Price Customization and Targeting in Matching Markets

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Matching Intermediaries

(Many-to-many) matching intermediaries:

- ad exchanges
- online retailers
- B2B platforms
- media platforms
- ...

Targeting and Price Customization

- Technological progress:
  - **targeting**

    (matching tailored to individual characteristics)
  - **price customization**

    (pricing tailored to individual characteristics)
- Example: ad exchanges and ad-supported platforms

  "compatibility" scores
- Example: online retailing

  pricing algorithms using third-party personal data
- Example: cable TV:

  price of additional channels determined by basic package – **bundling**

Policy Debate: Uniform Pricing

- Policy proposals aimed at curtailing targeting/price customization
  - **bans on price customization**
  - **privacy regulations** (e.g., GDPR and ePR)
- Obstacle to targeting/price customization: **mkt decentralization**
  - e.g., media markets
  - decentralization hinders bundling
- Policy debate lacks formal framework
- More generally,
  - distortions in matching markets?

This Paper

- Tractable, yet rich, model of mediated many-to-many matching:
  - third-degree price discrimination
  - second degree price discrimination
- Effects of uniform pricing obligations (mandated or due to mkt decentralization) on
  - targeting
  - consumer welfare
- Structural elasticities
- Analysis relevant for: ad-exchanges, online retailing, media markets

Related literature

- **Matching Design with Transfers**: Damiano and Li (2007), Johnson (2013), Gomes and Pavan (2016), Jeon, Kim and Menicucci (2021), Fershtman and Pavan (2017, 2020)...
- **Two-Sided Markets:** Rochet and Tirole (2006), Armstrong (2006), Weyl (2010), Ambrus, Calvano and Reisinger (2016), Jullien and Pavan (2019), Tan and Zhou (2020), Belleflamme and Peitz (2017, 2020, 2021)...
- **Price Discrimination:** Mussa and Rosen (1978), Myerson (1981), Maskin and Riley (1984), Armstrong (1996), Wilson (1998), Aguirre, Cowan, and Vickers (2010), Bergemann, Brooks, and Morris (2015)...
- **Bundling**: Armstrong (2013), Hart and Reny (2015)...
- **Targeting**: Bergemann and Bonatti (2011, 2015), Kox, Straathof, and Zwart (2017)...

Plan

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Model

Model

- Monopolistic platform
- Two sides $k \in \{a, b\}$
- Each side: unit-mass continuum of agents $i \in [0, 1]$
- Type of agent i from side k: $\theta_k^i = (v_k^i, x_k^i)$
  - $v_k^i \in V_k = [\underline{v}_k, \overline{v}_k]$: **vertical dimension**
  - $x_i^k \in [0, 1]$: **horizontal dimension** (location)
- Each θ_k^i drawn independently from cdf

$$F_k \quad \textit{with support} \quad \Theta \equiv V_k \times [0, 1]$$

Model

- Utility of type $\theta_k = (v_k, x_k)$ from match to type $\theta_l = (v_l, x_l)$:

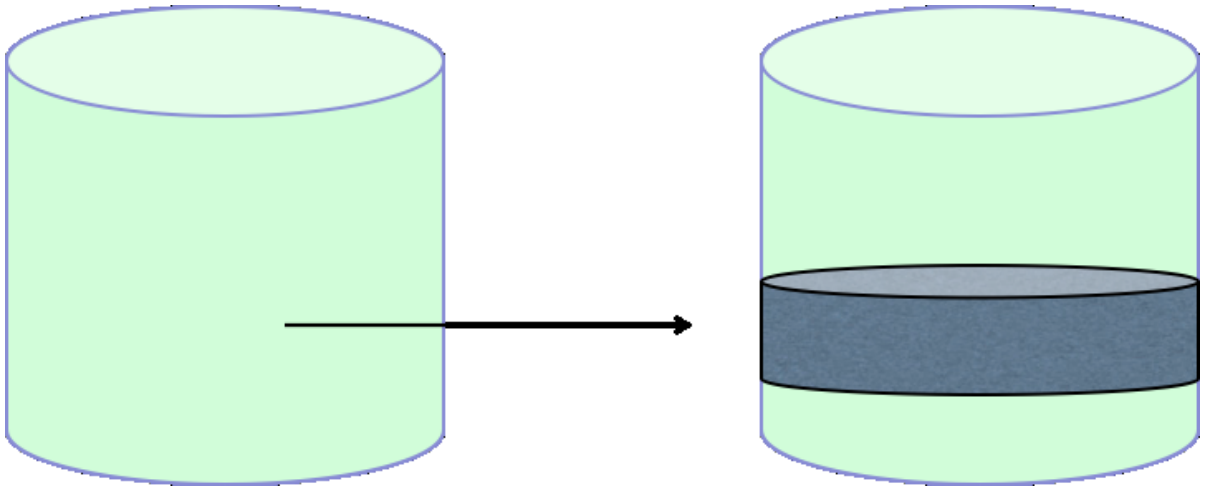
$$u_k(v_k, |x_k - x_l|)$$

  - increasing in v_k
  - decreasing in circular distance $|x_k - x_l|$

- Total utility from being matched, at price p, to set $\mathbf{s} \subset \Theta_l$:

$$\pi_k(\mathbf{s}, p; \theta) = \int_{\mathbf{s}} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - p.$$

Model



Example 1: Ad Exchanges

- Platform matches advertisers (side a) with publishers (side b).
- Advertiser $\theta_a = (v_a, x_a)$ obtains expected profit

$$u_a(v_a, |x_a - x_b|) = v_a \cdot \phi\left(|x_a - x_b|\right)$$

  from impression at publisher $\theta_b = (v_b, x_b)$, where:

  - v_a: profit per sale
  - ϕ: conversion probability
  - x_a: advertiser's profile
  - x_b: publisher's profile

- Heterogeneity in publishers' payoffs reflects differences in opportunity costs (as well as preferences over ad content)

Example 2: Media Platforms

- Media outlet matches viewers (side a) to content providers (side b)
- Viewer $\theta_a = (v_a, x_a)$ derives utility $u_a(v_a, |x_a - x_b|)$ from content of provider $\theta_b = (v_b, x_b)$, where:
  - x_a: viewer's preferred content
  - x_b: content provider's profile
  - $v_a > 0$: overall importance viewer attaches to media consumption
- Providers' payoff

$$u_b(v_b, |x_b - x_a|)$$

  may be positive (advertising) or negative (royalties)

Model: Information

- Vertical types: private information
- Horizontal types: public
  - Appendix deals with case locations are also private

Reciprocity

Definition. Tariffs T_k, $k = a, b$, **feasible** if induced matching demands s.t., for all $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l \in \{a, b\}$, $l \neq k$,

$$\theta_l \in \mathbf{s}_k(\theta_k) \Longleftrightarrow \theta_k \in \mathbf{s}_l(\theta_l).$$

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Price Customization

Customized Tariffs

Definition. Tariff T_k **customized** if there exists collection of **matching plans**

$$\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k)) : x_k \in [0,1]\},$$

- $\underline{\mathbf{s}}_k(x_k)$: baseline configuration
- $\underline{T}_k(x_k)$: baseline price
- $\rho_k(\cdot|\cdot; x_k)$: customizing price schedules

s.t. an agent with location x_k who selects customization $\mathbf{s}_k \in \Sigma(\Theta_l)$ is charged

$$T_k(\mathbf{s}_k|x_k) = \underline{T}_k(x_k) + \int_0^1 \rho_k(q_{x_l}(\mathbf{s}_k)|x_l; x_k)dx_l,$$

where $q_{x_l}(\mathbf{s}_k)$ is total mass of x_l-agents under customization $\mathbf{s}_k$

Price customization: $\rho_k(\cdot|\cdot; x_k)$ depends on x_k

Profit-Maximizing Tariffs

Lemma

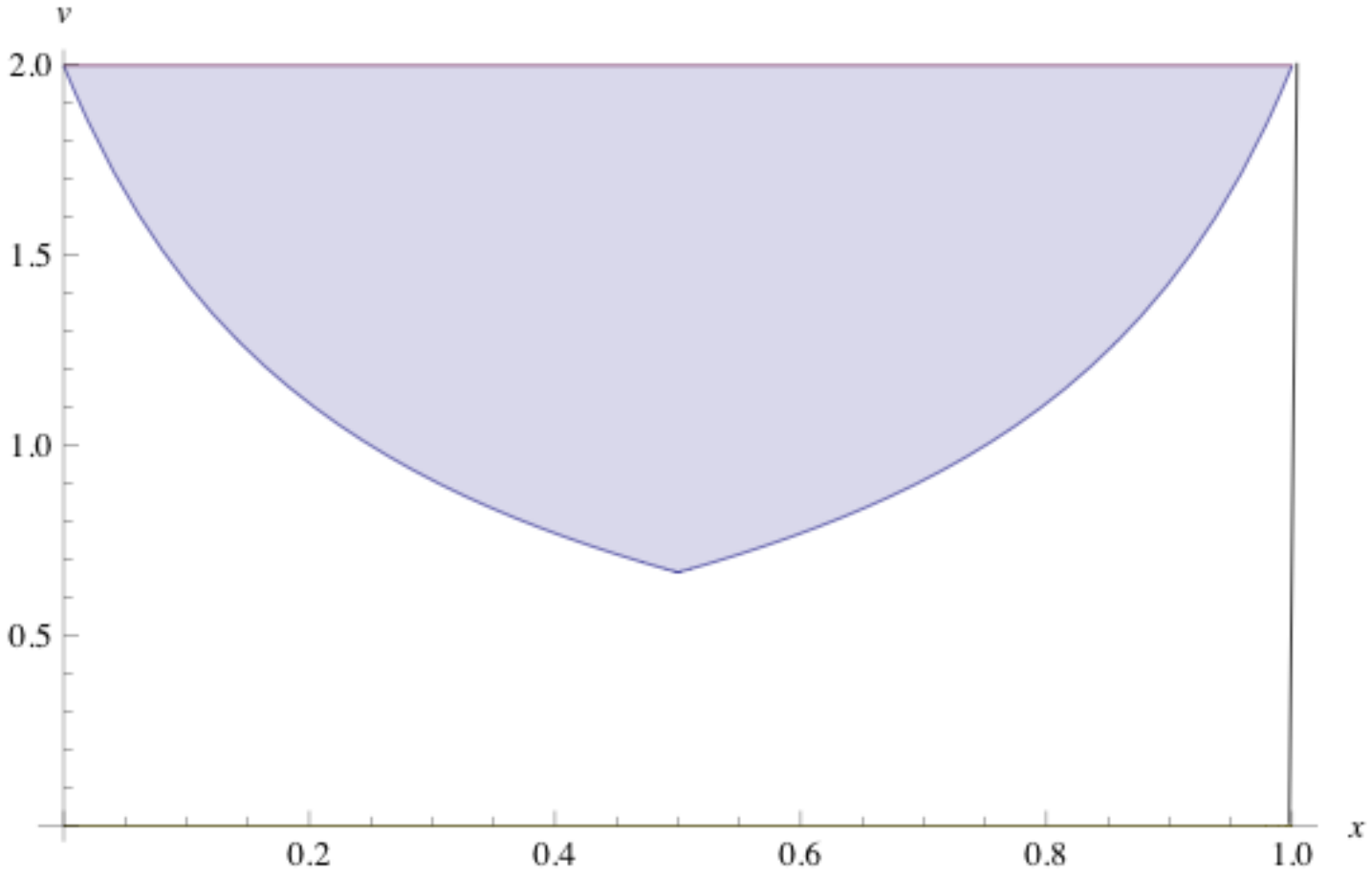
Following are true:

1. *there exists pair of customized tariffs* $(T_k^*)_{k=a,b}$ *that are profit-maximizing;*
2. *matching demands* $(\mathbf{s}_k^*)_{k=a,b}$ *under* $(T_k^*)_{k=a,b}$

$$\mathbf{s}_k^*(\theta_k) = \{(v_l, x_l) \in \Theta_l : \ v_l > t_k^*(\theta_k, x_l)\},$$

with $t_k^*(\cdot)$ *non-increasing in* v_k*, non-decreasing in* $|x_k - x_l|$.

Matching Sets



Elasticities of Matching Demands

- Elasticity of matching demand by x_k-agents wrt marginal price ρ'_k for q-th unit of x_l-agents:

$$\varepsilon_k\left(\rho'_k|x_l;x_k\right) \equiv -\frac{\partial D_k\left(\rho'_k|x_l,x_k\right)}{\partial\left(\rho'_k\right)}\cdot\frac{\rho'_k}{D_k\left(\rho'_k|x_l,x_k\right)}.$$

- Semi-elasticities of matching demands

$$\frac{\varepsilon_k\left(\rho'_k|x_l;x_k\right)}{\rho'_k}$$

Lerner-Wilson formula

Proposition

Fix x_a, x_b. For any q_a, q_b clearing market, that is, s.t.

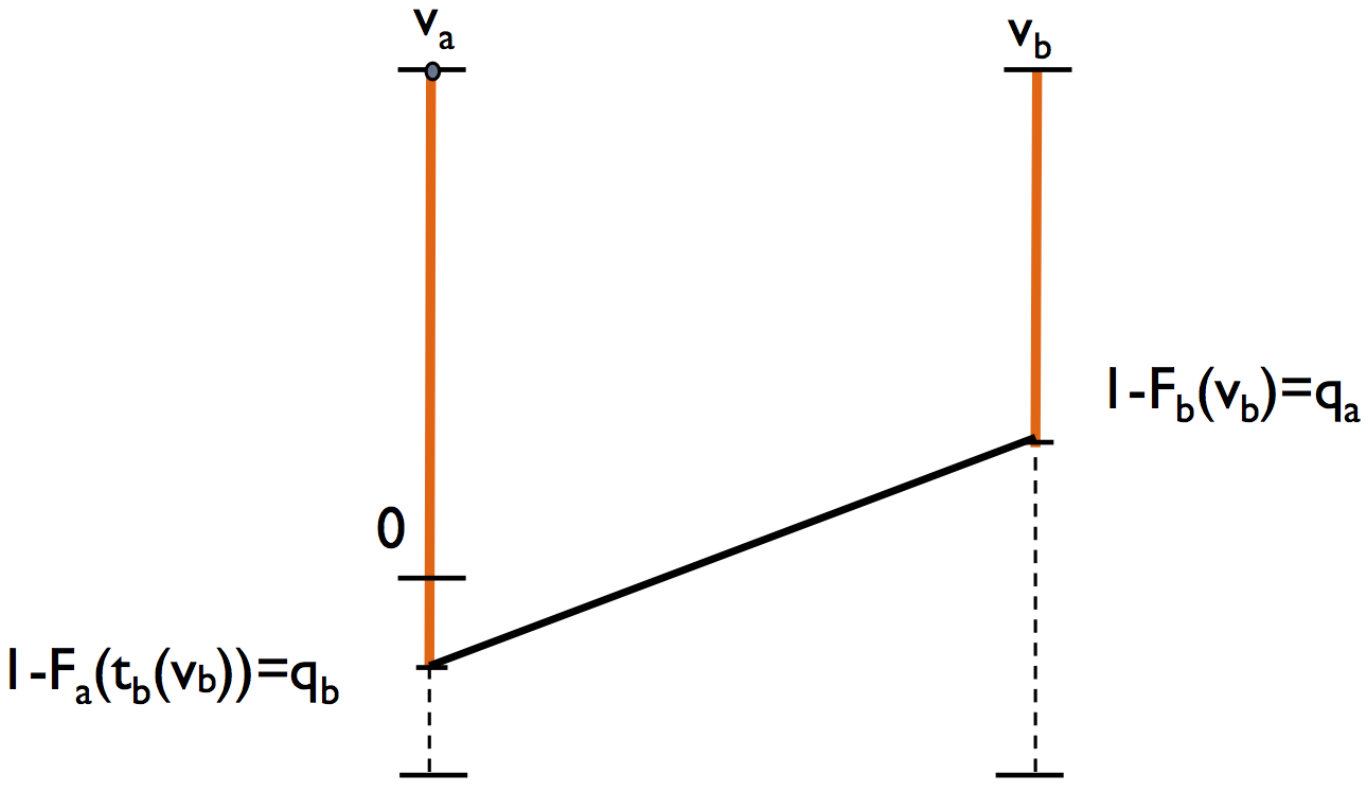
$$q_a = D_b\left(\rho_b^{*\prime}(q_b)\right) \text{ and } q_b = D_a\left(\rho_a^{*\prime}(q_a)\right),$$

profit-maximizing schedules satisfy:

$$\underbrace{\rho_a^{*\prime}(q_a)\left(1-\frac{1}{\varepsilon_a\left(\rho_a^{*\prime}(q_a)\right)}\right)}_{\text{net effect on side-a profits}} + \underbrace{\rho_b^{*\prime}(q_b)\left(1-\frac{1}{\varepsilon_b\left(\rho_b^{*\prime}(q_b)\right)}\right)}_{\text{net effect on side-b profits}} = 0.$$

Lerner-Wilson formula

- Fix locations (x_a, x_b)



Distortions and Horizontal Differentiation

Proposition

Either of following two sets of conditions suffice for side-k distortions to ***decrease*** *with distance:*

(1.a) side-k SE increasing in distance, decreasing in price; side-l SE increasing in both distance and price;

(1.b) u_k *submodular,* x_l *and* v_l *independent, hazard rate for* F_l^v *increasing in* v_l*,* u_l *submodular and concave in* v_l*.*

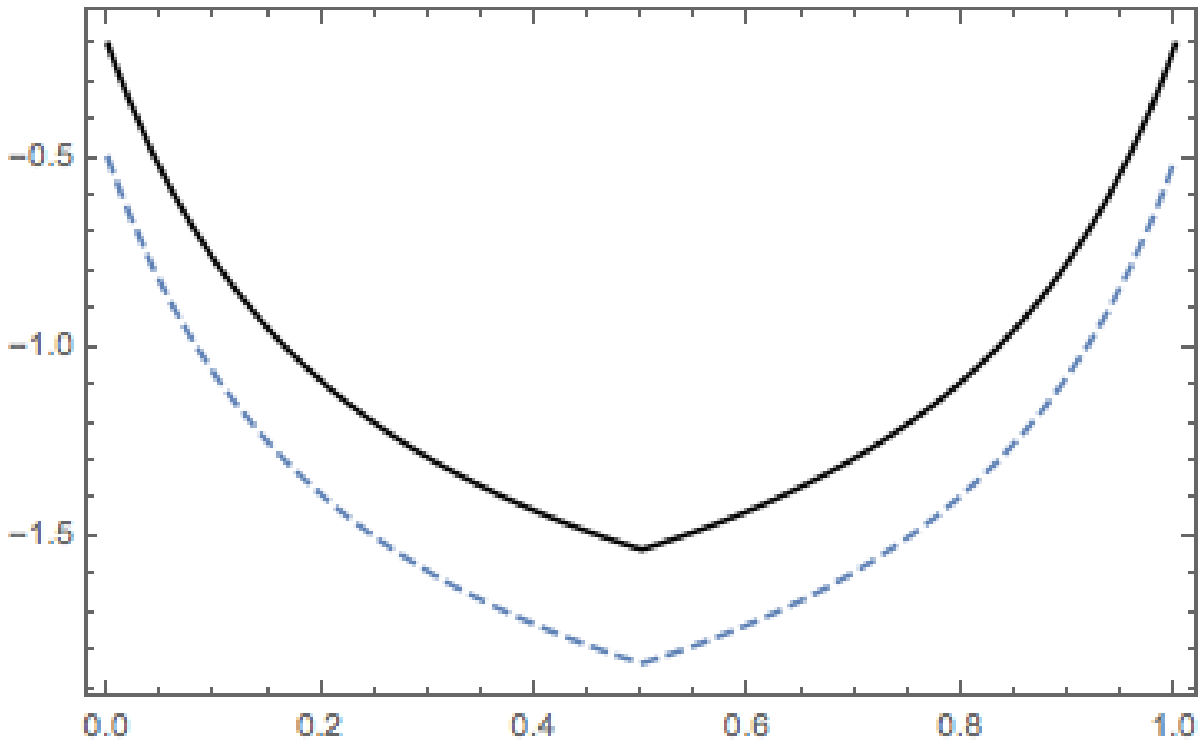
Distortions

Definition. Distortions on side $k \in \{a, b\}$ decrease with distance (alternatively, increase) iff, for all $\theta_k = (v_k, x_k)$,

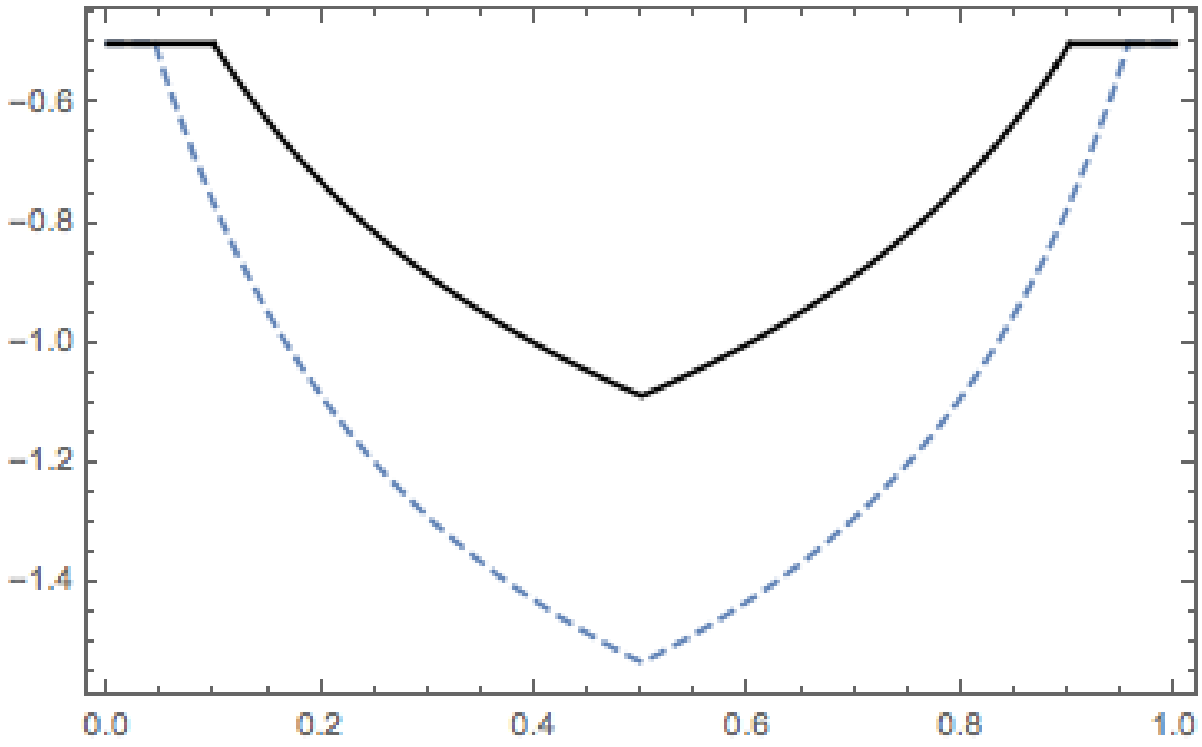
$$u_l\left(t_k^*(\theta_k, x_l), |x_l - x_k|\right) - u_l\left(t_k^e(\theta_k, x_l), |x_l - x_k|\right)$$

decreases (alternatively, increases) with $|x_k - x_l|$.

Constant Distortions



Distortions Decreasing in Distance



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Uniform Pricing

Uniform Pricing

- **Definition.** Tariff T_k consistent with **uniform pricing** if prices $p_k(q|x_l)$ invariant to agent's own characteristics, x_k.

- Mechanism design with novel constraint on implementing tariff

Uniform Pricing

- **Key property:** non-linear price $p_k(q|x_l)$ side-a agents pay to be matched to q side-b individuals from location x_l invariant to agents' own characteristics, θ_k

- e.g., price advertisers pay to place ad on ad-exchange invariant to ad's content

Uniform Pricing: Lerner-Wilson Revisited

Proposition 3. Profit-maximizing tariffs:

$$\underbrace{p_a'\left(1-\frac{1}{\bar{\varepsilon}_a(p_a')}\right)}_{\text{net effect on side-}a\text{ profits}} + \mathbb{E}_{H(\tilde{x}_a|x_b,p_a')}\left[\underbrace{\rho_b'(\hat{q}_b(\tilde{x}_a))\left(1-\frac{1}{\hat{\varepsilon}_b\left(\hat{\rho}_b'(\hat{q}_b(\tilde{x}_a))\right)}\right)}_{\text{net effect on side-}b\text{ profits}}\right] = 0,$$

where $H(x_a|x_b,p_a')$ is **distribution** over $X_a=[0,1]$ whose density given by

$$h\left(x_a|x_b,p_a'\right) \equiv \frac{\frac{\partial D_a(p_a'|x_b;x_a)}{\partial(p_a')}}{\frac{\partial \bar{D}_a(p_a'|x_b)}{\partial(p_a')}}.$$

Uniform Pricing: Average Virtual Values

- Mechanism design with **novel constraint on implementing payments**
- Under customized pricing, θ_a and θ_b matched iff

$$\varphi_a(\theta_a, \theta_b) + \varphi_b(\theta_a, \theta_b) \geq 0$$

- Under **uniform pricing**, θ_a and θ_b matched iff

$$\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\varphi_a\left(\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right),\tilde{x}_a\right),\theta_b\right)\right]\Big|_{p_a^{u\prime}=u_a(v_a,|x_b-x_a|)}$$

$$+\,\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\varphi_b\left(\theta_b,\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right),\tilde{x}_a\right)\right)\right]\Big|_{p_a^{u\prime}=u_a(v_a,|x_b-x_a|)} \geq 0$$

where $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)$ is unique solution to

$$u_a(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right),|x_a-x_b|) = u_a$$

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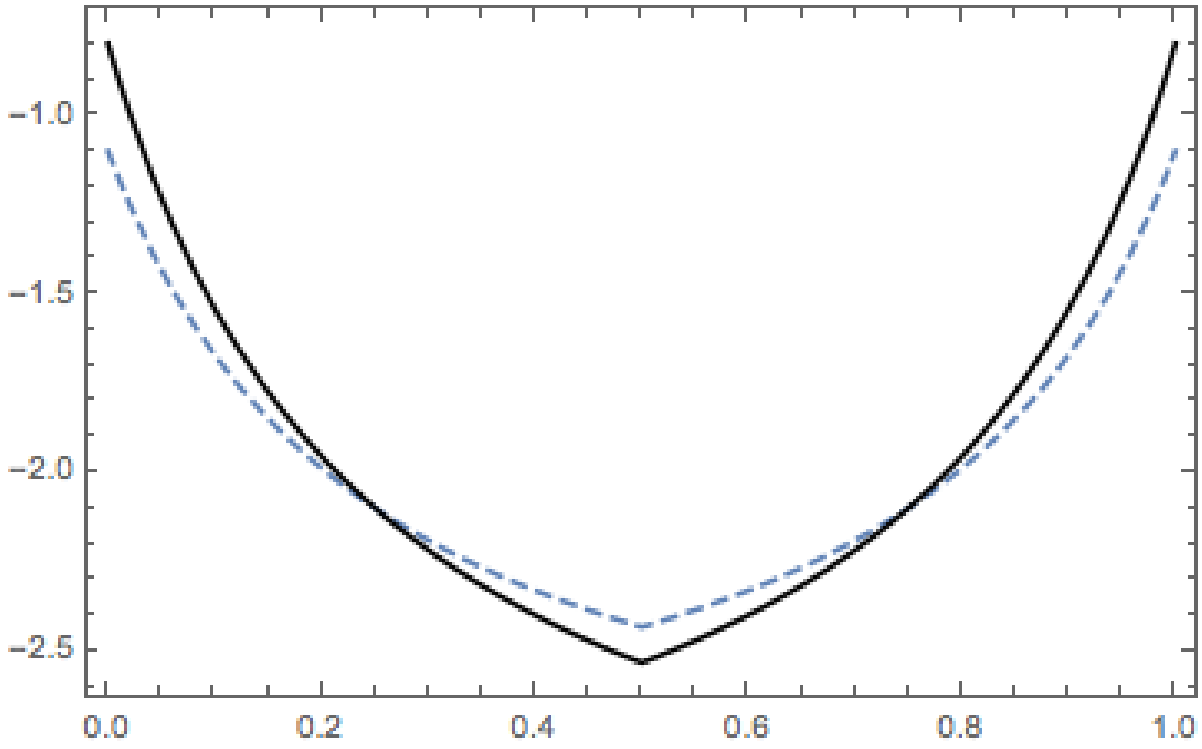
Targeting

Targeting

Definition. More targeting under customized pricing if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^c(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} < 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ > 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

When Customized Pricing Leads to More Targeting...



Threshold function $t_a^c(\theta_a, x_b)$ under customized pricing (black solid curve) and uniform pricing $t_a^u(\theta_a, x_b)$ (dashed blue curve) when customized pricing leads to more targeting than uniform pricing

Comparison: Targeting

Proposition

Suppose side-b preferences location-invariant (general case in paper)

1. Uniform pricing (on side a) leads to ***more targeting*** *than customized pricing (on both sides) when side-a SE* ***increasing*** *in both distance and price.*

2. Side-a SE increasing in both distance and price when

- x_a *and* v_a *independent*

- *hazard rate for* F_a^v *increasing in* v_a

- u_a *submodular and concave in* v_a

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Welfare

Convexity of Demands

Convexity of x_a-agents' demand for q-th unit of x_b-agents wrt p'_a:

$$CD_a\left(p'_a|x_b;x_a\right) = -\frac{\partial^2 D_a\left(p'_a|x_b;x_a\right)}{\partial\left(p'_a\right)^2}\left(\frac{\partial D_a\left(p'_a|x_b;x_a\right)}{\partial\left(p'_a\right)}\right)^{-1} p'_a$$

Condition [NDR] *Non Decreasing Ratio*:

$$\frac{p'_a}{2 - CD_a\left(p'_a|x_b;x_a\right)}$$

nondecreasing in p'_a.

Welfare Effect of Uniform Pricing

Proposition

Suppose Condition NDR holds, and either of following alternatives is satisfied:

1. targeting higher under uniform pricing and $CD_a(p'_a|x_b;x_a)$ *decreasing in* $|x_a - x_b|$.

2. targeting higher under customized pricing and $CD_a(p'_a|x_b;x_a)$ *increasing in* $|x_a - x_b|$.

Then welfare of side-a agents higher under uniform pricing.

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Decentralization

Decentralized Markets

- Many mkts transiting from centralized to decentralized structure (sellers post prices)
- E.g., Cable TV
- Decentralized mkt (+ private info on x_a) $\Rightarrow$ **uniform pricing** on side a
- Similar welfare analysis as for comparison between uniform and customized pricing
- **Extra welfare benefit** from decentralization: zero markup on sellers side

Conclusions

- Mediated many-to-many matching
  - vertically + horizontally differentiated preferences
- Customized vs uniform pricing
- Monotonicity of semi-elasticities (distance and price) key to
  - distortions
  - targeting under uniform and customized pricing
  - welfare implications of uniform pricing
  - welfare effects of mkt decentralization

THANKS!