Price Customization and Targeting in Matching Markets

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Matching Intermediaries

(Many-to-many) matching intermediaries:

- ad exchanges
- online retailers
- B2B platforms
- media platforms
- ...
Targeting and Price Customization

- Technological progress:
  - **targeting**
    (matching tailored to individual characteristics)
  - **price customization**
    (pricing tailored to individual characteristics)

- Example: ad exchanges and ad-supported platforms
  “compatibility” scores

- Example: online retailing
  pricing algorithms using third-party personal data

- Example: cable TV:
  price of additional channels determined by basic package – **bundling**
Policy Debate: Uniform Pricing

- Policy proposals aimed at curtailing targeting/price customization
  - bans on price customization
  - privacy regulations (e.g., GDPR and ePR)
- Obstacle to targeting/price customization: mkt decentralization
  - e.g., media markets
  - decentralization hinders bundling
- Policy debate lacks formal framework
- More generally,
  - distortions in matching markets?
This Paper

- Tractable, yet rich, model of mediated many-to-many matching:
  - third-degree price discrimination
  - second degree price discrimination

- Effects of uniform pricing obligations (mandated or due to mkt decentralization) on
  - targeting
  - consumer welfare

- Structural elasticities

- Analysis relevant for: ad-exchanges, online retailing, media markets
Related literature


- **Bundling**: Armstrong (2013), Hart and Reny (2015)...

- **Targeting**: Bergemann and Bonatti (2011, 2015), Kox, Straathof, and Zwart (2017)...
Plan

- Model
- Customized tariffs
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Model
Model

- Monopolistic platform

- Two sides $k \in \{a, b\}$

- Each side: unit-mass continuum of agents $i \in [0, 1]$

- Type of agent $i$ from side $k$: $\theta_k^i = (v_k^i, x_k^i)$
  - $v_k^i \in V_k = [v_k, v_k]$ : vertical dimension
  - $x_k^i \in [0, 1]$ : horizontal dimension (location)

- Each $\theta_k^i$ drawn independently from cdf

  \[
  F_k \quad \text{with support} \quad \Theta \equiv V_k \times [0, 1]
  \]
Model

- Utility of type $\theta_k = (v_k, x_k)$ from match to type $\theta_l = (v_l, x_l)$:
  \[ u_k(v_k, |x_k - x_l|) \]
  - increasing in $v_k$
  - decreasing in circular distance $|x_k - x_l|$

- Total utility from being matched, at price $p$, to set $s \subset \Theta_l$:
  \[ \pi_k(s, p; \theta) = \int_s u_k(v_k, |x_k - x_l|) \, dF_l(\theta_l) - p. \]
Model
Example 1: Ad Exchanges

- Platform matches advertisers (side $a$) with publishers (side $b$).
- Advertiser $\theta_a = (v_a, x_a)$ obtains expected profit

\[ u_a(v_a, |x_a - x_b|) = v_a \cdot \phi (|x_a - x_b|) \]

from impression at publisher $\theta_b = (v_b, x_b)$, where:

- $v_a$: profit per sale
- $\phi$: conversion probability
- $x_a$: advertiser’s profile
- $x_b$: publisher’s profile

- Heterogeneity in publishers’ payoffs reflects differences in opportunity costs (as well as preferences over ad content)
Example 2: Media Platforms

- Media outlet matches viewers (side $a$) to content providers (side $b$)
- Viewer $\theta_a = (v_a, x_a)$ derives utility $u_a(v_a, |x_a - x_b|)$ from content of provider $\theta_b = (v_b, x_b)$, where:
  - $x_a$: viewer’s preferred content
  - $x_b$: content provider’s profile
  - $v_a > 0$: overall importance viewer attaches to media consumption

- Providers’ payoff
  \[ u_b(v_b, |x_b - x_a|) \]
  may be positive (advertising) or negative (royalties)
Model: Information

- Vertical types: private information

- Horizontal types: public
  - Appendix deals with case locations are also private
Reciprocity

**Definition.** Tariffs $T_k$, $k = a, b$, **feasible** if induced matching demands s.t., for all $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l \in \{a, b\}$, $l \neq k$,

$$\theta_l \in s_k(\theta_k) \iff \theta_k \in s_l(\theta_l).$$
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Price Customization
**Customized Tariffs**

**Definition.** Tariff $T_k$ **customized** if there exists collection of **matching** plans

$$\{(s_k(x_k), T_k(x_k), \rho_k(\cdot|\cdot; x_k)) : x_k \in [0, 1]\},$$

- $s_k(x_k)$: baseline configuration
- $T_k(x_k)$: baseline price
- $\rho_k(\cdot|\cdot; x_k)$: customizing price schedules

s.t. an agent with location $x_k$ who selects customization $s_k \in \Sigma(\Theta_l)$ is charged

$$T_k(s_k|x_k) = T_k(x_k) + \int_0^1 \rho_k(q_{x_l}(s_k)|x_l;x_k)dx_l,$$

where $q_{x_l}(s_k)$ is total mass of $x_l$-agents under customization $s_k$

**Price customization:** $\rho_k(\cdot|\cdot; x_k)$ depends on $x_k$
**Lemma**

*Following are true:*

1. there exists pair of customized tariffs \((T^*_k)_{k=a,b}\) that are profit-maximizing;

2. matching demands \((s^*_k)_{k=a,b}\) under \((T^*_k)_{k=a,b}\)

\[ s^*_k(\theta_k) = \{(v_l, x_l) \in \Theta_l : v_l > t^*_k(\theta_k, x_l)\}, \]

with \(t^*_k(\cdot)\) non-increasing in \(v_k\), non-decreasing in \(|x_k - x_l|\).
Matching Sets
Elasticities of Matching Demands

- Elasticity of matching demand by $x_k$-agents wrt marginal price $\rho_k'$ for $q$-th unit of $x_l$-agents:

$$\varepsilon_k (\rho_k'|x_l;x_k) \equiv - \frac{\partial D_k (\rho_k'|x_l,x_k)}{\partial (\rho_k')} \cdot \frac{\rho_k'}{D_k (\rho_k'|x_l,x_k)}.$$

- Semi-elasticities of matching demands

$$\frac{\varepsilon_k (\rho_k'|x_l;x_k)}{\rho_k'}$$
Proposition

Fix $x_a, x_b$. For any $q_a, q_b$ clearing market, that is, s.t.

$$q_a = D_b \left( \rho_b^*'(q_b) \right) \quad \text{and} \quad q_b = D_a \left( \rho_a^*'(q_a) \right),$$

profit-maximizing schedules satisfy:

$$\rho_a^*'(q_a) \left( 1 - \frac{1}{\varepsilon_a(\rho_a^*'(q_a))} \right) = \rho_b^*'(q_b) \left( 1 - \frac{1}{\varepsilon_b(\rho_b^*'(q_b))} \right) = 0.$$
Lerner-Wilson formula

- Fix locations \((x_a, x_b)\)

\[ 1 - F_b(v_b) = q_a \]

\[ 1 - F_a(t_b(v_b)) = q_b \]
Proposition

Either of following two sets of conditions suffice for side-$k$ distortions to *decrease* with distance:

(1.a) side-$k$ SE increasing in distance, decreasing in price; side-$l$ SE increasing in both distance and price;

(1.b) $u_k$ submodular, $x_l$ and $v_l$ independent, hazard rate for $F_l^\gamma$ increasing in $v_l$, $u_l$ submodular and concave in $v_l$. 
Definition. Distortions on side $k \in \{a, b\}$ decrease with distance (alternatively, increase) iff, for all $\theta_k = (v_k, x_k)$,

$$u_l(t_k^*(\theta_k, x_l), |x_l - x_k|) - u_l(t_k^e(\theta_k, x_l), |x_l - x_k|)$$

decreases (alternatively, increases) with $|x_k - x_l|$. 
Constant Distortions
Distortions Decreasing in Distance
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Uniform Pricing
Uniform Pricing

- **Definition.** Tariff $T_k$ consistent with **uniform pricing** if prices $p_k(q|x_i)$ invariant to agent’s own characteristics, $x_k$.

- Mechanism design with novel constraint on implementing tariff
Uniform Pricing

- **Key property:** non-linear price $p_k(q|\chi_l)$ side-$a$ agents pay to be matched to $q$ side-$b$ individuals from location $\chi_l$ invariant to agents’ own characteristics, $\theta_k$

- e.g., price advertisers pay to place ad on ad-exchange invariant to ad’s content
**Uniform Pricing: Lerner-Wilson Revisited**

**Proposition 3.** Profit-maximizing tariffs:

\[
p'_a \left(1 - \frac{1}{\bar{\varepsilon}_a(p'_a)}\right) + \mathbb{E}_{H(\tilde{x}_a|x_b,p'_a)} \left[p'_b(\hat{q}_b(\tilde{x}_a)) \left(1 - \frac{1}{\hat{\varepsilon}_b(\hat{\rho}'_b(\hat{q}_b(\tilde{x}_a)))}\right)\right] = 0,
\]

where \(H(x_a|x_b,p'_a)\) is **distribution** over \(X_a = [0,1]\) whose density given by

\[
h(x_a|x_b,p'_a) \equiv \frac{\partial D_a(p'_a|x_b;x_a)}{\partial (p'_a)} \cdot \frac{\partial D_a(p'_a|x_b)}{\partial (p'_a)}.
\]
Uniform Pricing: Average Virtual Values

- Mechanism design with novel constraint on implementing payments

- Under customized pricing, \( \theta_a \) and \( \theta_b \) matched iff

\[
\varphi_a(\theta_a, \theta_b) + \varphi_b(\theta_a, \theta_b) \geq 0
\]

- Under uniform pricing, \( \theta_a \) and \( \theta_b \) matched iff

\[
\mathbb{E}_{H(\tilde{x}_a|x_b, p_{a}^{u'})} \left[ \varphi_a \left( \left( \hat{\nu}_{xb} \left( p_{a}^{u'} | \tilde{x}_a \right), \tilde{x}_a \right), \theta_b \right) \right] \bigg|_{p_{a}^{u'}=u_a(v_a, |x_b-x_a|)}
\]

\[
+ \mathbb{E}_{H(\tilde{x}_a|x_b, p_{a}^{u'})} \left[ \varphi_b \left( \theta_b, \left( \hat{\nu}_{xb} \left( p_{a}^{u'} | \tilde{x}_a \right), \tilde{x}_a \right) \right) \right] \bigg|_{p_{a}^{u'}=u_a(v_a, |x_b-x_a|)} \geq 0
\]

where \( \hat{\nu}_{xb} \left( p_{a}^{u'} | x_a \right) \) is unique solution to

\[
u_a(\hat{\nu}_{xb} \left( p_{a}^{u'} | x_a \right), |x_a - x_b|) = u_a
\]
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Targeting
Targeting

**Definition.** More targeting under customized pricing if, for each \( \theta_a = (v_a, x_a) \), there exists \( d(\theta_a) \in (0, \frac{1}{2}) \) such that

\[
 t^c_a(\theta_a, x_b) - t^u_a(\theta_a, x_b) \begin{cases} < 0 & \text{if } |x_a - x_b| < d(\theta_a) \\ > 0 & \text{if } |x_a - x_b| > d(\theta_a). \end{cases}
\]
When Customized Pricing Leads to More Targeting...

Threshold function $t^c_a(\theta_a, x_b)$ under customized pricing (black solid curve) and uniform pricing $t^u_a(\theta_a, x_b)$ (dashed blue curve) when customized pricing leads to more targeting than uniform pricing.
Comparison: Targeting

Proposition

Suppose side-b preferences location-invariant (general case in paper)

1. Uniform pricing (on side a) leads to more targeting than customized pricing (on both sides) when side-a SE increasing in both distance and price.

2. Side-a SE increasing in both distance and price when

- $x_a$ and $v_a$ independent
- hazard rate for $F^v_a$ increasing in $v_a$
- $u_a$ submodular and concave in $v_a$
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Welfare
Convexity of Demands

Convexity of $x_a$-agents’ demand for $q$-th unit of $x_b$-agents wrt $p'_a$:

$$CD_a (p'_a|x_b;x_a) = - \frac{\partial^2 D_a (p'_a|x_b;x_a)}{\partial (p'_a)^2} \left( \frac{\partial D_a (p'_a|x_b;x_a)}{\partial (p'_a)} \right)^{-1} p'_a$$

**Condition [NDR] Non Decreasing Ratio:**

$$\frac{p'_a}{2 - CD_a (p'_a|x_b;x_a)}$$

nondecreasing in $p'_a$. 
Welfare Effect of Uniform Pricing

Proposition

Suppose Condition NDR holds, and either of following alternatives is satisfied:

1. targeting higher under uniform pricing and \( CD_a(p'_a|x_b;x_a) \) decreasing in \( |x_a - x_b| \).

2. targeting higher under customized pricing and \( CD_a(p'_a|x_b;x_a) \) increasing in \( |x_a - x_b| \).

Then welfare of side-\( a \) agents higher under uniform pricing.
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Decentralization
Decentralized Markets

- Many markets transitioning from centralized to decentralized structure (sellers posting prices)

- E.g., Cable TV

- Decentralized market (+ private info on $x_a$) ⇒ **uniform pricing** on side $a$

- Similar welfare analysis as for comparison between uniform and customized pricing

- **Extra welfare benefit** from decentralization: zero markup on sellers side
Motivation

Model

Customized Tariffs

Uniform Pricing

Targeting

Welfare

Decentralization

Conclusions

- Mediated many-to-many matching
  - vertically + horizontally differentiated preferences

- Customized vs uniform pricing

- Monotonicity of semi-elasticities (distance and price) key to
  - distortions
  - targeting under uniform and customized pricing
  - welfare implications of uniform pricing
  - welfare effects of mkt decentralization
THANKS!