuals pledging is the unique rationalizable action under small perturbations of the original policy. Hence, provided that σ, ξ, δ are small, pledging is the unique rationalizable action for such individuals as well.

The policy maker can then extend the pass grade to some types to the left of $(\theta' + \theta'')/2$ and to some types to the right of $\theta'' + \delta/2$ while guaranteeing that pledging after hearing that the bank passed the test continues to be the unique rationalizable action for all agents. The construction sketched above can be iterated till one arrives at a new policy with a mesh smaller than $\mathcal{E}(\sigma)$. Because default under the new policy is smaller than under the original one, the new policy improves strictly over the original one.

Finally, one can show that, when σ is small, a pass grade can be given to all $\theta > \theta^{MS} + \varepsilon$, with $\varepsilon > 0$ small, while guaranteeing that all agents pledge after hearing that the bank passed the test.[38]
$\square$

3.3.1 Discussion: multiplicity of receivers and exogenous private information

It is worth contrasting the above results about the sub-optimality of monotone rules (when Condition M is violated) to those for economies featuring either a single privately-informed receiver, or multiple receivers with no exogenous private information.

Single Receiver. In this case, the optimal stress test is a simple monotone pass/fail policy with cut-off equal to $\theta^* = 0$. This is because, in this model, the policy maker's and the receiver's payoffs are aligned (they both want to avoid default when possible). Things are different when preferences are misaligned. To see this, suppose the policy maker's payoff is equal to W in case the bank avoids

[38]Formally, for any $\varepsilon > 0$, there exists $\sigma(\varepsilon)$ such that, for any $\sigma < \sigma(\varepsilon)$, given any pass/fail policy Γ satisfying the perfect-coordination property, there exists another pass/fail policy Γ' also satisfying the perfect-coordination property that agrees with Γ on any $\theta < \theta^{MS}$ and passes with certainty all banks with fundamentals $\theta \geq \theta^{MS} + \varepsilon$.

default, and L in case of default, with $W > L$ as in Examples 2 and 3 above. However, now suppose that the receiver's payoff differential between pledging and not pledging is equal to $-g$ in case of default and $-b$ in case of no default, with $g > 0 > b$. Such payoff may reflect the idea that the receiver is a speculator whose payoff is equal to zero when he refrains from speculating (equivalently, when he pledges). When, instead, he speculates, his payoff is positive in case speculation leads to default but negative in case the bank survives the speculative attack. Using the results in Guo and Shmaya (2019), one can then show that the optimal stress test in this case has the *interval structure*: each type x of the receiver is induced to play the action favorable to the policy maker (abstain from speculating) over an interval of fundamentals $[\theta_1(x), \theta_2(x)]$, with $\theta_1(x) < 1 < \theta_2(x)$, for all x, and with $\theta_1(x)$ decreasing in x and $\theta_2(x)$ increasing in x. Such a policy requires disclosing more than two signals and hence cannot be implemented through a simple pass/fail test. In contrast, with a continuum of heterogeneously informed receivers with the same payoffs as in the variant above, the optimal stress is a pass-fail policy that is typically non-monotone in θ, as shown in Examples 2 and 3 above.[39] Furthermore, when the optimal policy is not monotone, it does not have an interval structure, as each receiver with signal x is induced to pledge over a non-connected set of fundamentals. The reason for these differences is that, with a single receiver, to avoid an attack, the policy maker must persuade the receiver that the fundamentals are likely to be above 1, in which case the attack is unsuccessful. With multiple receivers, instead, the policy maker must persuade each receiver that enough other receivers are not attacking, which, as shown above, is best accomplished by a non-monotone policy that makes it difficult for the receivers to commonly learn the fundamentals, when the latter are above 0 but below θ^{MS}.

Multiple receivers with no exogenous private information. Because all receivers have the same posterior beliefs, under MARP, each of them plays the friendly action only if it is dominant to do so. The optimal policy is then again a simple monotone pass/fail policy with cut-off equal to $\theta^* = 0$ in case preferences are aligned and equal to some value $\theta^* \in (0, 1)$ in case they are mis-aligned. The reason why the optimal policy is monotone when the receivers possess no exogenous private information is that the policy maker needs to convince each of them that θ is above 1 with sufficiently high probability to induce them to play the friendly action. Interestingly, when the receivers possess no exogenous private information, the optimality of monotone rules extends to economies in which the policy maker can disclose different information to different receivers, as shown in Li et al (2020) and Morris et al (2020).

[39] This is because, under MARP, all agents play the friendly action if and only if it is iteratively dominant for them to do so, irrespective of the alignment in payoffs.

4 Robustness and Extensions

4.1 Generalizations

We now introduce a few generalizations whose value is twofold: (a) they permit us to discuss the robustness of the results in the baseline model, and (b) they introduce enrichments that we expect to be relevant for applications.

The fundamentals are given by (θ, z), with θ drawn from Θ according to F, and with z drawn from $[\underline{z}, \bar{z}]$ according to $Q_\theta(z)$, with the cdf $Q_\theta(z)$ weakly decreasing in θ, for any z (first order stochastic dominance).[40] The variable θ continues to parametrize the maximal information the policy maker can collect about the fundamentals. Likewise, any information the agents possess about z is encoded in the signals x they receive about θ.[41] The additional variable z parametrizes risk that the agents and the policy maker face at the time of the stress test (e.g., macroeconomic variables that are only imperfectly correlated with the bank's fundamentals, and/or the exogenous supply of funds to the bank from sources other than the agents under consideration, as in the next section).

There exists a function $R : \Theta \times [0, 1] \times [\underline{z}, \bar{z}] \to \mathbb{R}$ such that, given (θ, A, z), default occurs (i.e., $r = 0$) if, and only if, $R(\theta, A, z) \leq 0$. The function R thus implicitly defines the critical size of the pledge necessary for the bank to avoid default. It is continuous, strictly increasing in (θ, z, A), and such that $R(\underline{\theta}, 1, \bar{z}) = R(\bar{\theta}, 0, \underline{z}) = 0$, for some $\underline{\theta}, \bar{\theta} \in \mathbb{R}$, with $\underline{\theta} < \bar{\theta}$. The thresholds $\underline{\theta}$ and $\bar{\theta}$ thus define the "critical region" $(\underline{\theta}, \bar{\theta}]$ where the fate of the bank depends on the response of the market.[42] The policy maker's payoff is equal to

$$\hat{U}^P(\theta, A, z) = \begin{cases} \hat{W}(\theta, A, z) & \text{if } R(\theta, A, z) > 0 \\ \hat{L}(\theta, A, z) & \text{if } R(\theta, A, z) \leq 0. \end{cases} \tag{7}$$

The agents' payoff differential between playing the "friendly" action (which we continue to interpret as pledging to the bank, or abstaining from speculating against it) and the "adversarial" action (refusing to pledge, or speculating against the bank) is equal to

$$\hat{u}(\theta, A, z) = \begin{cases} \hat{g}(\theta, A, z) & \text{if } R(\theta, A, z) > 0 \\ \hat{b}(\theta, A, z) & \text{if } R(\theta, A, z) \leq 0. \end{cases}$$

We assume that the payoff differential is positive in case of no default and negative otherwise: $\hat{g}(\theta, A, z) > 0 > \hat{b}(\theta, A, z)$, for any (θ, A, z). With a slight abuse of notation, for any (θ, A), we then

[40]We assume that the support of each distribution $Q_\theta(z)$ is contained in a bounded interval $[\underline{z}, \bar{z}]$ only to ease the exposition. All the results below extend to the case where $Q_\theta(z)$ has unbounded support, for some θ.

[41]As in the baseline model, conditional on θ, the private signals $(x_i)_{i\in[0,1]}$ are i.i.d. draws from an (absolutely continuous) cumulative distribution function $P(x|\theta)$, with associated density $p(x|\theta)$ strictly positive over the interval $\varrho_\theta \in \mathbb{R}$.

[42]In the baseline model $R = A - 1 + \theta$ so that $\underline{\theta} = 0$ and $\bar{\theta} = 1$.

let $r(\theta, A) \equiv \Pr\{R(\theta, A, z) > 0|\theta, A\}$ denote the probability the bank avoids default,

$$g(\theta, A) \equiv \frac{\mathbb{E}\left[\mathbb{I}(R(\theta, A, z) > 0)\hat{g}(\theta, A, z)|\theta, A\right]}{r(\theta, A)} \text{ and } b(\theta, A) \equiv \frac{\mathbb{E}\left[\mathbb{I}(R(\theta, A, z) \leq 0)\hat{b}(\theta, A, z)|\theta\right]}{1 - r(\theta, A)}$$

the agents' expected payoff differential in case of no default and in case of default, respectively, and

$$W(\theta, A) \equiv \frac{\mathbb{E}\left[\mathbb{I}(R(\theta, A, z) > 0)\hat{W}(\theta, A, z)|\theta, A\right]}{r(\theta, A)} \text{ and } L(\theta, A) \equiv \frac{\mathbb{E}\left[\mathbb{I}(R(\theta, A, z) \leq 0)\hat{L}(\theta, A, z)|\theta, A\right]}{1 - r(\theta, A)}$$

the policy maker's expected payoff, again in case of no default and default, respectively. With this notation in hands, the agents' and the policy maker's expected payoffs can then be conveniently expressed as a function of θ and A only, by letting

$$u(\theta, A) \equiv r(\theta, A)g(\theta, A) + (1 - r(\theta, A))b(\theta, A)$$

denote a representative agent's expected payoff differential and

$$U^P(\theta, A) \equiv r(\theta, A)W(\theta, A) + (1 - r(\theta, A))L(\theta, A)$$

denoting the policy maker's expected payoff.

Hereafter, we assume that both $u(\theta, A)$ and $U^P(\theta, A)$ are non-decreasing in A and that $U^P(\theta, 1) > U^P(\theta, 0)$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$. That $u(\theta, A)$ is monotone in A implies that the continuation game among the agents remains supermodular. That $U^P(\theta, A)$ is non-decreasing in A implies that, for any Γ, MARP continues to coincide with the "smallest" rationalizable profile, that is, the one involving the smallest measure of agents pledging. Finally, that, for any θ in the critical region, the policy maker strictly prefers that all agents pledge to no agent pledging guarantees that, when the optimal policy has a pass/fail structure, it is obtained by maximizing the probability that a pass grade is given to banks whose fundamentals are in the critical range.

4.2 Results

Condition FB. *For any x, $u(\theta, 1 - P(x|\theta)) \geq 0$ (alternatively, $u(\theta, 1 - P(x|\theta)) \leq 0$) implies that $u(\theta'', 1 - P(x|\theta'')) > 0$ for all $\theta'' > \theta$ (alternatively, $u(\theta', 1 - P(x|\theta')) < 0$ for all $\theta' < \theta$).*

Condition FB is a single-crossing property requiring that, for any x, $u(\theta, 1 - P(x|\theta))$ changes sign only once, from negative to positive. This property clearly holds when $u(\theta, A)$, in addition to being non-decreasing in A as assumed above, is also non-decreasing in θ. It also holds when the default outcome is a deterministic function of (θ, A), as in the baseline model, because, for any (θ, A), $g(\theta, A) > 0 > b(\theta, A)$.

Given any common posterior $G \in \Delta(\Theta)$, for any x such that $\int p(x|\theta)G(\mathrm{d}\theta) > 0$, let

$$\bar{U}^G(x) \equiv \frac{\int u(\theta, 1 - P(x|\theta))p(x|\theta)G(\mathrm{d}\theta)}{\int p(x|\theta)G(\mathrm{d}\theta)}$$

denote the expected payoff differential of an agent with signal x who expects all other agents to pledge if their private signal exceeds x and to not pledge otherwise. Let ξ^G be the largest solution to $\bar{U}^G(x) = 0$ if such an equation admits a solution, $\xi^G = +\infty$ if $\bar{U}^G(x) < 0$ for all x such that $\int p(x|\theta)G(\mathrm{d}\theta) > 0$, and $\xi^G = -\infty$ if $\bar{U}^G(x) > 0$ for all x such that $\int p(x|\theta)G(\mathrm{d}\theta)$. Finally, let $\theta^G \equiv \inf\left\{\theta : u(\theta, 1 - P(\xi^G|\theta)) \geq 0\right\}$. The interpretation of ξ^G and θ^G is the following. Suppose that the policy maker induces a common posterior G over Θ, $p(x|\theta)$ is log-supermodular (i.e., satisfies MLRP), and Condition FB holds. Then, in the continuation game that starts after the realization s of the policy Γ induces the common posterior G, MARP is in threshold strategies and is defined by the cut-off ξ^G.[43] When agents play according to MARP given the induced posterior G, their expected payoff differential is non-positive for all $\theta \leq \theta^G$ and non-negative for all $\theta > \theta^G$.

Condition PC. *For any distribution $\Lambda \in \Delta(\Delta(\Theta))$ over posterior beliefs consistent with the common prior F (i.e., such that $\int G\Lambda(\mathrm{d}G) = F$), the following condition holds:*

$$\int \left(\int_{-\infty}^{\theta^G} U^P(\theta, 0)G(\mathrm{d}\theta) + \int_{\theta^G}^{+\infty} U^P(\theta, 1)G(\mathrm{d}\theta) \right) \Lambda(\mathrm{d}G) \geq \int \left(\int U^P(\theta, 1 - P(\xi^G|\theta))G(\mathrm{d}\theta) \right) \Lambda(\mathrm{d}G). \tag{8}$$

Condition PC trivially holds when the policy maker faces no aggregate uncertainty (i.e., when each distribution Q_θ is degenerate), and W and L are invariant in A, as in the baseline model. More generally, Condition PC accommodates for the possibility that both W and L depend on A, possibly non-monotonically, provided that, on average, the loss to the policy maker from having no agent pledging in states $\theta \leq \theta^G$ is more than compensated by the benefit from having all agents pledging in states $\theta > \theta^G$. The average is over both the posteriors induced by the policy maker and the fundamentals. The condition thus requires that the policy maker's and the agents' payoffs be not too misaligned.[44]

We then have the following result:[45]

Theorem 4. *(a) Given any (regular) policy Γ, there exists a (regular) policy Γ^* satisfying the perfect-coordination property and such that, when agents play according to MARP, at any θ, their expected payoff differential under Γ^* is at least as high as under Γ. Furthermore, when, under MARP, θ*

[43] The proof for this claim follows from arguments similar to those in the proof of Theorem 2.

[44] To see this, first observe that the right-hand side of (8) is the policy maker's expected payoff when the agents play according to MARP. Next, as explained above, when Condition FB holds and $p(x|\theta)$ is log-supermodular, the agents' expected payoff differential under MARP is negative for $\theta \leq \theta^G$ and positive for $\theta > \theta^G$. The left-hand side of (8) is thus the policy maker's expected payoff when, for any (G, θ), she induces all agents to (a) pledge, irrespective of their private signals, if the agents' expected payoff differential under MARP at (θ, G) is positive, and (b) refrain from pledging, irrespective of their signals, if their expected payoff differential is negative.

[45] Consistently with the definition in the baseline model, we say that a policy Γ is regular if MARP under Γ is well defined and the sign of the agents' expected payoff differential under MARP is measurable in θ (in the definition in Section 3, we required that the regime outcome is measurable in θ; because, in the baseline model, there is no aggregate uncertainty and payoffs depend only on θ and the regime outcome, in that version of the model, measurability of the regime outcome in θ is equivalent to measurability of the sign of the agents' payoff differential in θ).

perfectly predicts the default outcome (e.g., when, for any θ, Q_θ is a Dirac measure), the probability of default under Γ^* is the same as under Γ. (b) Suppose that $p(x|\theta)$ is log-supermodular and Condition FB holds. The policy Γ^* from part (a) is a pass/fail policy. (c) Suppose that, in addition to the conditions in part (b), Condition PC also holds. Then the policy maker's payoff under Γ^* is at least as high as under Γ. (d) Suppose that, in addition to the conditions in part (c), Condition M also holds. Then the policy Γ^* is monotone (that is, there exists a threshold θ^* such that, for any $\theta \leq \theta^*$, $\pi^*(\theta)$ assigns probability one to $s = 0$, whereas for any $\theta > \theta^*$, $\pi^*(\theta)$ assigns probability one to $s = 1$).

Proof of Theorem 4. The formal proof follows from arguments similar to those establishing Theorems 1-3 and is omitted for brevity.[46] Here we discuss the novel effects due to the enrichments introduced above and the role played by the conditions in the theorem.

First, consider part (a). When default depends on variables only imperfectly correlated with θ, perfect coordination cannot be induced by announcing to the agents the fate of the bank under MARP, as in the proof of Theorem 1. Perfect coordination, however, can still be induced by announcing, at any θ, the *sign* of the agents' expected payoff differential under the original policy. Arguments similar to those establishing Theorem 1 then imply that, when the agents learn that their expected payoff differential under the original policy was positive, under the new policy, they all pledge, irrespective of their signals. Likewise, when they hear their payoff was negative, they all refrain from pledging. That the new policy makes the agents better off then follows from the fact that the agents' payoff differentials are non-decreasing in the size of the aggregate pledge. In the special case in which θ is a perfect predictor of the default outcome, because the sign of the agents' expected payoff differential is determined by the default outcome, perfect coordination is obtained by informing the agents of the default outcome, as in the baseline model. In the online Supplement, we show that, in this case, the ability to coordinate perfectly the market while inducing the same default outcome as under the original policy extends to an even richer class of economies. In particular, economies in which (i) agents' prior beliefs need not be consistent with a common prior, nor be generated by signals drawn independently across agents, conditionally on θ, (ii) the number of agents is arbitrary (in particular, finitely many agents), (iii) agents' have a level-K degree of sophistication, (iv) payoffs may be heterogenous across agents, and (v) the designer may disclose different information to different agents.

Next, consider part (b). As explained above, when $p(x|\theta)$ is log-supermodular and $u(\theta, 1-P(x|\theta))$ has the single-crossing property, then, under MARP, the agents' strategies are monotone in their

[46] Because, in the generalized model, the default outcome need not be a deterministic function of θ, the definition of $x^*(\theta)$ and $\theta_0(x)$ in the proofs leading to Theorems 1-3 in the Appendix, must be amended as follows: $x^*(\theta)$ is the critical signal threshold such that, when agents pledge of $x > x^*(\theta)$ and do not pledge for $x < x^*(\theta)$, the agents' expected payoff differential $u(\tilde{\theta}, 1 - P(x^*(\theta)|\tilde{\theta}))$ changes sign at $\tilde{\theta} = \theta$; $\theta_0(x)$ is the critical fundamental threshold such that, when agents pledge of $\tilde{x} > x$ and do not pledge for $\tilde{x} < x$, the agents' expected payoff differential $u(\theta, 1 - P(x|\theta))$ changes sign at $\theta = \theta_0(x)$. As in the baseline model, we assume that these functions are continuous.

private signals. Arguments similar to those establishing Theorem 2 then imply that the new policy that perfectly coordinates the agents does not need to reveal anything more than the sign of the agents' expected payoff differential under the original policy.

Next, consider part (c). The pass/fail policy described above clearly makes all agents weakly better off. In general, it need not make the policy maker better off. However, when Condition PC also holds, possible losses to the policy maker from inducing fewer agents pledging in states in which the agents' expected payoff differential is negative are compensated by having more agents pledging in those states in which their expected payoff differential is positive. When this is the case, the new policy leads to a Pareto improvement.

Finally, consider part (d). As discussed in the previous section, in general, the optimal policy need not be monotone in θ. It is always monotone when, in addition to the conditions in part (c), Condition M also holds.

5 Micro-foundations and Comparative Statics

We conclude by showing how the general model of Section 4 accommodates as special cases an economy in which banks fund themselves with equity issuances, and one in which they fund themselves with debt. After showing how these two economies are nested into the general framework of the previous section, we illustrate how the model can be used for comparative statics analysis.

Consider a representative bank that, at the beginning of period 1, has former liabilities in the amount of D which need to be repaid by the end of the period for the bank to continue operating. The bank has legacy assets that deliver liquid funds $l(\theta) \in \mathbb{R}_+$ at the end of period 1 and, conditional on the bank repaying its period-1 liabilities, a cash flow $C(\theta) \in \mathbb{R}_+$ in period 2. In addition, in case of default, the liquidation of the bank's assets in period 1 delivers an extra cash flow equal to $\gamma(\theta)$, where the functions C and γ are bounded, differentiable, and Lipschitz continuous. Additionally, the bank has outstanding shares whose total amount is normalized to 1.

In order to pay for its former liabilities, the bank can either issue new shares or new short-term debt. We study each of these two cases separately. In both cases, we assume that each potential investor is endowed with 1 unit of capital and has to decide whether to "invest" by purchasing the security issued by the bank, "bet against" the bank by short-selling the security, or do nothing. Depending on the case of interest, the decision to do nothing may correspond to the decision to invest in other securities or, in case of an existing stakeholder, to maintain the existing portfolio. To keep the portfolio decision simple, we assume that each investor is constrained in the position he can take and let that position be normalized to 1. That is, each investor can either buy or sell at most one unit of the security issued (see Albagli et al. (2015) and Brunnermeier and Pedersen (2005) for similar assumptions). We also simplify the analysis by assuming that investors submit market orders (see, e.g., Kyle (1985)). This allows us to abstain from the role of the market as an aggregator of the investors' information which is beyond the scope of the analysis here.

Each investor $i \in [0,1]$ is endowed with an exogenous private signal $x_i = \theta + \sigma\epsilon_i$ of the bank's underlying fundamentals θ, with the noise ϵ_i drawn independently across investors (and independently of θ) from a log-concave distribution. For simplicity, assume here that $supp[P(\cdot|\theta)] = \mathbb{R}$ for all θ, in which case $\varrho_\theta = \mathbb{R}$ for all θ. We also assume that the policy maker confines attention to monotone policies, which, by virtue of Theorem 3, is without loss of optimality provided that the agents' expected payoffs differentials $u(\theta, 1 - P(x|\theta))$ between purchasing and selling the bank's securities are log-supermodular over $\{(\theta, x) \in [\underline{\theta}, \bar{\theta}] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\}$ and the policy maker's payoff satisfies Condition PC and part 3 of Condition M.

Finally, to simplify the exposition, we assume that θ is drawn from an improper uniform prior over $\mathbb{R}$. This assumption is inconsequential to our results. The agents' hierarchies of beliefs over θ (and hence their expected payoffs) are well defined despite the improperness of the prior. Furthermore, given the focus on monotone rules, optimal policies are also well defined (as discussed at the end of Section 4, a monotone policy is optimal if and only if its threshold θ^* satisfies Condition 3 and such a condition is well defined despite the improperness of the prior).

5.1 Equity issuances

The bank issues $q > 1$ new shares at a price p which is determined in equilibrium. After observing their private signals, all investors simultaneously decide whether to submit a *market order* to purchase one share of the bank ($a_i = 1$), short-sell the bank's equity ($a_i = 0$), or do nothing ($a_i = \emptyset$). Let A denote the amount of investors who decide to purchase the shares. We assume that the aggregate demand for the bank's shares is given by $A + Y_E(p, z)$, where $Y_E(p, z)$ represents additional demand coming from sources exogenous to the model (e.g., a combination of high-frequency traders submitting limit-orders and of short-term liquidity traders submitting market orders). The variable z parametrizes residual uncertainty that may correlate with the bank's fundamentals (e.g., the "amount" of liquidity traders, and/or the short-term value the high-frequency traders derive from purchasing the shares). We assume that $Y_E(\cdot, \cdot)$ is a non-increasing function of the price of the bank's shares, p, and a non-decreasing function of z.

Investors are risk-neutral. Along with the fact that investors submit market orders and face constraints on their positions, this last assumption implies that doing nothing is dominated by either purchasing or short-selling a share.[47] Because each investor who does not purchase a share, short-sells one, the total supply of shares is thus given by $1 - A + q$, where $1 - A$ is the amount of shares shorted by the investors. It follows that the equilibrium price of the shares, $p^\star_E(A, z)$, is implicitly determined by the market-clearing condition

$$1 - A + q = A + Y_E(p, z). \tag{9}$$

[47] Whether an investor sells a share he already owns or short-sells one that he borrows makes no difference in this settings. For simplicity, hereafter we focus on the case of short-selling.

Given the monotonicities of Y_E, $p_E^\star(A, z)$ is increasing in A and in z, and decreasing in q. Hereafter, we assume that a solution to (9) exists for any (z, A) and is bounded over $(A, z) \in [0, 1] \times [\underline{z}, \bar{z}]$.

The bank avoids default as long as the proceeds from the period-1 equity issuance are sufficient to cover the bank's liabilities D, that is, if and only if,

$$R_E(\theta, A, z) \equiv l(\theta) + \rho_S q p_E^\star(A, z) - D > 0,$$

where ρ_S is the short-term return on the cash $qp_E^\star$ collected through the equity issuance.

The investors' payoff differential (between buying and short-selling a share) in case of no default is then equal to

$$\hat{g}_E(\theta, A, z) \equiv 2\left(\frac{C(\theta) + \rho_L\left(l(\theta) + \rho_S q p_E^\star(A, z) - D\right)}{1 + q} - p_E^\star(A, z)\right)$$

whereas the payoff differential in case of default is equal to

$$\hat{b}_E(\theta, A, z) \equiv -2p_E^\star(A, z),$$

where ρ_L is the long-term return on the extra cash $l(\theta) + \rho_S q p_E^\star - D$ available to the bank at the end of period 1, after the bank pays its liabilities D. Note that, in writing $\hat{g}_E$ and $\hat{b}_E$, we used the fact that, in case of no default, the long-term equilibrium price of equity is equal to to the long-term cash flow $C(\theta)$ augmented by the long-term return on the funds $l(\theta) + \rho_S q p_E^\star - D$ invested at the end of period 1, divided by the amount of outstanding shares, $1 + q$. In case of default, instead, the long-term price of equity is equal to zero.[48] The payoff from short-selling the bank's shares is equal to the negative of the payoff from purchasing the shares and, therefore, the payoff differential between the two actions is equal to twice the payoff from purchasing the shares.

This economy is thus a special case of the general model in the previous section with the agents' expected payoff differential taking the form of

$$u_E(\theta, A) \equiv -2\int_{\underline{z}}^{\bar{z}} p_E^\star(A, z)\, dQ_\theta(z) + 2\int_{\hat{z}_E(\theta, A)}^{\bar{z}} \left[\frac{C(\theta) + \rho_L\left(l(\theta) + \rho_S q p_E^\star(A, z) - D\right)}{1 + q}\right] dQ_\theta(z) \qquad (10)$$

with $\hat{z}_E(\theta, A)$ denoting the critical level of z below which the bank defaults.[49] Provided that $u_E(\theta, A)$ is non-decreasing in A, and that, for any x, $u_E(\theta, 1 - P(x|\theta))$ has the single-crossing property of condition FB, all the conclusions from the previous section apply.[50]

[48] That, in case of default, the long-term price of equity is equal to zero reflects the fact that equity is junior to all other existing claims and the assets' liquidation value $\gamma(\theta)$ is small and hence insufficient to provide any funds to the equity holders.

[49] Formally, $\hat{z}_E(\theta, A)$ is implicitly defined by the solution to $l(\theta) + \rho_S q p_E^\star(A, z) = D$ whenever the equation has a solution, is equal to $\underline{z}$ when $l(\theta) + \rho_S q p_E^\star(A, \underline{z}) > D$, and is equal to $\bar{z}$ when $l(\theta) + \rho_S q p_E^\star(A, \bar{z}) < D$.

[50] Note that the first term in (10) is decreasing in A, as $p_E^\star(A, z)$ is increasing in A, for any z. However, the second term is increasing in A (the integrand is increasing in A and the threshold $\hat{z}(\theta, A)$ is decreasing in A). Hence, provided the effects from the second term prevail, $u_E(\theta, A)$ is increasing in A. When θ and z are independent, the first term

5.2 Debt issuances

Next, consider the case of debt issuances. The bank issues $q > 1$ bonds at the beginning of period 1. Each bond is a contract that specifies a payment of F_D in period 2, in case the bank does not default, and covenants L_D that discipline the way the proceeds from liquidation will be divided between old and new debt-holders in case of default. Investors either purchase ($a_i = 1$) or short-sell ($a_i = 0$) one unit of the bond by submitting a market order.[51]

Letting A denote the fraction of investors purchasing the bond and p its price, we then have that the total demand for the bond is equal to $A + Y_D(p, z)$, where $Y_D(p, z)$ represents the exogenous (net) demand for the bonds by high-frequency and noisy traders. As in the case of equity issuances, Y_D is non-increasing in p and non-decreasing in z. We assume that, for all $z \in [\underline{z}, \bar{z}]$, if $p > F_D$, then $Y_D(p, z) < 0$, which guarantees that the equilibrium price of debt, $p_D^\star$, is smaller than its face value F_D.

As in the case of equity issuances, the bank avoids default as long as its liquid funds at the end of period 1, $l(\theta) + \rho_S q p_D^\star$, exceed the amount of former liabilities, D. Formally, default is avoided if, and only if,

$$R_D(\theta, A, z) \equiv l(\theta) + \rho_S q p_D^\star(A, z) - D > 0,$$

where the equilibrium price for the newly issued bonds $p_D^\star(A, z)$ is implicitly given by the market-clearing condition

$$q + 1 - A = A + Y_D\left(p_D^\star, z\right). \tag{11}$$

As in the case of equity issuances, we assume that a solution to (11) exists for any (z, A) and is bounded over $(z, A) \in [\underline{z}, \bar{z}] \times [0, 1]$.

The payoff differential between purchasing the bond versus short-selling it is then equal to

$$\hat{g}_D(\theta, A, z) \equiv 2\left(\min\left\{F_D, \frac{C(\theta) + \rho_L\left(l(\theta) + \rho_S q p_D^\star(A, z) - D\right)}{q}\right\} - p_D^\star(A, z)\right)$$

in case the bank does not default, and is equal to

$$\hat{b}_D(\theta, A, z) \equiv 2\left(\frac{L_D}{qL_D + D}\left(\gamma(\theta) + l(\theta) + \rho_S q p_D^\star(A, z)\right) - p_D^\star(A, z)\right)$$

in case of default. That is, in case the bank is able to repay its short-term liabilities, investors that purchased the bond receive in period 2 the minimum between the bond's face value, F_D, and the bank's period-2 net cash-flows, $C(\theta) + \rho_L\left(l(\theta) + \rho_S q p_D^\star(A, z) - D\right)$, divided by the amount of

is invariant in θ whereas the second term is increasing in θ. When, instead θ and z are positively correlated, the first term may be decreasing in θ (this is because a higher a θ implies a FOSD shift in the distribution of z, i.e., $Q_\theta(z)$ is weakly decreasing in θ, for any z). However, provided the dependence of z on θ is small, $u_E(\theta, A)$ is increasing in θ. Clearly $u_E(\theta, A)$ being monotone in (θ, A) suffices for $u_E(\theta, 1 - P(x|\theta))$ to have the single-crossing property, but is not necessary.

[51] Investors may also do nothing ($a_i = \emptyset$). As in the case of equity issuances, such a decision is dominated by either purchasing or short-selling one unit of the bond. The arguments are the same as with equity issuances.

bonds issued, q. If, instead, the bank is unable to repay its short-term liabilities, and hence defaults, the amount that each debt-holder receives is equal to a fraction $qL_D/[qL_D + D]$ of the total cash $l(\theta) + \rho_S q p^{\star}_D$ available at the end of period 1, augmented by the additional funds $\gamma(\theta)$ obtained by liquidating the bank's assets, and divided by the amount of bonds issued, q. In other words, the available cash is divided between old and new debt holders in a pro-rated manner. Hereafter, we assume that, in case the bank does not default, the amount of cash $C(\theta)$ generated by the bank's legacy asset in period 2 is sufficiently large to cover the bond's face value F_D for all (θ, A, z).

This economy too is thus a special case of the general model of the previous section with the agents' expected payoff differential between purchasing and short-selling the bond taking the form of

$$\begin{aligned} u_D(\theta, A) = &-2\int_{\underline{z}}^{\bar{z}} p^{\star}_D(A, z)\, dQ_\theta(z) + 2F_D \int_{\hat{z}_D(\theta,A)}^{\bar{z}} dQ_\theta(z) \\ &+2\int_{\underline{z}}^{\hat{z}_D(\theta,A)} \frac{L_D}{qL_D+D} \left(\gamma(\theta) + l(\theta) + \rho_S q p^{\star}_D(A, z)\right) dQ_\theta(z) \end{aligned} \tag{12}$$

where $\hat{z}_D(\theta, A)$ is the critical level below which the bank defaults, defined in the same way as in the case of equity issuances. Provided that $u_D(\theta, A)$ is non-decreasing in A and that, for any x, $u_D(\theta, 1 - P(x|\theta))$ has the single-crossing property of Condition FB, all the conclusions from the previous section apply.[52]

5.3 Effects of market uncertainty on toughness of optimal stress tests

The results in the previous sections can also be used for comparative-statics analysis relating the properties of optimal stress tests to the primitives of the model. In this subsection, we show how the toughness of the optimal stress tests is affected by an increase in risk about the bank's fundamentals. Specifically, we use the two micro-foundations above to investigate how an increase in risk (formally captured by an increase in the parameter σ scaling the noise in the agents' signals $x_i = \theta + \sigma\epsilon_i$) affects the critical threshold θ^* below which the policy maker fails the bank under examination.

Let $\theta^*_E(\sigma)$ and $\theta^*_D(\sigma)$ denote the thresholds characterizing the optimal monotone policies when the precision of the agents' exogenous information is σ^{-2} and the bank funds itself with equity and debt, respectively. Also let θ^{MS}_E and θ^{MS}_D be the Laplacian thresholds for the two economies under consideration (defined by $\int_0^1 u_h(\theta^{MS}_h, A)dA = 0$, $h = E, D$) and recall that, in the absence of any public information disclosure, when $\sigma \to 0^+$, purchasing (alternatively, short-selling) security h is the unique rationalizable action for $x > \theta^{MS}_h$ (alternatively, for $x < \theta^{MS}_h$). Lastly, let $\theta^{\triangle}_h$ be defined by the solution to $u_h(\theta^{\triangle}_h, 1/2) = 0$, $h = D, E$, and note that the threshold $\theta^{\triangle}_h$ is the critical value of the fundamentals at which an investor who knows the fundamentals and expects 1/2 of the investors to purchase security h and 1/2 to short-sell it is indifferent between purchasing and short-selling the security, for $h = E, D$. Hereafter, we assume that the problem the policy maker faces is "severe"

[52] As in the case of equity issuances, the properties that $u_D(\theta, A)$ is non-decreasing in A and $u_D(\theta, 1 - P(x|\theta))$ has the single-crossing property of Condition FB may be consistent with the first term in (12) being decreasing in A.

in the sense that $\theta_h^{MS} \geq \theta_h^{\triangle}$, $h = E, D$. Under a few further simplifying assumptions that ease the derivations (stated in the next proposition), we then have the following result:

Proposition 1. *Suppose that (a) θ and z are independent, (b) $\rho_S = 1$, (c) $x_i = \theta + \sigma\epsilon_i$, with ϵ_i drawn from a standard Normal distribution, independently of (θ, z), (d) there exists $l \in \mathbb{R}_+$ such that $l(\theta) = l$ for all θ, and (e) $\gamma(\theta)$ and $C(\theta)$ are strictly increasing. There exists $\sigma^\dagger > 0$ such that, for any $\sigma, \sigma' \in (0, \sigma^\dagger]$, with $\sigma' > \sigma$, $\theta_E^*(\sigma') < \theta_E^*(\sigma)$ and $\theta_D^*(\sigma') > \theta_D^*(\sigma)$.*

The result in the proposition says the following. Take an economy in which the precision of the agents' exogenous information is sufficiently high (that is, σ is small) and consider the effect of an increase in risk (formally captured by the transition to $\sigma' > \sigma$) on the toughness of the optimal stress test (formally captured by the threshold in θ below which the policy maker fails the bank). More risk leads to a reduction in the toughness of the optimal stress test when the bank finances itself with equity and to an increase in the toughness of the optimal stress test when the bank finances itself with debt.

Intuitively, the reason why, under the assumed specification, risk is beneficial to the bank in case of equity financing but not in case of debt financing is the following. Under equity financing, investors are exposed to variations in fundamentals primarily through upside risk. Their payoff differential (between purchasing and short-selling equity) is increasing in θ in case of no default and is equal to $-2p_E^\star$ in case of default. Provided that the price of equity does not vary much with (θ, z), which is the case under the assumed specification, an increase in risk then makes investors more willing to purchase equity. The policy maker can then decrease the critical threshold θ_E^* below which she fails the bank while guaranteeing that, after announcing that the bank passed the test, the unique rationalizable profile continues to feature all investors pledging by purchasing equity.

Under debt financing, instead, investors are exposed to variations in fundamentals primarily through downside risk. When the liquidation value $\gamma(\theta)$ is increasing in θ and $p_D^\star$ is not very sensitive to (θ, z), the investors' payoff differential (between purchasing and short-selling debt) is increasing in θ in case of default but constant in fundamentals in case the bank survives. An increase in risk then makes investors less willing to pledge. The policy maker must then increase the critical threshold θ_D^* below which she fails the bank if she wants to guarantee that, after announcing that the bank passed the test, the unique rationalizable profile features all investors pledging by purchasing the newly issued debt.

That risk is beneficial to the bank in case of equity financing but detrimental in case of debt financing need not extend to alternative specifications of the investors' payoffs under the two securities. What appears to be true more generally is the following single-crossing property. Whenever more risk is beneficial to the bank in case of debt financing, the same tends to be true under equity financing.[53]

[53] Clearly, for this single-crossing result to hold, one needs to make sure that the two cases are comparable. This requires, among other things, that the exogenous demand for the bank's securities is the same in the two cases, i.e.,

6 Conclusions

We consider the design of optimal persuasion policies in coordination settings in which the receivers cannot be trusted to play favorably to the designer (e.g., pledging to a solvent but illiquid bank). We show that the optimal policy completely removes any strategic uncertainty, while retaining structural uncertainty: each agent can perfectly predict the actions of any other agent, but not the beliefs that rationalize such actions. We identify conditions under which the optimal policy has a pass/fail structure, as well as conditions under which the optimal policy is monotone, passing with certainty institutions with strong fundamentals and failing the others.

The results are worth extending in a few directions. The analysis assumes the policy maker knows how the distribution of market beliefs correlates with the banks' fundamentals. Such knowledge may come from previous experience with banks of similar characteristics, polls, data on professional forecasters, the IOWA betting markets, and the like. While this is a natural starting point, in future work it would be interesting to investigate how the structure of the optimal policy is affected by the policy maker's ambiguity about the joint distribution of the underlying fundamentals and market beliefs.[54]

The analysis in the present paper is static. Many applications of interest are intrinsically dynamic, with agents coordinating on multiple attacks and learning over time (see the discussion in Angeletos et al. (2007)). In future work, it would be interesting to consider dynamic extensions and investigate how the timing of information disclosures is affected by the agents' behavior in previous periods.[55]

Finally, the analysis is conducted by assuming that the maximal information that the designer can collect about the fundamentals (in the paper, θ) is exogenous. In future work, it would be interesting to accommodate for the possibility that part of the information is provided by the banks themselves. This creates an interesting screening+persuasion problem in the spirit of what is examined in the literature on privacy in sequential contacting (e.g., Calzolari and Pavan (2006a) Calzolari and Pavan (2006b), and Dworczak (2020)).[56]

Appendix

Proof of Theorem 1. Given any regular policy $\Gamma = (\mathcal{S}, \pi)$ and any $n \in \mathbb{N}$, let $T^{\Gamma}_{(n)}$ be the set of strategies surviving n rounds of IDISDS, with $T^{\Gamma}_{(0)}$ denoting the entire set of strategy profiles $a = (a_i(\cdot))_{i\in[0,1]}$, where for any $i \in [0,1]$, $a_i(x,s)$ denotes the probability agent i pledges, given (x,s). Let $a^{\Gamma}_{(n)} \equiv \left(a^{\Gamma}_{(n),i}(\cdot)\right)_{i\in[0,1]} \in T^{\Gamma}_{(n)}$ denote the most aggressive profile surviving n rounds of IDISDS (that is, the profile in $T^{\Gamma}_{(n)}$ that minimizes the policy maker's ex-ante payoff). The profiles $\left(a^{\Gamma}_{(n)}\right)_{n\in\mathbb{N}}$

that $Y_D(p,z) = Y_E(p,z)$, for any (p,z).

[54]For some recent work in this direction, see Dworczak and Pavan (2021).

[55]For some recent work in this direction, see Basak and Zhou (2020).

[56]See also Inostroza (2021) for recent developments in this direction.

can be constructed inductively as follows. The profile $a^{\Gamma}_{(0)} \equiv \left(a^{\Gamma}_{(0),i}(\cdot)\right)_{i\in[0,1]}$ prescribes that all agents refrain from pledging, irrespective of (x,s). Next, let $U^{\Gamma}_i(x_i,s;a)$ denote the payoff differential between pledging and not pledging for agent i when, under Γ, all other agents follow the strategy in a. Then, $a^{\Gamma}_{(n),i}(x_i,s)=0$ if $U^{\Gamma}_i\left(x_i,s;a^{\Gamma}_{(n-1)}\right)\le 0$ and $a^{\Gamma}_{(n),i}(x_i,s)=1$ if $U^{\Gamma}_i\left(x_i,s;a^{\Gamma}_{(n-1)}\right)>0$. MARP consistent with Γ is then the profile $a^{\Gamma}=(a^{\Gamma}_i(\cdot))_{i\in[0,1]}$ given by $a^{\Gamma}_i(\cdot)=\lim_{n\to\infty} a^{\Gamma}_{(n),i}(\cdot)$, all $i\in[0,1]$.

Next, consider the policy $\Gamma^+=(\mathcal{S}^+,\pi^+)$, $\mathcal{S}^+\equiv\mathcal{S}\times\{0,1\}$, that, for each θ, draws the score s from the same distribution $\pi(\theta)\in\Delta(\mathcal{S})$ as the original policy Γ, and then, for each s it draws, it also announces the regime outcome $r^{\Gamma}(\theta,s)$ that would have prevailed at θ when agents play according to MARP consistent with Γ; that is, for any θ, and any $s\in supp(\pi(\theta))$, it announces $\left(s,r^{\Gamma}(\theta,s)\right)$.

Define $T^{\Gamma+}_{(n)}$ and $a^{\Gamma+}_{(n)}$ analogously to $T^{\Gamma}_{(n)}$ and $a^{\Gamma}_{(n)}$, but with respect to the policy Γ^+ so defined.

The proof is in three steps. Steps 1 and 2 show that any agent i who, given (x_i,s), finds it dominant (alternatively, iteratively dominant) to pledge under Γ also finds it dominant (alternatively, iteratively dominant) to pledge under Γ^+ when receiving information $(x_i,(s,1))$. Step 3 uses the above property to establish that, because the game is supermodular and $a^{\Gamma+}$ is "less aggressive" than a^{Γ} (meaning that any agent who, given (x,s), pledges under a^{Γ} also pledges under $a^{\Gamma+}$ when receiving the information $(x,(s,1))$, then, under $a^{\Gamma+}$, all agents pledge (alternatively, refrain from pledging) when receiving information $(s,1)$ (alternatively, $(s,0)$).

Step 1. First, we prove that, for each $i\in[0,1]$,

$$\{(x_i,s): U^{\Gamma}_i(x_i,s;a)>0\ \forall a\}\subseteq\{(x_i,s): U^{\Gamma+}_i(x_i,(s,1);a)>0\ \forall a\}.$$

That is, any agent i who, under Γ, finds it dominant to pledge, given the information (x_i,s), also finds it dominant to pledge under Γ^+ when receiving the information $(x_i,(s,1))$.

To see this, first use the fact that the game is supermodular to observe that

$$\{(x_i,s): U^{\Gamma}_i(x_i,s;a)>0\ \forall a\}=\left\{(x_i,s): U^{\Gamma}_i\left(x_i,s;a^{\Gamma}_{(0)}\right)>0\right\}$$

and $\{(x_i,s): U^{\Gamma+}_i(x_i,(s,1);a)>0\ \forall a\}=\left\{(x_i,s): U^{\Gamma+}_i\left(x_i,(s,1);a^{\Gamma+}_{(0)}\right)>0\right\}$.

Now let $\Lambda^{\Gamma}_i(\theta,\mathbf{x}|x_i,s)$ denote the beliefs of agent $i\in[0,1]$ over the fundamentals, θ, and the cross-sectional distribution of signals, $\mathbf{x}\in\mathbb{R}^{[0,1]}$, when receiving information $(x_i,s)\in\mathbb{R}\times\mathcal{S}$ under Γ, and $\Lambda^{\Gamma+}_i(\theta,\mathbf{x}|x_i,(s,1))$ the corresponding beliefs under Γ^+. Bayesian updating implies that

$$\partial\Lambda^{\Gamma+}_i(\theta,\mathbf{x}|x_i,(s,1))=\frac{\mathbb{I}\left(r^{\Gamma}(\theta,s)=1\right)}{\Lambda^{\Gamma}_i(1|x_i,s)}\partial\Lambda^{\Gamma}_i(\theta,\mathbf{x}|x_i,s),\tag{13}$$

where $\mathbb{I}\left(r^{\Gamma}(\theta,s)=1\right)$ is the indicator function, taking value 1 if θ is such that $r^{\Gamma}(\theta,s)=1$, and 0 otherwise, and where $\Lambda^{\Gamma}_i(1|x_i,s)\equiv\int_{\{(\theta,\mathbf{x}):r^{\Gamma}(\theta,s)=1\}}\Lambda^{\Gamma}_i(\mathrm{d}(\theta,\mathbf{x})|x_i,s)$.

Next, observe that, under both $a^{\Gamma}_{(0)}$ and $a^{\Gamma+}_{(0)}$, default occurs if, and only if, $\theta\le 1$. Take any $i\in[0,1]$ and $(x_i,s)\in\mathbb{R}\times\mathcal{S}$ such that

$$U^{\Gamma}_i\left(x_i,s;a^{\Gamma}_{(0)}\right)=\int_{(\theta,\mathbf{x})}\left(b(\theta)\mathbb{I}\left(\theta\le 1\right)+g(\theta)\mathbb{I}(\theta>1)\right)\Lambda^{\Gamma}_i(\mathrm{d}(\theta,\mathbf{x})|x_i,s)>0.\tag{14}$$

The aforementioned property of Bayesian updating implies that

$$U_i^{\Gamma+}\left(x_i,(s,1);a_{(0)}^{\Gamma+}\right)=\frac{1}{\Lambda_i^{\Gamma}(1|x_i,s)}\int_{(\theta,\mathbf{x})}\left(b(\theta)\mathbb{I}\left(\theta\leq 1\right)+g(\theta)\mathbb{I}(\theta>1)\right)\mathbb{I}\left(r^{\Gamma}(\theta,s)=1\right)\Lambda_i^{\Gamma}(\mathrm{d}(\theta,\mathbf{x})|x_i,s)$$
$$\geq\frac{1}{\Lambda_i^{\Gamma}(1|x_i,s)}\int_{(\theta,\mathbf{x})}\left(b(\theta)\mathbb{I}\left(\theta\leq 1\right)+g(\theta)\mathbb{I}(\theta>1)\right)\Lambda_i^{\Gamma}(\mathrm{d}(\theta,\mathbf{x})|x_i,s)=\frac{1}{\Lambda_i^{\Gamma}(1|x_i,s)}U_i^{\Gamma}\left((x_i,s);a_{(0)}^{\Gamma}\right)>0,$$

where the first equality follows from (13), the first inequality from the fact that, for all θ such that $r^{\Gamma}(\theta,s)=0$, $b(\theta)\mathbb{I}\left(\theta\leq 1\right)+g(\theta)\mathbb{I}(\theta>1)=b(\theta)<0$, the second equality follows from the definition of $U_i^{\Gamma}\left(x_i,s;a_{(0)}^{\Gamma}\right)$, and the second inequality from (14).

This means that any agent for whom pledging was dominant after receiving information (x_i,s) under Γ, continues to find it dominant to pledge after receiving information $(x_i,(s,1))$ under Γ^+.

Step 2. Next, take any $n>1$. Assume that, for any $1\leq k\leq n-1$, any $i\in[0,1]$,

$$\left\{(x_i,s):U_i^{\Gamma}(x_i,s;a)>0\ \ \forall a\in T_{(k-1)}^{\Gamma}\right\}\subseteq\left\{(x_i,s):U_i^{\Gamma^+}(x_i,(s,1);a)>0,\ \ \forall a\in T_{(k-1)}^{\Gamma+}\right\}.\tag{15}$$

Arguments similar to those establishing the result in Step 1 above imply that

$$\left\{(x_i,s):U_i^{\Gamma}(x_i,s;a)>0\ \ \forall a\in T_{(n-1)}^{\Gamma}\right\}\subseteq\left\{(x_i,s):U_i^{\Gamma^+}(x_i,(s,1);a)>0,\ \ \forall a\in T_{(n-1)}^{\Gamma+}\right\}.\tag{16}$$

Intuitively, the result follows from the combination of the following two properties: (a) because the game is supermodular, $\left\{(x_i,s):U_i^{\Gamma}(x_i,s;a)>0\ \ \forall a\in T_{(n-1)}^{\Gamma}\right\}=\left\{(x_i,s):U_i^{\Gamma}\left(x_i,s;a_{(n-1)}^{\Gamma}\right)>0\right\}$ where recall that $a_{(n-1)}^{\Gamma}$ is the most aggressive profile surviving $n-1$ rounds of IDISDS (clearly, the same property holds for Γ^+); (b) $a_{(n-1)}^{\Gamma+}$ is "less aggressive" than $a_{(n-1)}^{\Gamma}$, in the sense that any agent who, given (x,s), pledges under $a_{(n-1)}^{\Gamma}$ also pledges under Γ^+ when receiving information $(x,(s,1))$; and (c) the observation that $r^{\Gamma}(\theta,s)=1$ removes from the support of the agents' posterior beliefs states in which default would have occurred under a^{Γ} and hence under $a_{(n-1)}^{\Gamma}$ as well (observe that $a_{(n-1)}^{\Gamma}$ is more aggressive that a^{Γ}, meaning that any agent who, given (x,s), pledges under $a_{(n-1)}^{\Gamma}$, also pledges under a^{Γ} when receiving the same information $(x,1)$).

Step 3. Equipped with the results in steps 1 and 2 above, we now prove that, for all $\theta\in\Theta$ and all $s\in supp(\pi(\theta))$ such that $r^{\Gamma}(\theta,s)=1$, for any $\mathbf{x}\in\mathbf{X}(\theta)$, and any $i\in[0,1]$, $a_i^{\Gamma^+}(x_i,(s,1))\equiv\lim_{n\to\infty}a_{(n),i}^{\Gamma+}(x_i,(s,1))=1$. This follows directly from the fact that, as shown above,

$$a_i^{\Gamma}(x_i,s)=1\Rightarrow a_i^{\Gamma^+}(x_i,(s,1))=1.\tag{17}$$

The announcement that θ is such that $r^{\Gamma}(\theta,s)=1$ thus reveals to each agent that, when agents play according to a^{Γ^+}, default does not occur. Because the payoff from pledging is strictly positive when default does not occur, any agent i receiving information $(s,1)$ under Γ^+ thus necessarily pledges, no matter x_i. Under the new policy Γ^+, all agents thus pledge when they learn that θ is such that $r^{\Gamma}(\theta,s)=1$. That they all refrain from pledging when they learn that θ is such that $r^{\Gamma}(\theta,0)=0$ follows from the fact that such an announcement makes it common certainty that $\theta\leq 1$.

We conclude that the new policy Γ^+ satisfies the perfect-coordination property and is such that, for any θ, the probability of default under Γ^+ is the same as under Γ. The result in the theorem then follows by taking $\Gamma^*=\Gamma^+$. Q.E.D.

Proof of Theorem 2. The proof is in 2 steps. Step 1 shows that, when $p(x|\theta)$ is log-supermodular, i.e., it satisfies MLRP, then, irrespective of Γ, MARP is in cut-off strategies. Step 2 then shows that, starting from any Γ satisfying the perfect-coordination property, one can drop any signal other than the predicted fate of the bank without changing the agents' behavior.

Step 1. Fix an arbitrary policy $\Gamma = (\mathcal{S}, \pi)$ and, for any pair $(x, s) \in \mathbb{R} \times \mathcal{S}$, let $\Lambda^\Gamma(\theta|x, s)$ represent the endogenous posterior beliefs over Θ of each agent receiving exogenous information x and endogenous information s.[57] Let $u(\theta, A) \equiv g(\theta)\mathbb{I}(A > 1-\theta) + b(\theta)\mathbb{I}(A \leq 1-\theta)$ be the payoff differential between pledging and not pledging when the fundamentals are θ and the aggregate size of the pledge is A.

Next, let $U^\Gamma(x, s|k) \equiv \int u(\theta, 1 - P(k|\theta))d\Lambda^\Gamma(\theta|x, s)$ denote the expected payoff differential of an agent with information (x, s), when all other agents follow a cut-off strategy with cut-off k (i.e., they pledge if their private signal exceeds k and refrain from pledging if it is below k). The following result establishes that, when the distribution $p(x|\theta)$ from which the signals are drawn satisfies MLRP, no matter Γ, MARP is in cut-off strategies:

Lemma 1. Suppose that $p(x|\theta)$ is log-supermodular. Given any policy $\Gamma = (\mathcal{S}, \pi)$, for any $s \in \mathcal{S}$, there exists $\xi^{\Gamma;s} \in \mathbb{R}$ such that MARP consistent with Γ is given by the strategy profile $a^\Gamma \equiv (a_i^\Gamma)_{i \in [0,1]}$ such that, for any $s \in \mathcal{S}$, $x \in \mathbb{R}$, $i \in [0, 1]$, $a_i^\Gamma(x, s) = \mathbb{I}\{x > \xi^{\Gamma;s}\}$ with $\xi^{\Gamma;s} \equiv \sup\{x : U^\Gamma(x, s|x) \leq 0\}$ if $\{x : U^\Gamma(x, s|x) \leq 0\} \neq \emptyset$, and $\xi^{\Gamma;s} \equiv -\infty$ otherwise. Moreover, the strategy profile a^Γ is a BNE of the continuation game that starts with the announcement of the policy Γ.

Proof of Lemma 1. Fix the policy $\Gamma = (S, \pi)$. For any $s \in \mathcal{S}$, let $\xi_{(1)}^{\Gamma;s} \equiv \sup\{x : \lim_{k\to\infty} U^\Gamma(x, s|k) \leq 0\}$. Given the public signal s, it is dominant for any agent with private signal x exceeding $\xi_1^{\Gamma;s}$ to pledge. Next, recall that, for any $n \in \mathbb{N}$, $T_{(n)}^\Gamma$ denotes the set of strategy profiles that survive the first n rounds of IDISDS and $a_{(n)}^\Gamma \equiv \left(a_{(1),i}^\Gamma\right)_{i\in[0,1]}$ denotes the most aggressive profile in $T_{(n)}^\Gamma$. Observe that the profile $a_{(1)}^\Gamma$ is given by $a_{(1),i}^\Gamma(x, s) = \mathbb{I}\{x > \xi_{(1)}^{\Gamma;s}\}$ for all $(x, s) \in \mathbb{R} \times \mathcal{S}$, and all $i \in [0, 1]$, and minimizes the policy maker's payoff not just in expectation but for any (θ, s). This follows from the fact that, when nobody else pledges, the expected payoff differential $\int u(\theta, 0)d\Lambda^\Gamma(\theta|x, s)$ between pledging and not pledging crosses 0 only once and from below at $x = \xi_{(1)}^{\Gamma;s}$. The single-crossing property of $\int u(\theta, 0)d\Lambda^\Gamma(\theta|x, s)$ in turn is a consequence of the fact that $u(\theta, 0)$ crosses 0 only once from below at $\theta = 1$ along with Property SCB below.

Property SCB. *Suppose that the function $h : \mathbb{R} \to \mathbb{R}$ crosses 0 only once from below at $\theta = \theta_0$ (that is, $h(\theta) \leq 0$ for all $\theta \leq \theta_0$ and $h(\theta) \geq 0$ for all $\theta > \theta_0$). Let $g : \mathbb{R}^2 \to \mathbb{R}_+$ be a log-supermodular function and suppose that, for any θ, there is an open interval $\varrho_\theta = (\underline{\varrho}_\theta, \bar{\varrho}_\theta) \subset \mathbb{R}$ containing θ such that $g(x, \theta) > 0$ for all $x \in \varrho_\theta$ and $g(x, \theta) = 0$ for (almost) all $x \in \mathbb{R} \setminus \varrho_\theta$, with the bounds $\underline{\varrho}_\theta, \bar{\varrho}_\theta$ non-decreasing in θ. Choose any (Lebesgue) measurable subset $\Omega \subseteq \mathbb{R}$ containing θ_0 and, for any $x \in \mathbb{R}$, let $\Psi(x; \Omega) \equiv \int_\Omega h(\theta)g(x, \theta)d\theta$. Suppose there exists $x^\star \in \varrho_{\theta_0}$ such that $\Psi(x^\star; \Omega) = 0$. Then,*

[57] Because $\Theta \subset \mathbb{R}$, $\Lambda^\Gamma(\theta|x, s)$ can be taken to be the cdf of the agent's posterior beliefs.

necessarily, $\Psi(x;\Omega) \geq 0$ for all $x \in \varrho_{\theta_0}$ with $x > x^\star$, and $\Psi(x;\Omega) \leq 0$ for all $x \in \varrho_{\theta_0}$ with $x < x^\star$, *with both inequalities strict if (a)* $\{\theta \in \Omega : h(\theta) \neq 0\}$ *has strict positive Lebesgue measure, (b) g is strictly log-supermodular over* $\mathbb{R}^2$. [58]

Proof of Property SCB. For any $x \in \mathbb{R}$, let $\Omega_x \equiv \{\theta \in \Omega : x \in \varrho_\theta\}$. The monotonicity of ϱ_θ in θ implies that Ω_x is monotone in x in the strong-order sense. Pick any $x' \in \varrho_{\theta_0}$ with $x' > x^\star$. That $x^\star$ and x' belong to ϱ_{θ_0} implies that $\theta_0 \in \Omega_{x^\star} \cap \Omega_{x'}$. Next, observe that

$$\begin{aligned}
\Psi(x';\Omega) &= \int_{\Omega_{x'}} h(\theta)g(x',\theta)d\theta \\
&= \int_{\Omega_{x'}\cap\Omega_{x^\star}} h(\theta)g(x',\theta)d\theta + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)g(x',\theta)d\theta \\
&= \int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(-\infty,\theta_0)} h(\theta)g(x^\star,\theta)\frac{g(x',\theta)}{g(x^\star,\theta)}d\theta + \int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(\theta_0,\infty)} h(\theta)g(x^\star,\theta)\frac{g(x',\theta)}{g(x^\star,\theta)}d\theta + \\
&\quad + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)g(x',\theta)d\theta \\
&\geq \frac{g(x',\theta_0)}{g(x^\star,\theta_0)}\left(\int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(-\infty,\theta_0)} h(\theta)g(x^\star,\theta)d\theta + \int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(\theta_0,\infty)} h(\theta)g(x^\star,\theta)d\theta\right) + \\
&\quad + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)g(x',\theta)d\theta \\
&\geq \frac{g(x',\theta_0)}{g(x^\star,\theta_0)}\underbrace{\Psi(x^\star;\Omega)}_{=0} + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)g(x',\theta)d\theta \geq 0.
\end{aligned}$$

The first equality follows from the fact that $g(x',\theta) = 0$ for almost all $\theta \in \Omega \setminus \Omega_{x'}$. The second equality follows from the fact that $\Omega_{x'}$ can be partitioned into $\Omega_{x'} \cap \Omega_{x^\star}$ and $\Omega_{x'} \setminus \Omega_{x^\star}$. The third equality follows from the fact that $g(x^\star,\theta) > 0$ for all $\theta \in \Omega_{x^\star}$. The first inequality follows from the fact that $g(x',\theta)/g(x^\star,\theta)$ is increasing over $\Omega_{x^\star} \cap \Omega_{x'}$ as a consequence of g being log-supermodular, along with the fact that $\theta_0 \in \Omega_{x^\star} \cap \Omega_{x'}$ and the assumption that h crosses 0 only once from below at $\theta = \theta_0$. The second inequality follows from the fact that, for any $\theta \in (\Omega_{x^\star} \setminus \Omega_{x'}) \cap (-\infty,\theta_0)$, $h(\theta) \leq 0$, along with the fact that $\Omega_{x^\star} \cap (\theta_0,+\infty) = \Omega_{x^\star} \cap \Omega_{x'} \cap (\theta_0,\infty)$, with the last property following from the fact that Ω_x are ranked in the strong-order sense. The last inequality follows from the observation that, for any $\theta \in \Omega_{x'} \setminus \Omega_{x^\star}$, $h(\theta) \geq 0$, which in turn is a consequence of (i) the monotonicity of the sets Ω_x in x, (ii) the assumption that h crosses 0 only once from below at $\theta = \theta_0$, and (iii) the assumption that $\theta_0 \in \Omega_{x^\star} \cap \Omega_{x'}$.

Similar arguments imply that, for $x < x^\star$, $\Psi(x;\Omega) \leq 0$. The same arguments also imply that, when (a) $\{\theta \in \Omega : h(\theta) \neq 0\}$ has strict positive Lebesgue measure and (b) g is strictly log-supermodular over $\mathbb{R}^2$, then $\Psi(x;\Omega) < 0$ for all $x < x^\star$ and $\Psi(x;\Omega) > 0$ for all $x > x^\star$. This completes the proof of Property SCB.■

The facts that (a) the continuation game is supermodular, (b) the density $p(x|\theta)$ is log-supermodular, and (c) when agents follow monotone strategies, the fate of the bank is monotone in θ imply that,

[58] That g is strictly log-supermodular over $\mathbb{R}^2$ also implies that $g(x,\theta) > 0$ for all $(x,\theta) \in \mathbb{R}^2$.

for any $s \in \mathcal{S}$, there exists a unique sequence $\left(\xi^{\Gamma;s}_{(n)}\right)_{n\in\mathbb{N}}$ such that, for any $n \geq 1$, $a^{\Gamma}_{(n)}$ is such that

$$a^{\Gamma}_{(n),i}(x,s) = \mathbb{I}\{x > \xi^{\Gamma;s}_{(n)}\},\ \text{all}\ (x,s) \in \mathbb{R}\times\mathcal{S}, \text{all}\, i \in [0,1],$$

with each $\xi^{\Gamma;s}_{(1)}$ as defined above, and with all other cut-offs $\xi^{\Gamma;s}_{(n)}$, $n > 1$, $s \in \mathcal{S}$, defined inductively by $\xi^{\Gamma;s}_{(n)} \equiv \sup\{x : U^{\Gamma}(x,s|\xi^{\Gamma;s}_{(n-1)}) \leq 0\}$.

Next, let $T^{\Gamma} \equiv \cap_{n=1}^{\infty} T^{\Gamma}_n$ denote the set of strategy profiles that are *rationalizable* for the agents under the policy Γ. The most aggressive strategy profile in T^{Γ} is then given by

$$a^{\Gamma}_i(x,s) \equiv \mathbb{I}\{x > \xi^{\Gamma;s}\}, \quad \text{all}\ (x,s) \in \mathbb{R}\times\mathcal{S}, \text{all}\, i \in [0,1],$$

where, for any $s \in \mathcal{S}$, $\xi^{\Gamma;s} \equiv \lim_{n\to\infty} \xi^{\Gamma;s}_{(n)}$. The sequence $(\xi^{\Gamma;s}_{(n)})_n$ is monotone and its limit is given by $\xi^{\Gamma;s} = \sup\{x : U^{\Gamma}(x,s|x) \leq 0\}$ if $\{x : U^{\Gamma}(x,s|x) \leq 0\} \neq \emptyset$, and $\xi^{\Gamma;s} \equiv -\infty$ otherwise. This establishes the first part of the lemma. That the profile a^{Γ} is a BNE for the continuation game that starts with the announcement of the policy Γ follows from the fact that, given any $s \in \mathcal{S}$, when all agents follow a cut-off strategy with cutoff $\xi^{\Gamma;s}$, the best response for each agent $i \in [0,1]$ is to pledge for $x_i > \xi^{\Gamma;s}$ and to refrain from pledging for $x_i < \xi^{\Gamma;s}$ (he is indifferent for $x_i = \xi^{\Gamma;s}$). This completes the proof of the lemma. ■

Step 2. Now take any policy $\Gamma = (\mathcal{S},\pi)$ satisfying the perfect-coordination property. Given the result in Theorem 1, without loss of generality, assume that $\Gamma = (\mathcal{S},\pi)$ is such that $\mathcal{S} = \{0,1\}\times\hat{S}$, for some measurable set $\hat{S}$, and is such that (a) when the policy discloses any signal $s = (\hat{s},1)$, all agents pledge and default does not happen, whereas (b) when the policy discloses any signal $s = (\hat{s},0)$, all agents refrain from pledging and default happens.

Equipped with the result in Lemma 1, we then show that, starting from $\Gamma = (\mathcal{S},\pi)$, one can construct a binary policy $\Gamma^* = (\{0,1\},\pi^*)$ also satisfying the perfect-coordination property and such that the probability of default under Γ^* is the same as under Γ. The policy $\Gamma^* = (\{0,1\},\pi^*)$ is such that, for any θ, $\pi^*(1|\theta) = \int_{\hat{\mathcal{S}}} \pi\left(\mathrm{d}\left(\hat{s},1\right)|\theta\right)$. That is, for each θ, the binary policy Γ^* recommends to pledge (equivalently, announces a "pass" grade) with the same total probability the original policy Γ discloses signals leading all agents to pledge.[59]

We now show that, under Γ^*, when the policy announces that $s = 1$, the unique rationalizable action for each agent is to pledge. To see this, for any $(x,1)$ that are mutually consistent given Γ^*, let $U^{\Gamma^*}(x,1|k)$ denote the expected payoff differential for any agent with private signal x, when the policy Γ^* announces $s = 1$, and all other agents follow a cut-off strategy with cut-off k.[60] From the law of iterated expectations, we have that

$$U^{\Gamma^*}(x,1|k) = \int_{\hat{\mathcal{S}}} U^{\Gamma}(x,(\hat{s},1)|k)\varsigma^{\Gamma}(\mathrm{d}\hat{s}|x,1) \tag{18}$$

[59]The notation $\int_{\hat{\mathcal{S}}} \pi\left(\mathrm{d}\left(\hat{s},1\right)|\theta\right)$ represents the total probability that the measure $\pi(\theta)$ assigns to signal $(\hat{s},r)$ such that $r = 1$.

[60]Recall that $(x,1)$ are mutually consistent under Γ^* if $p^{\Gamma^*}(x,1) \equiv \int p(x|\theta)\pi^*(1|\theta)dF(\theta) > 0$.

where $\varsigma^{\Gamma}(\cdot|x,1)$ is the probability measure over $\hat{S}$ obtained by conditioning on the event $(x,1)$, under the policy Γ. For any signal $s=(\hat{s},1)$ in the range of π, MARP consistent with Γ is such that $a_i^{\Gamma}(x,(\hat{s},1))=1$ all $x\in\mathbb{R}$, meaning that pledging is the unique rationalizable action after the policy Γ announces $s=(\hat{s},1)$. Lemma 1 in turn implies that, for all $s=(\hat{s},1)$ in the range of π, $\hat{s}\in\hat{S}$, all $k\in\mathbb{R}$, $U^{\Gamma}(k,(\hat{s},1)|k)>0$. From (18), we then have that, for all all $k\in\mathbb{R}$, $U^{\Gamma^*}(k,1|k)>0$. In turn, this implies that, given the new policy Γ^*, when $s=1$ is disclosed, under the unique rationalizable profile, all agents pledge, that is, $a_i^{\Gamma^*}(x,1)=1$ all x, all $i\in[0,1]$. It is also easy to see that, when the policy Γ^* discloses the signal $s=0$, it becomes common certainty among the agents that $\theta\leq 1$. Hence, under MARP consistent with Γ^*, after $s=0$ is disclosed, all agents refrain from pledging, irrespective of their private signals. The new pass/fail policy Γ^* so constructed thus (a) satisfies the perfect-coordination property, and (b) is such that, for any θ, the probability of default under Γ^* is the same as under Γ. Q.E.D.

Proof of Theorem 3. Without loss of generality, assume that the policy $\Gamma=(\mathcal{S},\pi)$ (a) is a (possibly stochastic) "pass/fail"policy (i.e., $\mathcal{S}=\{0,1\}$, with $\pi(1|\theta)=1-\pi(0|\theta)$ denoting the probability that signal $s=1$ is disclosed when the fundamentals are θ), (b) is such that $\pi(1|\theta)=0$ for all $\theta\leq 0$ and $\pi(1|\theta)=1$ for all $\theta>1$, and (c) satisfies the perfect-coordination property. Theorems 1 and 2 imply that, if Γ does not satisfy these properties, there exists another policy Γ' that satisfies these properties and yields the policy maker a payoff weakly higher than Γ. The proof then follows from applying the arguments below to Γ' instead of Γ.

Suppose that Γ is such that there exists no $\hat{\theta}$ such that $\pi(1|\theta)=0$ for F-almost all $\theta\leq\hat{\theta}$ and $\pi(1|\theta)=1$ for F-almost all $\theta>\hat{\theta}$.[61] We establish the result by showing that there exists a deterministic monotone policy $\Gamma^{\hat{\theta}}=(\{0,1\},\pi^{\hat{\theta}})$ satisfying the perfect-coordination property that yields the policy maker a payoff strictly higher than Γ.

Recall that, for the policy Γ to satisfy the perfect-coordination property, it must be that, when the policy discloses the signal $s=1$, $U^{\Gamma}(x,1|x)>0$ for all x such that $(x,1)$ are mutually consistent, where $U^{\Gamma}(x,1|x)$ is the expected payoff of an agent with signal x who hears that $s=1$ and who expects all other agents to follow a cut-off policy with cut-off x.

Now let $\mathbb{G}$ denote the set of policies $\Gamma'=(\mathcal{S},\pi')$ that, in addition to properties (a) and (b) above, are such that $U^{\Gamma'}(x,1|x)\geq 0$ for all x such that $(x,1)$ are mutually consistent. Observe that some policies Γ' in $\mathbb{G}$ need not satisfy the perfect-coordination property (namely, those for which there exists x such that $(x,1)$ are mutually consistent and $U^{\Gamma'}(x,1|x)=0$). For any Γ, let $\mathcal{U}^P[\Gamma]$ denote the policy maker's ex-ante expected payoff under MARP consistent with the policy Γ. Denote by

[61] Clearly, if the policy $\Gamma=(\{0,1\},\pi)$ is such that there does exists $\hat{\theta}\in[0,1]$ such that $\pi(1|\theta)=0$ for F-almost all $\theta\leq\hat{\theta}$ and $\pi(1|\theta)=1$ for F-almost all $\theta\geq\hat{\theta}$, then the deterministic monotone policy $\Gamma^{\hat{\theta}}=(\{0,1\},\pi^{\hat{\theta}})$ with cut-off $\hat{\theta}$ (that is, the policy such that $\pi^{\hat{\theta}}(1|\theta)=\mathbb{I}(\theta>\hat{\theta})$ for all θ) also satisfies the perfect-coordination property and yields the policy maker the same payoff as Γ, in which case the result trivially holds.

$\arg\max_{\tilde{\Gamma}\in\mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\}$ the set of policies that maximize the policy maker's payoff over the set $\mathbb{G}$.[62]

Step 1 below shows that any $\Gamma' \in \arg\max_{\tilde{\Gamma}\in\mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\}$ is such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$, where θ^* is the cut-off defined in (3).

We establish the result by showing that, given any policy $\Gamma' \in \mathbb{G}$ for which there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$, there exists another policy $\Gamma'' \in \mathbb{G}$ that that yields the policy maker a payoff strictly higher than Γ'. This property, together with the fact that any policy $\Gamma' = (\{0,1\}, \pi')$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$, for some $\hat{\theta}$, belongs to $\mathbb{G}$ only if $\hat{\theta} \in [\theta^*, 1]$ then gives the result.

Step 2 then shows that the policy maker's payoff under the optimal deterministic monotone policy $\Gamma^{\theta^*} = (\{0,1\}, \pi^{\theta^*})$ with cut-off θ^* can be approximated arbitrarily well by a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}}) \in \mathbb{G}$ that satisfies the perfect-coordination property (i.e., such that $U^{\Gamma^{\hat{\theta}}}(x, 1|x) > 0$ for all x such that $(x, 1)$ are mutually consistent), thus establishing the result in the theorem.

For brevity, the proof below considers the case where the prior F from which the fundamentals θ are drawn and the distribution P from which the agents' signals are drawn have unbounded support: $\Theta = \mathbb{R}$ and $(\underline{\varrho}_\theta, \bar{\varrho}_\theta) = \mathbb{R}$ for all $\theta \in \Theta$. In the online Supplement, we dispense with these restrictions.

Step 1. Take any policy $\Gamma' \in \mathbb{G}$ for which there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$. Let $X^{\Gamma'} \equiv \left\{x : U^{\Gamma'}(x, 1|x) = 0\right\}$. Clearly, if $X^{\Gamma'} = \emptyset$, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'.[63] Thus, assume that $X^{\Gamma'} \neq \emptyset$, and let $\bar{x} \equiv \sup X^{\Gamma'}$.[64]

For any x, let $\theta_0(x)$ be the fundamental threshold such that, when agents pledge when their private signal exceeds x and refrain from pledging otherwise, then their expected payoff $u(\theta, 1 - P(x|\theta))$ crosses zero from below at $\theta = \theta_0(x)$.[65] For any policy $\Gamma = \{\{0,1\}, \pi\} \in \mathbb{G}$, let $p^\Gamma(x, 1) \equiv \int_{-\infty}^{+\infty} \pi(1|\theta)p(x|\theta)dF(\theta)$ represent the joint probability density of observing the exogenous signal x and the endogenous signal $s = 1$. Let

$$\theta_H \equiv \sup\left\{\theta \in \Theta : \exists\delta > 0 \,\text{s.t.}\, \pi'(1|\theta') < 1 \,\text{for}\, F\text{-almost all}\, \theta' \in [\theta - \delta, \theta)\right\},$$

[62]That $\arg\max_{\tilde{\Gamma}\in\mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\} \neq \emptyset$ follows from the compactness of $\mathbb{G}$ and the upper hemi-continuity of $\mathcal{U}^P$ over $\mathbb{G}$.

[63]To see this, note that, because there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$, if $X^{\Gamma'} = \emptyset$, there must exists a set $(\theta', \theta'') \subseteq [0, 1]$ of F-positive probability over which $\pi'(1|\theta) < 1$. The policy Γ'' can then be obtained from Γ' by increasing $\pi'(1|\theta)$ over such a set. Provided the increase is small, the new policy is such that $U^{\Gamma''}(x, 1|x) \geq 0$ for all x, and hence $\Gamma'' \in \mathbb{G}$. Because $U^P(\theta, 1) > U^P(\theta, 0)$ over $[0, 1]$, the new policy improves over the original one.

[64]Clearly, $\bar{x}$ depends on the policy Γ'. We do not highlight the dependence to ease the notation.

[65]Because the sign of $u(\theta, 1 - P(x|\theta))$ is determined by the default outcome, $\theta_0(x)$ is implicitly defined by $P(x|\theta_0(x)) = \theta_0(x)$.

$$\theta_L \equiv \inf\{\theta \in \Theta : \exists\delta > 0 \,\text{s.t.}\, \pi'(1|\theta') > 0 \,\text{for}\, F\text{-almost all}\, \theta' \in [\theta, \theta+\delta)\}.$$

That $\Gamma' \in \mathbb{G}$ guarantees that θ_H and θ_L are well-defined. That, under Γ', there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$ implies that $\theta_L < \theta_H$. Furthermore, $u(\theta_L, 1 - P(\bar{x}|\theta_L)) < 0$.[66]

We distinguish between two cases.

Case 1: $\theta_0(\bar{x}) < \theta_H$.

Consider the policy $\Gamma^{\epsilon,\delta} = (\{0,1\}, \pi^{\epsilon,\delta})$ defined by $\pi^{\epsilon,\delta}(1|\theta) = \pi'(1|\theta)$ for all $\theta \leq \theta_0(\bar{x}+\delta)$, with $\delta > 0$ small so that $\theta_0(\bar{x}+\delta) < \theta_H$, and $\pi^{\epsilon,\delta}(1|\theta) = \min\{\pi'(1|\theta)+\epsilon, 1\}$ for all $\theta > \theta_0(\bar{x}+\delta)$, with $\epsilon > 0$ also small. To see that, when ϵ and δ are small, $\Gamma^{\epsilon,\delta} \in \mathbb{G}$, note that, by definition of $\theta_0(\cdot)$, for any x, and any $\theta > \theta_0(x)$, $u(\theta, 1 - P(x|\theta)) > 0$. This fact, together with the monotonicity of $\theta_0(\cdot)$, jointly imply that, for any $x \leq \bar{x}+\delta$,

$$\begin{aligned} U^{\Gamma^{\epsilon,\delta}}(x,1|x)p^{\Gamma^{\epsilon,\delta}}(x,1) &= \int_{-\infty}^{\theta_0(\bar{x}+\delta)} u(\theta, 1-P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta) + \\ &\quad + \int_{\theta_0(\bar{x}+\delta)}^{+\infty} u(\theta, 1-P(x|\theta))\min\{\pi'(1|\theta)+\epsilon, 1\})p(x|\theta)dF(\theta) \\ &> U^{\Gamma'}(x,1|x)p^{\Gamma'}(x,1) \geq 0. \end{aligned}$$

The strict inequality obtains from the fact that, for any $\theta \in [\theta_0(\bar{x}+\delta), \theta_H]$, $\pi^{\epsilon,\delta}(1|\theta) \geq \pi'(1|\theta)$, with the inequality strict over a subset of $[\theta_0(\bar{x}+\delta), \theta_H]$ of strictly positive F-measure, along with the fact that, because $x \leq \bar{x}+\delta$, $u(\theta, 1-P(x|\theta)) > 0$ for all $\theta \geq \theta_0(\bar{x}+\delta)$. That, when $\epsilon > 0$ is sufficiently small, $U^{\Gamma^{\epsilon,\delta}}(x,1|x) > 0$ also for all $x > \bar{x}+\delta$ follows from the fact that, by definition of $\bar{x}$, for any $x > \bar{x}+\delta$, $U^{\Gamma'}(x,1|x)$ is bounded away from 0 along with the fact that, for any $\delta > 0$, the function family $\left(U^{\Gamma^{\epsilon,\delta}}(\cdot,1|\cdot)\right)_\epsilon$ is continuous in ϵ in the sup-norm, in a neighborhood of 0.[67]

Case 2: $\theta_0(\bar{x}) \geq \theta_H$.

Consider the *monotone* policy $\Gamma^0 = \left\{\{0,1\}, \pi^0\right\}$ such that $\pi^0(1|\theta) \equiv \mathbb{I}(\theta \geq 0)$. Note that, for any $x \geq \bar{x}$,

$$\begin{aligned} U^{\Gamma'}(x,1|x)p^{\Gamma'}(x,1) &= \textstyle\int_0^{\theta_0(x)} u(\theta, 1-P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta) \\ &\quad + \textstyle\int_{\theta_0(x)}^{+\infty} u(\theta, 1-P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta) \\ &> \textstyle\int_0^{\theta_0(x)} u(\theta, 1-P(x|\theta))p(x|\theta)dF(\theta) + \int_{\theta_0(x)}^{+\infty} u(\theta, 1-P(x|\theta))p(x|\theta)dF(\theta) \\ &= U^{\Gamma^0}(x,1|x)p^{\Gamma^0}(x,1), \end{aligned}$$

[66]That $u(\theta_L, 1-P(\bar{x}|\theta_L)) < 0$ follows from the fact that, by definition of $\bar{x}$ and θ_L, $U^{\Gamma'}(\bar{x},1|\bar{x}) = \frac{1}{p^{\Gamma'}(\bar{x},1)}\int_{\theta_L}^{+\infty} u(\theta, 1-P(\bar{x}|\theta))\pi'(1|\theta)p(\bar{x}|\theta)dF(\theta) = 0$, together with the single-crossing property of $u(\theta, 1-P(\bar{x}|\theta))$ in θ.

[67]This means that, for any $z > 0$, there exists $\Delta > 0$ such that, for any $0 \leq \epsilon < \Delta$, and all x, $|U^{\Gamma^{\epsilon,\delta}}(x,1|x) - U^{\Gamma^{0,\delta}}(x,1|x)| \leq z$, where, by definition, $\Gamma^{0,\delta} = \Gamma'$.

where the inequality follows from the fact that (i) $u(\theta, 1-P(x|\theta)) < 0$ for any $\theta \leq \theta_0(x)$ along with (ii) the fact that $\pi'(1|\theta) = 1$ for F-almost all $\theta \geq \theta_0(x) \geq \theta_0(\bar{x}) \geq \theta_H$. As a result,

$$U^{\Gamma^0}(\bar{x}, 1|\bar{x}) \quad < \quad U^{\Gamma'}(\bar{x}, 1|\bar{x}) \frac{p^{\Gamma'}(x,1)}{p^{\Gamma^0}(x,1)} = 0.$$

We conclude that, necessarily, $\bar{x} < \bar{x}_G$, where $\bar{x}_G$ is the threshold defined in (1). This property in turn permits us to apply part (3) of Condition M to $\bar{x}$ in the arguments below.

For any $\gamma > 0$, let $\theta_L^\gamma \equiv \theta_L + \gamma$ and $\theta_H^\gamma \equiv \theta_H - \gamma$. Pick $\gamma, e_L, e_H > 0$ small such that (i) $\pi'\left(1|\theta_L^\gamma\right) > 0$ and $\pi'(1|\theta) > 0$ for $F-$almost $\theta \in \left(\theta_L^\gamma, \theta_L^\gamma + e_L\right)$, (ii) $\pi'\left(1|\theta_H^\gamma\right) < 1$ and $\pi'(1|\theta) < 1$ for $F-$almost all $\theta \in \left(\theta_H^\gamma - e_H, \theta_H^\gamma\right)$, and (iii) $\theta_L^\gamma + e_L < \theta_H^\gamma - e_H$.[68] Next, pick $\eta \in (0, \bar{x}_G - \bar{x})$ small such that $U^{\Gamma'}(x, 1|x) > \eta$ for all $x \geq \bar{x} + \eta$. Pick $\epsilon > 0$ also small and let $\delta(\epsilon, \eta)$ be implicitly defined by

$$\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u(\theta, 1-P(\bar{x}+\eta|\theta))\pi'(1|\theta)p(\bar{x}+\eta|\theta)dF(\theta) \tag{19}$$

$$= \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u(\theta, 1-P(\bar{x}+\eta|\theta))(1-\pi'(1|\theta))p(\bar{x}+\eta|\theta)dF(\theta).$$

Note that, for $\epsilon > 0$ small, $\theta_L^\gamma + \epsilon < \theta_H^\gamma - \delta(\varepsilon, \eta)$. Consider the policy $\Gamma^{\epsilon,\gamma,\eta} = \{\{0,1\}, \pi^{\epsilon,\gamma,\eta}\}$ defined by the following properties: (a) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = \pi'(1|\theta)$ for all $\theta \notin \left\{[\theta_L^\gamma, \theta_L^\gamma+\epsilon] \bigcup [\theta_H^\gamma - \delta(\epsilon,\eta), \theta_H^\gamma]\right\}$; (b) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = 0$ for all $\theta \in [\theta_L^\gamma, \theta_L^\gamma+\epsilon]$; and (c) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = 1$ for all $\theta \in [\theta_H^\gamma - \delta(\epsilon,\eta), \theta_H^\gamma]$. Note that Condition (19) implies that $U^{\Gamma^{\epsilon,\gamma,\eta}}(\bar{x}+\eta, 1|\bar{x}+\eta) = U^{\Gamma'}(\bar{x}+\eta, 1|\bar{x}+\eta) > 0$.

We now show that, under the new policy, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \geq 0$ for any x. Recall that, for any $\theta \in (0,1)$, $x^*(\theta)$ is the critical threshold such that, when agents pledge for $x > x^*(\theta)$ and do not pledge for $x < x^*(\theta)$, default occurs when fundamentals are below θ and does not occur when they are above θ, and hence $u(\tilde{\theta}, 1-P(x^*(\theta)|\tilde{\theta})$ turns from negative to positive at $\tilde{\theta} = \theta$.

Clearly, for any (ϵ, γ, η), and any $x \leq x^*(\theta_L)$, $U^{\Gamma'}(x,1|x), U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) > 0$. This is because, for any such x, $\theta_0(x) < \theta_L$ and hence $u(\theta, 1-P(x|\theta)) > 0$ for all $\theta > \theta_L$. The result then follows from the fact that, under both Γ' and $\Gamma^{\epsilon,\gamma,\eta}$,

$$\int_{-\infty}^{\theta_L} \pi'(1|\theta)\, dF(\theta) = \int_{-\infty}^{\theta_L} \pi^{\epsilon,\gamma,\eta}(1|\theta)\, dF(\theta) = 0$$

meaning that all agents assign probability one to the event that $\theta \geq \theta_L$. Furthermore, that

$$U^{\Gamma'}(x^*(\theta_L), 1|x^*(\theta_L)) > 0$$

along with the fact that $U^{\Gamma'}(x,1|x) > \eta$ for all $x \geq \bar{x}+\eta$ and the continuity of

$$U^{\Gamma'}(x,1|x)p^{\Gamma'}(x,1) = \int u(\theta, 1-P(x|\theta))\pi'(1|\theta)p(x|\theta)dF(\theta)$$

[68]If a single γ satisfying properties (i)-(iii) does not exist, let $\gamma = (\gamma_L, \gamma_H) \in \mathbb{R}^2_{++}$. The arguments below then apply verbatim by letting $\theta_L^\gamma = \theta_L + \gamma_L$ and $\theta_H^\gamma = \theta_H + \gamma_H$ and noting that a $\gamma = (\gamma_L, \gamma_H)$ satisfying properties (i)-(iii) always exists.

in x imply that there exists $\xi > 0$ such that, for any $x \in [x^*(\theta_L), x^*(\theta_L)+\xi] \cup [\bar{x}+\eta, +\infty)$, $U^{\Gamma'}(x,1|x)p^{\Gamma'}(x,1) > \xi$. Because, for any η, the function family $\left(U^{\Gamma^{\epsilon,\gamma,\eta}}(\cdot,1|\cdot)p^{\Gamma^{\epsilon,\gamma,\eta}}(\cdot,1)\right)_{\epsilon,\gamma}$ is continuous in (γ,ϵ) in the sup-norm, in a neighborhood of $(0,0)$.[69] and $x^*(\theta)$ is continuous in θ, there exist $\bar{\gamma}, \bar{\epsilon} > 0$ such that, when $\gamma \leq \bar{\gamma}$ and $\epsilon \leq \bar{\epsilon}$, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \geq 0$ for any $x \in (-\infty, x^*\left(\theta_L^\gamma+\epsilon\right)] \cup [\bar{x}+\eta, +\infty)$.

Next observe that, for any $x \in \left(x^*(\theta_L^\gamma+\epsilon), x^*\left(\theta_H^\gamma - \delta(\epsilon,\eta)\right)\right]$,

$$\begin{gathered} U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)p^{\Gamma^{\epsilon,\gamma,\eta}}(1,x) - U^{\Gamma'}(x,1|x)p^{\Gamma'}(1,x) \\ = -\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi'\left(1|\theta\right) dF(\theta) \\ + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \left(1-\pi'\left(1|\theta\right)\right) dF(\theta) > 0, \end{gathered}$$

where the inequality follows from the fact that the integrand in the first integral is negative, whereas that in the second integral is positive. Because $U^{\Gamma'}(x,1|x) \geq 0$ for all x, this implies that for any such x, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \geq 0$.

Next, consider $x \in \left(x^*\left(\theta_H^\gamma - \delta(\epsilon,\eta)\right), x^*(\theta_H^\gamma)\right)$. For any x, and any θ, let

$$q(\theta, x) \equiv \left|u\left(\theta, 1-P\left(x|\theta\right)\right)\right| p\left(x|\theta\right).$$

For any $x \leq \bar{x}+\eta$, let $\Delta U(x) \equiv U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)p^{\Gamma^{\epsilon,\gamma,\eta}}(1,x) - U^{\Gamma'}(x,1|x)p^{\Gamma'}(1,x)$. Note then that, for any $x \in \left(x^*\left(\theta_H^\gamma - \delta(\epsilon,\eta)\right), x^*(\theta_H^\gamma)\right)$,

$$\begin{aligned} \Delta U(x) &= \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\eta)} -u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &\quad + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_0(x)} -u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &\quad + \int_{\theta_0(x)}^{\theta_H^\gamma} -u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &\geq \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\eta)} \frac{q(\theta,x)}{q(\theta,\bar{x}+\eta)} q(\theta,\bar{x}+\eta) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &\quad + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_0(x)} \frac{q(\theta,x)}{q(\theta,\bar{x}+\eta)} q(\theta,\bar{x}+\eta) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &\quad + \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \int_{\theta_0(x)}^{\theta_H^\gamma} q(\theta,\bar{x}+\eta) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &\geq \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \int_{\theta_L^\gamma}^{\theta_H^\gamma} q(\theta,\bar{x}+\eta) f(\theta) \left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) d\theta \\ &= \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \Delta U(\bar{x}+\eta) \\ &= 0. \end{aligned}$$

[69]This means that, for any $z > 0$, there exists $\Delta > 0$ such that, for any (ϵ,γ) with $0 \leq \epsilon < \Delta$ and $0 \leq \gamma < \Delta$, and all x, $|U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) - U^{\Gamma^{0,0,\eta}}(x,1|x)| \leq z$, where, by definition, $\Gamma^{0,0,\eta} = \Gamma'$.

The first equality is by definition. The first inequality follows from the fact that (i) for any $\theta \le \theta_0(x)$, $u(\theta, 1-P(x|\theta)) < 0$, whereas, for any $\theta > \theta_0(x)$, $u(\theta, 1-P(x|\theta)) > 0$, along with the fact that (ii) for $\theta \in [\theta_0(x), \theta_H^\gamma]$, $\pi'(1|\theta) \le \pi^{\epsilon,\gamma,\eta}(1|\theta)$. Together, these two properties imply that

$$\int_{\theta_0(x)}^{\theta_H^\gamma} -u(\theta, 1-P(x|\theta))\, p(x|\theta)\, f(\theta) \left(\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)\right) d\theta$$

$$\ge 0 \ge \quad \frac{q(\theta_H^\gamma - \delta(\epsilon,\eta), x)}{q(\theta_H^\gamma - \delta(\epsilon,\eta), \bar{x}+\eta)} \int_{\theta_0(x)}^{\theta_H^\gamma} q(\theta, \bar{x}+\eta)\, f(\theta) \left(\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)\right) d\theta.$$

The second inequality follows from the fact that $\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)$ turns from positive to negative at $\theta = \theta_H^\gamma - \delta(\epsilon,\eta) \le \theta_0(x)$, along with the fact that, for any $\theta \in [\theta_L^\gamma, \theta_0(x)]$, the function $q(\theta, x)/q(\theta, \bar{x}+\eta)$ is non-increasing in θ as implied by the log-supermodularity of $|u(\theta, 1-P(x|\theta))|\, p(x|\theta)$ over $\{(\theta, x) \in [0,1] \times \mathbb{R} : u(\theta, 1-P(x|\theta)) \le 0\}$, by virtue of part 2 of Condition M. Finally, the last two equalities follow from the fact that $\theta_0(\bar{x}+\eta) > \theta_0(\bar{x}) > \theta_H \ge \theta_H^\gamma$, which implies that $u(\theta, 1-P(\bar{x}+\eta|\theta)) \le 0$ for all $\theta \le \theta_H^\gamma$, and hence that

$$\int_{\theta_L^\gamma}^{\theta_H^\gamma} q(\theta, \bar{x}+\eta)\, f(\theta) \left(\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)\right) d\theta = \Delta U(\bar{x}+\eta)$$

along with the fact that, by construction of the policy $\Gamma^{\epsilon,\gamma,\eta}$, $\Delta U(\bar{x}+\eta) = 0$. Hence, for any $x \in \left(x^*\left(\theta_H^\gamma - \delta(\epsilon,\eta)\right), x^*(\theta_H^\gamma)\right)$, $\Delta U(x) \ge 0$, which implies that $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \ge 0$.

Similar arguments imply that, for any $x \in \left[x^*(\theta_H^\gamma), \bar{x}+\eta\right]$,

$$\begin{aligned}
\Delta U(x) &= \int_{\theta_L^\gamma}^{\theta_H^\gamma} -u(\theta, 1-P(x|\theta))\, p(x|\theta)\, f(\theta) \left(\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)\right) d\theta \\
&= \int_{\theta_L^\gamma}^{\theta_H^\gamma - \delta(\epsilon,\gamma)} \frac{q(\theta, x)}{q(\theta, \bar{x}+\eta)} q(\theta, \bar{x}+\eta)\, f(\theta) \left(\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)\right) d\theta \\
&\quad + \int_{\theta_H^\gamma - \delta(\epsilon,\eta)}^{\theta_H^\gamma} \frac{q(\theta, x)}{q(\theta, \bar{x}+\eta)} q(\theta, \bar{x}+\eta)\, f(\theta) \left(\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)\right) d\theta \\
&\ge \frac{q(\theta_H^\gamma - \delta(\epsilon,\eta), x)}{q(\theta_H^\gamma - \delta(\epsilon,\eta), \bar{x}+\eta)} \Delta U(\bar{x}+\eta) = 0,
\end{aligned}$$

which implies that, for such x too, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \ge 0$.[70]

We conclude that, when ϵ, γ, η are small, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \ge 0$ for all x and hence $\Gamma^{\epsilon,\gamma,\eta} \in \mathbb{G}$.

We now show that, when property 3 in Condition M holds, the new policy yields the policy maker an expected payoff strictly higher than Γ'. To see this, observe that, fixing (γ, η), for any $\epsilon > 0$, the

[70] The first equality is by definition. The second equality follows from the fact that, for such x, $u(\theta, 1-P(x|\theta)) \le 0$ for all $\theta \le \theta_H^\gamma$. The inequality follows from the fact that $q(\theta, x)/q(\theta, \bar{x}+\eta)$ is non-increasing in θ over $[\theta_L^\gamma, \theta_H^\gamma]$ along with the fact that $\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)$ changes sign only once, turning from non-negative to non-positive at $\theta = \theta_H^\gamma - \delta(\epsilon,\gamma)$.

policy maker's payoff under the policy $\Gamma^{\epsilon,\gamma,\eta}$ is equal to

$$\begin{aligned}\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}] &= \int_{-\infty}^{\theta_L^\gamma+\epsilon} U^P(\theta,0)dF(\theta) + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} U^P(\theta,1)dF(\theta) \\ &+ \int_{(\theta_L^\gamma+\epsilon,\theta_H^\gamma-\delta(\epsilon,\eta))\cup(\theta_H^\gamma,+\infty)} \left\{\pi'(1|\theta)U^P(\theta,1) + (1-\pi'(1|\theta))U^P(\theta,0)\right\} dF(\theta).\end{aligned}$$

Differentiating $\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]$ with respect to ϵ, and taking the limit as $\epsilon \to 0^+$, we have that

$$\begin{aligned}\lim_{\epsilon\to0^+} \tfrac{d\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]}{d\epsilon} &= f(\theta_H^\gamma)(1-\pi'(1|\theta_H^\gamma))\left[U^P(\theta_H^\gamma,1)-U^P(\theta_H^\gamma,0)\right]\left(\lim_{\epsilon\to0^+}\tfrac{\partial\delta(\epsilon,\eta)}{\partial\epsilon}\right) \\ &\quad -f(\theta_L^\gamma)\pi'(1|\theta_L^\gamma)\left[U^P(\theta_L^\gamma,1)-U^P(\theta_L^\gamma,0)\right] \\ &= f(\theta_L^\gamma)\pi'(1|\theta_L^\gamma)\left(\left[U^P(\theta_H^\gamma,1)-U^P(\theta_H^\gamma,0)\right]\tfrac{p(\bar{x}+\eta|\theta_L^\gamma)u(\theta_L^\gamma,1-P(\bar{x}+\eta|\theta_L^\gamma))}{p(\bar{x}+\eta|\theta_H^\gamma)u(\theta_H^\gamma,1-P(\bar{x}+\eta|\theta_H^\gamma))} - \left[U^P(\theta_L^\gamma,1)-U^P(\theta_L^\gamma,0)\right]\right).\end{aligned}$$

Therefore, $\lim_{\epsilon\to0^+} \frac{d\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]}{d\epsilon} > 0$ if and only if

$$\frac{U^P\left(\theta_H^\gamma,1\right)-U^P\left(\theta_H^\gamma,0\right)}{U^P\left(\theta_L^\gamma,1\right)-U^P\left(\theta_L^\gamma,0\right)} > \frac{p\left(\bar{x}+\eta|\theta_H^\gamma\right)u\left(\theta_H^\gamma,1-P\left(\bar{x}+\eta|\theta_H^\gamma\right)\right)}{p\left(\bar{x}+\eta|\theta_L^\gamma\right)u\left(\theta_L^\gamma,1-P\left(\bar{x}+\eta|\theta_L^\gamma\right)\right)}.$$

Property 3 in Condition M, together with the fact that $\bar{x} \leq \bar{x}_G$ (as proved above), guarantee this is the case. We conclude that, when ϵ is small, the policy $\Gamma^{\epsilon,\gamma,\eta} \in \mathbb{G}$ strictly improves upon Γ'. Furthermore, the construction of $\Gamma^{\epsilon,\gamma,\eta}$ above can be iterated to arrive to a monotone deterministic policy. Because any monotone deterministic policy $\Gamma^{\hat{\theta}}$ with cut-off $\hat{\theta} > \theta^*$ yields the policy maker a payoff strictly smaller than the monotone deterministic policy with cut-off θ^* (and no monotone deterministic policy $\Gamma^{\hat{\theta}}$ with cut-ff $\hat{\theta} < \theta^*$ is in $\mathbb{G}$), we conclude that any policy $\Gamma' \in \arg\max_{\tilde{\Gamma}\in\mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\}$ is such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$.

Step 2. Take any policy $\Gamma' \in \arg\max_{\tilde{\Gamma}\in\mathbb{G}} \left\{\mathcal{U}^P[\tilde{\Gamma}]\right\}$. The result in step 1 implies that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$. The result in the theorem then follows from observing that, given Γ', there exists a nearby deterministic monotone policy $\Gamma^{\hat{\theta}} \in \mathbb{G}$ with cut-off $\hat{\theta} = \theta^* + \tilde{\varepsilon}$, for $\tilde{\varepsilon} > 0$ but small, such that $\Gamma^{\hat{\theta}}$ satisfies the perfect-coordination property (i.e., $U^{\Gamma^{\hat{\theta}}}(x,1|x) > 0$ all x) and yields the policy maker a payoff arbitrarily close to that under Γ'. Q.E.D.

Proof of Proposition 1. Given any threshold $\hat{\theta}$, and any signal x, let

$$\psi_h(x,\hat{\theta},\sigma) \equiv \int_\Theta u_h\left(\theta, 1-\Phi\left(\frac{x-\theta}{\sigma}\right)\right) d\Lambda\left(\theta|x,1;\sigma\right)$$

denote the payoff of an agent with signal x, of precision σ^{-2}, who, after hearing that the bank passed the test, learns that $\theta > \hat{\theta}$, and who expects all other agents to buy security h when their signal exceeds x and short-sell it otherwise, with $h = D$ in case the bank finances itself with debt, and $h = E$ in the case the bank finances itself with equity. Here $\Lambda\left(\cdot|x,1;\sigma\right)$ represents the posterior belief over Θ for an agent with exogenous signal x of precision σ^{-2} who learns that the bank passed the test (and hence that $\theta > \hat{\theta}$). We start with the following result:

Lemma 2. *For any $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$, any $x > \hat{\theta}$,*

$$\lim_{\sigma \to 0^+} \frac{\partial}{\partial \sigma} \psi_E(x, \hat{\theta}, \sigma) > 0 > \lim_{\sigma \to 0^+} \frac{\partial}{\partial \sigma} \psi_D(x, \hat{\theta}, \sigma).$$

Proof of Lemma 2. Note that, for any $x > \hat{\theta}$,

$$\begin{aligned} \tfrac{\partial}{\partial \sigma} \psi_h(x, \hat{\theta}, \sigma) &= \tfrac{\partial}{\partial \sigma} \int_{\hat{\theta}}^{\infty} u_h\left(\theta, 1 - \Phi\left(\tfrac{x-\theta}{\sigma}\right)\right) \frac{\phi\left(\frac{x-\theta}{\sigma}\right)}{\sigma \Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)} d\theta \\ &= \tfrac{\partial}{\partial \sigma} \left\{ \frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)} \int_{1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}^{1} u_h\left(x - \sigma \Phi^{-1}(1-A), A\right) dA \right\} \\ &= \frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)} \int_{1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}^{1} \frac{\partial u_h\left(x - \sigma \Phi^{-1}(1-A), A\right)}{\partial \theta} \left(-\Phi^{-1}(1-A)\right) dA \\ &+ \frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)} \left[\psi\left(x, \hat{\theta}, \sigma\right) - u_h\left(\hat{\theta}, 1 - \Phi\left(\tfrac{x-\hat{\theta}}{\sigma}\right)\right)\right] \phi\left(\tfrac{x-\hat{\theta}}{\sigma}\right)\left(\tfrac{x-\hat{\theta}}{\sigma^2}\right). \end{aligned} \tag{20}$$

The first equality follows from the change in variables, $A = 1 - \Phi\left((x-\theta)/\sigma\right)$, whereas the second equality follows from the chain rule of differentiation.

The proof proceeds in two steps. Step 1 shows that, for any $x > \hat{\theta}$, when $\sigma \to 0^+$, the second term in the right-hand side of the last equality in (20) vanishes. Step 2 shows that, for any $x > \hat{\theta}$, when $\sigma \to 0^+$, the first term in the right-hand side of the last equality in (20) is positive for equity but negative for debt.

Step 1. Because $g_h(\cdot)$ and $b_h(\cdot)$ are bounded, for any σ, $\psi_h\left(x, \hat{\theta}, \sigma\right) - u_h\left(\hat{\theta}, 1 - \Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)\right)$ is also bounded. Furthermore, for any $x > \hat{\theta}$, and any σ, $\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right) \in [1/2, 1)$. Finally, use L'Hopital's rule to observe that, for any $x > \hat{\theta}$, $\lim_{\sigma \to 0^+} \phi\left(\frac{x-\hat{\theta}}{\sigma}\right)\left(\frac{x-\hat{\theta}}{\sigma^2}\right) = 0$. Jointly, the above properties imply that, for any $x > \hat{\theta}$, the second term in the right-hand side of the last equality in (20) vanishes as $\sigma \to 0^+$.

Step 2. Because z and θ are independent,

$$u_h(\theta, A) = \int_{\underline{z}}^{\hat{z}_h(A)} \hat{b}_h(\theta, A, z)\, dQ(z) + \int_{\hat{z}_h(A)}^{\bar{z}} \hat{g}_h(\theta, A, z)\, dQ(z),$$

where $\hat{z}_h(A)$ is a shortcut for $\hat{z}_h(\theta, A)$ and is independent of θ because $l(\theta)$ is invariant in θ.[71] This means that

$$\frac{\partial u_h(\theta, A)}{\partial \theta} = \int_{\underline{z}}^{\hat{z}_h(A)} \frac{\partial \hat{b}_h(\theta, A, z)}{\partial \theta} dQ(z) + \int_{\hat{z}_h(A)}^{\bar{z}} \frac{\partial \hat{g}_h(\theta, A, z)}{\partial \theta} dQ(z)$$

where, for $h = E$,[72] $\partial \hat{b}_E(\theta, A, z)/\partial \theta = 0$ and $\partial \hat{g}_E(\theta, A, z)/\partial \theta = C'(\theta)/(1+q)$, implying that

$$\frac{\partial u_E(\theta, A)}{\partial \theta} = \frac{C'(\theta)}{1+q}\left(1 - Q\left(\hat{z}_E(A)\right)\right),$$

[71] Recall that $\hat{z}_h(\theta, A)$ is implicitly defined by the solution to $l(\theta) + \rho_S q p_h^\star(A, z) = D$ whenever the equation has a solution, is equal to $\underline{z}$ when $l(\theta) + \rho_S q p_h^\star(A, \underline{z}) > D$, and is equal to $\bar{z}$ when $l(\theta) + \rho_S q p_h^\star(A, \bar{z}) < D$.

[72] Note that we used the assumption that $\rho_S = 1$.

whereas, for $h = D$, $\partial \hat{b}_D(\theta, A, z)/\partial\theta = L_D\gamma'(\theta)/(qL_D + D)$ and $\partial \hat{g}_D(\theta, A, z)/\partial\theta = 0$, implying that

$$\frac{\partial u_D(\theta, A)}{\partial\theta} = \frac{L_D}{qL_D + D}\gamma'(\theta) Q(\hat{z}_D(A)).$$

Hence, the above results imply that, for any $x > \hat{\theta}$,

$$\begin{aligned}\lim_{\sigma\to 0^+}\frac{\partial}{\partial\sigma}\psi_h(x,\hat{\theta},\sigma) &= \lim_{\sigma\to 0^+}\frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\int_{1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}^{1}\frac{\partial u_h\left(x-\sigma\Phi^{-1}(1-A),A\right)}{\partial\theta}\left(-\Phi^{-1}(1-A)\right)dA \\ &= \int_0^1\frac{\partial u_h(x,A)}{\partial\theta}\left(-\Phi^{-1}(1-A)\right)dA\end{aligned}$$

where the second equality follows from Lebesgue dominated convergence theorem (that $|\partial u_h(\theta, A)/\partial\theta|$ is uniformly bounded follows from the derivations above along with the fact that C and γ are Lipschitz continuous).

Next, use the change in variables $\omega = -\Phi^{-1}(1-A)$ and the fact that, for any x, $\phi(x) = \phi(-x)$, to note that

$$\int_0^1\left(-\Phi^{-1}(1-A)\right)dA = \int_{-\infty}^{\infty}\omega\phi(\omega)\,d\omega = 0.$$

The last property implies that

$$\lim_{\sigma\to 0^+}\frac{\partial}{\partial\sigma}\psi_h(x,\hat{\theta},\sigma) = \int_0^1\frac{\partial u_h(x,A)}{\partial\theta}\left(-\Phi^{-1}(1-A)\right)dA = \text{cov}\left(\frac{\partial u_h(x,A)}{\partial\theta}, -\Phi^{-1}(1-A)\right).$$

Equity. Using the properties above, we have that, for any $x > \hat{\theta}$,

$$\text{cov}\left(\frac{\partial u_E(x,A)}{\partial\theta}, -\Phi^{-1}(1-A)\right) = \frac{C'(x)}{1+q}\text{cov}\left(1 - Q(\hat{z}_E(A)), -\Phi^{-1}(1-A)\right) > 0,$$

where the inequality follows from the fact that $\hat{z}_E(A)$ is decreasing in A.

Debt. Using the properties above, we have that, for any $x > \hat{\theta}$,

$$\text{cov}\left(\frac{\partial u_D(x,A)}{\partial\theta}, -\Phi^{-1}(1-A)\right) = \frac{L_D}{qL_D+D}\gamma'(\theta)\,\text{cov}\left(Q(\hat{z_D}(A)), -\Phi^{-1}(1-A)\right) < 0,$$

where the inequality follows again from the fact that $\hat{z}_D(A)$ is decreasing in A.

The lemma follows from combining the results from step 1 with those from 2. ■

Now observe that, for any precision of private information σ^{-2} and any monotone pass/fail policy with threshold $\hat{\theta}$, after the signal $s = 1$ is disclosed, purchasing the bank's security h is the unique rationalizable action for all investors if, and only if, $\psi_h\left(x, \hat{\theta}, \sigma\right) > 0$ for all $x \in \mathbb{R}$ (the arguments are analogous to the ones in the proof of Theorem 2). From the discussion following Theorem 3, then observe that the threshold $\theta_h^*(\sigma)$ defining the optimal monotone policy when the precision of the investors' information is σ^{-2} and the bank funds itself by issuing security $h = D, E$ is given by $\theta_h^*(\sigma) = \inf\left\{\hat{\theta} : \psi_h\left(x, \hat{\theta}, \sigma\right) \geq 0 \text{ for all } x \in \mathbb{R}\right\}$.

Next, for any $\sigma > 0$, let $x_h^*(\sigma) \equiv \arg\min_{x\in\mathbb{R}} \psi_h(x, \theta_h^*(\sigma), \sigma)$ and note that $x_h^*(\sigma)$ is a solution to the equation $\psi_h(x_h^*(\sigma), \theta_h^*(\sigma), \sigma) = 0$. Next, for any $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$, any $\tilde{\sigma} > 0$, let $\Psi_h(\hat{\theta}, \tilde{\sigma}) \equiv$

$\inf_{x\in\mathbb{R}}\ \psi_h(x,\hat{\theta},\tilde{\sigma})$ and, for any $(\hat{\theta},\tilde{\sigma})$ such that $\arg\min_{x\in\mathbb{R}}\ \psi_h(x,\hat{\theta},\tilde{\sigma})\neq\emptyset$, let $x_h^{**}(\hat{\theta},\tilde{\sigma})\in\arg\min_{x\in\mathbb{R}}\psi_h(x,\hat{\theta},\tilde{\sigma})$. Note that, when $\hat{\theta}=\theta_h^*(\sigma)$ and $\tilde{\sigma}=\sigma$, $x_h^{**}\left(\theta_h^*(\sigma),\sigma\right)=x_h^*\left(\sigma\right)$.

Now observe that $\lim_{\sigma\to 0^+}\theta_h^*\left(\sigma\right)=\theta_h^{MS}$, $h=E,D$. The definition of $\theta_h^{\triangle}$, along with the strict monotonicity of $u_h(\theta,1/2)$ in θ, imply that, for any $\theta>\theta_h^{\triangle}$, $u_h(\theta,1/2)>0$. Hence, for any $\hat{\theta}>\theta_h^{\triangle}$, $\sigma>0$, and $x\leq\hat{\theta}$,

$$\psi_h\left(x,\hat{\theta},\sigma\right)=\int_{\hat{\theta}}^{\infty}u_h\left(\theta,1-\Phi\left(\frac{x-\theta}{\sigma}\right)\right)\frac{\phi\left(\frac{x-\theta}{\sigma}\right)}{\sigma\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}d\theta>0. \tag{21}$$

The assumption that $\theta_h^{MS}>\theta_h^{\triangle}$, $h=E,D$, along with the continuity of $\theta_h^*\left(\sigma\right)$ in σ,[73] then imply that there exist $\hat{\sigma}>0$ such that, for any $\sigma\in(0,\hat{\sigma})$, $\theta_h^*(\sigma)>\theta_h^{\triangle}$, for $h=E,D$. The result in (21) then implies that, for any $\sigma,\tilde{\sigma}\in(0,\hat{\sigma})$, $x_h^*\left(\sigma\right),x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right)>\theta_h^*(\sigma)$.

Now observe that the arguments in the proof of Lemma 2 imply that there exists $\sigma^{\#},K>0$ such that, for any $\sigma,\tilde{\sigma}\in(0,\sigma^{\#})$, with $\tilde{\sigma}\geq\sigma$, and any $x>\theta_h^*(\sigma)$, $\psi_h\left(x,\theta_h^*(\sigma),\tilde{\sigma}\right)$ is partially differentiable in its third argument, $\tilde{\sigma}$, with the partial derivative continuous in $(x,\hat{\theta},\tilde{\sigma})$ and uniformly bounded over

$$\left\{(\sigma,\tilde{\sigma},x)\in(0,\sigma^{\#})\times(0,\sigma^{\#})\times\mathbb{R}:\tilde{\sigma}\geq\sigma,\ x-\theta_h^*(\sigma)\in(0,K)\right\},\quad h=E,D.$$

Also observe that, for any $\varepsilon>0$, there exists σ_ε such that, for any $\sigma,\tilde{\sigma}\in(0,\sigma_\varepsilon)$,[74]

$$|\theta_h^*(\sigma)-\theta_h^{MS}|,|x_h^*(\sigma)-\theta_h^{MS}|,|x_h^{**}(\theta_h^*(\sigma),\tilde{\sigma})-\theta_h^{MS}|<\varepsilon. \tag{22}$$

Now let $\bar{\sigma}\equiv\min\{\sigma^{\#},\hat{\sigma},\sigma_{K/2}\}$. The properties above, along with the envelope theorem of Milgrom and Segal (2002), imply that, for any $\sigma,\tilde{\sigma}\in(0,\bar{\sigma})$, with $\tilde{\sigma}\geq\sigma$, and $h=E,D$,

$$\frac{\partial}{\partial\tilde{\sigma}}\Psi_h(\theta_h^*(\sigma),\tilde{\sigma})=\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right),\theta_h^*(\sigma),\tilde{\sigma}\right).$$

The last property, along with the fact that $\Psi_h(\theta_h^*(\sigma),\sigma)=0$, imply that, for any $\sigma,\sigma'\in(0,\bar{\sigma})$, with $\sigma'>\sigma$,

$$\Psi_h(\theta_h^*(\sigma),\sigma')=\int_{\tilde{\sigma}=\sigma}^{\tilde{\sigma}=\sigma'}\frac{\partial}{\partial\tilde{\sigma}}\Psi_h(\theta_h^*(\sigma),\tilde{\sigma})d\tilde{\sigma}=\int_{\tilde{\sigma}=\sigma}^{\tilde{\sigma}=\sigma'}\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right),\theta_h^*(\sigma),\tilde{\sigma}\right)d\tilde{\sigma}.$$

The continuity of $\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x,\hat{\theta},\tilde{\sigma}\right)$ in $(x,\hat{\theta},\tilde{\sigma})$ around $\left(\theta_h^{MS},\theta_h^{MS},0\right)$, along with Condition (22), imply that there exists $\sigma^{\dagger}\in(0,\bar{\sigma})$ such that, for any $\sigma,\sigma'\in(0,\sigma^{\dagger})$,

$$\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right),\theta_h^*(\sigma),\tilde{\sigma}\right)\overset{sgn}{=}\lim_{\sigma\to 0^+}\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^*\left(\sigma\right),\theta_h^*(\sigma),\tilde{\sigma}\right)\Big|_{\tilde{\sigma}=\sigma}.$$

[73] The continuity of $\theta_h^*\left(\sigma\right)$ in σ in turn follows from the fact that $\theta_h^*\left(\sigma\right)$ and $x_h^*\left(\sigma\right)$ are such that $\psi_h\left(x_h^*\left(\sigma\right),\theta_h^*\left(\sigma\right),\sigma\right)=0$ along with the continuity of $\psi_h\left(x,\hat{\theta},\sigma\right)$ in $\left(x,\hat{\theta},\sigma\right)$ and the strict monotonicity of $\psi_h\left(x,\hat{\theta},\sigma\right)$ in $\hat{\theta}$ at any $\left(x,\hat{\theta},\sigma\right)$ such that $\psi_h\left(x,\hat{\theta},\sigma\right)=0$.

[74] The arguments for this claim are similar to those in other global-games papers and omitted for brevity.

The above properties, along with Lemma (2), thus imply that, for any $\sigma, \sigma' \in (0, \sigma^\dagger)$, with $\sigma' > \sigma$,

$$\Psi_E(\theta_E^*(\sigma), \sigma') > 0 > \Psi_D(\theta_D^*(\sigma), \sigma').$$

The result in the proposition then follows from the above conclusions along with the monotonicity of $\Psi_h(\cdot, \sigma')$ in the truncation point θ_h^*, $h = E, D$. Q.E.D.

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