Wedge Dynamics

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This paper

- General **formula** for dynamics of distortions under second-best
  - unifies all special cases in micro/macro literatures
- Novel forces governing dynamics of distortions:
  - "transaction costs" of recouping future rents
  - scope versus urgency of interventions to curtail future rents
- Novel implications for dynamics of distortions:
  - distortions can increase over lifecycle without increase in risk
  - endogenous processes: non-monotonicity of distortions

Plan

1. **Model**
2. First Best
3. Second Best and Wedges
4. Theorem
5. Conclusions

Model

Environment

- Time: $t = 1, ..., T$
- Period-t "productivity": θ_t
  - privately observed by agent at beginning of period t
- F_1: cdf of initial distribution (density f_1)
- $F_{t+1}(\cdot|\theta_t, y_t)$: cdf of θ_{t+1} (density f_{t+1})
  - endogenous process
  - dependence on past decisions: LBD, habit formation, etc.

Environment

- Agent's flow-payoff:

$$v(c_t) - \psi(y_t, \theta_t)$$

- Planner's flow-payoff:

$$v^P(y_t) - c_t$$

- Taxation literature: $\psi(y_t, \theta_t) = \frac{1}{1+\phi}\left(\frac{y_t}{\theta_t}\right)^{1+\phi}$ and $v^P(y_t) = y_t$

- $\theta^t = (\theta_1, ..., \theta_t) = (\theta^{t-1}, \theta_t)$; $\theta = \theta^T$

Environment

- Agent period-t continuation payoff (at θ^t):

$$V_t \equiv \mathbb{E}^{\theta^t, y_t} \left[\sum_{\tau=t} \delta^{\tau-t} \left(v(\tilde{c}_\tau) - \psi(\tilde{y}_\tau, \tilde{\theta}_\tau) \right) \right]$$

- "Redistribution" constraint:

$$(1-r)V_1(\theta_1) + r\mathbb{E}\left[q(V_1(\tilde{\theta}_1)) \right] \geq \kappa \qquad \forall \theta_1$$

- Planner maximizes

$$\mathbb{E}\left[\sum_{\tau=1} \delta^{\tau-1} \left(v^P \left(\tilde{y}_\tau \right) - \tilde{c}_\tau \right) \right]$$

s.t. IC + redistribution constraint

Plan

First Best

First Best: period-t output

- Principal's continuation payoff (at θ^t)

$$V_t^P \equiv \mathbb{E}^{\theta^t, y_t} \left[\sum_{\tau=t} \delta^{\tau - t} \left(v^P(\tilde{y}_\tau) - \tilde{c}_\tau \right) \right]$$

- Period-t output (at θ^t)

$$v^{P'}(y_t^*) + LD_t^* = \frac{\psi_y(y_t^*, \theta_t)}{v'(c_t^*)}$$

where

$$LD_t^* \equiv \delta \frac{\partial}{\partial y_t} \mathbb{E}^{\theta^t, y_t^*} \left[\tilde{V}_{t+1}^P + \frac{\tilde{V}_{t+1}}{v'(\tilde{c}_t^*)} \right]$$

captures impact of y_t on future surplus via effect on future types

Plan

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Second Best and Wedges

Second Best: Incentive Compatibility

IC requires that (at any θ^t)

$$V_t(\theta^{t-1}, \theta_t) = V_t(\theta^{t-1}, \underline{\theta}_t) - \int_{\underline{\theta}_t}^{\theta_t} \Delta_t(\theta^{t-1}, s) ds$$

where

$$\Delta_t(\theta^t) \equiv \mathbb{E}^{\theta^t, y_t(\theta^t)} \left[\sum_{\tau=t} \delta^{\tau-t} \tilde{I}_t^\tau \psi_\theta(\tilde{y}_\tau, \tilde{\theta}_\tau) \right]$$

with

$I_t^\tau = I_t^\tau(\theta^\tau, y^{\tau-1})$: **impulse response** of θ_τ to θ_t

and

$$\int_{\hat{\theta}_t}^{\theta_t} \Delta_t(\theta^{t-1}, s) ds \leq \int_{\hat{\theta}_t}^{\theta_t} \Delta_t^{\hat{\theta}_t}(\theta^{t-1}, s) ds$$

w. $\Delta_t^{\hat{\theta}_t}(\theta^t)$ analogous to $\Delta_t(\theta^t)$ w. period-t message replaced by $\hat{\theta}_t$

Second Best: Wedges

- Recursive approach
- Period-t wedge (at θ^t)

$$W_t \equiv v^{P'}(y_t) + LD_t - \frac{\psi_y(y_t,\theta_t)}{v'(c_t)}$$

- **Relative wedges** (unit-free):

$$\widehat{W}_t \equiv \frac{W_t}{\frac{\psi_y(y_t,\theta_t)}{v'(c_t)}}$$

- Taxation: wedges related to current marginal tax rates and future (expected) tax bills

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Theorem

Theorem

Under optimal allocations,

$$\boxed{\widehat{W}_t = [RA_t - D_t]\left[\widehat{W}_t^{RRN} + \Omega_t\right]}$$

- $\widehat{W}_t^{RRN}$: Rawlsian-risk-neutral benchmark
- Ω_t: effects of endogenous private information
- RA_t: correction due to non-trasferability of utility
- D_t: attenuation due to planner's inequality aversion

Theorem, cont'd: Rawlsian-risk-neutral benchmark

$$\widehat{W}_t^{RRN} \equiv \frac{\frac{\partial h_t}{\partial y_t}}{\psi_y(y_t, \theta_t)}$$

where period-t handicap

$$h_t \equiv -\frac{I_1^t}{\eta_1}\psi_\theta(y_t, \theta_t) \qquad \text{where} \quad \eta_1 \equiv \frac{f_1}{1 - F_1}$$

captures information rent

- DMD literature (transferable utility): dynamics driven by I_1^t
  - typically declining over time

Theorem, cont'd: endogenous types

$$\Omega_t \equiv \delta \frac{\frac{\partial}{\partial y_t} \mathbb{E}^{\theta^t, y_t} \left[\sum_{\tau = t+1} \delta^{\tau - t - 1} \tilde{h}_\tau \right]}{\psi_y(y_t, \theta_t)}$$

- monotonicity of handicaps in future types + FOSD:
  - extra benefit of lowering y_t
  - higher wedges
  - declining wedges from T-1 to T
- **scope vs urgency**:
  - Early t: more periods ahead to gain from reduction in rents
  - Early t: less urgency to act
  - non monotonicity of distortions over time

Theorem, cont'd: redistribution

$$D_t \equiv rv'(c_t)\mathbb{E}\left[\frac{1}{v'(\tilde{c}_1)}\right]\left(\frac{\mathbb{E}\left[q'(\tilde{V}_1)|\tilde{\theta}_1 \geq \theta_1\right]}{\mathbb{E}\left[q'(\tilde{V}_1)\right]}\right)$$

- Increasing consumption at θ^t relaxes redistribution constraint (when $r = 1$)
- Lower inequality aversion $\Rightarrow$ lower wedges

Theorem, cont'd: risk aversion and transaction costs

$$RA_t \equiv v'(c_t)\frac{\eta_1}{I_1^t}\sum_{\tau=1}^{t} I_\tau^t TC_\tau$$

where

$$TC_t = \frac{1-F_t}{f_t}\left\{\mathbb{E}^{\theta^{t-1},y_{t-1}}\left[\frac{1}{v'(\tilde{c}_t)}|\tilde{\theta}_t \geq \theta_t\right] - \mathbf{I_{t>1}}\mathbb{E}^{\theta^{t-1},y_{t-1}}\left[\frac{1}{v'(\tilde{c}_t)}\right]\right\}$$

Risk aversion and transaction costs

- Non-transferable utility (concavity in c): higher cost of future rents
- Risk increasing over time (e.g., rw) $\rightarrow$ wedges increasing over time
- Piling up of TCs over time can suffice for wedges to increase, even with decreasing risk
  - qualitative implications for wedge dynamics and consumption insurance provision
  - quantitative implications: **ongoing work**

Recouping rents

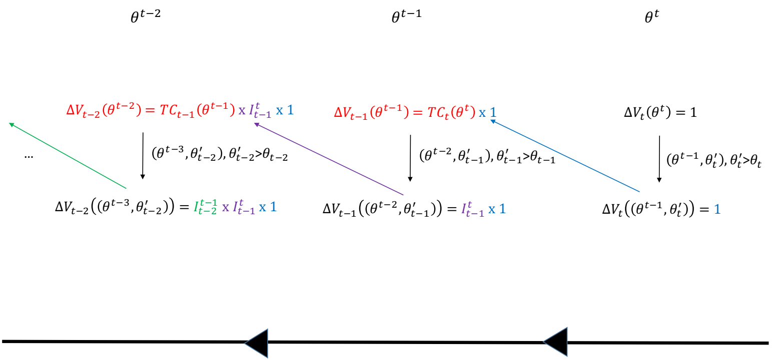

Figure: Accumulation of transaction costs

Recouping rents

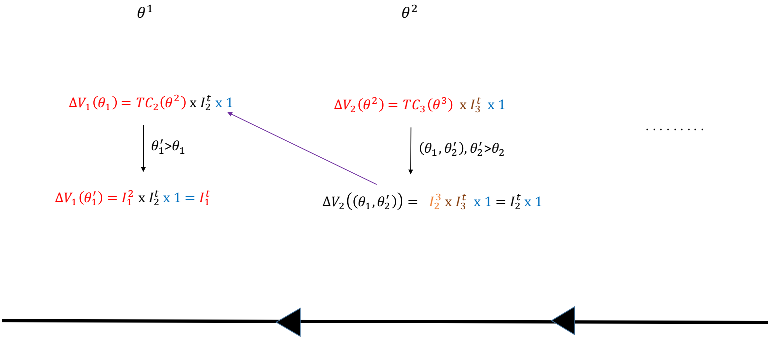

Figure: Accumulation of transaction costs

Conclusions

Conclusions

- general formula for wedge decomposition
  - interaction of forces driving dynamics of distortions
    - endogeneity and persistence of type process
    - risk aversion and accumulation of risk
    - transaction costs of moving rents over time
    - planner's preferences for redistribution
- Ongoing work
  - increasing distortions without increasing risk
  - quantitative implications
- Future work:
  - hidden savings
  - partial commitment
  - ...

Conclusions

THANKS!

Literature

- **Dynamic Mechanism Design**
  - Transferable utility
  - Declining distortions (higher quantity/quality) over time
  - Distortion dynamics driven by impulse responses of future types to initial ones
- **New Dynamic Public Finance**
  - Increasing distortions (and tax rates) over lifecycle
  - Distortions dynamics driven by risk aversion and evolution of risk
- **Taxation w. Human Capital Accumulation**
  - Special processes for productivity shocks

Go back

Recursive Problem

"Promised utility":

$$\Pi_{t+1}(\theta^t) \equiv \int V_{t+1}(\theta_{t+1}) dF_{t+1}(\theta_{t+1} \mid \theta_t, y_t(\theta^t))$$

"Marginal promise":

$$Z_{t+1}(\theta^t) \equiv \int I_t^{t+1}(\theta^{t+1}, y_t(\theta^t)) \times$$

$$\left[-\psi_\theta(y_{t+1}(\theta^{t+1}), \theta_{t+1}) + \delta Z_{t+2}(\theta^{t+1})\right] dF_{t+1}(\theta_{t+1} \mid \theta_t, y_t(\theta^t))$$

Recursive Problem

- With these definitions we have:

$$V_t(\theta^t) = v(c_t(\theta^t)) - \psi(y_t(\theta^t), \theta_t) + \delta \Pi_{t+1}(\theta^t)$$

and

$$\frac{\partial}{\partial \theta_t} V_t(\theta^t) = -\psi_\theta(y_t(\theta^t), \theta_t) + \delta Z_{t+1}(\theta^t)$$

Recursive Problem: $t > 1$

$$Q_t\left(\theta^t, y_{t-1}(\theta^{t-1}), \Pi_t(\theta^{t-1}), Z_t(\theta^{t-1})\right) \equiv \max_{y_t, \Pi_{t+1}, Z_{t+1}}$$

$$v^P(y_t) - v^{-1}\left(V_t(\theta^t) - \delta\Pi_{t+1}(\theta^t) + \psi(y_t, \theta_t)\right) +$$

$$\delta\mathbb{E}\left[Q_{t+1}\left(\theta^{t+1}, y_t(\theta^t), \Pi_{t+1}(\theta^t), Z_{t+1}(\theta^t)\right)|\theta^t, y_t(\theta^t)\right]$$

subject to

$$\frac{\partial}{\partial\theta_t}V_t(\theta^t) = -\psi_\theta(y_t, \theta_t) + \delta Z_{t+1}(\theta^t)$$

$$\Pi_t(\theta^{t-1}) \equiv \int V_t(\theta_t)dF(\theta_t \mid \theta_{t-1}, y_{t-1}(\theta^{t-1})),$$

and $\quad \Pi_{T+1}(\theta) \equiv 0$

Recursive Problem: $t > 1$

and

$$Z_t(\theta^{t-1}) \equiv \int I_{t-1}^{t}(\theta^t, y_{t-1}(\theta^{t-1}))\times$$
$$\left[-\psi_\theta(y_t(\theta^t), \theta_t) + \delta Z_{t+1}(\theta^t)\right] dF_t(\theta_t \mid \theta_{t-1}, y_{t-1}(\theta^{t-1}))$$

and $\quad Z_{T+1}(\theta) \equiv 0$

Recursive Problem: $t = 1$

$$Q_t\left(\theta^t, y_{t-1}(\theta^{t-1}), \Pi_t(\theta^{t-1}), Z_t(\theta^{t-1})\right) \equiv \max_{y_t, \Pi_{t+1}, Z_{t+1}}$$

$$v^P(y_t) - v^{-1}\left(V_t(\theta^t) - \delta \Pi_{t+1}(\theta^t) + \psi(y_t, \theta_t)\right) +$$

$$\delta \mathbb{E}\left[Q_{t+1}\left(\theta^{t+1}, y_t(\theta^t), \Pi_{t+1}(\theta^t), Z_{t+1}(\theta^t)\right) | \theta^t, y_t(\theta^t)\right]$$

subject to

$$\frac{\partial}{\partial \theta_t} V_t(\theta^t) = -\psi_\theta(y_t, \theta_t) + \delta Z_{t+1}(\theta^t)$$

$$\Pi_t(\theta^{t-1}) \equiv (1-r) V_t(\underline{\theta}_t) + r \int q(V_t(\theta_t)) dF(\theta_t \mid \theta_{t-1}, y_{t-1}(\theta^{t-1}))$$

Go back