Attention, Coordination, and Bounded Recall

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Motivation

- Many socioeconomic environments
  - large group of agents
  - actions under **dispersed information**

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- Most of the literature: **exogenous** information structure
- Many phenomena of interest: **attention (info. acquisition)** is **central**

This paper

- Flexible (yet rich) framework

  - complementarity or substitutability in actions

  - rich set of payoff interdependencies

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- **Equilibrium** and **efficient allocation** of attention

  - perfect recall

  - *bounded recall*

Questions

- What payoff interdependencies create inefficiency in eq. allocation of attention?
- How does inefficiency in attention relate to inefficiency in use of information?
- What is the effect of bounded recall?
- What policies can alleviate such inefficiencies? (related work)

Related literature (incomplete)

- **Efficient use of information and social value of information**
  Radner (1977), Vives (JET 1984, 2013)
  Morris and Shin (AER 2002)
  Angeletos and Pavan (AER, 2004, Ecma 2007, Jeea, 2009)
  ...
- **Information acquisition/(in)attention in coordination settings**
  Vives and Van Zandt (2007)
  Hellwig and Veldkamp (Restud, 2009)
  Amir and Lazzati (2014)
  Maćkowiak and Wiederholt (AER, 2009, 2012)
  ⟶ Myatt and Wallace (Restud 2012)
  Szkup and Trevino (2013), Yang (2013)
  ⟶ Colombo, Femminis and Pavan (Restud 2014)
  Tirole (2014), Denti (2016)
  ...
- **Memory**
  Benabou Tirole (JPE 2004)
  Wilson (2004), Kocer (2010)
  ...
- **Analogy-based equilibrium**
  Jehiel (JET 2005)

Plan

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Model

Actions and gross payoffs

$$u_i(\; k_i \;,\; \{k_j\}_{j\neq i},\; \theta \;)$$

Model

Actions and gross payoffs

- Continuum of agents with payoffs:

$$u\left(k \ , \ K \ , \ \theta, \ \sigma_k^2 \right)$$

where:

$k \in \mathbb{R}$ – individual action

$K = \int k' d\Psi(k')$ – aggregate action

$\sigma_k^2 = \int (k' - K)^2 d\Psi(k')$ – dispersion

$\theta \in \mathbb{R}$ – underlying uncertainty ("fundamentals")

- **Assumptions**:

$u(\cdot)$ quadratic in (k, K, θ), linear in σ_k^2

$u(\cdot)$ s.t. equilibrium and first-best unique and bounded

Examples

- **Investment spillovers** (Angeletos and Pavan *AER* 2004)

$$u_i = Rk_i - c(k_i)$$

$$R = (1-a)\,\theta + aK \qquad \text{and} \qquad c(k_i) = \frac{1}{2}k_i^2$$

- **Beauty contest** (Morris and Shin *AER* 2002)

$$u_i = -(1-r)\cdot(k_i-\theta)^2 - r\cdot(L(k_i)-\bar{L})$$

$$L(k_i) \equiv \int (k'-k_i)^2\, d\Psi(k') = (k_i-K)^2 + \sigma_k^2 \quad \text{and} \quad \bar{L} = \int L(k)d\Psi(k) = 2\sigma^2$$

Examples

- **Monetary economies** (Woodford 2005, Colombo, Femminis and Pavan, 2014, Llosa and Venkateswaran, 2015)

$$u(\theta, C_i, N_i) \equiv V(C_i) - N_i$$

$$C_i = \left(\int_{[0,1]} c_{hi}^{\frac{\nu-1}{\nu}} dh \right)^{\frac{\nu}{\nu-1}}$$

$$Y_i = \theta^{\alpha} N_i$$

$$\int_{[0,1]} p_h c_{hi} dh \leq p_i Y_i - T$$

- **Cournot and Bertrand games** (Vives *JET* 1984)

$$u_i = (a - \theta K) \cdot k_i - \frac{1}{2} k_i^2$$

Model

Information and attention

- Common prior:

$$\theta \sim N(0, \pi_\theta^{-1})$$

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- $N = 1,234,576$ sources of information:

$$y_l = \theta + \varepsilon_l \qquad \text{with} \qquad \varepsilon_l \sim N(0, \eta_l^{-1}) \qquad l = 1, ..., N$$

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- Agent i's "impressions" $x^i = (x_l^i)_{l=1}^N$ with

$$x_l^i = y_l + \xi_l^i \qquad \text{with} \qquad \xi_l^i \sim N\left(0, \left(z_l^i \cdot t_l\right)^{-1}\right) \qquad l = 1, ..., N$$

where

$$\eta_l \quad : \quad \textbf{accuracy}$$
$$t_l \quad : \quad \textbf{transparency/clarity}$$
$$z_l^i \quad : \quad \textbf{attention}$$

Model

Attention cost and net payoffs

- Attention cost: $C(z^i)$ where $z^i = (z_l^i)_{l=1}^N$

  $\cdot$ $C_n'\left(z^i\right) > 0$, all $z^i \neq 0$

  $\cdot$ $\lim_{z_n \to \infty} C_n'(z^i) = \infty$

  $\cdot$ convex (results extend to concave, e.g., entropy reduction)

- E.g. $C(z^i) = c\left(\sum_l z_l^i\right)$

- E.g. $C(z^i) = \sum_l g(z_l^i)$

- ...but also $C(z^i) = \mu(z^i; y)$ (entropy reduction)

- Net payoff

$$u\left(k_i, K, \sigma_k^2, \theta\right) - C(z^i)$$

Model
Timing

- agents allocate attention z^i
- update their beliefs based on x^i
- commit their actions k^i
- payoffs realized

Plan

1. Model (perfect recall)
2. **Equilibrium allocation of attention**
3. Efficient allocation of attention
4. Bounded Recall

Equilibrium use of information (Angeletos and Pavan, Ecma 2007)

- Optimality:

$$k_j = \mathbb{E}[\ \kappa + \alpha(K-\kappa)\ |\ \ x^j\ ;\ z^j]$$

where

$$\kappa = \kappa_0 + \kappa_1\theta \quad \text{(complete-info. equilibrium action)}$$

$$\boxed{\alpha \equiv \frac{u_{kK}}{|u_{kk}|} \quad \longrightarrow \quad \textbf{equilibrium degree of coordination}}$$

Equilibrium allocation of attention

Theorem

There exists a unique symmetric equilibrium. In this eq., the attention $\hat{z}$ that each agent assigns to the various sources of information is s.t., for any source $n = 1, ..., N$ that receives strictly positive attention,

$$\hat{z}_n = \kappa_1 \gamma_n \sqrt{\frac{|u_{kk}|}{2C_n'(\hat{z})t_n}}$$

where

$$\gamma_n \equiv \frac{\frac{(1-\alpha)\pi_n}{1-\alpha\rho_n}}{\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\pi_s}{1-\alpha\rho_s}}$$ *is "influence" of the source*

and where

$$\pi_s = \frac{\eta_s \hat{z}_s t_s}{\hat{z}_s t_s + \eta_s}$$ *is endogenous precision and* $\rho_s = \frac{\pi_s}{\eta_s}$ *is endogenous "publicity"*

Given equilibrium allocation of attention $\hat{z}$, equilibrium actions are given by

$$k^i = \kappa_0 + \kappa_1 \left(\sum_{n=1}^N \gamma_n x_n^i \right) \text{ all } i \in [0,1], \text{ almost all } x^i \in \mathbb{R}^N.$$

Private value of attention

- *Envelope reasoning*: hold $k(\cdot;\hat{z})$ fixed

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- Agent's eq. continuation payoff (fixing $k(\cdot;\hat{z})$):

$$U_i(z^i;\hat{z}) = \mathbb{E}[u(K,K,\sigma_k,\theta)] + \frac{u_{kk}}{2} Var[k_i - K \mid z^i, \hat{z}, k(\cdot;\hat{z})] - C(z^i)$$

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- **Private value of attention**

$$-\frac{|u_{kk}|}{2} \cdot \frac{\partial Var[k - K \mid z, k(\cdot;z)]}{\partial z_n}$$

private aversion to dispersion · reduction in dispersion
(fixing **eq. strategy** $k(\cdot;z)$)

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- Result generalizes Colombo, Femminis, Pavan (Restud 2014)

Plan

1. Model (perfect recall)
2. Equilibrium allocation of attention
3. **Efficient allocation of attention**
4. Bounded Recall

Efficiency

- Welfare : ex-ante utility of representative agent

Definition

Efficient allocation consists of attention z^* along with action rule $k^*(\cdot;z^*)$ that jointly maximize

$$\mathbb{E}[u(k,K,\sigma_k^2,\theta) \mid z] - C(z)$$

- *Team problem*
- *Planner's problem*: control incentives but cannot transfer information

Efficient use of information (Angeletos and Pavan, Ecma 2007)

- Given attention z, efficiency in *actions* requires that $k^*(\cdot;z)$ solves

$$k^*(x;z) = \mathbb{E}\left[\kappa^* + \alpha^*(K - \kappa^*) \mid x \,;\, z\right] \qquad \forall x,$$

where

$$\kappa^* = \kappa_0^* + \kappa_1^*\theta \quad \longrightarrow \quad \mathbf{FB}$$

$$\alpha^* \equiv \frac{u_{\sigma\sigma} - 2u_{kK} - u_{KK}}{u_{kk} + u_{\sigma\sigma}} = 1 - \frac{\text{aversion to volatility}}{\text{aversion to dispersion}}$$

socially optimal degree of coordination

Efficient allocation of attention

Theorem

Efficiency in attention requires that, for any n *for which* $z_n^* > 0$,

$$z_n^* = \kappa_1^* \gamma_n^* \sqrt{\frac{|u_{kk} + u_{\sigma\sigma}|}{2C_n'(z^*)t_n}}$$

where

$$\gamma_n^* \equiv \frac{\frac{(1-\alpha^*)\pi_n}{1-\alpha\rho_n}}{\pi_\theta + \sum_{s=1}^{N} \frac{(1-\alpha^*)\pi_s}{1-\alpha^*\rho_s}}$$ *is efficient "influence" of the source*

$$\pi_s = \frac{\eta_s z_s^* t_s}{z_s^* t_s + \eta_s}$$ *is endogenous precision and* $\rho_s = \frac{\pi_s^*}{\eta_s}$ *is endogenous publicity*

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- Recall that eq.

$$\hat{z}_n = \kappa_1 \gamma_n \sqrt{\frac{|u_{kk}|}{2C_n'(\hat{z})t_n}}$$

Efficient allocation of attention

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- Welfare under efficient use of information (for given attention z)

$$w^*(z) \equiv \mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)] - \mathscr{L}^*(z) - C(z),$$

where $u(\kappa^*, \kappa^*, 0, \theta)$ is welfare under FB allocation and

$$\mathscr{L}^*(\pi_x, \pi_z) \equiv \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} Var[K - \kappa^* \mid k^*(\cdot; z), z] + \frac{|u_{kk} + u_{\sigma\sigma}|}{2} Var[k - K \mid k^*(\cdot; z), z]$$

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- Holding $k^*(\cdot; z)$, $Var[K - \kappa^* \mid k^*(\cdot; z), z]$ independent of z

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- Holding $k^*(\cdot;z)$, $Var[K - \kappa^* \mid k^*(\cdot;z), z]$ independent of z
- **Social value of attention**

$$-\frac{|u_{kk} + u_{\sigma\sigma}|}{2} \cdot \frac{\partial Var[k - K \mid z, k^*(\cdot;z)]}{\partial z_n}$$

social aversion to dispersion · reduction in dispersion

(fixing **eff. strategy** $k^*(\cdot;z)$)

Equilibrium vs efficient allocation of attention

- **Private value of attention**

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Equilibrium vs efficient allocation of attention

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- **Social value of attention**

$$-\underbrace{\frac{|u_{kk}+u_{\sigma\sigma}|}{2}}_{\text{social aversion to dispersion}} \cdot \underbrace{\frac{\partial Var[k-K \mid z, k^*(\cdot;z)]}{\partial z_n}}_{\text{reduction in dispersion}}$$

(fixing **eff. strategy** $k^*(\cdot;z)$)

Efficient allocation of attention

- Efficiency in attention requires
  - efficiency in use of information: $k(\cdot;z) = k^*(\cdot;z)$
  - private = social aversion to dispersion $\Leftrightarrow u_{\sigma\sigma} = 0$

Plan

1. Model (perfect recall)
2. Equilibrium allocation of attention
3. Efficient allocation of attention
4. **Bounded Recall**

Bounded Recall

- Idea: posteriors correct, but agents cannot recall influence of individual sources
- Given attention z^j, posterior beliefs about θ continues to be Normal with mean

$$\bar{x}^j = \sum_{n=1}^{N} \delta_n x_n^i, \text{ with } \delta_n \equiv \frac{\pi_n}{\pi_\theta + \sum_{s=1}^{N} \pi_s} \text{ and } \pi_s \equiv \frac{\eta_s z_s t_s}{z_s t_s + \eta_s}$$

  and precision $\pi_\theta + \sum_{s=1}^{N} \pi_s$
- However, agent is unable to decompose $\bar{x}^j$ into various impressions $x^j \equiv (x_1^j, \ldots, x_N^j)$.
- Equivalently, unable to decompose his posteriors into

$$\tilde{\theta} \mid x_n^i$$

- **Measurability constraint** on $k(x^j; z^z)$
- Distinction relevant only in strategic setting

Bounded Recall

For simplicity: $\pi_\theta = 0$

Theorem

In unique symmetric equilibrium, given allocation $z^\#$, actions given by

$$k^i = \kappa_0 + \kappa_1 \vec{x}^i$$

For any source that receives strictly positive attention in eq.,

$$C_n'(z^\#) = -\frac{|u_{kk}|}{2}\frac{\partial Var\left[k - K; z^\#, k(\cdot; z^\#)\right]}{\partial z_n} - \frac{|u_{kk}|}{2}(1-\alpha)\frac{\partial Var\left[K - \kappa; z^\#, k(\cdot; z^\#)\right]}{\partial z_n}$$

Novel effect:

$$-\frac{|u_{kk}|}{2}(1-\alpha)\frac{\partial Var\left[K - \kappa; z^\#, k(\cdot; z^\#)\right]}{\partial z_n}$$

private aversion to volatility of own's average action · reduction in volatility (fixing **eq. strategy** $k^\#(\cdot; z)$)

Bounded vs Perfect Recall

Theorem

Let $\hat{z}$ be eq. allocation of attention with perfect recall. There exist publicity thresholds $\rho', \rho'' \in [0,1]$ s.t., starting from $\hat{z}$, any agent with bounded recall is better off by

(a) locally increasing attention to sources for which $\rho_n \in [\rho', \rho'']$;

(b) locally decreasing attention to sources for which $\rho_n \notin [\rho', \rho'']$.

Bounded vs Perfect Recall

- Reallocation of attention towards sources of average (endogenous) publicity

$$\rho_n = \frac{z_s t_s}{z_s t_s + \eta_s}$$

- Sources of low publicity: useful to forecast θ
- Sources of high publicity: useful to forecast K
- Sources of intermediate transparency: **good compromises**

Bounded vs Perfect Recall

- Previous result about best responses extends to equilibrium
- Suppose $C(z) = c\left(\sum_{s=1}^{N} z_s\right)$

Theorem

Let $\hat{z}$ be eq. attention with perfect recall and $z^{\#}$ eq. attention with bounded recall. There exist thresholds $t', t'' \in \mathbb{R}_{++}$ s.t. $z_n^{\#} > \hat{z}_n$ only if $t_n \in [t', t'']$. Furthermore for any n for which $t_n \in [t', t'']$, $z_n^{\#} < \hat{z}_n$ only if $z_n^{\#} = 0$.

Efficiency under Bounded Recall

- ...see paper!

Conclusions

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  (a) absence of externalities from action-dispersion
  (b) efficiency in use of information

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- **Attention** in large economies with
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  - rich set of payoff interdependencies
  - rich information structure

- **Efficiency** in allocation of attention requires
  (a) absence of externalities from action-dispersion
  (b) efficiency in use of information

- **Bounded recall**: reallocation of attention towards sources with intermediate transparency

Conclusions

- **Future work**

  - **endogenous sources / social learning**
  (e.g., capital mkts → information aggregation)

  - **"optimal" recall strategy**

  - **dynamics (optimal stopping)**

  - **fully flexible info. structures (attention-based correlated eq.)**

Thank You!